Dynamics of Compatibility under Switching Costs*

Doh-Shin Jeon† Domenico Menicucci‡ Nikrooz Nasr§

December 26, 2015

Abstract

We study firms' choices of compatibility in a dynamic setting. Current compatibility choice shapes the distribution of consumers' switching costs and thereby affects competition and compatibility choice in the future. Given today's market shares, the dynamics of compatibility is asymmetric in that firms are more likely to embrace compatibility tomorrow if products are compatible today but no such inertia exists for incompatibility. However, this asymmetry disappears when the market shares are endogenous. Contrary to what happens in a static setting, when consumer lock-in arises due to a significant switching cost, firms make their systems incompatible in order to soften future competition, which hurts consumers and tends to reduce welfare.

Key words: (In)Compatibility, Dynamics, Lock-in, Switching Cost

JEL Codes: D43, L13, L41.

---

*We thank the audience who attended our presentations at the Net Institute Conference 2015. We gratefully acknowledge financial support from the NET Institute.

†Toulouse School of Economics and CEPR. Manufacture de Tabacs, 21 allees de Brienne - 31000 Toulouse, France. E-mail: dohshin.jeon@gmail.com.

‡Dipartimento di Scienze per l'Economia e l'impresa, UniversitÃă degli Studi di Firenze. Via delle Pandette 9, I-50127 Firenze (FI), Italy. E-mail: domenico.menicucci@unifi.it.

§Toulouse School of Economics. Manufacture de Tabacs, 21 allees de Brienne - 31000 Toulouse, France. E-mail: me@nikrooz.com

1 Introduction

Will the future of the Internet be dominated by incompatible platforms? Will smartphone users be able to enjoy their preferred applications regardless of their platform choice? Back in the 90s' when the Internet was at its dawn, openness and compatibility seemed to be the future. For instance, during the times, Microsoft was the dominant player in the personal computer market but decided to bring two of its most successful software, Internet Explorer and Microsoft Office, to Macs. However, after the turn of the 21st century, we seem to enter a new era in which platforms try to lock-in customers by making it hard to move data across platforms or by providing some benefits exclusively to those who use all from the same ecosystem.

If a consumer purchased applications from iTunes (Google Play), she is more likely to choose an iPhone (an Android phone) instead of an Android phone (an iPhone).[1] In cloud computing, from the beginning, a major concern was vendor lock-in where due to incompatible technologies enterprises would incur a huge cost if they wish to transfer their data from one vendor to another[2].

This phenomena is known as *"walled garden"* or islands as Google CEO, Larry Page, describes:

> "The Internet was made in universities and it was designed to interoperate. And as we've commercialized it, we've added more of an island-like approach to it, which I think is a somewhat a shame for users."[3]

Furthermore, Apple and Google are expanding their businesses outside of the phone industry by leveraging their established smartphone platforms: smart car, smart TV, Google glasses, Apple watch, Apple pay etc. These new hardware and software bring the ultimate lock-in experience. For instance, Android Wear only works with Android phones and tablets. Apple Watch supports iPhones and iPads exclusively. We enter an era where working across multiple ecosystems deprive consumers of real benefit. This is why Drew Houston, CEO of Dropbox, says consumers are "trapped" in each platform.[4]

[1]The same logic applies to services like: iMessage and iCloud versus Hangouts and Google account, One Drive versus Google Drive, iCloud Keychain versus Chrome password syncing.

[2]http://fortune.com/2015/10/08/aws-lock-in-worry/

[3]http://fortune.com/2012/12/11/fortune-exclusive-larry-page-on-google/

[4]http://vimeo.com/70089044

In this paper, we attempt to offer a novel insight to understand the evolution of platforms' compatibility choices from a dynamic perspective, which is relevant because hardware must be upgraded and subscription to content/application often must be renewed. As the current compatibility choice shapes the distribution of consumers' switching costs, it in turn affects competition and compatibility choice in the future. As the main result, we show that when consumer lock-in arises due to a significant switching cost, firms make their systems incompatible in order to soften future competition, which hurts consumers and tends to reduce welfare. More generally, we find a strong conflict between the compatibility choice maximizing consumer surplus and the one maximizing industry profit. Our result is contrary to what happens in a static model in which symmetric platforms make their systems compatible (Matutes and Régibeau, 1988 and Economides, 1989)[5].

We extend the model of Matutes and Régibeau (1988) to two periods. Matutes and Régibeau (1988) study compatibility choices made by two symmetric firms (A, B) who compete to sell a system of complementary products (x and y). Therefore, under compatibility, four systems are available $((A, A), (A, B), (B, A), (B, B))$ while under incompatibility, only two pure systems (A,A) and (B,B) are available. Matutes and Régibeau (1988) study a two-stage game in which the first stage of non-cooperative choice between compatibility and incompatibility is followed by the second stage of price competition and find that the platforms choose compatibility since incompatibility intensifies competition. Our model is identical to theirs if there were no second period.

In the second period of our model, each platform competes to poach consumers by offering prices dependent on past purchase behavior (Chen, 1997 and Fudenberg and Tirole, 2000): there is price discrimination depending on which system a consumer bought in the first-period.[6] Under second-period compatibility, there are four submarkets: the market i_j refers to the market composed of the consumers who bought product $j = x, y$ from firm $i = A, B$ in the first period. We assume that each consumer discovers the value that she obtains from a product, which is a random draw from a uniform distribution, only after consuming it. But we assume for simplicity that all consumers incur the same switching cost s. For instance, in the market composed of the consumers who previously bought x from A, each consumer must incur a switching cost $s > 0$ if she wants to

[5]Economides (1989) generalizes Matutes and Régibeau (1988), who consider two firms, by showing that prices and pofits are higher under compatibility when n symmetric firms compete.

[6]For instance, apple launched Android device trade-in program to encourage iPhone upgrade: http://9to5mac.com/2015/03/16/applewillofferandroidswitchersgiftcardstotradeinrivalsmartphonesforiphones/

consume B's product x and therefore firm A is dominant in this market. Under second-period incompatibility, there are four submarkets if the firms chose compatibility in the first period: the market (x, y) refers to the the market composed of the consumers bought system $(x, y) = (A, A), (A, B), (B, A), (B, B)$. However, if the firms chose incompatibility in the first period, there are only two submarkets (A, A) and (B, B). In this case, the first-period compatibility choice dramatically affects the distribution of switching costs in that all consumers switching to (B, B) must incur a total switching cost of $2s$ under the first-period incompatibility whereas their total switching cost can be $2s$ or s under the first-period compatibility.

Given first-period market shares, the dynamics of compatibility is asymmetric in that firms are more likely to embrace compatibility tomorrow if products are compatible today but today's incompatibility does not necessarily imply that the firms are more likely to embrace incompatibility tomorrow. For instance, if the first-period market shares are symmetric, first-period compatibility leads to second-period compatibility no matter what the level of switching cost. In contrast, first-period incompatibility leads to second-period incompatibility if and only if the level of switching cost is high enough; otherwise, first-period incompatibility leads to second-period compatibility.

The above-described asymmetry in the dynamics of compatibility can be understood from the result of Hahn and Kim (2012) and Hurkens, Jeon and Menicucci (2013), who extend Matutes and Régibeau (1988) to asymmetric platforms (one is dominant and the other is dominated).[7] They consider a static model and find that when the level of dominance is large enough, both the dominant firm and the dominated firm prefer incompatibility since incompatibility softens competition. This mechanism is in place in our model. More precisely, it implies that in the market composed of the consumers who bought pure system (A, A) for instance, each firm's second-period profit is higher under second-period incompatibility than under compatibility if switching cost is large enough. This in turn suggests that for the switching cost large enough, the first-period incompatibility leads to the second-period incompatibility. However, under the first-period compatibility, symmetric market shares mean that many consumers bought hybrid systems $(A, B), (B, A)$. When the firms compete to poach these consumers, no firm is dominant

[7]Hahn and Kim (2012) consider uniform distribution of consumers on a Hotelling line as Matues and Régibeau do. Hurkens, Jeon and Menicucci (2013) consider the family of log-concave distributions which include the uniform distribution and offer a general insight for the result based on the property that the distribution of average location is more peaked than the original distribution for this family.

and hence second-period incompatibility intensifies the competition as in Matutes and Régibeau (1988). This is why the firms prefer second-period compatibility. Note that such an issue does not arise under first-period incompatibility as consumers buy only pure systems.

However, when first-period market shares are endogenous and the weight of the second-period profit is large, we find that the asymmetry in the dynamics of compatibility disappears. For switching costs low (high) enough, firms end up choosing compatibility (incompatibility) no matter what the first-period compatibility choice. In particular, when switching cost is high enough, under first-period compatibility, there exists no symmetric equilibrium and instead there exists a cornering equilibrium in which a firm corners both markets and chooses incompatibility in the second period. However, the cornered firm's overall profit is smaller than the profit it obtains under first-period incompatibility.

Therefore, when switching cost is small, firms choose compatibility both periods as compatibility softens competition in both periods. Furthermore, firms choose compatibility in both periods no matter what the switching cost as long as the weight of the second-period profit is small enough. However, when switching cost is high enough (and the weight of the second-period profit is large), firms choose incompatibility in both periods. Contrary to what happens in Matutes and Régibeau (1988) and Economides (1989), firms choose incompatibility and hence compete agressively in the first period in order to soften competition in the second period.

We complement our analysis by studying the case of non-negative pricing. When the weight of the second-period profit is large, the equilibrium first-period prices can be quite negative as the firms dissipate their second-period rent from consumer lock-in by lowering first-period prices. However, negative prices can induce consumers who are not interested in the products to buy them only to cash in the subsidies. Therefore, it makes sense to study the scenario of non-negative pricing. In this case, if the weight of the second-period profit is large enough, we find that the firms choose incompatibility (compatibility) in both periods if incompatibility (compatibility) generates a higher industry profit in the second period. In particular, we find a striking conflict between the firms' compatibility choice and the choice that maximizes consumer surplus; the two almost never coincides. The welfare analysis is more nuanced: conditional on that incompatibility arises, it reduces welfare compared to compatibility for switching cost high enough. This comparison of consumer surplus and welfare qualitatively carries over to the case of no restriction on prices.

Furtheremore, we extend the case of non-negative pricing by introducing some measure of new consumers in period two and find that the cut-off level of switching cost above which incompatibility is chosen in both periods increases with the measure of new consumers. This suggests that platforms are more likely to embrace in a growing market than in a mature market.

We merge two different strands of literature, the one on compatibility and the one on poaching. First, as incompatibility is equivalent to pure bundling, our paper is related to literature on bundling or incompatibility. The bundling (or tying) literature can be divided into three categories. The first one includes the papers that view bundling as a price discrimination device for a monopolist (Stigler, 1968, Schmalensee, 1984, McAfee et al. 1989, Salinger 1995, Armstrong 1996, Bakos and Brynjolfsson, 1999, Fang and Norman, 2006, Chen and Riordan, 2013). The second is about competitive bundling where entry and exit is not an issue (Matutes and Régibeau 1988, 1992, Economides, 1989, Carbajo, De Meza and Seidmann, 1990, Chen 1997, Denicolo 2000, Nalebuff, 2000, Armstrong and Vickers, 2010, Carlton, Gans, and Waldman, 2010, Thanassoulis 2011). The last is about leverage theory of bundling in which the main motive of bundling is to deter entry or induce the exit of rival firms in the competitive segment of the market (Whinston, 1990, Choi and Stefanadis 2001, Carlton and Waldman, 2002, Nalebuff, 2004, Peitz 2008, Jeon and Menicucci, 2006, 2012). Our paper is closely related to the second literature, in particular, Matutes and Régibeau (1988), Economides, (1989), Denicolo (2000) and the extension of Matutes and Régibeau (1988) to asymmetric firms by Hahn and Kim (2012) and Hurkens, Jeon and Menicucci (2013).

Second, our two-period game is similar to the games considered in poaching literature (Chen, 1997, Fudenberg and Tirole, 2000). The main difference is that we consider multi-product firms and their compatibility choices. This generates an interesting dynamics as the firms' first-period compatibility choice changes the distribution of switching costs and thereby affects the second-period competition to poach consumers and the second-period compatibility choice.

The paper is organized as follows. Section 2 describes the model. Section 3 analyzes second-period poaching competition and second-period compatibility choice given first-period compatibility choice and market shares. Section 4 analyzes first-period price competition and compatibility choice. Section 5 studies the case of non-negative pricing.

Section 6 conducts welfare and consumer surplus analysis. Section 7 presents an extension which studies the relationship between compatibility choice and market growth. All the proofs are gathered in Appendix.

2 Model

There are two firms, $i = A, B$, who produce two products, $j = x, y$, at constant marginal cost c. In our model, the two products can be perfect complements or can be independently consumed but our results are the same as long as each consumer obtains a high enough value from each product, which we assume. Since each firm produces each product, four systems are possible: $(x, y) = (A, A), (A, B), (B, A), (B, B)$. We consider two-period models in which consumers have switching costs in the second period.

Each firm's marginal cost of producing a product is $c > 0$. c is large enough such that if a consumer bought product $j = x, y$ from a firm, he or she has no incentive to buy the same product from the competing firm. Thus, at each period $t = 1, 2$, every consumer has a unit demand for each product. Given this assumption, we normalize c to zero, and interpret prices as the mark up above marginal cost without loss of generality. Therefore, a negative price in our model means that the real price is smaller than marginal cost, but it should still be positive.

In the beginning of each period, each firm simultaneously and non-cooperatively chooses between compatibility and incompatibility. If there is at least one firm which chooses incompatibility, incompatibility prevails. We consider the scenario in which no firm can commit in period one to its second-period compatibility choice.

In the first period, consumers have heterogeneous costs of learning to use each product as in Klemperer (1995). Precisely, a consumer' learning cost for product $j = x, y$ is uniformly distributed along a line segment $[0, 1]$. It is independently distributed across different products and different consumers. A consumer located at $(\theta_x, \theta_y) \in [0, 1]^2$ incurs a total learning cost of $t\theta_x + t\theta_y$ to use a system (A, A) or $t\theta_x + t(1 - \theta_y)$ to use system (A, B). In the beginning of period one, every consumer has the same expected valuation for each product, v^e. Therefore, the first-period utility of a consumer located at (θ_x, θ_y) from purchasing (A, A) under compatibility is:

$$U_1(A, A) = 2v^e - p^A_{1,x} - p^A_{1,y} - t\theta_x - t\theta_y, \tag{1}$$

where $p_{1,j}^{i}$[8] is the price for product j from firm i at time 1 under compatibility. The first-period utility of a consumer located at (θ_x, θ_y) from purchasing (A, A) under incompatibility is:

$$U_1(A, A) = 2v^e - P_1^A - t\theta_x - t\theta_y,$$

where P_1^i is the price of the system (i, i) at time 1 under incompatibility. If there is no second period, our model is identical to Matutes and Régibeau (1988).

Once a consumer uses firm i's product j in period one, she discovers her own true value v_j^i for the product, which is ex ante a random draw from a uniform distribution over $[\underline{v}, \overline{v}]$, with $\underline{v} > 0$. Hence, $(\underline{v} + \overline{v})/2 = v^e$. We assume that the distribution is independent across different products and different consumers.

Now let us describe what happens in the second period. Consider for instance a consumer who bought product x from A in period one. Then she has learnt her value v_x^A. Her choice is either to consume the same product and obtain v_x^A, or to switch to the rival product of B. In the latter case, her expected value is v^e and in addition she has to incur a switching cost of $s > 0$. The switching cost includes not only learning cost but also other costs due to a consumer's product-specific investment. For simplicity, we assume that the switching cost is the same for all consumers and products. In addition, we assume that each firm can engage in behaviour-based price discrimination to poach consumers: the price a firm charges to a consumer in period two can depend on the history of the consumer's purchase in period one. Suppose now that a consumer who bought (A, A) in the first period, switches to (A, B) in the second period (which is possible only under the second period compatibility). Then, her second period utility is given by:

$$U_2(A, B)_{|(A,A)} = v_x^A + v^e - p_{2,x}^A(A, A) - p_{2,y}^B(A, A) - s, \tag{2}$$

where $p_{2,j}^i(i, h)$ is the second-period price charged by firm i for product j under compatibility to the consumers who bought (i, h) in the first period with $i, h \in \{A, B\}$. Similarly, if a consumer who bought (A, A) in the first period switches to (B, B) in the second period, her second period utility under incompatibility is given by:

$$U_2(B, B)_{|(A,A)} = 2v^e - P_2^B(A, A) - 2s,$$

[8] We use lower-case letters for compatibility case and upper-case letters for incompatibility.

where $P_2^i(i,h)$ is the second-period price charged by firm i for its system under incompatibility to the consumers who bought (i,h) in the first period with $i,h \in \{A,B\}$.

All players have a common discount factor $\delta > 0$: δ can be larger than one since it represents the weight assigned to the second-period payoff. All firms have rational expectations. Regarding consumers' expectations, we consider the scenario of myopic consumers and relegate that of consumers with rational expectations to the future study.

We introduce the following assumption to guarantee a positive market share to each firm in each market in the second period competition:

Assumption 1: $s < \frac{3}{2}\Delta v$.

If the assumption is not satisfied, under the second period compatibility, no poaching occurs.

The timing in each period is given by:

- Stage 1: Each firm simultaneously and non-cooperatively chooses between compatibility and incompatibility.
- Stage 2: Each firm simultaneously and non-cooperatively chooses its prices. We allow for negative prices. The extension to the case in which firms cannot charge a negative price is done in Section 5.
- Stage 3: Consumers make purchase decisions.

At Stage 1, if at least one firm chooses incompatibility, then incompatibility prevails. Notice that there always exist an equilibrium in which both firms choose incompatibility. But we assume that each firm plays its weakly dominant action and therefore compatibility arises if and only if both firms prefer compatibility. In case a firm is indifferent between compatibility and incompatibility, we break the tie by assuming that the firm prefers making the same compatibility choice in both periods to changing its choices. This tie-breaking assumption is justified if there is a cost of changing compatibility. At stage 2, we allow for negative prices. The extension to the case in which firms cannot charge a negative price is done in Section 5.

In order to solve this two-period model, we first solve for firms' second-period equilibrium behavior and hence firms' second-period profits, for any given first-period compatibility choice and market shares. We find that equilibrium prices and profits, both under compatibility and under incompatibility, are linearly homogenous in Δv. Therefore sometimes it is useful to normalize $\Delta v = 1$; the model of $(\Delta v, s)$ is equivalent to that of $(1, s/\Delta v)$.

3 Second period: poaching competition and compatibility choice

In this section, we study the second-period competition for given first-period compatibility choice and market shares. We start by considering the case in which the firms chose compatibility in the first period.

3.1 Given the first-period compatibility

Suppose that the firms chose compatibility in the first period. We study the second-period price competition for given second-period compatibility choice, and then study the second-period compatibility choice.

Price competition given compatibility in both periods. Suppose first that there is compatibility in the second period. Consider the market composed of the consumers who have bought product j from i in the first period. We call it market i_j. Although there are four different markets i_j for $i \in \{A, B\}$, $j \in \{x, y\}$ in period 2, all four markets are alike. Hence, it is enough to analyze just one market i_j[9]. We normalize the total mass of consumers in market i_j to one. In this market, each firm offers poaching prices $p^i_{2,j}(i_j)$, and $p^h_{2,j}(i_j)$. To simplify notation, let us define:

$$p_2^+ \equiv p^i_{2,j}(i_j), p_2^- \equiv p^h_{2,j}(i_j), d_2^+ \equiv d^i_{2,j}\left(i_j\right), d_2^- \equiv d^h_{2,j}\left(i_j\right), \pi_2^+ = \pi^i_{2,j}(i_j), \pi_2^- = \pi^h_{2,j}(i_j),$$

where $d^i_{2,j}\left(i_j\right)$ is the demand for product j of firm i and $\pi^i_{2,j}(i_j)$ is firm i's profit in market i_j. A consumer of type v^i_j is indifferent between staying with i or switching to h if and only if $v^i_j - p_2^+ = v^e - p_2^- - s$. This implies that the demand for product j of firm i in this

[9]Without loss of generality, we can consider that the poaching price for product x (say) depends only on whether a given consumer bought x from A or B but not on whether he bought y from A or B.

market is given by:

$$d_2^+ = \begin{cases} 1 & : \qquad\qquad\qquad p_2^+ \;\leq\; s - \frac{\Delta v}{2} + p_2^- \\ \frac{1}{2} + \frac{1}{\Delta v}\left[s + p_2^- - p_2^+\right] & : s - \frac{\Delta v}{2} + p_2^- \;\leq\; p_2^+ \;\leq\; s + \frac{\Delta v}{2} + p_2^- \\ 0 & : s + \frac{\Delta v}{2} + p_2^- \;\leq\; p_2^+ \end{cases} \tag{3}$$

and the demand for product j of firm h is $d_2^- = 1 - d_2^+$. The equilibrium prices, $\left(p_2^{+*}, p_2^{-*}\right)$, maximize $p_2^+ d_2^+$ and $p_2^- d_2^-$, respectively. Note that this analysis is valid as long as there is second period compatibility, independently of the first period compatibility regime.

We have:

Lemma 1. *Suppose that compatibility was chosen in period two. Consider the period-two competition in the market i_j composed of the consumers who bought product j from firm i in period one. We normalize the total mass of consumers in market i_j to one. There exists a unique equilibrium.*

(i) The equilibrium prices in period 2 are

$$p_2^{+*} = \frac{\Delta v}{2} + \frac{s}{3}, \quad p_2^{-*} = \frac{\Delta v}{2} - \frac{s}{3}. \tag{4}$$

(ii) Each firm's second period profit from this market is given by:

$$\pi_2^{+*} = \frac{1}{\Delta v}\left(\frac{\Delta v}{2} + \frac{s}{3}\right)^2, \quad \pi_2^{-*} = \frac{1}{\Delta v}\left(\frac{\Delta v}{2} - \frac{s}{3}\right)^2. \tag{5}$$

(iii) Consumer surplus is given by $v^e - p_2^{+*} + \frac{1}{2}\pi_2^{-*}$.

Corollary 1. $\pi_2^{+*}(s) + \pi_2^{-*}(s)$ *strictly increases with s.*

The corollary says that an increase in the switching cost softens poaching competition such that the industry profit in a given product strictly increases with s. Note that the socially optimal switching requires $p_2^{+*} = p_2^{-*}$. As we have $p_2^{+*} > p_2^{-*}$, there always exists excessive switching from a social point of view.

Price competition when compatibility is followed by incompatibility. Suppose now that there is incompatibility in the second period. Then, we should distinguish two markets depending on the system that consumers bought in period one: the markets composed of the consumers who bought the hybrid system (A, B) or (B, A), and the markets composed of the consumers who bought the pure system (A, A) or (B, B).

Consider the market composed of the consumers who have bought a hybrid system (i, h) in the first period. We normalize the total mass of consumers in market (i, h) to one. A consumer with type $\left(v_x^i, v_y^h\right)$ is indifferent between buying (i, i) and (h, h) if and only if

$$v_x^i + v^e - P_2^i(i, h) - s = v^e + v_y^h - P_2^h(i, h) - s. \tag{6}$$

Therefore, the demand for firm i's system is given by

$$D_2^i(i, h) = \frac{1}{2(\Delta v)^2}\left[\Delta v - P_2^i(i, h) + P_2^h(i, h)\right]^2$$

and $D_2^h(i, h) = 1 - D_2^i(i, h)$ is the demand function for firm h's system.[10] As the two firms are symmetric in this market, we have a symmetric equilibrium. We now introduce simplifying notation:

$$P_2^{0*} = P_2^{i*}(i, h) = P_2^{h*}(i, h), \Pi_2^{0*} = \Pi_2^{i*}(i, h) = \Pi_2^{h*}(i, h).$$

We have:

Lemma 2. *Suppose that compatibility was chosen in period one while incompatibility was chosen in period two. Consider the period-two competition in the market composed of the consumers who bought the hybrid system (A, B) (or (B, A)) in period one. We normalize to one the total mass of consumers in this market. There exists a unique equilibrium, which is symmetric.*

(i) The equilibrium price for each firm is

$$P_2^{0*} = \frac{\Delta v}{2}.$$

[10] In fact, $D_2^i(i, h) = \frac{1}{2(\Delta v)^2}\left[\Delta v - P_2^i(i, h) + P_2^h(i, h)\right]^2$ is the demand function for firm i only if $P^h \leq P^i \leq P^h + \Delta v$, but this is the relevant portion of the demand function for equilibrium determination. A similar remark applies to (**??**) below.

(ii) Each firm's second period profit from this market is

$$\Pi_2^{0*} = \frac{\Delta v}{4}.$$

In the market composed of the consumers who bought a hybrid system in period one, neither firm has any advantage no matter what the level of switching cost, since the switching cost cancels out in equation (**??**) determining the marginal consumers' locations. Therefore, neither the equilibrium prices nor the profits depend on the level of switching cost.

Consider now the market composed of the consumers who purchased (i,i) in the first period. To simplify notation, let us define:

$$P_2^+ \equiv P_2^i(i,i), P_2^- \equiv P_2^h(i,i), D_2^+ \equiv D_2^i(i,i), D_2^- \equiv D_2^h(i,i), \Pi_2^+ = \Pi_2^i(i,i), \Pi_2^- = \Pi_2^h(i,i).$$

A consumer with type $\left(v_x^i, v_y^i\right)$ is indifferent between buying (i,i) and (h,h) if and only if

$$v_x^i + v_y^i - P_2^+ = 2v^e - P_2^- - 2s.$$

Therefore, the demand for firm h's system is given by

$$D_2^- = \frac{1}{2(\Delta v)^2}\left[\Delta v - 2s - P_2^- + P_2^+\right]^2 \tag{7}$$

and $D_2^+ = 1 - D_2^-$ is the demand function for the system of firm i.

We have:

Lemma 3. *Suppose that compatibility was chosen in period one while incompatibility was chosen in period two. Consider the period-two competition in the market composed of the consumers who bought a pure system (i,i) in period one. We normalize the total mass of consumers in market (i,i) to one. There exists a unique equilibrium.*

(i) The equilibrium prices in period 2 are

$$P_2^{+*} = \frac{1}{8}\left[3\sqrt{(2s-\Delta v)^2 + 8\Delta v^2} + 5(2s-\Delta v)\right], \; P_2^{-*} = \frac{1}{8}\left[\sqrt{(2s-\Delta v)^2 + 8\Delta v^2} - (2s-\Delta v)\right]$$

(ii) The firms' second period profits from this market are given by:

$$\Pi_2^{+*} = \left(1 - \frac{2\left(P_2^{-*}\right)^2}{\Delta v^2}\right) P_2^{+*}, \;\; \Pi_2^{-*} = \frac{2\left(P_2^{-*}\right)^3}{\Delta v^2}$$

(iii) Consumer surplus is given by $2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}$.

Corollary 2. $\Pi_2^{+*}(s) + \Pi_2^{-*}(s) = 2\Pi_2^{0*}$ *if* $s = 0$, *but* $\Pi_2^{+*}(s) + \Pi_2^{-*}(s)$ *strictly increases with* s *for* $s \in \left(0, \frac{3\Delta v}{2}\right)$ *while* $2\Pi_2^{0*}$ *does not depend on* s.

The corollary shows that an increase in the switching cost softens poaching competition in the market of pure system such that the industry profit strictly increases with s. As the socially optimal switching requires $P_2^{+*} = P_2^{-*}$ but we have $P_2^{+*} > P_2^{-*}$, there is always an excessive switching. In addition, for some consumer type the socially optimal switching involves switching only one product. Therefore, the incompatibility generates inefficiency as it forces a consumer either not to switch at all or to switch both products.

We now study how the second-period incompatibility affects the firms' profits with respect to the second-period compatibility. We have:

Corollary 3. *We normalize* $\Delta v = 1$ *without loss of generality. Then, there are three threshold values of switching costs,* $\overline{s}^1$, $\overline{s}^2$, $\overline{s}^3$ *with* $\overline{s}^1 < \overline{s}^2 < \overline{s}^3$, *such that*

$$\begin{aligned} 2\pi_2^{+*}(s) &\gtreqless \Pi_2^{+*}(s) \text{ if and only if } s \lesseqgtr \overline{s}^1; \\ 2\pi_2^{+*}(s) + 2\pi_2^{-*}(s) &\gtreqless \Pi_2^{+*}(s) + \Pi_2^{-*}(s) \text{ if and only if } s \lesseqgtr \overline{s}^2; \\ 2\pi_2^{-*}(s) &\gtreqless \Pi_2^{-*}(s) \text{ if and only if } s \lesseqgtr \overline{s}^3. \end{aligned}$$

More precisely, $\overline{s}^1 = 0.707$, $\overline{s}^2 = 0.825$ *and* $\overline{s}^3 = 1.187$.

Even if each of π_2^{+*}, π_2^{-*}, Π_2^{+*}, Π_2^{-*} depends on s, in what follows we do not highlight this dependence unless it is necessary. The next graph shows $2\pi_2^{+*}$, $2\pi_2^{-*}$, Π_2^{+*}, Π_2^{-*} and Π_2^{0*} as a function of $s \in (0, 3/2)$ where we set $\Delta v = 1$. There are three thresholds such that for $s > 0.701$ we have $\Pi_2^{+*} > 2\pi_2^{+*}$, for $s > 0.825$ we have $\Pi_2^{+*} + \Pi_2^{-*} > 2\pi_2^{+*} + 2\pi_2^{-*}$, and for $s > 1.187$ we have $\Pi_2^{-*} > 2\pi_2^{-*}$. From Corollary **??** and Corollary **??**, we know that as the switching cost increases, competition softens. Corollary **??** shows that for s high enough, switching cost softens competition more under incompatibility than under compatibility.

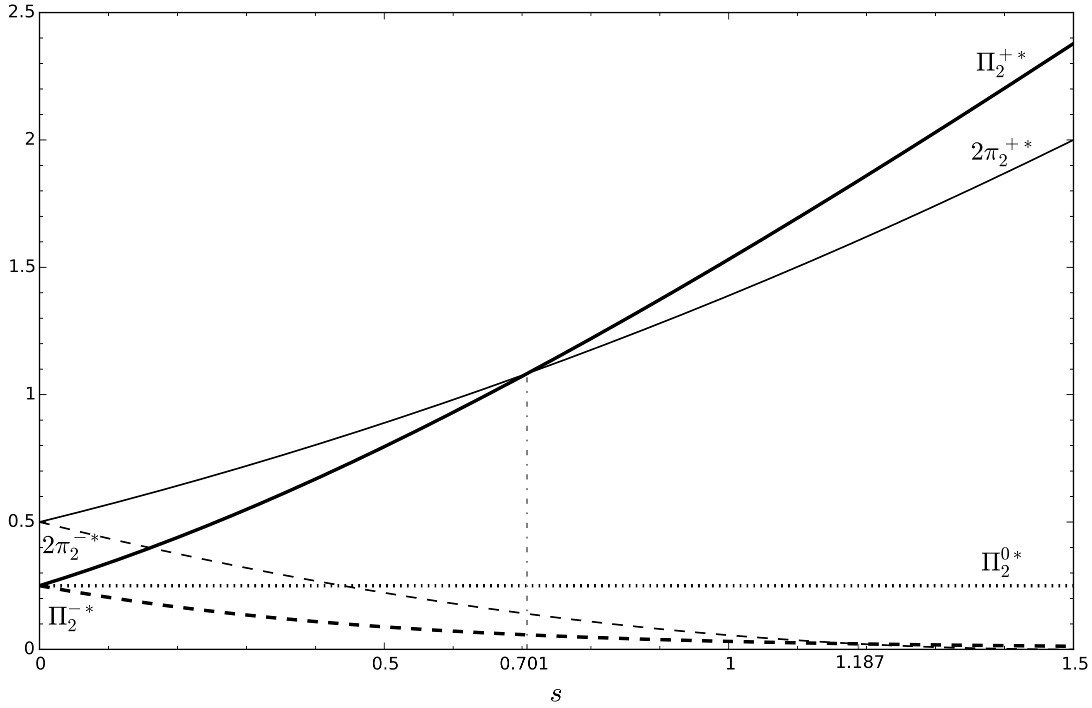

Figure 1: Second-period profits with or without compatibility

This result can be understood from the insight of Hahn and Kim (2012) and Hurkens, Jeon and Menicucci (2013), who extend Matutes and Régibeau (1988) to asymmetric platforms, such that one is dominant and the other is dominated. They show that as the level of dominance increases, incompatibility softens competition relative to compatibility. More precisely, they find two thresholds of dominance such that the dominant firm's profit is higher under incompatibility than under compatibility for a dominance level above the first threshold and the dominated firm's profit is higher under incompatibility for a dominance level above the second threshold, which is higher than the first one. The results suggest that there should be another threshold between the two such that the industry profit is higher under incompatibility if the dominance level higher than it. This mechanism is in place in our model as the dominance level of the dominant firm in a poaching market increases with the level of s.

Compatibility choice when compatibility was chosen in period one. Suppose that both firms chose compatibility in the first period. Let $\Delta v = 1$ without loss of generality. If both firms choose compatibility in the second period, then firm i's second-period profit is:

$$\left(d^i_{1,x}+d^i_{1,y}\right)\pi_2^{+-*}+\left(d^h_{1,x}+d^h_{1,y}\right)\pi_2^{-+*}, \tag{8}$$

where $d^i_{1,j}$ is the first-period market share of firm i for product j, and $d^i_{1,j}+d^h_{1,j}=1$. If any of the two firms chooses incompatibility in the second period, then i's profit is

$$d^i_{1,x}d^i_{1,y}\Pi_2^{+*}+\left(d^i_{1,x}d^h_{1,y}+d^h_{1,x}d^i_{1,y}\right)\Pi_2^{0*}+d^h_{1,x}d^h_{1,y}\Pi_2^{-*}. \tag{9}$$

Therefore, firm i chooses compatibility in the second period as long as (**??**) is at least as large as (**??**). Using Lemmas **??**, **??**, and **??**, we have the following result:

Lemma 4. *Suppose that compatibility was chosen in period one. Then compatibility choices in the second period are as follows:*

(i) If s is such that $2\pi_2^{+-*}-\Pi_2^{+*}\geq 0$ (i.e., $s\leq\overline{s}^1$), then both firms choose compatibility for any $(d^i_{1,x},d^i_{1,y})$ in $[0,1]^2$.

(ii) If s is such that $2\pi_2^{+-*}-\Pi_2^{+*}<0$ (i.e., $s>\overline{s}^1$), then at least one firm chooses incompatibility if and only if $d^i_{1,x}$ and $d^i_{1,y}$ are both close to 1 or both close to 0.

For $s>\overline{s}^1$ firm i prefers incompatibility if $d^i_{1,x}>\frac{\pi_2^{+-*}+\pi_2^{-+*}-\Pi_2^0}{\Pi_2^{+*}+\pi_2^{-+*}-\pi_2^{+-*}-\Pi_2^0}$ and $d^i_{1,y}>\frac{2\pi_2^{-+*}-\Pi_2^{-*}-(\Pi_2^0-\Pi_2^{-*}-\pi_2^{+-*}+\pi_2^{-+*})d^i_{1,x}}{(\Pi_2^0-\Pi_2^{-*}-\pi_2^{+-*}+\pi_2^{-+*})+(\Pi_2^{+*}+\Pi_2^{-*}-2\Pi_2^0)d^i_{1,x}}$, that is if $d^i_{1,x}$ and $d^i_{1,y}$ are close enough to one. This is quite intuitive, as when $d^i_{1,x}$ and $d^i_{1,y}$ are both close to 1, the profit comparison for firm i almost reduces to $2\pi_2^{+-*}$ vs Π_2^{+*}, and we have $2\pi_2^{+-*}<\Pi_2^{+*}$ given $s>\overline{s}^1$. Likewise, firm h prefers incompatibility if $d^h_{1,x}$ and $d^h_{1,y}$ are close to 1 (i.e., when $d^i_{1,x}$ and $d^i_{1,y}$ are close to zero). For instance, figure **??** shows firm i prefers incompatibility if $(d^i_{1,x},d^i_{1,y})$ is above the thin curve, and firm h prefers incompatibility if $(d^i_{1,x},d^i_{1,y})$ is below the thick curve, when $s=1.1$.

As s increases, the set of $(d^i_{1,x},d^i_{1,y})$ such that at least one firm prefers incompatibility becomes wider, but for no s it includes the point such that $d^i_{1,x}=d^i_{1,y}=\frac{1}{2}$, as the following corollary states.

Corollary 4. *Suppose that the firms chose compatibility in the first period. In the case of symmetric first-period market shares, that is $d^A_{1,x}=d^B_{1,x}=d^A_{1,y}=d^B_{1,y}=\frac{1}{2}$, both firms prefer compatibility to incompatibility for any $s\in(0,3/2]$.*

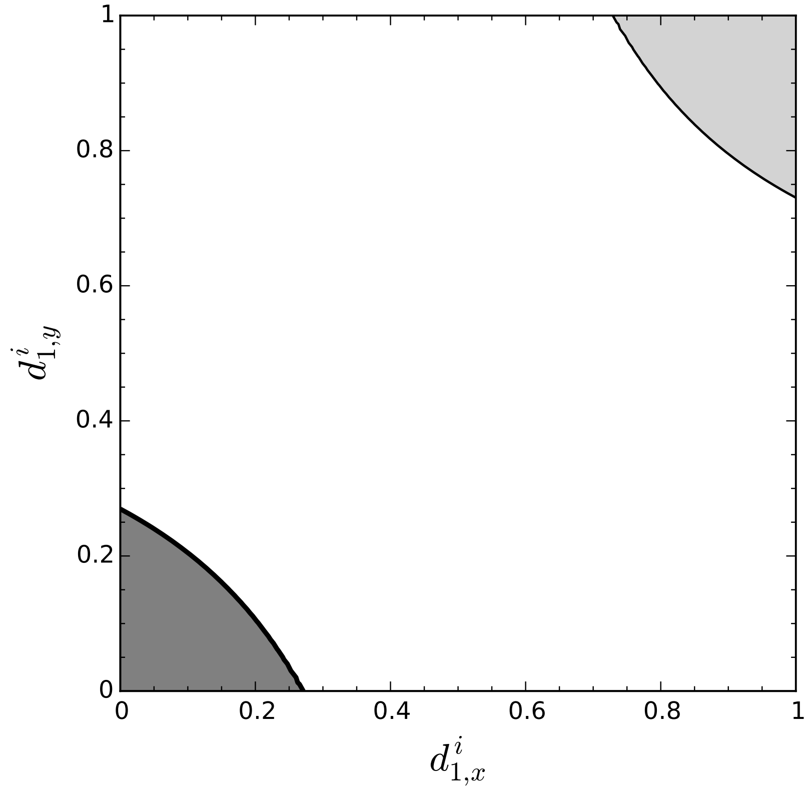

Figure 2: Second-period compatibility choice when compatibility was chosen in period one for $s = 1.1$

Given symmetric first-period market shares, we find for any $s \geq 0$

$$\pi_2^{+*}(s) + \pi_2^{-*}(s) > \frac{1}{4}(\Pi_2^{+*}(s) + \Pi_2^{-*}(s)) + \frac{1}{2}\Pi_2^{0*},$$

where the L.H.S. (the R.H.S) below is each firm's second period profit under compatibility (under incompatibility). Precisely, according to Matutes and Régibeau (1988) and Economides (1989), symmetric firms prefer compatibility over incompatibility since incompatibility intensifies competition: This implies $\pi_2^{+*}(s) + \pi_2^{-*}(s) > \Pi_2^{0*}$ for $s = 0$. In addition, as the former increases with s while the latter is independent of s, we have $\pi_2^{+*}(s) + \pi_2^{-*}(s) > \Pi_2^{0*}$ for any $s > 0$. This induces the firms to embrace compatibility no matter what the level of switching cost even if $\pi_2^{+*}(s) + \pi_2^{-*}(s) < \left(\Pi_2^{+*}(s) + \Pi_2^{-*}(s)\right)/2$ holds for $s > \overline{s}^2$.

3.2 Given first-period incompatibility

We now consider the case in which the firms chose incompatibility in the first period.

Suppose incompatibility in both periods. First-period incompatibility means there is no hybrid system in the first period. The equilibrium of the competition under incompatibility in each market (i,i) is given in Lemma **??**. Hence firm i's second-period profit is

$$D_1^i \Pi_2^{+*} + D_1^h \Pi_2^{-*}, \tag{10}$$

where D_1^i is i's first-period market share under incompatibility.

Suppose now that the first-period incompatibility is followed by the second-period compatibility. Then, as we wrote just before Lemma 1, the firms' second-period profits in each market i_j do not depend on the first-period compatibility regime. Therefore, Lemma 1 applies and firm i's second-period profit is

$$2\left(D_1^i \pi_2^{+*} + D_1^h \pi_2^{-*}\right). \tag{11}$$

In order to compare (**??**) and (**??**), we use $D_1^i = 1 - D_1^i$ and see that firm i prefers second-period compatibility if and only if

$$D_1^i 2\pi_2^{+*} + (1 - D_1^i) 2\pi_2^{-*} > D_1^i \Pi_2^{+*} + (1 - D_1^i)\Pi_2^{-*},$$

which is equivalent to $D_1^i < \frac{2\pi_2^{-*} - \Pi_2^{-*}}{\Pi_2^{+*} - \Pi_2^{-*} - (2\pi_2^{+*} - 2\pi_2^{-*})} \equiv \overline{D}$.[11] Likewise, firm h with $h \neq i$ prefers second-period compatibility if and only if

$$D_1^i 2\pi_2^{-*} + (1 - D_1^i) 2\pi_2^{+*} > D_1^i \Pi_2^{-*} + (1 - D_1^i)\Pi_2^{+*},$$

which is equivalent to $D_1^i > \frac{\Pi_2^{+*} - 2\pi_2^{+*}}{\Pi_2^{+*} - \Pi_2^{-*} - (2\pi_2^{+*} - 2\pi_2^{-*})} \equiv \underline{D}$. Then the following lemma is immediate.

Lemma 5. *Suppose that firms have chosen incompatibility in the first period (and let $\Delta v = 1$ w.l.o.g.). Then compatibility choices in the second period are as follows:*

(i) If $\Pi_2^{+*} < 2\pi_2^{+*}$ (that is, if $s < \overline{s}^1$), then $\overline{D} > 1$ and $\underline{D} < 0$. Hence compatibility

[11] Notice that the denominator $\Pi_2^{+*} - \Pi_2^{-*} - (2\pi_2^{+*} - 2\pi_2^{-*})$ is equal to $\frac{8s^2}{27 - 6s + 9\sqrt{4s^2 - 4s + 9}}$, which is positive for each $s \in (0, \frac{3}{2}]$.

emerges in the second period for any $D_1^i \in [0,1]$.

(ii) If $\Pi_2^{+*} + \Pi_2^{-*} \geq 2\pi_2^{+*} + 2\pi_2^{-*}$ (that is, if $s \geq \overline{s}^2$), then $\overline{D} \leq \underline{D}$. Hence incompatibility emerges in the second period for any $D_1^i \in [0,1]$.

(iii) If $\Pi_2^{+*} \geq 2\pi_2^{+*}$ and $\Pi_2^{+*} + \Pi_2^{-*} < 2\pi_2^{+*} + 2\pi_2^{-*}$ (that is, if $\overline{s}^1 \leq s < \overline{s}^2$), then $0 \leq \underline{D} < \frac{1}{2} < \overline{D} \leq 1$. Hence compatibility (incompatibility) emerges in the second period if $\underline{D} < D_1^i < \overline{D}$ (if $D_1^i \leq \underline{D}$, or $D_1^i \geq \overline{D}$).

We have shown previously that if the dominant firm prefers compatibility (i.e., if $\Pi_2^{+*} < 2\pi_2^{+*}$), then also the dominated firm prefers compatibility (i.e., $\Pi_2^{-*} < 2\pi_2^{-*}$). Therefore, as long as the dominant firm prefers compatibility (i.e., if $s < \overline{s}^1$), we have that (**??**) is greater than (**??**) for any D_1^i and both firms choose compatibility in the second period. As the switching cost increases enough above $\overline{s}^2$, the inequality $\Pi_2^{+*} + \Pi_2^{-*} \geq 2\pi^{+*} + 2\pi_2^{-*}$ holds. Then it is immediate that incompatibility emerges since this inequality means that the sum of the second-period profits is greater under incompatibility than under compatibility. Therefore at least one firm prefers incompatibility.

The following corollary covers the case of equal market shares.

Corollary 5. *Suppose that the firms chose incompatibility in the first period. In the case of symmetric market shares, i.e., $D_1^A = D_1^B = \frac{1}{2}$, the firms prefer incompatibility to compatibility in the second period if and only if $\Pi_2^{+*} + \Pi_2^{-*} \geq 2\pi_2^{+*} + 2\pi_2^{-*}$ (i.e., $s \geq \overline{s}^2$).*

The result in the corollary says that in the case of the symmetric market shares, the firms prefer incompatibility if and only if incompatibility generates a higher industry profit than compatibility, which occurs if and only if $s > \overline{s}^2$.

4 First-period price competition and compatibility choice

In this section, we study the first-period competition. We study the first-period price competition for given first-period compatibility choice and then study the first-period compatibility choice. As the case of first-period incompatibility is easier to study than that of the first-period compatibility, we start with the former.

4.1 Given incompatibility in the first period

If the firms have chosen incompatibility in period one, then Lemma 5 applies. As a consequence, for any $s \in [0, \overline{s}^1)$ (i.e., when $\Pi_2^{+*} < 2\pi_2^{+*}$) and for any pair of the first-period prices under incompatibility $\left(P_1^i, P_1^h\right)$, the firms will choose compatibility in the second period. Hence, we consider the following profit functions for any $\left(P_1^i, P_1^h\right)$:

$$\Pi^i = D_1^i(P_1^i + \delta 2\pi^{+*}) + (1 - D_1^i)(\delta 2\pi^{-*}), \qquad \Pi^h = (1 - D_1^i)(P_1^h + \delta 2\pi^{+*}) + D_1^i(\delta 2\pi^{-*}), \tag{12}$$

where D_1^i is firm i's market share in period one and is given by

$$D_1^i = \frac{1}{2}\left(1 + \frac{1}{2t}(P_1^h - P_1^i)\right)^2 - \frac{1}{2t}(P_1^h - P_1^i)\max\{0, \frac{1}{2t}(P_1^h - P_1^i)\}.$$

A similar argument can be applied to any $s \in [\overline{s}^2, \frac{3}{2}]$ (i.e., when $\Pi_2^{+*} + \Pi_2^{-*} \geq 2\pi_2^{+*} + 2\pi_2^{-*}$). In this case, for any pair of the first-period prices under incompatibility $\left(P_1^i, P_1^h\right)$, the second-period compatibility choice will be incompatibility. Therefore, for any $s \in [\overline{s}^2, \frac{3}{2}]$, we consider the following profit functions

$$\Pi^i = D_1^i(P_1^i + \delta\Pi_2^{+*}) + (1 - D_1^i)(\delta\Pi_2^{-*}), \qquad \Pi^h = (1 - D_1^i)(P_1^h + \delta\Pi_2^{+*}) + D_1^i(\delta\Pi_2^{-*}). \tag{13}$$

Proposition 1. *Suppose that incompatibility was chosen in the first period. Let $\Delta v = 1$ without loss of generality.*

(i) If $\Pi_2^{+*} < 2\pi_2^{+*}$ (i.e., if $s < \overline{s}^1$), there exists a unique equilibrium, which is symmetric. In the equilibrium, each firm charges the same price

$$P_1^{i*} = t - \delta 2\left(\pi_2^{+*} - \pi_2^{-*}\right) \quad \text{for } i = A, B.$$

Both firms choose compatibility in the second period. Each firm's total profit is

$$\Pi^* = \Pi^{i*} = \frac{t}{2} + \delta 2\pi_2^{-*} \quad \text{for } i = A, B.$$

(ii) If $2\pi_2^{+*} + 2\pi_2^{-*} \leq \Pi_2^{+*} + \Pi_2^{-*}$ (i.e., if $s \geq \overline{s}^2$), there exists a unique equilibrium, which

is symmetric. In the equilibrium, each firm charges the same price

$$P_1^{i*} = t - \delta\left(\Pi_2^{+*} - \Pi_2^{-*}\right) \quad \textit{for } i = A, B.$$

Both firms choose incompatibility in the second period. Each firm's total profit is:

$$\Pi^* = \Pi^{i*} = \frac{t}{2} + \delta\Pi_2^{-*}, \quad \textit{for } i = A, B. \tag{14}$$

In this equilibrium, consumer surplus is $2v^e - P_1^* - \frac{2}{3}t + \delta\left(2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}\right)$.

Under the first-period incompatibility, if $\delta = 0$ then it is well-known from Matutes and Régibeau (1988) that each firm charges a price for its system equal to t as in a two-dimensional Hotelling model. For $\delta > 0$, when $\Pi_2^{+*} < 2\pi_2^{+*}$, compatibility emerges in the second period regardless of the first-period market shares. Hence, if firm i attracts a consumer from the rival in the first period, its expected profit from the customer in the second period is $2\pi_2^{+*}$. But if the customer stays with the rival, firm i's expected profit from the customer in the second period is $2\pi_2^{-*}$. Therefore, each firm is ready to pay $\delta 2\left(\pi_2^{+*} - \pi_2^{-*}\right)$ to attract a consumer from the rival. This explains why the first-period equilibrium price is $t - \delta 2\left(\pi_2^{+*} - \pi_2^{-*}\right)$. As $\delta 2\left(\pi_2^{+*} - \pi_2^{-*}\right)$ is dissipated away because of competition, each firm's equilibrium profit is $\frac{t}{2} + \delta 2\pi_2^{-*}$.

When instead $2\pi_2^{+*} + 2\pi_2^{-*} \leq \Pi_2^{+*} + \Pi_2^{-*}$, incompatibility prevails in the second period for any first-period market shares. Then, the increase in the second-period profit from attracting an extra consumer from the rival is equal to $\delta\left(\Pi_2^{+*} - \Pi_2^{-*}\right)$. Therefore, each firm's equilibrium profit is $\frac{t}{2} + \delta\Pi_2^{-*}$.

Finally, as Lemma 5 states, if s is between $\overline{s}^1$ and $\overline{s}^2$ then the compatibility/incompatibility regime in period two depends on whether D_1^i belongs to the interval $(\underline{D}, \overline{D})$ or not: compatibility emerges if the first-period market shares are not too different. In the figure **??** below we represent $\overline{D}$ (the thin curve) and $\underline{D}$ (the thick curve), for $s \in [\overline{s}^1, \overline{s}^2)$.

In case there exists a symmetric pure-strategy equilibrium, then $D_1^i = \frac{1}{2}$ and thus the firms choose compatibility in the second period. However, a firm can induce incompatibility in period two by deviating in a way such that $D_1^i \leq \underline{D}$ or $D_1^i \geq \overline{D}$, holds. Therefore an equilibrium needs to be robust to this sort of deviations. A symmetric equilibrium candidate is not robust to such deviation if δ is large because a firm can profitably deviate by choosing a small price in period one in order to gain a large market share which allows

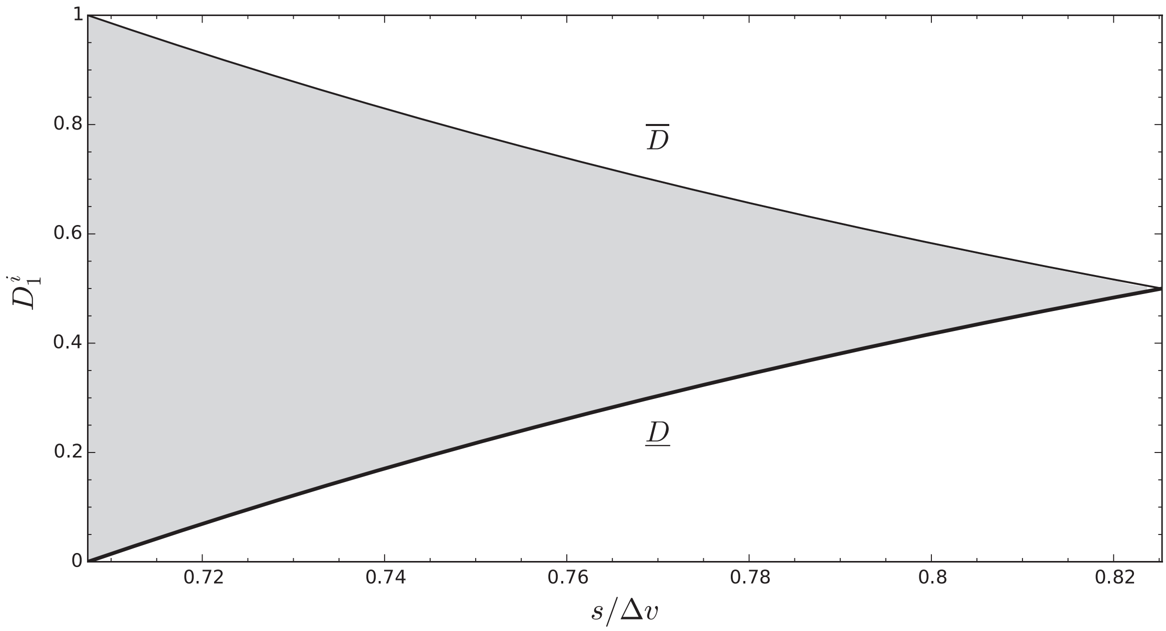

Figure 3: $(\underline{D}, \overline{D})$ under first-period incompatibility choice for $s \in [\overline{s}^1, \overline{s}^2)$.

to earn a large profit in period two. For δ large enough, such a deviation is profitable and therefore no symmetric pure-strategy equilibrium exists.[12] In fact, the next proposition shows that no asymmetric pure-strategy equilibrium exists either.

Proposition 2. *Suppose that incompatibility was chosen in the first period. Let $\Delta v = 1$ without loss of generality. If $\Pi_2^{+*} \geq 2\pi_2^{+*}$ and $2\pi_2^{+*} + 2\pi_2^{-*} > \Pi_2^{+*} + \Pi_2^{-*}$ (i.e., if s is between $\overline{s}^1$ and $\overline{s}^2$), then an equilibrium like the one described by Proposition* **??***(i) exists if and only if δ is sufficiently close to zero, otherwise no pure-strategy equilibrium exists (even allowing for asymmetric equilibria).*

4.2 Given compatibility in the first period

If the firms have chosen compatibility in period one, then Lemma 4 applies. Therefore, for $s \leq \overline{s}^1$, compatibility arises also in period two as no firm wants to choose incompatibility for any first-period market shares. Hence, we consider the following profit function for

[12] Precisely, the unique candidate equilibrium is such that each firm charges the price $P^* = t - \delta(2\pi_2^{+*} - 2\pi_2^{-*})$, but if firm i deviates with $P^i = P^* - 2t$ then firm i obtains full market share and profit $-t - \delta(2\pi_2^{+*} - 2\pi_2^{-*}) + \delta\Pi_2^{+*}$. This is greater than the candidate equilibrium profit $\frac{t}{2} + \delta 2\pi_2^{-*}$ if δ is large.

each firm i

$$\Pi^i = d^i_{1,x}(p^i_{1,x} + \delta\pi_2^{+*}) + (1 - d^i_{1,x})(\delta\pi_2^{-*}) + d^i_{1,y}(p^i_{1,y} + \delta\pi_2^{+*}) + (1 - d^i_{1,y})(\delta\pi_2^{-*}),$$

where $d^i_{1,j}$ represents demand for product j of firm i and is given by

$$d^i_{1,j} = \frac{1}{2} + \frac{1}{2t}(p^h_{1,j} - p^i_{1,j}).$$

A symmetric equilibrium is found in a straightforward way. However, the symmetric strategies which constitute an equilibrium for $s \leq \overline{s}^1$ may not be robust to deviations for $s > \overline{s}^1$. Then, firm i can deviate in a way to induce incompatibility in the second period, that is with $(d^i_{1,x}, d^i_{1,y})$ close to $(1,1)$ or $(0,0)$. The deviation with $(d^i_{1,x}, d^i_{1,y})$ close to $(1,1)$ is profitable if δ is large, because gaining a large market share in period one plus (second-period) incompatibility allows the deviating firm to obtain a large profit in period two, and this trade-off between the first-period profit and the second-period one is favorable when δ is large. Indeed, we have

Proposition 3. *Suppose that compatibility was chosen in the first period. Then, there exists a unique equilibrium, which is symmetric, if and only if $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$. In the equilibrium,*

(i) Each firm chooses compatibility in the second period.

(ii) The equilibrium prices in the first period are

$$p^{i*}_{1j} = t - \delta\left(\pi_2^{+*} - \pi_2^{-*}\right) = t - \frac{2}{3}\delta s, \quad \textit{for } i = A, B \textit{ and } j = x, y. \tag{15}$$

(iii) Each firm's total profit from market $j = x, y$ is:

$$\pi^* \equiv \pi^{i*} = \frac{t}{2} + \delta\pi_2^{-*} = \frac{t}{2} + \frac{\delta}{\Delta v}\left(\frac{\Delta v}{2} - \frac{s}{3}\right)^2 \quad \textit{for } i = A, B. \tag{16}$$

(iv) Consumer surplus in market j is $v^e - p_1^* - \frac{t}{4} + \delta\left(v^e - p_2^{+*} + \frac{1}{2}\pi_2^{-*}\right)$.

Given the second-period compatibility, the pricing in (**??**) is quite intuitive. For $\delta = 0$, each firm charges a price per product equal to t as in a standard Hotelling model. For $\delta > 0$, if firm i attracts a consumer from the rival in the first period in a given product market, its expected profit from the customer in the second period is π_2^{+*}. But if the customer stays with the rival, firm i's expected profit from the customer in the second period is π_2^{-*}. Therefore, each firm is ready to pay $\delta\left(\pi_2^{+*}-\pi_2^{-*}\right)$ to attract a consumer from the rival. The profit in (??) can be understood in a similar way. For $\delta=0$, each firm gets a profit of $t/2$ as in a standard Hotelling model. For $\delta>0$, each firm's profit from the second period is equal to π_2^{-*} as $\pi_2^{+*}-\pi_2^{-*}$ is dissipated away during the first-period competition.

Note that under the first-period incompatibility, each firm's total profit is $t/2$ when $\delta=0$. This means that the first-period incompatibility reduces the first-period profit from t to $t/2$ for $\delta=0$. This is well-known from Matutes and Régibeau (1988): incompatibility intensifies competition between symmetric firms in a one-period game.

As we mentioned previously, for s greater than $\overline{s}^1$ each firm can induce incompatibility in the second period by choosing low prices such that it corners both markets in the first period. Indeed, for δ large, cornering is the best deviation as there are economies of scale under second-period incompatibility. Suppose that firm A deviates by capturing $d_{1,x}^A=d_{1,y}^A=d_1^A\in(1/2,1]$ share of consumers.[13] Then, under the second period incompatibility, firm A's second-period profit is $\left(d_1^A\right)^2\Pi_2^{+*}+2d_1^A(1-d_1^A)\Pi_2^{0*}+(1-d_1^A)^2\Pi_2^{-*}$, which is strictly convex in d_1^A as $\Pi_2^{+*}+\Pi_2^{-*}-2\Pi_2^{0*}>0$ holds for $s>0$. To corner both markets, the deviating firm should charge p_1^*-t in period one and then obtains a profit of Π_2^{+*} in period two, which generates the total profit of $\delta\left[\Pi_2^{+*}-2\left(\pi_2^{+*}-\pi_2^{-*}\right)\right]$. Therefore, the cornering deviation is not profitable if $\Pi_2^{+*}-2\pi_2^{+*}\leq\frac{t}{\delta}$ holds. $\Pi_2^{+*}-2\pi_2^{+*}\leq\frac{t}{\delta}$ is obviously satisfied for $s\leq\overline{s}^1$. For $s\in(\overline{s}^1,\overline{s}^2)$ and δ satisfying $\delta\left(\Pi_2^{+*}-2\pi_2^{+*}\right)>t$, we find no pure strategy equilibrium.

However, there exists a cornering equilibrium when $s>\overline{s}^3$ and δ is large, in which a firm corners both markets, which leads to incompatibility in the second period. Precisely, in this equilibrium in each market one firm charges a price $\bar{p}$, and the other firm charges $\bar{p}+t$, such that the first firm gains full market share in both markets in period one, and therefore chooses incompatibility in the second period, earning a total profit of $2\bar{p}+\delta\Pi_2^{+*}$. The other firm has zero market share in both markets in period one, and earns a total profit of $\delta\Pi_2^{-*}$. The value of $\bar{p}$ must be small enough in order to deter the dominated firm from trying to earn a positive market share in period one, and high enough to deter the dominant firm from reducing its period one market share in order to reduce its period one

[13] Lemma 7 in the proof of Proposition 3 shows that the optimal deviation is characterized by symmetric market shares if the deviating firm expects incompatibility in period two.

losses. Let $C = \Pi_2^{+*} - \Pi_2^{-*} - 2\pi_2^{+*} + 2\pi_2^{-*}$.

Proposition 4. *Suppose that compatibility was chosen in the first period. Assume* $\Pi_2^{-*} - 2\pi_2^{-*} > 0$ *and* $\delta > \frac{4t}{C^2}\left(\Pi_2^{+*} + \Pi_2^{-*} - 2\pi_2^{+*} - 2\pi_2^{-*} - 2\sqrt{(\Pi_2^{+*} - 2\pi_2^{+*})(\Pi_2^{-*} - 2\pi_2^{-*})}\right)$. *Then there exists an asymmetric equilibrium such that*

(i) in the first period, one firm charges price $\bar{p} = t - \frac{\delta}{2}(\Pi_2^{+*} - \Pi_2^{-*})$, *the other firm charges price* $\bar{p} + t$, *and the first firm corners both markets;*

(ii) in the second period, incompatibility is chosen at least by the firm who cornered the markets, which earns a total profit of $2\bar{p} + \delta\Pi_2^{+*}$, *whereas the other firm's total profit is* $\delta\Pi_2^{-*}$.

4.3 First-period compatibility choice

Up to now we analyzed the first-period price competition given first-period compatibility or incompatibility. We now study the firms' first-period compatibility choices. Let us normalize Δv to one without loss of generality. If the switching cost is low enough (i.e., $s < \overline{s}^1$), then by Lemmas 4 and 5 the firms choose compatibility in period two no matter their compatibility choices in period one. As compatibility in period one generates a higher profit in period one than incompatibility (see Propositions 1(i) and 3), the firms will choose compatibility in both periods.

Suppose now that the switching cost is high enough (i.e., $s > \overline{s}^3$). Then, incompatibility in period one leads to incompatibility in period two, with the profit of $\frac{t}{2} + \delta\Pi_2^{-*}$ for each firm (see Proposition 1(i)). Regarding the scenario of compatibility in period one, we have to distinguish δ small from δ large. If δ is small (i.e., $\Pi_2^{+*} - 2\pi_2^{+*} \leq t/\delta$), compatibility in period one leads to compatibility in period two (see Proposition 3). In this case, each firm's profit is $t + \delta 2\pi_2^{-*}$. Therefore, the firms choose compatibility for δ small. If δ is large enough (see the inequality in Proposition 4), compatibility in period one leads to incompatibility in period two through the cornering equilibrium. In such an equilibrium, the dominated firm earns $\delta\Pi_2^{-*}$, which is less then $\frac{t}{2} + \delta\Pi_2^{-*}$. Therefore such a firm will choose incompatibility in period one.

Summarizing, we have

Proposition 5. *(first-period compatibility choice)* *(i) If either the switching cost is low enough (i.e., $s < \overline{s}^1$) to satisfy $2\pi_2^{+*}(s) > \Pi_2^{+*}(s)$ and/or δ is small enough to satisfy $\Pi_2^{+*} - 2\pi_2^{+*} \leq t/\delta$, then there is a unique equilibrium, which is symmetric and such that the firms choose compatibility in both periods.*

(ii) If both the switching cost and δ are high enough to satisfy $\Pi_2^{-*}(s) > 2\pi_2^{-*}(s)$ and $\delta > \frac{4t}{C^2}\left(\Pi_2^{+*} + \Pi_2^{-*} - 2\pi_2^{+*} - 2\pi_2^{-*} - 2\sqrt{(\Pi_2^{+*} - 2\pi_2^{+*})(\Pi_2^{-*} - 2\pi_2^{-*})}\right)$, then there is a unique equilibrium, which is symmetric and such that the firms choose incompatibility in both periods.

Proposition **??**(i) generalizes the finding of Matutes and Régibeau (1988) and Economides (1989) to our dynamic setting: as long as δ is small enough, the firms choose compatibility in the first period because this softens competition,in period one and the second-period profits are relatively unimportant. In this case, as the first period competition produces symmetric market shares, our Corollary 4 implies that they maintain compatibility in the second period.

Compatibility in both periods arises also if the switching cost is small, regardless of δ, because a small s leads to compatibility in period two, independently of the first period compatibility regime (see Lemma 4(i) and Lemma 5(i)). In addition, both firms prefer and compatibility in period one as it softens competition in period one.

Proposition **??**(ii) shows that the finding of Matutes and Régibeau (1988) and Economides (1989) is reversed if both the switching cost and the weight of the second period are large enough: then, the firms choose incompatibility in both periods. Even if the first-period incompatibility intensifies competition in period one, the firms choose it as this leads to the second-period incompatibility which softens competition in period two. Choosing compatibility in period one generates a corned allocation which leaves a low profit to the cornered firm. Since compatibility emerges only if both firms prefer compatibility, it follows that incompatibility prevails in period one.

5 Non-negative prices

Up to now, we have assumed that the firms can charge any negative prices in the first period. For instance, under the first period incompatibility, the symmetric equilibrium price of a system is either $t - \delta 2\left(\pi_2^{+*} - \pi_2^{-*}\right)$, for $s \leq \overline{s}^1$, or $t - \delta\left(\Pi_2^{+*} - \Pi_2^{-*}\right)$, for $s \geq \overline{s}^3$, which can be quite negative for δ/t high. Such high subsidy may not be feasible as it would induce consumers who are not interested in the products to buy them only to cash in the subsidy. Therefore, it would be interesting to study the case in which there is a limit on the amount of subsidy. We here study the case in which no firm can charge a negative price. We find that for small values of s, the firms are indifferent in period one between compatibility and incompatibility but prefer compatibility in period two regardless of the first period compatibility regime. In these cases, in order to break the indifference we assume that the firms prefer to have the same compatibility regime for both periods, and therefore they choose compatibility in period one. This occurs, for instance, if firms should incur any small cost to change compatibility over time. Then, we obtain the following results when we focus on the case in which δ/t is high enough and the switching cost is non-trivial.[14]

Proposition 6. *(non-negative prices) Suppose that the firms cannot charge any negative price. Assume* $\delta(\Pi_2^{+*} - 2\Pi_2^{0*} + \Pi_2^{-*}) \geq 4t$ *and the tie-breaking rule that if each firm is indifferent, the firms prefer maintaining the same compatibility choice for both periods to changing the choices across time.*

(i) Suppose that compatibility was chosen in the first period. Then there is a unique equilibrium, which is symmetric. In the equilibrium, each firm charges zero price for each product in the first period and choose compatibility in the second period.

(ii) Suppose that incompatibility was chosen in the first period. Then there is a unique equilibrium, which is symmetric. In the equilibrium, each firm charge zero price for its system in the first period, and chooses compatibility in the second period if $2\pi_2^{+*} + 2\pi_2^{-*} > \Pi_2^{+*} + \Pi_2^{-*}$, *incompatibility if* $2\pi_2^{+*} + 2\pi_2^{-*} \leq \Pi_2^{+*} + \Pi_2^{-*}$.

(iii) If $2\pi_2^{+*} + 2\pi_2^{-*} > \Pi_2^{+*} + \Pi_2^{-*}$, *under the tie-breaking rule, the firms choose compatibility in the first period, which is followed by compatibility in the second period. If* $2\pi_2^{+*} + 2\pi_2^{-*} \leq \Pi_2^{+*} + \Pi_2^{-*}$, *they choose incompatibility in the first period, which is followed by incompatibility in the second period.*

The lower bound on prices limits the dissipation of rents generated by switching costs: each firm charges zero price in period one and does not want to deviate by charging a

[14] If the switching cost is zero, we have $\Pi_2^{+*} = \Pi_2^{0*} = \Pi_2^{-*}$ and therefore the assumption in the proposition below is not satisfied.

strictly positive price when δ/t is high enough because its second period profit would be hurt. Proposition **??**(iii) is complementary to Proposition **??** for two reasons. First, it shows the robustness of the prediction we obtained without a lower bound on prices: given δ/t large, the prediction of Proposition **??** coincides with that of Proposition **??**(iii): compatibility in both periods if s is small, incompatibility in both periods if s is large. Second, Proposition **??**(iii) applies for any given switching cost, while Proposition **??** provides no prediction for middle values of switching costs.

6 Welfare and consumer surplus

We here make the welfare and consumer surplus analysis. Proposition **??** and Proposition **??**(iii) show that depending on parameters, we have either a symmetric equilibrium in which the firms choose compatibility in both periods or a symmetric equilibrium in which the firms choose incompatibility in both periods. Let us call the former CC equilibrium and the latter II equilibrium. We here provide comparison of the two in terms of welfare and consumer surplus.

We start our comparison by looking at the second period welfare and consumer surplus, which does not depend on whether we consider non-negative pricing or unrestricted pricing.

Lemma 6. *(i) In the CC equilibrium, the second period consumer surplus is given by*

$$2\left(v^e - p_2^{+*} + \frac{1}{2}\pi_2^{-*}\right);$$

the second period social welfare is given by

$$2\left(v^e - p_2^{+*} + \pi_2^{+*} + \frac{3}{2}\pi_2^{-*}\right).$$

(ii) In the II equilibrium, the second period consumer surplus is given by

$$2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*};$$

the second period social welfare is given by

$$2v^e - P_2^{+*} + \Pi_2^{+*} + \frac{5}{3}\Pi_2^{-*}.$$

(iii) The second period consumer surplus is higher in the II equilibrium than in the CC equilibrium if and only if $s < 0.876\Delta v$*. The second period industry profit is higher in the II equilibrium than in the CC equilibrium if and only if* $s > 0.825\Delta v$*. The second period social welfare is higher in the II equilibrium than in the CC equilibrium if and only if* $s \in (0.535\Delta v, 1.152\Delta v)$*.*

In the CC equilibrium, if we consider the market composed of the consumers who bought product x from A for instance and normalize the mass of consumers in this market to one, the consumer surplus is $v^e - p_2^{+*} + \frac{1}{2}\pi_2^{-*}$. Hence, the second period total consumer surplus in the CC equilibrium is $\left(v^e - p_2^{+*} + \frac{1}{2}\pi_2^{-*}\right) 2$. In the II equilibrium, if we consider the market composed of the consumers who bought both products x and y from A for instance and normalize the mass of consumers in this market to one, the consumer surplus is $2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}$. Hence, the second period total consumer surplus in the CC equilibrium is $2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}$.

Figure **??** shows consumer surplus, industry profit and welfare comparison for the second period when we normalize Δv to one. Note first that incompatibility increases consumer surplus relative to compatibility if and only if s is smaller than 0.876. We found in Section 3 that an increase in switching cost softens competition both under compatibility and under incompatibility but that it softens competition more under incompatibility than under compatibility. This implies that as the switching cost increases, the industry profit increases more under incompatibility than under compatibility. In fact, the graph shows a striking conflict between consumer surplus and industry profit. If the firms choose the second compatibility choice to maximize the industry profit, then they will make the choice which minimizes consumer surplus for all level of switching cost except for a very narrow middle range $s \in [0.825, 0.876]$.

Social welfare is lower under incompatibility than under compatibility either for s small enough or for s high enough. This is so for s small enough because consumers can make switching decision only at the system level (i.e., they cannot mix and match). For s high enough, no switching is socially optimal and therefore the compatibility choice which generates less switching gives a higher welfare. As incompatibility softens poaching competition such that more poaching arises under incompatibility, incompatibility generates lower welfare. This is reversed for an intermediate level of welfare where incompatibility induces the dominant firm firm to gain a higher market share than under compatibility because it competes more aggressively and benefits from the demand size effect of incompatibility.[15]

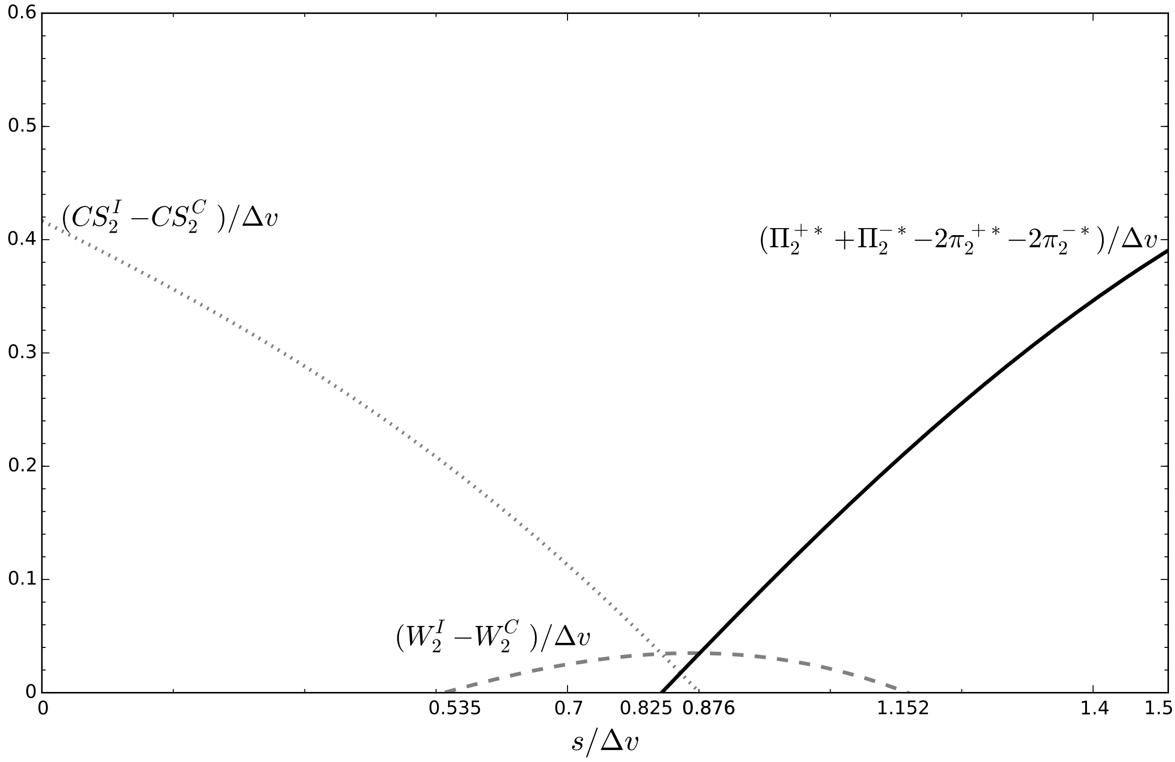

Figure 4: Second period: consumer surplus, profit and welfare comparison

We now turn to the overall consumer surplus and welfare comparison. Let us first consider the case of non-negative pricing with δ/t high enough. Then, as the firms' first period prices are zero, none among the first-period consumer surplus, profit and welfare depends on δ and hence is small enough. In addition, the firms' compatibility choices in both periods are completely guided by the second-period industry profit. They

[15]The demand size effect of incompatibility means that for any given symmetric prices, incompatibility increases the demand for the dominant firm. Hurkens-Jeon-Menicucci (2013) show that the demand size effect is positive for any symmetric log-concave distribution of consumer location.

choose compatibility in both periods if $2\pi_2^{+*} + 2\pi_2^{-*} > \Pi_2^{+*} + \Pi_2^{-*}$; otherwise, they choose incompatibility in both periods. Therefore, there is a strong conflict between the firms' compatibility choice and the choice maximizing consumer surplus. We have:

Proposition 7. *Suppose that no firm can charge any negative price. Consider δ/t high enough.*

(i) Then, the overall welfare and consumer surplus comparison is almost identical to the comparison based on second period welfare and consumer surplus in Lemma **??**.

(ii) In particular, the firms' compatibility choice is in strong conflict with the choice that maximizes consumer surplus: the two coincides only for a very narrow middle range of $s \in [0.825\Delta v, 0.876\Delta v]$.

(iii) The firms' compatibility choice is aligned with the choice that maximizes social welfare for $s \in (0, 0.535\Delta v]$ and $s \in [0.825\Delta v, 1.152\Delta v]$. The firms' compatibility choice is in conflict with the choice that maximizes social welfare for $s \in (0.535\Delta v, 0.825\Delta v)$ and $s \in (1.152\Delta v, 1.5\Delta v)$.

When there is no limit in prices, we have:

Proposition 8. *Suppose that there is no limit in the prices that the firms can charge.*

(i) In the CC equilibrium, the overall consumer surplus is given by

$$2\left[v^e - p_1^* - \frac{t}{4} + \delta\left(v^e - p_2^{+*} + \frac{1}{2}\pi_2^{-*}\right)\right];$$

the overall social welfare is given by

$$2\left[v^e - p_1^* + \frac{3t}{4} + \delta\left(v^e - p_2^{+*} + \frac{5}{2}\pi_2^{-*}\right)\right].$$

(ii) In the II equilibrium, the overall consumer surplus is given by

$$2v^e - P_1^* - \frac{2}{3}t + \delta\left(2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}\right);$$

the overall social welfare is given by

$$2v^e - P_1^* + \frac{1}{3}t + \delta\left(2v^e - P_2^{+*} + \frac{8}{3}\Pi_2^{-*}\right).$$

(iii) Consider δ/t large enough. The overall consumer surplus is higher in the II equilibrium than in the CC equilibrium if and only if $s < 1.168\Delta v$. The overall welfare is higher in the CC equilibrium than in the II equilibrium if and only if $s \in (0.535\Delta v, 1.152\Delta v)$. The firms play the CC equilibrium if $s < \overline{s}^1\Delta v = 0.707\Delta v$ and play the II equilibrium if $s > \overline{s}^3\Delta v = 1.187\Delta v$.

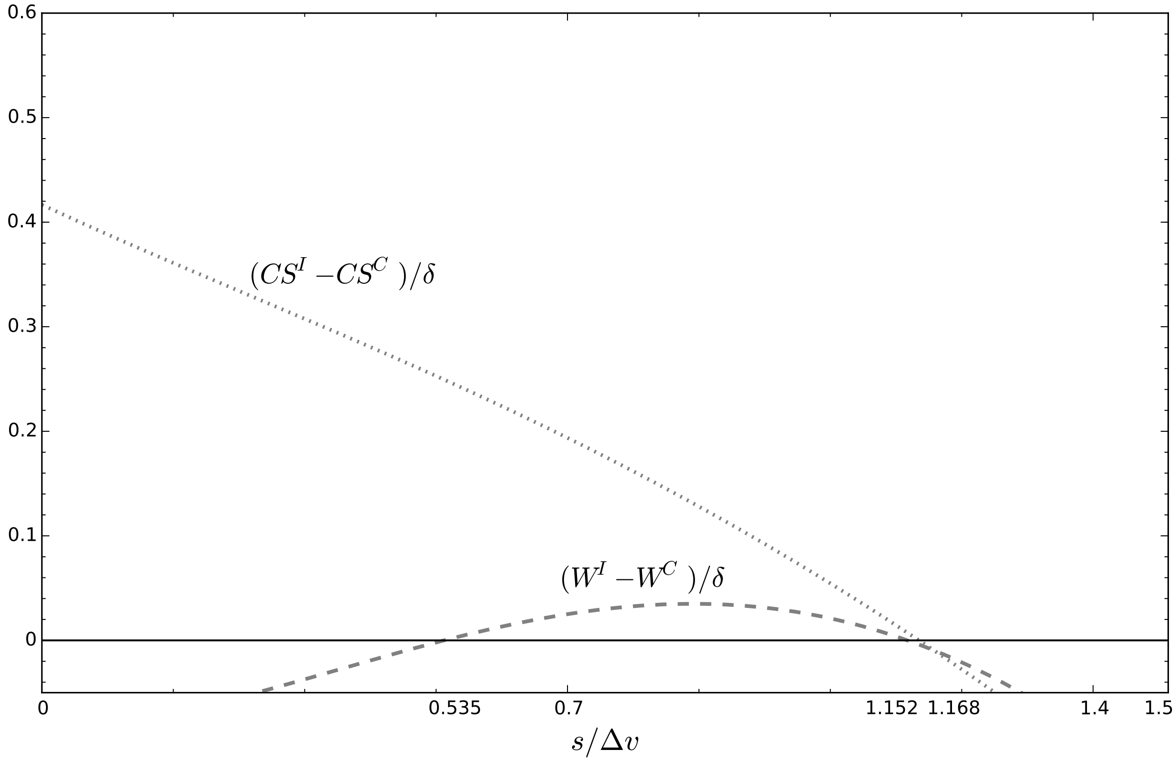

Figure 5: Overall welfare and consumer surplus comparison for δ/t large enough

Figure **??** shows overall welfare and consumer surplus comparison when we normalize Δv to one. In the graph, we represent only the terms which are multiplied by δ. Therefore, the welfare comparison is identical to that of the second period. Note also that the welfare comparison does not depend on whether or not there exists a lower bound on price since the lower bound affects only the distribution of the welfare between consumer surplus and profits. Hence, the main difference between Figure **??** and Figure **??** lies in consumer surplus comparison. In both figures, the difference decreases with s but the overall consumer surplus difference in Figure **??** decreases more slowly than the second period consumer surplus difference in Figure **??** such that the threshold level of s at which the difference is zero is larger in Figure **??** than Figure **??**. As the firms dissipate the rent they obtain from locked-in consumers by competing more aggressively in the first period, part of the rent is captured by consumers, which explains why the threshold is larger for the overall consumer surplus than for the second period one.

Despite this difference, the conflict between consumer surplus and the firms' choice of compatibility largely remains. More precisely, the firms choose compatibility for $s < 0.707$ (from Proposition **??**) but for this range of s, consumer surplus is higher with incompatibility: the firms choose incompatibility for $s > 1.187$ but for this range consumer surplus is higher with compatibility. Regarding social welfare, the firms' compatibility choice is aligned with the choice that maximizes social welfare for $s \in (0, 0.535]$ only; otherwise, the two are in conflict.

7 Extension: growing vs mature market

We here consider a simple extension in order to study how the compatibility choices are affected by whether the market is growing or mature. For this purpose, we consider the model of non-negative pricing and assume that in the second period, a measure $\sigma > 0$ of new consumers appears. As the firms can apply behavior-based price discrimination, the firms can offer targeted prices to these consumers. Therefore, each firm's profit from the new consumers is σt under second-period compatibility, is $\sigma t/2$ under second-period incompatibility. Hence, the firms will choose compatibility in both periods if $2\pi_2^{+*} + 2\pi_2^{-*} + \sigma t > \Pi_2^{+*} + \Pi_2^{-*}$, will choose incompatibility in both periods otherwise. Therefore, we have:

Proposition 9. *Suppose that there is a measure $\sigma > 0$ of new consumers in the second period and that the firms cannot charge any negative price. Assume $\delta(\Pi_2^{+*} - 2\Pi_2^{0*} + \Pi_2^{-*}) \geq 4t$, and hthe tie-breaking rule that if each firm is indifferent, the firms prefer maintaining the same compatibility choice for both periods to changing the choices across time. Then,* *the CC equilibrium is more likely in the growing market than in the mature market because the CC equilibrium is more likely as* σ *increases. Precisely, we have the CC equilibrium if* $2\pi_2^{+*} + 2\pi_2^{-*} + \sigma t > \Pi_2^{+*} + \Pi_2^{-*}$ *and the II equilibrium otherwise.*

The above proposition generates an interesting prediction that platforms are more likely to embrace compatibility in a growing market than in a mature market.

8 Conclusion

Our paper captures well Larry Page's claim of an "island-like" Internet. Consumer lock-in occurs naturally as consumers make product-specific investments. Then, platforms tend to embrace incompatibility today even if this intensifies today's competition only to maintain incompatibilty in order to soften future competition. Therefore, incompatibility hurts consumers and is likely to reduce welfare when consumer lock-in is substantial. Our results suggest that intense competition can be followed by lack of competition and that platforms are more likely to embrace compatibility in a growing market than in a mature market.

9 References

Armstrong, M. (1996). ”Multiproduct Nonlinear Pricing”, *Econometrica* 64, pp. 51-76.

Armstrong, M. and Vickers, J. (2010). ”Competitive Non-linear Pricing and Bundling”, *Review of Economic studies*, 77: 30-60.

Bakos, Yannis and Eric Brynjolfsson (1999). “Bundling Information Goods: Pricing, Profits and Efficiency” *Management Science*, 45, 1613-1630

Carbajo, Jose, David De Meza and Daniel J. Seidmann (1990). "A Strategic Motivation for Commodity Bundling", *Journal of Industrial Economics*, 38: 283-298

Carlton, Dennis W., Joshua S. Gans, and Michael Waldman (2010). "Why Tie a Product Consumers Do Not Use?," *American Economic Journal: Microeconomics* 2: 85-105

Carlton, Dennis, W. and Michael Waldman, Michael (2002). "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries," *RAND Journal of Economics*, 194-220

Chen, Yongmin, (1997). “Equilibrium Product Bundling,” *Journal of Business*, 70, pp. 85-103.

Chen, Yongmin, (1997). "Paying Customers to Switch", *Journal of Economics and Management Strategy*, 6: 877-897.

Chen, Yongmin and Michael Riordan (2013). “Profitability of Bundling,” *International Economic Review*, 54 (1): 35-57

Choi, Jay Pil and Stefanadis, Chris (2001). “Tying, Investment, and the Dynamic Leverage Theory”, *RAND Journal of Economics*, pp. 52-71.

Denicolo, Vincenzo (2000). “Compatibility and Bundling With Generalist and Specialist Firms", *Journal of Industrial Economics*, 48: 177-188

Economides, Nicholas (1989). “Desirability of Compatibility in the Absence of Network Externalities”, *American Economic Review*, 79, 1165-1181.

Fang, H. and P. Norman (2006). ”To Bundle or Not to Bundle”, *RAND Journal of Economics* 37, pp. 946-963.

Fudenberg, Drew and Jean Tirole (2000). "Customer Poaching and Brand Switching", The RAND Journal of Economics, 31: 634-657.

Hahn, Jong-Hee and Sang-Hyun Kim (2012). “Mix-and-Match Compatibility in Asymmetric System Markets”, *Journal of Institutional and Theoretical Economics*, 168, 311-338.

Hurkens, Sjaak, Doh-Shin Jeon and Domenico Menicucci (2013). "Dominance and Competitive Bundling", TSE working paper

Klemperer, Paul (1995). "Competition when Consumers have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics, and International Trade", *Review of Economic Studies*, 62: 515-539

Jeon, Doh-Shin and Domenico Menicucci (2006). "Bundling Electronic Journals and Competition among Publishers", *Journal of the European Economic Association*, vol.4 (5): 1038-83

Jeon, Doh-Shin and Domenico Menicucci (2012). "Bundling and Competition for Slots", *American Economic Review*, 102, 1957-1985.

Matutes, Carmen and Régibeau, Pierre. (1988). ””Mix and Match”: Product Compatibility without Network Externalities,” RAND Journal of Economics, vol. 19(2), pages 221-234.

Matutes, Carmen and Régibeau, Pierre. (1992). “Compatibility and Bundling of Complementary Goods in a Duopoly,” Journal of Industrial Economics, 40(1): 37–54.

McAfee, R.P, McMillan, J., Whinston, M. (1989) ”Multiproduct Monopoly, Commodity Bundling, and Correlation of Values”, *Quarterly Journal of Economics* 104, pp. 371-384.

Nalebuff, Barry, (2000). “Competing Against Bundles.” In *Incentives, Organization, Public Economics*, edited by Peter Hammond and Gareth D. Myles, London: Oxford University Press, pp. 323-336.

Nalebuff, Barry (2004). “Bundling as an Entry Barrier.” *Quarterly Journal of Economics*, 119, 159-188.

Peitz, Martin (2008). "Bundling May Blockade Entry," International Journal of Industrial Organization, 26: 41-58.

Salinger, Michael A. (1995). “A Graphical Analysis of Bundling.” *Journal of Business*, 68, 85-98.

Schmalensee, Richard L. (1984). “Gaussian Demand and Commodity Bundling.” *Journal of Business* 57. S211-S230.

Stigler, George J. (1968). “A Note on Block Booking.” in *The Organization of Industry,* edited by George J. Stigler. Homewood, Illinois: Richard D. Irwin.

Thanassoulis, John, (2011). “Is Multimedia Convergence To Be Welcomed?”, *Journal of Industrial Economics* 59(2), 225-253.

Whinston, Michael D. (1990). “Tying, Foreclosure and Exclusion.” *American Economic Review*, 80, 837-59.

10 Appendix A

10.1 Proof of Lemma 1

(i) We know p_2^{+*}, and p_2^{-*} maximize $p_2^+ d_2^+$, and $p_2^- d_2^-$. Hence,

$$\begin{aligned} -\frac{p_2^+}{\Delta v} + d_2^+ = 0 \Leftrightarrow \frac{\Delta v}{2} + s + p_2^- - 2p_2^+ &= 0 \\ -\frac{p_2^-}{\Delta v} + d_2^- = 0 \Leftrightarrow \frac{\Delta v}{2} - s - 2p_2^- + p_2^+ &= 0 \end{aligned}$$

This implies $p_2^{+*} = \frac{\Delta v}{2} + \frac{s}{3}$, $p_2^{-*} = \frac{\Delta v}{2} - \frac{s}{3}$.
(ii) Using (**??**), and the optimal prices, we have $\pi_2^{+*} = \frac{1}{\Delta v}\left(\frac{\Delta v}{2} + \frac{s}{3}\right)^2$, $\pi_2^{-*} = \frac{1}{\Delta v}\left(\frac{\Delta v}{2} - \frac{s}{3}\right)^2$.

(iii)

$$\begin{aligned}
CS_j &= \frac{1}{\Delta v}\int_{\underline{v}}^{\bar{v}}(v_j^i - p_2^{+*})dv_j^i - \frac{1}{\Delta v}\int_{\underline{v}}^{v^e+p_2^{+*}-p_2^{-*}-s}(v_j^i - p_2^{+*})dv_j^i + \frac{1}{\Delta v}\int_{\underline{\mathrm{V}}}^{v^e+p_2^{+*}-p_2^{-*}-s}(v^e - p_2^{-*} - s)dv_j^i \\
&= v^e - p_2^{+*} \\
&- \frac{1}{2\Delta v}\left(v^e + p_2^{+*} - p_2^{-*} - s - \underline{v}\right)\left(v^e - p_2^{+*} - p_2^{-*} - s + \underline{v}\right) \\
&+ \frac{1}{\Delta v}\left(v^e + p_2^{+*} - p_2^{-*} - s - \underline{v}\right)\left(v^e - p_2^{-*} - s\right) \\
&= v^e - p_2^{+*} + \frac{1}{2\Delta v}\left(\frac{\Delta v}{2} + p_2^{+*} - p_2^{-*} - s\right)^2 = v^e - p_2^{+*} + \frac{\Delta v}{2}(d_2^{-*})^2 = v^e - p_2^{+*} + \frac{1}{2}p_2^{-*}d_2^{-*} \\
&= v^e - p_2^{+*} + \frac{1}{2}\pi_2^{-*}
\end{aligned}$$

10.2 Proof of Lemma 2

It is straightforward to check the demand function in this case can be obtained by setting $s = 0$ in lemma **??**. Thus, we can get the prices and profits from lemma **??** by substituting $s = 0$.

10.3 Proof of Lemma 3

(i) The equilibrium prices maximize $D_2^+ P_2^+$, and $D_2^- P_2^-$, which implies $P_2^+ = 2s - \Delta v + 3P_2^-$, and $-8(P_2^-)^2 - 2(2s - \Delta v)P_2^- + \Delta v^2 = 0$. Consecutively,

$$P_2^{+*} = \frac{1}{8}\left[3\sqrt{(2s-\Delta v)^2 + 8\Delta v^2} + 5(2s - \Delta v)\right],\ P_2^{-*} = \frac{1}{8}\left[\sqrt{(2s-\Delta v)^2 + 8\Delta v^2} - (2s - \Delta v)\right]$$

(ii) Substituting the prices in the profit functions, and demands (**??**) would give us the second-period profits.

(iii)

$$\begin{aligned}
CS &= \frac{1}{\Delta v^2}\int_{\underline{v}}^{\bar{v}}\int_{\underline{v}}^{\bar{v}}(v_x^i + v_y^i - P_2^{+*})dv_x^i dv_y^i \\
&- \frac{1}{\Delta v^2}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-\underline{v}}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-v_x^i}(v_x^i + v_y^i - P_2^{+*})dv_x^i dv_y^i \\
&+ \frac{1}{\Delta v^2}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-\underline{v}}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-v_x^i}(2v^e - P_2^{-*} - 2s)dv_x^i dv_y^i \\
&= 2v^e - P_2^{+*} \\
&- \frac{1}{2\Delta v^2}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-\underline{v}}(2v^e - 2s - P_2^{-*} + P_2^{+*} - v_x^i - \underline{v})(2v^e - 2s - P_2^{-*} - P_2^{+*} + v_x^i + \underline{v})dv_x^i \\
&+ \frac{(2v^e - P_2^{-*} - 2s)}{\Delta v^2}\int_{\underline{v}}^{2v^e-2s-P_2^{-*}+P_2^{+*}-\underline{v}}(2v^e - 2s - P_2^{-*} + P_2^{+*} - v_x^i - \underline{v})dv_x^i \\
&= 2v^e - P_2^{+*} \\
&- \frac{(2v^e - 2s - P_2^{-*} + P_2^{+*} - 2\underline{v})^2}{2\Delta v^2}\left[-\frac{1}{3}(2v^e - 2s - P_2^{-*} + P_2^{+*} - 2\underline{v}) + (2v^e - 2s - P_2^{-*})\right] \\
&+ \frac{(2v^e - 2s - P_2^{-*} + P_2^{+*} - 2\underline{v})^2}{2\Delta v^2}(2v^e - 2s - P_2^{-*}) \\
&= 2v^e - P_2^{+*} \\
&+ \frac{1}{3}\frac{(2v^e - 2s - P_2^{-*} + P_2^{+*} - 2\underline{v})^2}{2\Delta v^2}(2v^e - 2s - P_2^{-*} + P_2^{+*} - 2\underline{v}) \\
&= 2v^e - P_2^{+*} + \frac{2}{3}D_2^{-*}P_2^{-*} = 2v^e - P_2^{+*} + \frac{2}{3}\Pi_2^{-*}
\end{aligned}$$

10.4 Proof of Lemma 4

(i) Firm i prefers compatibility if and only if

$$2\pi_2^{-*} - \Pi_2^{-*} \geq d_{1,x}^i d_{1,y}^i(\Pi_2^{+*} + \Pi_2^{-*} - \frac{1}{2}) + (d_{1,x}^i + d_{1,y}^i)(\frac{1}{4} - \Pi_2^{-*} - \pi_2^{+*} + \pi_2^{-*}) \qquad (17)$$

Likewise, firm h prefers compatibility if and only if

$$2\pi_2^{+*} - \Pi_2^{+*} \geq d_{1,x}^i d_{1,y}^i(\Pi_2^{+*} + \Pi_2^{-*} - \frac{1}{2}) + (d_{1,x}^i + d_{1,y}^i)(\frac{1}{4} - \Pi_2^{+*} + \pi_2^{+*} - \pi_2^{-*}) \qquad (18)$$

If $s \leq \bar{s}^1$, then we prove that (**??**) holds for each $(d_{1,x}^i, d_{1,y}^i) \in [0,1]^2$ is proved by showing that the maximum of the right hand side, considered as a function of $(d_{1,x}^i d_{1,y}^i) \in [0,1]^2$, is not larger than $2\pi_2^{-*} - \Pi_2^{-*}$. To this purpose, notice that the Hessian matrix of the right hand side is indefinite, hence no maximum point exists in the interior of $[0,1]^2$, and the four edges of the square $[0,1]^2$ have to be examined. For instance, if we consider the edge such that $d^i_{1,x} = 1$, then the right hand side of (**??**) is $d^i_{1,y}(\Pi_2^{+*} + \Pi_2^{-*} - \frac{1}{2}) + (1 + d^i_{1,y})(\frac{1}{4} - \Pi_2^{-*} - \pi_2^{+*} + \pi_2^{-*})$: this is a linear function of $d^i_{1,y}$ which is smaller than $2\pi_2^{-*} - \Pi_2^{-*}$ both at $d^i_{1,y} = 0$ and at $d^i_{1,y} = 1$. The other three edges of the square are dealt with similarly. The proof for (**??**) follows the same lines.

(ii) The proof follows from immediate manipulations of (**??**)-(**??**).

10.5 Proof of Proposition 1

(i) Given $s < \bar{s}^1$, we will have second period compatibility. Given the profit functions in (**??**), with $D^i_1 = \frac{1}{2}\left(1 + \frac{1}{2t}(P^h_1 - P^i_1)\right)^2 - \frac{1}{2t}(P^h_1 - P^i_1)\max\{0, \frac{1}{2t}(P^h_1 - P^i_1)\}$, the equilibrium first-period price, P^{i*}_1 solves the F.O.C. given by

$$\left[1 + \frac{1}{2t}(P^h_1 - P^{i*}_1)\right]\left[P^{i*}_1 + \delta\left(\pi_2^{+*} - \pi_2^{-*}\right)\right] - 2tD^i_1 = 0.$$

For firm h we obtain another condition which, combined with this one, implies $P^{i*}_1 = P^{h*}_1$, and thus $D^{i*}_1 = \frac{1}{2}$. Hence

$$P^{i*}_1 = P^{h*}_1 = t - \delta\left(2\pi_2^{+*} - 2\pi_2^{-*}\right)$$

Therefore the equilibrium profit for each firm is $\Pi^{i*} = \frac{t}{2} + \delta 2\pi_2^{-*}$.

(ii) Given $s > \bar{s}^2$, we will have second period incompatibility. Given the profit functions in (**??**), we can argue as in part (i) to obtain

$$P^{i*}_1 = P^{h*}_1 = t - \delta\left(\Pi_2^{+*} - \Pi_2^{-*}\right).$$

and therefore the equilibrium profit for each firm is $\Pi^{i*} = \frac{t}{2} + \delta\Pi_2^{-*}$.

The consumer surplus is

$$\begin{aligned} CS_1 &= 2\int_0^1 \int_0^{1-x} (2v^e - P_1^* - tx - ty)dydx \\ &= 2\int_0^1 \left(\frac{t}{2}x^2 - (2v^e - P_1^*)x - \frac{t}{2} + 2v^e - P_1^* \right) dx \\ &= 2v^e - P_1^* - \frac{2}{3}t \end{aligned}$$

10.6 Proof of Proposition 2

Given the strategies described by Proposition 1(i), it is immediate that there are no profitable deviations which lead to compatibility in period two, hence we consider here deviations which lead to incompatibility in period 2, that is such that $D^i \leq \underline{D}$, or $D^i \geq \underline{D}$.

10.6.1 Deviations with large P

Given that firms are symmetric, it suffices to consider the point of view of firm i at stage one, given incompatibility at stage one. Given $P_1^{h*} = P_1^* = t - \delta(2\pi_2^{+*} - 2\pi_2^{-*})$, let $\underline{P}$ be the price of firm i such that $D_1^i = \underline{D}$ (recall that for $s \in (\bar{s}^1, \bar{s}^2)$, we have $0 < \underline{D} < \frac{1}{2}$). Precisely, $\underline{P}$ belongs to (P^*, P^*+2t) and is such that $\frac{1}{2}(1+\frac{P^*-\underline{P}}{2t})^2 = \underline{D}$, that is $\underline{P} = P^*+2t-2t\sqrt{2\underline{D}}$. Since $P_1^i = \underline{P}$ induces incompatibility in period two, the profit of firm i from playing $P_1^i = \underline{P}$ is $\underline{D}\left(t - \delta(2\pi_2^{+*} - 2\pi_2^{-*}) + 2t - 2t\sqrt{2\underline{D}} + \delta\Pi_2^{+*}\right) + (1-\underline{D})\delta\Pi_2^{-*}$, that is

$$(3 - 2\sqrt{2\underline{D}})\underline{D}t + \delta\left(\Pi_2^{+*} - 2\pi_2^{+*} + \Pi_2^{-*}\right) \tag{19}$$

after using $\underline{D} = \frac{\Pi_2^{+*} - 2\pi_2^{+*}}{\Pi_2^{+*} - \Pi_2^{-*} - (2\pi_2^{+*} - 2\pi_2^{-*})}$. The difference between the equilibrium profit $\frac{t}{2} + \delta 2\pi_2^{-*}$ and (**??**) is

$$\left(\frac{1}{2} - (3 - 2\sqrt{2\underline{D}})\underline{D}\right)t + \delta(2\pi_2^{-*} - \Pi_2^{+*} + 2\pi_2^{+*} - \Pi_2^{-*})$$

which is positive for each $\underline{D} \in (0, \frac{1}{2})$ since $(3-2\sqrt{2\underline{D}})\underline{D} = 2\sqrt{2}(\frac{1}{4}\sqrt{2}+\sqrt{\underline{D}})\left(\frac{1}{2}\sqrt{2} - \sqrt{\underline{D}}\right)^2 >$ 0 and $2\pi_2^{-*} - \Pi_2^{+*} + 2\pi_2^{+*} - \Pi_2^{-*} > 0$ as $s < \bar{s}^2$.

In fact, firm i can achieve incompatibility in the second period by choosing any $P^i \in (\underline{P}, P^* + 2t]$, and in such a case the profit of firm i is $D_1^i(P^i + \delta(\Pi_2^{+*} - \Pi_2^{-*})] + \delta\Pi_2^{-*}$, with derivative

$$\frac{dD_1^i}{dP^i}(P^i + \delta(\Pi_2^{+*} - \Pi_2^{-*})) + D_1^i \tag{20}$$

Now consider the profit under compatibility in the second period, which is $D_1^i(P^i + \delta(2\pi_2^{+*} - 2\pi_2^{-*})]$, and its derivative with respect to P^i, $\frac{dD_1^i}{dP^i}(P^i + \delta(2\pi_2^{+*} - 2\pi_2^{-*})] + D_1^i$, which is larger than (??) because $\Pi_2^{+*} - \Pi_2^{-*} > 2\pi_2^{+*} - 2\pi_2^{-*}$. Since $\frac{dD_1^i}{dP^i}(P^i + \delta(2\pi_2^{+*} - 2\pi_2^{-*})] + D_1^i < 0$ for each $P^i > P_1^*$, in particular for $P^i > \underline{P}$, it follows that (??) is negative.

10.6.2 Deviations with small P

Given $P_1^{h*} = P_1^* = t - \delta(2\pi_2^{+*} - 2\pi_2^{-*})$, let $\bar{P}$ be the price of firm i such that $D_1^i = \bar{D}$ (recall that for $s \in (\bar{s}^1, \bar{s}^2)$, we have $\frac{1}{2} < \bar{D} < 1$). Precisely, $\bar{P}$ belongs to $(P^* - 2t, P^*)$ and is such that $1 - \frac{1}{2}(1 - \frac{P^* - \bar{P}}{2t})^2 = \bar{D}$, hence $\bar{P} = P^* - 2t + 2t\sqrt{2(1 - \bar{D})}$. Since $P_1^i = \bar{P}$ induces incompatibility in period two, the profit of firm i from playing $P_1^i = \bar{P}$ is $\bar{D}\left(t - \delta(2\pi_2^{+*} - 2\pi_2^{-*}) - 2t + 2t\sqrt{2(1 - \bar{D})} + \delta\Pi_2^{+*}\right) + (1 - \bar{D})\delta\Pi_2^{-*}$, that is

$$\left(2\sqrt{2(1 - \bar{D})} - 1\right)\bar{D}t + \delta 2\pi_2^{-*} \tag{21}$$

after using $\bar{D} = \frac{2\pi_2^{-*} - \Pi_2^{-*}}{\Pi_2^{+*} - \Pi_2^{-*} - (2\pi_2^{+*} - 2\pi_2^{-*})}$. The difference between the equilibrium profit $\frac{t}{2} + \delta 2\pi_2^{-*}$ and (??) is

$$\left(\frac{1}{2} - 2\bar{D}\sqrt{2(1 - \bar{D})} + \bar{D}\right)t$$

which is positive for each $\bar{D} > \frac{1}{2}$ since $\frac{1}{2} - 2\bar{D}\sqrt{2(1 - \bar{D})} + \bar{D} = \frac{\left(2\bar{D} - 1\right)^2\left(8\bar{D} + 1\right)}{2(1 + 2\bar{D} + 4\bar{D}\sqrt{2(1 - \bar{D})})} > 0$.

In fact, i can achieve incompatibility in the second period by choosing any $P^i \in [P^* - 2t, \bar{P})$, and in such a case the profit of firm i is $D_1^i(P^i + \delta(\Pi_2^{+*} - \Pi_2^{-*})] + \delta\Pi_2^{-*}$, with derivative (??) which is equal to

$$\frac{1}{4t^2}\left(-\frac{3}{2}(P^i)^2 - (2t + \Delta\Pi\delta + 4\Delta\pi\delta)P^i - \frac{4(\Delta\pi)^2\delta^2 - 7t^2 + 2\Delta\Pi t\delta + 4\Delta\pi t\delta + 4\Delta\Pi\Delta\pi\delta^2}{2}\right) \tag{22}$$

with $\Delta\Pi = \Pi_2^{+*} - \Pi_2^{-*}$, $\Delta\pi = 2\pi_2^{+*} - 2\pi_2^{-*}$. Notice that if (??) is positive at $P = \bar{P}$, then we can conclude that the profit from deviation is increasing with respect to P^i for $P^i \in [P^* - 2t, \bar{P})$, hence no profitable deviation exists, since we have already proved that deviating with $P^i = \bar{P}$ is unprofitable. Precisely, at $P = \bar{P}$ we find that (??) is equal to $-2-3\bar{D}+\frac{t+2\Delta\pi\delta-\Delta\Pi\delta}{2t}\sqrt{2(1-\bar{D})}$, which reduces to $-2+3\bar{D}+\frac{1}{2}\sqrt{2(1-\bar{D})}$ if $\delta=0$. Since $-2+3\bar{D}+\frac{1}{2}\sqrt{2(1-\bar{D})}\geq -2+3\bar{D}+1-\bar{D}=2\bar{D}-1>0$ for each $\bar{D}\in(\frac{1}{2},1)$, it follows that (**??**) is positive at $P=\bar{P}$ if $\delta>0$ is close to 0.

10.6.3 Non existence of other pure strategy equilibria

Suppose that $\bar{P}^i,\bar{P}^h$ is an equilibrium with $D_1^i>\overline{D}$. Then the profit functions are (**??**) in a neighborhood of $(\bar{P}^i,\bar{P}^h)$, hence from these profit functions the following focs are obtained

$$\begin{aligned}
-\frac{1}{2t}\left(1-\frac{P^h-P^i}{2t}\right)(P^i+\delta(\Pi_2^{+*}-\Pi_2^{-*}))+D^i &= 0 \text{ for firm } i\\
-\frac{1}{2t}\left(1-\frac{P^h-P^i}{2t}\right)(P^h+\delta(\Pi_2^{+*}-\Pi_2^{-*}))+1-D^i &= 0 \text{ for firm } h
\end{aligned}$$

which imply

$$\frac{D^i}{P^i+\delta(\Pi_2^{+*}-\Pi_2^{-*})}=1-\frac{P^h-P^i}{2t}=\frac{1-D^i}{P^h+\delta(\Pi_2^{+*}-\Pi_2^{-*})} \tag{23}$$

Notice that $D_1^i>\overline{D}$ implies $1-D_1^i<D_1^i$, $\bar{P}^i<\bar{P}^h$, and $\bar{P}^i+\delta(\Pi_2^{+*}-\Pi_2^{-*})<\bar{P}^h+\delta(\Pi_2^{+*}-\Pi_2^{-*})$. Hence (**??**) cannot hold.

Suppose that $\bar{P}^i,\bar{P}^h$ is an equilibrium with $D_1^i=\overline{D}$. Then the profit of firm i if $P^i<\bar{P}^i$ is Π^ι in (**??**), and its derivative needs to be ≥ 0 at $P^i=\bar{P}^i$, that is $\frac{dD_1^i}{dP_1^i}(P^i+\delta(\Pi_2^{+*}-\Pi_2^{-*}))+D_1^i\geq 0$; the profit if $P^i>\bar{P}^i$ is Π^ι in (**??**) and its derivative needs to be ≤ 0 at $P^i=\bar{P}^i$, that is $\frac{dD_1^i}{dP_1^i}(P^i+\delta(2\pi_2^{+*}-2\pi_2^{-*}))+D_1^i\leq 0$. However, these inequalities cannot both hold since $\frac{dD_1^i}{dP_1^i}<0$ and $\Pi_2^{+*}-\Pi_2^{-*}>2\pi_2^{+*}-2\pi_2^{-*}$.

Suppose that $\bar{P}^i,\bar{P}^h$ is an equilibrium with $D_1^i=1$. Then the derivative of the profit function of firm i is equal to one at $P^i=\bar{P}^i$, which implies that it is profitable for firm i to increase slightly P^i above $\bar{P}^i$.

10.7 Proof of Proposition ??

(i), (ii), (iii):

Step 1. In this step, we find the equilibrium given compatibility in both periods.
(a) Given compatibility in the both periods, the total profit of firm i for product j is:

$$\Pi_j^i = d_{1,j}^i p_{1,j}^i + \delta \left(d_{1,j}^i \pi_2^{+*} + (1 - d_{1,j}^i) \pi_2^{-*} \right) \tag{24}$$

where the first-period demand is $d_{1,j}^i = \frac{1}{2} + \frac{1}{2t}(p_{1,j}^h - p_{1,j}^i)$. The first-period equilibrium prices, $\overset{*}{p}{}_{1,j}^{i}$ and $\overset{*}{p}{}_{1,j}^{h}$, solve $d_{1,j}^i - \frac{1}{2t} p_{1,j}^i - \delta \frac{1}{2t} \left(\pi_2^{+*} - \pi_2^{-*} \right) = 0$, that is

$$d_{1,j}^i - \frac{1}{2t} p_{1,j}^i - \delta \frac{s}{3t} = 0$$

This and the h's F.O.C. imply $\overset{*}{p}{}_{1,j}^{i} = \overset{*}{p}{}_{1,j}^{h}$. Therefore, we have $\frac{1}{2} - \frac{1}{2t} p_{1,j}^i - \delta \frac{s}{3t} = 0$, which gives us the first-period equilibrium prices, $\overset{*}{p}{}_{1,j}^{i} = t - \frac{2}{3} \delta s$. The profit can be obtained by replacing prices in (**??**).

Moreover, thanks to corollary **??** we know that when $d_{1,x}^i = d_{1,y}^i = \frac{1}{2}$, both firm choose compatibility in the second period.

Step 2. Now, we show that if $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$, then there is no profitable deviation from the equilibrium candidate we found in the previous step.
(a) First, we know there exists no profitable deviation where we have compatibility in the second period. This implies that no profitable deviation exists if $\Pi_2^{+*} \leq 2\pi_2^{+*}$ (that is, if $s \leq \bar{s}^1$) because then Lemma 4(i) reveals that in any case compatibility emerges in the second period.
(b) Now consider the case in which $\Pi_2^{+*} > 2\pi_2^{+*}$. Then, by Lemma 4(ii), there exist deviations of firm i which lead to incompatibility in the second period (they need to be such that $d_{1,x}^i, d_{1,y}^i$ are both close to 1, or both close to 0), and we show that no profitable deviation exists for firm i if and only if $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$. Given incompatibility in the second period, the total profit of firm i is

$$\begin{aligned}
\Pi^i &= d_{1,x}^i p_{1,x}^i + d_{1,y}^i p_{1,y}^i \\
&+ \delta \left[d_{1,x}^i d_{1,y}^i \Pi_2^{+*} + d_{1,x}^i d_{1,y}^h \Pi_2^{0*} + d_{1,y}^i d_{1,x}^h \Pi_2^{0*} + d_{1,x}^h d_{1,y}^h \Pi_2^{-*} \right] \\
&= d_{1,x}^i p_{1,x}^i + d_{1,y}^i p_{1,y}^i + \delta B d_{1,x}^i d_{1,y}^i + \delta \left[\Pi_2^{0*} - \Pi_2^{-*} \right] (d_{1,x}^i + d_{1,y}^i) + \delta \Pi_2^{-*} \qquad (25)
\end{aligned}$$

where $d^i_{1,j} = \frac{1}{2} + \frac{1}{2t}(p^*_1 - p^i_{1,j})$ and $B = \Pi^{+*}_2 - \Pi^{0*}_2 - \Pi^{0*}_2 + \Pi^{-*}_2 > 0$. We first prove a useful lemma:

Lemma 7. *Suppose that the firms chose compatibility in the first period and that a deviating firm expects that the incompatibility would prevail in the second period. Then, without loss of generality, we can restrict attention to deviations with symmetric market shares in the first period.*

Proof of the lemma. Suppose that firm i deviates with $p^i_{1,x} \neq p^i_{1,y}$. Then consider $\hat{p}^i_{1,x}, \hat{p}^i_{1,y}$ such that $\hat{p}^i_{1,x} = \hat{p}^i_{1,y} = \frac{1}{2}(p^i_{1,x} + p^i_{1,y}) = \hat{p}^i_1$. Then the demand for the product of firm i is $\hat{d}^i_1 = \frac{1}{2}(d^i_{1,x} + d^i_{1,y})$ in each market and i's profit is[16]

$$2\hat{d}^i_1\hat{p}^i_1 + \delta B(\hat{d}^i_1)^2 + \delta\left[\Pi^{0*}_2 - \Pi^{-*}_2\right]2\hat{d}^i_1 + \delta\Pi^{-*}_2$$

which is larger than (**??**) because (i) $2\hat{d}^i_1\hat{p}^i_1 = 2\frac{1}{2}(d^i_{1,x} + d^i_{1,y})\frac{1}{2}(p^i_{1,x} + p^i_{1,y}) = d^i_{1,x}p^i_{1,x} + d^i_{1,y}p^i_{1,y} + \frac{1}{2}(p^i_{1,x} - p^i_{1,y})(d^i_{1,y} - d^i_{1,x}) = d^i_{1,x}p^i_{1,x} + d^i_{1,y}p^i_{1,y} + \frac{1}{4t}(p^i_{1,x} - p^i_{1,y})^2 > d^i_{1,x}p^i_{1,x} + d^i_{1,y}p^i_{1,y}$; (ii) $\delta B(\hat{d}^i_1)^2 > \delta B d^i_{1,x}d^i_{1,y}$ because $\delta B > 0$ and $(\hat{d}^i_1)^2 > d^i_{1,x}d^i_{1,y}$, given that $\hat{d}^i_1 = \frac{1}{2}(d^i_{1,x} + d^i_{1,y})$; (iii) $\delta\left[\Pi^{0*}_2 - \Pi^{-*}_2\right]2\hat{d}^i_1 = \delta\left[\Pi^{0*}_2 - \Pi^{-*}_2\right](d^i_{1,x} + d^i_{1,y})$. ■

Given a symmetric deviation such that $d^i_{1,x} = d^i_{1,y} = d$, we find that

$$\Pi^i = 2dp + \delta d^2 B + 2\delta d\left[\Pi^{0*}_2 - \Pi^{-*}_2\right] + \delta\Pi^{-*}_2,$$

where $d = 1 - \frac{\delta}{2t}\left(\pi^{+*}_2 - \pi^{-*}_2\right) - \frac{1}{2t}p$. Hence,

$$\Pi^i = -4t\left(1 - \frac{\delta}{4t}B\right)d^2 + 4t\left(1 - \frac{2\delta}{4t}A\right)d + \delta\Pi^{-*}_2, \tag{26}$$

where $A = -\Pi^{0*}_2 + \Pi^{-*}_2 + \pi^{+*}_2 - \pi^{-*}_2$. Then we need to consider a few cases, as a function of A and B.

Suppose that $1 - \frac{\delta}{4t}B \leq 0$, which implies that Π^i is convex in d. Then Π^i is maximized at $d = 0$ or (and) at $d = 1$. Precisely, the profit at $d = 1$ is $\delta\left(\Pi^{+*}_2 - 2\pi^{+*}_2 + 2\pi^{-*}_2\right)$, which is not larger than the equilibrium profit $t + 2\delta\pi^{-*}_2$ if and only if $\Pi^{+*}_2 - 2\pi^{+*}_2 \leq \frac{t}{\delta}$. The profit

[16] Notice that if incompatibility emerges in the second period given $d^i_{1,x}, d^i_{1,y}$, then it also emerges given the market shares $\hat{d}^i_1, \hat{d}^i_1$ in each market, given the shape of the set described in Lemma 4(ii), and immediately after the lemma.

at $d = 0$ is $\delta\Pi_2^{-*}$, which is not larger than the equilibrium profit $t + 2\delta\pi_2^{-*}$ if and only if $\Pi_2^{-*} - 2\pi_2^{-*} \leq \frac{t}{\delta}$, and since $\Pi_2^{-*} - 2\pi_2^{-*} < \Pi_2^{+*} - 2\pi_2^{+*}$, it follows that $\Pi_2^{+*} - 2\pi_2^{+*} \leq \frac{t}{\delta}$ implies $\Pi_2^{-*} - 2\pi_2^{-*} < \frac{t}{\delta}$.

Now suppose that $1 - \frac{\delta}{4t}B > 0$, which implies that Π^i is concave in d Then the maximum point for Π^i is either $d = 0$, or $d = 1$, or $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ if $0 < \frac{1}{2} - \frac{\delta}{4t}A < 1 - \frac{\delta}{4t}B$.[17] We have already dealt with the cases of $d = 0$ and $d = 1$. When $0 < \frac{1}{2} - \frac{\delta}{4t}A < 1 - \frac{\delta}{4t}B$, the deviation profit is:

$$\Pi^i = 4t\frac{\left[\frac{1}{2} - \frac{\delta}{4t}A\right]^2}{1 - \frac{\delta}{4t}B} + \delta\Pi_2^{-*}.$$

The gain from deviation is

$$\begin{aligned} t + 2\delta\pi_2^{-*} - \Pi^i &= \frac{1}{1 - \frac{\delta}{4t}B}\left[t(1 - \frac{\delta}{4t}B) + \delta(1 - \frac{\delta}{4t}B)C - 4t\left[\frac{1}{2} - \frac{\delta}{4t}A\right]^2\right] \\ &= \frac{\delta}{1 - \frac{\delta}{4t}B}\left[A - \frac{B}{4} + C - \frac{\delta}{4t}(BC + A^2)\right], \end{aligned}$$

where $C = 2\pi_2^{-*} - \Pi_2^{-*}$. In case $s = 0$ we have $A = B = 0$ and $C = \frac{\Delta v}{4}$ which imply the deviation is not profitable. Otherwise, from $\frac{1}{2} - \frac{\delta}{4t}A \leq 1 - \frac{\delta}{4t}B$, $0 < B - A$, and $0 < BC + A^2$ we can conclude

$$A - \frac{B}{4} + C - \frac{BC + A^2}{2(B - A)} \leq (2\overset{*}{\pi}{}_j^i - \Pi^i)(1 - \frac{\delta}{4t}B)\frac{1}{\delta}.$$

Since $0 < A - \frac{B}{4} + C - \frac{BC+A^2}{2(B-A)}$ for all $s > 0$ and Δv, there is no profitable deviation.

(iv)

$$CS_{1,j} = 2\int_0^{\frac{1}{2}}(v^e - p_1^* - tx)dx = v^e - p_1^* - \frac{t}{4}.$$

[17] Notice that $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ is the point where the derivative of Π^i vanishes. In fact, we should verify whether $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ induces incompatibility in period two, but in any case we can prove that Π^i in (**??**) at $d = \frac{\frac{1}{2} - \frac{\delta}{4t}A}{1 - \frac{\delta}{4t}B}$ is smaller than the equilibrium profit.

10.8 Proof of Proposition ??

We are considering the equilibrium candidate in which one firm, i corners the other, h, in the first period, in both markets. Precisely, firm i plays price $\bar{p}$ in each market, and firm h plays price $\bar{p}+t$ in each market, with $\bar{p}$ to be determined. As a consequence, the profit of firm i is $2\bar{p}+\delta\Pi_2^{+*}$, and the profit of firm h is $\delta\Pi_2^{-*}$.

10.8.1 Step 1

First, we focus on deviations given second-period incompatibility: this requires that the market share of the deviating firm is either close to 1 in both markets, or close to zero in both markets. Precisely, consider a deviation of firm i such that it plays price p^i in both markets, which implies that the quantity sold by firm i in each market is $d^i = \frac{1}{2}+\frac{1}{2t}(\bar{p}+t-p^i)$, and hence the profit of firm i is $f^i(d^i) = 2d^ip^i + \delta B(d^i)^2 + 2\delta\gamma d^i + \delta\Pi_2^{-*}$, with $B = \Pi_2^{+*} - 2\Pi_2^0 + \Pi_2^{-*}$ and $\gamma = \Pi_2^0 - \Pi_2^{-*}$. Using $p^i = 2t + \bar{p} - 2d^it$, we obtain $f^i(d^i) = (B\delta - 4t)\, d^2 + 2\,(2t+\bar{p}+\gamma\delta)\, d + \delta\Pi_2^{-*}$; of course, $f^i(1) = 2\bar{p}+\delta\Pi_2^{+*}$. No deviation of firm i which induces incompatibility in the second period is profitable for i if and only if $f^i(1) \geq f^i(0)$ and $df^i(1)/dd^i \geq 0$.[18] The two conditions are equivalent to $2\bar{p}+\delta\Pi_2^{+*} \geq \delta\Pi_2^{-*}$, and $2\,(B\delta-4t)+2\,(2t+\bar{p}+\gamma\delta) \geq 0$, respectively, that is

$$\alpha \equiv -\frac{\delta}{2}(\Pi_2^{+*}-\Pi_2^{-*}) \leq \bar{p} \qquad \text{and} \qquad \beta \equiv 2t - \delta(\Pi_2^{+*}-\Pi_2^0) \leq \bar{p}$$

Now consider firm h and a deviation such that firm h plays price p^h in both markets, which implies that the quantity sold by the firm in each market is $d^h = \frac{1}{2}+\frac{1}{2t}(\bar{p}-p^i)$, and hence its profit is $f^h(d^h) = 2d^hp^h + \delta B(d^h)^2 + 2\delta\gamma d^h + \delta\Pi_2^{-*}$. Using $p^h \triangleq t+\bar{p}-2d^ht$ we obtain $f_h(d^h) = (B\delta-4t)\,(d^h)^2 + 2\,(t+\bar{p}+\gamma\delta)\, d^h + \delta\Pi_2^{-*}$ (of course, $f^h(0) = \delta\Pi_2^{-*}$), and no deviation of firm h which induces incompatibility in the second period is profitable for h if and only if $f^h(0) \geq f^h(1)$ and $df^h(0)/dp^h \leq 0$, that is

$$\bar{p} \leq t - \frac{\delta}{2}(\Pi_2^{+*}-\Pi_2^{-*}) \equiv \mu \qquad \text{and} \qquad \bar{p} \leq -t - \delta(\Pi_2^0 - \Pi_2^{-*}) \equiv \sigma$$

[18] Precisely, the two conditions are necessary to rule out the profitability of a deviation such that $d^i = 0$, and the profitability of a deviation such that $d^i < 1$ but d^i is close to 1. Moreover, if $f^i(1) \geq f^i(0)$ and $df^i(1)/dd^i \geq 0$ then f^i is maximized at $d^i = 1$ both if f^i is concave and if f^i is convex.

Therefore, $\bar{p}$ needs to satisfy

$$\max\{\alpha, \beta\} \leq \bar{p} \leq \min\{\mu, \sigma\} \tag{27}$$

It is useful to notice that $\alpha \leq \mu$ holds given $t > 0$. Moreover, let $\bar{\delta}$ denote the lower bound on δ in the statement of Proposition 4. Then $\bar{\delta} > \frac{4t}{B}$, and then $\delta \geq \bar{\delta}$ implies $\beta < \alpha$ and $\mu < \sigma$. Hence (**??**) reduces to $\bar{p} \in [\alpha, \mu]$, that is any deviation which induces incompatibility is unprofitable if and only if $\bar{p} \in [\alpha, \mu]$.

10.8.2 Step 2

Now we consider deviations given second-period compatibility: this requires that in at least one market, the market share of the deviating firm is not close to 1, nor close to 0. Precisely, consider a deviation of firm h such that it plays price p^h in both markets, which implies that the quantity sold by the firm in each market is $d^h = \frac{1}{2} + \frac{1}{2t}(\bar{p} - p^i)$, and hence the variation of the profit of firm h with respect to the equilibrium profit is $g^h(d^h) = 2\left(d^h p^h + \delta(d^h \pi_2^{+*} + (1 - d^h)\pi_2^{-*})\right) - \delta\Pi_2^{-*}$. Using $p^h = t + \bar{p} - 2td^h$, we obtain

$$g^h(d^h) = -4t(d^h)^2 + 2\left(t + \bar{p} + \delta(\pi_2^{+*} - \pi_2^{-*})\right) d^h + \delta(2\pi_2^{-*} - \Pi_2^{-*})$$

If $t + \bar{p} + \delta(\pi_2^{+*} - \pi_2^{-*}) \leq 0$, then it is immediate that $g^h(d^h) < 0$ for each $d^h \in [0, 1]$. If $t + \bar{p} + \delta(\pi_2^{+*} - \pi_2^{-*}) > 0$, then g^h is maximized at $d^h = \frac{1}{4t}\left(t + \bar{p} + \delta(\pi_2^{+*} - \pi_2^{-*})\right)$ and $g^h(\frac{1}{4t}\left(t + \bar{p} + \delta(\pi_2^{+*} - \pi_2^{-*})\right)) = \frac{1}{4t}\left(t + \bar{p} + \delta(\pi_2^{+*} - \pi_2^{-*})\right)^2 + \delta(2\pi_2^{-*} - \Pi_2^{-*})$. Therefore $g^h(\frac{1}{4t}\left(t + \bar{p} + \delta(\pi_2^{+*} - \pi_2^{-*})\right)) \leq 0$ if and only if $\bar{p} \leq \bar{p}_{\max} \equiv -t - \delta(\pi_2^{+*} - \pi_2^{-*}) + \sqrt{4t(\Pi_2^{-*} - 2\pi_2^{-*})\delta}$, and the inequality $\delta \geq \bar{\delta}$ implies $\mu \leq \bar{p}_{\max}$. Therefore for each $\bar{p} \in [\alpha, \mu]$ there exists no profitable deviation for firm h, and we fix $\bar{p} = \mu$.

Regarding firm i, deviations which induce second-period compatibility imply that the variation of the profit of firm i with respect to the equilibrium profit is $g^i(d^i) = 2\left(d^i p^i + \delta(d^i \pi_2^{+*} + (1 - d^i)\pi_2^{-*})\right) - (2\mu + \delta\Pi_2^{+*})$, and using $p^i = 2t + \mu - 2td^i$ we obtain

$$g^i(d^i) = -4t(d^i)^2 + 2(3t - \frac{\delta}{2}C)d^i + 2\delta\pi_2^{-*} - 2t - \delta\Pi_2^{-*}$$

with $C = \Pi_2^{+*} - \Pi_2^{-*} - 2\pi_2^{+*} + 2\pi_2^{-*}$. Then g^i is maximized at $d^i = \frac{3t - \frac{\delta}{2}C}{4t}$ and $g^i(\frac{3t - \frac{\delta}{2}C}{4t}) = \frac{1}{4t}(3t - \frac{\delta}{2}C)^2 + \delta(2\pi_2^{-*} - \Pi_2^{-*}) - 2t$, which is decreasing with respect to δ, and negative for

$\delta = \bar{\delta}$. Hence no profitable deviation exists for firm i.

10.9 Proof of Proposition ??

The proof is straightforward and is omitted.

10.10 Proof of Proposition ??

As a deviating firm cannot charge a price below zero, the only deviation to consider consists in charging a strictly positive price, implying that the deviating firm's market share is smaller than a half. In our proofs of part (i) and part (ii), we show that such a deviation is not profitable, given δ/t larger than $\frac{4}{\Pi_2^{+*}-2\Pi_2^{0*}+\Pi_2^{-*}}$.

(i) Suppose that the deviating firm expects that compatibility would prevail in the second period. As compatibility holds for both periods, we can consider deviations only in the market for one product, say product x, without loss of generality. Then firm i's profit is given by

$$d_{1,x}^i(p_{1,x}^i + \delta\pi_2^{+*}) + (1 - d_{1,x}^i)\delta\pi_2^{-*} \tag{28}$$

where

$$d_{1,x}^i = \frac{1}{2} - \frac{p_{1,x}^i}{2t}.$$

given that $p_{1,x}^h = 0$. We now show that the first-order derivative of (**??**) with respect to $d_{1,x}^i$ is positive for any $d_{1,x}^i \in [0, 1/2]$, which implies that there is no profitable deviation. Precisely, we prove that

$$(p_{1,x}^i + \delta\pi_2^{+*}) - \delta\pi_2^{-*} - 2td_{1,x}^i > 0 \quad \text{for any } d_{1,x}^i \in [0, 1/2),$$

which is equivalent to

$$\delta(\pi_2^{+*} - \pi_2^{-*}) > 2td_{1,x}^i - p_{1,x}^i.$$

This last inequality holds since $\frac{\delta}{t} > \frac{4}{\Pi_2^{+*}-2\Pi_2^{0*}+\Pi_2^{-*}}$ implies $\delta\left(\pi_2^{+*} - \pi_2^{-*}\right) > t$ and $2td_{1,x}^i - p_{1,x}^i < t - p_{1,x}^i \leq t$.

Suppose that the firms chose compatibility in the first period and that the deviating firm expects that incompatibility would prevail in the second period. According to Lemma **??** in the proof of Proposition 3, we can restrict attention to symmetric deviations without loss of generality. Let firm i deviate by choosing $d_1^i = d_{1,x}^i = d_{1,y}^i$, and notice that – in view of Lemma 4, incompatibility arises in period two only if $2\pi_2^{+*} < \Pi_2^{+*}$ and d_1^i is small enough to violate (**??**): let $\bar{d}_1^i \in (0, \frac{1}{2})$ be such that (**??**) is violated if and only if $d_1^i \in [0, \bar{d}_1^i)$; hence, we consider d_1^i in the interval $[0, \bar{d}_1^i)$. We now illustrate why in this interval there exists no profitable deviation.

- If $2\pi_2^{-*} - \Pi_2^{-*} \geq 0$, then firm i prefers compatibility for each $d_1^i \in [0, \bar{d}_1^i)$ (i.e., (**??**) is satisfied for each $d_1^i \in [0, \bar{d}_1^i)$). Since we have proved above that, for any $d_1^i \in [0, 1/2]$, no deviation is profitable for i given the profit function (**??**) (more precisely, the profit is the double of (**??**)), it follows that no profitable deviation exists for i given incompatibility.

- If $2\pi_2^{-*} - \Pi_2^{-*} < 0$, then there exists $\tilde{d}_1^i$ in $(0, \bar{d}_1^i)$ such that firm i prefers compatibility for each $d_1^i \in (\tilde{d}_1^i, \bar{d}_1^i)$ (i.e., (**??**) is satisfied), but (weakly) prefers incompatibility for each $d_1^i \in [0, \tilde{d}_1^i]$ (i.e., (**??**) is violated). We can argue as above to conclude that no profitable deviation exists for i if $d_1^i \in (\tilde{d}_1^i, \bar{d}_1^i)$. If $d_1^i \in [0, \tilde{d}_1^i]$, then i's profit Π^i is given by (**??**) in the proof of Proposition 3, and the inequality $\frac{\delta}{t} > \frac{4}{\Pi_2^{+*} - 2\Pi_2^{0*} + \Pi_2^{-*}}$ implies that Π^i is convex in d_1^i. Hence, we only need to care about the extreme deviations: $d_1^i = 0$ leads to a profit of $\delta\Pi_2^{-*}$, which is smaller than the equilibrium profit $\delta(\pi_2^{+*} + \pi_2^{-*})$; choosing $d_1^i = \tilde{d}_1^i$ leads to a profit equal to the profit under compatibility (because (**??**) holds with equality if $d_1^i = \tilde{d}_1^i$), that is the double of (**??**) given $d_1^i = \tilde{d}_1^i$, and we already know it is not higher than the equilibrium profit.

Therefore, we conclude that no deviation is profitable for firm i whatever the compatibility regime it induces in period two. Given that each firm i plays $p_{1,x}^i = p_{1,y}^i = 0$, it follows that $d_{1,x}^i = \frac{1}{2}$, $d_{1,y}^i = \frac{1}{2}$ for each firm i, hence Corollary 4 implies that compatibility arises in period two and finally Lemma 1 applies in each market i_j.

(ii) Suppose that the firms chose incompatibility in the first period and $2\pi_2^{+*} + 2\pi_2^{-*} > \Pi_2^{+*} + \Pi_2^{-*}$. Then, with $P_1^A = 0$, $P_1^B = 0$ we have $D_1^A = \frac{1}{2}$, $D_1^B = \frac{1}{2}$ and Lemma 4 implies that compatibility emerges in the second period, with total profit of $\delta(\pi_2^{+*} + \pi_2^{-*})$ for each firm. If firm i deviates and expects that compatibility would prevail in the second period, then its profit (as $P_1^h = 0$) is given by

$$D_1^i(P_1^i + \delta 2\pi_2^{+*}) + (1 - D_1^i)\delta 2\pi_2^{-*}$$

with

$$D_1^i = \frac{1}{2}(1 - \frac{P_1^i}{2t})^2 \qquad \text{for each } P_1^i \in [0, 2t] \tag{29}$$

Hence the derivative of the profit with respect to P_1^i is

$$D_1^i + \frac{dD_1^i}{dP_1^i}(P_1^i + \delta 2\pi_2^{+*} - \delta 2\pi_2^{-*}) = \frac{1}{2}(1 - \frac{P_1^i}{2t})\left(1 - \frac{\delta}{t}(2\pi_2^{+*} - 2\pi_2^{-*}) - \frac{3P_1^i}{2t}\right)$$

which is negative for each $P_1^i \in [0, 2t)$, given that $\frac{\delta}{t} > \frac{4}{\Pi_2^{+*} - 2\Pi_2^{0*} + \Pi_2^{-*}}$ implies $1 - \frac{\delta}{t}(2\pi_2^{+*} - 2\pi_2^{-*}) < 0$. If the deviating firm expects incompatibility to prevail in the second period, then firm i is necessarily worse off than under compatibility in the second period because the analysis in Subsection 3.2, reveals that firm i prefers compatibility if $D_1^i < \bar{D}$, with $\bar{D} > \frac{1}{2}$, but $D^i \leq \frac{1}{2}$ given $P_1^h = 0$, $P_1^i > 0$. Since we have proved that no profitable deviation exists given compatibility in the second period, a fortiori no profitable deviation exists under incompatibility in the second period.

Now suppose that $2\pi_2^{+*} + 2\pi_2^{-*} \leq \Pi_2^{+*} + \Pi_2^{-*}$. Then Lemma 4 implies that incompatibility arises in period 2 for any D_1^i, and with $P_1^A = 0$, $P_1^B = 0$ we have that $D_1^A = D_1^B = \frac{1}{2}$ and profit $\delta(\Pi_2^{+*} + \Pi_2^{-*})$. Since any deviation induces incompatibility in the second period, the profit of firm i from playing $P_1^i > 0$ in period one is given by

$$D_1^i(P_1^i + \delta\Pi_2^{+*}) + (1 - D_1^i)\delta\Pi_2^{-*}.$$

with D_1^i given by (**??**) In this case we can show that no profitable deviation exists by arguing as above, given that $\frac{\delta}{t} > \frac{4}{\Pi_2^{+*} - 2\Pi_2^{0*} + \Pi_2^{-*}}$ implies $1 - \frac{\delta}{t}\left(\Pi_2^{+*} - \Pi_2^{-*}\right) < 0$. In period 2, Lemma 3 applies in markets (i, i) and (h, h).

(iii) The proof is straightforward and is omitted.

10.11 Proof of Lemma ??

The proof is omitted as it is proven in Lemma 1 and Lemma 3.

10.12 Proof of Proposition ??

The proof of consumer surplus is omitted as it is proven in Proposition 1 and Proposition 3. The welfare is simply obtained by adding consumer surplus to profits.

10.13 Proof of Proposition 9

The proof is a minor adaptation of the proof of Proposition **??**, after replacing the inequality $2\pi_2^{+*} + 2\pi_2^{-*} > \Pi_2^{+*} + \Pi_2^{-*}$ with $2\pi_2^{+*} + 2\pi_2^{-*} + \sigma t > \Pi_2^{+*} + \Pi_2^{-*}$.