Competing one-way essential complements: the forgotten side of net neutrality

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Abstract

We analyze the incentives of internet service providers (ISPs) to break net neutrality by excluding internet applications competing with their own products, a typical example being the exclusion of VoIP applications by telecom companies offering internet and voice services. Exclusion is not a concern when the ISP is a monopoly because it can extract the additional surplus created by the application through price rebalancing. When ISPs compete, it could lead to a fragmented internet where only one firm offers the application. We show that, both in monopoly and duopoly, prohibiting the exclusion of the app and surcharges for its use—a strong form of net neutrality—is not welfare improving.

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1. Introduction

In 2005, Madison River, a US internet service provider (ISP), excluded Vonage, a Voice over IP application (VoIP), from its network, which resulted in a conflict between stakeholders over the control of the bundle of services offered on the internet. Most ISPs offer multiple services—internet, phone, television and video, etc.—while applications such as Vonage are competing with these services. On the one hand, these applications create a business stealing effect and excluding them is a way for the ISP to limit unwanted competition. On the other hand, these applications create value for internet users who are willing to use and to pay for these new services. That value can possibly be extracted by the ISP through higher internet prices and, therefore, exclusion might not necessarily be optimal. The interplay of these two types of incentives is the main object of this article.

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This exclusion/no-exclusion problem is part of the larger “net neutrality” discussion. Because it is still a very lively debate, net neutrality does not have a definitive unitary definition. Still, Schuett (2010) summarizes it as “the principle that all data packets on an information network are treated equally”. The literature (Choi and Kim (2010); Economides and Hermalin (2012); Reggiani and Valletti (2012) for instance) has generally focused on two implications of this principle: the non-discrimination rule and the zero-price rule.

The first interpretation simply means that a bit is a bit and that contents should be treated similarly, regardless of their nature and origin. For example, there should be no prioritization: the bits sent by Youtube should not be transferred faster than those sent by Vimeo. Similarly, traffic management should be limited to isolated cases and the exclusion of particular applications –the most extreme form of discrimination– should be forbidden. Furthermore, the non-discrimination rule also implies that internet users can use the applications without paying an extra fee to the ISP. Stated differently, the ISP cannot condition the use of an application, a VoIP app for instance, to the payment of a surcharge. So, the non-discrimination rule prohibits the exclusion of competing apps and price surcharges for using such apps. In what follows, we distinguish two versions of net neutrality: a strong one and a weak one. An ISP complies with strong net neutrality if there is no exclusion of the app and no surcharge to use it. An operator complies with weak net neutrality if there is no exclusion but a surcharge to use the app.3

The zero-price rule prohibits financial transfers between residential ISPs and content producers (CP). On the internet, CP’s pay a backbone provider to be connected to the network and residential consumers pay to be connected to an ISP.4 According to the zero-price rule, the ISPs do not have the right to make CPs pay a termination fee for access to internet consumers. The zero-price rule implies that there is a “missing price”5 prohibiting financial transfers between CPs and ISPs.6 The zero-price rule and the non-discrimination rule have been criticized for prohibiting the emergence of value-added services on the internet.

Although the literature has generally focused on the implications of net neutrality on congestion (Choi and Kim (2010); Choi et al. (2014); Peitz and Schuett (2014) and Economides and Hermalin (2012)), innovation and investment (Reggiani and Valletti (2012); Bourreau et al. (forthcoming) and Choi et al. (2013)), we believe that the exclusion of competing applications is one of the most overlooked issues in the debate. Indeed, as highlighted in a BEREC report (BEREC, 2012), most of the alleged net-neutrality breaches are concentrated in two areas: data-intensive services, and applications competing with ISPs’ own services. We will focus on this second category where examples abound.

Let us consider, for instance, the first famous net neutrality breach, committed by Madison

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3Note that our definitions of weak and strong net neutrality differ from those of Gans (forthcoming) who states that net neutrality is strong if content-based price discrimination is outlawed both with regard to CPs and consumers, and that it is weak if discrimination is outlawed with regard to one group only.

4See Faratin et al. (2008) and Economides and Hermalin (2012) for more on the structure of the internet and net neutrality.

5For an analysis of net neutrality as a case of missing prices, see Jullien and Sand-Zantman (2012).

6Recently, Netflix decided to bypass the backbone provider and to connect directly to Comcast, a residential ISP. Many commentators (Rayburn (2014); The Economist (2014)) did not view the deal as a net neutrality breach because it focused on direct interconnection between Comcast and Netflix, not on an additional fee to reach end-users.
River, which we highlighted at the beginning of this introduction. After the blocking of Vonage, the FCC intervened, fined and made Madison River sign a consent decree to stop the throttling (FCC, 2005). Let us also consider the case which resulted in a net neutrality law in the Netherlands (International Telecommunications Union, 2012). In 2010, KPN, a Dutch ISP, started to develop a new strategy towards competing applications: users either had to pay to use Skype and WhatsApp or face blocking. The Dutch parliament reacted by enacting one of the first net neutrality laws in the world, effectively putting a halt to KPN’s strategy. The reaction has not been so prompt in Spain where ISP Yoigo is still making mobile users pay for access to VoIP applications: users have to pay a fee for mobile data and an additional fee if they want to use VoIP. The Swedish counterpart of Yoigo, Teliasonera, also tried to set the same pricing scheme but had to withdraw it after a public uproar (Grundberg, 2012). Hence, the intertwining of applications and ISPs’ own services, though overlooked by the literature, is a major issue.

To fully understand the issue, we build a model that focuses on the interaction of two markets: internet and voice services. The ISP has an installed network and offers internet and phone services to consumers. An alternative firm competes on the voice market by offering some VoIP software to internet users. Consumers thus are offered three products: the internet, the phone and the VoIP application (hereafter “the app” or “the application”).

Our model has four specific features. First, the app and the phone are horizontally differentiated substitutes. Second, the app needs the internet to work but the phone does not. The internet and the app therefore are one-way essential complements (Chen and Nalebuff, 2006) and the incentives of the ISP are complex because the app is complementary to one of its products, but it is a substitute for another. Third, the price of the app is zero and its profit comes from advertising, which is considered exogenous. Finally, consumers’ valuations for the internet are heterogeneous. Because most applications competing with ISP’s products are not data-intensive, congestion is not an issue and we do not incorporate it in our model (as Choi and Kim (2010) or Economides and Hermalin (2012) do for example). We also do not allow financial transfers between ISP’s and the app, the zero-price rule is always enforced. We believe this is the best way to focus on exclusion concerns because the zero-price rule is the worst-case scenario exclusion-wise: if ISP’s can extract money from apps, they have less incentives to exclude. Thus, if the app is available in our setting, it will a fortiori also be available if the zero-price rule is relaxed.

This paper is organized around three questions. First, does an ISP have incentives to exclude a competing application, thereby violating weak net neutrality? Second, should it charge a premium to consumers to use the app, thereby breaking strong net neutrality? Last, is net neutrality welfare improving? Each of these questions is considered in a monopolistic and a duopolistic setting.

We show that a monopoly ISP never finds it profitable to exclude the app. The reason is that the monopolist can rebalance its prices to benefit from the value added by the app. The competition created by the free app is thus compensated by a complementarity effect and additional revenues from the internet. We further show that selling the internet with the app at a premium price, thereby

\[7\] Note that we have picked these goods for illustration purposes but we could have picked Netflix and TV, WhatsApp and SMS, Spotify and music services, etc.

\[8\] The exception to this may be Video On Demand.
breaking strong net neutrality, is profitable for the ISP and, surprisingly also, for consumers as well as for welfare. With strong net neutrality, too many consumers use the free product—the app—at the expense of the costly competitor—the phone. When two versions of the internet are offered, a low quality/low price without the app and a high quality/high price with the app, consumers buy their preferred voice solution, thereby increasing their surplus as well as the firm’s profit.

When several ISPs compete, we first show that it is not possible to have, in equilibrium, exclusion of the app by both ISPs. Complete exclusion of the app, therefore, is not an issue, either under monopoly or duopoly. With competition between ISPs, offering the app is a way for firms to differentiate their products and, should one firm exclude the app, the other has no incentives to do so. Indeed, this other firm can escape fierce competition from the rival ISP by offering an improved product—the internet with the app. This product is a source of profit if the firm can sell it at a premium i.e. if the firm can ask a surcharge for the use of the app.

We then characterize the equilibrium under competition, considering both symmetric and asymmetric ISPs. ISPs are symmetric when they both offer the phone, they are asymmetric when only one offers the product competing with the app. We first show that only in the symmetric case, both firms offering the app for free (i.e. complying with the strong net neutrality), is a Nash equilibrium. This equilibrium leads to the highest welfare because competition drives down the phone price to marginal cost and therefore consumers’ choice is not biased towards the free app as in the monopoly case. Second, there always exist equilibria in which one ISP excludes the app and the other offers it at a premium price. This equilibrium configuration is unique in the asymmetric case and we show that, in this case, it leads to the highest welfare. In the asymmetric case, only one firm offers the phone and if strong net neutrality is imposed, too many consumers use the free app, thereby decreasing welfare. If only one firm offers the app on the internet, either we have a multi-product monopolist in the voice market or competition between differentiated products. In both cases, prices are higher but consumers choose their preferred voice solution. Internet fragmentation where the app is available at one ISP increases product differentiation and reduces competition between ISPs. But, this lower competition effect is more than compensated by a better match between consumers’ preferences and their actual choice. For this reason, strong net neutrality does not increase welfare in the asymmetric case.

Strong net neutrality thus can be seen as a “competition intensifier” which sometimes works well—the symmetric duopoly case—but sometimes quickens the pace too much—the monopoly and the asymmetric duopoly case. We therefore conclude that net neutrality should not be seen as a one-size-fits-all rule and that having a fragmented internet where apps are only available at some ISP does not necessarily hurt welfare. An ex-post kind of regulation assessing breaches case by case thus seems preferable to imposing a strong ex-ante rule on all market participants.

Our approach and its results are linked to different papers in the literature. Chen and Nalebuff (2006) study the competition between one-way essential complements that is, two goods that are complements but where one is essential for the other to be useful. They reach two interesting conclusions. First, if the firm producing the essential product (A) cannot enter the other firm’s (B) market, A has no incentive to degrade the quality of commodity B. Second, if A can enter B’s market, it will give away a substitute to product B for free and raise the price of A. Although
our set-up is different—we have three products—our results are somewhat similar. We show that competition from a free app does not hurt ISPs if they have the ability to increase the revenue from the complement—the internet. To be able to do that, the ISP needs to be sheltered from competition either through being a monopoly or the exclusive supplier of the app.

Kourandi et al. (forthcoming) study the problem of internet fragmentation whereby some applications are only available through a particular ISP because of bilateral exclusivity contracts, and not on the internet as a whole. They show that the zero-price rule cannot always prevent fragmentation while in our framework, fragmentation takes place unless the strong net neutrality rule is enforced. They also prove that having no fragmentation is always beneficial to consumers but not always to total welfare, which accords with our conclusion.

Dewenter and Rosch (2014) consider the incentives of a monopolistic ISP to exclude competing content providers from its network in a two-sided model where CPs compete for advertisers. They show that a monopolistic ISP may find it profitable to exclude the rivals’ content if there is little product differentiation on the content market and limited indirect network externalities. In that case, the competitor steals a large fraction of the ISP’s business on the advertising market because contents are close substitutes and it cannot be compensated by higher access fees because consumers little value the additional content due to limited network effects. Our paper focuses on a similar problem except for the fact that the app and the ISP compete on one side of the market. In that case, even if products are perfect substitutes, excluding the app is never profitable. Furthermore, we take a larger view of the problem by considering competing ISPs.

The paper is organized as follows. Section 2 presents the basic model we work with. Section 3 analyzes the case of a monopolistic ISP, selling the internet and another product, and a competing app provider. Section 4 extends the model to include competition between ISPs. Section 5 presents our conclusions. All omitted proofs are in the appendix.

2. Model

There are three products: the internet, the phone and the app. The application and the internet are one-way essential complements: the app cannot be used without the internet but the internet and the phone can be used on their own. The app is free and its producer, which we henceforth ignore, finances itself through exogenous sources such as advertising or the resale of personal data. The internet and the phone are offered by a monopolistic residential ISP. We relax this assumption in Section 4 where we introduce competition. Production costs are normalized to zero.

Consumer preferences are represented by a unit square. The app and the phone are horizontally differentiated substitutes. The horizontal axis is a unit Hotelling line with the app located at 0 and the phone at 1. Consumers obtain gross utility \( u \in [0, 1] \) from consuming either the app or the phone. A consumer located at \( x \) incurs a disutility \( tx \) when he consumes the app and a disutility \( t(1-x) \) when he consumes the phone. \( t \) is a measure of product differentiation and we assume that \( u \geq 2t \) (implying \( t \leq \frac{1}{2} \)) to guarantee full market coverage in the monopoly case without the app. The vertical axis is a unit line, representing consumers’ heterogeneous valuations \( \theta \) for the internet with \( \theta \in [0, 1] \). Consumers are uniformly distributed on the unit square. The population size is normalized to 1.
Let us denote the internet by $i$, the phone by $t$ and the app by $a$. The consumer can choose between four combinations of goods, as consumers will not use the app and the phone together because of our assumption on $u$ and $t$: $(i, t)$, $(i, a)$, $(i)$, $(t)$; they could also consume nothing ($\emptyset$).

The ISP offers the internet and the phone at prices $p_i \geq 0$ and $p_t \geq 0$, and consumers either subscribe or not to the services. The associated net utilities for a consumer located at $(x, \theta)$ are:

$$U(x, \theta) = \begin{cases} 
U(i, t) = \theta - p_i + u - (1 - x)t - p_t \\
U(i, a) = \theta - p_i + u - tx \\
U(i) = \theta - p_i \\
U(t) = u - (1 - x)t - p_t \\
U(\emptyset) = 0 
\end{cases}$$

We ignore the possibility for the ISP to bundle the phone and the internet but we consider the possibility for the ISP to offer a version of the internet where the app is disabled. In that case, there are two prices for the internet: $p_i$ without the app and $\tilde{p}_i$ with it. We also impose that the zero-price rule is always enforced. We define two versions of net neutrality: weak and strong.

**Definition 1.** A firm respects strong net neutrality if it allows the app on its network without any additional fee. A firm respects weak net neutrality if it allows the app on its network but charges a fee for its use.

In other words, net neutrality is strongly respected if there is no additional payment for the use of the app ($p_i = \tilde{p}_i$). Net neutrality is weakly respected if the app is available but consumers have to pay a surcharge to use it ($p_i < \tilde{p}_i$). When Yoigo asks a surcharge for VoIP applications, it respects weak net neutrality but not strong neutrality. When Madison River excludes Vonage, it does not even respect weak net neutrality.

### 3. Monopoly ISP

In this section, we consider a monopolistic ISP. If the ISP offers the internet and the phone at prices $p_i \geq 0$ and $p_t \geq 0$, its profit is $\Pi = d_i p_i + d_t p_t$, where $d_i$ and $d_t$ are respectively the demand for the internet and for the phone.

#### 3.1. Exclusion

Let us start with the case where the app is excluded by the ISP. The demand for internet at price $p_i$ is $d_i = 1 - p_i$ and the ISP’s profit is maximized for $p_i^{excl} = \frac{u}{2}$. The demand for the phone at price $p_t$ is $d_t = \min\left[\frac{u - p_t}{t}, 1\right]$. Under the assumption that $u \geq 2t$, the profit maximizing price is $p_t^{excl} = u - t$ and the market is fully covered ($d_t = 1$). The total profit of the ISP is $\Pi^{excl} = \frac{u}{4} + u - t$.

Figure 1 represents consumers’ product choice. Consumers with a high valuation of the internet buy both goods while those with a low valuation only buy the phone. As transportation cost is low enough ($t \leq u/2$), all consumers buy (at least) one product.

#### 3.2. No Exclusion

The impact of the app’s entry on the ISP’s profit is difficult to assess *a priori* because of two competing effects. On the one hand, there is a **complementarity** effect. Some users obviously
benefit from the availability of the free app. This higher utility, or higher willingness to pay, can be extracted through a rise in the price of the internet, which will increase profit. On the other hand, the app’s presence leads to a competition effect, whereby some consumers switch from the phone to the app. The impact of these two effects is a priori unclear.

Our first result, as summarized in proposition 1, is that the ISP can always rebalance its prices such that its profits when the app is available is at least as big as when the app is excluded. Hence, exclusion is never a profitable strategy.

**Proposition 1.** A monopolistic ISP never finds it profitable to exclude a competing application. Net neutrality is always weakly respected.

**Proof.** Suppose that the ISP sets \( p_i = \frac{1}{2} + u - t \) and \( p_t = u - t \). At these prices, given that \( u \geq 2t \), we have \( u(i, u) \geq u(i, t) \) and \( u(t) \geq u(\emptyset) \) for all consumers. Thus, internet users will use the app and not the phone and the market is fully covered. Consumers prefer the app and internet to the phone if \( \theta \geq 2xt - pt - t + pt = 2xt + \frac{1}{2} - t \). From that, we can compute the demand for internet and for phone (see Figure 2). At prices \( p_i = 1/2 + u - t \) and \( p_t = u - t \), \( d_i = d_t = \frac{1}{2} \) yielding a profit of \( \frac{1}{4} + u - t \) which is identical to the profit \( \Pi_{\text{excl}} \) in the case of exclusion. 

A monopolistic ISP will never exclude an application; therefore, net neutrality in its weak form is always respected. The ISP can always compensate for any loss on the phone market by rebalancing its prices to extract the extra surplus due to the app’s entry. To obtain the same profit as without the app, the ISP only has to increase the price of the internet by \( u - t \), which is exactly the price of the phone without the app. Note also that by increasing the price of the internet, the ISP reduces competition on the voice market. Indeed, because the app and the internet are one-way essential complements, a rise in the price of the essential good is similar to one in the price of the non-essential product.

To push the analysis further, we derive the optimal prices \((p_{\text{noexcl}}^i, p_{\text{noexcl}}^t)\) in the monopoly case.

**Proposition 2.** Define \( \bar{\ell} = \frac{1}{3}(5 - \sqrt{17}) \approx 0.219 \) and \( \bar{u} = \frac{2\ell^2 - 3\ell}{2\ell - 1} \). If \( 0 < t \leq \bar{\ell} \) and \( \bar{u} \leq u \), the market is fully covered at equilibrium and the equilibrium prices are \( p_{\text{noexcl}}^i = 1/2 + u - t \) and \( p_{\text{noexcl}}^t = u - t \).
If these conditions are not satisfied, the market is not fully covered at equilibrium, \( p_t > u - t \) and \( \Pi > \Pi^{excl} \).

Proposition 2 defines the optimal prices under the weak net neutrality constraint. We observe that if the market is covered, the internet price increases by \( p_t \): \( p_{i}^{\text{noexcl}} = p_{i}^{\text{excl}} + p_t \) and losses on the voice market due to competition are compensated by an increase in the internet price. In that case, all internet users switch to the free app. If products are sufficiently differentiated, the ISP increases the phone price above \( u - t \). The market is no longer be fully covered at equilibrium and the ISP can extract strictly higher profits.

We now consider the possibility for the ISP to ask for a surcharge to use the app, breaching in this case strong net neutrality. We show that selling two versions of the internet, one where the app is enabled at price \( \tilde{p}_i \) and one where it is disabled at price \( p_i \) increases consumers segmentation and therefore profits. Indeed, the lower-quality internet at low price \( p_i \) entices some consumers –who would only buy the internet otherwise– to also buy the phone. We show that this situation benefits both consumers and the ISP and therefore increases welfare.

**Proposition 3.** A monopolistic ISP always finds it profitable to ask internet users for a surcharge for the use of the free application. Net neutrality, therefore, is always weakly – never strongly – respected.

**Proof.** Suppose that the ISP rebalances its product bundle and its tariff to offer a premium version of the internet, enabling the app, at price \( \tilde{p}_i = p_{i}^{\text{noexcl}} \), a standard version of the internet without the app at price \( p_i = p_{i}^{\text{noexcl}} - p_{i}^{\text{excl}} \) and the phone at price \( p_t = p_{t}^{\text{noexcl}} \). Internet users will keep the app if they are located at \( x < \frac{1}{2} \) and switch to the phone if \( x > \frac{1}{2} \), leaving the profits unchanged. In addition, for some phone users \( u(i, t) > u(t) \) and there will be an expansion of the demand for internet leading to a strictly higher profit. Thus a monopolistic ISP has no incentives to exclude the app but it has incentives to ask for a premium for using it.

Notice that if the market is fully covered at equilibrium with two prices \( (p_{i}^{\text{noexcl}}, p_{t}^{\text{noexcl}}) = (\frac{1}{2} + u - t, u - t) \), then the equilibrium prices without strong net neutrality are \( (\tilde{p}_i, p_i, p_t) = (\frac{1}{2} + u - t, \frac{1}{2}, u - t) \) and the corresponding profit is equal to \( \Pi = \frac{1}{4} + u - t + \frac{1}{4} \).

**Proposition 4.** If the market is covered \( (0 < t \leq \bar{t} \text{ and } u \leq u) \) imposing strong net neutrality decreases consumer surplus and welfare.
The intuition is easy to understand. With strong net neutrality, consumers are partitioned in two sets, phone users and internet + app users. Without strong net neutrality, consumers can buy \((i, a)\) and \((i, t)\) at the same price and they therefore choose their preferred product. Therefore, consumer surplus strictly increases. Moreover, there is an additional demand expansion effect that increases further consumer surplus and the ISP’s profit. We therefore conclude that net neutrality rules are not necessary when there is a monopoly ISP.

4. Duopoly

We now consider that there are two competing ISPs, ISP\(_1\) and ISP\(_2\). We consider two different cases. In the first, the symmetric case, ISPs are both offering the internet and the phone. In the second, the asymmetric case, ISP\(_1\) offers the internet and the phone while ISP\(_2\) only offers the internet. Although the first case is more likely if we think of examples such as the phone and a VoIP app, the second aims at representing interactions of goods such as Netflix and ISPs’ VOD products that are not offered by all ISPs.\(^9\)

We will use the concept of fragmented internet (Kourandi et al., forthcoming) to refer to a situation where the application is available at one ISP but not at the other. The concepts of fragmentation and net neutrality are closely linked: fragmentation implies that one ISP is breaking net neutrality by excluding the app.

4.1. Symmetric ISPs

We first consider two symmetric ISPs competing à la Bertrand. We assume that consumers cannot multihome and buy internet at one ISP and the phone at the other. The game is played in the following way:

**Timing of the game.**

1. ISPs decide to exclude or allow the application,
2. ISPs set the prices of the internet \((p_i)\), the internet with the app \((\tilde{p}_i)\) and the phone \((p_t)\).

Let us start with the analysis of the second stage of the game. If both ISPs adopt the same policy towards the app—exclusion or no exclusion—, they are perfectly symmetric, Bertrand competition leads to marginal cost pricing and profits are zero. All the consumers are buying the internet and, if the app is available, consumers choose the closest voice solution, the app for consumers located at \(x \in [0, 1/2]\) and the phone for consumers located at \(x \in [1/2, 1]\). If the app is not available, all consumers buy the phone.

But homogeneity is not a definitive curse: one ISP could exclude the app from its network. In this case, the internet with the app is only offered by one firm. Still, the internet without the app and the phone are offered by the two firms leading to \(p_i = p_t = 0\). The firm offering the app chooses

\(^9\)The ISPs may decide in a previous stage on the bundles of services they want to offer. Our two structures can thus be endogenized.
a surcharge equals to $\tilde{p}_i = t/2$, consumers located at $x \in [0, \frac{1}{4}]$ buy the app and the ISP realizes a profit equal to $t/8$.

Through this exclusion, the ISP creates differentiated internet products catering to different consumers. One could see these products as a “high-quality” internet with the app and a “low-quality” internet without the app. Allowing the app enables the ISP to sell the internet with the app at a positive price, $\tilde{p}_i = t/2$, yielding a positive profit. Notice that because $p_i \neq \tilde{p}_i$, only the firm offering the app complies with the weak net neutrality criterion.

The above results are summarized in the pay-off matrix in table 3. The details of the computations are relegated to the appendix.

Turning to the first stage of the game, it is clear that there are three Nash equilibria: one where both firms allow the app and offer it for free, and two where only one firm allows the app and offers it at a premium price while the other excludes it. Thus, as in the monopoly case, the application will not be totally excluded. But it is possible that the internet becomes fragmented with only one ISP offering the app. Intuitively, if one ISP excludes the app, the other can offer a differentiated product, thereby making positive profits by charging a positive price for this good. Compared to the monopoly case, there is no longer a competition effect created by the app, as competition already exists on the phone market. Thus there remain the complementarity effect and the possibility to monetize the value created by the app. This possibility only exists if there is reduced competition, i.e. if only one firm offers the app. Therefore, total exclusion is never an equilibrium.

**Proposition 5.** If two symmetric ISPs compete à la Bertrand, there are two classes of equilibrium: one where the internet is not fragmented and both firms respect strong net neutrality, and another where the internet is fragmented and one firm respects weak net neutrality while the other excludes the app.

Computing the welfare effects of each class of equilibrium, we show that:

**Proposition 6.** Welfare and consumer surplus are highest under the strong net neutrality equilibrium.

Strong net neutrality is pro-competitive and drives down all prices to marginal cost. Consumers then choose their preferred voice solution, thereby minimizing transportation costs. If only one firm offers the app, the internet will be offered at a premium price and fewer consumers will buy it, thereby increasing transportation cost and decreasing welfare. With symmetric ISP, the strong net neutrality rule is pro-competitive and welfare enhancing. As we will see this result only holds true under the symmetry assumption.

Figure 3: Pay-off Matrix in the Symmetric Case
4.2. Asymmetric ISPs

Let us now suppose that ISP₁ sells the phone and the internet while ISP₂ only sells the internet. First, because of the asymmetric situation, ISP₁ has the upper hand: whatever the choice of the other, it will always get a positive pay-off because it has one differentiated good, the phone. Second, the only way for ISP₂ to have a positive pay-off is to offer the app when ISP₁ excludes it. The equilibrium in the pricing game is represented by the pay-off matrix in table 4.

Proposition 7. If two asymmetric ISPs compete à la Bertrand, the only pure Nash equilibria are those with fragmentation.

The logic is the same as before. Because they want to avoid strong competition, each ISP is willing to differentiate its products by offering the app when the other excludes it. But there is an additional effect implying that having both firms offering the app is no longer an equilibrium. Indeed, when ISP₂ offers the app, the rival prefers to exclude it. With both firms offering the app, competition for the internet is intense which, in turn, creates a strong competitive pressure on the phone which is offered exclusively by ISP₁. If ISP₁ excludes the app, it relaxes competition on the voice market as it is now costly to buy the internet with the app at ISP₂. ISP₁ can then charge a higher price for the phone. This effect—which was not present in the symmetric case because of Bertrand competition for the phone—implies that the strong net neutrality outcome is not an equilibrium in the asymmetric case. Strong net neutrality will never be enforced and the internet will always be fragmented when ISPs are asymmetric. Interestingly, we can show that imposing the strong net neutrality rule increases consumer surplus but not welfare.

Proposition 8. Consumer surplus is highest under strong net neutrality. Total welfare on the other hand is highest under fragmentation.

With strong net neutrality, only one product—the phone—is offered at a price above its marginal cost with, consequently, too many consumers using the free app and an increase in transportation costs. When the internet is fragmented, there is less competition between ISPs: either the two ISPs compete with differentiated products, ISP₁ offering the internet and the phone and ISP₂ offering the internet with the app, or ISP₁ is the exclusive provider of the phone and the app. Lower competition in a fragmented internet has two impacts on consumers. This has an obvious negative impact as consumers pay a higher price but also a positive impact as consumers will buy their preferred voice solution thereby minimizing transportation costs. In terms of welfare, higher prices are compensated by higher profits and welfare is highest in the fragmented internet thanks to the reduction in transportation costs. In terms of consumer surplus, it is highest under
strong net neutrality with the impact of higher transportation costs being more than compensated by lower prices. Finally, let us notice that, for consumers the worst situation is that where one ISP monopolizes both the phone and the app. Hence, while the strong net neutrality rule is pro-competitive and beneficial to consumers, it does not lead to the highest welfare.

4.3. Exclusivity

In both the symmetric and the asymmetric cases, the profit of an ISP is highest if it is the sole provider of the app. Therefore, ISPs may compete to obtain the exclusivity to offer the app. Exclusivity can be obtained either by paying the rival in exchange of a commitment to block the app or by signing an exclusivity contract with the app developer (as in Kourandi et al. (forthcoming)). Exclusivity in the asymmetric case will lead to a monopolization on the voice market with ISP$_1$ being the sole provider of both the phone and the app. As we have shown, such an outcome is detrimental to consumers but not to welfare and therefore, if total welfare is the chosen criterion, such arrangements should perhaps be legalized.

Though in principle the zero-price rule prohibits financial transfers between ISPs and CPs, hence also exclusivity contracts, telecom operators in Belgium and in France have signed contracts with Netflix to include its VOD catalogue on their internet/TV box, thereby allowing subscribers to watch Netflix on their TV. For these operators, it is expected that the value-added by Netflix to their internet service can compensate the losses created by an intensified competition.

5. Conclusion

We have focused on what we believe to be a forgotten aspect of net neutrality: the interaction of ISPs and applications which are complementary to one of the ISP’s products are a substitute for another. We have argued that weak net neutrality will always be respected without any need for legislation. We have also shown that imposing strong net neutrality decreases welfare in a monopoly setting and possibly also in a duopoly. Net neutrality regulations, despite their pro-competitive features, do not seem to be welfare improving even though they may benefit consumers.

For the sake of simplicity, we have overlooked a number of important issues. First, our model is static and does not encompass investments issues. Although investment is not a problem in the monopoly case—the ISP is always weakly better off after entry—the duopoly scenario offers a different view. Imposing net neutrality through regulation may decrease profits so much, especially in the asymmetric case, that investment could plummet. This argument against net neutrality has been put forward in the literature by, for instance, Economides and赫ralin (2012) and Choi and Kim (2010) (for some parameter ranges). Our view on total welfare may thus be slightly biased upwards: if we take investment into account, imposing net neutrality in a duopoly framework could lead to further decreases in welfare (or smaller increases depending on the case considered).

Second, while we have also assumed that applications are financed through ads, we have taken it as a black box without any further explanations. We think the most important conclusions in the model would not change because the zero-price rule is always enforced and hence ISPs cannot extract ad money from CPs. Given that our model focuses on a single app, even if the zero-price rule is relaxed, mechanisms of ad competition between CPs as in Kourandi et al. (forthcoming) are
not applicable and our results should hold. A crucial assumption is that the competing product provided by the ISP is not financed by ads but by a positive price. Therefore, our results are not subject to the ad competition between the good of the ISP and the app as in Dewenter and Rosch (2014).

Finally, it might also be interesting to generalize the model to allow for network effects. Indeed, while a phone user can be reached via Skype and vice versa, a WhatsApp user cannot send a message to someone who does not own the application. Thus a fragmented internet might lower users’ willingness to pay and the profits of ISPs, thereby modifying incentives to exclude.
6. Bibliography

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Appendix

Proof of proposition 2. To prove the proposition, we derive the equilibrium prices \((p_{i\text{noexcl}}^*, p_{t\text{noexcl}}^*)\) in the case the application is available for free on the internet and we show that the corresponding profit is always higher or equal to \(\Pi_{\text{excl}}\). We prove the proposition in three steps. First, we identify a candidate equilibrium where the market is fully covered and consumers buy the internet (with the app) or the phone. Second, we show that there is no equilibrium in which consumers buy the internet and the phone. Last, we fully characterize the equilibrium by considering market configurations where the market is not fully covered.

• Step 1. If \(p_t \geq t\), \(u(i,t) \leq u(i,a)\) for all \(x \in [0,1]\). We will start our analysis by searching for candidate equilibrium prices satisfying \(t \leq p_t \leq u - t\). Under these conditions, the market is fully covered and consumers buy either the internet and the app or the phone. The firm’s profit is given by:

\[
\Pi = p_i d_i + p_t d_t.
\]

Solving \(u(i,a) = u(t)\), we identify those consumers who are indifferent to the two options.

\[
\theta(x) = (p_i - p_t) - t(1 - 2x).
\]

From this equation, we can identify two boundary values: \(\theta(0) = (p_i - p_t) - t\) and \(\theta(1) = (p_i - p_t) + t\) that will be used to derive the demand functions. Four configurations (see figure .5) for the demands need to be considered:

(i) \(0 \leq \theta(0) \leq 1\) and \(0 \leq \theta(1) \leq 1\)
(ii) \(0 \leq \theta(0) \leq 1\) and \(\theta(1) \geq 1\)
(iii) \(\theta(0) \leq 0\) and \(0 \leq \theta(1) \leq 1\)
(iv) \(\theta(0) \leq 0\) and \(\theta(1) \geq 1\).

Note first that case (iv) can be ruled out because the conditions for \(\theta(0) \geq 0\) and \(\theta(1) < 1\) to be simultaneously respected are respectively \((p_i - p_t) \geq t\) and \((p_i - p_t) \leq 1 - t\), which is impossible given \(t < 1/2\). If \(t = 1/2\), case (iv) is just a boundary case of the others.

In case (i), the demands are given by:

\[
d_i = \frac{(1 - \theta(0)) + (1 - \theta(1))}{2},
\]

\[
d_t = 1 - d_i.
\]

Profit maximizing prices are \((p_i, p_t) = (\frac{1}{2} + u - t, u - t)\) and they satisfy \(t \leq p_i - p_t = \frac{1}{2} \leq 1 - t\). The corresponding demands are given by \((d_i, d_t) = (\frac{1}{2}, \frac{1}{2})\). and the firm’s profit is equal to \(\Pi = \frac{1}{4} + u - t = \Pi_{\text{excl}}\).

In case (ii), \(\theta(1) > 1\) and the demands are given by:

\[
d_i = \frac{(1 - \theta(0))}{2} \tilde{x},
\]

\[
d_t = 1 - d_i.
\]
where \( \hat{x} \) is the solution of \( 1 = (p_i - p_t) - t(1 - 2\hat{x}) \). Given that \((p_i - p_t) \geq 1 - t\), the profit maximizing prices are \((p_i, p_t) = (1 + u - 2t, u - t)\), the demands are \((d_i, d_t) = (t, 1 - t)\) and the firm’s profit is \(\Pi = u - t^2 < \frac{1}{4} + u - t\).

In case (iii), \( \theta(0) < 0 \) and the demands are given by:

\[
d_i = 1 - d_t, \quad \theta(0) \leq 0 \leq \theta(1) \leq 1
\]

\[
d_t = \frac{\theta(1)}{2}(1 - \hat{x}), \quad \theta(1) \geq 1
\]

where \( \hat{x} \) is the solution of \( 0 = (p_i - p_t) - t(1 - 2\hat{x}) \). The profit maximizing prices are \((p_i, p_t) = (u - t, u - 2t)\), the demands are \((d_i, d_t) = (1, 0)\) and the firm’s profit is \(\Pi = u - t < \frac{1}{4} + u - t\).

There is thus a unique equilibrium candidate for the covered market situation: \((p_i, p_t) = (\frac{1}{2} + u - t, u - t)\).

\* \textbf{Step 2}. Next, we show that there is no equilibrium candidate with \( p_t \leq t \). If this condition holds true, some consumers will buy the internet and the phone. Solving the equation \( u(i, a) = u(i, t) \), we can identify the indifferent consumer \( x^* \) defined as:

\[
x^* = \frac{1}{2} + \frac{p_t}{2t}.
\]

From this, we can derive the demand functions. Two cases must be considered depending on
whether \( \theta(0) \) is positive or negative. Say first \( \theta(0) > 0 \). Then
\[
d_i = \frac{1 - \theta(0)}{2} + \frac{(1 - \theta(x^*))}{2} x^* + \frac{x^*}{2} (1 - p_i), \tag{7}
\]
\[
d_t = 1 - \frac{(1 - \theta(0)) + (1 - \theta(x^*))}{2} x^*. \tag{8}
\]
In Equation .7, the first term is the internet demand of consumers using the app; the second term is the internet demand of those buying the phone. Let us notice that in this case, \( d_i + d_t \geq 1 \).

The first order conditions of the profit maximization problem write as follows:
\[
\frac{\partial \Pi}{\partial p_t} = -\frac{3p_t^2 + (2 + 4p_i - t)p_t + p_t(6p_i - 4(1 + t))}{4t} = 0 \tag{9}
\]
\[
\frac{\partial \Pi}{\partial p_i} = 1 - 2p_i + \frac{(p_t + t)(3p_t + t)}{4t} = 0 \tag{10}
\]
Combining .9 and .10 and simplifying, we have:
\[
9p_t^3 + 6p_t^2 + 2(8 - t) - p_t t(4 + 5t) = 0 \tag{11}
\]
It can be checked that for all \( t \leq \frac{1}{2} \), there is no solution \( p_t \) to (.11) in \([0, t]\). Thus, the solution is a corner solution. (i) If \( p_t = 0 \), then \( p_i = \frac{t}{2} + \frac{t}{8} \) by (.10), \( d_i = \frac{1}{2} + \frac{t}{8} \), \( d_t = \frac{3(4 - t)}{16} \) and \( \Pi = \frac{(4 + t)^2}{64} \). (ii) If \( p_t = t \), then \( p_i = \frac{1}{2} + t \), \( d_i = \frac{1}{2} + \frac{t}{2} \), \( d_t = \frac{1}{2} \) and \( \Pi = \frac{1}{4} + t \). In both cases, the profit is strictly smaller than \( \Pi = \frac{1}{4} + u - t \).

Say now that \( \theta(0) \leq 0 \). Then, demands are
\[
d_i = (1 - x^*)(1 - p_i) + x^* (1 - p_i) + \frac{x^* + \frac{t}{2}}{2} p_i \tag{12}
\]
\[
d_t = 1 - x^* (1 - p_i) - \frac{x^* + \frac{t}{2}}{2} p_i \tag{13}
\]
The first-order conditions are
\[
\frac{\partial \Pi}{\partial p_t} = -\frac{4p_i + 3p_t^2 + 2t}{4t} = 0 \tag{14}
\]
\[
\frac{\partial \Pi}{\partial p_i} = \frac{6p_i p_t + 3p_t^2 + 4t - 4p_t t}{4t} = 0 \tag{15}
\]
It can be checked that this system has no solution for \( p_t \) in \([0, t]\) and for \( t \leq \frac{1}{2} \) and we have again a corner solution. If \( p_t = 0 \), there is no real solution to (.15). If \( p_t = t \), \( p_i = \sqrt{\frac{2t}{3}} \) and \( \Pi = \frac{1}{18}(5\sqrt{6t} + 3t) \) which is strictly smaller than \( \frac{1}{4} + u - t \) for \( t \leq \frac{1}{2} \) and \( u \geq 2t \).

There is thus no price equilibrium in which consumers buy the phone and the internet together.

• **Step 3.** In the last step, we consider the case of a non-covered market by assuming that \( p_t > u - t \). In this case, consumers have three options: (i,a), (t) and (\emptyset). Solving the
equations $u(i, a) = 0$ and $u(t) = 0$, we have the indifferent consumers defined as:

$$\hat{\theta}(x) = p_i - u + tx,$$

and

$$\hat{x} = 1 - \frac{u - p_t}{t}$$

Under the conditions $\hat{\theta}(0) \in [0, 1]$, $\theta(1) \in [0, 1]$ and $\hat{x} \in [0, 1]$, the demands are given by:

$$d_i = \frac{1 - \hat{\theta}(0)) + (1 - \theta(\hat{x}))}{2} \hat{x} + \frac{(1 - \theta(\hat{x})) + (1 - \theta(1))}{2} (1 - \hat{x}), \quad (16)$$

$$d_t = \frac{\theta(\hat{x}) + \theta(1)}{2} (1 - \hat{x}). \quad (17)$$

And in this case, $d_i + d_t \leq 1$.

It is important to note that the candidate equilibrium with covered market $(p_i, p_t) = (\frac{1}{2} + u - t, u - t)$ corresponds to the limit case where $\hat{x} \rightarrow 0$. This means that if we use the demand functions defined in (16) and (17) to compute the profit and if $\frac{\partial \Pi}{\partial p_t} \bigg|_{p_i=\frac{1}{2}+u-t, p_t=u-t} < 0$, increasing $p_t$ above $u - t$ does not increase the profit. Consequently, $(p_i, p_t) = (\frac{1}{2} + u - t, u - t)$ is the unique equilibrium. The reasoning is illustrated in figure .6.

Conversely, if $\frac{\partial \Pi}{\partial p_t} \bigg|_{p_i=\frac{1}{2}+u-t, p_t=u-t} > 0$, then increasing the price above $p_t$ gives a strictly higher profit. Therefore $(p_i, p_t) = (\frac{1}{2} + u - t, u - t)$ is not an equilibrium. Performing the adequate computations we obtain that

**Lemma 1.** $\frac{\partial \Pi}{\partial p_t} \bigg|_{p_i=\frac{1}{2}+u-t, p_t=u-t} > 0$ if one of the following two conditions is met:

1. $0 < t \leq \frac{1}{4}(5 - \sqrt{17}) = \bar{t}$ and $0 < u < \frac{2\bar{t} - \bar{u}}{2\bar{t} - 1} = \bar{u}$
2. $\bar{t} < t$

Equilibrium when the market is not covered can be derived numerically but a complete analytical characterization is particularly complex as many cases should be considered.
Proof of proposition 5. In the case where both exclude or neither does, prices and profits are zero and everyone consumes the internet (the left half of the square with the app, the other half with the phone). Let us consider now that ISP$_1$ allows the app and ISP$_2$ does not. The price of the internet alone is zero, $p_i = 0$ and therefore, demand will be divided among internet with the app and internet with the phone. Comparing utilities in each case yields an indifferent consumer located at

$$x = \frac{1}{2} - \frac{\bar{p}_i}{2t}$$

The profit of ISP$_2$ is zero and the ISP$_1$’s profit is

$$\Pi = \left( \frac{1}{2} - \frac{\bar{p}_2}{2t} \right) \bar{p}_i$$

This yields a unique solution, $\bar{p}_i = \frac{t}{2}$ and $\Pi_1 = \frac{t}{8}$. ISP$_2$ will make no profit since $p_i = p_t = 0$ because of Bertrand competition. When ISP$_2$ allows the app, the proof is exactly similar.

Proof of proposition 6.

$$W_{sym}^{nf} = \int_{0}^{1} \int_{0}^{1} \theta + u - xt \, d\theta \, dx + \int_{1}^{2} \int_{0}^{1} \theta + u - t + xt \, d\theta \, dx = \frac{1}{2} + u - \frac{t}{4}$$

In the exclusion/no-exclusion or no-exclusion/exclusion cases, it is equal to

$$W_{sym}^{n/e} = \int_{0}^{1} \int_{0}^{1} \theta + u - xt - \frac{t}{2} \, d\theta \, dx + \int_{1}^{2} \int_{0}^{1} \theta + u - t + xt \, d\theta \, dx + \frac{t}{8} = \frac{1}{2} + u - \frac{5t}{16}$$

It is easily seen that $W_{sym}^{nf} > W_{sym}^{n/a}$.

Proof of proposition 7.

- Both exclude. Let’s first consider that both exclude. $p_i = 0$ because both ISPs supply the internet and homogeneous Bertrand competition takes place. Therefore, all consumers buy the internet and some also buy the phone. Comparing utilities in each case, we find the indifferent consumer located at

$$x = 1 - \frac{u - p_t}{t}$$

Thus, the profit of ISP$_1$ can take two forms:

$$\Pi_1 = \begin{cases} p_t = u - t & \text{if } p_t \leq u - t \\ p_t \left( \frac{u - p_t}{t} \right) & \text{if } p_t > u - t \end{cases}$$

The first order condition of the second form of profit yields $p_t = u/2$ which can not be since $u/2 < u - t$. Therefore, the corner solution, $p_t = \Pi_1 = u - t$ is the only solution.

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10Throughout the appendix, the superscripts a and e respectively stand for allow and exclude. The first letter refers to ISP$_1$ and the second letter to ISP$_2$. “nf” refers to no fragmentation that is, no ISP excludes.
Both allow. The only product offered for a price other than zero is the phone, for the usual Bertrand reasons. Everyone consumes the internet and demand is separated between those who consume it with the phone and those who consume it with the app. The indifferent consumer is located at
\[ x = \frac{p_t}{2t} + \frac{1}{2} \]

The profit of the ISP_1 is
\[ \Pi = p_t \left( \frac{p_t}{2t} + \frac{1}{2} \right) \]

The first-order condition yields \( p_t = t/2 \) and \( \Pi_1 = t/8 \).

ISP_1 excludes, ISP_2 allows. The price of the internet is zero again. Consumers are divided between those who consume it with the phone and those who consume it with the application. The indifferent consumer is located at
\[ x = \frac{1}{2} + \frac{p_t - \tilde{p}_i}{2t} \]

Firm’s profits are
\[ \Pi_1 = \tilde{p}_i \left( \frac{1}{2} + \frac{p_t - \tilde{p}_i}{2t} \right) \]
\[ \Pi_2 = p_t \left( \frac{1}{2} - \frac{p_t - \tilde{p}_i}{2t} \right) \]

Computing firms’ best responses yields the equilibrium \( \tilde{p}_i = p_t = t \) and \( \Pi_2 = \Pi_1 = \frac{t}{4} \). Another possible candidate equilibrium would be higher prices with some consumers consuming the internet only. In that case, the indifferent consumer (between the internet and the app and the internet alone) and the profit of ISP_2 is
\[ x = \frac{u - \tilde{p}_i}{t} \]
\[ \Pi_2 = \tilde{p}_i \left( \frac{u - \tilde{p}_i}{t} \right) \]

This yields an optimal price of \( \tilde{p}_i = \frac{u}{2} \), implying that ISP_2 will want to cover the whole market with the app. Because firms are symmetric, at the equilibrium no consumer will consume the internet alone and our previous result is the unique equilibrium.

ISP_1 allows, ISP_2 excludes.

Because it only supplies the internet, the profit of ISP_2 is zero. ISP_1 thus sets the prices of the internet with the app and of the phone freely. Two cases are possible: consumers are divided among app-users and phone users or some consumers choose to buy the internet alone. This second option can immediately be discarded for the same reason as in the exclusion/no-exclusion case. Let us consider therefore that no one consumes the internet alone. One can think of the problem as finding the highest prices such that the consumers indifferent between
the internet and the internet with the app—or between the internet and the internet with the phone—are located at $x = \frac{1}{2}$. This implies

$$\frac{u - \tilde{p}_t}{t} = \frac{1}{2}$$

$$1 - \frac{u - p_t}{t} = \frac{1}{2}$$

This yields $\tilde{p}_t = p_t = u - \frac{t}{2}$ and $\Pi_t = u - \frac{t}{2}$.

**Proof of proposition 8.** Under no fragmentation, consumer surplus and welfare are:

$$CS_{asym}^{nf} = \int_0^\frac{1}{2} \int_0^1 \theta + u - xt \, d\theta \, dx + \int_0^\frac{1}{2} \int_0^1 \theta + u - t + xt - \frac{t}{2} \, d\theta \, dx$$

$$= \frac{1}{2} + u - \frac{7t}{16}$$

$$W_{asym}^{nf} = \frac{1}{2} + u - \frac{7t}{16} + \frac{t}{8} = \frac{1}{2} + u - \frac{5t}{16}$$

Under the exclusion/no-exclusion case, they are

$$CS_{asym}^{e/a} = \int_0^\frac{1}{2} \int_0^1 \theta + u - xt - t \, d\theta \, dx + \int_0^\frac{1}{2} \int_0^1 \theta + u - (1 - x)t - t \, d\theta \, dx$$

$$= \frac{1}{2} + u - \frac{20t}{16}$$

$$W_{asym}^{e/a} = \frac{1}{2} + u - \frac{20t}{16} + \frac{t}{8} = \frac{1}{2} + u - \frac{4t}{16}$$

Under the no-exclusion/exclusion case, they are

$$CS_{asym}^{a/e} = \int_0^\frac{1}{2} \int_0^1 \theta + u - xt - u + \frac{t}{2} \, d\theta \, dx + \int_0^\frac{1}{2} \int_0^1 \theta + u - (1 - x)t - u + \frac{t}{2} \, d\theta \, dx$$

$$= \frac{1}{2} + \frac{4t}{16}$$

$$W_{asym}^{a/e} = \frac{1}{2} + \frac{4t}{16} + \frac{u - t}{2} = \frac{1}{2} + u - \frac{4t}{16}$$

It is then easily seen that $CS_{asym}^{nf} > CS_{asym}^{e/a} > CS_{asym}^{a/e}$ and $W_{asym}^{e/a} = W_{asym}^{a/e} > W_{asym}^{nf}$. 

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