Optimal Crowdfunding Design*

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Abstract

We characterize optimal reward-based crowdfunding where production is contingent on an aggregate funding threshold. Crowdfunding adapts project-implementation to demand (market-testing) and its multiple prices enhance rent-extraction via pivotality, even for large crowds, indeed arbitrarily large if tastes are correlated. Adaptation raises welfare. Rent-extraction can enhance adaptation, but sometimes distorts production and lowers welfare. Threshold commitment, central to All-Or-Nothing platforms, raises profits but can lower consumer welfare. When new buyers arrive ex-post, crowdfunding's market-test complements traditional finance and optimizes subsequent pricing. We prove that crowdfunding is a general optimal mechanism in our baseline.

Keywords: Crowdfunding, mechanism design, entrepreneurial finance, market-testing, adaptation, rent-extraction.
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‡This version extends and replaces Ellman and Hurkens (2014).

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1 Introduction

Crowdfunding is a recent and rapidly growing phenomenon with a major promise: to help bring more socially beneficial projects to fruition.[1] Crowdfunding platforms have sharply reduced the cost for entrepreneurs to pitch their projects to a wide range of potential funders before having to sink the costs of production.[2] We build a model of crowdfunding to investigate how the strategic interaction between entrepreneurs and funders determines consumer, producer and total welfare. We locate the main social advantage of crowdfunding in its ability to adapt production to the "crowd's information" about market demand. Entrepreneurs also use crowdfunding to extract consumer rent, often further improving adaptation, but possibly lowering consumer and even total welfare relative to standard finance and selling mechanisms. We identify how platform design can limit these negative effects. We also show how crowdfunding, as credible market signal, actually complements standard finance and optimizes pricing, when new buyers arrive later on.

We study the prominent case of reward-based crowdfunding where funders are compensated with the project's product. So the funders are buyers. Each buyer chooses a bid after the entrepreneur sets a funding threshold and a minimal price. Production occurs in the "success" event where the aggregate funds, the sum of bids, reaches the funding threshold. The entrepreneur then receives these funds and has to sink her production costs and deliver her product to all buyers who bid at least the minimal price. Buyers can rest assured that (a) they pay nothing in the event of funding failure and (b) they pay exactly their bids in the event of success. That is, buyers know prices (BKP) since refund property (a) implies any (above-minimum) bid is a price. Together with the simple aggregate funds rule (AFR), these reassuring properties explain why so many small funders are willing to participate in crowdfunding.[3]

Crowdfunding is attractive to entrepreneurs as a tool for adaptation (market-testing) and rent-extraction (price-discrimination), as we now explain in our baseline model of buyers with independent high and low valuations. Adaptation is simplest when high types are frequent. The entrepreneur then sets the high valuation as minimal price and her fixed cost as threshold. This threshold perfectly adapts production to actual demand: she sinks her fixed cost precisely in the demand states that are profitable. Crowdfunding effectively combines production finance with sales marketing in an ex-ante mechanism: the entrepreneur

[1] E.g., the recent U.S. JOBS act facilitated crowdfunding as a tool for creating jobs and consumer surplus.

[2] Digital search and referral tools further help match funders with projects of likely interest.

[3] In 2014, 3.3 million backers pledged 529 million dollars, generating over 22,000 successfully financed projects on the major reward-based platform Kickstarter, alone. Overall, Massolution (2015) estimate that global crowdfunding raised 16.2 billion dollars and predict that 2015 will double this figure.

offers a sales contract *before* producing any goods. Her offer, to the buyers as a "crowd," is explicitly contingent on their aggregate bids reaching her threshold. This creates an incentive-compatible market test, unavailable in standard finance and selling where the entrepreneur first finances and implements her project and only *then* posts a price offer to buyers. Crowdfunding strictly raises welfare here.

Rent-extraction enters the picture when high types are less frequent, inducing the entrepreneur to set the low valuation as minimal price. High types are then willing to bid strictly above this minimum to help reach the funding threshold. Caring more for project success, high types pay more than low types for the exact same good. This rent-extraction *via multiple bids* is important. For high fixed cost projects, the price-discrimination certainly raises welfare by augmenting the parameter range with beneficial production. But the entrepreneur might set her threshold too high since threatening not to produce in states with few high bids extracts rent at the cost of wasting some production opportunities. This can even lower welfare relative to standard finance but only when fixed costs are moderately low (and high types infrequent).

Before describing further results, we respond to two concerns. First, one might worry that free-riding among funders and equilibrium coordination or computation difficulties render the pivotality motive weak, at best. Second, aggregation over large numbers might trivialize market uncertainty and with it, the need for adaptation.

Despite the media hype, a typical crowdfunding project actually attracts bids from only about 50 active funders on Kickstarter (see Section 6). Moreover, pivotality concerns and adaptation gains prove significant for surprisingly large crowds, thanks to refund property (a) above. We now illustrate this by demonstrating the above results with 500 active funders. Later, we also prove that market uncertainty and adaptation gains remain important for arbitrarily large crowds in two natural extensions with correlated preferences.

Esther, a musician, wants to record a CD but only if demand is sufficient to cover the recording studio fixed cost of €2650. Some people are fans who value her music at €20, others are only willing to pay €5 and most simply have no interest. Concretely, 500 people have some interest: their values are *i.i.d.* draws from $\{5, 20\}$, each being a fan with probability 1/5. So aggregate demand is uncertain. Standard finance is unprofitable: if Esther paid the fixed cost to produce, she would optimally sell at €5, yielding a loss of €150. Now crowdfunding can help her out: offering the CD at €20 and making production contingent on raising at least €2650 gives her a 0.02% chance of success and no risk of losing money. This adaptation gain is some progress, but the expected profit of about 1 cent may not justify even quite small effort costs of pitching her project to funders. Fortunately, she can do much better raising her threshold above her actual cost to €2736 to also exploit crowdfunding's pivotal incentives: if she then offers the CD at €4.99, while suggesting true fans pledge €7.21, fans optimally do so to raise the chance of project success and getting the CD.[4] The project now has a 17% chance to succeed and gives Esther a net expected profit of €16.52. So this rent-extraction improves social and private welfare, but rent-extraction can lower welfare at a lower fixed cost such as €2350. Then Esther would set a minimum price of €4.99, ask fans to pay €5.55 and set a threshold of €2547. This yields a higher expected profit than selling to all at the unique price of €5 (or €4.99), but reduces the probability of success to 80% from 100% under standard finance.

For the bulk of projects with around 50 interested funders, each funder is more likely to be pivotal, so rent-extraction is more substantial, raising profits and success rates. In addition, success on small projects can be critical stepping stones in an entrepreneur's career path, amplifying their social importance and also changing the relevant short-run objective from profit-maximization towards project success or audience-maximization.[5] Rent-extraction is again vital in the 500 person example with Esther now a budding musician who aims to demonstrate her talent, in order to move on to a greater stage. By selling at €4.99, asking fans to pay €6.50 and setting her threshold at her cost of €2650, she achieves a 38.6% chance of recording the CD with no risk of financial loss.

We argue below that these examples are representative, because entrepreneurs self-select into pitching their projects on crowdfunding platforms precisely when these adaptation and extraction gains compensate for the costs of pitching projects to funders and the substantial platform charges. For instance, Kickstarter charges 5% on top of transaction fees on the funds raised on successful projects. So when Esther's fixed cost is far below €2500, making rent-extraction unprofitable as well as guaranteeing profitability, she strictly prefers to sink the cost and sell at €5, to avoid the additional 5% fee. If Esther needs credit, she could raise it on crowdfunding, but standard finance at a risk-free market rate would be cheaper.

Overall, our model shows that the "funding" in crowdfunding is not fundamental. An entrepreneur with no credit constraints uses crowdfunding purely to adapt to demand and to extract rent. Buyers do typically pay their bids in advance instead of just committing to buy, but this is a simple way to enforce payment of high bids even if not needed for funding the fixed cost. Of course, Esther would use crowdfunding purely for credit if banks were less informed and did not trust her. But crowdfunding can also complement standard finance (as in Mollick and Kuppuswamy, 2014, Table 3) by providing a credible market test: in our

[4] This strategy is part of a pure and strict Nash equilibrium and bid suggestions facilitate coordination and computation. Also this is the unique Pareto-optimal equilibrium.

[5] Section 8.1 investigates the use of crowdfunding to achieve such alternative objectives. Early crowdfunding platforms like SellaBand catered to creative entrepreneurs whose main objective was often to get their work out (project success) without getting into debt.

Section 7 extension, funds from crowdfunding buyers signal the profitability of an ex-post market of new buyers with correlated preferences.[6] This signal also helps to optimize pricing and generates interesting dynamics.

Can general mechanism design improve on crowdfunding? Building on Cornelli's (1996) analysis in Section 5, we prove that in our baseline model, crowdfunding is already optimal in the class of general mechanisms (*i.e.*, relaxing the above simplicity restrictions, BKP and AFR). This also proves that crowdfunding is optimally reward-based, not investment-based, in our setting. Extending to three or more buyer types, the general optimum *can* still be implemented in a mechanism satisfying BKP, but not with BKP *and* AFR: crowdfunding tends to be soft on intermediate buyer types to achieve general optimality. This extended setting also delivers three new results about crowdfunding: the entrepreneur may set a threshold strictly below her fixed cost (committing to produce in some loss states to reward the highest types); her minimal price may exceed the optimal posted price; and bid restrictions (chosen by the entrepreneur) may matter. The last two results reflect how crowdfunding is too simple to deal with higher dimensions optimally. Returning to the two-type setting, but removing threshold commitment, generates parallel findings as we now explain.

Can a platform raise welfare by preventing threshold commitment? The entrepreneur's cost is then the only credible threshold and we showed that profit-maximizing entrepreneurs tend to set excessive thresholds, so this could be useful. But we also showed how above-cost thresholds can raise welfare. So perhaps platforms and regulators should instead work to support threshold commitment? To answer these questions, we compare our baseline crowdfunding mechanism, known as "all-or-nothing" (AON), against the same mechanism with no threshold commitment (NTC).[7]

As NTC effectively lowers the entrepreneur's threshold to equal cost, production rises at fixed buyer bids but high type buyers bid less. At intermediate high type frequencies, the entrepreneur instead raises high type bids by substituting for her reduced threshold with a *higher* minimal price. This excludes low types and reduces consumer and total welfare. But with infrequent high types, exclusion remains too costly and we observe a surprising effect. The entrepreneur actually substitutes with a *lower* minimal price, below the low type valuation. This maintains inclusion and encourages high type bids by raising the gap between her at-cost threshold and the sum of low type bids, each equal to the minimal price. NTC then yields higher total and consumer welfare than AON. We also show that bid restrictions are neutral in AON, but help the entrepreneur extract rents in NTC.

[6]The entrepreneur must be unable to manipulate this signal via a pseudonym or confederate as discussed below. The crowdfunding threshold then also reassures funders that only viable projects will go ahead.

[7]Only the commitment to *not* produce when the threshold is not reached ever binds, except in Section 5.

To enforce AON, regulators or platforms must prevent entrepreneurs with above-cost thresholds from self-bidding, via pseudonyms or confederates, so as to access funds that fail to reach their thresholds but still exceed costs.[8] Platforms can also use their gatekeeper power, in project ranking and search tools, to influence threshold choices, and some directly oblige NTC by offering only a "flexible" or "keep-it-all" crowdfunding model where the entrepreneur can always take the funds and produce, even on failing to reach her threshold (which becomes cheap-talk). Platforms' percentage cut on funds raised motivates them to attract both sides of their market: buyers and entrepreneurs. Where AON raises even consumer surplus, platforms would certainly gain from facilitating threshold commitment, since it raises both consumer and entrepreneur participation. This may explain AON's current predominance. But NTC cannot be ruled out for parameters where it delivers higher consumer welfare; then platforms might restrict to NTC or do nothing to prevent self-bidding, if consumer participation is sufficiently elastic relative to that of entrepreneurs.

Related Literature.

The field of crowdfunding has become quite crowded, but Agrawal et al. (2014) and Belleflamme et al. (2015) offer useful surveys on empirics and theory. Our work stands out from all related models because they all restrict to a unique crowdfunding price, whereas we show that this is generally suboptimal. We thereby capture the salient feature that buyers pay multiple prices for the exact same reward, but we first relate our main results to the prior work of Belleflamme et al. (2014) who argued that a non-standard motivation of funders was necessary to understand crowdfunding. Crowdfunders enjoy community benefits proportional to their standard valuation of the good and the entrepreneur price discriminates by having high valuation buyers buy in advance at a higher price. We show that community benefits, while plausible especially for artistic and charitable projects, are certainly not crucial. Indeed, we assume consumers and entrepreneurs have purely standard self-interest motivations, except on generalizing in 8.1.

The first key contribution of our work is to demonstrate crowdfunding's role as a market-test for adapting production (and later pricing) by modeling a finite crowd; Belleflamme et al. (2014) treat a continuum of buyers so there is no aggregate uncertainty to which to adapt. The second key contribution is to model the pivotality motive and its implications for rent-extraction via price-discrimination; this again requires the finite crowd. We also analyze threshold commitment; while production is contingent on sufficient advance-purchase revenues in Belleflamme et al. (2014), the threshold is simply the fixed cost. Our paper is the only one to provide a non-trivial, rational analysis of threshold setting.

Two contemporaneous papers also model pivotality-based price-discrimination, but with

[8] *E.g.*, Kickstarter discourages self-bidding but enforcement is imperfect (see Mollick, 2014, p.6).

suboptimal designs. Like Belleflamme et al. (2014), they focus on price-discrimination with a high crowdfunding price ex-ante and a low regular price ex-post. Sahm (2015) treats a finite crowd, arguing that it must be small for price discrimination via pivotality. We show that in fact the crowd need not be so small, even for pivotality. We also show that his mechanism is suboptimal since multiple crowdfunding prices deliver better outcomes. Sahm (2015) assumes the entrepreneur has no credit at all and must set her threshold at cost, despite the ex-post revenues.[9] Kumar et al. (2015) do, like us, let the entrepreneur choose the threshold, but they work with a continuum of independent consumers. This has two major drawbacks: consumers are not in fact pivotal and again there is no aggregate demand uncertainty.[10]

More recently, Strausz (2015) considers a simplified version of our baseline model with low valuation set at zero ($v_L = 0$) so that only the exclusive option is relevant. His contribution is to model the moral hazard that entrepreneurs take the funds and run, instead of producing and delivering the rewards. Chang (2015) also analyzes this moral hazard, but in a common value environment.[11] We assume that reputational concerns and ex-post sales enforce production and delivery commitments, consistent with Mollick (2014) who estimates less than 4% delivery failure on successful Kickstarter projects.

All these papers restrict to a unique price in crowdfunding, but multiple prices matter. Without them, threshold choice is trivialized, unlike in reality and in our paper. Multiple prices often substantially increase efficiency by enhancing demand adaptation. They also introduce the risk of excessive extraction. These effects are crucial to our results comparing platforms with and without threshold commitment. Multiple bids are also necessary to minimize credit constraints and to fully capture price discrimination and price dynamics.[12]

Closely related to our paper is Cornelli (1996) who considers the problem of a monopolist selling an excludable good with fixed cost of production. Her general mechanism design anal-

[9]Sahm (2015) does consider standard finance but only as an alternative, implicitly assuming they are always substitutes; in fact, combining with crowdfunding is often optimal as we prove.

[10]In their model, all crowdfunding projects exactly reach the entrepreneur's threshold. This is inconsistent with the fact that over half the projects fail to reach the threshold. We view these failures as a sign of market-test in action, filtering out projects with too little demand, so that costs are only sunk on viable projects.

[11]Common values are important for studying investment-based crowdfunding as in Hakenes and Schlegel (2014); exploiting the "wisdom of the crowd" is a complementary benefit of crowdfunding.

[12]Prices fall over time in all the three papers that feature price discrimination: the unique pre-sale or crowdfunding price is high and the ex-post price is low. We show that prices may in fact rise or fall, depending on the crowdfunding signal; see Section 7. An earlier literature derives advance-purchase *discounts* under capacity constraints and aggregate demand uncertainty (*e.g.*, Dana, 2001; Gale and Holmes, 1993) or when consumers have ex-ante preference uncertainty (*e.g.*, Courty and Li, 2000; Möller and Watanabe, 2010; Nocke et al., 2011). In those papers, production is not contingent on the level of pre-sale revenues. In more related, recent work on inter-personal bundling, price offers to one buyer can depend on other buyers' choices, but fixed costs are sunk. So fixed costs, which are crucial to all our results, are effectively zero. Nonetheless, insights on word-of-mouth advertising suggest a complementary attraction (see *e.g.*, Chen and Zhang, 2015).

ysis treats weak implementation and a continuous type distribution. She does not consider crowdfunding; in particular, her mechanisms break the aggregate funds rule (AFR). Barbieri and Malueg (2010) investigate whether Cornelli's optimal design ever takes the form of a crowdfunding mechanism but they can only solve with two buyers ($N = 2$). Even then, the necessary conditions are highly restrictive. AFR makes the problem extremely difficult to solve for a continuum of types and no solutions exist for $N > 2$ (see also Alboth et al., 2001). Our general mechanism analysis in Section 5 makes progress using discrete type distributions; in particular, in our baseline model, the optimal crowdfunding design implements the general optimal outcome for any number of buyers.

In crowdfunding, each buyer's bid helps cover the fixed cost, benefitting other buyers, so the literature on private provision of public goods is relevant. One strand uses general mechanism design to provide implementability results, including characterizations of interim incentive efficient allocations for discrete excludable public goods in environments with independent, private valuations (*e.g.*, Cornelli, 1996; Ledyard and Palfrey, 2007). A second strand considers indirect mechanisms with simple, intuitive rules, such as voluntary contribution, satisfying AFR, and subscription games, satisfying BKP and AFR.[13] With excludability via a minimum price, subscription games are equivalent to reward-based crowdfunding. Unfortunately, as we just noted, with or without excludability, subscription games have proven extremely hard to analyze with more than two players. In contrast, we fully characterize equilibria for any number of bidders, albeit in a binary type space.

The rest of the paper is organized as follows. Section 2 introduces the baseline model, analyzed in Section 3 under full commitment. Section 4 analyzes commitment, comparing AON and NTC. Section 5 investigates the optimality of crowdfunding in a general mechanism framework and extends beyond two types. Section 6 demonstrates crowdfunding's relevance to large markets. Section 7 introduces ex-post buyers and studies credit and price dynamics. We extend to alternative, not-for-profit objectives and sequential bidding in Section 8, concluding in Section 9 with a discussion on applying the results. Proofs are collected in Appendix A.

2 Baseline model

We present a streamlined model, deferring justification and extensions. A single entrepreneur has a project for producing a good at fixed cost $C > 0$ and a constant marginal cost, nor-

[13]Theory and experiments demonstrate the benefit of such threshold mechanisms and the advantage of refund property (a) for voluntary contributions for pure public goods (see *e.g.*, Palfrey and Rosenthal, 1984; Croson and Marks, 2000), and also with excludability, (see *e.g.*, Menezes et al., 2001; Swope, 2002).

malized to zero. N buyers have unit demand for the good. Their valuations are independent draws from the 2-type distribution, with probability q on v_H and $1-q$ on $v_L < v_H$. Buyers each learn their private values during crowdfunding; learning after production suffices in standard finance (SF). The number k of the N buyers with high demand, v_H, defines the demand state and has the binomial (N, q) distribution, $f_k^N(q) = \binom{N}{k} q^k (1-q)^{N-k}$; we follow the conventions, $\binom{M}{k} = 0$ if $k < 0$ or $k > M$ and $\binom{0}{0} = 1$ and we suppress q where unconfusing. To ensure that production is sometimes profitable, we assume: $C < N v_H$. Finally, we define $\hat{q} = v_L/v_H$; this ratio plays an important and recurrent role in our analysis.

We first solve the benchmark case of standard finance, where the entrepreneur decides project finance before learning demand state k and simply posts a price to all buyers. Then we illustrate the potential for gains from an ex-ante selling mechanism, setting the stage for (optimal) crowdfunding design. We normalize to no time discounting and a risk-free interest rate of zero.

2.1 Standard finance and optimal posted-prices

In standard finance (SF), the entrepreneur and any financiers decide production before setting up their selling mechanism. If production goes ahead, the entrepreneur sinks her cost C and then designs her selling mechanism. At that point, she can do no better than use a simple posted-price p.[14] Her expected revenue is then Np for $p \le v_L$, qNp for $p \in (v_L, v_H]$ and 0 for higher p. So she chooses between the "exclusive" price $p = v_H$ that excludes L-types, extracting all H-type rent, and the "inclusive" price $p = v_L$ that includes L-types, extracting all their rent. Exclusion is optimal if $q > \hat{q}$ and inclusion is optimal if $q \le \hat{q}$. The entrepreneur indeed produces when her fixed cost C is low enough: $C < \max\{N v_L, q N v_H\}$. So in the standard finance mechanism, she earns expected profit,

$$\pi^{SF} = \begin{cases} (N v_L - C)_+ & \text{if } q \le \hat{q} \\ (q N v_H - C)_+ & \text{if } q > \hat{q} \end{cases} \tag{1}$$

where x_+ denotes $\max(x, 0)$ for any x.

2.2 Crowdfunding and other ex-ante selling mechanisms

As her fixed cost C is strictly positive, the entrepreneur may improve on standard finance and a posted-price, with a pre-production or "ex-ante" mechanism. First, having buyers evaluate the good and credibly express their demand for it in a pre-production purchase commitment

[14] Probabilistic offers and interpersonal bundling are useless given independent valuations.

allows the entrepreneur to avoid producing at a loss. She can adapt her production decision to actual, instead of only expected, demand. Second, she may use non-production threats to induce buyers to pay more than in standard finance SF. In SF's inclusive strategy, the price $p = v_L$ gives H-types a rent, so they would voluntarily pay more to raise the probability of trade. The entrepreneur can therefore extract rent by using an ex-ante mechanism to make production and sales contingent on buyers' expressed demands. In crowdfunding, this contingency takes a simple form: the entrepreneur's production decision – project implementation – responds to buyer purchase commitments or funds via a simple aggregate rule. With no threshold commitment, NTC, this arises endogenously, but in the baseline model, AON crowdfunding, the entrepreneur often extracts more by committing against producing when too few buyers agree to pay a good price.

Concretely, the entrepreneur commits to respond to bids from buyers by producing when the sum of bids (aggregate revenues from production) exceeds a funding threshold T that she chooses along with a minimum price $p > 0$. She can additionally restrict the set of feasible bids $\mathcal{B} \subseteq \mathbb{R}_+$ for buyers who choose to make a bid. So the entrepreneur's crowdfunding offer consists of the triple $(T, \mathcal{B}, p)$. Bids are only payable when production occurs and only bids above the minimum price are rewarded with a unit of the good.[15]

The timing is straightforward. First, the entrepreneur chooses her offer $(T, \mathcal{B}, p)$. Second, buyers simultaneously and independently choose their bids b from $\mathcal{B}$ or make no bid. Since $p > 0$, we denote not bidding by $b = 0$. If the sum of these bids is at least T, the entrepreneur produces, incurring a fixed cost C, and each buyer pays his bid to the entrepreneur and receives one unit of the good if his bid exceeds the minimum price p. If the bid sum fails to reach T, the project fails: there is no production and no buyer pays anything.

2.3 Equilibrium concept and outcomes

The entrepreneur sets the mechanism and then buyers move simultaneously in the bidding game. The outcome induced by a particular strategy profile depends on the realizations of the valuation of each buyer. For a profile of valuations $\mathbf{v} \in \{v_L, v_H\}^N$, the outcome specifies for each buyer i, i's probability $p_i(\mathbf{v})$ of getting the good and i's transfer $t_i(\mathbf{v})$ to the entrepreneur. As we treat buyers symmetrically, the outcome depends on i's type, L or H and the demand state, k. The outcome determines, for each state, whether the entrepreneur produces and how much she receives.

The standard mechanism design approach seeks the entrepreneur's preferred Bayesian Nash equilibrium (BNE) of the bidding game, that is her preferred Bayesian incentive com-

[15] To guarantee equilibrium existence we restrict $\mathcal{B}$ to be a closed set.

patible (BIC) outcome. We also go beyond this by investigating *full implementation* of this outcome under a mild equilibrium refinement:[16] we seek a mechanism for which *all* BNE satisfying the refinement generate this outcome. We find that in the baseline model and extensions maintaining threshold commitment, the entrepreneur's optimal BIC outcome can be uniquely implemented in Pareto-undominated BNE: one BNE Pareto dominates all the others and this BNE delivers the preferred outcome.[17]

Several crowdfunding mechanisms may yield the same profit to the entrepreneur, but the profit-maximizing BIC outcome is generically unique in our study. To cover even the non-generic parameter sets where distinct BIC outcomes deliver the same maximized entrepreneurial profit, we define the optimal crowdfunding outcome to be that which maximizes production among BIC outcomes that maximize the entrepreneur's profit. This always gives a unique optimal crowdfunding outcome, albeit not necessarily a unique crowdfunding mechanism.

3 Optimal design with full commitment

In this section, we analyze the baseline model. The entrepreneur has full commitment to any desired threshold T and bid restriction $\mathcal{B}$. As we prove below, the entrepreneur need only consider crowdfunding mechanisms that induce symmetric pure strategy equilibria with L and H type buyers bidding $b_L \leq b_H$, respectively, and $\mathcal{B} = \{b_L, b_H\} \setminus \{0\}$. The entrepreneur's expected profit is zero if $T > Nb_H$, $Nb_L - C$ if $b_H = b_L \geq T/N$ and otherwise,

$$\pi(b_L, b_H, T, p) = \sum_{k=n}^{N} f_k^N (kb_H + (N-k)b_L - C)$$

where $n = \left\lceil \frac{T - Nb_L}{b_H - b_L} \right\rceil$ is the pivotal number of H-types triggering production.[18]

The threshold T and minimum price p only affect the profit function via the pivotal number n and the incentive compatibility of bids. So we can denote the entrepreneur's profit function by $\pi_n(b_L, b_H)$ and optimize the choice of pivot n together with compatible bids b_L and b_H. Denoting the project success rate by $S_n^M = \sum_{k=n}^{M} f_k^M$, we can write (see Lemma B.1(iii)),

$$\pi_n(b_L, b_H) = S_n^N \big(Nb_L - C + \mathbb{E}[k|k \geq n](b_H - b_L)\big) \tag{2}$$

[16]See *e.g.*, Palfrey (1992) and Jackson (2001) on mechanism design, implementation and multiple equilibria.

[17]This concept is also called interim incentive efficiency in the mechanism design literature (Ledyard and Palfrey, 2007). Other refinements such as strict payoff dominance and risk dominance are equally effective.

[18]We use $\lceil x \rceil$ to denote the smallest non-negative integer larger than or equal to x.

This expected profit captures the tradeoff between project success (production probability) and the production-contingent expectation of revenue minus cost.

With $b_L > 0$, the entrepreneur maximizes profit subject to the individual rationality constraints, $p \leq b_L \leq v_L$, $b_H \leq v_H$ and incentive compatibility of H-type buyers bidding b_H instead of b_L:[19]

$$(v_H - b_H)S_{n-1}^{N-1} \geq (v_H - b_L)S_n^{N-1} \tag{IC}$$

In equilibrium, each H-type believes that if he bids b_L instead of b_H, he only gets the good when at least n of the other $N-1$ buyers are H-type, whereas if he bids b_H, only $n-1$ other buyers need be H-type. So he trades off the higher net gain $v_H - b_L$ against lower success S_n^{N-1}. Defining hazard ratio,

$$h_n = \frac{f_{n-1}^{N-1}}{S_{n-1}^{N-1}} = 1 - \frac{S_n^{N-1}}{S_{n-1}^{N-1}}$$

we can rewrite (IC) as,

$$b_H \leq h_n v_H + (1 - h_n)b_L \tag{3}$$

or as $\delta \leq h_n(v_H - b_L)$ where $\delta = b_H - b_L$ is the H-type voluntary additional bid.

With $b_L = 0$ representing no bid by L-types, the entrepreneur simply maximizes (2), subject to $b_H \leq v_H$. We first characterize the solution for this simple case of exclusive strategies, denoted script E, and then characterize inclusive strategies, denoted script I. We then compare the two optimized profits to find the optimal strategy. We conclude with a welfare analysis.

3.1 Exclusion

When the entrepreneur excludes low type buyers ($b_L = 0$), she readily extracts all H-type rent by setting $b_H = v_H$ and $p \leq b_H$. She can then dedicate T or n to adapting implementation to demand: $T = C$ ensures production cost C is sunk precisely in the profitable demand states, k with $kv_H - C \geq 0$. Equivalently, she picks $n_E = \lceil \tilde{n}_E \rceil$ where $\tilde{n}_E = C/v_H$; any $T \in ((n_E - 1)v_H, n_E v_H]$ will do, including C. This gives her optimized expected profit as,

$$\pi_{n_E}^E = \sum_{k=n_E}^{N} f_k^N (kv_H - C)$$

Given exclusion, crowdfunding perfectly reveals the aggregate high type demand and perfectly adapts the production decision to this information.

[19] The converse incentive compatibility for L-types cannot bind since the entrepreneur prefers the high bid.

3.2 Inclusion

Turning to inclusive strategies, $p \le b_L \le v_L$, for a given n, the entrepreneur maximizes profit (2) subject to the binding incentive constraint (IC), which guarantees $b_H \le v_H$. Raising b_L relaxes the incentive constraint and raises profits, so the entrepreneur sets $b_L = v_L$, efficiently extracting all rent from L-types; any b_L will do when $n = N$. As b_H raises profit, (IC) binds giving $b_H = \bar{b}_n$ where,

$$\bar{b}_n \quad = \quad h_n v_H + (1 - h_n) v_L \tag{4}$$

This inclusive n-type strategy gives profit,

$$\pi_n^I = S_n^N \big(N v_L - C + \mathbb{E}[k | k \ge n] h_n (v_H - v_L) \big) \tag{5}$$

The maximal threshold compatible with an n-type inclusive strategy is $\overline{T}_n = N v_L + n \bar{\delta}_n$ where $\bar{\delta}_n = \bar{b}_n - v_L = h_n (v_H - v_L)$. The next section shows that this maximal threshold is the unique optimum in the absence of bid restrictions. The hazard ratio h_n determines the fraction of H-type's rent from buying at v_L that can be extracted using the pivotality motive. As h_n is strictly increasing from $h_0 = 0$ to $h_N = 1$ (see Lemma B.1(vii)), $\bar{b}_n$ and $\bar{\delta}_n$ increase strictly with n, as does $\mathbb{E}[k | k \ge n]$. But the probability of project success S_n strictly falls with n, so there is a tradeoff. At $n = 0$, there is no pivotal motive. All buyers pay v_L and production takes place for sure. At the other extreme, with $n = N$, production requires all buyers to be H-type, so all H-types are fully pivotal, removing free-riding. The entrepreneur then extracts all their informational rent with $\bar{b}_N = v_H$, but production only occurs with probability q^N.

The tradeoff between higher rent extraction from high n against a lower probability of production generates a single-peaked sequence π_n^I and the following lemma characterizes the entrepreneur's optimal choice, n_I, from inclusive strategies n.

Lemma 1. *π_n^I is increasing for $n \le n_I$ and decreasing for $n \ge n_I$, where $n_I = \lceil \tilde{n}_I \rceil$ and*

$$\tilde{n}_I = \frac{C - N v_L + q(N v_H - C)}{v_H - v_L} \tag{6}$$

Since

$$1 - \frac{\tilde{n}_I}{N} = \frac{1 - q}{1 - \hat{q}} \left(1 - \frac{C}{N v_H} \right) \quad \text{and} \quad 1 - \frac{\tilde{n}_E}{N} = 1 - \frac{C}{N v_H}$$

we have $\tilde{n}_I = \tilde{n}_E$ along the line $q = \hat{q}$ and $\tilde{n}_I < \tilde{n}_E$ for $q < \hat{q}$. Notice that $\tilde{n}_I$ (and thus also n_I) is increasing in both the cost C and probability q of H-types. This is intuitive:

the entrepreneur loses less from failing to produce in the event of few high types as her production cost rises and as that event becomes less likely.

To understand the comparative statics graphically, we define for each n,

$$C_n(q) = \frac{N(v_L - qv_H) + n(v_H - v_L)}{1 - q} \tag{7}$$

On the curve $C = C_n(q)$, $n_I = \tilde{n}_I = n$ and the entrepreneur is indifferent between the inclusive strategies of types n and $n+1$. Recall from Section 2.3 that she then picks the more efficient one, type n_I. So by Lemma 1, $n_I = \arg\min_n \{C \leq C_n(q)\}$. It follows that the entrepreneur never produces at a loss:

Lemma 2. *Under inclusion, the entrepreneur never produces at a loss; the maximal optimal threshold strictly exceeds cost.*

3.3 Overall optimum

To find the entrepreneur's overall optimum strategy, we compare the profit expressions from her optimal exclusive and inclusive strategies. This reveals that exclusion is optimal if and only if $q > \hat{q} = v_L/v_H$. It is intuitive that the relative profitability of exclusion increases with q, since the excluded L-types become less common. But it is not obvious why the cut-off value of q at which exclusion becomes optimal should be exactly the same as for a posted-price. Indeed, we later show that this is not true when the entrepreneur lacks threshold commitment. The reason why it is true with commitment is that $n_E = n_I$ at $q = \hat{q}$, as we noted after Lemma 1. So production takes place for exactly the same demand realizations under optimal exclusion and inclusion. This implies an identical expected cost term under both strategies, making their relative profitabilities independent of C at $q = \hat{q}$. Now at $C = 0$ and $q = \hat{q}$, a posted price of v_L is as profitable as a posted price of v_H, so $\pi_0^E = \pi_0^I$. It follows that $\pi_{n_E}^E = \pi_{n_I}^I$ at $q = \hat{q}$ for any C, with exclusion optimal above the line $q = \hat{q}$ and inclusion optimal below it. We summarize the overall solution as Proposition 1.

Proposition 1. *The optimal crowdfunding outcome is characterized as follows:*

(a) For $q > \hat{q} = v_L/v_H$, L-types are excluded and H-types get the good, paying v_H, if and only if the number of H-types $k \geq n_E = \lceil C/v_H \rceil$.

(b) For $q \leq \hat{q} = v_L/v_H$, both L and H types get the good, paying respectively v_L and $\bar{b}_{n_I}$, where n_I is as in Lemma 1.

The sets of mechanisms with the tightest bid restrictions that uniquely implement the *respective optimal exclusive and inclusive outcomes in Pareto-undominated BNE are:*

$$\mathcal{M}_E = \{(T, \{v_H\}, p) : 0 \leq p \leq v_H \text{ and } (n_E - 1)v_H < T \leq n_E v_H\}$$
$$\mathcal{M}_I = \{(T, \{v_L, \bar{b}_{n_I}\}, p) : 0 \leq p \leq v_L \text{ and } \overline{T}_{n_I} - \bar{\delta}_{n_I} < T \leq \overline{T}_{n_I}\}.$$

The multiplicity of optimal mechanisms owes to the fact that bid restrictions are superfluous under threshold commitment in our baseline model. In the next subsection, we will derive unique optimal mechanisms when bid restrictions are not possible. But notice already that the optimal exclusion outcome can be simply implemented by setting the minimal price at the high valuation and the break-even funding threshold $T = C$, while the optimal inclusive outcome can be implemented using $p = v_L$ and $T = \overline{T}_{n_I} > C$.

The bidding game induced by any of the optimal mechanisms does have other equilibria besides the one preferred by the entrepreneur, but they are all Pareto dominated. Namely, the equilibria with buyers not bidding or always bidding v_L yield a zero payoff for any type of buyer. There also exists a mixed strategy equilibrium in which H-types sometimes bid v_L. Such a mixed strategy leads to a lower probability of project success and therefore to a strictly lower payoff for H-types. Hence, the mixed strategy equilibrium is also Pareto dominated.

The probability that an optimally chosen mechanism leads to project success is equal to $S^N_{n_E}(q)$ if $q > \hat{q}$ and equal to $S^N_{n_I}(q)$ if $q \leq \hat{q}$. The success probability decreases with C but is non-monotonic in q because a small increase in q can lead to a discrete jump up in n_I. Nevertheless, the profit of the entrepreneur is strictly increasing in q (and decreasing in C).

Proposition 2. *n_E is increasing in C and constant in q. n_I is increasing in C and q. It follows that optimal entrepeneur profit strictly decreases with C and increases with q.*

[Figure 1 about here.]

Figure 1 shows the regions in the (C, q)-space where different outcomes are optimal for $N = 5$, $v_L = 1$ and $v_H = 1.6$. Note that $\pi^I_5 = \pi^E_5$ as in both cases one only sells when all buyers have high valuation. For $\hat{q} = v_L/v_H = 0.625$ and $C \in [(i-1)v_H, iv_H]$, $\pi^I_i = \pi^E_i$ (for $i = 1, ..., 4$). For $q > \hat{q}$, it is optimal to sell only at high price v_H; for $q \leq \hat{q}$, it is optimal to include low valuation buyers. For a given $q < \hat{q}$, the maximal threshold $\overline{T}$ goes up in steps as C increases. For example, with $q = 0.5$: $\overline{T}_0 = 5$, $\overline{T}_1 = 5.04$, $\overline{T}_2 = 5.32$, $\overline{T}_3 = 5.98$, $\overline{T}_4 = 6.92$, $\overline{T}_5 = 8$. For a given fixed cost C, the minimal number of high valuation buyers goes up in steps as q increases. The corresponding maximal threshold $\overline{T}$ makes upward jumps as the optimal outcome switches from producing when there are at least n to $n+1$ H-type buyers, but decreases in q in the interior region where production occurs when there are at least n

H-type buyers. Although setting a threshold $T < C$ may be in the entrepreneur's optimal set, she never has a strict incentive to do so: she can always achieve her optimal outcome with a threshold $T \geq C$. See Figure 2 for an illustration.

[Figure 2 about here.]

3.4 Welfare gains and losses from crowdfunding

We now analyze the welfare effects of introducing crowdfunding in comparison with a posted-price, as used with standard finance.

In the case, $q > \hat{q}$, where exclusion is optimal for the entrepreneur, crowdfunding is always beneficial as we now explain. First, note that while exclusive strategies are always inefficient in excluding L-types, standard finance is equally exclusive because $p^{post} = v_H$ on this region. To see how crowdfunding strictly raises welfare by adapting production to demand, recall that under a posted-price, production occurs if and only if expected profits are positive, that is, when $qNv_H - C \geq 0$. Figure 3 illustrates. Standard finance never results in production on the orange triangular region where $C/q > Nv_H$ and always involves production in the other orange region, even though ex-post demand from H-types sometimes does and sometimes does not cover the fixed cost. In contrast, crowdfunding generates positive welfare (equal to profits) on the former region by producing in the high demand states, $n \geq n_E$. Similarly, by avoiding production in the low demand states, $n < n_E$, it generates welfare and profit gains for parameters in the latter region; recall that $n_E \geq 1$ (the vertical dashed white lines distinguish $n_E = 1, 2, 3, 4, 5$). So, on both regions, posted-prices involve strict ex-post production inefficiencies that crowdfunding completely avoids by adapting to demand.

Crowdfunding is clearly not welfare-maximizing if production occurs with (excluded) L-types present, as with $n_E < N$. In addition, n_E tends to exceed the welfare optimal minimum which we denote by n^*: in the welfare-maximizing allocation, all buyers consume if production occurs (inclusion) and production is optimal in all states k with $kv_H + (N - k)v_L - C \geq 0$, that is, $k \geq \tilde{n}^* = (C - Nv_L)/(v_H - v_L)$. So, whenever $n^* = \lceil \tilde{n}^* \rceil < N$, inclusion of L-types is necessary and exclusion is suboptimal. However, taking exclusion as given, crowdfunding *does* maximize total welfare.

We now turn to the case of $q \leq \hat{q}$. Here the entrepreneur adopts an inclusive strategy, both under standard finance (posted prices) and crowdfunding. For $C \leq Nv_L$, standard finance leads to production for sure and this maximizes both consumer and total welfare because $n^* = 0$ on this low cost range. Given this low cost, crowdfunding can only do harm: to extract rent, the entrepreneur may raise the threshold T above C, thereby restricting production to states $k \geq n_I > 0$. On the convex green region with low C and q, crowdfunding would involve $n_I =$

0, so the impact is neutral; in any case, here the entrepreneur would be content with posted-prices rather than crowdfunding. But on the adjacent purple region, crowdfunding lowers welfare by restricting production. For higher costs, $C > Nv_L$, depicted by the blue rectangle, crowdfunding is strictly advantageous because standard funding implies no production. The H-type buyers are strictly better off, as is the entrepreneur. Total and consumer surplus may not be maximized, because the inequality $n_I \geq n^*$ may be strict.[20]

[Figure 3 about here.]

4 Relaxing threshold commitment and bid restrictions

In the analysis so far, we assumed that the entrepreneur could commit to any production threshold T and could restrict buyers' bids. In this section, we investigate how our results change on weakening these commitment powers. Fixing bidding behavior, removing threshold commitment increases welfare because adaptation improves, but weakening either commitment power shifts buyers' bidding incentives. Removing commitment powers obviously cannot help the entrepreneur, but we identify when profits are strictly reduced and how consumer and total welfare change. We find that, in our model, threshold commitment is more important than bid restrictions.

Threshold commitment is nontrivial for two reasons. First, the entrepreneur might use a pseudonym or a confederate to bid up the funds on her own project as a way to ensure that she can implement the project and use the sum of funds from legitimate crowdfunding when this sum exceeds her cost. If the platform cannot prevent the entrepreneur from doing this, the effective threshold triggering production is equal to cost and the stated threshold is mere cheap talk. Second, the entrepreneur may try to use the crowdfunding information to implement the project independently.[21]

Bid restrictions, standard in general mechanism design, seem relatively easy to impose. Indeed, crowdfunding platforms often permit entrepreneurs to offer multiple purchase options (even for a fixed reward) at different prices. But it could be difficult to prevent buyers from bidding for additional units using pseudonyms. Even with unit demand, buyers might buy extra units to increase the probability of production (as in Romano, 1991). Recall that in our analysis of the inclusion option with two permitted prices, the H-type buyers pay the high price exactly for this reason. The question is whether allowing more than two options,

[20] It is straightforward to verify that $1 - \tilde{n}_I/N = (1-q)(1-\tilde{n}^*/N)$, which implies $\tilde{n}_I > \tilde{n}^*$.

[21] The entrepreneur might implement the project on her own or using standard finance if her crowdfunding proposal fails to attract enough funds, but this is less problematic if the entrepreneur then forfeits the right to sell to all the participating funders.

specifically allowing bids to be unrestricted, would upset the equilibrium analyzed under bid restrictions.[22] After setting the minimal price equal to the low bid, shaving down from the high bid is the main concern, as opposed to shaving up from the low bid.

Of course, the entrepreneur can still use the minimal price p. Does she perhaps raise p, making consumers worse off? Then strategies and policies supporting the bid restriction option would certainly raise social surplus. Or do bid restrictions make consumers worse off? Some platforms, like Kickstarter, do always allow the buyer to voluntarily pay more, which implies bids are unrestricted above the minimum price. Imposing unrestricted bids in this way would make sense if bid restrictions hurt the platform's long-run interests by harming consumers. Another reason could be enforcement costs or simplicity and possibly unawareness of the benefits. Our analysis helps to identify which motives are plausible.

Before we analyze in detail how the lack of threshold commitment and bid restrictions may change the outcome, we first identify the cases where these powers are in fact irrelevant.

The case $n_I = N$ arises when $C > C_{N-1}(q)$ and $q < \hat{q}$. Though not excluded by the minimal price, no L-type buyer ever consumes because production requires all N buyers to be H-type. So it is equivalent for the entrepreneur to directly exclude L-type buyers with $p = v_H$ and to set $T = C$, which here implies $n_E = N$. In sum, the inclusive outcome with $n_I = N$ is equivalent to the exclusive one with $n_E = N$.

In the case of $q \geq \hat{q}$ or $q < \hat{q}$ but $n_I = 0$, the optimal solution with full commitment has a singleton bid restriction, $\mathcal{B} = \{b\}$ with $b = v_H$ and $b = v_L$, respectively. Meanwhile, as noted above when $n_I = N$, the optimal solution was equivalent to exclusion with $n = N$ (since production never arises with any L-type). Now a singleton bid restriction $\mathcal{B} = \{b\}$ can be perfectly substituted for by a minimal price $p = b$ because this dissuades underbidding and higher bids were never a problem. Threshold commitment is also irrelevant in these three cases, because $T = C$ was already an optimal solution under exclusion and under inclusion with $n_I = 0$.

Next, we need to consider the cases where under full commitment inclusion is optimal with $0 < n_I < N$. So the rest of this section treats $q < \hat{q}$ and the intermediate cost range $(C_0(q), C_{N-1}(q)]$. We first show in Section 4.1 that bid restrictions are actually irrelevant so long as the entrepreneur has threshold commitment. However, when the entrepreneur lacks threshold commitment it proves critical to distinguish the cases of unrestricted and restricted bids. We do so in Sections 4.2 and 4.3, respectively.

[22] If bid restriction is only partially compromised, as by pseudonyms permitting all positive integer combinations of legitimate bids, reducing bids from v_H without reaching v_L is still possible for $2v_L < v_H$.

4.1 Threshold commitment, no bid restrictions

Recall that under full commitment, profit-maximizing mechanisms required bids in $\mathcal{B} = \{v_L, \bar{b}_{n_I}\}$, a minimum price not exceeding v_L and a threshold in the range $\left(\overline{T}_{n_I} - \bar{\delta}_{n_I}, \overline{T}_{n_I}\right]$. Without bid restrictions, it is necessary to use precisely the maximal threshold $\overline{T}_{n_I}$ and the maximum minimal price, v_L. It is then still an equilibrium for high type buyers to bid $\bar{b}_{n_I}$ and for low type buyers to bid v_L. Clearly, L-type buyers have no profitable deviation and H-types bid at least v_L. As before, H-types are indifferent between bidding v_L and $\bar{b}_{n_I}$. Now bidding something in between v_L and $\bar{b}_{n_I}$ is strictly worse than bidding v_L. Proposition 3 proves that bidding above $\bar{b}_{n_I}$ can also be ruled out.

Moreover, the optimal outcome is still uniquely implemented by pure strategy Pareto undominated equilibrium. The only candidates for alternative Pareto undominated equilibria are where $n \neq n_I$ H-types are needed who all bid $b'_n = (\overline{T}_{n_I} - (N-n)v_L)/n$. It is readily verified that when $n < n_I$ this candidate is not an equilibrium because it does not satisfy H's IC. When $n > n_I$, the candidate is an equilibrium but it is worse for the entrepreneur and for H-types, and thus Pareto dominated. Namely, a H-type buyer expects to obtain $(v_H - b'_n)S^{N-1}_{n-1} < (v_H - v_L)S^{N-1}_{n-1}$ (as $b'_n > v_L$) while in the optimal equilibrium he obtains $(v_H - \bar{b}_{n_I})S^{N-1}_{n_I-1} = (v_H - v_L)S^{N-1}_{n_I} \geq (v_H - v_L)S^{N-1}_{n-1}$.

Proposition 3. *With threshold commitment, bid restrictions are superfluous.*

We used bid restrictions, which are standard in mechanism design, for expositional clarity; this result proves that the specific choices of maximal threshold ($T = \overline{T}_{n_I}$) and maximal minimum price ($p = v_L$) makes bid restrictions unnecessary for achieving the optimal profit. In practice, bid restrictions or suggested prices may help buyers coordinate.

4.2 No threshold commitment, no bid restrictions

We now consider NTC (no threshold commitment), defined as the case where the entrepreneur can neither restrict bids nor make production commitments; in particular, she cannot commit against producing when the total sum of bids covers the fixed cost C and will produce precisely when this happens.[23]

[23] This is distinct from having no threshold, or more precisely, a threshold forced to equal zero. There, the entrepreneur always keeps the funds even when they do not reach the stated goal. Indiegogo is a prominent example offers this format known as "keep-it-all" or flexible funding. This format obliges a credit-constrained entrepreneur to break the production commitment in our fixed-cost setting, but it can make sense when projects are scalable, in that a lower quality good could be produced at a lower cost; see Mollick (2014) who emphasizes charitable projects and other not-for-profits, and Cumming et al. (2014)for a comparison of "keep-it-all" and AON models in the context of scalable projects and risk averse funders and entrepreneurs.

The entrepreneur is strictly worse off. The optimal payoff under commitment requires H-types to bid $\bar{b}_{n_I}$ (and L-types to bid v_L) but this is no longer an equilibrium without commitment. An H-type can shave his bid down to $\bar{b}_{n_I} - \varepsilon$, knowing that the entrepreneur still produces with n_I or more H-types because $n_I \bar{b}_{n_I} - \varepsilon + (N - n_I)v_L = \overline{T}_{n_I} - \varepsilon > C$.

Non-commitment therefore strictly reduces profits. To quantify by how much, we impose the *local* incentive compatibility of not shaving down from b_H which requires equality in the n-pivotality condition with $T = C$ (from lack of threshold commitment): $nb_H + (N-n)b_L = C$. Surprisingly, it is no longer optimal to extract maximal rents from L-types by using $p = v_L$ to force $b_L = v_L$. Lowering p to $p' < v_L$, with L-types willingly bidding $b_L = p'$, raises the gap, $T - Nb_L = C - Nb_L$, that induces H-type's to raise their bids. Lowering p substitutes for the inability to raise T above C. So, denoting the corresponding high bid b_H and its excess over p', by b'_n and δ'_n, *local* incentive compatibility and n-pivotality require $nb'_n + (N-n)p' = C$, or equivalently,

$$\delta'_n = (C - Np')/n \tag{8}$$

which falls with p'. From (2), the entrepreneur's profit for $n \geq 1$ is now,

$$\pi'_n \quad = \quad S_n^N \left(\mathbb{E}[k|k \geq n] - n\right) \delta'_n$$

For a given n, the entrepreneur chooses p' to maximize δ'_n, under the individual rationality constraint of the L-type, $p' \leq v_L$, and incentive compatibility of H-types not deviating to bid p',

$$\delta'_n \leq h_n(v_H - p') \tag{IC$'$}$$

In this case, solving (8) and the binding (IC$'$) gives the optimal p':

$$p'_n = \frac{C - nh_n v_H}{N - nh_n} \tag{9}$$

which is readily seen to decrease with n. This n-type strategy is feasible only if $p'_n \leq v_L$ which in turn is equivalent to $C \leq Nv_L + nh_n(v_H - v_L) = \overline{T}_n$. Of course, the entrepreneur may prefer to switch to the inclusive strategy $n = 0$ with $b_H = p'_0 = v_L$, yielding profit $\pi'_0 = Nv_L - C$. The entrepreneur's optimal n from all these feasible inclusive strategies, denoted n'_I, trades off higher rent extraction against a lower probability of production. The entrepreneur's overall optimal strategy then compares the payoff from this optimal inclusive strategy against the optimized exclusive payoff.

Proposition 4. *Suppose that under full commitment, maximal profit equals $\pi^I_{n_I}$ with $0 <$*

$n_I < N$. With unrestricted bids and no threshold commitment, the entrepreneur compares the optimal profit from exclusion ($\pi^E_{n_E}$) with that from inclusion ($\pi'_{n'_I}$). Compared to full commitment (threshold plus bid restrictions), profit is strictly lower and:
(a) if inclusion remains optimal, consumer surplus is strictly higher and total surplus and the success rate also rise – $n'_I \leq n_I \leq n_E$ – strictly if $n'_I \leq n_I$;
(b) if exclusion becomes optimal, consumer surplus is strictly lower and total surplus and the success rate also fall – strictly if $n_I < n_E$.

The subtle and technical part of the proof of this proposition lies in showing that the entrepreneur's optimal inclusion strategy has a lower minimal number of H-types: $n'_I \leq n_I$. The simplest intuition behind this result is that, unable to commit to a threshold above her fixed cost, the entrepreneur's new constraint implied by (8) encourages her to lower n. Taking into account her optimal choice of the minimal price p complicates the argument, but as p'_n is decreasing, her incentive to reduce n does turn out to dominate.[24]

The reduced minimal number of H-types reveals that, in this case, the lack of threshold commitment is good for total welfare, at least weakly. Since the entrepreneur cannot gain from lost commitment power and in fact strictly loses, consumer surplus is strictly higher. Indeed, even L-types now get a strict positive surplus because they pay less than v_L. So total and consumer welfare rise provided that inclusion remains optimal, which holds for low values of q and C.[25] If instead, q or C is relatively high, exclusion becomes optimal and consumer surplus falls to zero.

The results are illustrated in Figure 4. The dotted black curves represent the loci $C = \overline{T}_1$ through $C = \overline{T}_4$. These indicate which inclusive strategies are feasible without threshold commitment. Note that type $n = 4$ is often feasible, but never optimal because it is dominated by exclusion. Clearly, in the orange subregion below $q = \hat{q}$ where exclusion becomes optimal, consumer surplus falls to zero and profits also fall since $\pi^I_{n_I}$ was strictly preferred here. In the other regions below $q = \hat{q}$, consumer surplus is strictly higher because H-types pay strictly less, L-types pay weakly less, and the probability of production is weakly higher.[26]

[24]A mild further complication is that there now exist multiple Pareto undominated equilibria with $p < v_L$. H-types prefer the equilibrium where L-types bid $b_L = v_L$ (and n is minimized at a value weakly lower than n'_I) whereas the entrepreneur and the L-types prefer the equilibrium with n'_I in which L-types bid p. Standard mechanism design assumes that the entrepreneur selects her preferred equilibrium, but note that even if the H-type's preferred equilibrium is played, our key results are robust: $n \leq n_I$ and NTC raises welfare provided the entrepreneur does not switch to exclusion.

[25]Of course, inclusion never arises when exclusion was optimal with commitment because threshold commitment has no impact on the optimal exclusion strategy.

[26]In fact, generically low type buyers pay strictly less when $n'_I \geq 1$: L-types pay v_L only along the dotted curve, $C = \overline{T}_{n'_I}$. In the green region marked π^I_0, all buyers pay v_L so NTC does not benefit L-typed strictly.

[Figure 4 about here.]

4.3 No threshold commitment, restricted bids

Finally, we consider the case where the entrepreneur cannot commit to not produce when the pledged sum strictly exceeds the cost C, but is able to restrict the feasible bids. Allowing for bid restrictions the entrepreneur need not worry about the local incentive constraint because buyers cannot shave their bids downwards. The entrepreneur may therefore do better than under no commitment at all. In fact, in some cases the entrepreneur can obtain the outcome optimal under full commitment, even with no threshold commitment. When restricting bids to $\mathcal{B} = \{v_L, \bar{b}_n\}$, it is still an equilibrium strategy for a H-type buyer to bid $\bar{b}_n$ if and only if C is a threshold in some mechanism implementing this outcome, that is, when

$$\overline{T}_n \geq C > \overline{T}_n - \bar{\delta}_n \tag{10}$$

In particular, when (10) holds for $n = n_I$, the entrepreneur can obtain the same maximal payoff as under full commitment. In other cases, this is not possible because a H-type then has an incentive to bid v_L since such a unilateral deviation does not affect the probability that the sum of bids is below cost. The entrepreneur then either chooses to exclude L-types, or to use one of two alternative inclusive strategies.

The first alternative inclusive strategy is to implement the payoff π_n^I for some $n \neq n_I$ where n satisfies (10). This implies $n < n_I$.[27] Clearly, in this case consumer and total surplus are higher than under full commitment, because success rate is higher and H-types pay less.

The second alternative inclusive strategy is similar to the one considered in Section 4.2. In an n-type equilibrium, L-types will pay some $p'' \leq v_L$ and H-types will pay some b''_n, but, because of the bid restrictions, H-types cannot shave down their bids and no local incentive compatibility constraint need be imposed. Instead, n-pivotality means, as always,

$$(n-1)b''_n + (N-n+1)p'' < T \leq nb''_n + (N-n)p''$$

but lack of threshold commitment implies that production must occur when the pledged sum strictly exceeds C, so that

$$(n-1)b''_n + (N-n+1)p'' \leq C \tag{11}$$

These conditions are compatible by setting T slightly above C such that no N bids from

[27] For example, it will be optimal to implement payoff π_n^I when $n = n_I - 1$ satisfies (10) and where $\pi_{n_I}^I$ is only slightly higher than π_n^I.

$\{0, p'', b''_n\}$ combine to an amount in (C, T). For given n, the entrepreneur chooses p'' so as to maximize $\delta''_n = b''_n - p''$. Both (11) and the global incentive constraint $\delta''_n \leq h_n(v_H - p'')$ must be binding, so that the optimal minimum price is given by

$$p''_n = \frac{C - (n-1)h_n v_H}{N - (n-1)h_n}$$

Feasibility of this n-type strategy requires $p''_n \leq v_L$ which is equivalent to $C \leq \overline{T}_n - \bar{\delta}_n$. Note that $p''_n > p'_n$. This implies that the optimal inclusive strategy will now have $n''_I \geq n'_I$ so that total welfare can be reduced by allowing for bid restrictions when there is no threshold commitment. On the other hand, because profits are higher when using bid restrictions, there will be less exclusion in this case, so bid restrictions may also improve total welfare. In comparison with the full commitment case, we have $n_I \geq n''_I$ so that lack of threshold commitment is again beneficial for total welfare as long as the entrepreneur does not exclude L-types. Both types of buyers gain when $p''_n < v_L$ because H-types pay $b''_n = h_n v_H + (1 - h_n)p''_n < \bar{b}_n$.

In summary, bid restrictions are sometimes a powerful – and in some cases even perfect – substitute for threshold commitment. They help the entrepreneur extract more consumer surplus. This also generates additional total surplus when the project succeeds more often. Of course, there are regions in the parameter space where the entrepreneur still opts for exclusion. In this case consumer and producer surplus are strictly reduced by the lack of threshold commitment, even when bid restrictions are feasible. However, these regions are smaller than those when bid restrictions are not feasible.

Figure 5 illustrates these findings. Consider for example the region where $n_I = 3$, that is between the curves C_2 and C_3. For relatively high cost (above the dotted line indicating $C = \overline{T}_3 - \bar{\delta}_3$) the entrepreneur can use the bid restrictions to obtain her optimal payoff, despite the lack of threshold commitment. Below this line she cannot and must switch to an alternative strategy. For relatively low cost (close to the curve C_2) she switches to π^I_2 or π''_2 (depending on which one is feasible) or to π''_3 (if that is more profitable), increasing consumer surplus. For high q she switches to exclusion, reducing consumer and total welfare.

[Figure 5 about here.]

5 Multiple types and general mechanisms

By restricting the baseline model to two types of buyer, we were able to fully characterize the optimal crowdfunding mechanism, both with and without threshold commitment and bid restrictions. We now: (1) generalize to settings with more than two types ($J > 2$) and (2) solve for the optimal general mechanism. The general mechanism removes crowdfunding's twin restrictions, that buyers know their prices (BKP) and aggregate funds determine production. We first characterize this general optimum for a general number of types J. With this benchmark, we verify that crowdfunding is in fact optimal within the general class of mechanisms in our baseline setting, that is, for $J = 2$. We state necessary and sufficient conditions for crowdfunding to achieve the general optimum. Example 1 illustrates how crowdfunding profits can be strictly lower. We also show: crowdfunding may set a below-cost threshold, requiring a positive production commitment and entrepreneur liquidity or alternative finance (example 2); bid restrictions can matter even with threshold commitment (example 3); crowdfunding may set a higher price than the general optimum and standard selling (example 4).

5.1 Notation

Each buyer's valuation is now an independent draw from $\mathbf{v} = (v_1, ..., v_J)$ with probabilities $\mathbf{q} = (q_1, ..., q_J)$ where $\sum_{j=1}^{J} q_j = 1$, each $q_j \in (0, 1)$, and $0 \leq v_1 < \ldots < v_J$; bold letters denote $1 \times J$ vectors. The demand state is now summarized by $\mathbf{k}$ where k_j is the number of buyers with valuation v_j for each $j = 1, ..., J$. We also define: cumulative probabilities, $Q_j = \sum_{j' \leq j} q_{j'}$; the j'th unit vector, $\mathbf{e_j}$; for $M = N-1, N$, $\Omega_M = \{\mathbf{k} \in \mathbb{N}^J : \mathbf{k} \cdot \mathbf{1} = M\}$ where $\mathbf{1} = (1, ..., 1)$.[28] The probability of a state $\mathbf{k} \in \Omega_M$ is given by:

$$f_{\mathbf{k}}^{M}(\mathbf{q}) = \binom{M}{\mathbf{k}} \prod_{j}^{J} q_j^{k_j} \tag{12}$$

where $\binom{M}{\mathbf{k}} = {}^{M!}/_{k_1! \ldots k_J!}$, $\binom{0}{0,\ldots,0} = 1$ and $\binom{M}{\mathbf{k}} = 0$ if any $k_j < 0$ or $> M$. We suppress $\mathbf{q}$ where not confusing and for non-trivial production, we assume: $C < N v_J$.

5.2 Optimal general mechanism

Our analysis uses virtual valuations, following techniques developed by Myerson (1981) and applied by Cornelli (1996) to the case of a seller with a fixed cost. As our type space is discrete instead of a continuum, type j's virtual valuation is defined by,[29]

$$w_j = v_j - (v_{j+1} - v_j)\frac{1 - Q_j}{q_j}.$$

[28] States in Ω_N represent realized demands; states in Ω_{N-1} are relevant for a buyer estimating how other buyers behave. Both sets lie in $\mathbb{N}^J$ and $\forall j$, $\mathbf{k} \in \Omega_{N-1} \implies \mathbf{k} + \mathbf{e_j} \in \Omega_N$.

[29] See Bergemann and Pesendorfer (2007) for details.

We assume strict monotonicity: $\mathbf{w} = (w_1, ..., w_J)$ has $w_1 < w_2 < ... < w_J$.[30] Noting that $w_J = v_J > 0$, we define $j^* = \min\{j : w_j \geq 0\}$ and $\mathbf{w}_+$ replaces negative values in $\mathbf{w}$ by zero.

In the general optimal solution, types $j < j^*$ are excluded and pay nothing, higher types get the good in states $\mathbf{k}$ in the production set, $K^* = \{\mathbf{k} \in \Omega_N : \mathbf{w}_+ \cdot \mathbf{k} \geq C\}$. Maximal expected transfers T_j from types $j \geq j^*$ then follow recursively from incentive compatibility and the equilibrium probabilities P_j with which a type j buyeranticipates getting the good. $P_j = 0$ for all $j < j^*$. For $j \geq j^*$, defining $K^*_{-j} = \{\mathbf{k} \in \Omega_{N-1} : \mathbf{k} + \mathbf{e_j} \in K^*\}$, the set of other buyer demands for which production occurs if j plays the equilibrium,

$$P_j = \sum_{\mathbf{k} \in K^*_{-j}} f^{N-1}_{\mathbf{k}}(\mathbf{q}).$$

Without loss of generality, P_j is strictly increasing on $j \geq j^*$.[31] For all $j < j^*$, individual rationality implies $T_j = 0$ as $P_j = 0$. For $j \geq j^*$, T_j is given recursively by: $T_{j^*} = v_{j^*} P_{j^*}$ (individual rationality) and $T_{j+1} = T_j + (P_{j+1} - P_j) v_{j+1}$ (incentive compatibility that $j+1$ is just willing to not masquerade as type j).

These expected payoffs can always be implemented with buyers stating bids they pay when production occurs. So, imposing the crowdfunding restriction BKP alone does not preclude the general optimal outcome. An optimal indirect mechanism satisfying BKP has buyers choose bids from the set $\{0, b^*_{j^*}, b^*_{j^*+1}, \ldots, b^*_J\}$ where $\mathbf{b}^*$ is the unique bid strategy $\mathbf{b}$: $b_j = 0$ for $j < j^*$ as $P_j = 0$; $b_{j^*} = v_{j^*}$; $b_j = T_j/P_j$ for $j > j^*$. Defining $H_j = 1 - {}^{P_{j-1}}/_{P_j}$ on $j > j^*$, $b_j = b_{j-1}(1 - H_j) + H_j v_j > b_{j-1}$ as P_j's monotonicity implies $H_j > 0$; higher types pay strictly higher prices. The profile of N bids exactly reveals the demand state $\mathbf{k} \in \Omega_N$ and the entrepreneur must produce if and only if $\mathbf{k} \in K^*$, then providing the good to all buyers with a strictly positive bid. But in crowdfunding, some threshold on aggregate funds must determine production. This potentially loses relevant demand information. The next subsection investigates whether crowdfunding precludes implementing K^*.

5.3 Optimal crowdfunding

In addition to BKP, crowdfunding requires the production set to satisfy a threshold rule on aggregate funds. So it is possible to implement the general optimum if and only if $\{\mathbf{k} \in \Omega_N : \mathbf{b}^* \cdot \mathbf{k} \geq T\} = K^*$ for some T; any price in $(0, b_{j^*})$ then allocates goods optimally, given production. If any such T exists, $T = \min_{\mathbf{k} \in K^*}\{\mathbf{b} \cdot \mathbf{k}\}$ is one such threshold. Together with $\mathcal{B} = \{b_{j^*}, \ldots, b_J\}$ and $p = b_{j^*}$, this defines a crowdfunding mechanism that achieves

[30] Ironing techniques readily deliver similar results for the general case.

[31] Virtual valuations are increasing, so $\mathbf{k}+\mathbf{e_j} \in K^* \Rightarrow \mathbf{k}+\mathbf{e_{j+1}} \in K^*$ so $P_{j+1} \geq P_j\ \forall j \geq j^*$ and if $P_{j+1} = P_j$, these types' identical pivotalities imply that combining all $j+1$'s into j's gives identical outcomes.

the general optimum. As we prove in the Appendix, this is always possible in our baseline setting of $J = 2$:[32]

Proposition 5. *Crowdfunding can achieve the general optimum when $J = 2$.*

This result proves that two of Cornelli's (1996) claims do not apply to the two-type case. Namely, Cornelli (1996) states that the production rule must depend on the composition of individual bids, not just the bid aggregate, and she states that the optimum must sometimes commit to ex-post losses in certain states (the optimality of $T \geq C$ in Section 3.3 precludes losses in optimal crowdfunding and hence in the general optimum for $J = 2$). However, her claim does apply to our discrete type scenario on just raising J to 3.

Example 1: Crowdfunding cannot always achieve the general optimum

Let $J = 3$, $N = 2$, $\mathbf{v} = (0, 1, 2)$ and $\mathbf{q} = (1/4, 1/2, 1/4)$, with $1 < C < 2$. Then $\mathbf{w} = (-3, 1/2, 2)$ so the general optimum excludes type 1 and has production in states in $K^* = \{(0, 0, 2), (1, 0, 1), (0, 1, 1)\}$. Type-wise bids are $\mathbf{b} = (0, 1, 7/4)$ and the expected profit is $(18-7C)/16$. Crowdfunding cannot implement this outcome because threshold $T = \min_{\mathbf{k}\in K^*}\{\mathbf{b}\cdot\mathbf{k}\} = 7/4$ just reached in state $\mathbf{k} = (1, 0, 1)$ also generates production success in state $(0, 2, 0)$. Figure 6 illustrates this.

[Figure 6 about here.]

Notice that the general optimum requires the entrepreneur to produce in state $(1, 0, 1)$, despite making a loss if $C > 7/4$ as she then earns only $7/4$ from the single type 3 buyer. The positive commitment to produce in this state allows her to extract a high rent in other states in the production set K^*. This feature of incurring a loss in some states can also occur with crowdfunding now that $J > 2$, as we show next.

Example 2: Crowdfunding may involve losses

Let $J = 3$, $N = 2$, $\mathbf{v} = (0, 7, 10)$ and $\mathbf{q} = (2/5, 2/5, 1/5)$, with $9.4 < C < 10$. Then $\mathbf{w} = (-10.5, 5.5, 10)$ so the general optimum excludes type 1's and has production set $K^* = \{(0, 0, 2), (1, 0, 1), (0, 1, 1), (0, 2, 0)\}$. Type-wise bids are $(0, 7, 9.4)$. The crowdfunding mechanism with $p = 7$, $T = 9.4$ and $\mathcal{B} = \{7, 9.4\}$ is optimal since it can implement this. Notice that it has the entrepreneur produce at a loss, $b_3 - C < 0$, in state $(1, 0, 1)$.

[32] We provide a constructive proof in the Appendix, but the intuition for why crowdfunding rules are not restrictive when $J = 2$ is that aggregating bids then loses no relevant information: Ω_N is then 1-dimensional, so the production set K^* is too, and can be represented by a threshold rule on aggregate funds $\mathbf{b}^* \cdot \mathbf{k}$, as both are monotone increasing in k_2 (general optimal exclusion is also monotone decreasing in j).

Example 3: Crowdfunding may be more exclusive

Let $J = 3$, $N = 2$, $\mathbf{v} = (0, 1, 2)$ and $\mathbf{q} = (^{19}/_{64}, ^{26}/_{64}, ^{19}/_{64})$, with $C = 15/16$. Then $\mathbf{w} = (-^{45}/_{19}, ^{7}/_{26}, 2)$. K^* is as in example 1 and again crowdfunding cannot achieve the general optimum. To better extract rent from type 3 buyers under the crowdfunding restrictions, the entrepreneur optimally excludes type 2's, setting $T = p = v_3$ and earning profit $q_3^2(2v_3 - C) + 2q_3(1 - q_3)(v_3 - C)$. Under standard posted-prices, the entrepreneur would sell at v_2 since $2(q_2 + q_3)v_2 - C > 2q_3v_3 - C$. Indeed, a posted-price of v_3 is unprofitable, unlike crowdfunding with minimal price v_3 which avoids producing when there are no type 3 buyers.

Example 4: Bid restrictions may matter for crowdfunding when $J > 2$

Let $J = 3$, $N = 3$, $\mathbf{v} = (2, 3, 14)$ and $\mathbf{q} = (0.5, 0.49, 0.01)$, with $C = 6$. Then $\mathbf{w} = (1, 136/49, 14)$, so the general optimum sells to all types when production occurs, namely in all states except $(3, 0, 0)$ and $(2, 1, 0)$. Type-wise bids are $\mathbf{b} = (2, 199/75, 549/100)$ and the expected profit is $33/40$. Crowdfunding with $p = 2, \mathcal{B} = \{b_1, b_2, b_3\}$ and threshold $T = b_1 + 2b_2$ implements this outcome. Bid restrictions are necessary as type 3 buyers would prefer to bid just below $b_3 = 5.49$.[33] Note that even with bids restricted to $\mathcal{B}$, a type 3 buyer might try to set up two bidding accounts to buy two units of the good at price $b_1 = 2$. As the other two buyers pay at least 2 each, this would guarantee reaching the threshold at lower cost $2b_1 < b_3$. Crowdfunding platforms must then try to prevent multiple pseudonyms.

6 Market size

The popular press and many crowdfunding platforms draw attention to projects, such as PebbleWatch and Star Citizen, that attract contributions from huge numbers of funders. Such projects starkly demonstrate how small contributions from many people can add up to significant amounts, but they are far from representative, lying in a small upper tail of the distribution for number of funders. The representative crowdfunding project is far smaller. For Kickstarter projects, the average number of funders is 101.3 and this number falls to 56.2 on excluding the top one percent.[34] So most projects are moderate-sized. Nonetheless, we want to know which of our results are relevant for moderately large and large projects.

In our baseline model with independent individual demands, crowdfunding's advantage

[33] The minimal type 3 probability, chosen to stay close to the baseline, limits the expected gain from bid restriction, but losses can be substantial, for example, with $\mathbf{q} = (1/2, 3/8, 1/8)$, $\mathbf{v} = (2, 3, 8)$ and $C = 21/6$.

[34] To be precise, this excludes projects with over a thousand funders, constituting 1.3 percent of projects. We use the Kickstarter data from 2005-2014 that U.C. Berkeley's Fung Institute made publicly available at http://rosencrantz.berkeley.edu/crowdfunding/index.php.

over standard finance disappears in the limit, because aggregate uncertainty becomes insignificant relative to expected demand. Entrepreneurs with mass-interest projects of this type would self-select into standard finance to avoid the fees, plus usual transactions costs, of using crowdfunding platforms. However, as illustrated in the introduction, we find that crowdfunding *is* important for projects that expect to interest moderately large crowds, especially if their net expected profits from standard finance are low or negative.

We also show that when the crowd is composed of a small number of large groups of buyers with intra-group correlated valuations (but valuations that are independent across groups), aggregate uncertainty remains and crowdfunding can adapt to it, even though rent-extraction is then no longer helpful. In fact, this latter point can be sustained in a very general model of buyer demand; by contrast, our baseline model assumes only two types and Belleflamme et al. (2014), Sahm (2015), and Kumar et al. (2015) assume uniform distributions. Moreover, we show that even rent-extraction is feasible with large groups when group members have a group-ethic. We borrow this notion from models successfully used to explain turnout data in large elections. It has some relevance for groups of fans with a strong group identity, but the adaptation result has much more general relevance.

6.1 A large crowd with independent valuations

When the number of buyers is very large, there is little aggregate demand uncertainty and the advantage of crowdfunding over standard finance by avoiding fixed costs in particularly bad states of demand or producing in particularly high states of demand is limited because those are very unlikely events. Furthermore, the probability of being pivotal is very small and while the rent-extracting advantage of crowdfunding over standard finance depends instead on h_n thanks to the refund property, this also converges to zero, albeit slowly. Proposition 6 makes this precise.

Proposition 6. *The advantage of crowdfunding over standard finance, $\pi^{CF} - \pi^{SF}$, converges exponentially to zero as $N \to \infty$.*

Although the advantage of crowdfunding over standard finance disappears in the limit, it is always strictly positive. This would suggest that all products with fixed cost of production should be marketed through crowdfunding. Of course, this is not the case because crowdfunding campaigns have costs and disadvantages we have ignored sofar. Most importantly, platforms usually charge a fee to entrepreneurs and buyers. For example, Kickstarter pockets 5 per cent of revenues of successful projects and buyers pay a commission for paying with credit cards. Pitching and maintaining a campaign is also time-consuming for the entrepreneur as she has to keep updating to inform backers on project progress. Hence, one should expect some selection bias under entrepreneurs who do choose to use crowdfunding. First of all, entrepreneurs with tight credit constraints and no easy access to external funding must choose crowdfunding as they lack an alternative. Similarly, projects with negative net expected profits under standard finance only have a chance to see the light when it is possible to produce by adapting to demand. Similarly, entrepreneurs will choose crowdfunding for projects that are viable under standard finance but only deliver moderate expected profits. This all suggests a bias towards projects with relatively high fixed costs.

To be more specific, let us now consider the case of a moderately large crowd. For large N the number of buyers with high value is approximately distributed according a normal distribution with mean $\mu = qN$ and standard deviation $\sigma = \sqrt{Nq(1-q)}$. Projects with relatively high fixed cost require a number of high types, n_E or n_I, above the mean. Those projects are not viable under standard finance so crowdfunding is the only alternative. Projects with small negative expected profits under standard finance require a number of high buyers above but close to the mean, and thus have a chance of almost 50 per cent chance of success. If the required number is less than two standard deviations above the mean, there is at least a 2.5 per cent chance that the project will succeed. Projects with huge costs that require, four standard deviations above the mean have such a small chance of succeeding (about 0.05 per cent) that it is not worthwhile for the entrepreneur to do the effort of pitching her project. The platform may also not like to flood her site with many projects that will almost surely fail and may set an upper bound on the threshold.

On the other hand, projects with relatively low cost require a number of high types below the mean, and are thus viable under standard finance as well. Entrepreneurs may choose crowdfunding for these projects in order to adapt to demand (when $q > \hat{q}$) and to extract more rents (when $q < \hat{q}$). The gain from doing so is small for projects where the required number of high types is far below the mean. In the case of exclusion and low cost so that the required fraction of high types is two standard deviations below the mean, crowdfunding only brings a small expected benefit: it avoids investing the low fixed cost in the 2.5 per cent chance event that demand is insufficient. In the case of inclusion, crowdfunding comes with the risk of failure in order to try to extract rents from relatively few high types. For example, a project that requires the fraction of high types to be one standard deviation from the mean, has a chance of failing of 16 per cent.

6.2 A large crowd with correlated valuations

We now consider the possibility that the crowd consists of N large groups with m members each. Let the valuation of each member of a group be drawn from some distribution G on

$\mathbb{R}_+$. We assume that there is perfect intra-group correlation but independence across groups. Rent-extraction through crowdfunding is difficult when m is large, as seen previously.[35] For simplicity we now abstract away from this possibility and focus exclusively on adaptation to demand. The entrepreneur can now only set a minimal price p and must set $T = C$. When valuations across groups are independent, aggregate uncertainty remains. Crowdfunding can still adapt to demand and yield substantial private and social benefits compared to standard finance. This is most easily verified in the case where the valuations are taken again from a two point distribution, as in our baseline model. The entrepreneur can then either set the inclusive price $p = v_L$ yielding $Nmv_L - C$, or the exclusive price $p = v_H$, yielding π^E. Whenever exclusion is optimal, conditional production yields substantial gains over standard finance.

Because the entrepreneur now has only one strategic instrument, the minimal price p, we can characterize the optimal price for any (well-behaved) distribution from which the valuations for each group are drawn. This does not go only far beyond our baseline model, but also beyond the ones employing a uniform distribution. When the entrepreneur sets minimum price equal to p, the demand will be equal to $k \cdot m$ with probability $f_k^N(q)$ where $q = 1 - G(p)$. The profit for the entrepreneur is then

$$\pi(p) = \sum_{k=n(p)}^{N} f_k^N(kmp - C)$$

where $n(p) = \lceil C/(mp) \rceil$. In a region where $n(p) \equiv n$, the optimal minimum price must satisfy the first-order condition, which can be written as

$$0 = q + pq' + h_n q'\big((n-1)p - cN\big)$$

where $c = C/Nm$ denotes the per capita cost. It follows that, in this region, the minimal price strictly increases in cost, in contrast to the case of standard finance with a posted-price that is independent of cost. Under standard finance, the entrepreneur would set the price $p^{PP} = \arg\max pq(p)$ as long as this yields a positive expected profit, and would not produce otherwise. The minimal price in crowdfunding is strictly higher than the posted-price for any $c > 0$. As c increases, the entrepreneur finds it optimal to increase n at some cutoff, at which point the minimal price jumps down, to compensate for the reduced probability of success.

[35] Cremer and McLean (1988) show that with correlated valuations the optimal mechanism is able to extract full surplus, but this is not possible using a mechanism with the attractive features of crowdfunding where buyers know what they pay and never suffer a loss.

Proposition 7. *For N groups of buyers, each with m members sharing a common valuation that is drawn, independently for each group, from $G(\cdot)$ on $\mathbb{R}_+$, the entrepreneur is effectively restricted to use single price crowdfunding mechanism as m gets large. She then uses $T = C$, optimally adapting production to demand, given her price. Her price strictly exceeds the optimal posted-price, but total welfare and profits are strictly higher than under standard finance. This price rises with cost except for discrete downward jumps, despite which, production is always decreasing in cost.*

Notice, as a corollary, that the results of Proposition 7 hold for any m, including the baseline model where $m = 1$, if the entrepreneur chooses to only allow a single price in her crowdfunding mechanism. This might be relevant for moderately large N if buyers greatly value simplicity or approximate small pivotality probabilities by zero; note however that people also appear to over-estimate pivotality probabilities.

This group-preference model demonstrates that crowdfunding can generate important adaptation benefits for projects of arbitrarily large scope: that is, for goods that are attractive to any size of market. The point is even more general: so long as aggregate uncertainty is significant, crowdfunding has a valuable role to play in market-screening to adapt production to demand. We offer another instance of this general point in Section 7 where projects differ in their likely attractiveness, q. But before leaving the group-preference model, two remarks are in order.

First, note that the perfect intra-group correlation assumed here is purely to simplify the algebra. The insights would continue to hold if member types were noisy variations on a group prototype. Second, rent-extraction may be significant in this group model even with large m if group members identify so strongly with their group's interest as to remove intra-group free-riding on the cost of bidding to trigger production. Indeed, the formalization of this group-based model by Coate and Conlin (2004) for tackling costly turnout in elections re-generates exactly our baseline model.[36]

7 Ex-post market

Many buyers are completely unaware of potentially attractive projects until crowdfunded. Entrepreneurs naturally want to sell to such buyers *after* running the crowdfunding mechanism. We call this the "ex-post" or after-market. Even when all potential buyers are present "ex-ante," as crowdfunding participants, the entrepreneur, as monopoly seller of a durable

[36] Coate and Conlin (2004) builds on Feddersen and Sandroni (2006); see Feddersen (2004) for a survey, including related models with group leaders and experimental support (*e.g.*, Schram and Sonnemans, 1996).

good, may be tempted to sell ex-post at a lower price.[37] Extending the model to include ex-post sales offers several new insights.

First, without price commitment, the entrepreneur may switch to more inclusive crowdfunding; multiple prices during funding substitute for a falling minimum price; exclusive crowdfunding becomes less effective both for rent-extraction and market-screening.[38] Second, new buyers arriving ex-post can motivate an entrepreneur to produce even when ex-ante sales revenues fail to cover her fixed cost. So crowdfunding, even with advance payments, may complement standard finance. Third, the demand signals from crowdfunding can inform the entrepreneur's subsequent pricing strategy. We establish a pricing dynamic beyond the well-known decreasing price of a durable good monopolist: the entrepreneur actually has an incentive to *raise* her price ex-post whenever crowdfunding reveals sufficiently high demand. Crowdfunding now serves to adapt price, as well as production, to demand.

Two selling periods. We extend the model to two selling periods.[39] There are N_1 crowdfunding participants or "funders" who can fund by buying in period one, ex-ante, and N_2 "new buyers" who can only buy in period two, ex-post. Note that the labels refer to ability, not choice: funders can buy in period two or not buy instead of funding. Neither funders nor new buyers ever buy more than one unit of the product as it is durable.[40]

7.1 Baseline with ex-post sales

The baseline model revisited. The baseline model set $N_2 = 0$ and admitted no ex-post selling. Nothing can be gained by opening an ex-post market when all potential buyers are present ex-ante, have the same time-discount rate, and can choose from multiple prices. Nonetheless, before turning to non-commitment, it is instructive to study the hypothetical scenario where crowdfunding must set a unique price, as other studies implicitly assume.

Recall that, for $q < \hat{q}$, our baseline inclusive solution featured two crowdfunding prices, $\bar{b}_{n_I}$ and v_L, paid ex-ante by H and L type funders. The entrepreneur can generate the same

[37] The baseline model implicitly assumed there is no after-market. This is justified when the entrepreneur's reputation lets her commit against price reduction or the good is non-durable (as when fans care to "hear it first"), or if she already preferred an inclusive crowdfunding strategy (see below).

[38] Also, thanks to multiple ex-ante prices, assumed away in Sahm (2015) and Kumar et al. (2015), we prove that the entrepreneur always prefers to commit against ex-post sales if able to reach all buyers ex-ante.

[39] Additional periods further pressure towards inclusive crowdfunding.

[40] We also make two standard assumptions: (1) the entrepreneur lacks the information or legal right to price-discriminate ex-post between funders who did not yet buy and new buyers; (2) the entrepreneur cannot sell to funders if the project fails. (2) rules out renegotiation. Even under NTC where the entrepreneur can effectively lower her threshold to ensure success, she cannot explicitly break or cancel her crowdfunding contract and still access funders to offer a new sales contract. Renegotiation with funders complicates rent-extraction. It would also allow entrepreneurs to evade paying the platform its share of sales revenues. So platforms have a strong enforcement incentive.

payoff outcome with the unique crowdfunding price $p_1 = \bar{b}_{n_I}$ if she adjusts to threshold $T = n_I p_1$, and sells ex-post at $p_2 = v_L$ when her project succeeds. But imposing a single crowdfunding price is, in general, distortionary. First, $J > 2$ buyer types may require more than two prices. Second, the entrepreneur cannot use ex-post sales revenues to L-types to fund her fixed cost. So, if alternative credit is restricted and $C > n_I \bar{b}_{n_I}$, the optimum needs multi-price crowdfunding. Moreover, even without credit constraints, ex-post sales may inefficiently delay consumption.

Price commitment problems in the baseline. Ex-post selling might occur anyway when the entrepreneur is unable to commit against lowering her price ex-post. Sahm (2015) and Kumar et al. (2015) model this *no-price-commitment* scenario; we denote it NPC to distinguish from *no-threshold-commitment*, NTC.[41] The inclusive solution is unaffected by ex-post sales as funders were already buying at $p = v_L$ ex-ante, but NPC does bind when $q > \hat{q}$. We now study $q > \hat{q}$ with threshold commitment under NPC.

In a baseline exclusive equilibrium, only L-types remain with a demand ex-post, so the entrepreneur would want to set $p_2 = v_L$ after selling to the $k < N$ ex-ante H-type funders. But anticipating this price cut, H-types would delay to buy ex-post. This forces the entrepreneur to switch to an inclusive strategy or accept a mixed strategy equilibrium in which H-types only buy ex-ante with a probability $r < 1$, low enough to make $p_2 = v_H$ credible.

In the mixed strategy solution, each funder is: a H-type who funds, with probability qr; a H-type who buys ex-post, with probability $q(1-r)$; a L-type, who buys ex-post if $p_2 = v_L$, with probability $1 - q$. Conditional on not buying ex-ante, a given funder is H-type with probability $\frac{q(1-r)}{1-qr}$; this rises with q, falls with r and is independent of the number of funders the entrepreneur observes buying ex-ante, denoted k' to distinguish from the actual number of H-type funders, k. So ex-post, exclusion remains credible for any $r \leq \hat{r} = \frac{1-\hat{q}/q}{1-\hat{q}} \in (0, 1)$ if she sets $p_2 = v_H$ when indifferent ex-post. The entrepreneur prefers r as close to $\hat{r}$ as possible, so she induces $r = \hat{r}$. She thereby maintains full exclusion, but at a cost: she can only adapt production to the noisy signal k' of the true number k of high types.[42] The distorted adaptation reduces both welfare and profits under exclusion. On the other hand, when NPC lowers exclusion profits by enough, as when q just exceeds $\hat{q}$, the entrepreneur switches to inclusion so that the price commitment problem can instead raise total welfare.

[41] NPC and NTC imply an effective threshold of $T = C$ in Sahm (2015), but combining crowdfunding with external finance or personal wealth, this would be C minus expected ex-post revenues or $T = C - (N - k)v_L$.

[42] The optimal exclusive solution has a crowdfunding minimal price, $p_1 = v_H$ and a minimal number of ex-ante buyers $n_E^{NPC} = \lceil \frac{C}{v_H} \frac{1-\hat{q}(N_1 v_H/C)}{1-\hat{q}} \rceil$, below $n_E = \lceil \frac{C}{v_H} \rceil$ of the baseline because of anticipated ex-post sales to H-types funders who delay. The noise and inefficiency is decreasing in $\hat{r}$ and non-trivial since $\hat{r} < 1$.

7.2 New buyers

Price dynamics with new buyers. As is now well-understood from the literature on Coasian pricing dynamics, the arrival of new buyers facilitates price commitment. Indeed, in the relevant case, $q > \hat{q}$ of our model, the entrepreneur needs no reputational power to commit to $p_2 = v_H$ if enough new buyers arrive ex-post; that is, when N_2 is large enough. More interestingly, we now show how crowdfunding can also help the entrepreneur choose her price. That is, we reveal a market testing role for adapting price, as well as production, to demand, even under no price commitment, NPC.[43]

Case I. *New buyers are independent of, and identically distributed to, the funders.*

We first consider $q > \hat{q}$ and the entrepreneur seeking an exclusive solution with $p_1 = v_H$ and H-types buying ex-ante with probability r. If k' funders buy ex-ante, then $N_1 + N_2 - k'$ buyers have a demand ex-post and an expected fraction $q\left(\frac{(N_1-k')(1-r)/(1-qr)+N_2}{N_1-k'+N_2}\right)$ of these are H-type. If the ex-post market is relatively large, $N_2/N_1 \geq \hat{q}/(q-\hat{q})$, then $p_1 = p_2 = v_H$ is feasible even after $k' = 0$ with $r = 1$. Exclusion is then optimal as well as credible.

Complementarity with standard finance. The price dynamic here is trivial but this scenario offers a simple demonstration of how crowdfunding can complement standard finance, because the maximal optimal threshold is $T = C - N_2 q v_H < C$. All funders buy ex-ante so $k' = k$ but additional finance is strictly valued to help cover the fixed cost in states $\lceil \frac{C}{v_H} - N_2 q \rceil < k < \lceil \frac{C}{v_H} \rceil$. Moreover, this can arise for $(N_1 + N_2) v_H q < C$, where standard selling implies non-production, so crowdfunding raises the demand for standard finance. We state this formally as a simple case of strict complementarity.[44]

Proposition 8. *Crowdfunding strictly complements standard finance when $N_2 > 0$ and C is relatively large, e.g., when $q > \hat{q}$, $N_2/N_1 \geq \hat{q}/_{q-\hat{q}}$ and $C/(qv_H) \in (N_1 + N_2, N_1/q + N_2)$.*

An endogenous crowdfunding-contingent price. When instead the ex-post market is relatively small, $N_2/N_1 < \hat{q}/(q-\hat{q})$, if all H-types still buy in advance, $r = 1$, and L-types are still initially excluded, then the ex-post price depends non-trivially on k' or equivalently the funds raised in crowdfunding. Using n_H to denote the cut-off value of k' at which p_2 switches from v_L to v_H, we have $n_H = \lceil N_1 - \frac{q-\hat{q}}{\hat{q}} N_2 \rceil$, which exceeds 0 here. The entrepreneur must now reduce her ex-ante minimum price to p_1 given by a H-type incentive compatibility condition,

$$(v_H - p_1) S^{N_1-1}_{n_E(N_2)-1} = (v_H - v_L)(S^{N_1-1}_{n_E(N_2)} - S^{N_1-1}_{n_H})$$

[43] We focus on NPC, since the interesting results with price commitment are qualitatively similar, committing either to a lower bound on the ex-post price or a price contingent on total funds raised during crowdfunding.

[44] We discuss debt and equity -based crowdfunding below. As Footnote 42 makes clear, the entrepreneur may also strictly value additional finance when N_2 is low and a mixed strategy solution is optimal.

where we make the dependence of n_E on N_2 explicit simply to distinguish from its definition in the baseline model; n_H depends on N_1 and N_2. This represents the binding incentive compatibility constraint on H-types: paying p_1 raises project success and buying cheap ex-post is not always an option.[45]

This price dynamic reflects how a greater number of funders not buying in period 1 signals a higher fraction of low types among the pool of possible ex-post buyers. Setting a high price ex-post is more attractive, the more funds the entrepreneur collects during crowdfunding. We now study a similar, but economically more important pricing dynamic (one that is significant even when $N_2 \gg N_1$) by allowing for correlation between the demands of funders and new buyers. In this extension, buyer valuations are correlated through their common dependence on the state of the project.[46]

Case II. *Again all buyers, both funders and new buyers, have independent and identically distributed valuations, but now q depends on whether the entrepreneur's project is good G or bad B: $0 < q^B < q^G < 1$. The good and bad states of the project have equal probability and are not observed by entrepreneur or buyers, but each buyer observes his valuation on inspecting the project (ex-ante for funders, ex-post for new buyers).*

Adaptation. Before moving on to pricing, notice how crowdfunding is again valuable for arbitrarily large markets for adapting to demand. If $\max\{q^B v_H, v_L\} < C/N_2 < q^G v_H$ and $N_1 \ll N_2$, the entrepreneur would like to produce in the good state and not the bad state. By setting a crowdfunding mechanism with $p = v_H$ and threshold $T = nv_H$, if $p_2 = v_H$ is credible, then all H-types buy ex-ante and the entrepreneur produces whenever $k \geq n$. Now the number k of H-types signals a posterior probability that the project is good, given by

$$\eta^G(k) = \frac{1/2 f_k^{N_1}(q_G)}{1/2 f_k^{N_1}(q_G) + 1/2 f_k^{N_1}(q_B)} = \frac{f_k^{N_1}(q_G)}{f_k^{N_1}(q_G) + f_k^{N_1}(q_B)} \tag{13}$$

which is always increasing in k. Picking the least k for which this posterior exceeds $(C - kv_H)/(N_2 v_H)$ gives the optimal crowdfunding solution. Crowdfunding is valuable for adapting production no matter how large is N_1. Indeed, a higher N_1 raises the accuracy of crowdfunding as a signal of the project's type, allowing better adaptation.

Under the plausible assumption that buyers find it harder to inspect a crowdfunding project's good ex-ante than to learn its value once the good has been produced and put up for sale ex-post, entrepreneurs may have to attract crowdfunders by offering a discount. Extending the model to capture this point would allow one to endogenize the relative sizes

[45] The entrepreneur potentially prefers to have H-types set $r < 1$ so that she can credibly commit to a lower n_H, but for a range of parameter values, this is an optimal solution.

[46] Technically, buyer valuations are still private (other buyer's signals have no impact on a given buyer's expected value given his signal), but their signals are affiliated.

of N_1 and N_2. We expect the following tradeoff: lowering the minimal price in crowdfunding may reduce average revenues per funder but attract more inspection and permit more accurate market-screening or adaptation.[47]

To illustrate how crowdfunding can affect **price dynamics** when there is an after-market, we now suppose $q^B < (q^B + q^G)/2 < \hat{q} < q^G$ and a cost so small that, with ex-post sales at a fixed price, the entrepreneur cannot do better than sell to all buyers at v_L, as in standard finance, and obtain a profit of $(N_1 + N_2)v_L - C$. There is no role for demand adaptation, because the entrepreneur wants to produce for sure and costs are so low that rent-extraction is undesirable. But crowdfunding can be used as a means of market testing and learning how to price when many buyers are present ex-ante and ex-post. For example, the entrepreneur can offer prices v_L and $b_H > v_L$, and make production contingent on at least one high type buyer, thereby learning the number of high types.

To satisfy the IC of the high type, $b_H \leq \bar{b}_1 = h_1 v_h + (1 - h_1)v_L$. When N_1 is large, h_1 and rent-extraction are minimal while production occurs with probability close to one. What the entrepreneur gains, however, is the signal provided by the pledged sum. For large N_1 this signal is very accurate and she will learn whether her project is G or B and set her ex-post price optimally. Her overall expected profit is approximately $N_1 v_L + \frac{1}{2} N_2(v_L + v_H) - C$, much larger than $(N_1 + N_2)v_L - C$ when N_2 is large.

Interestingly, the adaptation of price in this example may actually lead to a higher price ex-post for instance, PicoBrew Zymathic's automatic beer brewing appliance sold at 1599 dollars or less during crowdfunding on Kickstarter and then sold ex-post for 1999 dollars.

In general, the optimal crowdfunding design must carefully trade off demand adaptation, rent-extraction and market-testing. Characterizing the optimal design is beyond the scope of the present paper.

8 Extensions

In this section we discuss two extensions of our baseline model. First, we show that crowdfunding is useful for entrepreneurs who do not seek to maximize profits, but instead, are interested in maximizing success, audience or welfare under various liquidity constraints. We also show that our baseline model readily applies to the case where backers do not consider themselves as mere buyers but receive warm-glow from participating in crowdfunding project. Second, we argue that our qualitative insights about demand-adaptation and rent-extraction are robust to having buyers bid sequentially rather than simultaneously, although

[47] Platforms play an important role in this strategic interaction because pre-inspection uncertainty introduces additional commitment difficulties, placing funders at risk of entrepreneurial hold-up.

rent-extraction is reduced. We also show that when buyers can choose when to bid, they would all bid in the same period, justifying our assumption of simultaneous bidding.

8.1 Alternative objectives

So far, we analyzed how crowdfunding works when entrepreneurs are profit-maximizers and buyers are purely self-interested. We now explain how crowdfunding is helpful when entrepreneurs or buyers have other motives.

We start with buyers. Suppose they consider their primary role to be active participants, supporters, vital contributors, into a creative process rather than mere buyers. Do our key effects of adaptation and extraction apply when buyers are thus motivated? It turns out that the answer is an emphatic "yes." We describe a quick if highly specialized case in point, that works well even when the good is fully public, being non-excludable as well as non-rival.

Suppose buyers value the project at v_H or v_L, but only if they contribute to the project with a bid of at least the minimum. So buyers value participating in the project. They may no longer receive a physical consumption good, as when the project is to deliver charitable goods to distant, third-party beneficiaries. Or the project may produce a good like news or music that will be subsequently distributed freely to everyone. In the charity example, buyers are donors who get a warm-glow from giving. In the news or music example, one can view the private value from contributing as a "community benefit" (Belleflamme et al., 2014). Donors and contributors may also be motivated by receiving recognition, as in film credits or donor list publications. With these assumptions, our model, despite having excludability, applies exactly, because contribution to a successful project becomes the private "good" that can be produced and is excludable even if the project's basic good is entirely public.

We now show how entrepreneurs with non-profit motivations can also gain from crowdfunding. We consider three new cases: success, audience and welfare maximizers.[48] Without a new constraint, the first pair make production always optimal. We treat two canonical constraints: ex-post budget balance (BB) requires that funds raised always cover costs C (i.e., no loss in any state k); non-negative (expected) profit (NNP) requires that the entrepreneur breaks even on average. BB is appropriate when the entrepreneur, such as an artist, has no access to credit and no personal wealth, but also when she is unable or unwilling to risk suffering a loss in any demand state. NNP, sometimes called ex-ante budget balance, is appropriate when the entrepreneur can access credit subject to breaking even on average. NNP and welfare-maximization can capture an ideal not-for-profit, but publicity motivations make audience or success maximization plausible too. Relatedly, success-maximization with

[48] Hansmann (1981) argues that not-for-profit organizations in the performing arts may seek to maximize audience or quality. Arguably, quality can be interpreted as success probability in our context.

BB can capture an unproven entrepreneur who aims to maximize long-run profits but must break even with her project as vital stepping stone to an entrepreneurial career.

We treat the baseline case with full commitment and $J = 2$. So neither constraint BB nor NNP affects outcomes under profit-maximization. That is, the new constraints do not make profit-maximization infeasible. In fact, for a given n, the entrepreneur will set prices so as to maximize profits, as it relaxes the additional constraints. But in determining the optimal n-type strategy, the entrepreneur will be guided by her not-for-profit motives. To study BB, we assume $C > Nv_L$ else the entrepreneur can guarantee maximum success, audience and total welfare, by setting $p = v_L$ and $T = C$. For NNP, we assume $C > \max\{Nv_L, qNv_H\}$ else a posted-price can guarantee maximum success and audience.

8.1.1 Balanced Budget (BB)

If the entrepreneur cannot make a loss in any demand state k, then she can only produce if raised funds cover her fixed costs C. Fixing whether a strategy is inclusive or exclusive, it is then optimal to set p to maximize profit to relax her credit constraint and minimize n subject to maintaining $T \geq C$. This clearly maximizes success and expected audience, but also expected welfare: the budget balance constraint implies non-negative profit, ensuring that production is welfare optimal in all demand states with $T \geq C$. So $n = n_E$ is optimal among exclusive strategies and $n = n_I^{BB} = \min\{n : C \leq \overline{T}_n\}$ gives the optimal inclusive strategy.

Success maximization. Success-maximizers only care about minimizing n. This makes them more inclusive than profit-maximizers: they certainly select inclusion for any $q \leq \hat{q}$, since then $n_I^{BB} \leq n_I \leq n_E$. If both latter inequalities are weak, inclusion is favored as it produces more goods. When either is strict, the overall optimal strategy is also inclusive for a range of q strictly above $\hat{q}$. Exclusion may remain optimal for sufficiently high q. In sum, we predict more inclusion when the entrepreneur seeks to maximize success under a balanced-budget constraint.

Audience and welfare maximization. For a given success rate, audience maximizers always strictly prefer inclusion of L-types into their audience, and welfare maximizers do so too once cost C is sunk. So, in either case, inclusion becomes yet more likely and is certainly optimal if $n_I^{BB} \leq n_E$.[49] The two cases can differ on the inclusion/exclusion choice for $n_I^{BB} > n_E$: (a) pure audience-maximizers gain more from inclusion as they value L-types as much as H-types, unlike welfare-maximizers who value H-type consumption more;

[49]Of course, inclusion is always optimal for the unconstrained welfare-maximizer; see Section 3.4 where we derived n^*, readily seen to be lower than both n_I^{BB} and n_E.

(b) welfare-maximizers care about the expected cost of production which is higher under exclusion given $n_I^{BB} > n_E$. We can express expected audience and welfare, respectively, as $qNS_{n_E-1}^{N-1}$ and π^E under exclusivity, and as $NS_{n_I^{BB}}^N$ and $S_{n_I^{BB}}^N \left(Nv_L - C + \mathbb{E}[k|k \geq n_I^{BB}]v_H\right)$ under inclusivity. From this, we characterize when these qualitative differences are strict:

Proposition 9. *All three alternative objectives raise inclusivity compared to the profit-maximizing baseline where inclusion was optimal on $q \leq \hat{q}$. The inclusion region expands to $n_I^{BB} \leq n_E$ for success-maximizers and further expands to include parameters with $n_I^{BB} < n_E$ and, respectively, $qS_{n_E-1}^{N-1} < S_{n_I^{BB}}^N$ for audience maximizers and $qS_{n_E-1}^{N-1} - \hat{q}S_{n_I^{NNP}}^N - qS_{n_I^{NNP}-1}^{N-1} \leq c(S_{n_E}^N - S_{n_I^{NNP}}^N)$ for welfare maximizers.*

In addition, for all three objectives, success rates rise if $q \leq \hat{q}$, strictly so if $n_I^{BB} < n_E$. In our introductory example with $N = 500$, $q = 1/5$, $v_L = 5$, $v_H = 20$ and $C = 2650$, $n^{BB} = n_I^{BB} = 103$, yielding a success rate of 38.6% . This more than doubles the 17% rate under profit-maximization. What about welfare? All three objectives also raise welfare, perhaps weakly, except possibly in a special case that has to have audience-maximizers select inclusion and welfare-maximizers prefer exclusion.

8.1.2 Non-negative profit (NNP)

NNP is a weaker constraint than BB. So entrepreneurs advance their objectives more effectively. The qualitative implications are similar, but there is one important twist in the analysis compared to that for BB: welfare-maximizers no longer minimize n among exclusion strategies, nor potentially among inclusion strategies. We again compute the minimal feasible values for n. For exclusion, this can now be less than n_E, so we denote it n_E^{NNP}, $n_E^{NNP} = \min\{n : C \leq \mathbb{E}[k|k \geq n]v_H\}$. For inclusion, the value is now $n_I^{NNP} = \min\{n : C \leq Nv_L + \mathbb{E}[k|k \geq n](v_H - v_L)\}$. Observe that given the more relaxed constraint, $n_I^{NNP} \leq n_I^{BB}$ and $n_E^{NNP} \leq n_E^{BB}$.

Success and audience maximization. Success-maximizers only care about minimizing n which makes them again more inclusive than profit-maximizers: they certainly select inclusion for any $q \leq \hat{q}$, since then $n_I^{NNP} \leq n_E^{NPP}$. If the inequality is weak, inclusion is favored as it produces more goods. When the inequality is strict, the overall optimal strategy is also inclusive for a range of q strictly above $\hat{q}$. Exclusion may remain optimal for sufficiently high q. In sum, we predict more inclusion when the entrepreneur seeks to maximize success under a non-negative profit constraint.

Audience-maximizers choose inclusion whenever $qNS_{n_E^{NNP}}^N < NS_{n_I^{NNP}}^N$. Clearly, audience-maximizers choose inclusion more often than success-maximizers do.

Welfare maximization. A welfare-maximizer can obtain the first-best in the (rare) event when $\pi^I_{n^*} \geq 0$. If $\pi^I_{n^*} < 0$, she chooses inclusion of type $n_I^{NNP} > n^*$ whenever $S^N_{n_I^{NNP}}(Nv_L - C + \frac{qNv_H}{1-(1-q)h_{n_I^{NNP}}}) \geq S^N_{n_E}(-C + \frac{qNv_H}{1-(1-q)h_{n_E}})$. Note that the welfare-maximizer prefers exclusive strategy of type n_E over the one of type $\hat{n}_E$. Hence, if she optimally chooses exclusion under NNP, then she would also under BB and the more relaxed constraint does not help. Also, she may choose exclusion when success- or audience-maximizers choose inclusion. However, if inclusion is optimal under BB, then it is also optimal under NNP.

8.2 Sequential bidding

Our baseline model assumes that buyers bid simultaneously. In practice, crowdfunding campaigns last for several weeks and buyers receive information about the sum of money pledged so far. Kuppuswamy and Bayus (2015) show that most pledges occur at the start and near the end of a campaign. Agrawal et al. (2015) find that family and friends of the entrepreneur are among the first to contribute, possibly providing a signal of quality to other potential backers. Potters et al. (2007), using experimental methods, show that in a setting with informed and uninformed contributors to a public good, (endogenous) sequential provision is better because of social learning. In our model, valuations are private and independent, so the role of learning is limited but timing may affect pivotality. Varian (1994) shows that in a complete information setting of voluntary donations to a public good, bidding reduces the amount of the public good provided in comparison with simultaneous bidding, as it exacerbates free-riding. Bag and Roy (2011) characterize when sequential bidding increases voluntary donations in a private information setting. Romano and Yildirim (2001) show that sequentiality can be better in a warm-glow setting.[50]

In this section, we analyze how and when sequentiality matters in our setting with private information and a discrete excludable public good. We show that adaptation and rent-extraction remain valuable instruments of crowdfunding. Rent-extraction is reduced because late-movers will not pay more than the minimal price once the threshold is reached. We also argue that our simultaneous bidding assumption is justified when buyers decide when to pledge. This is relevant in practice because buyers can easily delay bidding or cancel pledges before the deadline and threshold are reached.[51]

Suppose buyers bid sequentially rather than simultaneously. In particular, assume that buyers enter sequentially and observe the total amount pledged up to the point in time they are required to decide, and that buyers know their place in the queue (that is, they know

[50]Note that in their setting strategies are strategic complements while in ours they are strategic substitutes.
[51]Kickstarter facilitates both options, for instance, by offering automated deadline reminders.

how many buyers come after them).

The exclusive outcomes are not affected by sequential bidding. When the minimum price is set at $p > v_L$ and the threshold equals C, all H-type buyers bidding p, independent of their place in the queue and the amount pledged sofar, constitutes a subgame perfect equilibrium. Hence, the entrepreneur will set $p = v_H$ in this case.

It is also clear that the $\bar{\pi}_0$ outcome is not affected by sequential bidding. With the minimal price set at v_L and the threshold at $C \leq Nv_L$, all buyers will bid the minimal price, independent of the place in the queue. However, in general the inclusive outcomes where L and H-types pay different prices, are affected by sequential bidding. Late moving H-type buyers will not bid the high price when the threshold is already reached (or when the threshold will for sure be reached given that all buyers after him bid at least v_L). Moreover, early moving H-type buyers may bid low and free-ride on late movers. Hence, the H-type buyers have different incentives under sequential bidding, and the entrepreneur must change the mechanism to take these altered incentives into account.

To illustrate these issues let us analyze the case of $N = 2$ in detail. We assume the entrepreneur has threshold commitment and sets a minimum price p.[52] We already argued that the optimal exclusive outcome under simultaneous bidding can be replicated under sequential bidding by setting the minimal price equal to $p = v_H$. In case of inclusion the minimal price will be $p = v_L$. We now analyze how the bidding will proceed for different threshold values.

We restrict attention to $T \leq 2v_H$ so that the threshold can be reached and the entrepreneur can make a profit. Clearly, if $T \leq 2v_L$, buyers foresee that the threshold will be reached and so everybody bids the minimal price v_L, and the entrepreneur collects profits $\pi_0^I = 2v_L - C$. If the threshold is set strictly above $T > v_L + v_H$, buyers foresee that the threshold can only be reached when both buyers have high type. The first buyer (when H-type) thus bids $T - v_H$, knowing that the second buyer (when H-type), will bid v_H. The entrepreneur then collects expected profits $q^2(T - C)$. The entrepreneur sets the threshold at the maximum value and obtains profit π_2^E.

Now let us consider the remaining case where $2v_L < T \leq v_L + v_H$. If the first buyer bids $b_1 \geq v_L$, the threshold is reached (exactly) if and only if the second buyer is H-type, because he is willing to bid $T - b_1 \leq v_H$. In order to increase the probability of production the first buyer needs to bid (at least) $T - v_L$, in which case the threshold will be reached for sure. A H-type buyer thus bids $T - v_L$ if $v_H - (T - v_L) > (v_H - v_L)q$, or, equivalently, if $T - v_L < \bar{b}_1$. The entrepreneur's expected profit in this case equals $(T - C)(1 - (1 - q)^2)$.

[52] It can be shown that bid restrictions do not matter in this case, and the exposition is in fact easier in the case without bid restrictions.

In the complementary case where $T - v_L > \bar{b}_1$, the H-type buyer bids v_L and the expected profit of the entrepreneur is then $(T - C)q$. Assuming that the buyer bids $T - v_L$ also in case of indifference, the entrepreneur will choose the highest T consistent with these cases and thus sets either $T = v_L + \bar{b}_1$ (letting late H-types free-ride sometimes), yielding $\bar{\pi}_1 = (v_L + \bar{b}_1 - C)(1 - (1 - q)^2)$ or $T = v_L + v_H$ (letting early H-types free-ride), yielding expected profit $\hat{\pi}_1 = q(v_L + v_H - C)$.

The entrepreneur compares the payoffs from the different inclusive and exclusive options and chooses the one that yields the highest profit. Under sequential bidding the payoff π_1^I is not attainable. Hence, for the (C, q)-pairs where π_1^I was the optimal payoff under simultaneous bidding, the entrepreneur must change the mechanism. Figure 7 illustrates. The entrepreneur strictly loses in comparison with the simultaneous bidding. For relatively low values of q and C, the entrepreneur switches to selling to all at v_L. This improves consumer surplus and total welfare. For relatively high values of q and low values of C, the entrepreneur switches to the exclusive strategy yielding payoff π_1^E. This lowers consumer surplus but leaves total welfare unaffected because the project succeeds whenever at least one buyer is H-type. For lower values of q, the entrepreneur uses the same threshold as under simultaneous bidding (as $\overline{T}_1 = v_L + \bar{b}_1$) but her profit is lower because a second H-type free-rides. Consumer surplus is higher but total welfare is again unaffected. Finally, for very high values of C the entrepreneur sets threshold $T = v_L + v_H$, which lets a first-moving H-type free-ride (and bid v_L) but extracts the full rent from late moving H-types. This lowers the expected consumer surplus and total welfare, because the project succeeds only if the second buyer has H-type.

Note that under the "late-comers free-ride" strategy, the project succeeds when at least one buyer has H-type. That is, it succeeds in the same demand states as the exclusive strategy with threshold $C \leq v_H$. This explains the horizontal line segment dividing the areas marked π_1^E and $\bar{\pi}_1$ in Figure 7. Note that the curve separating the areas marked $\hat{\pi}_1$ and π_1^E is the same as the one that separates the areas where π_1^I and π_2^I are optimal under simultaneous bidding. Namely, $\hat{\pi}_1 = \pi_2^E$ if and only if $q = (v_L + v_H - C)/(2v_H - C)$, while straightforward calculations show that $\pi_1^I = \pi_2^E$ also if and only if $q = (v_L + v_H - C)/(2v_H - C)$.

[Figure 7 about here.]

In the discussion so far, we assumed that buyers enter one by one in an exogenously given order. Suppose now that buyers endogenously decide on whether to move early or late.[53] It is obvious that this endogeneity does not affect the outcome when the entrepreneur implements

[53] See Hamilton and Slutsky (1990) for a general discussion of endogenous timing, and Romano and Yilderim (2001) for an application to charities.

an exclusive outcome, or the outcome where everybody just pays v_L. If the entrepreneur sets a threshold strictly above $\bar{b}_1 + v_L$, H-types want to move first in order to free-ride. However, all H-types moving first and trying to free-ride leads to a very undesirable outcome, namely failing to reach the high threshold and the project will necessarily fail. On the other hand, if the entrepreneur sets a threshold equal to (or below) $\bar{b}_1 + v_L$, all buyers will want to move late and the outcome is as under simultaneous bidding. In particular, if both buyers happen to have H-type they will both pay $\bar{b}_1$ and the entrepreneur's profit equals $\bar{\pi}_1$. Hence, the model with endogenously moving buyers replicates the results under of our baseline model with simultaneous moves.

9 Concluding remarks

We have characterized the optimal design of crowdfunding in a private value environment, showing that reward-based crowdfunding is optimal for entrepreneurs who may be either profit-maximizing or success-maximizing (or other not-for-profits), even if consumers have standard motivations. We demonstrated the twin roles of the crowdfunding threshold mechanism in adapting production and pricing to the crowd's revealed demand, and in price-discrimination, which usually further improves adaptation but can involve an excessive threshold that wastes trade opportunities. We found that crowdfunding, as a market-test, can inform subsequent pricing and also provides a signal of future profitability to traditional financiers. So that with many buyers arriving later, crowdfunding tends to complement rather than substitute for traditional finance.

An investment-based element of crowdfunding can also increase funds, but standard financiers may have advantages in centralizing monitoring (Diamond, 1984) and providing expert advice (Gompers and Lerner, 2001) that complement credible information from crowdfunding, plus funders' feedback and "word of mouth" advertising. The most important part of crowdfunding is "crowd," not "funding". "Crowd-commitment" can suffice: enough people committing to buy the product at high enough bids provide a valuable signal to motivate finance. Of course, the most effective way to enforce a purchase commitment is to have buyers put their money where their mouth is, and platforms like Kickstarter do require buyers to pay their bids in advance, so that it is natural to have buyers fund the entrepreneurs. So funding is a typical feature of crowdfunding, albeit not fundamental.

While no other paper models multiple prices within crowdfunding, the idea that consumers volunteer to pay different prices for the same good is not new. Hansmann (1981) marshals the evidence in Baumol and Bowen (1968) to argue convincingly that this "voluntary price discrimination" is critical in the world of culture and performing arts for the survival of theatres, museums and opera. We formalize his verbal argument that self-interested buyers pay extra to raise the chance that the theatre survives.[54] He also argues that the not-for-profit organizational form of many cultural institutions helps achieve this goal, consistent with our results in 8.1. Hansmann (1981) and Baumol and Bowen (1968) also give evidence that not-for-profit organizations set very low minimal prices, in effect subsidizing low valuation consumers, in order to convince high valuation buyers to pay extra, thereby guaranteeing sufficient funds to cover the fixed costs. This intentional underpricing is consistent with our results in 4.2 without threshold commitment.

We made a number of simplifications that, while common in mechanism design, may appear restrictive. First, we assumed that N, the number of people participating in crowdfunding, is known. All that is actually crucial is that there is uncertainty about aggregate demand, which we generate from preferences.[55] Second, we assumed that the fixed cost C is known. This is innocuous in our baseline model as buyers only care about the cost in so far as it affects the threshold, but they observe the threshold directly. Finally, we assumed that product quality is known and that valuations are private. In particular, buyers know their value for the product conditional on project success. Despite entrepreneur's goodwill and effort in reducing buyers' uncertainty about quality by providing product videos, sample songs, and relevant personal information, some doubt about delivery of the good may remain. In fact, the majority of successful projects suffers from delivery delay, although receiving no good at all is very rare (Mollick, 2014). So moral hazard is in practice not much of a problem and delivery delays can easily be accounted for with net present private valuations. The private value assumption is related to this. Variable quality affects all buyers, generating a common value element when buyers observe private signals of quality. Fortunately, crowdfunding can generate important benefits in the common value setting that complement those analyzed here.[56]

In the model, we have taken participation in crowdfunding as exogenous, but participation requires entrepreneurs to invest in creating and presenting their projects and it requires buyers to spend time searching and inspecting projects. Since both entrepreneurs and funders are crucial to project success, it is necessary to ensure that both parties earn sufficient rents after sinking their costs of participation. Entrepreneurs may have difficulty committing to give buyers a reasonable rent because they are tempted to hold up buyers once their

[54] His reduced-form model simply assumes that buyers bid above the minimal price in proportion to their net value from consumption at the minimal price.

[55] Adding a third type of buyer who values the good at zero generates an uncertain number of non-trivial participants. As shown in Section 5, our main insights remain valid when there are more than two types.

[56] Case II in Section 7.2 considered an extension with correlated preferences. See also Hakenes and Schlegel (2014) and Chang (2015) on crowdfunding in a common value environment.

search and inspection costs are sunk. The sequential nature of crowdfunding, omitted from our baseline model, helps solve this problem since the dynamic funding process tends to punish entrepreneurs who offer too little rent: their projects attract fewer early buyers and later potential buyers who observe this can respond by staying away without having to sink substantial inspection costs.

Crowdfunding platforms can help further by promoting projects that seem more promising. This is immediate when simply observing minimal price and threshold is costly for buyers. When the important inspection costs are those of evaluating common value elements of projects, platforms may find evaluation difficult too, but they can exploit entrepreneurs' past behavior and learn from and complement the dynamic processes in the sequential case. Notice that platforms have a stronger incentive to build a reputation for doing this than do individual entrepreneurs to promise high rents, because they typically host large numbers of entrepreneurs. Another simple policy that would limit rent-extraction would be to rule out threshold commitments, but our results show that this is risky since it can even lower buyer welfare, even aside from dissuading entrepreneurial participation and reducing innovation incentives.

In sum, we have explained the success of reward-based (and charitable) crowdfunding via its potential for market-testing and rent-extraction, on top of its simplicity and the reassuring property that buyers lose no money when thresholds are not reached. Our results on threshold commitment and welfare have clear policy implications for regulators and platform design but a full-fledged two-sided model is needed to draw precise conclusions. A possible concern for the future of this two-sided market is that strong network effects may prevent healthy ongoing competition between crowdfunding platforms and a dominant platform like Kickstarter may be able to charge inefficiently high fees, perhaps shifting the fee structure which also merits analysis. An exciting related topic is the study of equity- and lending-based crowdfunding which are currently growing even faster than the reward-based model. Extending our analysis to a pure common value environment will be valuable in this respect. Given that multiple prices are also a feature of investment-based crowdfunding, our binary model may be very useful for representing buyers with good and bad pieces of information.

Appendix A Proofs

We let $S_n^M(q) = \sum_{k=n}^{M} f_k^M(q)$ so that $h_n(q) = f_{n-1}^{N-1}(q)/S_{n-1}^{N-1}(q)$. Mostly omitting argument q, we state and prove some simple relations between these expressions for use in later proofs.

Lemma B.1.

(i) $f_k^N = qf_{k-1}^{N-1} + (1-q)f_k^{N-1}$

(ii) $S_n^N = S_{n-1}^{N-1} - (1-q)f_{n-1}^{N-1}$

(iii) $\sum_{k=n}^{N} kf_k^N = qNS_{n-1}^{N-1}$, *for all* $N \geq 1$ *and* $0 \leq n \leq N$.

(iv) $\sum_{k=n}^{N}(N-k)f_k^N = (1-q)NS_n^{N-1}$, *for all* $N \geq 1$ *and* $0 \leq n \leq N$.

(v) $\frac{\partial f_k^M(q)}{\partial q} = f_k^M \frac{k-Mq}{q(1-q)}$

(vi) $\frac{\partial S_n^N(q)}{\partial q} = Nf_{n-1}^{N-1}$

(vii) h_n *is strictly increasing in* n *for* $0 \leq n \leq N$, *with* $h_0 = 0$ *and* $h_N = 1$.

(viii) *For* $0 < n < N$, $\frac{\partial h_n(q)}{\partial q} < 0$.

(ix) $n(1-q)h_n \geq n - qN$ *where the inequality is strict when* $q > 0$ *and* $n < N$

Proof of Lemma B.1.

(i) is immediate on expanding on any one draw and $N-1$ other independent draws.

(ii) Summing (i) from $k = n$ to N and recalling that $f_N^{N-1} = 0$

$$\begin{aligned} S_n^N &= qS_{n-1}^{N-1} + (1-q)S_n^{N-1} \\ &= qS_{n-1}^{N-1} + (1-q)\left(S_{n-1}^{N-1} - f_{n-1}^{N-1}\right) \\ &= S_{n-1}^{N-1} - (1-q)f_{n-1}^{N-1} \end{aligned}$$

(iii)

$$\begin{aligned} \sum_{k=n}^{N} kf_k^N &= \sum_{k=n}^{N} kq^k(1-q)^{N-k}\frac{N!}{(N-k)!k!} \\ &= Nq\sum_{k=n}^{N} q^{k-1}(1-q)^{N-1-(k-1)}\frac{(N-1)!}{(N-1-(k-1))!(k-1)!} \\ &= Nq\sum_{k=n}^{N} f_{k-1}^{N-1} = Nq\sum_{k=n-1}^{N-1} f_k^{N-1} = NqS_{n-1}^{N-1} \end{aligned}$$

(iv) Using (ii) and (iii),

$$\begin{aligned}\sum_{k=n}^{N}(N-k)f_k^N(q) &= N\left((1-q)\left(S_{n-1}^{N-1}-f_{n-1}^{N-1}\right)\right)-NqS_{n-1}^{N-1} \\ &= N(1-q)(S_{n-1}^{N-1}-f_{n-1}^{N-1}) \\ &= N(1-q)S_n^{N-1}\end{aligned}$$

(v) Differentiating, $$\partial f_k^M(q)/\partial q = \binom{M}{k}q^{k-1}(1-q)^{M-k-1}\Big[k(1-q)-(M-k)q\Big]$$
$$= \frac{\binom{M}{k}q^k(1-q)^{M-k}}{q(1-q)}(k-Mq) = f_k^M\frac{k-Mq}{q(1-q)}$$

(vi) Differentiating the summation that defines S_n^N using (v) gives,

$$\begin{aligned}\partial S_n^N(q)/\partial q &= \sum_{k=n}^{N}(k-Nq)f_k^N\Big/q(1-q) \\ &= \left(NqS_{n-1}^{N-1}-NqS_n^N\right)\Big/q(1-q) \text{ (from (iii))} \\ &= \left(S_n^N+(1-q)f_{n-1}^{N-1}-S_n^N\right)N\Big/(1-q) \text{ (from (ii))} \\ &= Nf_{n-1}^{N-1}\end{aligned}$$

(vii) From the definition it is clear that $h_0=0$ and $h_N=1$. We will show that h_n is strictly increasing by induction. As a first step, note that $h_N=1>h_{N-1}$ since $f_{N-1}^N>0$, for all $q\in(0,1)$. Now suppose that $h_N>h_{N-1}>...>h_{n+2}>h_{n+1}$ for $N-1\geq n+1\geq 0$. We have to show that $h_{n+1}>h_n$ follows.

Note that
$$h_{n+2}>h_{n+1}\Leftrightarrow\frac{f_{n+1}^{N-1}}{S_{n+1}^{N-1}}>\frac{f_n^{N-1}}{S_n^{N-1}}\Leftrightarrow\frac{S_n^{N-1}}{f_n^{N-1}}>\frac{S_{n+1}^{N-1}}{f_{n+1}^{N-1}} \tag{*}$$

Next observe that for any $N-1\geq k\geq 0$,

$$\frac{f_{k+1}^{N-1}}{f_k^{N-1}}=\frac{\binom{N-1}{k+1}q^{k+1}(1-q)^{N-k-2}}{\binom{N-1}{k}q^k(1-q)^{N-k-1}}=\frac{q}{1-q}\frac{N-k-1}{k+1}$$

This is clearly decreasing in k so that in particular,

$$\frac{f_n^{N-1}}{f_{n-1}^{N-1}}>\frac{f_{n+1}^{N-1}}{f_n^{N-1}}$$

Combined with the induction hypothesis expressed as (*), we have

$$\frac{S_n^{N-1}}{f_{n-1}^{N-1}} > \frac{S_{n+1}^{N-1}}{f_n^{N-1}}$$

Adding 1 to both sides of the inequality yields

$$\frac{S_{n-1}^{N-1}}{f_{n-1}^{N-1}} > \frac{S_n^{N-1}}{f_n^{N-1}}$$

which is precisely $1/h_n > 1/h_{n+1}$, completing the proof by induction.

(viii)
$$\begin{aligned}\frac{\partial h_n(q)}{\partial q} &= \left[(\partial f_{n-1}^{N-1}/\partial q)\sum_{k=n-1}^{N-1} f_k^{N-1} - f_{n-1}^{N-1}\sum_{k=n-1}^{N-1}(\partial f_k^{N-1}/\partial q)\right]\Big/(S_{n-1}^{N-1})^2 \\ &= f_{n-1}^{N-1}\left[(n-1-(N-1)q)\sum_{k=n}^{N-1} f_k^{N-1} - \sum_{k=n}^{N-1} f_k^{N-1}(k-(N-1)q)\right]\Big/ q(1-q)(S_{n-1}^{N-1})^2 \\ &= \frac{f_{n-1}^{N-1}\sum_{k=n}^{N-1} f_k^{N-1}(n-1-k)}{q(1-q)(S_{n-1}^{N-1})^2} < 0\end{aligned}$$

The inequality follows from the facts that the summation is over $k > n-1$, $f_k^{N-1} > 0$ on the summation range and $f_{n-1}^{N-1} > 0$ for $n \geq 1$ and the summation range is non-trivial for $n \leq N-1$. Note that when n takes its extremal values of $n = 0$ and $n = N$, the derivative equals zero since h_n is then fixed at 0 and 1, respectively.

(ix) Clearly the inequality holds when $q = 0$ or $n = N$. Observe next that for all $n < N$, $n < \mathbb{E}[k|k \geq n] = (\sum_{k=n}^N k f_k^N)/S_n^N$. Hence, using Lemma B.1(iii) and (ii)

$$\frac{qNS_{n-1}^{N-1}}{S_n^N} > n \Leftrightarrow qNS_{n-1}^{N-1} < nS_n^N = n(S_{n-1}^{N-1} - (1-q)f_{n-1}^{N-1}) \Leftrightarrow qN > n(1-(1-q)h_n)$$

■

Proof of Lemma 1.

From (5) and $\mathbb{E}[k|k \geq n] = qNS_{n-1}^{N-1}/S_n^N$ we have

$$\begin{aligned}\pi_n^I - \pi_{n+1}^I &= (Nv_L - C)f_n^N + (v_H - v_L)qN\left(f_{n-1}^{N-1} - f_n^{N-1}\right) \\ &= q(Nv_H - C)\left(f_{n-1}^{N-1} - f_n^{N-1}\right) + (Nv_L - C)\,f_n^{N-1} \quad \text{by Lemma B.1(i)} \\ &> 0 \Leftrightarrow \frac{n(1-q)}{(N-n)q} = \frac{f_{n-1}^{N-1}}{f_n^{N-1}} > \frac{C - Nv_L + q(Nv_H - C)}{q(Nv_H - C)}\end{aligned}$$

The statements follow because $n(1-q)/((N-n)q)$ is increasing in n. ■

Proof of Lemma 2.

We show that for all $n < N$ and $q > 0$, $C_n(q) < \overline{T}_n$.

$$\begin{aligned} & \frac{N(v_L - qv_H) + n(v_H - v_L)}{1-q} < Nv_L + nh_n(v_H - v_L) \\ \Leftrightarrow \quad & N(v_L - qv_H) + n(v_H - v_L) < N(1-q)v_L + nh_n(1-q)(v_H - v_L) \\ \Leftrightarrow \quad & n(v_H - v_L)(1 - h_n(1-q)) < qN(v_H - v_L) \end{aligned}$$

The result follows from Lemma B.1(ix). ■

Proof of Proposition 1.

It remains to show that $\pi^I_{n_I}(q) \geq \pi^E_{n_E}(q)$ if and only if $q < \hat{q}$ with a strict inequality on $q < \hat{q}$ when $n_I < N$ and on $q > \hat{q}$ when $n_E < N$. Below we prove the stronger claim that for any N and any $0 \leq n < N$, $\pi^I_n(q) > \pi^E_n(q)$ if and only if $q < \hat{q}$. The result for optimized strategies follows quickly from this. Consider the case with $q < \hat{q}$. If $n_I < N$ and $n_E = N$ then $\pi^I_{n_I} > \pi^I_N = (Nv_H - C)q^N = \pi^E_N = \pi^E_{n_E}$.[57] If $n_I, n_E < N$ then $\pi^I_{n_I} \geq \max_{n<N}\{\pi^I_n\} > \max_{n<N}\{\pi^E_n\} = \pi^E_{n_E}$. The proof for $q > \hat{q}$ is an exact parallel. We now prove the stronger claim.

$$\begin{aligned} \pi^I_n(q) - \pi^E_n(q) &= \sum_{k=n}^{N} f^N_k(q)\Big[(N-k)v_L - k(v_H - \bar{b}_n)\Big] \\ &= N\Big[(1-q)S^{N-1}_n v_L - qS^{N-1}_{n-1}(1-h_n)(v_H - v_L)\Big] \\ &\qquad \text{(using respectively Lemma B.1(iv),(iii) and Section 3.2)} \\ &= NS^{N-1}_n\Big[(1-q)v_L - q(v_H - v_L)\Big] \\ &= NS^{N-1}_n(v_L - qv_H) \\ &> 0 \Leftrightarrow q < \hat{q} \end{aligned}$$

for any $n \in \{0, 1, ..., N-1\}$ since then $S^{N-1}_n > 0$. ■

Proof of Proposition 2.

Part (i). For parameter regions where exclusion is optimal, the highly intuitive result that

[57] This statement holds generically, but the inequality is replaced by an equality at the knife-edge case where $n_I = N - 1$ and $n = N$ deliver identical payoffs. That is, the statement holds almost everywhere, but not at the atom where $\tilde{n}_I = N - 1$. This trivial complication is just a result of the fact that profits are continuous in C, q but the integer-valued n_I is not.

profits are decreasing in C and increasing in q is easily verified from the profit expression:

$$\pi^E_{n_E} = \sum_{k=n_E}^{N} f^N_k (kv_H - C) = E_k[\max\{0, kv_H - C\}]$$

where E_k denotes the expectation operator. Since an increase in q induces a first-order stochastic dominating distribution of k, and the expectation is taken over an increasing (utility) function, the expectation is increasing in q. The impact of C is more immediate: profits fall at the rate $S^N_{n_E}$.

From the proof of Lemma 1 we know that $\pi^I_n = (Nv_L - C)S^N_n + (v_H - v_L)qNf^{N-1}_{n-1}$. Taking derivatives with respect to q yields

$$\begin{aligned}
\frac{\partial \pi^I_n}{\partial q} &= (Nv_L - C)(\frac{\partial S^N_n}{\partial q}) + N(v_H - v_L)\left(f^{N-1}_{n-1} + \frac{\partial f^{N-1}_{n-1}}{\partial q} q\right) \\
&= (Nv_L - C)Nf^{N-1}_{n-1} + N(v_H - v_L)f^{N-1}_{n-1}\left(1 + q\frac{n-1-(N-1)q}{q(1-q)}\right) \\
&\qquad\qquad\qquad\qquad\qquad\qquad \text{(by Lemmas B.1(vi) and (v))} \\
&= (Nv_L - C)Nf^{N-1}_{n-1} + N(v_H - v_L)f^{N-1}_{n-1}\left(\frac{n - Nq}{1-q}\right) \\
&= \frac{Nf^{N-1}_{n-1}}{1-q}\Big((Nv_L - C)(1-q) + (v_H - v_L)(n - Nq)\Big) \\
&= \frac{N(v_H - v_L)f^{N-1}_{n-1}}{1-q}\Big(n - \tilde{n}_I\Big)
\end{aligned}$$

Recall that $\tilde{n}_I = \frac{C - Nv_L + q(Nv_H - C)}{v_h - v_L}$ and $n_I = \lceil \tilde{n}_I \rceil$. So $n_I > \tilde{n}_I$ except at critical values of q at which $n_I = \tilde{n}_I$. These exceptional values have measure zero; they occur on the boundary between strategy types. It follows that the maximal profit $\pi^I_{n_I}$ is strictly increasing in q. ■

Proof of Proposition 3.

In general, bidding above $p = v_L$ can be attractive only if it increases the probability of production. In a candidate equilibrium where L-types bid $p = v_L$, H-types bid $\bar{b}_n$ and threshold equals $\overline{T}_n = n\bar{\delta}_n + Nv_L$, an individual buyer bidding $b \geq p$ generates the project success rate S^{N-1}_ℓ where $\ell = \lceil \frac{b-p}{\bar{\delta}_n} \rceil$. Bidding above p reduces by ℓ the number of the other $N-1$ buyers who need to be H-type for the project to succeed. Notice that bid increments that do not raise ℓ are weakly dominated, so we need only consider bids of the form $b = v_L + \ell\bar{\delta}_n$ for integer values of ℓ. For the same equilibrium choices to remain valid without bid restrictions, we need to check that H-types are willing to set $\ell = 1$. Deviating to $\ell = 0$ is not a problem by incentive compatibility in the full-commitment solution. It remains to verify that deviating to a bid $b = v_L + \ell\bar{\delta}_n$ is weakly inferior for integer values of

$\ell \geq 2$ in the case of $n = n_I$, but it is as simple to prove it for all n so we do.

In the putative equilibrium without bid restrictions, with $p = v_L$ and $T = Nv_L + n\bar{\delta}_n$, if the two types continue to make respective bids, $b_H = v_L + \bar{\delta}_n$ and b_L, then the production probability is S_n^N. From the perspective of a single buyer of the H-type playing the equilibrium strategy, this probability is higher at S_{n-1}^{N-1}, and falls to S_n^{N-1} if he deviates to bid v_L, but rises to $S_{n-\ell}^{N-1}$ if he deviates to the proposed bid with some $\ell \geq 2$. The first two options give this buyer the same expected utility because inequality (IC) binds as Section 3.2; this payoff is $(v_H - v_L)\, S_n^{N-1}$ The deviation option gives,

$$\left(v_H - v_L - \ell\bar{\delta}_n\right) S_{n-\ell}^{N-1}$$

So, substituting for $\bar{\delta}_n = h_n\,(v_H - v_L)$ and dividing by $(v_H - v_L)\, S_{n-\ell}^{N-1}$, we seek to show that,

$$(1 - \ell h_n) \leq S_n^{N-1}/S_{n-\ell}^{N-1}, \qquad \forall \ell \geq 2$$

Now the right-hand side can be written as the product of $(1 - h_n)\,(1 - h_{n-1})\, ... \,(1 - h_{n-\ell})$, but h_n is increasing in n, so this expression weakly exceeds $(1 - h_n)^{\ell}$. Now $h_n \in [0, 1]$ so defining $a = 1 - h_n$, we have $a \in [0, 1]$, so for any $\ell \geq 1$,

$$1 - a^{\ell} = (1 - a)\left(1 + ... + a^{\ell-1}\right) \leq (1 - a)\ell$$

Rearranging terms and substituting back for a, this gives $1 - \ell h_n \leq (1 - h_n)^{\ell}$, concluding the proof. ■

Proof of Proposition 4. We prove that $n_I' \leq n_I$. The statements about profits, consumer and total welfare follow from this.

Recall that $n_I = \arg\min_n\{C \leq C_n(q)\}$ where $C_n(q)$ was defined in (7). Also recall that feasibility of the n-type strategy required $C \leq \overline{T}_n$. In particular, feasibility is guaranteed for all $n \geq 1$ when $C = Nv_L$. It follows from Lemma 2 that the n_I-type strategy is feasible.

Next we show that there exist unique values $0 < q_1' < \cdots < q_{N-1}'$ so that the entrepreneur is indifferent between strategies of type n and $n+1$ (as long as both are feasible) when $q = q_n'$, independently of C. Note that $\pi_n' = (qNS_{n-1}^{N-1} - nS_n^N)\delta_n'$ where $\delta_n' = h_n(v_H - p_n') = h_n(Nv_H - C)/(N - nh_n)$. Hence, $\left[\pi_{n+1}' - \pi_n'\right]/(Nv_H - C)$ is independent of C. There must exist a q_n' where the entrepreneur is indifferent, because the difference is strictly negative when $q > 0$ is very small while it is strictly positive when $q < 1$ is close to one. Tedious calculations show that a marginal increase in q above q_n' increases the difference $\pi_{n+1}' - \pi_n'$, and the uniqueness result follows.

Some more tedious calculations show that at $q_n = n/N$, $\pi_n'(q_n) > \pi_{n+1}'(q_n)$, which implies that $q_n' > q_n$. It then follows that the optimal inclusive strategy is of type n_I' where n_I' is the smallest n such that both $q \leq q_n'$ and $C \leq \overline{T}_n$.

■

Proof of Proposition 5. The general optimum has exclusion ($j^* = 2$) when $0 > w_1 = v_1 - (v_2 - v_1)q_2/q_1$, which is equivalent to $v_1/v_2 > q_2$, in which case production occurs in state $(N-k, k)$ if and only if $kv_2 \geq C$, that is when $k \geq C/v_2$. This is readily implemented by crowdfunding with $p = v_2$ and $T = C$.

The general optimum has inclusion ($j^* = 1$) when $v_1/v_2 \leq q_2$. Production occurs in states $(N-k, k)$ for which $(N-k)w_1 + kw_2 \geq C$. That is, when $(N-k)(v_1 - (v_2 - v_1)q_2/q_1) + kv_2 \geq C$. Substituting $q_1 = 1 - q_2$ this can be rewritten as

$$k \geq \frac{C - Nv_1 + q_2(Nv_2 - C)}{v_2 - v_1} = \tilde{n}_I,$$

so that production occurs when at least $n_I = \lceil \tilde{n}_I \rceil$ are of the high type v_2. The probabilities of obtaining the good for type $j = 1, 2$ are $P_1 = S_{n_I}^{N-1}(q_2)$ and $P_2 = S_{n_I-1}^{N-1}(q_2)$. Expected transfers are $T_1 = v_1 P_1$ and $T_2 = T_1 + (P_2 - P_1)v_2$. Type-wise bids are thus $b_1 = v_1$ and $b_2 = h_{n_I} v_2 + (1 - h_{n_I})v_1$. Crowdfunding can implement this with threshold $T = n_I b_2 + (N - n_I)v_1$ and minimal price $p = v_1$. ■

Proof of Proposition 6.

To simplify notation let us normalize $v_H = 1$ and define $c = C/N$ as the cost per potential buyer. Note that now $\hat{q} = v_L$ and $0 \leq c \leq 1$.

Let us first consider the case $q > \hat{q}$ where exclusion is optimal. Recall that $n_E = \lceil C/v_H \rceil = \lceil Nc \rceil$.

The profit from a posted-price mechanism using standard funding is equal to $\pi^{SF} = \max\{0, (q-c)N\}$. The profit from crowdfunding is equal to $\pi^{CF} = \pi_{n_E} = \sum_{k=n_E}^{N} f_k^N (k - cN) = \sum_{k=n_E}^{N} k f_k^N - S_{n_E}^N cN = qNS_{n_E-1}^{N-1} - S_{n_E}^N cN$. Using Lemma B.1 we can write $\pi^{CF} = N(q-c)S_{n_E-1}^{N-1} + Nc(1-q)f_{n_E-1}^{N-1}$. We thus see that, if $c < q$

$$\pi^{CF} - \pi^{SF} = (c-q)N(1 - S_{n_E-1}^{N-1}) + (1-q)cNf_{n_E-1}^{N-1}.$$

For large N (and $q(N-1) > 5$, $(1-q)(N-1) > 5$) we can use the normal approximation to the Binomial to quantify this advantage. In particular, let $\Phi(\cdot)$ denote the CDF of the Normal distribution with mean $\mu = q(N-1)$ and standard deviation $\sigma = \sqrt{(N-1)q(1-q)}$.

Let $\phi(\cdot)$ denote the corresponding density function. Then

$$\begin{aligned}
\pi^{CF} - \pi^{SF} &\approx (q-c)N\Phi(n_E - 1.5) + (1-q)cN\big(\Phi(n_E - 0.5) - \Phi(n_E - 1.5)\big) \\
&= (q-c)N \int_{-\infty}^{cN-1.5} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx + (1-q)cN \int_{cN-1.5}^{cN-0.5} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\
&= \frac{(q-c)N}{\sqrt{(N-1)q(1-q)2\pi}} \int_{-\infty}^{cN-1.5} e^{-\frac{(x-q(N-1))^2}{2(N-1)q(1-q)}} dx \\
&\quad + \frac{(1-q)cN}{\sqrt{(N-1)q(1-q)2\pi}} \int_{cN-1.5}^{cN-0.5} e^{-\frac{(x-q(N-1))^2}{2(N-1)q(1-q)}} dx \\
&= \frac{(q-c)N}{\sqrt{(N-1)q(1-q)2\pi}} \int_{-\infty}^{c+\frac{c-1.5}{N-1}} (N-1) e^{-(N-1)\frac{(y-q)^2}{2q(1-q)}} dy \\
&\quad + \frac{(1-q)cN}{\sqrt{(N-1)q(1-q)2\pi}} \int_{cN-1.5}^{cN-0.5} e^{-\frac{(x-q(N-1))^2}{2(N-1)q(1-q)}} dx = O(N^{3/2}e^{-N}) \to 0
\end{aligned}$$

Similarly, if $c \geq q$,

$$\pi^{CF} - \pi^{SF} \approx (q-c)N(1 - \Phi(n_E - 1.5)) + cN(1-q)\big(\Phi(n_E - 0.5) - \Phi(n_E - 1.5)\big) \to 0$$

Let us now consider the case with $q \leq \hat{q}$ so that inclusion is optimal. The profit from standard funding is $\pi^{SF} = \max\{0, (v_L - c)N\}$. The profit from crowdfunding equals $\pi^{CF} = N(v_L - c)S^N_{n_I} + qNS^{N-1}_{n_I-1}(v_H - v_L)h_{n_I} = N(v_L - c)S^N_{n_I} + qN(1-v_L)f^{N-1}_{n_I-1} = N(v_L - c)S^{N-1}_{n_I-1} + Nf^{N-1}_{n_I-1}(q - \hat{q} + (1-q)c)$.

Note that $\tilde{n}_I/N = \big(c - v_L + q(v_H - c)\big)/(v_H - v_L)$ so that if $c < v_L = \hat{q}$, then $n_I < qN$ while if $c > v_L = \hat{q}$, $n_I > qN$, for large N. Hence, if $c < v_L$

$$\pi^{CF} - \pi^{SF} \approx -(v_L - c)N\Phi(n_I - 1.5) + N(q - \hat{q} + (1-q)c)\big(\Phi(n_I - 0.5) - \Phi(n_I - 1.5)\big)$$

and the advantage converges to zero at an exponential rate. If $c > v_L$

$$\pi^{CF} - \pi^{SF} \approx (v_L - c)N(1 - \Phi(n_I - 1.5)) + N(q - \hat{q} + (1-q)c)\big(\Phi(n_I - 0.5) - \Phi(n_I - 1.5)\big)$$

and the advantage converges to zero at an exponential rate. ■

Proof of Proposition 7.

Using Lemma B.1 (iii) and (ii) one can rewrite the profit expression as

$$\pi(p) = -S^N_n C + mpqNS^{N-1}_{n-1} = \big(mpqN - C\big)S^N_n + mpqN(1-q)f^{N-1}_{n-1}$$

Using Lemma B.1 (vi) and (v), the optimal p must satisfy———- FOC...

$$\begin{aligned} 0 = \frac{\partial \pi}{\partial p} &= \big(mpqN - C\big) N f_{n-1}^{N-1} q' + (mqN + mpq'N) S_n^N \\ &\quad + mpqN(1-q) f_{n-1}^{N-1} \frac{n-1-(N-1)q}{q(1-q)} q' \\ &\quad + \big(mqN(1-q) + mpN(1-2q)q'\big) f_{n-1}^{N-1} \end{aligned}$$

Using again Lemma B.1 (ii), and defining $c = C/Nm$, this is equivalent to

$$\begin{aligned} 0 &= \big(pqN - cN\big) f_{n-1}^{N-1} q' + (q + pq')\big(S_{n-1}^{N-1} - (1-q) f_{n-1}^{N-1}\big) \\ &\quad + p f_{n-1}^{N-1} \big(n-1-(N-1)q\big) q' \\ &\quad + \big(q(1-q) + p(1-2q)q'\big) f_{n-1}^{N-1}. \end{aligned}$$

Taking out a factor S_{n-1}^{N-1} and rearranging yields

$$0 = q + pq' + h_n q' \big((n-1)p - cN\big).$$

Note that $n = \lceil cN/p \rceil$ so that $(n-1)p - cN < 0$ while $q' = -g(p) < 0$ when G has a positive density. Hence, a marginal increase in c that leaves n fixed leads to a strict increase in p. ∎

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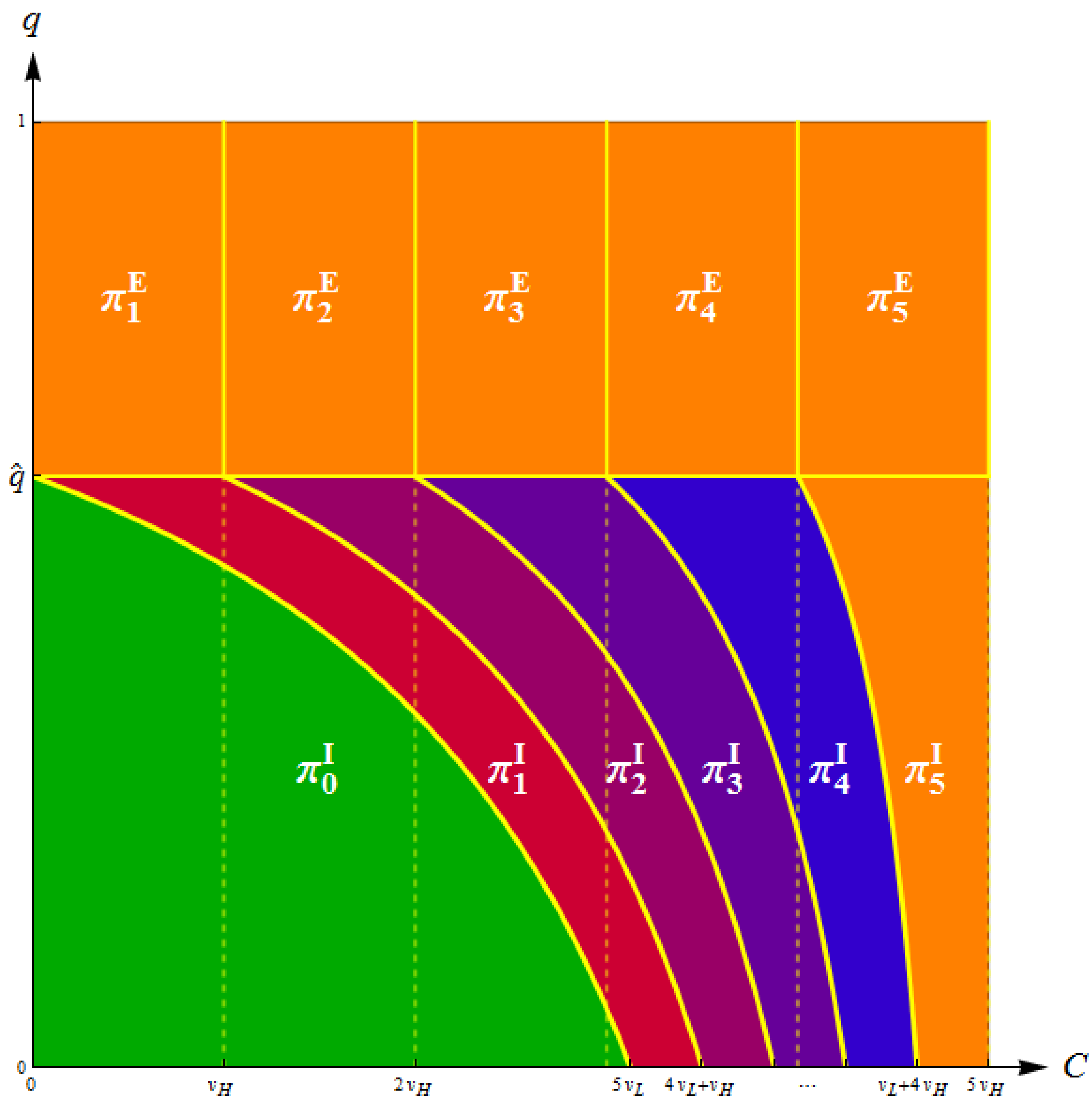

Figure 1: Optimal selling strategies in (C, q)-space ($N = 5$, $v_L = 1$, $v_H = 1.6$).

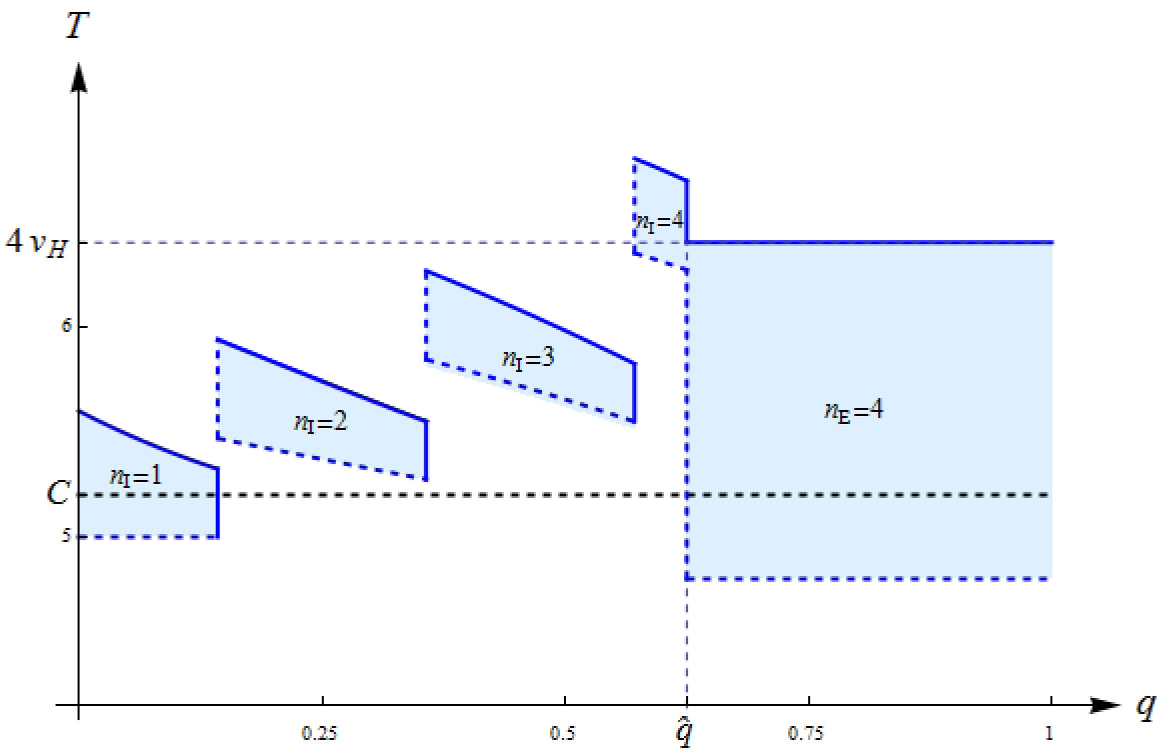

Figure 2: Optimal funding threshold correspondence of q for $C = 5.2$, $N = 5$, $v_L = 1$, $v_H = 1.6$.

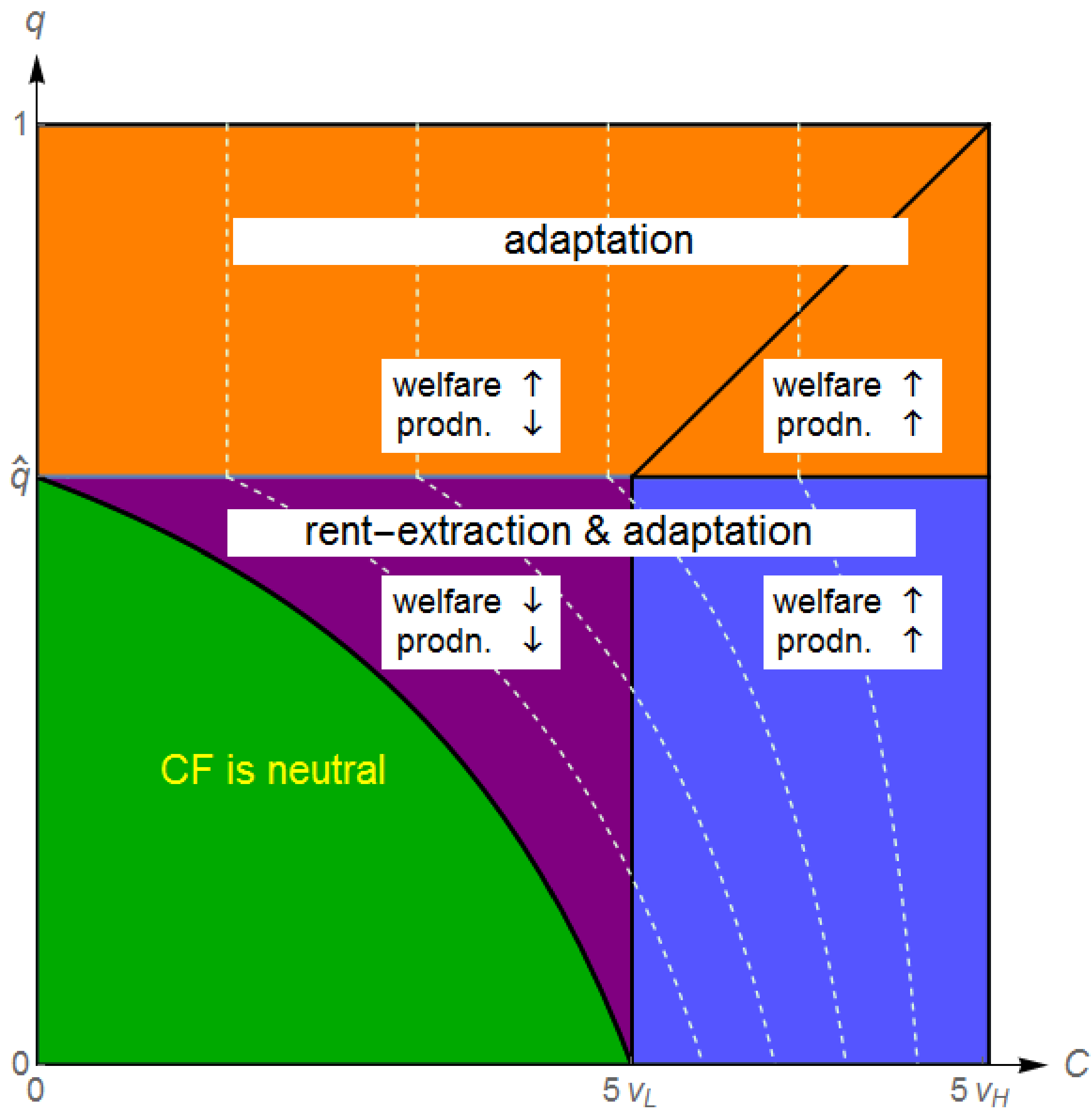

Figure 3: Welfare effects of crowdfunding relative to standard finance, with adaptation of production (to higher demand states to avoid losses) and rent-extraction (where threshold effect induces price-discrimination) indicated.

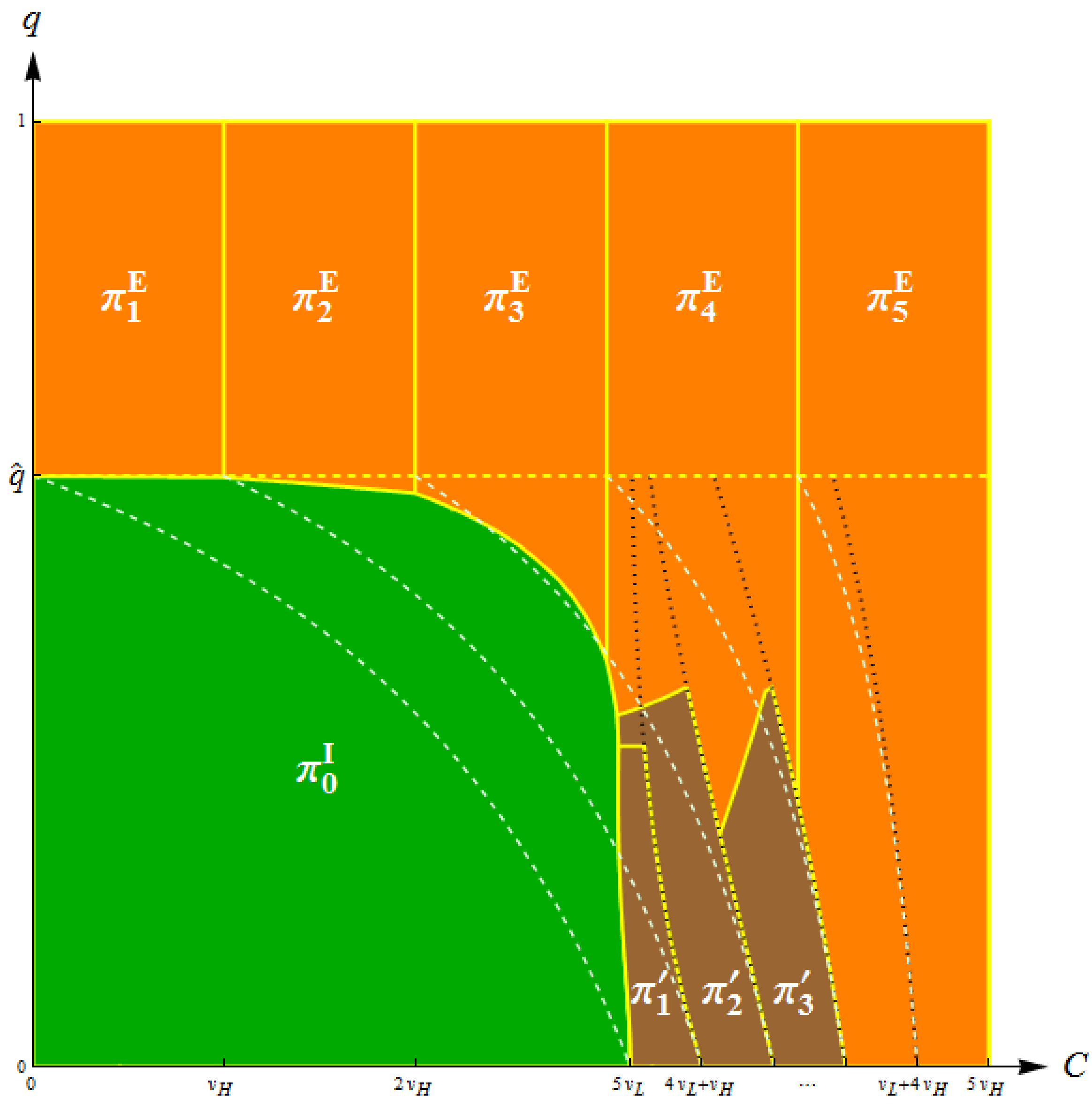

Figure 4: Optimal selling strategies in (C, q)-space under no threshold commitment with just a minimal bid ($N = 5$, $v_L = 1$, $v_H = 1.6$).

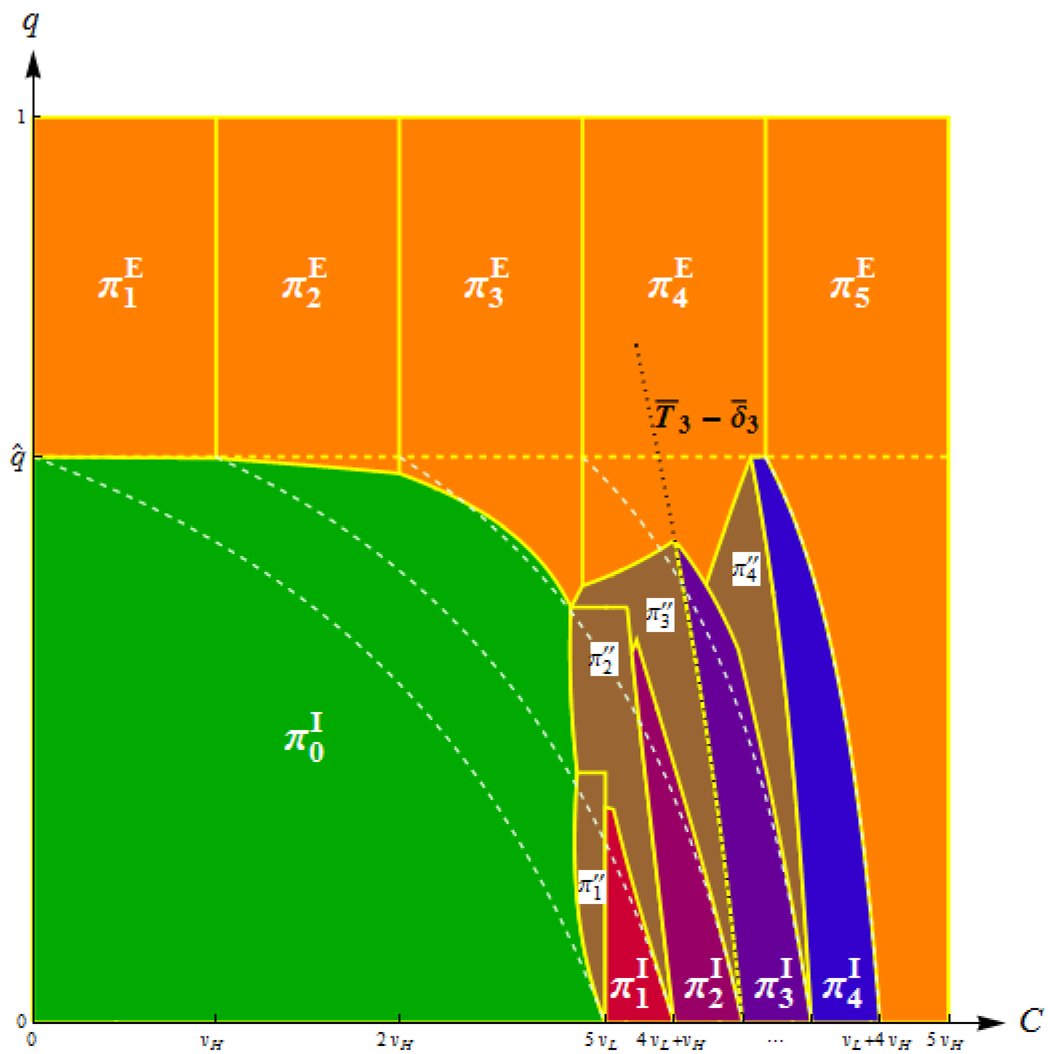

Figure 5: Optimal selling strategies in (C, q)-space under no threshold commitment but with bid restrictions ($N = 5$, $v_L = 1$, $v_H = 1.6$).

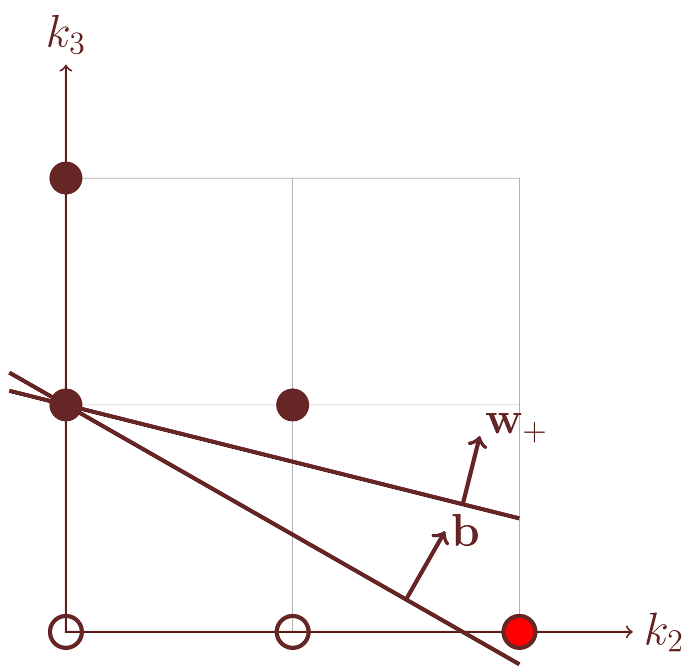

Figure 6: Crowdfunding cannot implement the general optimum.

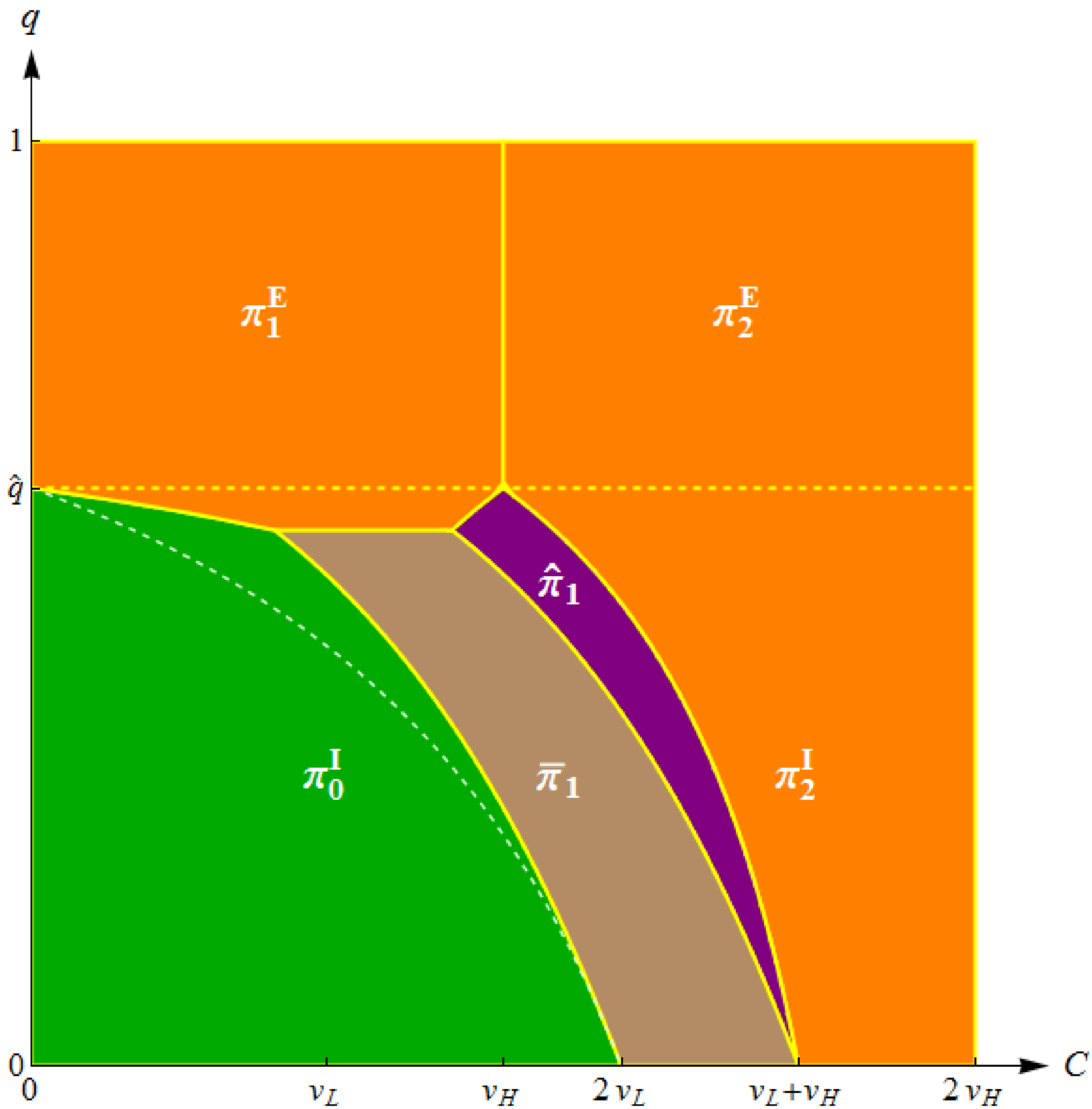

Figure 7: Optimal selling strategies for sequential bidding in (C, q)-space ($N = 2$, $v_L = 1$, $v_H = 1.6$).