Price Discrimination by a Two-sided Platform: with Applications to Advertising and Privacy Design*

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Abstract

We study price discrimination by a monopoly two-sided platform who mediates interactions between two different groups of agents. We adapt a canonical model of second-degree price discrimination à la Mussa and Rosen (1978) to a two-sided platform by focusing on *non-responsiveness*, a clash between the allocation the platform wants to achieve and the incentive compatible allocations. In this framework we address the key question of when a price discrimination on one side complements or substitutes a price discrimination on the other side. We offer two applications on advertising platforms and also highlight the role of commitment in eliciting personal information for targeted advertising.

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1 Introduction

Second-degree price discrimination of a monopolist (Maskin and Riley, 1984; Mussa and Rosen, 1978) has been intensively studied and become one of the best-known applications of the principal-agent theory.[1] However, little has been known about second-degree price discrimination of a two-sided monopoly platform who earns its profit by mediating interactions between two different groups. In this paper we adapt a canonical model of second-degree price discrimination à la Mussa and Rosen to a two-sided platform and address the following questions. When does the price discrimination on one side complement or substitute the price discrimination on the other side? How does the lack of commitment on one side affect the set of allocations implementable on the other side? We apply our insight to an advertising platform and also examine the role of a platform's commitment in eliciting personal information from content users for better targeted advertising.

A central concept in our paper is *non-responsiveness* (Guesnerie and Laffont, 1984; Laffont and Martimort, 2002), which refers to a situation of clash between the allocation the principal wants to achieve and the incentive compatible allocations. In a standard principal-agent model, this conflict may arise when the agent's type directly affects the principal's utility. For instance, suppose that the principal is a benevolent regulator who takes care of not only the production cost of the regulated firm (i.e., the agent) but also the amount of pollution that the firm emits. The incentive compatibility requires that the low-cost type produce more than the high-cost type. However, if the reason why the high-cost type has a higher cost is that it produces less pollution, then the principal may want to induce the high-cost type to produce more than the low-cost type. As such a non-monotonic schedule clashes with the incentive compatibility condition, the principal ends up adopting a pooling allocation.

A two-sided platform can frequently face non-responsiveness situations due to the very nature of its business: it mediates cross-group interactions. Consider, for instance, a two-sided online media platform mediating consumers and advertisers. Suppose that there are two types, H and L, of consumers who have different degree of privacy aversion: an H type is more averse to disclosing personal information to advertisers than an L type. Then, the incentive compatibility implies that an H type reveals less personal information than an L

[1]There is a vast literature on nonlinear pricing; e.g., see Armstrong (2015) and Wilson (1993) for in-depth reviews.

type. Suppose further that H types are on average richer than L types so that, conditional on watching a well-targeted advertisement, an H type is more likely to consume the advertised good than an L type. Then, the platform may want to induce an H type consumer to reveal more personal information than an L type in order to improve the targeting accuracy but it cannot because of the incentive compatibility condition. Starting from this situation of non-responsiveness, we show the platform can mitigate the incentive compatibility constraint on the consumer side by designing appropriate price discrimination on the advertiser side.

Our model applies well to online media platforms whose business model primarily relies on advertising. Platforms such as YouTube[2] and Kindle have been interested in a tiered-service on the consumer side such that consumers have an option to avoid advertising at some price. In addition, platforms may want to sell a tiered-targeting service to the advertisers as well. For example, Facebook recently proposed a new service called 'Instant Articles' to selected newspapers such that their content can be hosted directly within the Facebook website, which can be interpreted as providing a menu of quality on the content side because a given content consumed in a seamless way is more convenient than the same content whose link to the original newspaper must be clicked.

In our canonical model, the monopoly two-sided platform offers a menu of price-quality pairs to each side. There is a mass one of agents on each side. The utility that an agent i of side k $(= A, B)$ obtains from interacting with an agent j of side $k' \neq k$ (and $k' = A, B$) is given by a multiplicative form of $\theta^k_{ij} u^k(q^k_i, q^{k'}_j)$ where θ^k_{ij} is a parameter depending on both agents' types and q^k_i $(q^{k'}_j)$ is the quality that agent i (j) receives. We consider a two-type model: each agent on a given side can have either an H type or an L type. Starting from no price discrimination on the other side k', an H type of side k is assumed to have a larger (expected) *private benefit* than an L type of side k from interacting with the agents on the other side k'. However, this does not necessarily imply that an H type on side k generates more positive *externalities* to the agents of the other side k' than an L type on side k does. In this environment, we show that the first-best allocation on side k can entail a *non-monotonic* quality schedule on side k if an L type generates sufficiently large positive externalities than an H type. Furthermore, if an H type obtains less private benefit than an L type when

[2]See Tom Huddleston, Jr. (2015) "YouTube plans video subscription service without those annoying ads." *Fortune*, April 8. Available at http://fortune.com/2015/04/08/youtube-subscription-service/

interacting with an L type agent on the other side, we say that there is a *type reversal.*

In order to address the question of when price discriminations on both sides are complements or substitutes,[3] we start by considering the case of asymmetric information on side A only. We first show that without a type reversal on side A, the implementability condition on side A coincides with the monotonicity condition regardless of whether a price discrimination is applied to side B . We find, however, that in the presence of a type reversal, a non-monotonic schedule can be implemented on side A when some appropriate price discrimination is introduced onto side B unless the utility function $u^A(q^A, q^B)$ is separable. More specifically, we find that if the qualities are complements, it requires a non-monotonic schedule on side B; if they are substitutes, it requires a monotonic schedule on side B.

The intuition behind this result is in what follows. The implementability condition means that given the quality schedule on side B, an L type (on side A)'s gain from choosing q_L^A instead of q_H^A must be larger than an H type's gain from choosing q_L^A instead of q_H^A. Hence, without discrimination on side B, a non-monotonic schedule cannot be implementable on side A. However, if the qualities are complements and $q_H^B < q_L^B$, under type reversal, an L type's gain from choosing q_L^A instead of $q_H^A (< q_L^A)$ can be larger than an H type's gain. This is because an L type benefits much more from the quality than an H type when interacting with an L type on side B. Symmetrically, if the qualities are substitutes and $q_H^B > q_L^B$, then an L type suffers much less from the substitution than an H type when interacting with an L type on side B.

To answer the initial question of when price discriminations on both sides are complements or substitutes, we consider a symmetric model with asymmetric information on both sides and study the implementable allocations with symmetric mechanisms. When the qualities on both sides are complements, we find that the implementable set includes all monotonic schedules, and may also include some non-monotonic schedules. By contrast, when the qualities are substitutes, we find that the implementable set includes only monotonic schedules,

[3]Note that the complements and substitutes between the price discriminations are used in different meaning from the complements and substitutes between qualities offered to each side of the market. The complementarity (substitutability) between the two price discriminations is said to exist when a price discrimination on one side is more (less, respectively) likely to induce a price discrimination on the other side. By contrast, the complementarity (substitutability) between the two qualities means the cross-partial derivative of the utility with respect to the two qualities is positive (negative, respectively). This distinction should be more clear in our analysis.

and possibly consists of a strict subset of monotonic schedules. Therefore, we can conclude that price discriminations on both sides become complements (substitutes) when qualities are complements (substitutes, respectively).

Interestingly, as we solve for the optimal symmetric mechanism in the symmetric setting, we find two different kinds of non-responsiveness. The first is the one discovered by Guesnerie and Laffont (1984) in one-sided market: a necessary condition for such non-responsiveness is to have a non-monotonic first-best schedule. However, we also find a new kind of non-responsiveness arising from cross-group interactions in a two-sided market. When there exists a type reversal, extracting information rent from an H type on the side k requires the platform to introduce an upward distortion in the quality allocated to an L type on the other side k' and a downward distortion in the quality allocated to an H type on the same side k. If these two distortions lead to a non-monotonic schedule which clashes with the implementability condition, then a pooling contract can be optimal even if the first-best schedule is monotonic.

After the analysis of the canonical principal-agent model, we provide two applications to online media platforms who earn profits from consumers' content consumption and from advertisers' advertising to the content users. In the first application, we focus on replicating the main insight of the canonical model: we show how a price discrimination on advertiser side helps to implement a non-monotonic advertising schedule on the consumer side. Specifically, an H type consumer may choose to receive more advertising than an L type consumer even though the former dislikes advertising more than the latter on average. This can occur in the presence of type reversal as the former dislikes less some particular types of ads than the latter.

The second application adopts more general framework in that consumers now care about the privacy (the amount of personal information collected by the platform) as well as the advertising annoyance. This application studies the role played by the platform's commitment (in terms of the usage of personal information) in eliciting personal information for better targeted advertising. Precisely, we find that with no commitment to the mechanism to be used on advertising side, the platform ends up choosing either pooling or a monotonic disclosure schedule. However, with commitment, the platform can implement a non-monotonic disclosure schedule such that an H type (i.e. high aversion to disclosure) releases more personal information to the platform than an L type. In addition, we point out that when the platform can sell the collected personal information to a third-party, a platform without commitment power may end up eliciting no personal information at all. In such situation, requiring the platform to obtain consent from each consumer prior to the sale of the collected information can partially mitigate the commitment problem, which gives support to the European Commission's recent data protection reform.

The rest of the article is organized as follows. We set up the model in Section 2, and derive first-best allocations in Section 3. In Section 4, we study an intermediate situation in which there is asymmetric information on one side only with complete information on the other side. Then, we consider asymmetric information on both sides in Section 5. After the canonical model analyses (Sections 2-5), we offer two applications in the context of advertising with no targeting (Section 6) and privacy design with targeting (Section 7). Section 8 concludes.

■ **Related Literature**

This article is related to several strands of literature. First, our paper is closely related to the second-degree price discrimination from the principal-agent framework (e.g., Maskin and Riley, 1984; Mussa and Rosen, 1978) and to the concept of non-responsiveness developed by Guesnerie and Laffont (1984) and then explored by Caillaud and Tirole (2004) in the context of financing an essential facility and by Jeon and Menicucci (2008) in the context of allocation of talent between the private sector and the science sector. To our knowledge, however, non-responsiveness has never been previously explored in two-sided markets; our paper is the first in this direction.

Although the literature on two-sided platforms has been expanding rapidly,[4] to our knowledge, there is little work that studies price discrimination in a two-sided market by explicitly addressing type-dependent interactions. One exception is Gomes and Pavan (2014) who consider heterogeneous agents on both sides in a centralized many-to-many matching setting. They provide conditions on the primitives under which the optimal matching rule is a threshold rule that each agent on one side is matched with agents on the other side having a large enough type. They also provide a precise characterization of the thresholds. We do not consider matching as in our model, all agents on one side interact with all agents on the other side. Since we are particularly interested in the role of type reversal in the cross-side inter-

[4]The literature is vast including Anderson and Coast (2005), Armstrong(2006), Caillaud and Jullien(2001, 2003), Hagiu(2006), Hagiu and Jullien(2011), Jeon and Rochet(2010), Rochet and Tirole(2003, 2006), Rysman(2009), and Weyl(2010).

actions and its impact on the optimal quality schedules, our work complements theirs. Choi, Jeon and Kim (2015) study a second-degree price discrimination of a two-sided monopoly platform in the context of network neutrality, and compares it with the case of no discrimination. However, they consider heterogeneous agents only on the content side and assumes homogeneous agents on the consumer side. In this paper we consider heterogeneous agents on both sides.

Our applications are closely related to the literature of the economics of privacy (Acquisti, Taylor, and Wagman, 2015) and of Internet media (Peitz and Reisinger, 2014). Since we provide a model of privacy design and targeted advertising, this paper is related to Johnson (2013) who analyze targeted advertising by firms when consumers prefer blocking irrelevant ads, and to Casadesus-Mansaanell and Hervas-Drane (2015) who analyze firms' competition in disclosure levels for consumer information. Distinguished from both, we study privacy design from a mechanism design approach. Our paper delivers an important implication for the recent data protection reform in Europe: giving users control over their personal information already disclosed to the platform may play as a critical commitment mechanism so that the platform can elicit useful personal information ex ante for better targeting. This implication is consistent with Miller and Tucker (2014) who find empirically that limiting redisclosure without the individual's consent incentivizes individuals to obtain genetic tests. Lastly, our application to privacy design is related to Hagiu (2006) who also considers sequential offers by two-sided platforms. Without commitment, platforms announce their prices to sellers before their prices to buyers; with commitment both prices are simultaneously announced. In his model the lack of commitment is about the pricing on the buyer side, whereas in our model the lack of commitment is about the use of collected personal information on the advertising side.

2 A canonical principal-agent model in two-sided markets

We here consider a canonical principal-agent model (Laffont and Martimort, 2002; Mussa and Rosen, 1978) and adapt it to a two-sided market where a monopoly platform (i.e., the principal) designs a mechanism to mediate interactions between agents from two sides, $k = \mathrm{A}, \mathrm{B}$. On each side, there is a mass one of agents. Let θ_i^k represent the type of agent i on side k. $\theta_j^{k'}$ is similarly defined. For simplicity, we consider a two-type model: on each side, an agent has either an H type or an L type, i.e., $\theta_i^k \in \{H, L\}$ and $\theta_j^{k'} \in \{H, L\}$. Although we consider two types, two-sided interactions make the model very rich and involved. Let $\nu^k \in (0,1)$ represent the fraction of H-types on side k. The platform chooses quality q_i^k for each agent i of side k and quality $q_j^{k'}$ for each agent j of side $k'(\neq k)$. When an agent i of side k interacts with an agent j of side k', the utility the former obtains can be represented as

$$U^k(\theta_i^k, \theta_j^{k'}, q_i^k, q_j^{k'}).$$

As this is a very general formulation, in what follows we consider a particular class of utility functions in which the types interact in a multiplicative way with qualities:

$$U^k(\theta_i^k, \theta_j^{k'}, q_i^k, q_j^{k,'}) = \theta_{ij}^k u^k(q_i^k, q_j^{k'}),$$

where θ_{ij}^k represents the consumption intensity of a side k agent as a function of both agents' types. We assume that the utility function $u^k : \mathbb{R}_+^2 \to \mathbb{R}$ is strictly increasing and strictly concave, and denote with u_l^k the partial derivative of u^k with respect to its l-th variable, for $l = 1, 2$. Moreover, we define u_{12}^k as follows:

$$u_{12}^k(q_i^k, q_j^{k'}) \equiv \frac{\partial^2 u^k(q_i^k, q_j^{k'})}{\partial q_i^k \partial q_j^{k'}}.$$

We assume that u_{12}^k has the same sign for each q_i^k and $q_j^{k'}$; for side k the qualities are said to be independent if $u_{12}^k = 0$, complements if $u_{12}^k > 0$, and substitutes if $u_{12}^k < 0$. The costs of producing q_i^A and q_j^B are respectively denoted by $C^A(q_i^A)$ and $C^B(q_j^B)$. We assume that both cost functions are strictly increasing and convex.

Depending on the match of types, we may have the following four parameters of consumption intensity on side k:

$k \backslash k'$	H	L
H	θ_{HH}^k	θ_{HL}^k
L	θ_{LH}^k	θ_{LL}^k

Even with a two-type model and with the multiplicative specification, our model is still rich enough as we have many parameters given exogenously: $\Theta^k \equiv \{\theta_{HH}^k, \theta_{HL}^k, \theta_{LH}^k, \theta_{LL}^k\}$, ν^k, and the utility function u^k, for $k = \mathrm{A}, \mathrm{B}$. For this reason, when we characterize the second-best mechanism, we focus on a symmetric model (see Section 5). Sometimes we consider the case in which $u^k(q^k, q^{k'})$ is separable such that there exist two one-variable functions w^A, w^B satisfying

$$u^A(q^A, q^B) = u^B(q^B, q^A) = w^A(q^A) + w^B(q^B). \tag{1}$$

Of course, in this case we have $u^A_{12} = u^B_{12} = 0$. In particular, if $w^A(\cdot) = w^B(\cdot)$, $u^A(\cdot) = u^B(\cdot)$ is symmetric. Such restriction allows us to isolate the distortions in quality schedule generated by cross-group interactions through types as $q^{k'}$ does not affect the marginal utility from q^k. Before we move on to our analysis, let us introduce notation for the differences in the quality preference parameters as follows:

$$\begin{aligned}
\Delta^{b,k}_H &= \theta^k_{HH} - \theta^k_{LH}, \quad \Delta^{b,k}_L = \theta^k_{HL} - \theta^k_{LL}, \\
\Delta^{e,k}_H &= \theta^k_{HH} - \theta^k_{HL}, \quad \Delta^{e,k}_L = \theta^k_{LH} - \theta^k_{LL},
\end{aligned}$$

where the superscript b means private *benefit* and the superscript e means *externality*. The component $\Delta^{b,k}_j$ captures the change in the benefit of an agent on side k when his type changes from L to H for a given type of the side $k'(\neq k)$ agent which is fixed at $\theta^{k'}_j = H, L$. In other words, the superscript b stands for difference in private benefit generated by the own side type change. In contrast, the component $\Delta^{e,k}_i$ captures the change in the benefit of an agent on side k when the type of the side k' agent changes from L to H, given that side k agent's type is fixed at $\theta^k_i = H, L$.

3 First-best allocations

In this section we characterize the monopoly platform's first-best quality schedule on both sides of the market. Let $\mathbf{q} \equiv (q^A_H, q^A_L, q^B_H, q^B_L)$ denote the vector of quality specifications. The first-best quality schedule maximizes the total surplus which is given as follows:

$$\begin{aligned}
\Pi^{FB}(\mathbf{q}) = {} & \nu^A\nu^B \left[\theta^A_{HH}u^A(q^A_H, q^B_H) + \theta^B_{HH}u^B(q^B_H, q^A_H)\right] + \nu^A(1-\nu^B)\left[\theta^A_{HL}u^A(q^A_H, q^B_L) + \theta^B_{LH}u^B(q^B_L, q^A_H)\right] \\
& + (1-\nu^A)\nu^B\left[\theta^A_{LH}u^A(q^A_L, q^B_H) + \theta^B_{HL}u^B(q^B_H, q^A_L)\right] \\
& + (1-\nu^A)(1-\nu^B)\left[\theta^A_{LL}u^A(q^A_L, q^B_L) + \theta^B_{LL}u^B(q^B_L, q^A_L)\right] \\
& - \nu^A C^A(q^A_H) - (1-\nu^A)C^A(q^A_L) - \nu^B C^B(q^B_H) - (1-\nu^B)C^B(q^B_L).
\end{aligned}$$

Given our assumptions, Π^{FB} is concave, hence we can use the FOCs with respect to q_H^k and q_L^k in order to characterize the first-best quality schedule for side k as follows:

$$\nu^{k'}\left[\theta_{HH}^k u_1^k(q_H^k, q_H^{k'}) + \theta_{HH}^{k'} u_2^{k'}(q_H^{k'}, q_H^k)\right] + (1-\nu^{k'})\left[\theta_{HL}^k u_1^k(q_H^k, q_L^{k'}) + \theta_{LH}^{k'} u_2^{k'}(q_L^{k'}, q_H^k)\right] = \frac{dC^k(q_H^k)}{dq_H^k};$$

$$\nu^{k'}\left[\theta_{LH}^k u_1^k(q_L^k, q_H^{k'}) + \theta_{HL}^{k'} u_2^{k'}(q_H^{k'}, q_L^k)\right] + (1-\nu^{k'})\left[\theta_{LL}^k u_1^k(q_L^k, q_L^{k'}) + \theta_{LL}^{k'} u_2^{k'}(q_L^{k'}, q_L^k)\right] = \frac{dC^k(q_L^k)}{dq_L^k}.$$

That is, the first-best quality vector $\mathbf{q}^{FB}$ is determined as a solution to the system composed of the four FOCs, two from each $k = A$ and $k = B$.

In what follows, we assume:

Assumption 1. $\nu^B \Delta_H^{b,A} > \max\left\{-(1-\nu^B)\Delta_L^{b,A}, 0\right\}$ *and* $\nu^A \Delta_H^{b,B} > \max\left\{-(1-\nu^A)\Delta_L^{b,B}, 0\right\}$.

Assumption 1 means that (i) the expected change in the private benefit on side k when an L type is replaced by an H type on the same side k is positive, i.e., $\nu^B \Delta_H^{b,A} + (1-\nu^B)\Delta_L^{b,A} > 0$ and $\nu^A \Delta_H^{b,B} + (1-\nu^A)\Delta_L^{b,B} > 0$ (ii) an H type has a larger private benefit than an L type for his interaction with an H type in the other side, i.e., $\Delta_H^{b,A} > 0$ and $\Delta_H^{b,B} > 0$. Thus, Assumption 1 extends the standard meaning of an H type and of an L type to a two-sided market. Note however that Assumption 1 does not restrict the sign of $\Delta_L^{b,A}$ and of $\Delta_L^{b,B}$: if they are negative, the L type on side k can have a larger private benefit than the H type on side k for her interaction with the L type in side $k'(\neq k)$.

As a benchmark, consider the case of quality independence such that u^A and u^B satisfy condition (1). The quality schedule on side B does not affect the FOCs determining the quality schedule on side A, and *vice versa*. Hence, the quality schedule for each side is determined independently from that of the other side. We find that the first-best quality schedule is non-monotonic on side A (i.e., $q_H^{A,FB} < q_L^{A,FB}$) if and only if

$$\nu^B \Delta_H^{b,A} + (1-\nu^B)\Delta_L^{b,A} < -\left[\nu^B \Delta_H^{e,B} + (1-\nu^B)\Delta_L^{e,B}\right]. \tag{2}$$

The L.H.S. of the inequality (2) represents the expected change in the private benefit on side A when the focal agent's type changes from L to H on side A, whereas the bracketed term in the R.H.S. represents the expected change in the externality on side B from the same change. If $\nu^B \Delta_H^{e,B} + (1-\nu^B)\Delta_L^{e,B} > 0$, the H type of side A generates more (positive) externality to side B than the L type of side A. This, together with Assumption 1, implies $q_H^{A,FB} > q_L^{A,FB}$. In contrast, if $\nu^B \Delta_H^{e,B} + (1-\nu^B)\Delta_L^{e,B} < 0$, then the L type generates more (positive) externality than H does. Thus, we have $q_H^{A,FB} < q_L^{A,FB}$ if an H type's relative gain in terms of private benefit on side A is smaller than an L type's relative contribution in terms of externality on side B, which is exactly meant by (2).

Proposition 1. *(First-best) Suppose that u^A and u^B satisfy the separability condition of (1). Then we have $q_H^{A,FB} < q_L^{A,FB}$ if and only if an H type's relative gain in terms of private benefit on side* A *is smaller than an L type's relative contribution in terms of externality on side* B *(i.e., inequality (2) holds). A similar statement holds for the condition to have $q_H^{B,FB} < q_L^{B,FB}$.*

4 Information asymmetry on one side only

Let us begin our analysis by studying an intermediate situation in which there is asymmetric information on one side only, say side A, but complete information on side B. Introducing private information only on side A allows us to analyze how a given quality schedule on side B affects the set of implementable quality schedules on side A, which applies as well to the case of asymmetric information on both sides. In addition, we can identify possible quality distortions generated by cross-side interactions separately from standard own-side quality distortions.

The platform's maximization problem under asymmetric information only on side A, is stated as

$$\max_{\{(q_H^k,p_H^k),(q_L^k,p_L^k)\}} \nu^A \left[p_H^A - C^A(q_H^A)\right] + (1-\nu^A)\left[p_L^A - C^A(q_L^A)\right] + \nu^B \left[p_H^B - C^B(q_H^B)\right] + (1-\nu^B)\left[p_L^B - C^B(q_L^B)\right]$$

subject to

$$(\mathrm{IC}_H^A)\ \nu^B\theta_{HH}^A u^A(q_H^A, q_H^B) + (1-\nu^B)\theta_{HL}^A u^A(q_H^A, q_L^B) - p_H^A \geq \nu^B\theta_{HH}^A u^A(q_L^A, q_H^B) + (1-\nu^B)\theta_{HL}^A u^A(q_L^A, q_L^B) - p_L^A$$

$$(\mathrm{IC}_L^A)\ \nu^B\theta_{LH}^A u^A(q_L^A, q_H^B) + (1-\nu^B)\theta_{LL}^A u^A(q_L^A, q_L^B) - p_L^A \geq \nu^B\theta_{LH}^A u^A(q_H^A, q_H^B) + (1-\nu^B)\theta_{LL}^A u^A(q_H^A, q_L^B) - p_H^A$$

$$(\text{IR}_H^A) \qquad \nu^B\theta_{HH}^A u^A(q_H^A, q_H^B) + (1-\nu^B)\theta_{HL}^A u^A(q_H^A, q_L^B) - p_H^A \geq 0$$

$$(\text{IR}_L^A) \qquad \nu^B\theta_{LH}^A u^A(q_L^A, q_H^B) + (1-\nu^B)\theta_{LL}^A u^A(q_L^A, q_L^B) - p_L^A \geq 0$$

$$(\text{IR}_H^B) \qquad \nu^A\theta_{HH}^B u^B(q_H^B, q_H^A) + (1-\nu^A)\theta_{HL}^B u^B(q_H^B, q_L^A) - p_H^B \geq 0$$

$$(\text{IR}_L^B) \qquad \nu^A\theta_{LH}^B u^B(q_L^B, q_H^A) + (1-\nu^A)\theta_{LL}^B u^B(q_L^B, q_L^A) - p_L^B \geq 0.$$

Let $\mathbf{q}^{SB}$ represent the profit-maximizing quality vector. Obviously, on side B, the platform will set its tariffs p_H^B and p_L^B such that both participation constraints IR_H^B and IR_L^B are binding. Matters are less obvious on side A, and it is useful to distinguish the case of $\Delta_L^{b,A} \geq 0$ from that of $\Delta_L^{b,A} < 0$.

Definition. *When $\Delta_L^{b,A} < 0$ holds, we say that there is a* _type reversal_ *on side A.*

Type reversal means that an H type of side A gets more benefit than an L type of side A when each of them is matched with an H type on side B, but the reverse holds when each is matched with an L type on side B. We show that the design problem is standard when $\Delta_L^{b,A} \geq 0$, but the analysis becomes more involved when a type reversal exists. Below we offer the analysis for each case starting from no type reversal.

4.1 The case of no type reversal: $\Delta_L^{b,A} \geq 0$

Given $\Delta_L^{b,A} \geq 0$, we can use standard arguments to prove that (i) IR_L^A and IC_H^A make IR_H^A redundant so that IR_H^A can be safely neglected; (ii) IR_L^A and IC_H^A bind in the optimal menu of contracts; (iii) given that IC_H^A binds, IC_L^A reduces to

$$(\mathbf{I}^A) \quad \Phi^A := \nu^B\Delta_H^{b,A}\left[u^A(q_H^A, q_H^B) - u^A(q_L^A, q_H^B)\right] + (1-\nu^B)\Delta_L^{b,A}\left[u^A(q_H^A, q_L^B) - u^A(q_L^A, q_L^B)\right] \geq 0 \tag{3}$$

which we call the *implementability condition* on side A. Because $\Delta_H^{b,A} > 0$ from Assumption 1 and currently we consider the case of $\Delta_L^{b,A} \geq 0$, $(\mathbf{I}^A)$ is equivalent to the standard monotonicity constraint $q_H^A \geq q_L^A$ irrespective of the quality schedule on side B. From IR_L^A and IC_H^A we obtain the prices charged to the agents of side A:

$$p_L^A = \nu^B\theta_{LH}^A u^A(q_L^A, q_H^B) + (1-\nu^B)\theta_{LL}^A u^A(q_L^A, q_L^B), \tag{4}$$

$$p_H^A = \nu^B\theta_{HH}^A u^A(q_H^A, q_H^B) + (1-\nu^B)\theta_{HL}^A u^A(q_H^A, q_L^B) - \Omega_H^A, \tag{5}$$

where Ω_H^A in the expression p_H^A represents the *information rent* of the H type agent on side A and is given by:

$$\Omega_H^A := \nu^B \Delta_H^{b,A} u^A(q_L^A, q_H^B) + (1-\nu^B)\Delta_L^{b,A} u^A(q_L^A, q_L^B). \tag{6}$$

Substituting (4) and (5) into the objective of the platform yields

$$\hat{\Pi}(\mathbf{q}) = \Pi^{FB}(\mathbf{q}) - \nu^A \Omega_H^A \tag{7}$$

which we need to maximize subject to ($\mathbf{I}^A$), that is, subject to $q_H^A \geq q_L^A$. Our analysis of the first-best in Section 3 has shown that the first-best quality schedule can be non-monotonic, i.e., $q_H^{k,FB} < q_L^{k,FB}$ when an L type's contribution in terms of externality is larger than an H type's gain in terms of private benefit. If this phenomenon occurs on side A, then the allocation the platform wants to achieve on side A may clash with the allocations implementable given ($\mathbf{I}^A$), yielding a *non-responsiveness* situation. Let $\hat{\mathbf{q}}$ denote the maximizer of $\hat{\Pi}$ when the constraint $q_H^A \geq q_L^A$ is neglected. While the information rent does not depend on q_H^A as in the standard model, in general we no longer obtain the standard result of *no distortion at top* for q_H^A. That is, $\hat{q}_H^A = q_H^{A,FB}$ is not warranted in the two-sided framework. This is because the distortions in q_H^B and q_L^B from their first-best values change the marginal value of q_H^A. In particular, if $\hat{q}_H^B > q_H^{B,FB}$ and $\hat{q}_L^B > q_L^{B,FB}$ and the qualities are complements (substitutes) for agents of both sides, then we get a upward distortion at the top on side A, i.e., $\hat{q}_H^A > q_H^{A,FB}$ (then a downward distortion of $\hat{q}_H^A < q_H^{A,FB}$); these conclusions are reversed if $\hat{q}_H^B < q_H^{B,FB}$ and $\hat{q}_L^B < q_L^{B,FB}$.

Regarding the other variables, q_L^A, q_H^B, and q_L^B, it is immediate that the partial derivatives of $\hat{\Pi}$ with respect to each of these variables is negative at $\mathbf{q} = \mathbf{q}^{FB}$ because of the term Ω_H^A. Therefore, if for instance we fix $(q_H^A, q_H^B, q_L^B) = (q_H^{A,FB}, q_H^{B,FB}, q_L^{B,FB})$, then q_L^A is distorted downward with respect to $q_L^{A,FB}$, as in a standard setting of a one-sided market. Likewise, if $(q_H^A, q_L^A, q_L^B) = (q_H^{A,FB}, q_L^{A,FB}, q_L^{B,FB})$, then we conclude that q_H^B is distorted downward compared to $q_H^{B,FB}$. Similarly if $(q_H^A, q_L^A, q_H^B) = (q_H^{A,FB}, q_L^{A,FB}, q_H^{B,FB})$, q_L^B is distorted downward from $q_L^{B,FB}$. Unlike the distortion on q_L^A, the distortions in the qualities in side B are generated because of the two-sided market interactions as the information rent of the H type on side A increases with both q_H^B and q_L^B from (6).

Notice however that $\hat{\mathbf{q}}$ is the second-best optimum only if it satisfies $\hat{q}_H^A \geq \hat{q}_L^A$. If this is not the case, then a pooling contract must be implemented on side A, and the pooling quality is determined by maximizing $\hat{\Pi}(q_L^A, q_L^A, q_H^B, q_L^B)$ with respect to (q_L^A, q_H^B, q_L^B).

For an explicit condition, let us consider the case in which u^A and u^B satisfy the separability condition of (1). Then, $\hat{\mathbf{q}}$ exhibits a non-monotonic schedule such that $\hat{q}_H^A < \hat{q}_L^A$ if and only if

$$\nu^B \frac{\Delta_H^{b,A}}{1-\nu^A} + (1-v^B)\frac{\Delta_L^{b,A}}{1-\nu^A} < -\left[\nu^B \Delta_H^{e,B} + (1-v^B)\Delta_L^{e,B}\right]. \tag{8}$$

This condition is similar to the condition for the first-best allocation (2). The difference arises only for the L.H.S.: $\Delta_H^{b,A} = \theta_{HH}^A - \theta_{LH}^A$ is replaced by $\theta_{HH}^A - \left[\theta_{LH}^A - \frac{\nu^A}{1-\nu^A}\Delta_H^{b,A}\right]$ where $\theta_{LH}^A - \frac{\nu^A}{1-\nu^A}\Delta_H^{b,A}$ is the virtual valuation for θ_{LH}^A and similarly for $\Delta_L^{b,A} = \theta_{HL}^A - \theta_{LL}^A$. If (8) is violated, then $\hat{\mathbf{q}}$ are confirmed to be the optimal second-best quantities, and given separable u^A and u^B , they are such that

$$q_H^{A,SB} = q_H^{A,FB}, \qquad q_L^{A,SB} < q_L^{A,FB}, \qquad q_H^{B,SB} < q_H^{B,FB}, \qquad q_L^{B,SB} < q_L^{B,FB}.$$

If (8) is satisfied, then $\hat{\mathbf{q}}$ yields a non-monotonic schedule on side A, which clashes with the constraint $q_H^A \geq q_L^A$. Then agents on side A are offered a pooling contract.

4.2 The case of type reversal: $\Delta_L^{b,A} < 0$

If $\Delta_L^{b,A} < 0$, then the platform's design problem is more involved because satisfying IC_H^A and IR_L^A does not imply that IR_H^A is satisfied, and because IC_L^A does not necessarily reduce to $q_H^A \geq q_L^A$. In what follows, we examine the set of points (q_H^A, q_L^A) such that satisfies the implementability condition for given (q_H^B, q_L^B). In particular, we have seen above that the platform may want to achieve a non-monotonic schedule on side A, and therefore we investigate what kind of quality schedule on side B allows the platform to implement a non-monotonic schedule on side A by distinguishing the case of complements from the case of substitutes.

Consider first the maximization problem in which IR_H^A is neglected. Then, as in Subsection 4.1, we can prove that IR_L^A and IC_H^A bind in the optimum, therefore (4)-(5) hold and the profit function is still $\hat{\Pi}$ in (7); moreover, IC_L^A is equivalent to $(\mathbf{I}^A)$.

We denote with F the set of (q_H^A, q_L^A) satisfying $(\mathbf{I}^A)$, given (q_H^B, q_L^B).[5] In order to describe

[5]Hence, F depends on (q_H^B, q_L^B) even though our notation does not make it explicit.

F, we let

- M (from "monotonic") denote the set of (q_H^A, q_L^A) such that $q_H^A > q_L^A \geq 0$;
- N (from "non-monotonic") denote the set of (q_H^A, q_L^A) such that $0 \leq q_H^A < q_L^A$;
- D (from "diagonal") denote the set of (q_H^A, q_L^A) such that $0 \leq q_H^A = q_L^A$.

Since $\Phi^A = 0$ at each point satisfying $q_H^A = q_L^A$, it follows that $D \subseteq F$. Moreover it is immediate to identify F if Φ^A is strictly monotone with respect to q_H^A. Precisely, if Φ^A is strictly increasing in q_H^A then $F = M \cup D$; if Φ^A is strictly decreasing with respect to q_H^A, then $F = N \cup D$.

If u^A satisfies (1), then $\Phi^A = \left(\nu^B \Delta_H^{b,A} + (1-\nu^B)\Delta_L^{b,A}\right)\left(w^A(q_H^A) - w^A(q_L^A)\right)$, which is strictly increasing in q_H^A by Assumption 1, and thus $F = M \cup D$. In the case of complements, we have that Φ^A is strictly increasing in q_H^A if $q_H^B \geq q_L^B$ (since in this case $u_1^A(q_H^A, q_H^B) \geq u_1^A(q_H^A, q_L^B)$ and Assumption 1 holds) or if $q_H^B < q_L^B$ and $|(1-v^B)\Delta_L^{b,A}|$ is close to zero and/or the effect of complementarity is small. Conversely, if $|(1-v^B)\Delta_L^{b,A}|$ is close to $v^B \Delta_H^{b,A}$ and the effect of complementarity is strong, then Φ^A is strictly decreasing with respect to q_H^A.

In the case of substitutes, we obtain opposite results: Φ^A is strictly increasing in q_H^A if $q_H^B \leq q_L^B$ (again $u_1^A(q_H^A, q_H^B) \geq u_1^A(q_H^A, q_L^B)$ and Assumption 1 holds), or if $q_H^B > q_L^B$ and $|(1-v^B)\Delta_L^{b,A}|$ is close to zero and/or the effect of substitution is small. Conversely, if $|(1-v^B)\Delta_L^{b,A}|$ is close to $v^B \Delta_H^{b,A}$ and the effect of substitution is strong, then Φ^A is strictly decreasing with respect to q_H^A. The following proposition summarizes the results.

Proposition 2. *Consider the setting with private information on A side only under type reversal ($\Delta_L^{b,A} < 0$) for a given quality schedule on side B, $\left(q_H^B, q_L^B\right)$.*

(i) If u^A satisfies the separability condition of (1), the implementable set is equal to the set of the weakly monotonic schedules (i.e., $F = M \cup D$) no matter what the quality schedule on side B $\left(q_H^B, q_L^B\right)$.

(ii) Suppose that qualities are complements on side A (i.e. $u_{12}^A(q^A, q^B) > 0$).

(a) If $q_H^B \geq q_L^B$, then the implementable set on side A is equal to the set of the weakly monotonic schedules (i.e., $F = M \cup D$).

(b) If $q_H^B < q_L^B$, the complementarity is sufficiently strong and $|(1-v^B)\Delta_L^{b,A}|$ is not close to 0, F includes some strictly non-monotonic schedules only (i.e. $F \subset N$).

(iii) Suppose that qualities are substitutes on side A (i.e. $u^A_{12}(q^A, q^B) < 0$*).*

> *(a) If* $q^B_H \leq q^B_L$*, then the implementable set on side A is equal to the set of the weakly monotonic schedules (i.e.,* $F = M \cup D$*).*
>
> *(b) If* $q^B_H > q^B_L$*, the substitution is sufficiently strong and* $|(1 - v^B)\Delta^{b,A}_L|$ *is not close to zero,* F *includes some strictly non-monotonic schedules only (i.e.* $F \subset N$ *).*

The implementability condition (3) says that given the quality schedule on side B, an H type (on side A)'s gain from choosing q^A_H instead of q^A_L must be larger than an L type's gain from choosing q^A_H instead of q^A_L. Therefore, absent price discrimination on side B, a non-monotonic schedule (i.e., $q^A_H < q^A_L$) is not implementable because an H type suffers more than an L type when quality is reduced from q^A_L to q^A_H. However, if the qualities are complements on side A and $q^B_H < q^B_L$, under type reversal, a non-monotonic schedule (i.e., $q^A_H < q^A_L$) can be implemented as an L type's loss can be larger than an H type's one when the quality is reduced from q^A_L to q^A_H. This is because an L type enjoys a high marginal utility from interacting with an L type because of the complementarity between qualties, $q^B_H < q^B_L$ and type reversal (i.e., $\theta^A_{HL} < \theta^A_{LL}$). Symmetrically, if qualities are substitutes, implementing a non-monotonic schedule requires $q^B_H > q^B_L$ as an L type enjoys a high marginal from interacting with an L type because of the substitutability, $q^B_H > q^B_L$ and type reversal.

Note that both for complements and for substitutes there exist intermediate cases in which Φ^A is non-monotone and thus F includes both some points in M and some points in N. Appendix B provides a more detailed description of F under an additional assumption which implies that Φ^A is strictly concave in q^A_H.[6] The discussions of the case with no type reversal and the above proposition tell us when the implementable set is equal to the set of the weakly monotonic schedules:

Corollary 1. *The implementable set on side A is equal to the set of the weakly monotonic schedules if any of the following conditions is satisfied.*

(i) There is no type reversal ($\Delta^{b,A}_L \geq 0$*).*

(ii) u^A *satisfies the separability condition of (1).*

(iii) There is no price discrimination on side B (i.e., $q^B_H = q^B_L$*).*

[6]Note that these findings hold even when there is asymmetric information on both sides.

Finally, the constraint IR_H^A is satisfied if

$$\nu^B \Delta_H^{b,A} u^A(q_L^A, q_H^B) + (1-\nu^B)\Delta_L^{b,A} u^A(q_L^A, q_L^B) \geq 0. \tag{9}$$

Given Assumption 1, IR_H^A is satisfied either when $q_H^B \geq q_L^B$, or when $q_H^B < q_L^B$ and $|(1-\nu^B)\Delta_L^{b,A}|$ is small.

5 Information asymmetry on both sides: symmetric model

We now consider asymmetric information on both sides. In order to address the question of whether price discriminations on both sides are complements or substitutes, we will consider a symmetric model. Although a symmetric model is detached from the real world, its simplicity helps us to isolate the main driving forces that determine the answer to the question. We augment our analysis by providing real world applications based on asymmetric models in later sections. We introduce the following notation for the symmetric model: $\theta_{HH}^A = \theta_{HH}^B \equiv \theta_{HH}$, $\theta_{HL}^A = \theta_{HL}^B \equiv \theta_{HL}$, $\theta_{LH}^A = \theta_{LH}^B \equiv \theta_{LH}$, $\theta_{LL}^A = \theta_{LL}^B \equiv \theta_{LL}$, $\Delta_H \equiv \theta_{HH} - \theta_{LH}$, $\Delta_L \equiv \theta_{HL} - \theta_{LL}$, $v^A = v^B \equiv v$ and $u^A = u^B \equiv u$. We focus on a symmetric mechanism with $q_H^A = q_H^B = q_H$ and $q_L^A = q_L^B = q_L$.

The profit function under complete information for this symmetric model is given by

$$\begin{aligned}\pi^{FB}(q_H, q_L) \quad = \quad & 2\nu^2\theta_{HH}u(q_H,q_H) + 2\nu(1-\nu)(\theta_{HL}u(q_H,q_L) + \theta_{LH}u(q_L,q_H)) \\ & +2(1-\nu)^2\theta_{LL}u(q_L,q_L) - 2\nu C(q_H) - 2(1-\nu)C(q_L).\end{aligned}$$

Consider now the standard approach in which we assume that only IR_L and IC_H bind. Substituting the transfers obtained from the binding constraints into the platform's objective gives the following profit function:

$$\begin{aligned}\widehat{\pi}(q_H, q_L) \quad \equiv \quad & \pi^{FB}(q_H,q_L) - 2\nu\left[\nu\Delta_H u(q_L,q_H) + (1-\nu)\Delta_L u(q_L,q_L)\right] \\ = \quad & 2\nu^2\theta_{HH}u(q_H,q_H) + 2\nu(1-\nu)\left[\theta_{HL}u(q_H,q_L) + \theta_{LH}^v u(q_L,q_H)\right] \\ & +2(1-\nu)^2\theta_{LL}^v u(q_L,q_L) - 2\nu C(q_H) - 2(1-\nu)C(q_L)\end{aligned} \tag{10}$$

where $\theta_{LH}^v = \theta_{LH} - \frac{\nu}{1-\nu}\Delta_H (< \theta_{LH})$ and $\theta_{LL}^v = \theta_{LL} - \frac{\nu}{1-\nu}\Delta_L$. Hence $\widehat{\pi}$ differs from π^{FB} only because θ_{LH} and θ_{LL} are replaced, respectively, by θ_{LH}^v and θ_{LL}^v. Regarding Δ_L, we distinguish the case of no type reversal (i.e., $\Delta_L \geq 0$) from the case of type reversal (i.e., $\Delta_L \in (-\frac{v}{1-v}\Delta_H, 0)$). Note that

$$\theta_{LL} \gtreqless \theta_{LL}^v \text{ if and only if } \Delta_L \gtreqless 0.$$

The first-order condition with respect to q_H is given by

$$\nu\theta_{HH}\left[u_1(q_H,q_H)+u_2(q_H,q_H)\right]+(1-\nu)\left[\theta_{HL}u_1(q_H,q_L)+\theta_{LH}^v u_2(q_L,q_H)\right]=C'(q_H), \quad (11)$$

As $\theta_{LH}^v < \theta_{LH}$, given $q_L = q_L^{FB}$, the q_H that satisfies (11) is smaller than q_H^{FB}. This downward distortion arises because of the cross-group interactions in a two-sided market. If an H type on side k chooses the contract designed for type L, then he obtains a utility from interacting with an H type on side k', and therefore reducing the quality allocated to an H type on side k' helps to extract the information rent from an H type on side k. The first-order condition with respect to q_L is given by

$$\nu\left[\theta_{HL}u_2(q_H,q_L)+\theta_{LH}^v u_1(q_L,q_H)\right]+(1-\nu)\theta_{LL}^v\left[u_1(q_L,q_L)+u_2(q_L,q_L)\right]=C'(q_L). \quad (12)$$

In the case of no type reversal, given $q_H = q_H^{FB}$, the q_L that satisfies (12) is smaller than q_L^{FB}. In this case, the well-known downward distortion in one-sided market is reinforced by the downward distortion due to cross-group interactions in a two-sided market. In the case of type reversal, as $\theta_{LL} < \theta_{LL}^v$, the q_L that satisfies (12) given $q_H = q_H^{FB}$ can be higher or lower than q_L^{FB}. On the one hand, when we neglect the cross-group interactions and consider one-sided market, there is a downward distortion as $\nu\Delta_H + (1-\nu)\Delta_L > 0$ from Assumption 1. On the other hand, the cross-group interaction induces an upward distortion. Type reversal implies that when interacting with an L type on side k', an H type on side k obtains a smaller utility than an L type on side k, all other things being equal. Hence, increasing the quality allocated to an L type on side k' allows to extract rent from an H type on side k.

Consider now a pooling contract such that $q_H = q_L = q$. Then, from the binding IR_L, we find that the platform's profit is given by

$$\widehat{\pi}(q,q) \equiv 2\left[\nu\theta_{LH} + (1-v)\theta_{LL}\right]u(q,q) - 2C(q).$$

Let q^p be the maximizer of $\widehat{\pi}(q, q)$.

Non-responsiveness refers to a situation in which there is a clash between the allocation the platform wants to achieve and the incentive compatibility constraints. More precisely, the platform wants to implement a non-monotonic schedule, but she cannot due to incentive constraints and ends up choosing a pooling contract. In our model, we can distinguish two kinds of non-responsiveness depending on whether the first-best schedule is monotonic or not (i.e., q_H^{FB} is greater or smaller than q_L^{FB}). In a one-sided market, non-responsiveness occurs only if the first-best schedule is non-monotonic. To see this point, start from a monotonic first-best schedule such that $q_H^{FB} > q_L^{FB}$. This, combined with the standard result of "no distortion at the top and downward distortion at the bottom," implies $q_H^{SB} > q_L^{SB}$. Hence, a pooling cannot be optimal in one-sided market whenever $q_H^{FB} > q_L^{FB}$. However, this is no longer the case in a two-sided framework in which there are additional distortions due to cross-group interactions. In particular, denoting by $(\widehat{q}_H, \widehat{q}_L)$ the maximizer of $\widehat{\pi}(q_H, q_L)$, in the case of type reversal we can have $q_H^{FB} > \widehat{q}_H$ and $q_L^{FB} < \widehat{q}_L$ such that $\widehat{q}_L > \widehat{q}_H$ may hold even if $q_H^{FB} > q_L^{FB}$. Then, a pooling contract can be optimal even when the first-best requires a monotonic schedule, $q_H^{FB} > q_L^{FB}$.

In what follows, we first study the implementable set of allocations when there is asymmetric information on both sides. As we have seen in Section 4.1, in the case of no type reversal, the implementable set coincides with the monotonic schedules. In the case of type reversal, the result crucially depends on whether the qualities on the two sides are substitutes or complements. If they are substitutes, the implementable set is a subset of monotonic schedules, such that some monotonic schedules may not be implementable. If they are complements, the implementable set includes all monotonic schedules and possibly also some non-monotonic schedules.

Therefore, finding the second-best quality schedule can be complicated when there is type reversal and the qualities are complements. If $\widehat{q}_H \geq \widehat{q}_L$ holds, then we have $q_H^{SB} = \widehat{q}_H$ and $q_L^{SB} = \widehat{q}_L$. However, if $\widehat{q}_H < \widehat{q}_L$ holds, then $(\widehat{q}_H, \widehat{q}_L)$ can be implementable or not. If it is implementable and satisfies IR_H, then we have $q_H^{SB} = \widehat{q}_H$ and $q_L^{SB} = \widehat{q}_L$. Otherwise, we should compare the profit from the optimal pooling contract $\widehat{\pi}(q^p, q^p)$ and the highest profit from implementable non-monotonic schedules. When we solve for the latter, we should pay particular attention to IR_H as the best outcome from the implementable non-monotonic schedules may not satisfy IR_H. We then illustrate these points by analyzing a quadratic setting and the detailed results are relegated to Appendix B.

5.1 The simple case of no type reversal: $\Delta_L \geq 0$

As in Subsection 4.1, in the absence of type reversal, we find that IR_L and IC_H both bind and the optimal contracts are found by maximizing $\hat{\pi}$ with respect to (q_H, q_L) subject to $q_H \geq q_L$, since IC_L is equivalent to $q_H \geq q_L$. We then find the maximizer $(\hat{q}_H, \hat{q}_L)$ of $\hat{\pi}$ neglecting $q_H \geq q_L$. If $\widehat{q}_H \geq \widehat{q}_L$ then IC_L is satisfied by $(\widehat{q}_H, \widehat{q}_L)$, thus we have obtained the solution: $q_H^{SB} = \widehat{q}_H, q_L^{SB} = \widehat{q}_L$. Conversely, if $\widehat{q}_L > \widehat{q}_H$ then IC_L must be taken into account when maximizing $\hat{\pi}(q_H, q_L)$ with respect to (q_H, q_L). Then, the optimal quality schedule entails pooling: $q_H^{SB} = q_L^{SB} = q^P$. This approach extends to the asymmetric model as long as $\Delta_L^{b,A} \geq 0$ and $\Delta_L^{b,B} \geq 0$.

5.2 The case of type reversal: $\Delta_L < 0$

In the presence of type reversal, if IR_L and IC_H bind, then IC_L reduces to

$$(\mathbf{I}) \qquad \nu\Delta_H[u(q_H, q_H) - u(q_L, q_H)] + (1-\nu)\Delta_L[u(q_H, q_L) - u(q_L, q_L)] \geq 0. \qquad (13)$$

Here it is convenient to define $r \equiv \frac{(1-v)\Delta_L}{v\Delta_H}$, such that $r \in (-1, 0)$. Hence, the implementability condition in (13) can be written as

$$(\mathbf{I}) \quad A + rB \geq 0 \qquad (14)$$

where

$$A = \int_{q_L}^{q_H} u_1(t, q_H)dt \qquad \text{and} \qquad B = \int_{q_L}^{q_H} u_1(t, q_L)dt.$$

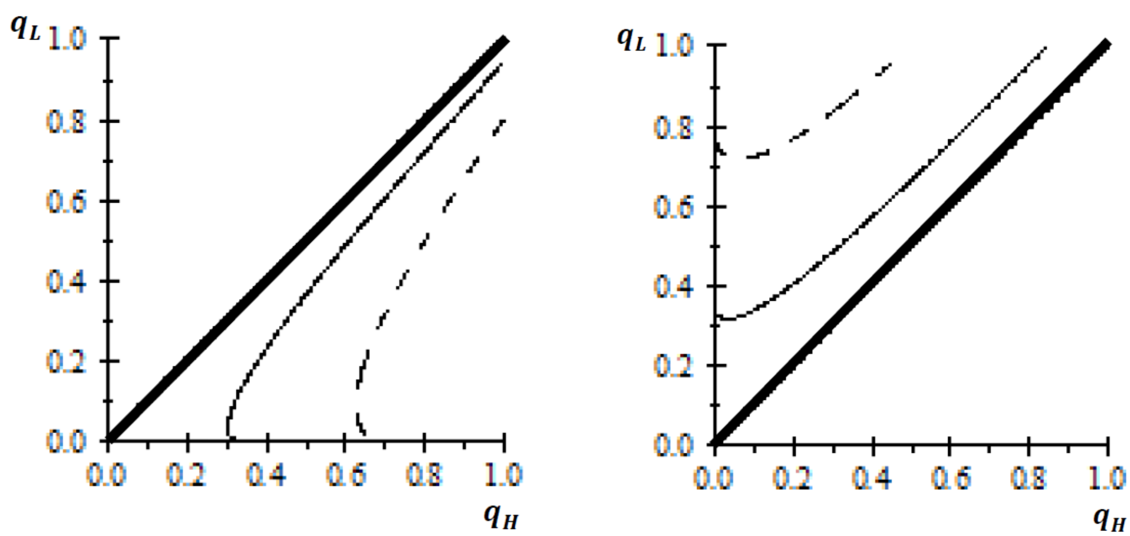
We below study the set of (q_H, q_L) that satisfies $(\mathbf{I})$ by distinguishing the case of complements from that of substitutes.

■ The case of substitutes

Consider the case in which the qualities on the two sides are substitutes. Suppose $q_H > q_L$. Because of the substitution, we have $u_1(t, q_H) < u_1(t, q_L)$, implying $B > A > 0$. Therefore, $(\mathbf{I})$ is satisfied at (q_H, q_L) for $r \geq -A/B$ and is violated for $r < -A/B$. In particular, any pair (q_H, q_L) with $q_H > q_L$ satisfies the implementability condition $(\mathbf{I})$ for $r = 0$, and no pair (q_H, q_L) such that $q_H > q_L$ satisfies the implementability condition $(\mathbf{I})$ for $r = -1$ from $B > A$.

Next, consider now $q_H < q_L$. Because of the substitution, we have $u_1(t, q_H) > u_1(t, q_L)$, implying $A < B < 0$. Hence, for any $r \in (-1, 0)$, no pair (q_H, q_L) with $q_H < q_L$ satisfies the implementability condition (**I**). Therefore the implementable set does not include any non-monotonic schedule; some monotonic schedules are also excluded from the implementable set if r is close to -1.

Consider the case in which $u(q_H, q_L) = 4\sqrt{q_H} - q_H q_L + 4\sqrt{q_L}$, with $q_H \in [0, 1]$, $q_L \in [0, 1]$ in order for u to be concave. Let $r^* = -\frac{6.88}{10}$. If $r \in (r^*, 0)$, then each (q_H, q_L) such that $q_H > q_L$ satisfies (**I**). Conversely, if $r \in (-1, r^*)$ then there exist some (q_H, q_L) with $q_H > q_L$ which do not satisfy (**I**). For instance, if $r = -\frac{8.5}{10}$ then (**I**) fails to hold for the points to the right of the dashed curve in Figure 1-(a); if $r = -\frac{9.5}{10}$, then (**I**) fails to hold for the points to the right of the thin curve in Figure 1-(a).



(a) The case for substitutes (b) The case for complements

Figure 1: The incentive compatible allocations for $u(q_H, q_L) = 4\sqrt{q_H} - q_H q_L + 4\sqrt{q_L}$, with $q_H \in [0, 1]$, $q_L \in [0, 1]$

This suggests to maximize $\hat{\pi}$ in the set between the dashed curve (if $r = -\frac{8.5}{10}$) and the 45 degree line.[7] If we have $\widehat{q}_L > \widehat{q}_H$, then the second best contracts are given by the pooling solution: $q_H^{SB} = q_L^{SB} = q^p$. Conversely, if $\widehat{q}_L < \widehat{q}_H$ (the "standard" case) then $(\widehat{q}_H, \widehat{q}_L)$ may be not feasible and, if so, the platform needs to maximize $\hat{\pi}$ subject to the constraint that (q_H, q_L) belongs to the dashed curve. However, IR$_H$ boils down to $u(q_L, q_H) + ru(q_L, q_L) \geq 0$.

[7]Notice that IR$_H$ boils down to $u(q_L, q_H) + ru(q_L, q_L) \geq 0$, hence it is satisfied by each point such that $q_H \geq q_L$, that is by each point in the feasible set.

Hence it is satisfied by each point such that $q_H \geq q_L$, that is by each point in the feasible set. This makes the mechanism design problem standard in the case of substitutes.

Recall from Corollary 1 that if there is price discrimination only on one side, then the implementable set on that side coincides with all monotonic schedules. When price discrimination occurs on both sides and qualities are substitutes, we see that with a symmetric mechanism the feasible set shrinks, as the platform cannot implement any non-monotonic schedule, and some monotonic schedules are not implementable either.

■ **The case of complements**

Consider now the case in which the qualities on the two sides are complements. Suppose $q_H > q_L$. Because of the complementarity, we have $u_1(t, q_H) > u_1(t, q_L)$ for any t, implying $A > B > 0$. Therefore, for any $r \in (-1, 0)$ any pair (q_H, q_L) satisfying $q_H > q_L$ satisfies the implementability condition (**I**). Suppose now $q_H < q_L$. Because of the complementarity, we have $u_1(t, q_H) < u_1(t, q_L)$ for any t, implying $B < A < 0$. Therefore, (14) is satisfied for $r \leq -|A|/|B|$ and is violated for $r > -|A|/|B|$. In particular, any pair (q_H, q_L) satisfying $q_H < q_L$ meets the implementability condition for $r = -1$ and no pair (q_H, q_L) with $q_H < q_L$ satisfies the implementability condition for $r = 0$. The previous arguments imply that the implementable set consists of all points below the 45 degree line (i.e., $q_H \geq q_L$), and possibly some points which are above the 45 degree line.

Consider for instance $u(q_H, q_L) = 4\sqrt{q_H} + q_H q_L + 4\sqrt{q_L}$, with $q_H \in [0, 1]$, $q_L \in [0, 1]$ in order for u to be concave. Let $r^* = -\frac{7.77}{10}$. If $r \in (r^*, 0)$, then no (q_H, q_L) such that $q_L > q_H$ satisfies (**I**). If $r \in (-1, r^*)$, then there exist some (q_H, q_L) such that $q_L > q_H$ which satisfy (**I**). For instance, if $r = -\frac{8.5}{10}$ then they are the points above the dashed curve in Figure 1-(b); if $r = -\frac{9.5}{10}$, then they are the points above the thin curve in Figure 1-(b).

Therefore, when qualities are complements, the price discrimination on both sides enlarges the feasible set with respect to the case of price discrimination on a single side: with a symmetric mechanism, the platform can implement any monotonic schedule, and possibly also some non-monotonic schedule on both sides. In this sense, price discriminations on both sides are complements.

Summarizing, we have

Proposition 3. *Consider the symmetric model with asymmetric information on both sides, and* $\Delta_L \in \left(-\frac{v}{1-v}\Delta_H, 0\right)$.

(i) *When qualities on both sides are substitutes,*

(a) *the price discrimination on both sides are substitutes in the sense that with a symmetric mechanism, the platform can never implement any non-monotonic schedule and can implement only monotonic schedules with relatively small gap $q_H - q_L$ if $|\Delta_L|$ is not close to 0;*

(b) *The implementable set shrinks with $|\Delta_L|$.*

(ii) *When qualities on both sides are complements,*

(a) *the price discrimination on both sides are complements in the sense that with a symmetric mechanism, the platform can always implement any monotonic schedule and can implement also non-monotonic schedules with relatively large gap $q_L - q_H$ if $|\Delta_L|$ is not close to 0.*

(b) *The implementable set expands with $|\Delta_L|$.*

6 Application I: advertising without targeting

We here provide a simple application of the insight from the canonical model to an advertising platform. The primary goal of this application is to demonstrate our main result in one of real-world two-sided markets. We aim to show clearly how a price discrimination on advertiser side helps to implement a non-monotonic advertising schedule on the consumer side. Since we consider asymmetric information on the consumer side only, it fits to Section 4 in our canonical model.

On side A, there is a mass one of consumers, who have two different types H and L. On side B there is a mass one of advertisers who also have two different types, H and L. Let us consider the symmetric case such that $\nu^A = \nu^B = 1/2$. As in the canonical model, on the consumer side there is asymmetric information. The platform offers a menu q_H and q_L with $(q_H, q_L) \in \{0, 1\}^2$ where '1' means advertising and '0' means no advertising. Targeted advertising is not considered here in the sense that each consumer i receives all advertising ($q_i = 1$) or no advertising ($q_i = 0$); next section we will consider a different model with targeted advertising. On side B we consider complete information on the advertising side; the platform offers an advertising level depending on the type of an advertiser, a_H or a_L. A consumer earns a constant utility u_0 from consuming content from the platform if he does not receive any advertising and thus no advertising nuisance. When the disutility of consumer i from watching advertisement of advertiser j is given by $\theta^A_{ij}\psi(a_j)$ where we assume ψ is increasing and convex. Then, with advertising a consumer i's utility is given by

$$\begin{cases} u_0 - \frac{1}{2}\theta^A_{HH}\psi(a_H) - \frac{1}{2}\theta^A_{HL}\psi(a_L) \text{ if } \theta^A_i = H \\ u_0 - \frac{1}{2}\theta^A_{LH}\psi(a_H) - \frac{1}{2}\theta^A_{LL}\psi(a_L) \text{ if } \theta^A_i = L \end{cases}$$

Similarly, the advertising revenue of advertiser j from consumer i is given by $\theta^B_{ji}R(a_j)$ where R is increasing and concave. Then, an advertiser j's expected revenue from joining the platform is given by

$$\begin{cases} \frac{1}{2}\theta^B_{HH}R(a_H) + \frac{1}{2}\theta^B_{HL}R(a_H) \text{ if } \theta^B_j = H \\ \frac{1}{2}\theta^B_{LH}R(a_L) + \frac{1}{2}\theta^B_{LL}R(a_L) \text{ if } \theta^B_j = L \end{cases}$$

We can reinterpret ψ and R in terms u^A and u^B in the canonical model as follows:

$$u^A(q,a) = -\psi(a) \cdot 1_{[q=1]}, \quad u^B(q,a) = R(a) \cdot 1_{[q=1]}$$

We impose the following assumptions on the parameters for the two-sided interactions:

$$\begin{cases} \theta^A_{HH} + \theta^A_{HL} > \theta^A_{LH} + \theta^A_{LL}; \quad \theta^A_{HH} < \theta^A_{LH}, \quad \theta^A_{HL} > \theta^A_{LL}; \\ \theta^B_{HH} > \theta^B_{LH}, \quad \theta^B_{HL} > \theta^B_{LL} \end{cases}$$

The assumptions in the first line are made to satisfy Assumption 1 and to introduce type reversal. In other words, an H type consumer dislikes more advertising than an L type consumer on average; however, there is type reversal such that, conditional on receiving the ads from H type advertisers, an H type consumer's nuisance is smaller than an L type consumer's nuisance. The second line implies that both types of advertisers find that an H type consumer is more valuable than an L type consumer for their revenues.

To highlight the role of price discrimination on side B, we assume that the L type advertisers' parameters θ^B_{LH} and θ^B_{LL} are so low that the first-best allocation requires $q^{FB}_L = 0$ and $q^{FB}_H = 1$, and hence a^{FB}_H and a^{FB}_L are determined by

$$\theta^A_{HH}\psi'(a_H) = \theta^B_{HH}R'(a_H), \qquad \theta^A_{HL}\psi'(a_L) = \theta^B_{LH}R'(a_L).$$

Consider now asymmetric information on side A. If there is no price discrimination on side B (i.e., $a_L = a_H = a$), it is impossible to implement an allocation satisfying $q_L = 0$ and $q_H = 1$. To see it, suppose that the platform offers a menu $\{(q_L = 0, t_L), (q_H = 1, t_H)\}$. Then, the incentive constraints are given as

$$\begin{aligned}
\text{IC}_L &: \quad u_0 - t_L \geq u_0 - \frac{1}{2}\theta^A_{LH}\psi(a) - \frac{1}{2}\theta^A_{LL}\psi(a) - t_H \\
\text{IC}_H &: \quad u_0 - \frac{1}{2}\theta^A_{HH}\psi(a) - \frac{1}{2}\theta^A_{HL}\psi(a) \geq u_0 - t_L.
\end{aligned}$$

Summing the two constraints lead to $\theta^A_{HH} + \theta^A_{HL} \leq \theta^A_{LH} + \theta^A_{LL}$, which is a contradiction. Yet the same logic allows us to show that $q_L = q_H = 1$ and $(q_H = 0,\ q_L = 1)$ are implementable. Hence, if we postulate that the platform is not viable without selling any advertising, the platform will choose either $q_L = q_H = 1$ or $(q_H = 0, q_L = 1)$.

Suppose now that the platform offers $a_L < a_H$. Then, it may be possible to implement $(q_H = 1,\ q_L = 0)$ as the implementability condition is given by

$$\frac{\theta^A_{LH} - \theta^A_{HH}}{\theta^A_{HL} - \theta^A_{LL}} \geq \frac{\psi(a_L)}{\psi(a_H)}.$$

Therefore, if a_H is large enough relative to a_L, the non-monotonic schedule $(q_H = 1, q_L = 0)$ can be implemented. The intuition is simple. The H type consumer dislikes less the H type ads than the L type consumer. In order to achieve a non-monotonic schedule such that the H type consumers receive advertising while the L types do not, the platform should move from $a_H = a_L$ to $a_H > a_L$.

From the canonical model analysis, this application clearly shows Proposition 2-(iii). To see this point, we note that the marginal disutility from a is zero when $q = 0$, but it is positive when $q = 1$. In other words, from $u^A(q, a) = -\psi(a) \cdot 1_{[q=1]}$, u^A_{12} is given by $u^A_2(1, a) - u^A_2(0, a) = -\psi'(a) < 0$ and thus (q, a) are substitutes. According to Proposition 2-(iii)-(b), implementing a non-monotonic schedule $(q_H = 1, q_L = 0)$ on side A requires a monotonic schedule on side B.

The above result can provide some insight about the actual or potential business practices by real-world platforms such as YouTube and Kindle. Currently, YouTube adopts the pooling with no subscription fee on the consumer side. But, as the plan is discussed (see footnote 2), we may see soon that H type consumers pay a certain fee to avoid the ads while only L types watch the ads with no payment; in this sense, the platform would implement a monotonic schedule. Also, Kindle users already can choose "Special Offers" to avoid ads with some price ($15-$20), which still corresponds to a monotonic schedule in our model. However, our results show that it may be possible and profitable that the ads are sent only to H type consumers if they generate more advertising revenues than L types consumers and there exists a type reversal such that for certain ads the H type consumers get less nuisance than the L types. Then implementing such non-monotonic schedule on the consumer side would require to increase those ads leading to the type reversal than other ads.

7 Application II: privacy design and targeted advertising

In this section, we consider the second application by extending the first application in Section 6. Here we focus on the role of commitment in eliciting personal information for better targeted advertising. Although this application stands itself as an independent interest, we try to keep a tight connection to the key insight of the canonical model.

7.1 The model

As in the first application, again we consider a mass one of consumers (or content users) and a mass one of advertisers. There are two types of consumers, $\theta \in \{H, L\}$. Let $v \in (0,1)$ denote the proportion of H type consumers. Each H type consumer has a higher disutility from releasing his personal information and from receiving advertising than each L type consumer.[8] Let $a \in [0,1]$ represent the amount of advertising, where $a = 1$ is the maximum possible amount of advertising, and $a = 0$ is ads-free environment. Let $\gamma \in [0,1]$ represent the level of personal information released to the platform. The case of $\gamma = 0$ corresponds to *perfect privacy* or *perfect anonymity*, and $\gamma = 1$ implies *no privacy* or *maximum disclosure*. Alternatively, $1-\gamma$ measures the level of privacy. A consumer of type θ earns the utility from joining the platform given (γ, a) as follows:

$$u_0 - f_\theta(\gamma) - g_\theta(a),$$

[8] According to the recent Eurobarometer Special Survey 431 on "Data Protection", 67% respondents are concerned about not having complete control over the information they provide online (Source: `http://ec.europa.eu/public_opinion/archives/ebs/ebs_431_en.pdf`). In our model those concerned are represented by the H type and those not by the L type.

where $f_\theta(\gamma)$ and $g_\theta(a)$ measure the disutility from releasing personal information and from receiving advertising, respectively. We assume that both f_θ and g_θ is increasing and convex, and that the marginal disutility is higher for an H type than for an L type in each dimension:

$$f'_H(\gamma) > f'_L(\gamma), \qquad g'_H(a) > g'_L(a), \tag{15}$$

Assuming $f_\theta(0) = g_\theta(0) = 0$, this implies $f_H(\gamma) + g_H(a) > f_L(\gamma) + g_L(a)$.

For given (γ, a), each advertiser's surplus from a consumer θ is given by

$$q_\theta(\gamma) + s_\theta(a)$$

where both q_θ and s_θ are concave. We assume that q_θ is increasing in γ because more personal information can improve the targeting accuracy and thus elevate the targeting effectiveness. On the contrary, we assume that s_θ is initially increasing but start to decrease as a gets close to one.[9] Each advertiser obtains a greater marginal revenue from an H type than from an L type:

$$q'_H(\gamma) > q'_L(\gamma), \qquad s'_H(a) > s'_L(a), \tag{16}$$

Assuming $q_\theta(0) = s_\theta(0) = 0$, this implies $q_H(\gamma) + s_H(a) > q_L(\gamma) + s_L(a)$. For analytical simplicity, the platform is assumed to extract all the surplus on the advertising side, which is the case if all advertisers are ex ante symmetric.[10] Also, the separable disutility function and separable advertising surplus function are assumed for simplicity and our main insight would carry out for non-separable functions, provided that for these functions the cross partial second order derivatives are not too large in absolute value.

We distinguish two scenarios depending on whether the platform has a commitment power or not. With commitment, the platform proposes a mechanism $\{t(\theta), \gamma(\theta), a(\theta)\}$ to consumers and $\{p, a(\theta)\}$ to advertisers where $t(\theta)$ is the payment from a θ type consumer to the platform,

[9] This assumption is supported in the literature of online media and advertising (Peitz and Reisinger, 2014). For instance, as the advertising amount increases, advertisers are more likely to be engaged in product competition, reducing the profit that the platform can extract.

[10] We can introduce some rent due to asymmetric information by specifying a micro-foundation. More precisely, given a type of consumer, each advertiser can receive a signal about the revenue it can generate by showing an advertisement to the consumer. The platform can decide how many advertising slots to auction off. Our main insight will carry over to such extension even if it will be much more technically involved than the current model.

$1-\gamma(\theta)$ the level of privacy to a θ type, $a(\theta)$ is the amount of advertising to a θ type and p is the price paid by each advertiser. Without commitment, the platform first proposes $\{t(\theta), \gamma(\theta)\}$ to consumers and then $\{p, a(\theta)\}$ to advertisers. Hence, consumers expect that the platform will choose $a(\theta)$ ex post to maximize its payoff. More precisely, the timing of events are as follows.[11]

Under commitment,

- Stage 0: Each consumer discovers his type.
- Stage 1: The platform proposes $\{t(\theta), \gamma(\theta), a(\theta)\}$ and $\{p, a(\theta)\}$
- Stage 2: Each consumer and advertiser simultaneously decides to accept or reject the offer. If a consumer accepts the offer, he chooses one of the two contracts in the menu and accordingly, pays $t(\theta)$ and disclose the amount of personal information $\gamma(\theta)$ and the platform chooses the advertising level $a(\theta)$.

Under no commitment,

- Stage 0: Each consumer discovers his type.
- Stage 1: The platform proposes $\{t(\theta), \gamma(\theta)\}$ to consumers.
- Stage 2: Each consumer decides to accept or reject the offer. If a consumer accepts the offer, he chooses one of the two contracts in the menu and accordingly pays $t(\theta)$ and disclose the amount of personal information $\gamma(\theta)$.
- Stage 3: The platform proposes $\{p, a(\theta)\}$ to advertisers.
- Stage 4: Each advertiser accepts or rejects the offer.

We assume that the transfer $t(\theta)$ cannot be negative: otherwise, consumers may cash in and run without any interest in consuming the content. We say that a disclosure schedule is monotonic (non-monotonic) if it satisfies $\gamma_L > \gamma_H$ ($\gamma_L < \gamma_H$). Similarly, an advertising schedule is monotonic (non-monotonic) if it satisfies $a_L > a_H$ ($a_L < a_H$).

[11] Haigu (2006) also consider two-sided platforms' sequential pricings. Without commitment, platforms announce their prices to sellers before their prices to buyers. By contrast, with commitment they can announce the prices to buyers at the same time they announce the prices to sellers even if buyers arrive to the market later than sellers. In his model the commitment is to the price to buyers whereas in our model the commitment is on the advertising amount.

7.2 First-best

Define the first-best privacy level and advertising level for type θ as a solution to maximizing the total surplus:

$$\max_{\gamma,a}\ q_\theta(\gamma)+s_\theta(a)-f_\theta(\gamma)-g_\theta(a)$$

The first-order conditions for γ and for a yield:

$$\begin{cases} q'_\theta(\gamma)-f'_\theta(\gamma)=0, \\ s'_\theta(a)-g'_\theta(a)=0. \end{cases}$$

To focus on more interesting cases, we further assume that

$$q'_H(\gamma)-f'_H(\gamma)>q'_L(\gamma)-f'_L(\gamma), \tag{17}$$

$$q'_L(0)-f'_L(0)>0, \quad q'_H(1)-f'_H(1)<0.$$

Assumption (17) combined with (15)-(16) implies that the first-best level of γ is higher for a H type than for a L type. The second line in (17) ensures an interior solution for $\gamma_\theta^{FB}\in(0,1)$.

Regarding the first-best advertising level, similarly we assume that

$$s'_H(a)-g'_H(a)<s'_L(a)-g'_L(a) \tag{18}$$

$$s'_\theta(0)-g'_\theta(0)>0, \quad s'_\theta(1)-g'_\theta(1)<0.$$

Under this assumption (18), the socially efficient advertising requires that an H type receives less advertising than an L type. Therefore, we have

$$\gamma_H^{FB}>\gamma_L^{FB} \quad \text{and} \quad a_H^{FB}<a_L^{FB}.$$

We can envision H types as consumers with higher income and greater time opportunity cost compared to L types. Then, an H type is likely to demand more anonymity because of a positive correlation between income and privacy (e.g. Michael, Fuchs and Scott, 1980) and to show greater advertising annoyance. However, an H type's personal information is more valuable than an L type to the advertisers' business such that the first-best outcome requires an H type to disclose more personal information than an L type. Note that we implicitly assume that the targeting accuracy is high enough. If the targeting accuracy were low such that personal information is not much helpful to improve targeting, then the first-best outcome would satisfy $\gamma_H^{FB} < \gamma_L^{FB}$, $a_H^{FB} < a_L^{FB}$. As the analysis of this situation is straightforward, we focus on more interesting situation of high targeting accuracy in which the first-best disclosure schedule is non-monotonic, $\gamma_H^{FB} > \gamma_L^{FB}$.

7.3 Second-best under commitment

Here we examine the second-best mechanism design under commitment; the platform offers contracts $\{t(\theta), \gamma(\theta), a(\theta)\}$ to consumers and $\{p, a(\theta)\}$ to advertisers. Under full surplus extraction from the advertisers, the platform maximizes the following objective

$$\nu \{t_H + q_H(\gamma_H) + s_H(a_H)\} + (1-\nu)\{t_L + q_L(\gamma_L) + s_L(a_L)\}$$

subject to

$$\begin{aligned}
\text{IR}_L &: \; u_0 - f_L(\gamma_L) - g_L(a_L) - t_L \geq 0 \\
\text{IR}_H &: \; u_0 - f_H(\gamma_H) - g_H(a_H) - t_H \geq 0 \\
\text{IC}_L &: \; u_0 - f_L(\gamma_L) - g_L(a_L) - t_L \geq u_0 - f_L(\gamma_H) - g_L(a_H) - t_H \\
\text{IC}_H &: \; u_0 - f_H(\gamma_H) - g_H(a_H) - t_H \geq u_0 - f_H(\gamma_L) - g_H(a_L) - t_L.
\end{aligned}$$

As usual, IC_L and IR_H jointly implies IR_L because

$$u_0 - f_L(\gamma_L) - g_L(a_L) - t_L \geq u_0 - f_L(\gamma_H) - g_L(a_H) - t_H \geq u_0 - f_H(\gamma_H) - g_H(a_H) - t_H \geq 0 \; .$$

Thus we can neglect IR_L, but IR_H and IC_L must bind; otherwise it is possible to increase profitably t_H and/or t_L. Hence

$$\begin{aligned}
t_H &= u_0 - f_H(\gamma_H) - g_H(a_H), \\
t_L &= u_0 - f_L(\gamma_L) - g_L(a_L) - (f_H(\gamma_H) + g_H(a_H) - f_L(\gamma_H) - g_L(a_H))
\end{aligned}$$

and IC_H is equivalent to

$$\int_{\gamma_H}^{\gamma_L} (f_H'(t) - f_L'(t))dt + \int_{a_H}^{a_L} (g_H'(t) - g_L'(t))dt \geq 0 \qquad (19)$$

Since $f'_H(t) > f'_L(t)$ and $g'_H(t) > g'_L(t)$ for each t, it follows that the inequalities $\gamma_L \geq \gamma_H$ and $a_L \geq a_H$ are sufficient to satisfy (19), although not necessary. The platform maximizes

$$u_0 + \nu \{q_H(\gamma_H) + s_H(a_H) - f_H(\gamma_H) - g_H(a_H)\}$$
$$+(1-\nu)\{q_L(\gamma_L) + s_L(a_L) - f_L(\gamma_L) - g_L(a_L) - (f_H(\gamma_H) + g_H(a_H) - f_L(\gamma_H) - g_L(a_H))\}.$$

Let us use hat ($\widehat{\ }$) to denote the solution to the max problem when neglecting (19). Then we have no distortion at the top: $\widehat{\gamma}_L = \gamma_L^{FB}$ and $\widehat{a}_L = a_L^{FB}$, but there are downward distortions at the bottom: $\widehat{\gamma}_H < \gamma_H^{FB}$ and $\widehat{a}_H < a_H^{FB}$, since $\widehat{\gamma}_H$ satisfies (24) and $\widehat{a}_H$ satisfies

$$s'_H(a_H) = g'_H(a_H) + \frac{1-\nu}{\nu}\left(g'_H(a_H) - g'_L(a_H)\right)$$

If $\widehat{\gamma}_H \leq \gamma_L^{FB}$ holds, then (19) is satisfied and the optimal second-best contract is given by

$$\gamma_\theta^{SB} = \widehat{\gamma}_\theta, \quad a_\theta^{SB} = \widehat{a}_\theta.$$

Therefore, the question is to find the second-best optimal contracts when $\widehat{\gamma}_H > \gamma_L^{FB}$. In this case, the first term in (19) is negative, but the second term is positive since $\widehat{a}_H < a_L^{FB}$. This means that (19) may or may not be satisfied at the solution $(\widehat{\gamma}_H, \widehat{a}_H, \gamma_L^{FB}, a_L^{FB})$. If it is satisfied, then again

$$\gamma_\theta^{SB} = \widehat{\gamma}_\theta, \quad a_\theta^{SB} = \widehat{a}_\theta.$$

If it is not satisfied at $(\widehat{\gamma}_H, \widehat{a}_H, \gamma_L^{FB}, a_L^{FB})$, then (19) affects the solution, as described in the second part of the following proposition.

Proposition 4. *(Commitment) Consider the model of privacy design in which the platform mediates interactions between consumers and advertisers. Suppose that the platform has commitment power. Suppose that $\widehat{\gamma}_H > \gamma_L^{FB}$ holds. Then, the second-best contract always entails a non-monotonic disclosure schedule ($\gamma_L^{SB} < \gamma_H^{SB}$) and a monotonic advertising schedule ($a_H^{SB} < a_L^{SB}$). In other words, by committing to a monotonic advertising schedule, the platform implements a non-monotonic disclosure schedule. More precisely we have:*

(i) if (19) is satisfied at $(\gamma_H, a_H, \gamma_L, a_L) = (\widehat{\gamma}_H, \widehat{a}_H, \gamma_L^{FB}, a_L^{FB})$, then $\gamma_H^{SB} = \widehat{\gamma}_H, a_H^{SB} = \hat{a}_H$ and $\gamma_L^{SB} = \gamma_L^{FB}, a_L^{SB} = a_L^{FB}$;

(ii) if (19) is not satisfied at $(\gamma_H, a_H, \gamma_L, a_L) = (\widehat{\gamma}_H, \widehat{a}_H, \gamma_L^{FB}, a_L^{FB})$*, then the optimal contracts are such that* $\gamma_L^{FB} < \gamma_L^{SB} < \gamma_H^{SB} < \widehat{\gamma}_H$ *and* $a_H^{SB} < \hat{a}_H < a_L^{FB} < a_L^{SB}$*. That is, the platform commits to an advertising schedule in which the difference* $a_L - a_H$ *is greater than* $a_L^{FB} - \hat{a}_H$ *in order to induce the H type to reveal more personal information than (19) allows given* $(\hat{a}_H, a_L^{FB})$*.*

If (19) is not satisfied at $(\gamma_H, a_H, \gamma_L, a_L) = (\widehat{\gamma}_H, \widehat{a}_H, \gamma_L^{FB}, a_L^{FB})$, the platform should make the disclosure schedule less non-monotonic by reducing the gap $\gamma_H - \gamma_L$. This together with the binding (19) implies that the gap $a_L - a_H$ should increase.

7.4 Second-best with no commitment

Suppose that the platform has no commitment; this implies the platform can choose the advertising amount to maximize its *ex post* profit.

7.4.1 Pooling contracts

Consider a pooling contract such that $\gamma_H^P = \gamma_L^P = \gamma^P$. Since the platform is offering a unique contract (γ^P, t^P) to both types, it does not learn any information about the consumer type θ upon acceptance of the contract. Hence, it chooses a^P to maximize the advertisers' surplus from advertising:

$$\max_a \quad v s_H(a) + (1 - v) s_L(a).$$

Because of the separability assumption, a^P does not depend on γ^P which is chosen in Stage 1. As t^P is determined by the binding H type's participation constraint,

$$t^P = u_0 - f_H(\gamma^P) - g_H(a^P) \geq 0, \tag{20}$$

γ^P is determined as the maximizer of

$$v q_H(\gamma) + (1 - v) q_L(\gamma) - f_H(\gamma).$$

Then an L type gets an information rent equal to

$$U_L^P = f_H(\gamma^P) - f_L(\gamma^P) + g_H(a^P) - g_L(a^P) > 0.$$

The platform's profit is

$$\pi^P = u_0 - f_H(\gamma^P) - g_H(a^P) + v(q_H(\gamma^P) + s_H(a^P)) + (1-v)(q_L(\gamma^P) + s_L(a^P))$$

In a pooling equilibrium there is a socially excessive advertising (i.e. $a^P > a^{FB}$) because consumers' advertising annoyance is not internalized by the platform's decision.

7.4.2 Separating contracts

Let us now study separating contracts such that $\gamma_H \neq \gamma_L$. We start from the platform's choice of advertising at Stage 3 when each consumer has chosen a contract $\{t(\theta), \gamma(\theta)\}$. After learning θ from the contract, the platform will choose a_θ^N such that $a_\theta^N \in \arg\max_a s_\theta(a)$ for $\theta = L, H$, and assumption (16) implies $a_H^N > a_L^N$. Therefore, even though formally the platform offers two pairs $\{(\gamma_L, t_L), (\gamma_H, t_H)\}$ to consumers, it is implicitly offering two triplets $\{(\gamma_L, t_L, a_L^N), (\gamma_H, t_H, a_H^N)\}$. Consequently, the participation and incentive compatibility constraints are obtained from those in the commitment case by replacing a_L by a_L^N and a_H by a_H^N. Proceeding as in the commitment case, we can show that IR_H and IC_L must bind. Hence, we have:

$$\begin{aligned} t_H &= u_0 - f_H(\gamma_H) - g_H(a_H^N), && (21) \\ t_L &= u_0 - f_L(\gamma_L) - g_L(a_L^N) - \big(f_H(\gamma_H) + g_H(a_H^N) - f_L(\gamma_H) - g_L(a_H^N)\big) && (22) \end{aligned}$$

Moreover, IC_H reduces to

$$\int_{\gamma_H}^{\gamma_L} (f_H'(t) - f_L'(t))dt + \int_{a_H^N}^{a_L^N} (g_H'(t) - g_L'(t))dt \geq 0 \qquad (23)$$

in which both the integrand functions are positive because of (15). This together with $a_H^N > a_L^N$ implies that the second term in (23) is negative. Therefore, $\gamma_L > \gamma_H$ is necessary to satisfy (23): a non-monotonic disclosure schedule can never be implemented with separating contracts.

Using (21) and (22), we can write the profit of the platform as follows:

$$\begin{aligned} \pi = \; & u_0 + v[q_H(\gamma_H) + s_H(a_H^N) - f_H(\gamma_H) - g_H(a_H^N)] \\ & +(1-v)[q_L(\gamma_L) + s_L(a_L^N) - f_L(\gamma_L) - g_L(a_L^N) - (f_H(\gamma_H) + g_H(a_H^N) - f_L(\gamma_H) - g_L(a_H^N))] \end{aligned}$$

which we need to maximize with respect to γ_H, γ_L subject to (23). Let $\left(\gamma_H^S, \gamma_L^S\right)$ denote the solution and π^S the resulting maximum value of π.

We identify the optimal contracts by comparing π^S with π^P. The comparison will tell us whether we have a pooling equilibrium or a separating equilibrium. We denote with $\hat{\gamma}_L, \hat{\gamma}_H$ the maximum point for π, neglecting (23), and find that $\hat{\gamma}_L = \gamma_L^{FB}$, and $\hat{\gamma}_H$ satisfies

$$q_H'(\gamma_H) = f_H'(\gamma_H) + \frac{1-\nu}{\nu}\left(f_H'(\gamma_H) - f_L'(\gamma_H)\right) \tag{24}$$

hence $\hat{\gamma}_H < \gamma_H^{FB}$. In words, we find that there is no distortion for an L type's disclosure level from the first-best level, while a downward distortion arises for an H type's disclosure level from its first-best counterpart.

Suppose to fix the ideas that $\hat{\gamma}_H > \hat{\gamma}_L$ (and recall that $\gamma_L^{FB} < \gamma_H^{FB}$). Then, in the space (γ_L, γ_H), there are points on the diagonal (i.e., such that $\gamma_L = \gamma_H$) which are closer to $(\hat{\gamma}_L, \hat{\gamma}_H)$ than any point satisfying (23), and thus it may seem intuitive that the optimal pooling contract is superior to the best separating contracts. However, this is not necessarily the case, essentially because when $\gamma_H = \gamma_L$ the platform loses the ability to learn the type of the consumer, and thus chooses $a_L = a_H$. Intuitively, the possibility to choose $a_L \neq a_H$ is more valuable when a_H^N is sufficiently higher than a_L^L, which is confirmed in the numerical example offered in Appendix C.

In summary, we have:

Proposition 5. *(No Commitment) Consider the model of privacy design in which the platform mediates interactions between consumers and advertisers. Suppose that the platform has no commitment.*

(i) The platform can never implement any non-monotonic disclosure schedule of personal information satisfying $\gamma_H > \gamma_L$.

(ii) The optimal privacy design entails either pooling ($\gamma_H^P = \gamma_L^P = \gamma^P$) accompanied by pooling level of advertising ($a_H^P = a_L^P = a^P$) or a strictly monotonic disclosure schedule ($\gamma_L^S > \gamma_H^S$) accompanied by a strictly non- monotonic advertising schedule ($a_H^N > a_L^N$).

Comparing Proposition 5 with Proposition 4, we draw our key statement: With commitment, the platform chooses a non-monotonic disclosure schedule and a monotonic advertising schedule. However, with no commitment the platform ends up choosing either pooling in both schedules or a monotonic disclosure schedule followed with a non-monotonic advertising schedule.

7.5 Implication for the EU data protection reform

A recent document by European Commission on data protection reform[12] proposes many agenda to improve data protection in the EU. At least two are notable from our application model perspective. First, the reform will allow people to ask for deletion of their data when they want and there are no legitimate grounds to deny such request. Another action item is to require data operators to seek each individual's consent before his or her personal information is redisclosed to any third party.[13]

Note that within our model a consumer in general has no incentive to exercise such right as each individual rationality constraint is satisfied *ex post* regardless of whether the platform has commitment power or not. However, the platform, in addition to making revenue from advertising, might be able to generate a certain revenue by selling the collected personal information to a third party. If this sale leads to increased nuisance possibly because of the increased advertising and risk of data breach, the individual rationality constraint can be violated *ex post* when the platform has no commitment power. In other words, after disclosing personal information, even if the individual rationality constraint within the platform is satisfied, the constraint can be violated when such additional nuisance from a third-party starts to be taken into account. In particular, it would be impossible for a consumer to monitor whether any given nuisance experiencing outside the platform is originated from the backdoor sale of the personal information that the individual initially disclosed to the platform. Therefore, the possibility to trade the personal information to a third-party can significantly exacerbate the commitment problem such that the platform may not be able to elicit any personal information without commitment power, which in turn makes the ad

[12] http://ec.europa.eu/justice/data-protection/document/review2012/factsheets/1_en.pdf

[13] These agenda are related to broadly-defined "the right to be forgotten". An individual can request search engines such as Google, Bing, and Yahoo to remove allegedly inadequate, irrelevant or no longer relevant, or excessive information in search results upon the individual's name (Kim and Kim, 2015). Also, people may have the right to ask data operators to delete self-provided personal information at any time and to ask the removal of even reposted content when the original content provider wants. See Rosen (2012) for further details.

targeting difficult.

In this situation, if the platform is required to obtain consent from each consumer prior to its disclosure personal information to any third-party, then the consumer is likely to veto such activity. Therefore, the platform without commitment power can achieve the outcome described by Proposition 5 when the data protection reform gives consumers more control right over personal data even after its initial disclosure. As we mentioned in the introduction, this implication is consistent with Miller and Tucker (2014), albeit the empirical setting is different.

8 Concluding Remarks

Our results show that when designing price discrimination on one side, a two-sided platform should pay particular attention to how it would affect the incentives of the agents on the other side. In particular, price discriminations on both sides can be substitutes or complements. If they are substitutes, the optimal mechanism can involve no price discrimination (i.e., pooling) at least on one side. No price discrimination is likely to be optimal on the side generating large externalities to the other side if the type generating large externalities receives less private benefit than the type generating smaller externalities. However, if price discriminations are complements, price discrimination on the side that benefits from the externalities can help screen agents on the side generating externalities.

Our results also point out the role of commitment in the platform's ability to discriminate. In particular, in the context of eliciting personal information to improve targeted advertising, the platform is unlikely to be able to commit not to profitably use the collected information once it is collected. In other words, the platform is likely to choose too high advertising level *ex post*. This lack of commitment on the advertising side in turn affects consumers' incentive to disclose personal information such that the platform may find pooling optimal. In addition, when the platform cannot commit not to sell the collected information to a third-party, consumers may disclose no personal information at all, which significantly reduces the platform's ability to target advertising. Then, empowering consumers by requiring the platform to obtain each consumer's consent prior to sale of the personal information may partially mitigate its commitment problem.

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Appendices

A Mathematical Proofs

The proofs for Proposition 1, Proposition 3, and Proposition 4 are discussed in the text. Thus, here we provide mathematical proofs for Proposition 2 and Proposition 5.

A.1 Proof of Proposition 2

Here we prove a more detailed version of Proposition 2, and for that purpose we let

$$\phi(q_H^A) = \nu^B \Delta_H^{b,A} u_1^A(q_H^A, q_H^B) + (1-\nu^B)\Delta_L^{b,A} u_1^A(q_H^A, q_L^B)$$

denote the derivative of Φ^A with respect to q_H^A: notice that ϕ does not depend on q_L^A.

Refined version of Proposition 2

(i) Suppose that $u_{12} > 0$ and $u_{112} \geq 0$ (not needed for part (a)).

(a) When $q_H \geq q_L$, we have $F = M \cup D$.

(b) When $q_H < q_L$, we have that

(b1) If $\phi(0) \leq 0$, then $F = N \cup D$ if $\phi(0) \leq 0$.

(b2) If $\phi(0) > 0 > \lim_{q_H^A \to +\infty} \phi(q_H^A)$, then let $\overline{q}_H^A$ be uniquely defined by $\phi(\overline{q}_H^A) = 0$. The set F has the shape of a sandglass, such that it includes some points in M if $q_L < \overline{q}_H^A$, and some points in N if $q_L > \overline{q}_H^A$.

(b3) If $\lim_{q_H^A \to +\infty} \phi(q_H^A) \geq 0$,then $F = M \cup D$.

(ii) Suppose that $u_{12} < 0$ and $u_{112} \leq 0$ (not needed for part (a)).

(a) when $q_H \leq q_L$, we have $F = M \cup D$.

(b) When $q_H > q_L$, we have that (b1-b3) from part (i) hold.

Proof of part (i): Complements: $u_{12}^A(q^A, q^B) > 0$ and $u_{112}^A(q^A, q^B) \geq 0$ for each q^A, q^B

1. If $q_H^B \geq q_L^B$, then $u_1^A(q_H^A, q_H^B) \geq u_1^A(q_H^A, q_L^B)$ and $\phi(q_H^A) \geq (\nu^B \Delta_H^{b,A} + (1-\nu^B)\Delta_L^{b,A}) u_1(q_H^A, q_L^B) > 0$. Since $\Phi^A(q_L^A, q_L^A) = 0$, it follows that $(\mathbf{I}^A)$ is equivalent to $q_H^A \geq q_L^A$.

2. If $q_H^B < q_L^B$, then assume $u_{112}^A \geq 0$, that is u_{11}^A is increasing with respect to q^B, or equivalently u_{12}^A is increasing with respect to q^A. Then $\phi'(q_H^A) = \nu^B \Delta_H^{b,A} u_{11}^A(q_H^A, q_H^B) + (1-\nu^B)\Delta_L^{b,A} u_{11}^A(q_H^A, q_L^B) \leq (\nu^B \Delta_H^{b,A} + (1-\nu^B)\Delta_L^{b,A}) u_{11}^A(q_H^A, q_H^B) < 0$. Therefore ϕ is strictly decreasing.

   - If $\phi(0) \leq 0$, then $\phi(q_H^A) < 0$ for each $q_H^A > 0$. Since $\Phi^A(q_L^A, q_L^A) = 0$, it follows that $\Phi^A(q_H^A, q_L^A) < 0$ for each $q_H^A > q_L^A$, but $\Phi^A(q_H^A, q_L^A) \geq 0$ for each $q_H^A \leq q_L^A$. Hence $(\mathbf{I}^A)$ is equivalent to $q_H^A \leq q_L^A$.
   - If $\phi(0) > 0 > \lim_{q_H^A \to +\infty} \phi(q_H^A)$, then let $\overline{q}_H^A$ be uniquely defined by $\phi(\overline{q}_H^A) = 0$.

     Now fix q_L^A, and consider $q_L^A < \overline{q}_H^A$. Then $\phi(q_H^A) > 0$ in $(0, q_L^A)$ and $\Phi^A(q_H^A, q_L^A) < 0$ for each $q_H^A < q_L^A$. Conversely, $\Phi^A(q_H^A, q_L^A) > 0$ at least for $q_H^A \in (q_L^A, \overline{q}_H^A]$, because Φ^A is increasing in q_H^A for $q_H^A \in (q_L^A, \overline{q}_H^A)$. Since $\phi(q_H^A) < 0$ for $q_H^A > \overline{q}_H^A$, it is possible that $\Phi^A(q_H^A, q_L^A) < 0$ for q_H^A sufficiently larger than $\overline{q}_H^A$.

     Now consider $q_L^A > \overline{q}_H^A$. Then $\phi(q_H^A) < 0$ for each $q_H^A > q_L^A$, hence $\Phi^A(q_H^A, q_L^A) < 0$ for each $q_H^A > q_L^A$. Conversely, $\Phi^A(q_H^A, q_L^A) > 0$ at least for $q_H^A \in [\overline{q}_H^A, q_L^A)$ because Φ^A is decreasing in q_H^A for $q_H^A \in (\overline{q}_H^A, q_L^A)$. Since $\phi(q_H^A) > 0$ for $q_H^A < \overline{q}_H^A$, it is possible that $\Phi^A(q_H^A, q_L^A) < 0$ for q_H^A sufficiently smaller than $\overline{q}_H^A$.

In this case the feasible set is non convex, and has vaguely the shape of a sandglass.

- If $\lim_{q_H^A \to +\infty} \phi(q_H^A) \geq 0$, then Φ^A is strictly increasing in q_H^A, hence ($\mathbf{I}^A$) is satisfied if and only if $(q_H^A, q_L^A) \in M \cup D$.

Proof of part (ii): Substitutes: $u_{12}^A(q^A, q^B) < 0$ **and** $u_{112}^A(q^A, q^B) \leq 0$ **for each** q^A, q^B

1. If $q_H^B \leq q_L^B$, then $u_1^A(q_H^A, q_H^B) \geq u_1^A(q_H^A, q_L^B)$ and $\phi(q_H^A) \geq (\nu^B \Delta_H^{b,A} + (1-\nu^B)\Delta_L^{b,A}) u_1^A(q_H^A, q_L^B) > 0$. Therefore ($\mathbf{I}^A$) is equivalent to $q_H^A \geq q_L^A$.

2. If $q_H^B > q_L^B$, then assume $u_{112}^A \leq 0$, that is u_{11}^A is decreasing with respect to q^B, or equivalently u_{12}^A is decreasing with respect to q^A. Then $\phi'(q_H^A) = \nu^B \Delta_H^{b,A} u_{11}^A(q_H^A, q_H^B) + (1-\nu^B)\Delta_L^{b,A} u_{11}^A(q_H^A, q_L^B) \leq (\nu^B \Delta_H^{b,A} + (1-\nu^B)\Delta_L^{b,A}) u_{11}^A(q_H^A, q_L^B) < 0$. Therefore ϕ is strictly decreasing and we obtain a feasible set similar to the case 2 above: (i) $N \cup D$ if $\phi(0) \leq 0$; (ii) a sandglass if $\phi(0) > 0 > \lim_{q_H^A \to +\infty} \phi(q_H^A)$; (iii) $M \cup D$ if $\lim_{q_H^A \to +\infty} \phi(q_H^A) \geq 0$. ■

A.2 Proofs of Proposition 5

1. Case (i): Discussed in the text.

2. Case (ii): Since (19) is violated at $(\gamma_H, a_H, \gamma_L, a_L) = (\widehat{\gamma}_H, \widehat{a}_H, \gamma_L^{FB}, a_L^{FB})$, the optimal contracts satisfy (19) with equality and we can earn insights about them using the lagrangian function

$$\mathcal{L} = \pi + \lambda \left(\int_{\gamma_H}^{\gamma_L} (f'_H(t) - f'_L(t))dt + \int_{a_H}^{a_L} (g'_H(t) - g'_L(t))dt \right)$$

since $(\gamma_H^{**}, a_H^{**}, \gamma_L^{**}, a_L^{**})$ satisfy the following equalities

$$\frac{\partial \pi}{\partial \gamma_L} + \lambda(f'_H(\gamma_L) - f'_L(\gamma_L)) = 0, \qquad \frac{\partial \pi}{\partial \gamma_H} - \lambda(f'_H(\gamma_H) - f'_L(\gamma_H)) = 0 \quad \text{(A.1)}$$

$$\frac{\partial \pi}{\partial a_L} + \lambda(g'_H(a_L) - g'_L(a_L)) = 0, \qquad \frac{\partial \pi}{\partial a_H} - \lambda(g'_H(a_H) - g'_L(a_H)) = 0 \quad \text{(A.2)}$$

The value of λ cannot be zero, otherwise we obtain $(\gamma_H, a_H, \gamma_L, a_L) = (\widehat{\gamma}_H, \widehat{a}_H, \gamma_L^{FB}, a_L^{FB})$, which violates (19) by assumption. Hence $\lambda > 0$, and from (A.1)-(A.2) we can conclude that $\gamma_L^{**} > \gamma_L^{FB}, \gamma_H^{**} < \widehat{\gamma}_H, a_L^{**} > a_L^{FB}, a_H^{**} < \hat{a}_H$. Moreover, since $a_H^{**} < a_L^{**}$, it follows from (19) written with equality that $\gamma_L^{**} < \gamma_H^{**}$. ■

B Symmetric quadratic setting: complements with type-reversal

When qualities on both sides are complements and there is type-reversal, the implementable set is composed of all monotonic schedules and possibly some non-monotonic schedules with relatively large gap $q_L - q_H$. Thus, the implementable set itself may not be a convex set and thus finding the optimal mechanism can be challenging. We here analyze the optimal mechanism for a symmetric quadratic setting with the complementarity in the qualities. This analysis confirms the general insight in a more visible manner through explicit solutions. Let us begin by specifying the utility function:

$$\tilde{u}(q^A, q^B) = q^A - \frac{1}{2}(q^A)^2 + q^B - \frac{1}{2}(q^B)^2 + \alpha q^A q^B$$

with $\alpha \in [0, 1)$. We assume that $\Delta_L \in (-\Delta_H, 0)$, $\nu = \frac{1}{2}$ and $C(q) = q^2/2$.

Unfortunately, $\tilde{u}$ is not monotone increasing in q^A, q^B, as it has a global max point at $(q^A, q^B) = (\frac{1}{1-\alpha}, \frac{1}{1-\alpha})$; this suggests to consider $\tilde{u}$ as defined in the square $S = [0, \frac{1}{1-\alpha}] \times [0, \frac{1}{1-\alpha}]$. Even in this refined domain, $\tilde{u}$ is not monotone increasing in q^A, q^B: for instance, it is decreasing with respect to q^B for $q^B > 1 + \alpha q^A$. For this reason, we consider the function u defined below, after introducing a suitable partition of the set S:

$$\begin{aligned}
R_1 &= \{(q^A, q^B) : q^B \in [0, \frac{1}{1-\alpha}), \, q^A \in (1 + \alpha q^B, \frac{1}{1-\alpha}]\}, \\
R_2 &= \{(q^A, q^B) : q^B \in [0, \frac{1}{1-\alpha}), \, q^A \in [q^B, 1 + \alpha q^B]\}, \\
R_3 &= \{(q^A, q^B) : q^A \in [0, \frac{1}{1-\alpha}), \, q^B \in (q^A, 1 + \alpha q^A]\}, \\
R_4 &= \{(q^A, q^B) : q^A \in [0, \frac{1}{1-\alpha}], \, q^B \in [1 + \alpha q^A, \frac{1}{1-\alpha}]\}
\end{aligned}$$

The Figure B.1 illustrates the partitions of the domain set S.

Then we define u in S as follows:

$$u(q^A, q^B) = \begin{cases} \tilde{u}(1 + \alpha q^B, q^B) & \text{if } (q^A, q^B) \in R_1 \\ \tilde{u}(q^A, q^B) & \text{if } (q^A, q^B) \in R_2 \cup R_3 \\ \tilde{u}(q^A, 1 + \alpha q^A) & \text{if } (q^A, q^B) \in R_4 \end{cases}$$

In order to understand this definition, consider for instance $q^A \in [0, \frac{1}{1-\alpha})$, and recall that $\tilde{u}$ is strictly decreasing with respect to q^B if $q^B > 1 + \alpha q^A$. Then, for $q^B > 1 + \alpha q^A$, $u(q^A, q^B)$

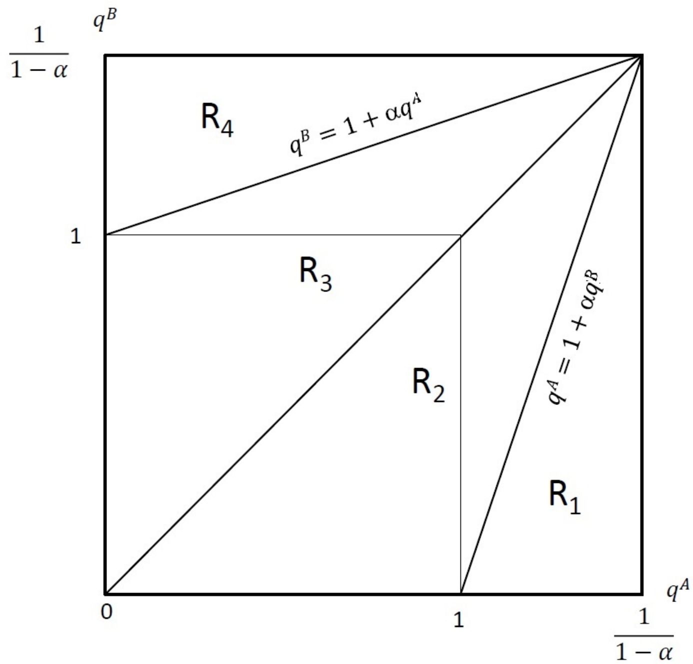


Figure B.1: The domain with partitions for the symmetric quadratic setting

is defined as $\tilde{u}(q^A, 1+\alpha q^A)$, such that u is constant with respect to q^B in the set R_4.

For this setting, it is interesting to notice the following:

$$q_H^{FB} \geq q_L^{FB} \quad \text{if} \quad \theta_{HH} \geq \theta_{LL}, \qquad q_H^{FB} < q_L^{FB} \quad \text{if} \quad \theta_{HH} < \theta_{LL}$$

Under incomplete information, if we assume that IR_L and IC_H bind, and neglect IR_H and IC_L, then we find $\hat{\pi}$ in (10) and $\hat{q}_H, \hat{q}_L$ is such that

$$\widehat{q}_H \geq \widehat{q}_L \quad \text{if} \quad \theta_{HH} \geq \theta_{LL}^v, \qquad \widehat{q}_H < \widehat{q}_L \quad \text{if} \quad \theta_{HH} < \theta_{LL}^v$$

Since $\Delta_L < 0$, we have $\theta_{LL}^v > \theta_{LL}$. Therefore, if the first-best schedule is non-monotonic, then $(\widehat{q}_H, \widehat{q}_L)$ is non-monotonic as well. Moreover, $(\widehat{q}_H, \widehat{q}_L)$ can be non-monotonic even if the first-best schedule is monotonic. As we explained previously, this is because $\theta_{LL}^v > \theta_{LL}$ can create an upward distortion in $\widehat{q}_L$.

Under incomplete information, $\left(q_H^{SB}, q_L^{SB}\right)$ does not necessarily coincide with $(\widehat{q}_H, \widehat{q}_L)$ because $(\widehat{q}_H, \widehat{q}_L)$ may fail to satisfy IC_L and/or IR_H. Precisely, given that IR_L and IC_H

bind, IC_L and IR_H reduce to

$$(I) \quad [u(q_H,q_H)-u(q_L,q_H)]+r[u(q_H,q_L)-u(q_L,q_L)] \;\geq\; 0 \tag{B.1}$$

$$u(q_L,q_H)+ru(q_L,q_L) \;\geq\; 0 \tag{B.2}$$

with $r=\frac{\Delta_L}{\Delta_H}\in(-1,0)$. Next lemma identifies the subset of S in which (B.1) is satisfied, as a function of $\alpha\in[0,1)$.

Lemma 1.

Let
$$\alpha_1 = \frac{1-|r|}{1+|r|}, \qquad \alpha_2=\frac{1-|r|}{2|r|} \qquad \textit{and}$$
$$b = \sqrt{\frac{1}{2}\left(1-\frac{1}{|r|}+\frac{1+|r|}{|r|}\alpha\right)}, \qquad c=\frac{2(1-|r|)}{1+|r|(2\alpha-1)}>0, \qquad d=\frac{(2\alpha-1)+|r|}{1+|r|(2\alpha-1)}.$$

(i) If $r<-\frac{1}{3}$, then $\alpha_2<1$ and the set of (q_H,q_L) which satisfy (B.1) depends on α as follows:

$$\begin{cases} R_1\cup R_2 & \text{if } \alpha\in[0,\alpha_1] \\ R_1\cup R_2\cup\{(q_H,q_L): q_H\in[0,\frac{1}{1-\alpha}) \text{ and } q_L\in[\frac{1-b}{1-\alpha}+bq_H,\frac{1}{1-\alpha}]\} & \text{if } \alpha\in(\alpha_1,\alpha_2) \\ R_1\cup R_2\cup\{(q_H,q_L): q_H\in[0,\frac{1}{1-\alpha}) \text{ and } q_L\in[c+dq_H,\frac{1}{1-\alpha}]\} & \text{if } \alpha\in[\alpha_2,1) \end{cases}$$

(ii) If $r\geq-\frac{1}{3}$, then $\alpha_2\geq 1$ and the set of (q_H,q_L) which satisfy (B.1) depends on α as follows:

$$\begin{cases} R_1\cup R_2 & \text{if } \alpha\in[0,\alpha_1] \\ R_1\cup R_2\cup\{(q_H,q_L): q_H\in[0,\frac{1}{1-\alpha}) \text{ and } q_L\in[\frac{1-b}{1-\alpha}+bq_H,\frac{1}{1-\alpha}]\} & \text{if } \alpha\in(\alpha_1,1) \end{cases}$$

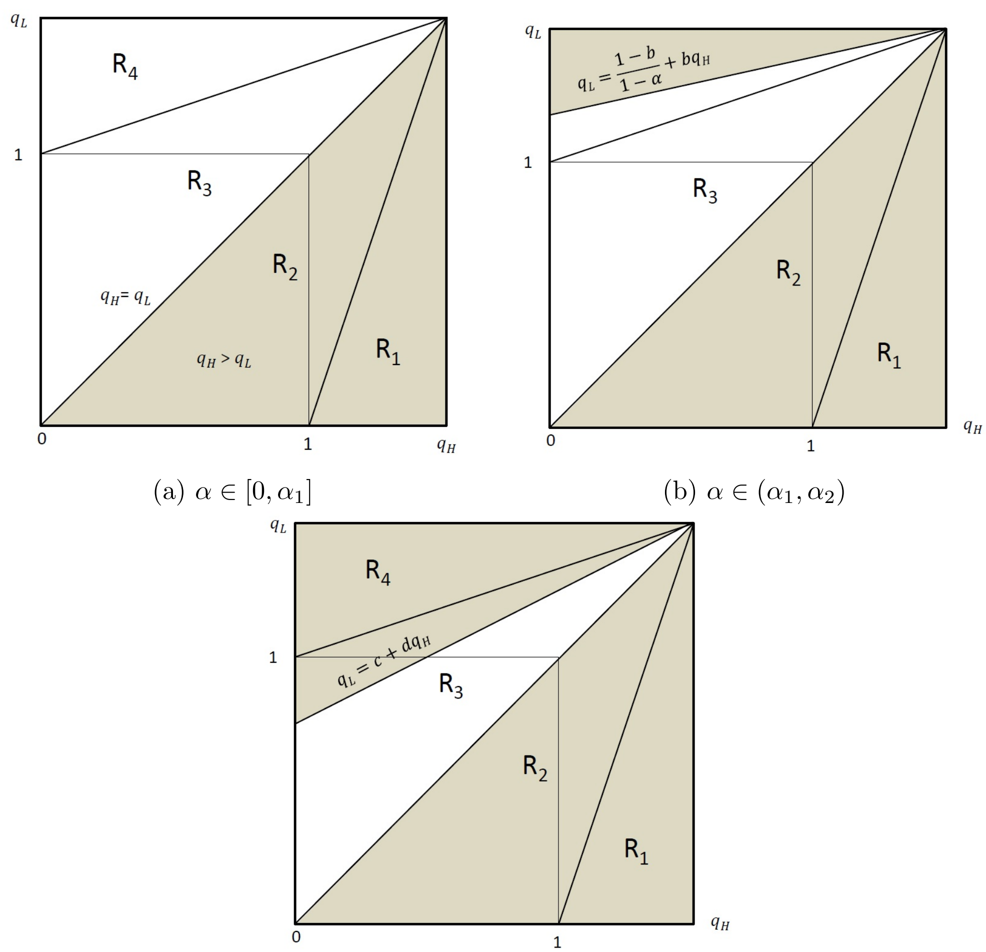
The inequality (B.1) has a different expression depending on whether we consider (q_H,q_L) in R_1, or in $R_2\cup R_3$, or in R_4. Precisely, it is equivalent to

$$\tilde{u}(q_H,q_H)-\tilde{u}(q_L,1+\alpha q_L)+r[\tilde{u}(1+\alpha q_L,q_L)-\tilde{u}(q_L,q_L)] \;\geq\; 0 \text{ if } (q_H,q_L)\in R_1 \tag{B.3}$$

$$(q_L-q_H)(q_L-c-dq_H) \;\geq\; 0 \text{ if } (q_H,q_L)\in R_2\cup R_3 \tag{B.4}$$

$$\tilde{u}(q_H,q_H)-\tilde{u}(1+\alpha q_H,q_H)+r[\tilde{u}(q_H,1+\alpha q_H)-\tilde{u}(q_L,q_L)] \;\geq\; 0 \text{ if } (q_H,q_L)\in R_4 \tag{B.5}$$

Figure B.2 represents this set in the three cases of $\alpha\in[0,\alpha_1]$, $\alpha\in(\alpha_1,\alpha_2)$, and $\alpha\in[\alpha_2,1)$. Notice that for $\alpha\in(\alpha_1,\alpha_2)$, the line $q_L=\frac{1-b}{1-\alpha}+bq_H$ lies above the line $q_L=1+\alpha q_H$, that is it is entirely in R_4 and the feasible set consists of the points in S which are on or below the diagonal, plus a subset of R_4. For $\alpha = \alpha_2$, the two lines $q_L = \frac{1-b}{1-\alpha} + bq_H$ and $q_L = c + dq_H$ both coincide with $q_L = 1 + \alpha q_H$, and for $\alpha > \alpha_2$, the line $q_L = c + dq_H$ is included in R_3, but is bounded away from the line $q_L = q_H$ even as $\alpha \to 1$: when α tends to 1, the line $q_L = c + dq_H$ tends to the line $q_L = 2\alpha_1 + q_H$. Thus the set $R_1 \cup R_2 \cup \{(q_H, q_L) : q_H \in [0, \frac{1}{1-\alpha})$ and $q_L \in [c + dq_H, \frac{1}{1-\alpha}]\}$ is a strict subset of S for each $\alpha \in [\alpha_2, 1)$.



(a) $\alpha \in [0, \alpha_1]$

(b) $\alpha \in (\alpha_1, \alpha_2)$

(c) $\alpha \in [\alpha_2, 1)$

Figure B.2: The set of feasible allocations for the symmetric quadratic setting: complements

The proof for Lemma 1 is in what follows.

Step 1 (B.1) holds for each point in $R_1 \cup R_2$.

For each $(q_H, q_L) \in R_2$, we find that (B.4) holds because $q_L - q_H \leq 0$ and $q_L - c - dq_H \leq q_H - c - dq_H < 0$, given that $q_H \in (0, \frac{1}{1-\alpha})$. Hence, each $(q_H, q_L) \in R_2$ satisfies (B.1).

Regarding R_1, the term $\tilde{u}(q_H, q_H)$ in the left hand side in (B.3) is at least as large as $\tilde{u}(1 +$

$\alpha q_L, 1+\alpha q_L)$, therefore the left hand side in (B.3) is at least as large as $\frac{1}{2}(1+r+2\alpha)(1-q_L+\alpha q_L)^2>0$. Hence IC_L holds at each point in R_1.

Step 2 The subset of $R_3\cup R_4$ in which (B.1) is satisfied depends on α as follows

$$\begin{cases} \varnothing & \text{if } \alpha\in[0,\alpha_1] \\ \{(q_H,q_L): q_H\in[0,\frac{1}{1-\alpha}) \text{ and } q_L\in[\frac{1-b}{1-\alpha}+bq_H,\frac{1}{1-\alpha}]\} & \text{if } \alpha\in(\alpha_1,\alpha_2) \\ \{(q_H,q_L): q_H\in[0,\frac{1}{1-\alpha}) \text{ and } q_L\in[c+dq_H,\frac{1}{1-\alpha}]\} & \text{if } \alpha\in[\alpha_2,1) \end{cases}$$

Step 2.1 $\alpha\in[0,\alpha_1]$**.**

For each $(q_H,q_L)\in R_3$, (B.1) is equivalent to $q_L-c-dq_H\geq 0$, but $q_L-c-dq_H\leq -(1-q_H+\alpha q_H)\frac{1+r+2\alpha r}{1+r-2\alpha r}<0$, in which the first inequality follows from $q_L\leq 1+\alpha q_H$, and the second inequality follows from $\alpha\leq\alpha_1$. Regarding R_4, if $\alpha\leq\alpha_1$ then the left hand side in (B.5) has a unique maximizer at $q_H=\frac{1}{1-\alpha}$, $q_L=\frac{1}{1-\alpha}$, and the maximum value is 0. Hence (B.1) is violated in $R_3\cup R_4$ for each $\alpha\in[0,\alpha_1]$.

Step 2.2 $\alpha\in(\alpha_1,\alpha_2)$**.**

For $\alpha\in(\alpha_1,\alpha_2)$, we can argue as in the proof of Step 2.1 to establish that (B.1) is violated in R_3. Regarding R_4, the left hand side in (B.5) is non negative at $(q_H,q_L)\in R_4$ if and only if $\frac{1-b}{1-\alpha}+bq_H\leq q_L\leq\frac{1}{1-\alpha}$.

Step 2.3 $\alpha\in[\alpha_2,1)$**.**

Regarding R_3, for each $q_H\in[0,\frac{1}{1-\alpha}]$ the inequality $c+dq_H\leq 1+\alpha q_H$ holds given that $\alpha>\alpha_2$,[14] hence (B.1) is satisfied in R_3 if and only if $q_L\geq c+dq_H$. Regarding R_4, the term $\tilde{u}(q_L,q_L)$ in the left hand side in (B.5) is at least as large as $\tilde{u}(1+\alpha q_H,1+\alpha q_H)$, therefore the left hand side in (B.5) is at least as large as $-\frac{1}{2}\left(1-q_H+\alpha q_H\right)^2(1+r+2\alpha r)$, which is non-negative because $\alpha\geq\alpha_2$. Hence (B.1) holds at each point in R_4. ■

Proposition 6. *Consider the symmetric quadratic setting with type reversal. Suppose that the qualities are complements.*

(i) If $\theta_{HH}\geq\theta^v_{LL}$*, then* $q^{SB}_H=\hat{q}_H, q^{SB}_L=\hat{q}_L$*.*

(ii) Assume $\theta_{HH}<\theta^v_{LL}$*. Then,*

(a) If $\alpha\in[0,\alpha_1]$*, then* $q^{SB}_H=q^p$*,* $q^{SB}_L=q^p$*.*

[14] The inequality holds at $x=0$ and at $x=\frac{1}{1-\alpha}$, hence it holds for each $x\in(0,\frac{1}{1-\alpha})$.

(b) If $\theta_{HH} \geq \frac{3}{4}\theta_{HL} - \frac{3}{4}$, then $q_H^{SB} = q^p$, $q_L^{SB} = q^p$ for each $\alpha \in (\alpha_1, \alpha_2)$. If $\theta_{HH} < \frac{3}{4}\theta_{HL} - \frac{3}{4}$, then there exist parameter values (with α close to α_2) such that the q_H^{SB}, q_L^{SB} belong to region R_4, implying $q_H^{SB} < q_L^{SB}$.

The result in this proposition is immediate, as $\theta_{HH} \geq \theta_{LL}^v$ implies $\hat{q}_H > \hat{q}_L$, which satisfies (B.1) because $R_1 \cup R_2$ is the set of points in S such that $q_H \geq q_L$. Moreover, from (B.2) it is immediate that $\hat{q}_H \geq \hat{q}_L$ makes IR$_H$ satisfied, given that $r \in (-1, 0)$. Hence $q_H^{SB} = \hat{q}_H, q_L^{SB} = \hat{q}_L$.

The case of $\theta_{HH} < \theta_{LL}^v$ is more difficult to deal with, since we have $\hat{q}_H < \hat{q}_L$, and precisely $(\hat{q}_H, \hat{q}_L) \in R_3$, and we know from Lemma 1 that IC$_L$ is violated at some points in $R_3 \cup R_4$.

Part (ii)-(a) holds since when $\alpha \in [0, \alpha_1]$, the feasible set is $R_1 \cup R_2$ hence $(\hat{q}_H, \hat{q}_L)$ is infeasible. Then we maximize $\hat{\pi}(q, q) = (\theta_{LH} + \theta_{LL})\, u(q, q) - 2C(q)$ with respect to q, and find the maximizer $q^p = \frac{\theta_{LH}+\theta_{LL}}{(1-\alpha)(\theta_{LH}+\theta_{LL})+1}$ (with $\hat{\pi}(q^p, q^p) = (\theta_{LH} + \theta_{LL})q^p$). Since also IR$_H$ is satisfied when $q_L = q_H$, it follows that $q_H^{SB} = q^p$, $q_L^{SB} = q^p$.

Part (ii)-(b) is about the case in which some non monotonic allocation is feasible. Precisely, if $\alpha \in (\alpha_1, \alpha_2)$, then the feasible set consists of $R_1 \cup R_2$, and a subset of R_4. Yet, it is still the case that $(\hat{q}_H, \hat{q}_L)$ is infeasible, since our assumptions (included $\theta_{LL}^v > \theta_{HH}$) imply $(\hat{q}_H, \hat{q}_L) \in R_3$. In order to find q_H^{SB}, q_L^{SB} we need to evaluate $\max_{q_H} \hat{\pi}(q_H, \frac{1-b}{1-\alpha} + bq_H) \equiv \hat{\pi}_{R_4}$, and compare it with $\hat{\pi}(q^p, q^p)$. If $\hat{\pi}(q^p, q^p) \geq \hat{\pi}_{R_4}$, then $q_H^{SB} = q^p, q_L^{SB} = q^p$; if $\hat{\pi}(q^p, q^p) < \hat{\pi}_{R_4}$, then (q_H^{SB}, q_L^{SB}) belongs to R_4, as it is possible to prove that IR$_H$ is satisfied. Characterizing exactly when $\hat{\pi}(q^p, q^p) \geq \hat{\pi}_{R_4}$ as α varies in (α_1, α_2) is possible in principle, as we can always obtain closed form solutions, but those closed forms are quite complicated. Part (ii)-(b) establishes that if $\theta_{HL} - \theta_{HH}$ is negative, or not too positive, then $\hat{\pi}(q^p, q^p) > \hat{\pi}_{R_4}$ for each $\alpha \in (\alpha_1, \alpha_2)$, whereas if θ_{HL} is sufficiently larger than θ_{HH}, then for some parameters $\hat{\pi}(q^p, q^p) < \hat{\pi}_{R_4}$ if α is close to α_2.[15]

We now move to consider $\alpha \in [\alpha_2, 1)$, and we find that dealing with this case is quite difficult. In detail, it is possible that $(\hat{q}_H, \hat{q}_L)$ is infeasible, and then we need to compare the optimal pooling contract with the optimal (q_H, q_L) in $R_3 \cup R_4$, which is found by maximizing $\hat{\pi}(q_H, c + dq_H)$ with respect to q_H. Precisely, let $\tilde{q}_H = \arg\max_{q_H} \hat{\pi}(q_H, c + dq_H)$, and $\tilde{q}_L = c + d\tilde{q}_H$. If $\hat{\pi}(q^p, q^p) \geq \hat{\pi}(\tilde{q}_H, \tilde{q}_L)$, then $q_H^{SB} = q^p, q_L^{SB} = q^p$, but if $\hat{\pi}(q^p, q^p) < \hat{\pi}(\tilde{q}_H, \tilde{q}_L)$, then $q_H^{SB} = \tilde{q}_H, q_L^{SB} = \tilde{q}_L$, provided that $\tilde{q}_H, \tilde{q}_L$ satisfies IR$_H$. However, it is also possible that

[15] This is the case, for instance, if $\theta_{HH} = 3, \theta_{HL} = 5.1, \theta_{LH} = 1.6, \theta_{LL} = 5.7$, and $\alpha = \frac{2}{3}$.

$(\hat{q}_H, \hat{q}_L) \in R_3$, and thus it is feasible. In this case $q_H^{SB} = \hat{q}_H, q_L^{SB} = \hat{q}_L$ if IR$_H$ is satisfied. We are no longer able to cover these cases for general parameter values; instead, we offer a particular numeric example with full characterization for every possible $\alpha \in [0, 1)$ below.[16]

Consider parameter values such that $\theta_{HH} = 0.8$, $\theta_{HL} = 0.81$, $\theta_{LH} = 0.6$, $\theta_{LL} = 1$. Then, we can compute $\theta_{LH}^v = 0.4$, $\theta_{LL}^v = 1.19$ and $r = -\frac{19}{20}$, $\alpha_1 = \frac{1}{39}$, $\alpha_2 = \frac{1}{38}$, $b = \sqrt{\frac{39}{38}\alpha - \frac{1}{38}}$, $c = \frac{2}{38\alpha+1}$, $d = \frac{40\alpha-1}{38\alpha+1}$.

(i) If $\alpha \in [0, \frac{1}{38}]$, then $q_H^{SB} = q^p$, $q_L^{SB} = q^p$;

(ii) If $\alpha \in (\frac{1}{38}, \frac{1}{6}]$, then $q_H^{SB} = q^p$, $q_L^{SB} = q^p$;

(iii) If $\alpha \in (\frac{1}{6}, 0.1913]$, then q_H^{SB}, q_L^{SB} is such that $q_L^{SB} = c + dq_H^{SB}$ and such that IR$_H$ binds;

(iv) If $\alpha \in (0.1913, \frac{40}{123}]$, then $q_H^{SB} = \tilde{q}_H$, $q_L^{SB} = \tilde{q}_L$;

(v) If $\alpha \in (\frac{40}{123}, 0.8671]$, then $q_H^{SB} = \hat{q}_H$, $q_L^{SB} = \hat{q}_L$;

(vi) If $\alpha \in (0.8671, 1)$, then q_H^{SB}, q_L^{SB} is obtained by maximizing $\hat{\pi}$ subject to IR$_H$ binding.

Part (i) is a corollary of Proposition 6 to the case of $\alpha \in (0, \frac{1}{38})$, since $\theta_{HH} \geq \frac{3}{4}\theta_{HL} - \frac{3}{4}$ is satisfied. The remaining parts can be distinguished between (ii)-(iv), which refer to the case in which $(\hat{q}_H, \hat{q}_L)$ is infeasible, and (v)-(vi), which refers to the case in which $\hat{q}_H, \hat{q}_L$ belongs to R_3.

When $\alpha \in (\frac{1}{38}, \frac{40}{123}]$, $(\hat{q}_H, \hat{q}_L)$ is infeasible. Therefore we need to identify the best (q_H, q_L) on the line $q_L = c + dq_H$, denoted $(\tilde{q}_H, \tilde{q}_L)$, and to compare it with the pooling contract. It turns out that the pooling contract is superior for $\alpha \in (\frac{1}{38}, \frac{1}{6}]$, whereas $(\tilde{q}_H, \tilde{q}_L)$ is superior for $\alpha > \frac{1}{6}$. However, $\tilde{q}_H, \tilde{q}_L$ satisfies IR$_H$ only for $\alpha \in (0.1913, \frac{40}{123}]$, but violates IR$_H$ for $\alpha \in (\frac{1}{6}, 0.1913]$; in such a case the optimal contract is such that all the four constraints bind.

For $\alpha > \frac{40}{123}$, $(\hat{q}_H, \hat{q}_L)$ is feasible (i.e., it satisfies IC$_L$), therefore it is the optimal contract if it satisfies IR$_H$, which occurs if $\alpha \in (\frac{40}{123}, 0.8671)$. For greater values of α, we need to take into account also IR$_H$ to find the optimal contracts.

C A numeric example for Proposition 5

Consider the following explicit functions of

[16]Detailed mathematical derivations are available upon request.

- $q_\theta(\gamma) = \lambda_\theta \gamma - \frac{1}{2}\gamma^2$, with $0 < \lambda_L < \lambda_H < 1$;
- $s_\theta(a) = \rho_\theta a - \frac{1}{2}a^2$, with $0 < \rho_L < \rho_H < 1$;
- $f_\theta(\gamma) = \beta_\theta \gamma$, with $\beta_L < \beta_H$;
- $g_\theta(a) = \delta_\theta a$, with $\delta_L < \delta_H$.

with the following parameters of $v = \frac{1}{2}$, $\lambda_L = \frac{1}{4}$, $\lambda_H = \frac{3}{4}$, $\beta_L = \frac{1}{10}$, $\beta_H = \frac{3}{10}$, $\delta_L = \frac{9}{20}$, $\delta_H = \frac{11}{20}$. Notice that ρ_L and ρ_H have not been specified.

In the separating case, we derive $a_H^N = \rho_H$, $a_L^N = \rho_L$ and (23) reduces to $2(\gamma_L - \gamma_H) + (\rho_L - \rho_H) \geq 0$. Once we solve for the unconstrained max point, the solution $\gamma_H = 0.25$, $\gamma_L = 0.15$ does not satisfy (23). Thus, we impose the constraint as binding $2(\gamma_L - \gamma_H) + (\rho_L - \rho_H) = 0$, and derive the maximized profit under the optimal separating contrast as follows:

$$\pi^S = u_0 + \frac{7}{32}\rho_H^2 + \frac{1}{16}\rho_H \rho_L - \frac{27}{80}\rho_H + \frac{7}{32}\rho_L^2 - \frac{17}{80}\rho_L + \frac{1}{50}$$

In the case of pooling case, we derive $a^P = \frac{1}{2}\rho_H + \frac{1}{2}\rho_L$ and $\gamma^P = \frac{1}{5}$. Hence, substituting these into the value, we get

$$\pi^P = u_0 + \frac{1}{8}\rho_H^2 + \frac{1}{4}\rho_H \rho_L - \frac{11}{40}\rho_H + \frac{1}{8}\rho_L^2 - \frac{11}{40}\rho_L + \frac{1}{50}$$

The difference $\pi^S - \pi^P$ is equal to

$$\frac{3}{32}\left(\rho_H - \rho_L - \frac{2}{3}\right)(\rho_H - \rho_L)$$

and recall that $\rho_H > \rho_L$. Hence $\pi^S - \pi^P < 0$ if ρ_H is only slightly larger than ρ_L, but is positive if $\rho_H > \rho_L + \frac{2}{3}$, for instance if $\rho_L = 0.1$ and $\rho_H = 0.8$.