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Jean-Pierre Amigues, Gilles Lafforgue and Michel Moreaux

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Jean-Pierre Amigues*, Gilles Lafforgue†
and
Michel Moreaux‡

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Abstract

Using a standard Hotelling model of resource exploitation, we determine the optimal consumption paths of three energy resources: dirty coal, which is depletable and carbon-emitting; clean coal, which is also depletable but carbon-free thanks to an abatement technology (CCS: Carbon Capture and Storage), and solar energy which is renewable and carbon-free. Carbon emissions are released into the atmosphere and we assume that the atmospheric carbon stock cannot exceed a given ceiling. We consider learning-by-doing in the abatement technology, implying that the marginal CCS cost is decreasing in the cumulative consumption of clean coal. We show the following results. i) Learning-by-doing does not imply "early" capture, i.e. the clean coal exploitation must begin at the earliest once the carbon cap is reached. ii) The energy price path can evolve non-monotonically over time. iii) When the solar cost is low enough, there may exist unusual energy consumption sequence along which solar energy is interrupted for some time and replaced by clean coal.

Keywords: Climate change; Energy substitution; Carbon Capture and Storage; Learning-by-doing.

JEL classifications: O44, Q31, Q42, Q54, Q55.

*Toulouse School of Economics (INRA and LERNA). E-mail address: amigues@toulouse.inra.fr.

†Corresponding author. University of Toulouse, Toulouse Business School. 20 bd Lascrosses - BP 7010 - 31068 Toulouse Cedex 7 - France. E-mail address: g.lafforgue@tbs-education.fr. We thank the French Energy Council (CFE) for financial support.

‡Toulouse School of Economics (IDEI and LERNA). E-mail address: michel.moreaux@tse-fr.eu

1 Introduction

Carbon dioxide capture and storage (CCS) is a process consisting of the separation of CO_2 from the emissions stream from fossil fuel combustion, transporting it to storage location, and storing it in a manner that ensures its long-run isolation from the atmosphere (IPCC, 2005). Currently, the major CCS effort focuses on the removal of CO_2 directly from industrial or utility plants and storing it in secure geological reservoirs. Given that fossil fuels supply over 85% of all primary energy demands, CCS appears as the only technology that can substantially reduce CO_2 emissions while allowing fossil fuels to meet the world's pressing needs (Herzog, 2011). Moreover, CCS technology may have considerable potential to reduce CO_2 at a "reasonable" social cost, given the social costs of carbon emissions predicted for a business-as-usual scenario (Islegen and Reichelstein, 2009). According to Hamilton *et al.* (2009), the mitigation cost for capture and compression of the emissions from power plants running with gas is about \$52 per metric ton CO_2. Adding the transport and storage costs[1] in a range of \$5-15 per metric ton CO_2, a carbon price of about \$60-65 per metric ton CO_2 is needed to make these plants competitive.

This CCS technology has motivated a large number of empirical studies, mainly developing complex integrated assessment models (e.g. Edenhofer *et al.*, 2005, Gerlagh and van der Zwaan, 2006, Grimaud *et al.*, 2011, McFarland *et al.*, 2003). In these models, the motivation for using CCS technologies is to reduce CO_2 emissions[2] and, in this sense, climate policies are essential to create a significant market for these technologies. These empirical models generally conclude that an early introduction of sequestration can lead to a substantial decrease in the social cost of climate change. However a high level of complexity for such models, aimed at defining some specific climate policies and energy scenarios, may be required so as to take into account the various interactions at the hand.

The theoretical economic literature on CCS is more succinct. For instance, Lafforgue *et al.* (2008-a) characterize the optimal CCS policy in a model of energy substitution when carbon emissions can be stockpiled into several reservoirs of finite size. Ayong Le kama *et al.* (2013) develop a growth model aiming at exhibiting the main driving forces that determine the optimal CCS policy when the command variable of such a policy is the sequestration rate instead of the sequestration flow. Grimaud and Rouge (2009) study the

[1] As explained in Hamilton *et al.* (2009), the transport and storage costs are very site specific.

[2] As mentioned by Herzog (2009), the idea of separating and capturing CO_2 from the flue gas of power plants did not originate out of concern about climate change. The first commercial CCS plants that have been built in the late 1970s in the United States to achieve enhanced oil recovery (EOR) operations, where CO_2 is injected into oil reservoirs to increase the pressure and thus the output of the reservoir.

implications of the CCS technology availability on the optimal use of polluting exhaustible resources and on optimal climate policies within an endogenous growth model. However, the outcomes of these models cannot be easily compared since they strongly vary according to the properties of the environmental damage and abatement cost functions. In the case where a carbon stabilization cap is considered instead of a standard damage function, i.e. under a cap-and-trade approach, the crucial question of the timing of the CCS policy arises. In particular, is it optimal to wait for being constrained by this carbon cap before starting sequestration, or not? Assuming a carbon ceiling constraint and a constant average CCS cost, Lafforgue *et al.* (2008-a)[3] conclude that, for any storage capacity, it is never optimal to deploy CCS before the atmospheric carbon cap is attained. Using the same type of "ceiling model", Amigues *et al.* show that an "earlier" CCS development – that is before the constraining ceiling is reached – can be induced by assuming heterogeneity in carbon emitters regarding the cost of the abatement technology they have access to (Amigues *et al.* 2013-a), or by assuming decreasing returns to scale in the CCS technology (Amigues *et al.* 2013-b).

In the present study, we address the question of the qualitative impacts of learning-by-doing in the CCS technology on the optimal timing of the abatement policy. Since, as pointed out by Gerlagh (2006) or by Manne and Richels (2004), the cumulative experience in carbon capture generates in most cases some beneficial learning tending to reduce the involved costs, the average cost function may be decreasing in the cumulative sequestration. When such a learning-by-doing process is considered, the intuition would suggest that CCS may be deployed as soon as possible to benefit from this learning. To check this point, we extend the model of Lafforgue *et al.* (2008-a) in which the marginal sequestration cost is constant, by assuming that this cost is decreasing in the cumulative abatement, and we characterize the optimal time profiles of the energy price, the energy consumption, the carbon emissions and the flow of abatement within this new framework.

The sketch of the model is the following. The energy needs can be supplied by three types of energy resources that are perfect substitutes. The first resource is non-renewable and carbon-emitting (dirty coal), the second resource is also non-renewable but carbon-free thanks to a CCS device (clean coal) and the last resource is renewable and clean (solar energy). Hence, we consider two alternative mitigation options allowing to relax the carbon cap constraint: the exploitation of the solar energy and of the clean coal. The

[3]Note that this kind of Hotelling model of fossil resource extraction with a critical threshold that the atmospheric carbon stock should not exceed has been pioneered by Chakravorty *et al.* (2006).

design of the optimal energy consumption path thus results from the comparison of the respective marginal costs of these three energy sources. Both the marginal extraction cost of coal and the marginal production cost of the solar energy are assumed to be constant, the former been lower than the later. However, producing clean coal requires an additional marginal CCS cost which is decreasing in the cumulative clean coal consumption to justify learning-by-doing. We show the following results. The clean coal exploitation must begin at the earliest once the carbon cap is reached. Moreover, the energy price path can evolve non-monotonically over time. When the solar cost is low enough, this last case can give rise to an unusual sequence of energy consumption along which the solar energy consumption is interrupted for some time and replaced by the clean coal exploitation.

The paper is organized as follows. Section 2 presents the model. Section 3 introduces the optimal program of the social planner and derives the first-order conditions. Section 4 describes the typical optimal paths by distinguishing different scenarios for the solar energy depending upon its relative cost as compared with the clean coal exploitation. Finally, we conclude in Section 5 by discussing in particular the main qualitative dynamical properties of the carbon tax required to enforce the carbon cap constraint.

2 The Model

We consider an economy in which the energy services can be supplied by two primary resources, a potentially carbon-emitting non-renewable resource (coal) and a clean renewable resource (solar). These energy sources are assumed to be perfect substitutes for the final users.

Polluting non-renewable resource

Let $X(t)$ be the available stock of coal at time t, X^0 be the initial endowment, with $X(0) \equiv X^0 > 0$, and $x(t)$ the instantaneous extraction rate so that:

$$\dot{X}(t) = -x(t), \ X(t) \geq 0 \tag{1}$$

$$x(t) \geq 0. \tag{2}$$

The average delivery cost of coal, denoted by c_x, is assumed to be constant, hence equal to the marginal cost. This cost includes all the costs that must be incurred to supply ready-for-use energy services to the final users, that is the extraction cost, the cost of industrial processing and the transportation cost.

Burning and consuming coal generates some carbon emissions that are proportional to its use. Let ζ be the unitary pollutant content of coal so that, without any abatement policy, the pollution flow which would be released into the atmosphere amounts to $\zeta x(t)$. However, the effective flow of carbon emissions can be lower than this potential pollution flow thanks to the carbon capture and sequestration option.

Clean versus dirty energy services

Instead of expressing the CCS control variable by the share of the potential emission flow which is captured, we proceed formally otherwise by considering in fact two types of fossil energies allowing to produce final energy services together with the clean renewable substitute. We define the clean coal as the part of the coal consumption whose emissions are captured and the dirty coal as the other part whose emissions are directly released into the atmosphere. Denoting respectively by $x_c(t)$ and $x_d(t)$ the instantaneous consumption rates of clean and dirty coals, with $x_c(t) + x_d(t) = x(t)$, (1) and (2) have to be rewritten as:

$$\dot{X}(t) = -\left[x_c(t) + x_d(t)\right], \ X(t) \geq 0 \tag{3}$$

$$x_c(t) \geq 0 \quad \text{and} \quad x_d(t) \geq 0. \tag{4}$$

Let $S(t)$ be the cumulative clean coal consumption from time 0 up to time t. Assuming that $S(0) = 0$ for the sake of simplicity, we thus have:

$$S(t) = \int_0^t x_c(\tau)d\tau \text{ and } \dot{S}(t) = x_c(t). \tag{5}$$

CCS cost and learning-by-doing

Producing energy services from clean coal is costlier than from dirty coal since an additional CCS cost must be incurred. Let c_s be this additional cost, per unit of clean coal, so that the average cost of the clean fossil energy amounts to $c_x + c_s$. We assume that this additional cost depends on the cumulative clean coal consumption S and that the larger S, the larger the cumulative experience in carbon capture and sequestration, hence the lower the average additional cost c_s. The objective of the paper being to isolate a pure learning effect, we neglect any possible locking of the learning experience by reservoir limited capacity constraints, even if such constraints are most probably empirically relevant.[4] Thus the type

[4]Due to the scarcity of the most accessible sites into which the carbon can be sequestered, the average CCS cost should also increase with S up to possibly some upper bound $\bar{S}$ corresponding to the global capacity of the geological carbon sinks. This is the type of stock effect that is thoroughly examined in Lafforgue *et al.* (2008-a and 2008-b). In case of multiple carbon sinks, they show that the different reservoirs must be filled by increasing order of their respective sequestration costs.

of CCS cost functions is such that $c_s : [0, X^0] \to \mathbb{R}_+$ is a $\mathcal{C}^2$ function, strictly decreasing and strictly convex, $c_s'(S) < 0$ and $c_s''(S) > 0$ for any $S \in (0, X^0)$, with $\lim_{S\downarrow 0} c_s(S) \equiv \bar{c}_s < \infty$ and $c_s(X^0) \equiv \underline{c}_s > 0$.

Atmospheric pollution stock

Let $Z(t)$ be the level of the atmospheric carbon concentration at time t, and Z^0 the initial concentration inherited from the past: $Z(0) \equiv Z^0 \geq 0$. This pollution stock is assumed to be self-regenerating at a constant proportional rate α, $\alpha > 0$.[5] Since only the dirty coal feeds the atmospheric carbon stock, the dynamics of Z is:

$$\dot{Z}(t) = \zeta x_d(t) - \alpha Z(t). \tag{6}$$

Pollution damages

We assume that, as far as the atmospheric pollution stock does not overshoot some critical level $\bar{Z}$, the damages due to the atmospheric carbon accumulation are negligible. However, for pollution stocks that are larger than $\bar{Z}$, the damages would be immeasurably larger than the sum of the discounted gross surplus generated along any path triggering this overshoot.[6] By doing that, we assume a lexicographic structure of the preferences over the set of the time paths of energy consumption and pollution stock. Technically, this lexicographic structure translates into two constraints, the first one on the pollution stock Z and the second one on the pollution flow ζx_d.

Since the overshoot of this critical cap would destroy all that could be gained otherwise, then we must impose:

$$\bar{Z} - Z(t) \geq 0 \quad \text{and} \quad \bar{Z} - Z^0 > 0. \tag{7}$$

An implication of this constraint is that, when the ceiling is reached, the maximum quantity of dirty coal which can be consumed is the exact quantity whose emissions are balanced by the natural regeneration of the atmosphere. Denoting by $\bar{x}_d$ this maximum consumption rate of dirty coal, (6) implies that $\bar{x}_d = \alpha\bar{Z}/\zeta$.

[5] Similar developments would be obtained under a self-regeneration marginal rate which is positive but decreasing with Z. As pointed out in Toman and Withagen (2000), more difficult problems have to be solved when the marginal rate is first positive and next negative, due to inherent non-convexity.

[6] See Amigues *et al.* (2011) for a model in which the both types of effects – small and drastic damages – are explicitly taken into account, showing that the main qualitative properties of the optimal policy of the pure ceiling model are preserved.

Clean renewable primary resource

The other primary resource, clean and renewable, can be processed at a constant average cost c_y. As for the non-renewable resource this cost includes all the costs that must be incurred to supply ready-for-use energy services to the final users. We denote by y^n the constant natural flow of solar energy and by $y(t)$ the solar energy consumption at time t, with the usual non-negativity constraint:

$$y(t) \geq 0. \tag{8}$$

We assume that y^n is sufficiently large to provide all the energy needs of the society at the marginal cost c_y so that no rent has ever to be charged for an efficient exploitation of this resource. Last, we assume that c_y is larger than c_x to justify the use of coal during some initial period. Since relaxing the ceiling constraint can be achieved by using either clean coal or solar energy, the relative competitiveness of these two options may depend upon their respective costs. That is why we will distinguish the cases of a "high" or a "low" solar energy cost in the following analysis. What we mean by "high" or "low" will be explicitly precised in the next section.

Gross surplus generated by energy consumption

Clean coal, dirty coal and solar energy being assumed to be perfect substitutes, we define the aggregate energy consumption as $q(t) = x_c(t)+x_d(t)+y(t)$. This consumption generates an instantaneous gross surplus $u(q)$. The function $u(\cdot)$ is assumed to satisfy the following standard assumptions: $u : \mathbb{R}_+ \to \mathbb{R}$ is a $\mathcal{C}^2$ function, strictly increasing and strictly concave verifying the Inada condition: $\lim_{q \downarrow 0} u'(q) = +\infty$.

We denote by $p(q)$ the marginal gross surplus function $u'(q)$, and by $q(p)$ its inverse, i.e. the energy demand function. When the solar energy is the unique energy source, then its optimal consumption amounts to $\tilde{y}$ solution of $u'(\tilde{y}) = c_y$, provided that y^n is not smaller than $\tilde{y}$, what we mean by assuming that y^n is sufficiently large.

3 The social planner program

The social planner problem consists in determining the path $\{(x_c(t), x_d(t), y(t)), t \geq 0\}$ that maximizes the sum of the discounted net surplus while keeping away the catastrophic events that would be triggered by a too high pollution stock. Denoting by ρ the instantaneous social discount rate (constant and positive), the corresponding program writes:[7]

$$\max_{x_c,x_d,y} \int_0^{\infty} \{u(x_c + x_d + y) - c_x[x_c + x_d] - c_s(S)x_c - c_y y\} e^{-pt} dt$$

subject to the state equations (3), (5) and (6), and to the inequality constraints (4), (7) and (8). Let λ_S, λ_X and $-\lambda_Z$ be the co-state variables of S, X and Z respectively,[8] by ν's the Lagrange multipliers associated with the inequality constraints on the state variables and by γ's those corresponding to the inequality constraints on the control variables, hence the current valued Hamiltonian, $\mathcal{H}$, and Lagrangian, $\mathcal{L}$:

$$\begin{aligned} \mathcal{H} &= u(x_c + x_d + y) - c_x[x_c + x_d] - c_s(S)x_c - c_y y - \lambda_X[x_c + x_d] - \lambda_Z[\zeta x_d - \alpha Z] + \lambda_S x_c \\ \mathcal{L} &= \mathcal{H} + \nu_X X + \nu_Z[\bar{Z} - Z] + \gamma_c x_c + \gamma_d x_d + \gamma_y y. \end{aligned}$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial x_c} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_x + \lambda_X + c_s(S) - \lambda_S - \gamma_c \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial x_d} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_x + \lambda_X + \zeta\lambda_Z - \gamma_d \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_y - \gamma_y \tag{11}$$

$$\dot{\lambda}_S = \rho\lambda_S - \frac{\partial \mathcal{L}}{\partial S} \quad \Rightarrow \quad \dot{\lambda}_S = \rho\lambda_S + c_s'(S)x_c \tag{12}$$

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial \mathcal{L}}{\partial X} \quad \Rightarrow \quad \dot{\lambda}_X = \rho\lambda_X - \nu_X \tag{13}$$

$$\dot{\lambda}_Z = \rho\lambda_Z + \frac{\partial \mathcal{L}}{\partial Z} \quad \Rightarrow \quad \dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z \tag{14}$$

together with the usual complementary slackness conditions and the following transversality conditions:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_S(t)S(t) = 0 \tag{15}$$

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) = 0 \tag{16}$$

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0. \tag{17}$$

As it is well known, with a constant marginal extraction cost, the mining rent λ_X must grow at the social rate of discount as long as the stock of coal is not exhausted. Denoting by $\bar{t}_X$ the time at which exhaustion occurs, we must have:

$$t < \bar{t}_X \Rightarrow \lambda_X(t) = \lambda_{X0}e^{\rho t}, \text{ with } \lambda_{X0} \equiv \lambda_X(0).$$

[7] We drop the time index for notational convenience as far as possible.

[8] Using $-\lambda_Z$ as the co-state variable of Z, we can directly interpret $\lambda_Z \geq 0$ as the unitary tax on the pollution emissions generated by dirty oil consumption.

Hence from the transversality condition (16), if coal have some positive initial value, $\lambda_{X0} > 0$, then it must be exhausted along the optimal path.

Initially, ν_Z is nil as long as the ceiling constraint is not binding yet. Denoting by $\underline{t}_Z$ the time at which the atmospheric carbon cap $\bar{Z}$ is reached, (14) implies:

$$t \leq \underline{t}_Z \Rightarrow \lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}, \text{ with } \lambda_{Z0} \equiv \lambda_Z(0).$$

Once the ceiling constraint is no longer active, λ_Z must be nil. Denoting by $\bar{t}_Z$ the time at which this constraint ceases to be active, it comes:

$$t \geq \bar{t}_Z \Rightarrow \lambda_Z(t) = 0.$$

Last, denoting respectively by $\underline{t}_c$ and by $\bar{t}_c$ the date at which the clean coal production begins and the date at which it ceases, (12) implies:

$$t \leq \underline{t}_c \Rightarrow \lambda_S(t) = \lambda_{S0}e^{\rho t}, \text{ with } \lambda_{S0} \equiv \lambda_S(0).$$

A common component of the costs of the two types of coal is the processing cost c_x augmented by the mining rent λ_X. We denote by p^F (F for free of tax and free of cleaning cost) this common component:

$$p^F(t) = c_x + \lambda_{X0}e^{\rho t} \Rightarrow \dot{p}^F(t) = \rho\lambda_{X0}e^{\rho t} > 0. \quad (18)$$

In addition to this common component, the full marginal cost of the dirty coal, which is denoted by $c_m^d(x_d)$, must also include the imputed social marginal cost of the carbon emissions generated by its consumption:

$$c_m^d(x_d) = p^F(t) + \zeta\lambda_Z(t). \quad (19)$$

The full marginal cost $c_m^c(x_c)$ of the clean coal must include the marginal abatement cost $c_s(S_c)$ reduced by the marginal value of a larger cumulative clean coal production λ_S:

$$c_m^c(x_c) = p^F(t) + c_s(S) - \lambda_S(t). \quad (20)$$

As we shall see, λ_S is positive and it can be interpreted as a unitary subsidy on clean coal consumption, owing to the learning-by-doing process on CCS technology.

The dynamics of exploitation of the two types of coal and of the solar energy are driven by their respective instantaneous full marginal costs. Given that we assume a constant marginal cost of the solar energy, we may organize the discussion depending on whether this marginal cost is “high” or “low” relatively to the other carbon-free option, i.e. the clean coal. As it will be identified later, this technically amounts to compare c_y with the marginal gross surplus $u'(\bar{x}_d)$ obtained when only dirty coal is consumed at the ceiling and the two possible scenarios are either $c_y > u'(\bar{x}_d)$ or $c_y < u'(\bar{x}_d)$. For the problem be meaningful, we also assume that the initial coal endowment X^0 is large enough to ensure that the ceiling constraint (7) binds in finite time along the optimal path.

4 The qualitative properties of the optimal paths

We first determine the general properties of the optimal paths. We restrict the analysis to the cases where clean coal has to be exploited along these optimal paths. We show that the exploitation of the clean coal and of the solar energy may never begin before the ceiling constraint is active and that the exploitation of these two carbon-free energy sources may never be done simultaneous. Next we consider successively the cases of a high and a low solar energy cost. In both cases, it may happen that the energy price increases non-monotonically through time if the effect of learning-by-doing is strong enough. Moreover, in the case of a low solar cost, it can be optimal to consume solar energy during two disconnected time intervals.

4.1 General properties of the optimal paths

Thanks to learning from cumulative experience, the more the clean coal is presently used, the lower its future marginal abatement cost. This suggests that λ_S – which captures the positive externality of the clean coal production upon its future additional costs – must be positive. However this positive externality has to be taken into account as far as clean coal is consumed in the future. Once the clean coal exploitation definitively ceases, this externality trivially disappears.

Proposition 1 *The co-state variable associated with the cumulative clean coal consumption is positive as far as its exploitation runs and nil thereafter:*

$$\lambda_S(t) \begin{cases} > 0, & \text{for } t < \bar{t}_c \\ = 0, & \text{for } \bar{t}_c \leq t. \end{cases} \tag{21}$$

Proof: Solving the differential equation (12) results in $\lambda_S = \left[\lambda_{S0} + \int_0^t c_s'(S) x_c e^{-\rho\tau} d\tau\right] e^{\rho t}$, where $\lambda_{S0} = -\int_0^\infty c_s'(S) x_c e^{-\rho t} dt$ from the transversality condition (15). Substituting for

λ_{S0} we obtain:

$$\lambda_S(t) = -\int_t^\infty c_s'(S)x_c e^{-\rho(\tau-t)}d\tau, \tag{22}$$

which is strictly positive as long as $\int_t^\infty x_c d\tau > 0$, that is as long as $t < \bar{t}_c$. ■

Integrating by parts (22) we get the following alternative expression of λ_S which will turn out to be useful in the proofs of Propositions 2, 3 and 4:

$$\lambda_S(t) = c_s(S) - \rho\int_t^\infty c_s(S)e^{-\rho(\tau-t)}d\tau. \tag{23}$$

The following Propositions 2 and 3 show that the exploitation of the clean coal cannot begin before the ceiling constraint is binding, i.e. before time $\underline{t}_Z$, and it must be closed before the end of the ceiling period, i.e. before time $\bar{t}_Z$. However, as we shall see later (cf. sections 4.2 and 4.3), it can be optimal to begin the clean coal exploitation strictly after the time at which the ceiling is attained.

Proposition 2 *The clean coal exploitation may not begin before the ceiling constraint is binding:* $\underline{t}_c \geq \underline{t}_Z$.

Proof: Assume that the clean cost is exploited while the ceiling constraint is not binding yet: $\underline{t}_c < \underline{t}_Z$. Then: i) Either only the clean coal is used during the time interval $(\underline{t}_c, \underline{t}_Z)$; ii) or there exists a sub-interval (t_c', t_Z'), $\underline{t}_c \leq t_c' < t_Z' \leq \underline{t}_Z$, during which both the clean and the dirty coals are simultaneously exploited; iii) or there exists a sub-interval (t_c'', t_Z''), $\underline{t}_c \leq t_c'' < t_Z'' \leq \underline{t}_Z$, during which the clean coal and the solar energy are simultaneously exploited.

i) Assume first that only the clean coal is consumed during the interval $(\underline{t}_c, \underline{t}_Z)$, then from $Z(\underline{t}_c) < \bar{Z}$ and $\dot{Z}(t) = -\alpha Z(t) < 0$ for $t \in (\underline{t}_c, \underline{t}_Z)$, we conclude that $Z(\underline{t}_Z) < \bar{Z}$, a contradiction.

ii) Assume next that both the dirty and the clean coals are exploited during some interval (t_c', t_Z'). Equating their respective full marginal costs results in $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_s(S) - \lambda_S(t)$, $t \in (t_c', t_Z')$. Substituting the R.H.S. of (23) for λ_S, we get:

$$\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = \rho\int_t^\infty c_s(S)e^{-\rho(\tau-t)}d\tau. \tag{24}$$

Time differentiating (24) yields to $\zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)t} = -\rho c_s(S) + \rho^2\int_t^\infty c_s(S)e^{-\rho(\tau-t)}d\tau$ and, using (24) again, we finally get:

$$0 < \zeta\alpha\lambda_{Z0}e^{(\rho+\alpha)t} = -\rho c_s(S) < 0, \quad t \in [t_c', t_Z'],$$

a contradiction.

iii) Last we show in Proposition 4 that clean coal and solar energy may never be exploited simultaneously. ■

Proposition 3 *Clean coal exploitation must cease before the end of the ceiling period:* $\bar{t}_c \leq \bar{t}_Z$.

Proof: Assume that at $\bar{t}_Z$, both types of coal are still used, that is $x_c(\bar{t}_Z) > 0$ and $x_d(\bar{t}_Z) = \bar{x}_d$. Equating their full marginal costs and taking into account that $\lambda_Z(\bar{t}_Z) = 0$, we get $p^F(\bar{t}_Z) = p^F(\bar{t}_Z) + c_s(S(\bar{t}_Z)) - \lambda_S(\bar{t}_Z)$. Substituting the R.H.S. of (23) for $\lambda_S(\bar{t}_Z)$ results in:

$$p^F(\bar{t}_Z) = p^F(\bar{t}_Z) + \rho \int_{\bar{t}_Z}^{\infty} c_s(S) e^{-\rho(\tau - t)} d\tau > p^F(\bar{t}_Z),$$

a contradiction. ■

Proposition 4 *The clean coal and the solar energy may never be exploited simultaneously.*

Proof: Assume that the clean coal and the solar energy are simultaneously used during some interval (t_1, t_2). Equating their full marginal costs yields $c_y = c_x + \lambda_{X0} e^{\rho t} + c_s(S) - \lambda_S(t)$, $t \in (t_1, t_2)$. Substituting the R.H.S. of (23) for λ_S, we get:

$$c_y - c_x = \lambda_{X0} e^{\rho t} + \rho \int_t^{\infty} c_s(S) e^{-\rho(\tau - t)} d\tau. \tag{25}$$

Time differentiating, we obtain: $0 = \rho \left[\lambda_{X0} e^{\rho t} - c_s(S)\right] + \rho^2 \int_t^{\infty} c_s(S) e^{-\rho(\tau - t)} d\tau$ and, taking (25) into account: $0 = \rho[c_y - c_x] - \rho c_s(S)$. Time differentiating again, we finally get:

$$0 = -\rho c_s'(S) x_c(t) > 0, \quad t \in (t_1, t_2)$$

a contradiction. ■

From Propositions 2, 3 and 4 we conclude that, if it is optimal to use clean coal, its exploitation must occur during some time interval $(\underline{t}_c, \bar{t}_c)$ strictly included within the ceiling period $(\underline{t}_Z, \bar{t}_Z)$. During this interval the both types of coal are used, $q(t) = x_c(t) + \bar{x}_d$, so that from (9):

$$u'(x_c(t) + \bar{x}_d) = p^F(t) + c_s(S) - \lambda_S(t).$$

Time differentiating and substituting the R.H.S. of (12) for $\dot{\lambda}_S$ results in:

$$\dot{x}_c(t) = \frac{\rho[\lambda_{X0}e^{\rho t} - \lambda_S(t)]}{u''(x_c(t) + \bar{x}_d)}. \tag{26}$$

Hence $\dot{x}_c(t)$ may be of either sign, positive of negative. However we can show that x_c, and consequently also p, can follow only two types of paths.

Remark that, from (21), λ_S tends towards 0 at the end of the clean coal exploitation period. Thus, since λ_S is necessarily continuous in such a deterministic model, there must exist some terminal sub-interval $(\bar{t}_c - \Delta, \bar{t}_c)$, $0 < \Delta \leq \bar{t}_c - \underline{t}_c$, during which $\dot{x}_c$ is negative and the energy price p is increasing. We have now to determine what could happen at the beginning of the clean coal consumption period when this terminal sub-interval is strictly shorter than the entire phase, that is when $\Delta < \bar{t}_c - \underline{t}_c$. Proposition 5 below states that the sign of $\dot{x}_c$ may change at most only once within the interval.

Proposition 5 *During a phase of simultaneous exploitation of clean and dirty coal at the ceiling, the energy price is either monotonically increasing, or first decreasing and next increasing. Equivalently, the clean coal production is either monotonically decreasing or first increasing and next decreasing.*

Proof: Assume that $\lim_{t\downarrow \underline{t}_c} \dot{x}_c(t) > 0$. Define t_0 as the first date at which $\dot{x}_c(t)$ alternates in sign since, in this case, the sign is changing at least once: $t_0 = \inf\{t : \dot{x}_c(t) \leq 0,\ t \in [\underline{t}_c, \bar{t}_c)\}$ implying $\dot{x}_c(t_0) = 0$. From (26), we have:

$$u''(x_c(t) + \bar{x}_d)\dot{x}_c(t) = \rho[\lambda_{X0}e^{\rho t} - \lambda_S(t)].$$

Defining $\phi(t) \equiv \lambda_{X0}e^{\rho t} - \lambda_S(t)$, the concavity of $u(.)0$ implies:

$$\dot{x}_c(t) > / = / < 0 \ \Leftrightarrow\ \phi(t) < / = / > 0.$$

Time differentiating $\phi(t)$ and using (12), we get: $\dot{\phi}(t) = \rho\phi(t) - c_s'(S)x_c(t)$. Integrating over $[t_0, t]$, $t_0 < t \leq \bar{t}_c$, and taking into account that $\phi(t_0) = 0$, we obtain:

$$\phi(t) = -e^{\rho t}\int_{t_0}^{t} c_s'(S)x_c e^{-\rho\tau}d\tau > 0, \quad t \in (t_0, \bar{t}_c].$$

We conclude that, if the sign of $\dot{\phi}$, hence the sign of $\dot{x}_c$ and $\dot{p}$, is changing over $[\underline{t}_c, \bar{t}_c)$, it is only once. ■

The next Proposition 6 shows that the shadow marginal value λ_S of the learning in abatement is decreasing when the clean coal production is also decreasing. Define $(t'_c, \bar{t}_c)$, $t'_c < \bar{t}_c$, as the maximum time interval during which x_c is decreasing, then:

Proposition 6 λ_S *is decreasing over the time interval* $(t'_c, \bar{t}_c)$.

Proof: Let t_i and t_{i+1}, $t'_c \leq t_i < t_{i+1} < \bar{t}_c$, be two successive dates at which the sign of λ_S alternates. Since $\dot{\lambda}_S = 0$ at these dates, we get from (12):

$$\zeta\lambda_S(t_h) = -c'_s(S(t_h))x_c(t_h), \quad h = i, i+1.$$

Because $S(t_{i+1}) > S(t_i)$ and $c''_s > 0$, then $-c'_s(S(t_i)) > -c'_s(S(t_{i+1}))$ which, together with $x_c(t_i) > x_c(t_{i+1})$, implies that $\lambda_S(t_i) > \lambda_S(t_{i+1})$. Thus $\lambda_S(t_i)$ is a local maximum while $\lambda_S(t_{i+1})$ is a local minimum.

Let t_j and t_{j+1}, $t_j < t_{j+1} < \bar{t}_c$, be the two last successive dates at which λ_S attains a maximum followed by a minimum before $\bar{t}_c$. Since $\lim_{t\uparrow\bar{t}_c} \lambda_S(t) = 0$, there must exist a third date t_{j+2}, $t_{j+1} < t_{j+2} < \bar{t}_c$, at which λ_S attains a local maximum, so that $\lambda_S(t_{j+1}) < \lambda_S(t_{j+2})$, which is in contradiction with the above result. ■

However, whatever the type of path followed by the energy price during the period at the ceiling, there exists one and only one type of path of the shadow cost λ_Z of the pollution stock during this period.

Proposition 7 *During the period at the ceiling, the shadow marginal cost of the pollution stock* λ_Z *must be decreasing.*

Proof: Assume first that the both types of coal are exploited. Using the clean coal option implies that $p(t) = p^F(t) + c_s(S) - \lambda_S(t)$. Time differentiating and substituting the R.H.S. of (12) for $\dot{\lambda}_S$ results in:

$$\dot{p}(t) = \dot{p}^F(t) + c'_s(S)x_c(t) - \dot{\lambda}_S(t) = \dot{p}^F(t) - \zeta\lambda_S(t). \quad (27)$$

Since the dirty coal is exploited simultaneously, then we must also have $p(t) = p^F(t) + \zeta\lambda_Z(t)$. Time differentiating and taking (27) into account, we get $\dot{p}(t) = \dot{p}^F(t) + \zeta\dot{\lambda}_Z(t) = \dot{p}^F(t) - \zeta\lambda_S(t)$, hence:

$$\dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_S(t) < 0.$$

Assume now that both the dirty coal and the solar energy are exploited at the same time, then: $p(t) = p^F(t) + \zeta\lambda_Z(t) = c_y$. Time differentiating and using (18) implies:

$$\dot{p}^F(t) + \zeta\dot{\lambda}_Z(t) = \zeta\lambda_{X0}e^{\rho t} + \zeta\dot{\lambda}_Z(t) \quad \Leftrightarrow \quad \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_{X0}e^{\rho t} < 0.$$

Finally, assume that only the dirty coal is exploited, then: $p(t) = u'(\bar{x}_d) = p^F(t) + \zeta\lambda_Z(t)$. Hence according to (18):

$$\dot{\lambda}_Z(t) = -\frac{1}{\zeta}\dot{p}^F(t) = -\frac{\rho}{\zeta}\lambda_{X0}e^{\rho t} < 0.$$

Although $\dot{\lambda}_Z$ is not necessarily continuous, λ_Z is continuous, which concludes the proof. ■

The last common characteristics shared by all the possible optimal paths concerns their respective behavior during the pre-ceiling phase, i.e. before the ceiling constraint begins to be active. From Proposition 2, we know that this phase must also occur before the beginning of the clean coal exploitation, that is over the time interval $[0, \underline{t}_Z] \subseteq [0, \underline{t}_c]$. During this initial phase the full marginal cost of the clean coal option amounts to:

$$c_m^c = c_x + \bar{c}_s + (\lambda_{X0} - \lambda_{S0})e^{\rho t},$$

which may be either increasing or decreasing depending on whether the initial shadow marginal cost of coal λ_{X0} is larger or smaller than the initial shadow marginal value λ_{S0} of the cumulative experience in abatement. Such a formulation could prove to be paradoxical since no experience has been accumulated yet. But λ_{S0} must be read as the marginal value of a zero-experience and this marginal value may be very high.

The sign of $\lambda_{X0} - \lambda_{S0}$, which is endogenous, determines the position of the phase of a simultaneous exploitation of clean and dirty coals in the optimal sequence of phases. However these types of optimal sequences depend upon whether the solar energy cost is high or low.

4.2 The high solar cost case: $c_y > u'(\bar{x}_d)$

In the high solar cost case, since $c_y > u'(\bar{x}_d)$, solar energy is necessarily used after the period at the ceiling. Hence, the argument that determines the different possible types of scenarios is the scarcity of coal, which is measured by λ_{X0}, relative to the shadow marginal value of the learning in sequestration, λ_{S0}.

High marginal shadow cost of coal: $\lambda_{X0} > \lambda_{S0}$

Since $\lambda_{X0} > \lambda_{S0}$, the clean coal exploitation must begin precisely at the time at which the pollution cap $\bar{Z}$ is attained: $\underline{t}_c = t_Z$. This case is illustrated in Figure 1. At the intersection of the trajectories $p^F(t) + \bar{c}_s - \lambda_{S0}e^{\rho t}$ and $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ (remind that $p^F(t) = c_x + \lambda_{X0}e^{\rho t}$) either the common full marginal cost of coal is lower than $u'(\bar{x}_d)$, as illustrated in Figure 1, or it is higher (not depicted) so that the clean coal is never competitive. Thus the unique possible optimal sequence of phases is the following: i) only dirty coal up to the time at which the ceiling is attained and, simultaneously, the clean coal becomes competitive, ii) both the dirty and clean coals at the ceiling, iii) only dirty coal at the ceiling, iv) only dirty coal during a first post-ceiling phase, and v) the infinite phase of solar energy use.

[Figure 1 here]

The other implication of $\lambda_{X0} > \lambda_{S0}$ is that at time $\underline{t}_c^+$, i.e. at the beginning of the phase of a joint exploitation of the both types of coal, due to the continuity of $\lambda_S(t)$, then:

$$\lambda_{X0}e^{\rho \underline{t}_c^+} - \lambda_S(\underline{t}_c^+) \simeq (\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c^+} > 0.$$

From (26), we conclude that $\dot{x}_c(\underline{t}_c^+) < 0$ and, from Proposition 5, that $\dot{x}_c(t) < 0$ for all $t \in [\underline{t}_c^+, \bar{t}_c)$. As a result, the energy price is increasing during this phase.

Low marginal shadow cost of coal: $\lambda_{X0} < \lambda_{S0}$

In this case, the marginal value of the CCS experience is higher than the scarcity rent of coal. This gives rise to some new types of optimal paths, not only because what is happening during the phase of a joint exploitation of the both types of coal is different, but also because the position of this phase within the optimal sequence of phases may be different.

Figure 2 illustrates why the time profiles of the energy price and of the energy consumption are different during the phase of joint exploitation although the optimal sequence of phases is the same as the sequence of the previous case where $\lambda_{X0} > \lambda_{S0}$.

[Figure 2 here]

Since $(\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c} < 0$, then at the beginning of the joint exploitation phase we may have $\lambda_{X0} e^{\rho \underline{t}_c^+} - \lambda_S(\underline{t}_c^+) < 0$ so that $\dot{x}(\underline{t}_c^+) > 0$. From Proposition 5 we know that, in this case, the energy price must be first decreasing and next increasing as illustrated in Figure 2, thus implying an unusual increase in the total coal consumption once the pollution cap is attained to capitalize on the learning effects.

Finally, a last case has to be considered. In Figure 3, the optimal sequence of phases is modified in the following terms. The clean coal begins to be competitive after the beginning of the period at the ceiling so that $\underline{t}_c$ no longer coincides with $\underline{t}_Z$. Consequently, the phase of joint exploitation of the both types of coal still occurs during the ceiling period, but it is now flanked by two phases of exclusive dirty coal use: $\underline{t}_Z < \underline{t}_c < \bar{t}_c < \bar{t}_Z$. Contrary to the previous cases, the exploitation of the clean coal begins here smoothly: $\lim_{t \downarrow \underline{t}_c} x_c(t) = 0$. Hence, there is not an abrupt change anymore in dirty coal consumption at time $\underline{t}_c$, contrary to the case where $\underline{t}_c = \underline{t}_Z$, as illustrated in Figure 2 for instance.

[Figure 3 here]

Analysis

In the high solar cost case, solar energy is never competitive before the depletion of the coal energy source. The optimal timing and scale of the clean coal option thus depends only on the relative competitiveness of the dirty and clean coal options. Proposition 2 states that the cleaning option is never competitive before the economy is constrained by the atmospheric carbon ceiling, irrespective of the importance of future learning opportunities. It shows that an early introduction of the cleaning option, that is before $\underline{t}_Z$, is always dominated by a reduction in the use of dirty coal allowing to delay the attainment of the ceiling.

However, this does not mean that the cleaning option has to be introduced right from the beginning of the ceiling phase. $u'(\bar{x}_d)$ gives a measure of the opportunity cost of the pollution constraint during the ceiling period. This cost has to be balanced with the highest opportunity cost of clean coal energy generation given by $p^F(t) + \bar{c}_s - \lambda_S(t)$. This cost increases over time with a high shadow marginal cost of coal, implying an introduction of the cleaning option right from $\underline{t}_Z$. It decreases over time with a low shadow marginal cost of coal resulting into two possibilities at the beginning of the ceiling phase. Either it is lower than $u'(\bar{x}_d)$, the opportunity cost of the constraint on dirty coal production, in which case the cleaning option has to be introduced immediately when attaining the ceiling. Or it is higher than $u'(\bar{x}_d)$ at $\underline{t}_Z$ and abatement should be introduced only after some time period at the ceiling with only dirty coal burning before that time when the two opportunity costs are equalized.

As we are going to examine now, in the low solar cost case, clean coal will be in competition not only with dirty coal but also with the carbon-free solar energy, thus resulting in new optimal scenarios of energy consumption.

4.3 The low solar cost case: $c_y < u'(\bar{x}_d)$

Since $c_y < u'(\bar{x}_d)$ then, during the ceiling period, dirty coal is necessarily used together with either solar energy or clean coal. Concerning the solar energy, the problem is to determine whether its exploitation may begin before the pollution cap $\bar{Z}$ is attained or not. Proposition 8 shows that, like the clean coal exploitation, the exploitation of the solar energy may not begin before the ceiling constraint is binding.

Proposition 8 *The solar energy exploitation may not begin before the ceiling constraint is binding:* $t_y \geq \underline{t}_Z$.

Proof: The logic of the proof is the same as those developed for the clean coal in Proposition 2. Assume that solar energy is used while the ceiling constraint is not binding yet: $t_y < \underline{t}_Z$. Then over the time interval $(t_y, \underline{t}_Z)$, only solar energy must be used since $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} > c_y$ and clean coal may not be exploited according to Proposition 2. Hence $Z(t)$ decreases during the interval, implying $Z(\underline{t}_Z) < \bar{Z}$, a contradiction. ■

As in the high solar cost case, various types of optimal paths are possible according to whether the initial mining rent, λ_{X0}, is larger or smaller than the initial shadow marginal value of learning, λ_{S0}.

High marginal shadow cost of coal: $\lambda_{X0} > \lambda_{S0}$

According to the arguments developed in the previous subsections, the phase of joint exploitation of the two types of coal must begin when the ceiling is attained and the energy price must be increasing during this phase although the shadow marginal cost of the pollution stock is decreasing, up to the time at which this price equals c_y instead of $u'(\bar{x}_d) < c_y$, time at which the solar energy becomes competitive (see Figure 4). Then, according to Proposition 4, the exploitation of the clean coal must cease precisely at this date. The production of solar energy thus substitutes for the production of clean coal while staying at the ceiling up to the time at which $p^F(t) = c_y$. Then the dirty coal exploitation is closed, the coal reserves must be exhausted and the solar energy supplies the totality of the energy needs. Note that in this case, the total coal consumption shuts down $\underline{t}_c = t_y$ when the solar energy becomes competitive.

[Figure 4 here]

Low marginal shadow cost of coal: $\lambda_{X0} < \lambda_{S0}$

First, the period of joint exploitation of the two types of coal may precede the period of competitiveness of the solar energy. The associated price and consumption paths are illustrated in Figure 5.

[Figure 5 here]

However, as illustrated in Figure 6, the phase of competitiveness of the clean coal may also take place once the solar energy is competitive, that is at a date at which the solar energy is already used: $t_y = \underline{t}_Z < \underline{t}_c < \bar{t}_c < \bar{t}_Z$. In this case, the exploitation of the solar energy must be interrupted since the energy price falls down the trigger price c_y during the time interval $[\underline{t}_c, \bar{t}_c]$ of joint exploitation of the both kinds of coal. At time $t = \bar{t}_c$, the solar energy becomes competitive again and its production replaces the production of the clean coal. Then, the dirty coal and the solar energy are simultaneously used, as in the first phase of the ceiling period, up to the time $t = \bar{t}_Z$ at which $p^F(t) = c_y$ and at which the stock of coal is exhausted.

[Figure 6 here]

Analysis

In the low solar case, the energy price cannot be higher than c_y, the (constant) marginal cost of solar energy. When the shadow cost of coal is large, the highest opportunity cost being permanently rising, either it is higher than c_y at $\underline{t}_Z$, meaning that the CCS option is irrelevant with respect to the cheaper solar alternative, either it is lower, and the CCS option eliminates temporarily the use of solar energy. Learning being presently unable to prevent the continuous rise of the full marginal cost of clean coal, its exploitation ceases after its full marginal cost becomes higher than the solar energy cost. In this scenario, learning abilities only opens a transitory window for clean coal energy generation before the introduction of carbon-free solar energy.

High learning opportunities with respect to coal scarcity opens new possibilities. As in the high solar cost case, the highest opportunity cost of the cleaning option declines before the introduction of clean coal. If it is lower than c_y when the ceiling is attained, the cleaning option eliminates, at least temporarily, the solar option. In the reverse case, solar energy is introduced once the atmospheric carbon constraint begins to be binding, but the continuous fall of the cleaning highest opportunity cost allows the cleaning option to be introduced at the expense of the solar option after some time at the ceiling. However the competitive advantage of clean coal can only be transitory because of coal depletion and solar energy ultimately replace clean coal energy generation. In such scenarios, solar energy is used first at the ceiling in combination with dirty coal energy waiting for the clean coal option to become competitive. It appears once again when the competitive advantage of clean coal has been sufficiently reduced by the increasing scarcity of fossil fuels.

5 Concluding remarks

We have shown that it is optimal to wait that the ceiling constraint be effective before beginning to abate some part of the potential pollution flow when the instantaneous average abatement cost is linear in the flow of sequestration, even if learning effects are at work and, maybe more surprising, whatever strong these learning effects are: our results do depend only upon the qualitative properties of the CCS cost function. Two main reasons can explain this result. First, the abatement cost function can be broken up in the sense of Uzawa (1965): $C(S, x_c) = c(S)x_c$. This implies constant returns to scale in abatement technology and then, particular marginal effects. More precisely, whatever the learning function, it is always the first unit of sequestration that is the costlier. From a discounting argument, it is thus always optimal to start abatement as late as possible, that is once the ceiling becomes really constraining for the economy. A more general decomposable form such as $C(S, x_c) = C_0(S)C_1(x_c)$, with $C_1(.)$ increasing and convex, would lead to different results. Amigues et al (2013-b) show that with decreasing returns to scale but without learning effect, it is optimal to deploy abatement before the ceiling is reached. A possible extension would be to extend this result by also considering learning-by-doing.

A second reason is that the learning function is assumed to be linear in our model: $\dot{S} = x_c$. A more general assumption would be to set $\dot{S} = g(x_c)$, with $g(.)$ increasing. If we assume in addition that $g(.)$ is concave, thus reflecting decreasing returns in the learning function, the smaller the abatement, the larger the marginal acquisition of experience. In such a case, the intuition suggests that it would be optimal to start the learning as soon as possible. Again, this argument constitutes a line for future research. Note that, in all these possible extensions, the timing of CCS policies must be defined relatively to the (endogenous) date at which the ceiling is reached. In the absolute, learning-by-doing has also an effect on this date.

The optimal policy has to be supported by both a pollution tax upon the carbon released into the atmosphere and by a subsidy to the clean coal alternative. The time profile of the pollution tax rate λ_Z has the well know U inverted profile obtained in most studies of the warming problem, along the lines pioneered by Ulph and Ulph (1994) and Tahvonen (1997). This time profile is also the standard profile of the models with ceiling constraints pioneered by Chakravorty *et al.* (2006) and the time profile of the mixed model of Amigues *et al.* (2011) in which both *small* and *catastrophic* damages are taken into account.

The time profile of the unitary subsidy to the clean coal production is first increasing during the preliminary phase preceding the beginning of its production and decreasing down to zero during the time interval at the ceiling within which its production decreases. It is less easy to characterize during the phase of increasing clean coal production if such a phase exists, the time path being not necessarily monotone.

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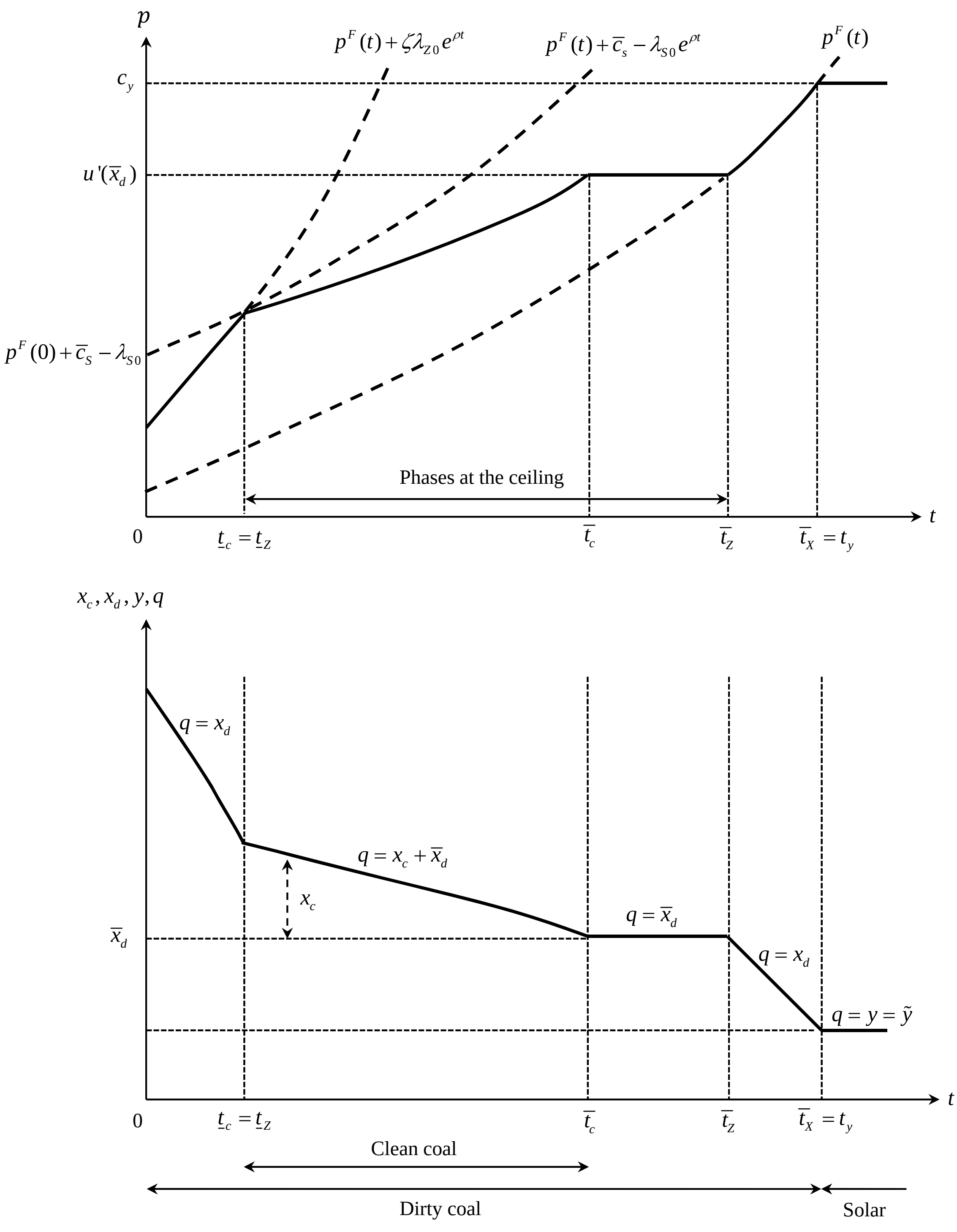


Figure 1: Optimal paths when $c_y > u'(\bar{x}_d)$ and $\lambda_{X0} > \lambda_{S0}$

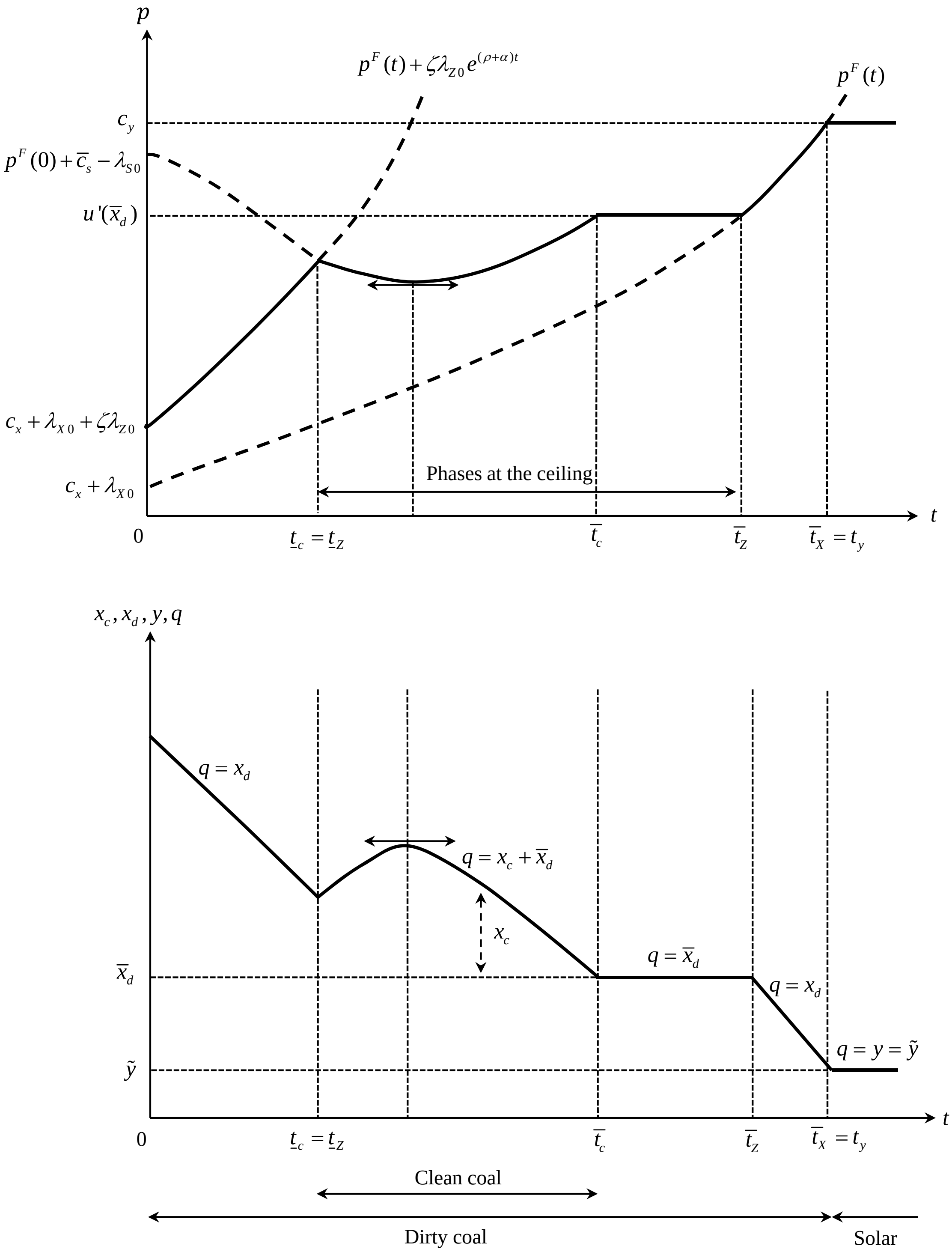


Figure 2: Optimal paths when $c_y > u'(\bar{x}_d)$, $\lambda_{X0} < \lambda_{S0}$ and $\underline{t}_c = \underline{t}_Z$

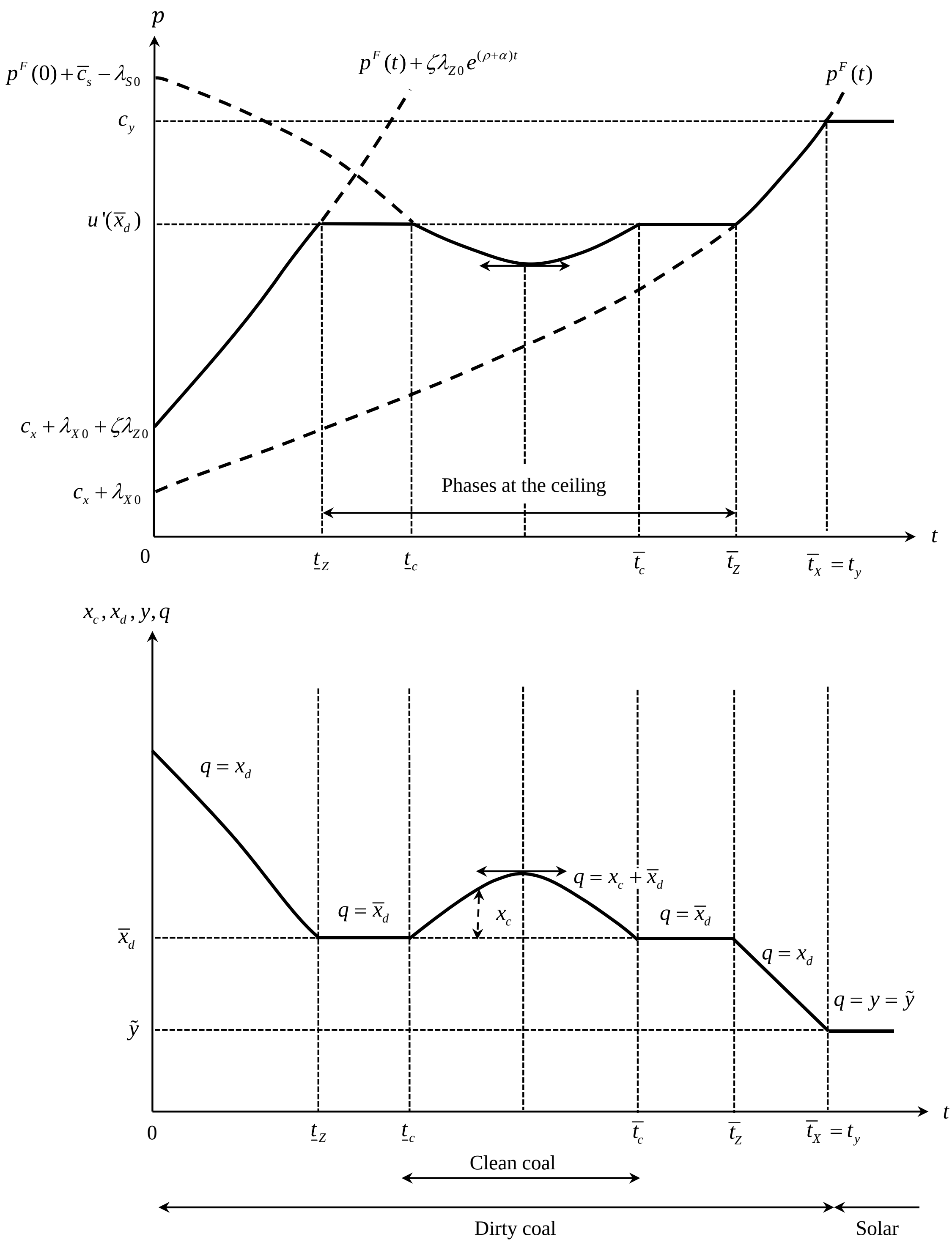


Figure 3: Optimal paths when $c_y > u'(\bar{x}_d)$, $\lambda_{X0} < \lambda_{S0}$ and $\underline{t}_c > \underline{t}_Z$

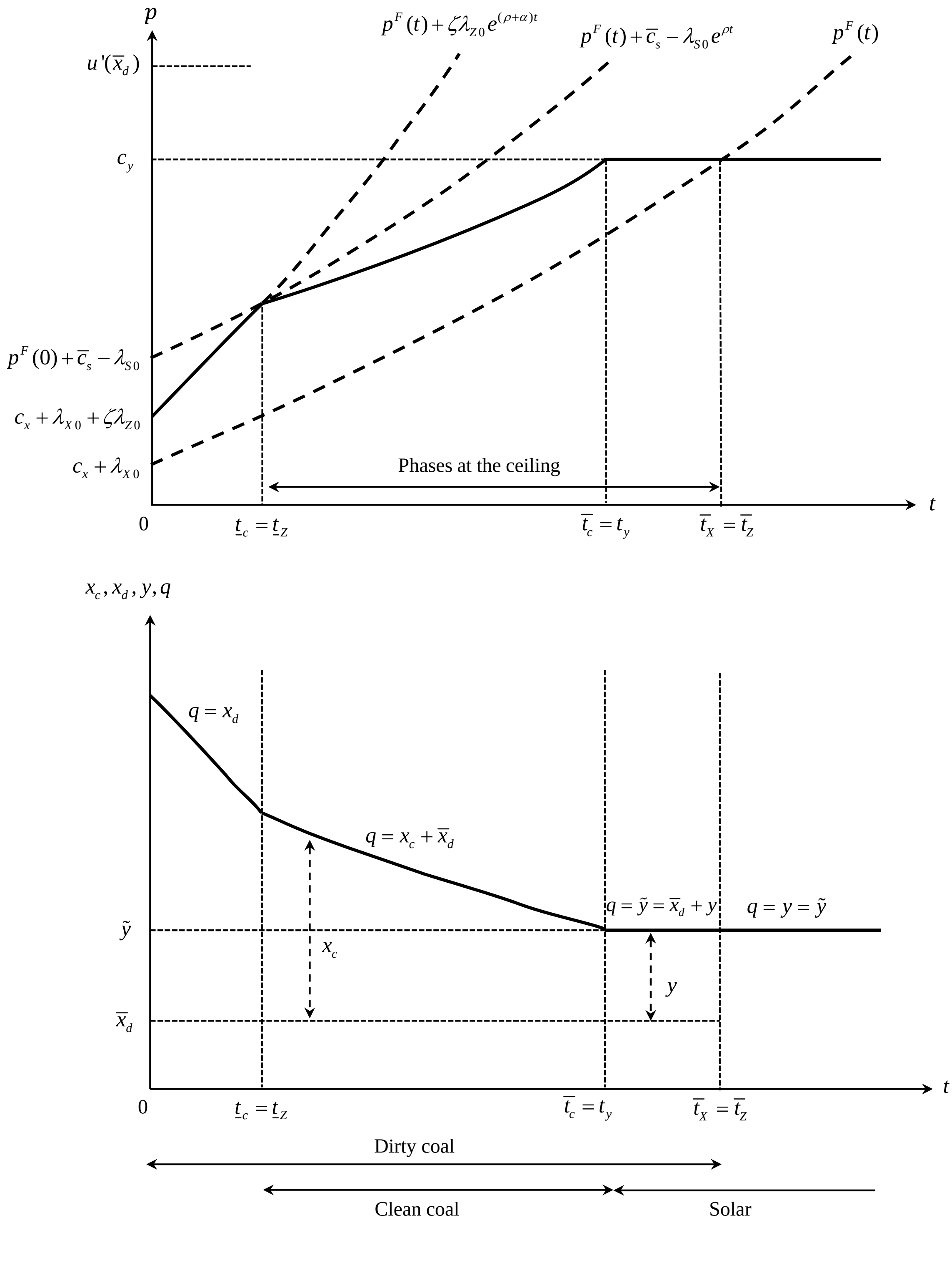
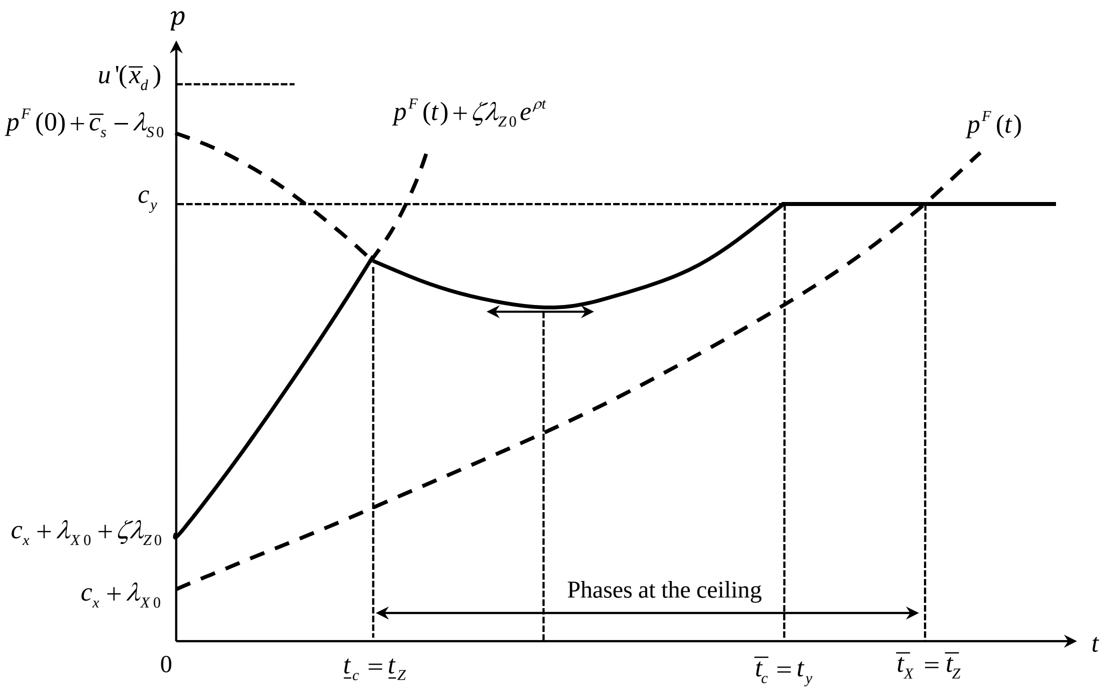


Figure 4: Optimal paths when $c_y < u'(\bar{x}_d)$ and $\lambda_{X0} > \lambda_{S0}$



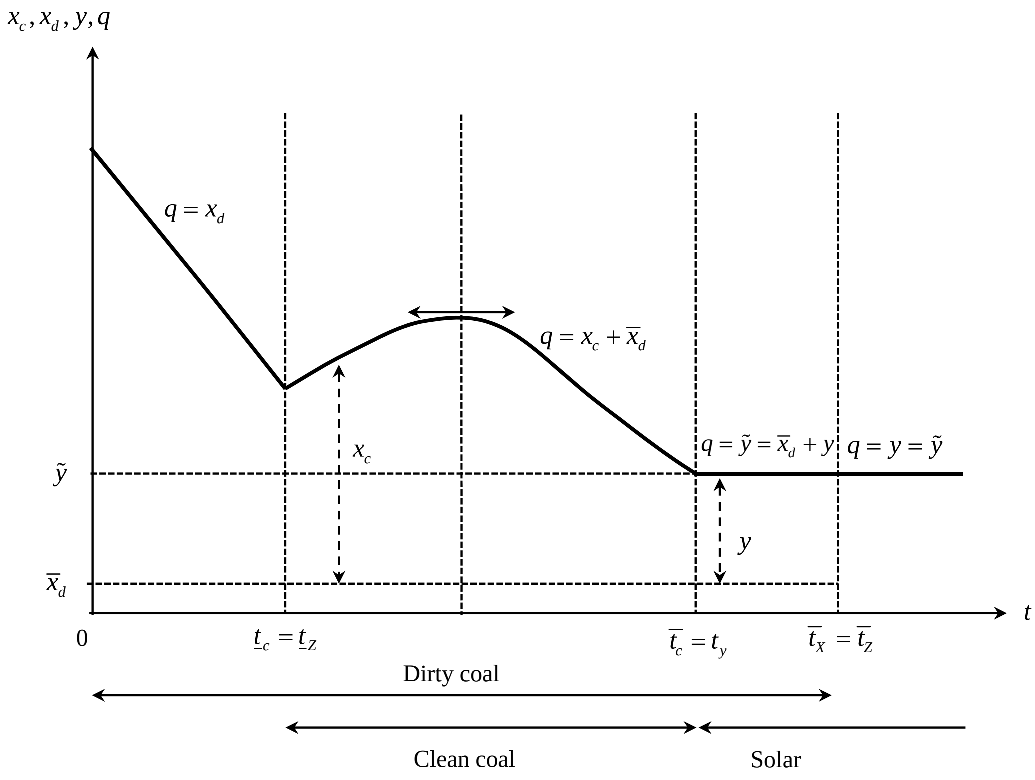


Figure 5: Optimal paths when $c_y < u'(\bar{x}_d)$, $\lambda_{X0} < \lambda_{S0}$ and $\underline{t}_c = \underline{t}_Z$

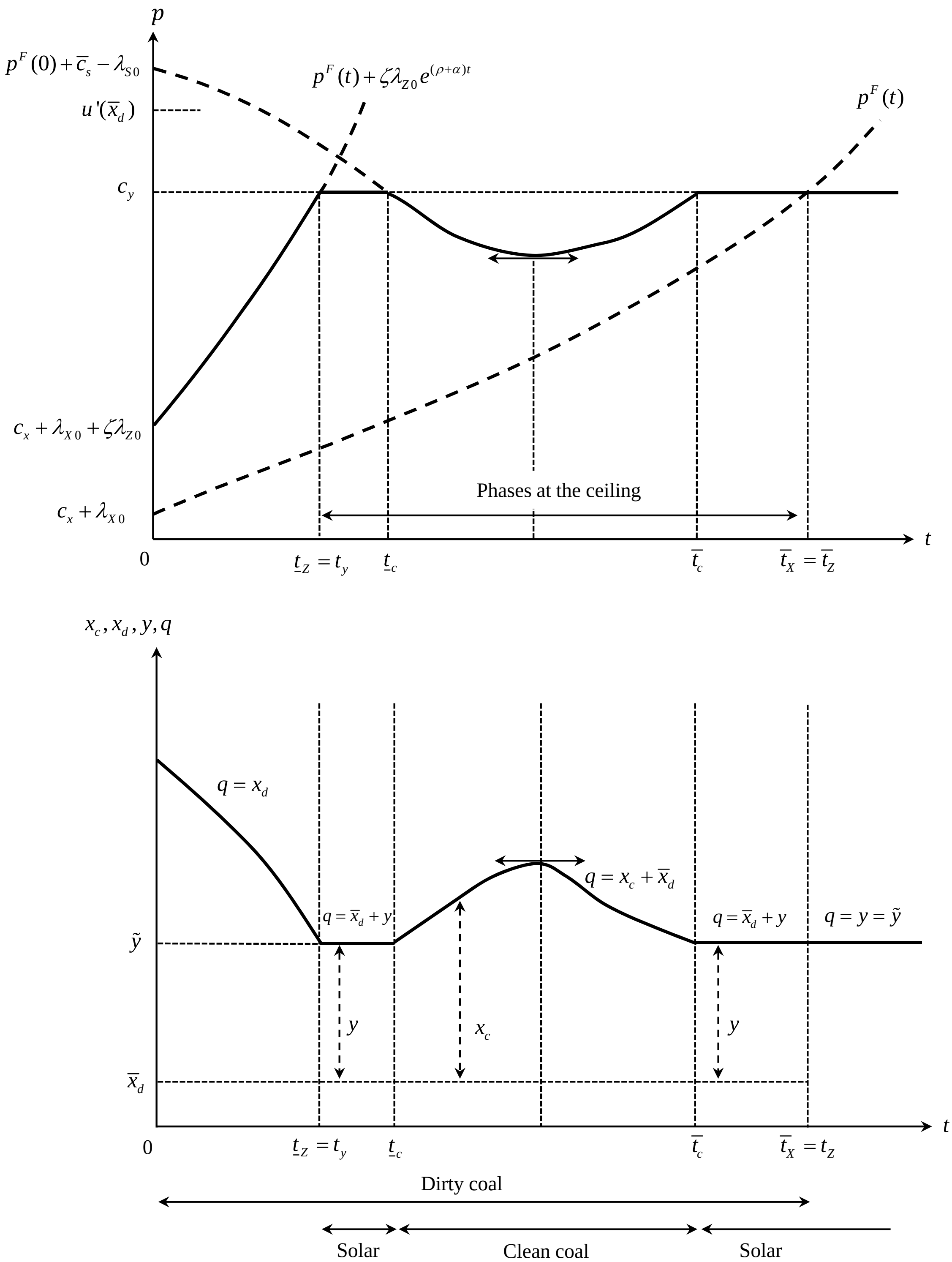


Figure 6: Optimal paths when $c_y < u'(\bar{x}_d)$, $\lambda_{X0} < \lambda_{S0}$ and $\underline{t}_c > \underline{t}_Z$