Optimal Selling Mechanisms for On-line Services: When do Posted Price and Auction-like Selling Mechanisms co-exist?*

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Abstract

We study the economic rationale of the use of auctions and posted price selling mechanisms for the simultaneous allocation of on-line services having the infrastructure as a service public cloud computing market as a motivating example. By offering both selling mechanisms, the monopolist of a homogeneous good, under certain conditions can discriminate among buyers of different private valuations and maximize in this way her expected payoff. Auctioning the services can be designed so as to incorporate the risk for the winners of losing access to their service while it is still in operation. The posted price mechanism can by construction eliminate that risk. Buyers of high valuations prefer to pay a risk premium and get the service through the posted price mechanism while buyers of low valuations unable to meet the price level of the risk premium enter the auction.

1 Introduction

1.1 A Motivating Example: The Infrastructure as a Service Public Cloud Computing Market

Infrastructure as a Service (IaaS) public cloud computing is a fast growing market, leaping up 33% in 2015 to become an estimated $16.5 billion market, according to research firm Gartner[1]. In IaaS market, providers provide physical or virtualized hardware in the form

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[1] http://www.gartner.com/newsroom/id/3055225

of storage, servers, network, firewalls and load balancers. This is very useful and popular for small scale businesses and startups as they cannot afford to buy such costlier hardware components (built on premise).

The dominant provider of IaaS services is Amazon[2] which enjoyed in the past extraordinary high market shares of more than 80% of the IaaS market (due to its first mover advantage). Amazon's Elastic Compute Cloud (EC2) has hosted numerous well-known internet companies and websites, such as Expedia, Netfix and Adope Systems. The basic unit of computation on EC2 is a virtual machine, known as an instance. Users can specify certain parameters about the hardware and location where their instances will run, and also have several available purchasing options.

Initially, EC2 only offered a posted price selling mechanism, so that buyers could have guaranteed access to the virtual machines by paying a fixed non-discriminatory hourly rate. With only a posted price mechanism , Amazon clearly had frequent slack capacity, and to utilize this in December 2009 introduced spot instances.

Spot instances allow Amazon to auction of excess capacity. To use spot Instances, buyers place a spot instance request, specifying, the number of spot instances they want to run, and the maximum price they are willing to pay per instance hour. Amazon changes the spot price periodically based on supply and demand. When a user's bid is above the current spot price, her instances get scheduled, and run until either they complete, or until the spot price rises above the bid, in which case the instances are terminated.

Amazons description of How Spot Instances Work[3] reveals that spot prices are set through a $(q+1)$th uniform price, sealed-bid auction, in which q are the available units in auction.

As in an price auction of multiple goods, each client bids for the desired number of goods (spot instances). The seller/provider chooses the top q bidders. She may set q up-front on the basis of available capacity, or, she might retroactively set q after receiving the bids,to maximize revenue. In any case, q cannot exceed the available capacity. The provider sets the uniform price to the price declared by the highest bidder who did not win the auction (bidder number $q+1$) and publishes it. The q winning bidders pay the published price and their instances start running. In this case, the published price is a price bid by an actual client.

The provider may also decide to ignore bids below a publicly known minimal price or below a hidden reserve price or equivalently to reduce the units q offered in the spot market, to prevent the goods from being sold cheaply, or to give the impression of increased demand.

Note an important difference between the posted price selling mechanism and the spot market: If the buyer chooses the posted price, she enjoys a constant price over time, and its instance(s) will never be terminated against its will. In contrast, spot users bear the risk of price fluctuations and having their running instances terminated whenever the spot

[2]Amazon was the first to initiate its Cloud services in August 2006 offering access on a first-come, first-served basis.

[3]http://aws.amazon.com/ec2/spot-instances/

price rises above their bids.

Moreover, buyers know neither the seller's capacity constraints nor the number of buyers who choose the posted price mechanism. So, the number of units q in the auction can be considered as a choice variable of the auctioneer.

While the motivation for the introduction of the spot market was to eliminate the waste of slack resources, the combination of a simultaneous posted price mechanism with the spot market raises given that each buyer knows only privately her valuation and does not observe the bids of the other buyers in the (sealed-bid) auction raise the question how the provider can design the selling mechanism that accommodates both selling options in an incentive-compatible and revenue maximizing way.

Note that the Amazon's EC2 is currently the only provider of IaaS cloud services that use a hybrid of posted price and spot market pricing scheme. The other providers including the fast growing Google's Compute Engine and Microsoft's Azure who entered the market in 2013[4] adopt a posted price mechanism only. So, a natural question to ask is why the other providers did not adopt the spot market pricing option and whether the large market share of Amazon can explain this difference in pricing options.

1.2 Motivation and Contribution

This paper inspired by the IaaS cloud computing market studies when the simultaneous allocation of goods through a posted price and an auction is revenue maximizing. We develop a seller's revenue maximizing mechanism which allows the seller to price discriminate between high valuation buyers who select the posted price option and the low valuation buyers who select to participate in the auction in an environment where the seller chooses the posted price and the number of units that are available in the auction and where the buyers that go to the auction are uncertain about whether the usage of the good will be terminated while still in operation.

When a service is suddenly interrupted while it is still in use by the buyer, there is a termination cost which decreases the valuation of the buyer for the service. For example, buyers can be considered as downstream firms that use upstream services for transactions with final consumers. Due to interruption of the services[5], buyers will not be in the position to serve efficiently the market and they will incur some loses (e.g. damaged reputation, inability to meet commitments and deadlines which sometimes enforced by contracts with the final consumers). Such incurred losses generated by the uncertainty over the continuation of offering the service make buyers to have lower valuations for the same service when it is auctioned á la IaaS spot market and higher valuation when it is offered through a posted mechanism that guarantees access to the server for as long as the buyer wants. The auctioneer can affect the uncertainty over the usage of the service

[4]Long after the introduction of the spot market by Amazon, where the term "long" is stated with respect to the very dynamic nature and growth of the market.

[5]An example from the IaaS market: Consider a downstream firm that needs to have access to a cloud server (virtual machine) for running a service. If it looses the access to the server, it is unable to run the service anymore.

in the auction through the choice of the number of auctioned units. Fewer available units makes interruption more probable and it leads to higher expected cost of interruption. The provider, by decreasing the number of auctioned units can make the auction less attractive option for the high valuation buyers who prefer the posted price mechanism even if the posted price is relatively high. The selling mechanism with high posted price and sufficiently low number of auctioned units is incentive compatible in that all the buyers with valuations higher than the posted price, prefer to buy from the posted mechanism avoiding the risky auction (even under risk neutrality) and all the buyers with valuations lower than the posted price participate in the auction, since they do not find profitable to buy the good using the posted price option.

The comparison between different selling mechanisms and in particular between posted prices and auctions has been studied in the literature in a various of settings both by economic and computer science literature. To begin with, Harris and Ravin (1981b) consider a multi-unit selling procedure under a uniform distribution of buyers' valuations and concludes that the optimal mechanism is a posted price selling procedure. Maskin and Riley (1989) generalizes this result for general distribution functions under very mind conditions over the buyers' valuations. Riley and Zeckhauser (1983) finds that sequential search (or posted selling) mechanism under commitment is always optimal. This result is generalized by Skreta (2006) in the case that the seller cannot commit to a particular selling mechanism. In contrast, Wang (1993) compares the seller's revenue from auctions and posted price mechanisms by considering a seller who meets buyers with exogenously given (Poisson arrival) probabilities and finds that when there are no auctioning costs, auctioning is always optimal. If auctioning costs are present, the steepness of the marginal-revenue curve associated with the distribution of buyers' valuations determines whether the optimal selling option. When this steepness is large, auctioning is still preferable (for the seller) to the posted price mechanism. Kultti (1999) considers agents who choose whether to participate in markets where goods are sold in auctions or markets where goods are sold through a posted price mechanism and concludes that both mechanisms are totally equivalent. Julien et al. (2002) develop a model with two buyers and sellers who offer homogeneous goods and consider the choice of sales mechanism from three possibilities: posted prices, and auctions with and without reserve prices. They find that sellers' expected revenues are highest when both sellers use auctions with reserve prices.

Hammond (2010) motivated by the finding of Harris and Raviv (1981a) that if the number of goods in a monopolist's inventory exceeds potential demand, a posted price is optimal as well as the analysis of Zeithammer and Liu (2008) who consider the possibility that the inventory is heterogeneous and conclude that a monopolist with a heterogeneous inventory prefers the auction mechanism while a monopolist with a homogeneous inventory prefers the posted price, investigates empirically these theoretical claims based on data collected on compact disc sales. While he finds that the size of the inventory has a significant impact on the choice of the selling procedure by compact-disc sellers, he does not find any sufficient support for the impact of heterogeneity of the inventory on the mechanism's choice. The empirical study of Vakrat and Seidmann (1999) compares prices paid through online auctions and catalogs for the same product. They observe that auctions result in average prices 25% below the catalog ones. They build a simple model of single-unit auctions with a deterministic number of bidders, but ignoring consumer choice behavior.

As far as studies that consider the simultaneous use of posted price selling and online auctions are concerned, Budish and Takeyama(2001)consider a single seller and two types of risk-averse buyers and show that the English auction with a buy price can raise the revenue of the seller. Similar result is found by Reynolds and Wooders (2009). Etzion et al.(2006)show that the simultaneous use of posted price selling and online auctions leads to a significant increase in sellers revenue. The buyers in this study take into account the fact that the level of competition is higher in the online auction market. The model has a number of unique features, including the arrival rate of the buyers to the website. By taking a buyers discounting of the expected utility of auctions into account, Sun(2008)shows that the achievement of market segmentation is a rationale for the simultaneous use of posted price selling and online auctions. Sun argues that, in the case of a posted price sale, there is little or no uncertainty regarding the price of the product but the number of units sold is subject to uncertainty. The reverse is true in the case of auctions and hence there is no dominant selling mechanism. Sun further argues that the choice of selling mechanism depends on factors such as the sellers inventory cost and the buyers discount factor. Suns analysis is based on near-optimal approximation of the sellers profits. Hammond(2013)argues that differences across buyers do not explain the simultaneous use of auctions and posted price selling. He finds that the simultaneous use of auction and posted price sale decreases the level of competition among the sellers. Sellers with high value items prefer posted price sale, even though it leads to fewer sales, because the items can be sold at a higher price. Celis et al.(2014) further present an analysis of a randomized mechanism that they call buy-it-now or take-a-chance in which bidders have the option of first buying an object at a posted price, and if nobody buys the object at a posted price, the object is then sold at random to one of the top d bidders. They conclude that this mechanism, when only two different types (of valuations) of buyers are available, outperforms a second price auction with optimal reserve price. However, when we move to an environment where the distribution of buyers' valuations is continuous, this is not generally true, but, it depends on the specific values of the parameters of their model. In the infinite-horizon model of van Ryzin and Vulcano (2004), the seller operates auctions and posted prices simultaneously, and replenishes her stock in every period. However, the streams of consumers for both channels are independent, and the seller decides how many units to allocate to each of the channels separately. Etzion et al. (2006) study the profitability of selling consumer goods on-line using posted price and open ascending-bid uniform-price auction simultaneously. They develop a model of consumer behavior when faced with the choice between the two channels. The model is simulated in order to identify the best designs of the dual channel regime and compare its performance with that of the only posted price regime. They find that the best designs of dual channels with open-bid auctions differ from those of dual channels with sealed-bid auctions previously studied. In addition, when optimally designed, the dual channel regime outperforms the posted price regime.

There are several studies that try to explain the pricing options of eBay and related markets and specifically, to provide an economic rationale for the buy-it-know option that is followed by an efficient auction (which can be viewed as a hybrid of a posted price and an auction offered simultaneously)[6]. The majority of the relevant theoretical studies find that such kind of mechanism price discriminates between high and low valuation buyers as the allocation of the good is not simultaneous. Buyers that prefer the buy-it-know option get the object immediately by paying the respective posted price while buyers that go to the auction are subject to delays until they get the goods on sale (if they win the auction). These delays create opportunity costs (especially as some of the buyers may be very impatience). High valuation buyers that find the buy-it-know price affordable they prefer this option in order to avoid the opportunity costs involved with the auction options. The buyers with lower valuation find preferable the auction selling mechanism due to the higher buy-it-know price. A similar reasoning holds if buyers are differentiated with respect to how impatient or how risk averse (as already pointed above) they are.

Our novel approach is to study posted price and auction selling mechanisms for goods that are not only simultaneously offered but also simultaneously allocated (regardless the choice of the mechanism the buyer chooses), sharing in this way a main feature of IaaS market. Hence, opportunity costs or arguments about buyers' impatience cannot be relevant for the justification of the coexistence of the two selling procedures. Moreover, we consider that the good sold is a service. As we will show below, when the nature of the good sold is a service and for risk neutral buyers, the seller can adopt an optimal mechanism which maximize the seller's revenue under the simultaneous use of posted price and auctions by introducing sufficient risk in the auction. We allow the seller to select how many units she will allocate through the auction as her capacity is her private information. The seller can exploit this informational advantage to price discriminate among buyers with different valuations. There may be other markets that our model can be applied. For example the market for transmission of electricity by generators. However, in such network industries which are heavily regulated.

The rest of the paper is organized as follows: In section 2 we discuss the model and the underlying assumptions and we study the coexistence of posted price and auction mechanisms when there is not any risk factor involved. As we show, the simultaneous consideration of both mechanisms for selling services does not beat optimal standalone mechanisms in which only an auction or a posted mechanism is used. By introducing the risk of interruption in the auction, we illustrate how the seller can use this risk factor in order to price discriminate and increase her payoff to higher levels than any standalone mechanism. Section 4 concludes and discusses our future agenda of relevant research.

[6]See for example Wang et. al. (2004), Hummel (2015), Mathews (2004), Anwar and Zheng (2015), Onur and Tomak (2009), Kirkegaard and Overgaard (2008), Caldentey and Vulcano (2007), Chen et al. (2013), Peters and Severinov (2006), Hidvegi et al. (2006), Ackerberg et al. (2006), Ambrus et al. (2014), Ockenfels and Roth (2006) and Roth and Ockenfels (2002) to name a few.

2 Model

Consider a seller (she) who offers multiple indivisible units of the same homogeneous good and N potential buyers each of which considers either to purchase one unit of the good or not to buy at all. The buyers can be divided into two groups, the ones with high valuation v_H for one unit of the good and the others with low valuation v_L for one unit of the good (i.e. $v_H > v_L > 0$). Let a and $1-a$ be the proportion of the high and the low valuation buyers, respectively, where $0 < a < 1$. The valuation of each buyer (he) is her private information while the proportion a is a common knowledge. The number of buyers is N and seller can offer up to Q units of the goods where Q is her private information.

If the seller knew the valuation of each buyer, she could maximize her expected payoff by selling one unit to each buyer at a price equal to his valuation (first best). Her payoff in that case would have been $\Pi_S^{FB} = aNv_H + \min\left(Q - aN, (1-a)N\right) v_L$ while the buyers would have had zero payoff (if $Q < N$ the proportion $N - Q$ of the low valuation buyers remains unserved).

Since the valuation of each buyer is his private information, the seller is unable to extract all the generated from the trade surplus from each of the buyers. If $aNv_H \geq \min(N, Q)v_L$, then the optimal mechanism is the one in which the seller trades only with the high valuation buyers and gets all the generated surplus from trade. Otherwise, she prefers to trade with all the buyers to maximize her payoff:

$$\Pi_S = \begin{cases} aNv_H & \text{if } aNv_H \geq \min(N, Q)v_L \\ \max(Q, N)v_L & \text{if } aNv_H < \min(N, Q)v_L \end{cases} \quad (1)$$

We will examine the seller's payoff under three different mechanisms:

1. Standalone posted mechanism in which the seller chooses the price p that each of the offered units is sold and the buyer after observing sellers offer decides whether to buy.

2. A standalone uniform price auction $A(r, q)$ in which the seller auctions q units at a reserve price r and the buyer after observing these characteristics of the auction decides whether to participate and how much he will bid.

3. A hybrid of a posted price p and an auction of q units at the reserve price r. An additional decision by the buyer in this case is which of the two options he will take if he wants to buy a unit.

In the standalone posted mechanism the seller announces the price per unit, p and the buyer buys only if his valuation (weakly) exceeds the posted price and does not buy otherwise.

In the standalone uniform auction $A(r, q)$, the seller announces the reserve price r and the number of available units for sale, q and the buyer participates only if his valuation (weakly) exceeds the reserve price r and does not participate otherwise. Each of the $q-th$

highest bidders win one unit and pays a uniform price that equals the $(q+1)th$ highest bid, if that bid exceeds the reserve price r and the reserve price r, otherwise. It is easy to see that under such a design, the participants find optimal to bid their true valuations[7].

Lemma 1. *A standalone posted price mechanism with*

$$p^* = \begin{cases} v_H & \text{if } aNv_H \geq \min(N,Q)v_L \\ v_L & \text{if } aNv_H < \min(N,Q)v_L \end{cases}$$

and a standalone uniform price auction with

$$r^* = \begin{cases} v_H & \text{if } aNv_H \geq \min(N,Q)v_L \\ v_L & \text{if } aNv_H < \min(N,Q)v_L \end{cases}$$

and

$$q^* = \begin{cases} aN & \text{if } aNv_H \geq \min(N,Q)v_L \\ \min(Q,N) & \text{if } aNv_H < \min(N,Q)v_L \end{cases}$$

generate the same maximum revenue for the seller.

Proof. Consider firstly the standalone posted mechanism. By setting a price $p < v_L$, all the buyers buy one unit of the good by paying price p. The seller can do better by increasing the price at $p = v_L$ as she serves the same number of consumers but at higher price. For $v_L < p < v_H$, only the high valuation buyers wants to buy by paying p. Again the seller can do better by increasing the price at $p = v_H$ serving all the high valuation buyers at the maximum price they can afford. For $p > v_H$, no buyer wishes to buy, so the seller generates zero payoff.

So, the optimal standalone posted mechanism is either setting $p = v_L$ or $p = v_H$. When the proportion of the high valuation buyers is sufficiently high ($aNv_H \geq \min(N,Q)v_L$), the seller maximizes her payoff by only serving the high valuation buyers at the optimal price $p^* = v_H$. Otherwise, she prefers to serve all the consumers at the optimal price $p^* = v_L$.

Moving to the standalone auction $A(r,q)$, for $r < v_L$, all the buyers participate in the auction. For given q, the seller can do better by increasing the reserve price to $r = v_L$. In order to maximize her payoff, she serves as many buyers as possible, so, $q = \min(Q,N)$. If $v_L < r < v_H$, only the high valuation buyers participate in the auction. For given q, the seller maximizes her payoff by increasing the reserve price to $r = v_H$ and serving all the high valuation buyers, $q = aN$. If the sellers prefers to only serve the high valuation buyers, she chooses $r^* = v_H$ and $q^* = aN$. Otherwise, she chooses $r^* = v_L$ and $q^* = \min(Q,N)$ which complete the proof. □

[7] This is true no matter what value for r and q the seller chooses. Let us abstract for a moment from the choice of r by considering that $r = 0$. By bidding above his valuation, the buyer faces a positive probability that the $(q+1)th$ highest bid is above his valuation but below his bid. In this case, the buyer wins a unit but incur losses. In all the other cases, the buyer has the same payoff with the one when he bids his own valuation. If now the buyer bids below his valuation, then with positive probability the $(q+1)th$ highest bid is in between the buyer's valuation and his bid. In such a case, the buyer does not win the unit while by winning he would have had positive payoff. In all the other cases, the buyer gets the same payoff as when he bids his valuation. Hence, bidding his true valuation is a weakly dominant strategy. This is obviously true for any value or r and q.

The fact that these standalone mechanisms maximize the sellers payoff with respect to other standalone mechanisms does not mean that the seller cannot do better. The seller can alternatively use a hybrid of announcing a posted price p and a uniform price auction $A(r, q)$. The buyers choose whether to rent their units from the posted mechanism by paying a price p or participate in the auction $A(r, q)$. The hybrid makes sense only if both selling options are selected by some buyers. Otherwise, one of the two standalone mechanisms could easily be proven that perform better.

First we will derive the revenue maximizing hybrid mechanism when interruption risk is absent. This will help us to show that the risk of interruption is needed to justify the preference of the seller to the hybrid mechanism instead of the revenue maximizing standalone mechanisms

In the hybrid mechanism, the seller chooses optimally the price p of the posted mechanism, the number of units to be sold in the auction, q as well as the reserve price r of the auction. So, she has three choice variables. We start by determining the optimal values for these choice variables. The next lemma derives the optimal reserve price for given p and q.

Lemma 2. *In the optimal hybrid mechanism, the seller sets* $r^* = v_L$ *and* $p^* > v_L$

Proof. For any given $q > 0$, if the seller chooses $r \geq p$, then no buyer selects to go to the auction and they all prefer the posted mechanism, so, the hybrid makes sense only if $r < p$. Then, if $r < v_L$, the seller can do (weakly) better by setting $r = v_L$. If $v_L < r$, then no low valuation buyer buys from the hybrid mechanism. The seller can maximize her payoff by setting $r = v_H$ and $p > v_H$ by serving all the high valuation buyers. But, in this case, all the buyers buy from the auction. Therefore, hybrid makes sense only if both high and low buyers find profitable to trade with the seller. So, the optimal hybrid requires that $r^* = v_L$ and $p^* > v_L$. □

Note that without loss of generality we can assume that the reserve price r is hidden, so that the seller reports only the values of p and q of the hybrid mechanism before buyer makes his purchase decision. Since the optimal value for r is unique in the hybrid mechanism when $p > v_L$, the buyers can infer the value of r even when it is not reported but it is a private information of the seller. In this sense, it is totally equivalent to consider a variant of the uniform price auction $A(q)$ in which the seller only reports the number of available units in the auction, q.

The low valuation buyers only have the option to go to the auction. The high valuation buyers have two options. They either buy from the posted mechanism at price p or they participate in the auction $A(r^*, q)$. The select the option that leaves them with the highest expected payoff.

Lemma 3. *In the optimal hybrid mechanism* $q^* \leq aN$

Proof. If $q > aN$ and given that $p^* > v_L$, all high valuation bidders prefer the auction as in the efficient uniform price auction they expect to win one unit with probability 1 and pay a price equal to v_L which is lower with they would have had to pay in the posted mechanism. Hence, the hybrid makes sense only if $q \leq aN$. □

When $q \leq aN$, each high valuation buyer, when he considers what buying option to select should consider what the other buyers will choose. If a buyer i believes that all the other $aN - 1$ high valuation buyers select the auction, then, given that $q < aN$, he knows that his expected payoff from the auction will be zero as the equilibrium price, the $(q+1)th$ bid, will be v_H. Hence, in this case if $p \leq v_H$, the buyer i prefers the posted mechanism[8]

Let z be the mixed strategy probability with which a buyer select the auction for purchasing a unit and $1 - z$ the probability that he selects the posted mechanism. The expected payoff of a high valuation buyer from participation in the auction will be[9]:

$$B(z;q) = \left(\sum_{k=0}^{q-1} \frac{(aN-1)!}{k!(aN-1-k)!} z^k (1-z)^{aN-1-k} \right) (v_H - v_L)$$

The high valuation buyer by knowing that the hybrid mechanism is characterized by posted price p and auction $A(r^*, q)$, where $q \leq aN$, selects his strategy z given his beliefs for the strategies of the other high valuation buyers. In the symmetric equilibrium, each high valuation buyer adopts the same strategy $z^*(p, q)$. The following proposition identifies the symnmetric Bayesian Nash equilibria z^* of the hybrid game

Proposition 4. *In the symmetric Bayesian Nash equilibria z^* satisfies one of the following conditions:*

$$v_H - p = B(z;q) \rightarrow z^*(p, q) \tag{2}$$

$$z^* = \frac{q}{aN} \tag{3}$$

The multiplicity of equilibria obviously reduce the applicability of the hybrid mechanism only if the seller cannot reach higher revenue than the standalone mechanisms for all the equilibria by setting optimally her choice variables [10].

Note that from (2), the higher the posted price is, the more attractive the auction will be for the buyers or the higher will be the equilibrium value of z^* in the marginal case that $p = v_L$, the strategies $z^* = 0$, so all the high valuation buyers prefer the posted mechanism. But, in such a case, the posted mechanism would have also been accessible by the low valuation buyers and the hybrid would not have made any sense. In the other marginal case, with $p = v_H$, then $z^* = 1$, so all the high valuation buyers go to the auction and again the hybrid would not have made any sense. So, hybrid makes sense only if the payoff of the seller is maximized under $v_L < p^* < v_H$ or equivalently under $0 < z^* < 1$.

The seller can exploit the strategic interactions among high valuation buyers in order to price discriminate. And she will do it to the extent to find it profitable. By decreasing the

[8]In case of indifference, we assume that the buyer selects the posted mechanism.

[9]Since, all high valuation buyers are symmetric to each other, we will focus on the symmetric Bayesian Nash equilibrium of the hybrid game. Hence, without the loss of generality in equilibrium, each high valuation buyer selects the same symmetric z probability of participating in the auction.

[10]The potential existence of asymmetric equilibria makes the analysis even more complicated reducing the applicability of the hybrid mechanism.

available units q in the auction, she increases the competition of the high valuation buyers for winning an auctioned unit, decreasing their expected payoff from the auction. As a result, high valuation buyers find more attractive the posted mechanism option which in turn makes profitable for the seller to increase the posted price p without loosing customers in the posted mechanism. Such a price discrimination device comes at cost as the amount of units sold to low valuation buyers decrease.

As for (3), it certifies that the proportion z^* will go to the auction. In fact, this equilibrium proportion of the high valuation buyers corresponds to the highest possible number of them that can generate positive payoff if they go to the auction. For higher number of high valuation buyers in the auction, the $(q+1)th$ equilibrium price becomes v_H, so, all the buyers that go to the auction get zero payoff.

Given the equilibrium strategies by the buyers and the trade off of reducing q and increasing p, the seller selects $p^* > v_L$ and $q_* \leq aN$, so as to maximize her payoff,

$$\begin{aligned} \Pi_S^{Hybrid} &= (1-z^*)aNp + (1-(1-z^*)a)q\left[Prob(z^* \leq \frac{q}{aN})v_L + \left(1 - Prob(z^* \leq \frac{q}{aN})\right)v_H\right] \\ &= (1-z^*)aNp + (1-(1-z^*)a)q\left[v_H - (v_H - v_L)Prob(z^* \leq \frac{q}{aN})\right] \end{aligned} \tag{4}$$

where the second equality comes from (2).

In the case of (3), from (4) we have: $Prob(z^* \leq \frac{q}{aN}) = 1$ and that the revenue of the seller is maximized under

$$p^* = v_H$$

and

$$q^* = \begin{cases} \frac{a}{1+a}N\frac{v_H}{v_L} & \text{if } v_H \leq (1+a)v_L \\ aN & \text{if } v_H > (1+a)v_L \end{cases}$$

and corresponds to:

$$\Pi_{S^*}^{Hybrid} = \begin{cases} \frac{a}{1+a}Nv_H\frac{2(1+a)v_L - v_H}{(1+a)v_L} & \text{if } v_H \leq (1+a)v_L \\ aNv_L & \text{if } v_H > (1+a)v_L \end{cases}$$

Obviously, for $q^* = aN$, the hybrid mechanism evolves to a standalone auction mechanism, as all the high valuation buyers go to the auction.

The following proposition applies:

Proposition 5. *Under the symmetric Bayesian Nash equilibrium* (3) *the seller prefers the revenue maximizing standalone mechanisms from the hybrid one with optimal values of* $r^* = v_L$, $p^* = v_H$ *and* $q^* = \begin{cases} \frac{a}{1+a}N\frac{v_H}{v_L} & \textit{if } v_H \leq (1+a)v_L \\ aN & \textit{if } v_H > (1+a)v_L \end{cases}$

Proof. If $q^* = aN$ in the hybrid mechanism, then by definition we are in the revenue maximizing standalone auction mechanism. So, it suffices to compare the profitability of the hybrid mechanism with $r^* = v_L$, $p^* = v_H$ and $q^* = \frac{a}{1+a}N\frac{v_H}{v_L}$, when $v_H \leq (1+a)v_L$ with the revenue maximizing standalone posted mechanism if $av_H > v_L$ and with the revenue maximizing auction mechanism, otherwise. It is easy to see that $\Pi_{S^*}^{Hybrid} < aNv_H$ in the former case and that $\Pi_{S^*}^{Hybrid} < Nv_L$ in the latter case. □

Since under (3) only the high valuation buyers are served, the seller cannot do better than the revenue standalone mechanisms. The hybrid mechanism can only do better than the standalone mechanisms if both type of buyers have access to the service and pay different prices for buying the units.

Such a price distrimination selling procedure can be the case through (2). The one-to-one relationship between p and z allows us to determine some features of the hybrid mechanism at the optimal choice of p^*. To begin with, note that

$$\frac{d\Pi_S^{Hybrid}}{dz}|_{z=1} = a(q-N)v_H < 0$$

, $\forall q$.

This implies that in the neighborhood of $p = v_H$, when, $z^* = 1$ and the sales are made through the auction only, the seller's expected revenue increases as the price p decreases and some of buyers go to the posted mechanism so that the sellers payoff is maximized.

In the neighborhood of $z^* = 0$ where all the high valuation buyers go to the buy from the posted mechanism. We have:

$$\frac{d\Pi_S^{Hybrid}}{dz}|_{z=0} = a[N(aN-1) - (N-q)v_L]$$

which is positive for sufficiently large number of buyers and for any given q.

Hence, for sufficient number of buyers there exists at least a $z^* \in (0,1)$ such that the expected revenue of the seller reaches its maximum.

The seller will choose the hybrid mechanism only if it generates higher payoff than the optimal standalone mechanisms for all possible equilibrium strategies of the buyers. Multiplicity of equilibria and the fact that as we showed above some of them are inferior to the standalone mechanisms makes more difficult to justify why a seller would prefer a hybrid mechanism. As we are going to show in the next section, the seller, by introducing the risk of interruption of services in the auction and under certain conditions, can kill the multiplicity of equilibria and induce the high valuation buyers to choose the posted mechanism leaving the available units in the auction for the low valuation buyers.

3 Hybrid Mechanism with Risk of Interruption

Let the seller introduce a hybrid mechanism with a posted price p and a uniform price auction $A(r,q)$ but this time the buyers that win the auction face a risk to loose the unit the won because of interruption of access to the network. Specifically, the sellers offer in the auction incorporate the possibility that the auctioned units will be asked back from the winners before they have completed their operations with the used units. Such interruption has a specific expected cost which depends on the probability that it will take place and the damage it will occur in the operation of the winner.

Under specific assumptions over the cost of interruption, the seller can induce the emergence of a unique Bayesian Nash equilibrium in which the high valuation buyers to purchase from the posted mechanism leaving the available units in the auction for the the low valuation buyers.

Let the cost of interruption from a purchase through the auction be $C(q, v_i)$, where $i = L, H$. The probability of interruption is set by the seller such that $C_q(\cdot, v_i) < 0$, which means that the lower the available units in the auction are, they more probable will be for the winner to loose access before the end of his operation. We assume that the magnitude of the interruption cost depends on how valuable is the service for the holder and specifically that $C(q, v_H) > C(q, v_L)$, $\forall q$ and we impose the normalization $C(N, v_L) = 0$.

Hence, A high type buyer has a valuation v_H for getting a unit through the posted mechanism and valuation $v_H - C(q, v_H)$ for participating in the auction. In the same way, the low type buyer has valuation v_L for a posted price good and $v_L - C(q, v_L)$ for an auctioned good. Let $v_H > C(q, v_H)$ and $v_L \geq C(q, v_L)$ and $v_H - v_L - (C(q, v_H) - C(q, v_L)) > 0$, $\forall q$.

In line with Lemma 2, the optimal value of the reserve price in the auction is

$$r^*(q) = v_L - C(q, v_L)$$

Then,

Proposition 6. *If the seller sets p and q such that $p \leq v_L + C(q, v_H) - C(q, v_L)$, then there is a unique Bayesian Nash equilibrium in which all the high valuation buyers participate in the posted mechanism and all the low valuation buyers go to the auction.*

Proof. The high valuation buyers get payoff $v_H - p$ from the posted mechanism. They would prefer this mechanism only if it generates for them weakly higher payoff than the auction. In the auction, the $(q+1)th$ highest bid will be either $v_H - C(q, v_H)$, which corresponds to q units allocated to high types with each of them getting zero payoff or $v_L - C(q, v_L)$ when the number of the high types going to the auction is not so high and their payoff is $v_H - v_L - (C(q, v_H) - C(q, v_L)) > 0$.

If the seller sets $p \leq v_L + C(q, v_H) - C(q, v_L)$ the expected payoff of a high type buyer from the auction cannot exceed what he gets from the posted mechanism. Therefore, in equilibrium, the high type sets $z^* = 0$ and they all go to the posted price. In fact this is the unique (weakly) dominant strategy for each high type and corresponds to the unique Bayesian Nash equilibrium. Consequently, only the low types participate in the auction by bidding their true valuation $v_L - C(q, v_L) = r^*(q)$. The q of them win a unit. □

They payoff of the seller under r^* will be:

$$\Pi_S^{Hybrid}(p, q) = aNp + q\left(v_L - C(q, v_L)\right)$$

The optimal value of posted price, corresponds to the maximum possible value for which the high type buyers remain to the posted mechanism and do not switch to the auction, namely, $p^*(q) = v_L + C(q, v_H) - C(q, v_L)$, Hence, the sellers payoff becomes:

$$\Pi_S^{Hybrid}(q) = aNv_L + aN\left(C(q, v_H) - C(q, v_L)\right) + q\left(v_L - C(q, v_L)\right) \tag{5}$$

If we consider the case that the seller sets the interruption probability such that $\frac{\partial(C(q,v_H)-C(q,v_L))}{\partial q} \geq 0$, then, the number of auctioned units primary affects the high type buyers. The higher the number of auctioned units, the lower the expected cost of interruption will be for both types. But, the cost of the low type reduces in a greater rate than the cost of the high type. This allows the risk of interruption to be the dominant factor of price discrimination and the seller does not feel constrained in increasing the units allocated through the auction. For revenue maximization, the seller chooses $q^* = (1-a)\min(Q, N)$. For favor of comparison, let us assume that $Q > N$[11]. Despite that high values of q reduce the cost of interruption, it is the relevant effect that an increase of q has on the costs of the high and low type that allows the price discrimination between the two buyers. The maximum profit for the seller will be:

$$\Pi_S^{Hybrid}(q^*) = Nv_L + aNC((1-a)N, v_H)$$

Note that, in the case that $v_L > av_H$, the optimal hybrid mechanism with risk of interruption generates higher payoff than the optimal standalone mechanism. So, it is preferred by the seller.

When in contrast $v_L < av_H$, the seller prefers the hybrid mechanism instead of the optimal standalone one, only when the interruption cost of high type buyer at q^* is sufficiently high, namely, only if

$$C((1-a)N, v_H) > \frac{av_H - v_L}{a} \tag{6}$$

for every $a \in [\frac{v_L}{v_H}, 1]$.

As the proportion of the high valuation buyers increases, their expected cost of interruption in the auction increases as well. If the cost of interruption of the high types is sufficiently high and/or sufficiently convex in q, the seller can effectively price discriminate between the different types no matter how high the proportion of haigh valuation buyers is and therefore, she strictly prefers to serve both types in the optimum though the hybrid mechanism.

The seller has an incentive to introduce a risk of interruption in the auction in such a way that by choosing the number of units q in the auction, it can affect the cost of interruption by facilitating the price discrimination among the different types of buyers.

[11]The results below about the comparison among the revenue under the hybrid mechanism and the standalone auctions are not affected by this assumption.

From (5) it becomes clear that if she can set q such as $\frac{\partial(C(q,v_H)-C(q,v_L))}{\partial q} \geq 0$, then she can reach the maximum possible payoff. If instead, $\frac{\partial(C(q,v_H)-C(q,v_L))}{\partial q} < 0$, then, it may be the case that the optimal value of q cannot reach the proportion of the low type buyers, $(1-a)N$ but it should be smaller, as high q makes harder for the seller to price discriminate. Hence, in such a case, she generates moderate profits. Whether the seller can by increasing q to facilitate price discrimination is crucial for the maximization of her revenue in the hybrid mechanism.

Numerical examples that satisfy $\frac{\partial(C(q,v_H)-C(q,v_L))}{\partial q} < 0$ over specific interruption cost functions can show that the profit of the seller under the hybrid mechanism evaluated at the optimal value of q can generate higher payoff than the respective standalone mechanisms for given values of parameters of the model. But, in all of these examples the generated profit does not exceed the profits of the optimal hybrid mechanism under $\frac{\partial(C(q,v_H)-C(q,v_L))}{\partial q} \geq 0$.

We summarize the results and insights in the following lemma and proposition:

Lemma 7. *The seller's revenue under the hybrid mechanism with risk of interruption is maximized when a change in the number of auctioned units decreases the interruption cost of the low type further than the respective cost of the high type buyer.*

Proof. This follows immediately from the objective function (5). This lemma is true in the weak sense. Even if $\frac{\partial(C(q,v_H)-C(q,v_L))}{\partial q} < 0$, it may be the case that in the optimum the seller gets as much as when $\frac{\partial(C(q,v_H)-C(q,v_L))}{\partial q} \geq 0$, but, never more than that □

Proposition 8. *The seller can generate higher revenue by using the hybrid mechanism with risk of interruption with $r^* = v_L$, $p^* = v_L + C((1-a)N, v_H)$ and $q^* = (1-a)N$ if $\frac{\partial(C(q,v_H)-C(q,v_L))}{\partial q} \geq 0$ and either $av_H < v_L$ or condition* (6) *are satisfied than the optimal standalone mechanisms.*

The higher the proportion of the high valuation buyers will be, the easier for the seller to set higher price will be as fewer units will be available in the auction Hence, there will be greater competition among the high types and lower expected payoff from the auction.

4 Conclusion: Preliminary

Motivated by the IaaS cloud computing market we provide an economic rationale of its currently using pricing schemes. Specifically, we illustrate under what conditions the seller can increase her revenue by offering simultaneously a posted price scheme and an auction for selling services. What we find is that when the seller introduces a risk of interruption for the holders of the goods, she can price discriminate among buyers of different valuations and generate higher payoff.

There may be other markets that our model can be applied. For example the market for transmission of electricity by generators and the risk of congestion rents has some similarities with our environment. However, in such network industries which are heavily regulated, the objective is often the maximization of total welfare and not the profit of the generators.

An interesting question is why while Amazon EC2 uses a hybrid mechanism, other providers use only a posted mechanism. While we find evidence that the decision to use of the hybrid depends on the number of buyers, we do not consider how the entry of competitors affects the choice of the mechanism of each of the competitors. This is a nice extension which we are going to explore.

We decided to focus on a static version of the problem in order to investigate whether and when the hybrid is chosen by the seller neglecting the complications that could be in place from a more dynamic considerations. However, we plan to check our idea and main results in more dynamic frameworks to see how robust is our rationale for the pricing options.

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