Signaling quality in vertical relationships

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Abstract

This paper addresses the issue of price signaling in a model of vertical relationship between a manufacturer and a retailer who share the same information about quality, unlike consumers who do not observe it *a priori*. We show that delegating the price setting task to a retailer and controlling it through a vertical contract (two-part tariff) helps drastically reduce the number of price signaling equilibria available to the retailer. The outcome of a unique price charged to consumers obtains without invoking the consumer sophistication usually required by selection criterions. The vertical contract turns to be the most efficient way for the vertical chain to tie its hands on a unique final price. This price may disclose or not information to consumers depending on their initial optimism about quality. We prove that there also exists circumstances where consumers prefer *ex ante* not to learn the true quality through price.

JEL : L12, L15, D82.

Key-words: quality signalling, vertical relationship, information disclosure.

1 Introduction

After Spence (1973) stated that firms could signal product quality by taking some costly actions, a number of theoretical papers have explored the idea that prices have a specific role to play in signaling quality. Following Bagwell and Riordan (1991), one important feature of the market for experience goods is that firms set prices above the full information level to signal high-quality new products. Indeed, like most signaling behaviors, price signaling may entail some inefficiencies. To credibly inform consumers that quality is high, the firm often needs to distort its price, either upward or downward, compared to what would prevail under full information.[1] More precisely, when a monopolist uses price as a signal of quality, Bagwell and Riordan (1991) prove the need and existence of an upward distortion in price to fully reveal that quality is high *at the Riley separating equilibrium outcome*.[2]

Building on this model, we open the black box of the monopoly to allow for a vertical contract in the form of a two-part tariff between a manufacturer and a retailer. Both of them know product quality better than consumers do, hence the vertical contract specifies a consumer price which serves the same task of signaling quality as that investigated by Bagwell and Riordan (1991). Our predictions somewhat complement their findings but also differ in several respects. We still find that an upward-distorted price may fully reveal that quality is high. Nevertheless, we show that such an information disclosure occurs provided that consumer prior information does not involve too much optimism about quality. Moreover, in this separating equilibrium outcome, the vertical contract does implement a unique price signaling quality without the standard use of equilibrium refinement —specifically, the additional restrictions imposed by Bagwell and Riordan (1991) on out-of-equilibrium beliefs in the spirit

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[1] Besides Bagwell and Riordan (1998), Daughety and Reinganum (2008) also show that firms need to distort prices upward to signal high quality in a continuum-type model where price is the only means of signaling quality. When quality is environmental, Mahenc (2008) states that the price distortion due to signaling goes upward provided that marginal costs increase with environmental quality. Nevertheless, prices may be driven below the full-information levels when the firm uses an additional signaling instrument such as advertising expenditures (see Milgrom and Roberts, 1986, Linnemer, 1998, Hertzendorf and Overgaard, 2001, and Fluet and Garella, 2002).

[2] After Riley (1979), the so called Riley equilibrium is the only separating equilibrium outcome that is Pareto undominated from the informed agent's standpoint.

of the "intuitive criterion" (see Cho and Kreps, 1987)— It turns out that the vertical contract makes it possible to ignore consumer out-of-equilibrium beliefs, and the manufacturer uses the franchise to commit the retailer on the (most profitable) Riley outcome.

However, the Riley outcome is Pareto dominated by pooling equilibrium outcomes when consumers, before purchase, rather believe quality to be high. We show that, in this way, the vertical contract implements the most profitable pooling price, thereby concealing information about quality. Although this result departs from the traditional concentration on the Riley separating equilibrium outcome, it is economically appealing in the following sense. If the vertical contract allows for a commitment on a particular signaling behavior, it seems reasonable that the manufacturer ties the retailer's hands on the signaling equilibrium that Pareto dominates the other equilibria, even though it is pooling. The cost of disclosing information through separating equilibrium prices may be a burden to the manufacturer, especially when consumers are highly optimistic about the product quality. In that event, the manufacturer is better off concealing information, contrary to standard predictions based on the refinement of signaling equilibria.

Moreover, the vertical contract restores the continuity in pricing behaviors between games of incomplete and complete information. If there is a very slight chance that quality is low, the pooling price implemented through the vertical contract does not differ significantly from that in a situation with no chance at all of low quality. Mailath et *alii* (1993) raise the same continuity argument to motivate the selection of "undefeated" equilibria in signaling games. Thanks to the vertical contract, we predict signaling behaviors similar to those selected by their refinement, but once again, these predictions do not require restrictions on out-of-equilibrium beliefs. In other terms, we do not need the assumption that uninformed consumers are highly sophisticated in the way they build and update their beliefs in order to get economically intuitive signaling behaviors. The only assumption made on consumer inference here is that they revise their beliefs according to Bayes' rule when observing a price. Delegating the price setting task to the retailer and controlling it through the vertical

contract drastically reduces the number of price signaling equilibria available to the retailer. Finally, the analysis provides insight on the circumstances under which consumers prefer *ex ante* to learn or not the true quality through price.

The paper is organized as follows. In the section 2, we lay down the model and the assumptions. Then in section 3, we characterize the equilibria under full information and asymmetric information in the case of no delegation. In section 4, we then examine the case of delegation. Section 5 concludes. An appendix contains most of proofs.

2 Assumptions and notations

Consider a good produced by a manufacturer and sold by a retailer, both firms acting as a monopolist. In the following, we will consider two situations in turn: in the first one, both firms form a vertically integrated structure and in the second one, both firms act independently. The former case can be interpreted as a situation where the manufacturer totally controls the distribution level while the latter one represents a situation where the manufacturer chooses to delegate distribution to an independent seller.

Using the terminology of Nelson (1970), the product is assumed to be an "experience good" in the sense that consumers cannot observe its actual quality before purchase. We denote q the quality characteristic of the product and we further assume, for simplicity, that q may be either high ($q = H$) or low ($q = L$) with $H > L \geq 0$. Higher quality is more costly to produce so that the manufacturer incurs constant marginal costs of production denoted c_q for $q = H, L$, with $c_H = c > 0$ and $c_L = 0$. In addition, the retailing activity does not generate any specific cost related to the quality so that we normalize the cost of retailing to zero.

A priori, consumers perceive the quality to be high with probability $\mu_0 \equiv prob(q = H)$, and low with probability $1 - \mu_0 \equiv prob(q = L)$, $\mu_0 \in (0, 1)$. When deciding to purchase the good, consumers observe the price p, they try to infer some information about quality and thereby they update their beliefs. Let $\mu(p) : R^+ \to [0, 1]$ denote the consumers' posterior

belief that quality of product 0 is H upon seeing p. For any belief μ, we denote $\tilde{q}(p) = \mu(p)\Delta + L \in [L, H]$ the expectation of quality formed by consumers, where $\Delta = H - L$ is the quality gap between a high and low-quality product. As there is a one to one relationship between expectation $\tilde{q}$ and belief μ, we will use only $\tilde{q}$ in the following, to save on notations. In particular, we denote $q_0 \equiv \mu_0 \Delta + L \in [L, H]$ the prior quality expectation.

We assume further that aggregate demand is a smooth and strictly decreasing function of price when positive, cutting both axes, and is an increasing function of perceived quality, that we denote $D(p, \tilde{q})$.[3] Note that our assumption of a common prior and posterior beliefs on quality does not impede heterogeneity in consumers' preferences.

For the rest of this section, we do as if the perceived quality $\tilde{q}$ were constant and thus independent of the price p. This is done in order to define some functions of interest for the analysis. First, denote by $p_{\tilde{q}}^{\max}$ the reservation price for the perceived quality $\tilde{q}$, such that $D(p, \tilde{q}) = 0$ for any $p \geq p_{\tilde{q}}^{\max}$. Assume that $p_H^{\max} > c$ and $p_L^{\max} > 0$, and it follows that:

$$D(c, H) > 0 \text{ and } D(0, L) > 0 \tag{1}$$

which ensures that both qualities priced at marginal cost can get a positive demand under complete information about their characteristic, making their presence on the market socially efficient.[4]

We denote:

$$\pi(p; x, \tilde{q}) = (p - x)D(p, \tilde{q}) \tag{2}$$

the (variable) profit that can be made on the market, as a function of the price p in $\left[x, p_{\tilde{q}}^{\max}\right]$, the marginal cost $x \leq p_{\tilde{q}}^{\max}$ and the quality $\tilde{q} \in [L, H]$ expected by consumers. The marginal

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[3] Note that our general specification of demand encompasses several possibilities for how quality expectation may affect the slope of demand with respect to price. Indeed, consider the popular Mussa Rosen preferences specification: assume a continuum, with unitary mass, of heterogenous consumers indexed by $y \in [0, 1]$ following the uniform distribution, each one buying at most one unit. Then the individual utility is $u = y\tilde{q} - p$ and the aggregate demand is $D(p, \tilde{q}) = 1 - \frac{p}{\tilde{q}}$. In that case, increasing quality makes the demand less price elastic. Consider now a linear city where the product is located in 0 and consumers are indexed by their location y in $[-a, a]$, still following the uniform distribution and where $a > 0$ is large. Here the utility can be specified as $u = \tilde{q} - ty - p$ where t is the transportation cost parameter. And the aggregate demand writes $D(p, \tilde{q}) = \frac{2}{t}(\tilde{q} - p)$. where quality affects only the reservation price.

[4] As we will show in the next section, the condition $D(0, L) > 0$ is actually necessary for separation in the signaling game with the vertically integrated firm.

cost x will correspond to the marginal cost of production in a vertically integrated firm and to the wholesale price when the manufacturer and the retailer are independent. As long as demand $D(p, \tilde{q})$ is positive, we assume strict concavity of $\pi(p; x, \tilde{q})$ with respect to price.[5]

We also define the (unique) optimal price $\hat{p}(x, \tilde{q})$ as:

$$\hat{p}(x, \tilde{q}) \equiv \arg \max_{p \geq x} \pi(p; x, \tilde{q}) \tag{3}$$

that is increasing in x and $\tilde{q}$, and we denote:

$$\hat{\pi}(x, \tilde{q}) \equiv \pi(\hat{p}(x, \tilde{q}); x, \tilde{q}) \tag{4}$$

the corresponding maximized profit level.

In the analysis, we will often meet the following function:

$$f(p; x, \tilde{q}) \equiv \pi(p; x, \tilde{q}) - \hat{\pi}(x, L) \tag{5}$$

for any p in $\left[x, p_{\tilde{q}}^{\max}\right]$, for any $x \leq p_{\tilde{q}}^{\max}$ and for any $\tilde{q} \in [L, H]$. The function $f(p; x, \tilde{q})$ represents the difference between the (variable) profit a seller, perceived as a $\tilde{q}$ quality seller, can make by facing the cost x and when charging p, and the maximum profit $\hat{\pi}(x, L)$ it can earn in the worst situation, that is, when consumers perceived it as a low-quality seller. A number of interesting properties of f recorded in the following lemma will be useful for the analysis.

Lemma 1 *For any $x \leq p_{\tilde{q}}^{\max}$ and $\tilde{q}$ in $[L, H]$, the function $f(.; x, \tilde{q})$ has exactly two roots on the interval $\left[x, p_{\tilde{q}}^{\max}\right]$, that we denote $\underline{p}(x, \tilde{q})$ and $\overline{p}(x, \tilde{q})$ with $\underline{p}(x, \tilde{q}) < \overline{p}(x, \tilde{q})$. The function $f(.; x, \tilde{q})$ is also positive between the roots and negative either. Moreover, both roots are strictly interior when $\hat{\pi}(x, L) > 0$ and they are increasing in x. When $\hat{\pi}(x, L) = 0$ then $\underline{p}(x, \tilde{q}) = x$ and $\overline{p}(x, \tilde{q}) = p_{\tilde{q}}^{\max}$.*

Proof: See Appendix A. ∎

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[5] The weakest sufficient condition for strict concavity is the mark-up in absolute terms, $m(p) = p - x \equiv -\frac{D(p, \tilde{q})}{\frac{\partial D}{\partial p}(p, \tilde{q})}$ to be decreasing or not too increasing in price, that is $m'(p) \leq 1$.

As will be clear below, the threshold prices identified in Lemma 1 help characterize the set of separating and pooling price equilibria in the signaling game. Specifically, this will enable us to clearly define the Riley separating equilibrium price and determine whether it is Pareto dominated or not by pooling equilibrium prices.

3 Signaling quality in the vertically integrated structure: the case of no delegation

We consider in this section the situation where the manufacturer and the retailer form an integrated vertical structure, that acts as a monopolist on the final market. Under complete information, consumers are able to distinguish perfectly between both qualities and the price $\hat{p}(c, H)$ (respectively $\hat{p}(0, L)$) denotes the optimal price for a high-quality (resp. for a low-quality) product. The monopolist is making $\hat{\pi}(c, H)$ if of high quality and $\hat{\pi}(0, L)$ otherwise.

Under asymmetric information, the integrated firm may choose either to disclose his private information on quality through separating prices, or to conceal this information by setting pooling prices. The monopolist maximizes profit with respect to price, given the beliefs held by consumers after observing this price. This objective defines a signaling game similar to that investigated by Bagwell and Riordan (1991). The monopolist's pricing strategies must be supported as a Perfect Bayesian Equilibrium (PBE). Many of the proof techniques used by Bagwell and Riordan (1991) are readily adapted for characterizing the PBE in our setting. We will consider in turn the issue of separating price equilibria and of pooling price equilibria.

3.1 The set of separating outcomes

Consider a putative separating equilibrium $\left(p_H^I, p_L^I\right)$ in the integrated structure. As usual, such an equilibrium must satisfy two kinds of constraints: first, individual rationality (IR) constraints and, second, incentive compatibility (IC) constraints. The IR constraints ensure that the monopolist finds it profitable to choose an equilibrium price rather than the optimal price associated with the worst belief that consumers can hold from the monopolist's

standpoint, i. e., $\tilde{q} = L$.

The IR constraints are given for H and L, respectively, by:

$$\pi\left(p_H^I; c, H\right) \geq \hat{\pi}\left(c, L\right) \tag{IR$_H$}$$

$$\pi\left(p_L^I; 0, L\right) \geq \hat{\pi}\left(0, L\right). \tag{IR$_L$}$$

The IC constraints require that the price set for one quality would not be worth mimicking if the quality were different. They are given for H and L, respectively, by:

$$\pi\left(p_H^I; c, H\right) \geq \pi(p_L^I; c, L) \tag{IC$_H$}$$

$$\pi(p_L^I; 0, L) \geq \pi(p_H^I; 0, H). \tag{IC$_L$}$$

We simplify the analysis of the set of prices satisfying the above constraints by noting the following two claims.

Claim 1 *The constraint (IR$_L$) is necessarily binding.*

Indeed, by definition, the monopolist correctly identified as selling a low-quality product cannot get more than by charging the price $\hat{p}(0, L)$. It follows that, at a separating equilibrium, the low-quality price is equal to the low-quality price under complete information:

$$p_L^I = \hat{p}(0, L).$$

Claim 2 *The constraint (IC$_H$) is satisfied once (IR$_H$) holds.*

Indeed, this is because by definition $\hat{\pi}\left(c, L\right) \geq \pi(p_L^I; c, L)$ for all p_L^I.

Both results imply that the set of high-quality separating prices is described by the remaining constraints, (IR$_H$) and (IC$_L$) taken into account that $p_L^I = \hat{p}(0, L)$ and $\pi(p_L^I; c, L) = \hat{\pi}\left(c, L\right)$. It follows that (IR$_H$) and (IC$_L$) can be rewritten respectively as:

$$f(p_H^I; c, H) \geq 0 \text{ and } f(p_H^I; 0, H) \leq 0. \tag{6}$$

We fully characterize the set of prices p_H^I that satisfy (6), in the proof of the following Proposition, that only exhibits the least-cost separating price equilibrium.

Proposition 1 *The least-cost separating price equilibrium for an integrated firm is a pair of prices $\left(p_L^I, p_H^I\right)$ given by:*

$$p_L^I = \hat{p}(0, L)$$

$$p_H^I = \left\{ \begin{array}{l} \overline{p}(0, H) \ \text{whenever } c < \overline{c} \\ \hat{p}(c, H) \ \text{otherwise.} \end{array} \right.$$

where $\overline{c} > 0$ denotes the (unique) cost value such that $\overline{p}(0, H) = \hat{p}(\overline{c}, H)$.

Proof: See Appendix B ∎

Proposition 1 characterizes the separating equilibrium outcome that entails the minimum loss in profit needed to fully reveal that quality is high: this is the so-called Riley separating equilibrium outcome (after Riley, 1979). As a result, the equilibrium price of a low-quality good is never distorted compared to the full information situation. By contrast, when $c < \overline{c}$, the separating price for high quality involves an upward distortion compared to the full information price, that is, $p_H^I = \overline{p}(0, H)$ exceeds $\hat{p}(c, H)$. This upward distorted price is such that it makes the low-quality monopolist indifferent between mimicking the high-quality counterpart by charging $\overline{p}(0, H)$ and being correctly identified by consumers by choosing p_L^I.

However, whenever $c \geq \overline{c}$, then $\hat{p}(c, H)$ belongs to the set of separating prices and hence it is always possible for the high-quality seller to signal its quality without cost. More precisely, if $p_H^{\max}$ is sufficiently large so that $\hat{p}(p_H^{\max}, H) > \overline{p}(0, H)$, then there always exists a range of costs c for which signaling quality entails no distortion.

Throughout the proof of Proposition 1, we show that there exists an infinity of separating equilibrium prices. The presence of multiple equilibria is not surprising in a standard monotonic signaling game such as the one investigated here. Clearly, this is a weakness in terms of generating behavioral predictions. The solution proposed by the literature to this problem is to employ refinements which impose additional restrictions on the beliefs held by consumers out of equilibrium. For instance, a standard exercise here would be either to single out the Riley outcome with the "intuitive criterion" proposed by Cho and Kreps (1987), or to show that the Riley outcome "defeats" all the other separating equilibrium outcomes in

the sense of Mailath et *alii* (1993). We do not need to perform the refinement exercise here for one reason that will clearly appear in the next section: the vertical contract makes it possible to single out a unique signaling price without invoking the consumer sophistication in building and updating beliefs usually required by selection criterions.

Note that our assumption that $D(0, L) > 0$, i.e., consumers value positively the low quality charged at marginal cost, is necessary for separation. Indeed, otherwise $\hat{\pi}(0, L) = 0$ and hence (IC_L) cannot hold with a strictly positive price p_H^I.

3.1.1 The set of pooling outcomes

We now turn to the characterization of pooling equilibria if they exist. Let p^I denote a pooling equilibrium price in the integrated structure. Since the price charged by the monopolist is the same regardless of quality, consumers' posterior beliefs after observing this price are the same as their prior beliefs. Hence, the integrated firm with quality q earns $\pi(p^I; c_q, q_0)$ in equilibrium. To conceal information in equilibrium, any pooling price p^I must give no less profit than what the monopolist could get at best if its product were thought to be of low quality with certainty, that is:

$$\pi(p^I; c_q, q_0) \geq \hat{\pi}(c_q, L), \text{ for } q = H, L. \tag{7}$$

The set of prices p^I such that (7) holds is the set $P(q_0)$ of pooling equilibrium prices.

Proposition 2 *Whenever $\underline{p}(c, q_0) \leq \overline{p}(0, q_0)$, any price p^I that belongs to $P(q_0) = \left[\underline{p}(c, q_0), \overline{p}(0, q_0)\right]$ is a pooling equilibrium price for the integrated firm.*

Note that the condition for existence of pooling equilibria can be interpreted as an upper bound of admissible cost c for high quality because $\underline{p}(c, q_0)$ is increasing in c. Intuitively, if c is high enough *ceteris paribus*, then any equilibrium of the signalling game involves separation.

Let us show that there exist circumstances under which a pooling equilibrium Pareto dominates the Riley separating equilibrium, from the perspective of both types of manufacturer. Let us examine when this situation occurs. As $\pi(p^I; 0, L) \geq \hat{\pi}(0, L)$ by definition of

a pooling equilibrium, the low-quality monopolist that chooses p^I will always make at least the equilibrium profit under separation.

The same is not always true for the high-quality monopolist but happens to be true when the two following conditions are met. First, it must be that the price charged for the high-quality product is distorted upward relative to the full information situation, i. e., $p_H^I = \overline{p}(0, H)$, which arises when $\overline{p}(0, H) > \hat{p}(c, H)$. Otherwise, signalling quality would entail no cost and the high-quality monopolist would never benefit from pooling since $\hat{\pi}(c, q_0) < \hat{\pi}(c, H)$ as long as $q_0 < H$. The second condition is that the high-quality monopolist is indeed better off with the uninformative price p^I than with the separating price $\overline{p}(0, H)$:

$$\pi(p^I; c, q_0) \geq \pi(\overline{p}(0, H); c, H). \tag{8}$$

Note that condition (8) holds for sufficiently high values of q_0. Indeed, because $\hat{\pi}(c, q_0)$ is strictly increasing in q_0, we can define $\hat{q}_0$ as the unique threshold in quality expectation such that

$$\hat{\pi}(c, \hat{q}_0) = \pi(\overline{p}(0, H); c, H). \tag{9}$$

The left-hand side of (9) is the maximum profit that the high-quality monopolist can make by holding back information, when consumers believe the expected quality to be $\hat{q}_0$. The critical value $\hat{q}_0$ is the quality level such that the high-quality producer is indifferent between signaling quality with the upward-distorted price $\overline{p}(0, H)$ and concealing information about quality in the optimal way. We sum up our discussion of pooling equilibria with the following result.

Proposition 3 *For all q_0 higher than $\hat{q}_0$, there exists at least one pooling equilibrium (actually an infinity of) that Pareto dominates the least-cost separating equilibrium regardless of the actual quality.*

Figure 1 depicts the pooling and separating equilibria for values of c lower than $\overline{c}$, first when q_0 is lower than $\hat{q}_0$ in the upper panel, and then when q_0 is larger than $\hat{q}_0$ in the
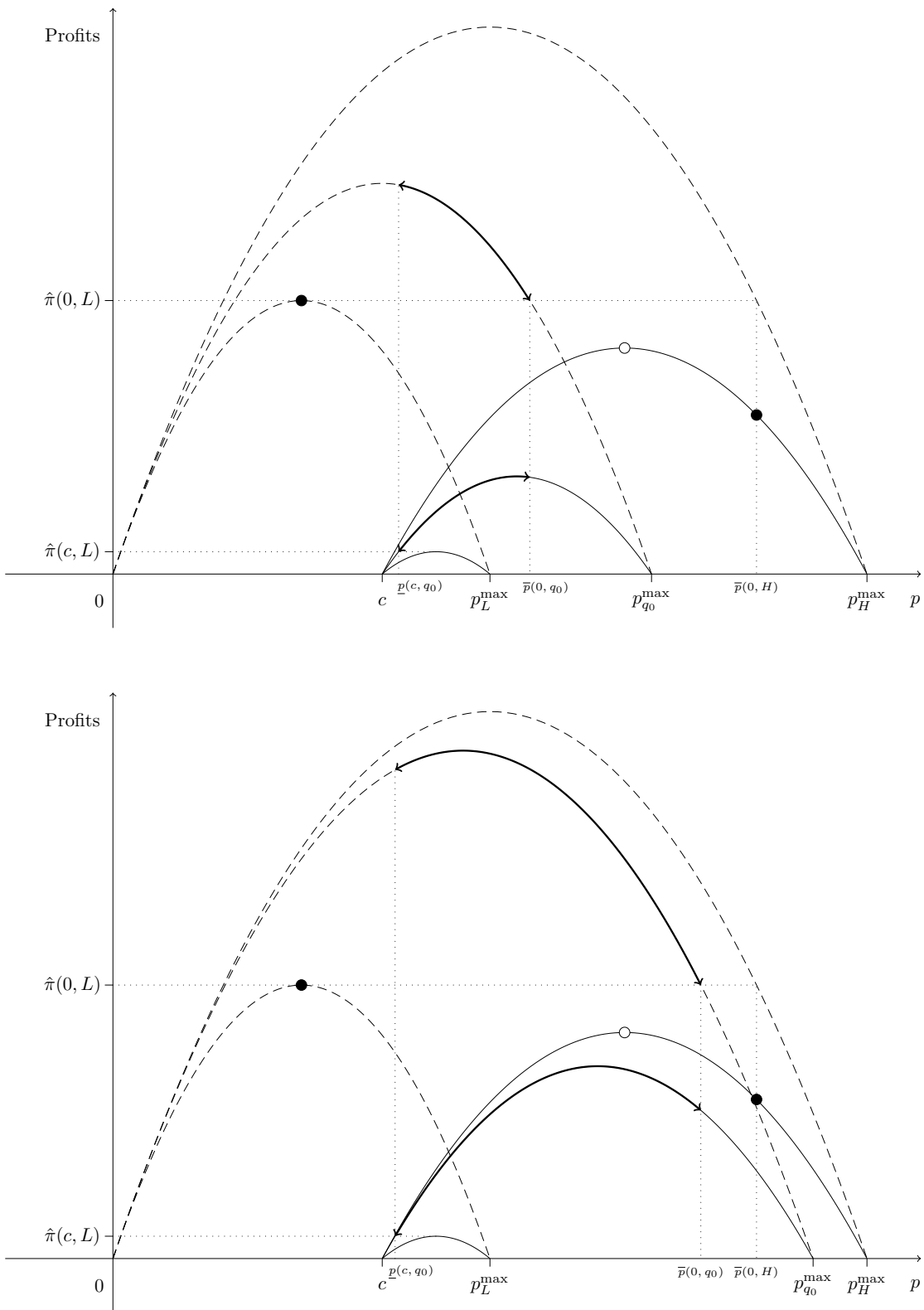
Figure 1: Pooling (thick curves with arrows) and separating (black points) equilibria for both low and high quality firms when $q_0 < \hat{q}_0$ (upper panel) and when $q_0 > \hat{q}_0$ (lower panel).

lower panel, in the special case of a linear demand $D(p, \tilde{q}) = 1 - p/\tilde{q}$.[6] In both panels
and for both the low and high quality, the thick curves with arrows indicate the range of
pooling profits while the black points indicate the least-cost separating profits at the Riley
equilibrium outcome. The white point indicates the perfect information profit for a high-
quality monopolist.Clearly, no pooling equilibrium allows a high-quality monopolist to earn
more than at the least-cost separating price in the upper panel. On the contrary, there is
clearly a range of pooling equilibria that give more revenue to the high-quality monopolist
than the least cost separating price in the lower panel. Note that in both cases, the low-quality
monopolist is always better off when playing a pooling equilibrium price.

As previously mentioned, the presence of multiple equilibria, whether separating or pool-
ing, raises an issue which various authors address making use of equilibrium refinements. If
we were applying here the logic of the criterion proposed by Cho and Kreps (1987) to the
multitude of pooling equilibria depicted in Figure 1, we would eliminate all of them as be-
ing "unintuitive". As usual, the Riley equilibrium outcome is the only signaling equilibrium
robust to the intuitive criterion regardless of the prior beliefs held by consumers. However,
there is some question of selecting the Riley equilibrium outcome as the most plausible, espe-
cially when it is Pareto dominated (from the monopolist's standpoint) by pooling equilibria
as it happens here when $q_0 > \hat{q}_0$. Roughly speaking, the idea is that both the high- and the
low-quality types should not be wastefully competitive with themselves. Another concern is
the following. When consumers are almost sure that quality is high, one expects the high-
quality monopolist to choose a price fairly close to the full information price $\hat{p}(c, H)$. In such
circumstances however, the intuitive criterion happens to emphasize signaling behaviors that
significantly differ from those prevailing under full information. As stated in Proposition 1,
if there is a very slight chance that quality is low, the Riley equilibrium outcome predicts
that the monopolist will set the upward-distorted price $p_H^I = \overline{p}(0, H)$ to signal high quality

[6]In both panels, the dashed curves depict the family of profits curves for a low-quality monopolist, $\pi(p; 0, \tilde{q})$
for $\tilde{q} \in \{L, q_0, H\}$. The solid curves depict the family of profit curves for a high-quality monopolist, $\pi(p; c, \tilde{q})$
for $\tilde{q} \in \{L, q_0, H\}$.

whenever $c \geq \bar{c}$, which exceeds $\hat{p}(c, H)$. By contrast, the pooling price $\hat{p}(c, q_0)$ which Pareto dominates the two prices involved in the Riley equilibrium outcome, tends to $\hat{p}(c, H)$ as q_0 comes close to H. The interested reader will find further grounds for criticism of the intuitive criterion in Mailath et *alii* (1993). The alternative refinement they propose would discard the Riley equilibrium outcome when $q_0 > \hat{q}_0$, because it is "defeated" by the Pareto dominating pooling equilibrium outcomes in this parameter configuration. Further examination of the vertical contract will show how it greatly simplifies the issue of selecting a reasonable price in equilibrium, with no restriction on out-of-equilibrium beliefs.

To conclude the analysis, it is worth noting that consumers also benefit *ex ante* from pooling equilibria under some circumstances. In such cases, there must be unanimity among all participants (consumers and firms) not to distort price in order to reveal quality, but rather to conceal information via a pooling price. For this, consumer surplus before purchase must be higher at the pooling equilibrium price than at the expected price involved by the Riley separating equilibrium outcome with costly signaling. We examine this possibility throughout the following example. Assume again that preferences are of Mussa-Rosen type, i.e., consumer with taste y has utility $u = y\tilde{q} - p$ when buying one unit of perceived quality $\tilde{q}$ at price p and 0 either. Assuming that y is distributed uniformly on $[0, 1]$ leads to a linear aggregate demand $D(p, \tilde{q}) = 1 - p/\tilde{q}$. Consider now the parameter configuration such that, first, the separating price intended to signal high quality is the upward-distorted price $p_H^I = \bar{p}(0, H)$, and second, there exists a pooling price at which the monopolist earns more profit than at the least-costly separating prices. The first condition requires that $c < \bar{c}$, that is, $c < \sqrt{H\Delta}$ with Mussa-Rosen preferences. The second condition requires that the pooling equilibrium price $\hat{p}(c, q_0)$ provides the high-quality monopolist with profit $\hat{\pi}(c, q_0)$ higher than $\pi(\bar{p}(0, H); c, H)$, that is, $q_0 \geq \hat{q}_0$. Is it possible that, under these conditions, consumers find it *ex ante* cheaper to buy the product at price $\hat{p}(c, q_0)$, rather than at the Riley separating equilibrium prices?

Facing $\hat{p}(c, q_0)$, a consumer with taste y has expected utility $yq_0 - \hat{p}(c, q_0)$ that depends on the prior quality expectation $q_0 = \mu_0 H + (1 - \mu_0)L$. By contrast, the expected utility in the

14

situation of costly revelation of quality is $y(\mu_0 H + (1 - \mu_0)L) - \mu_0 \bar{p}(0, H) - (1 - \mu_0)\hat{p}(0, L)$. Indeed, *ex ante*, with probability μ_0, the consumer is charged $\bar{p}(0, H)$ that correctly reveals that the product is of high quality and, with probability $1 - \mu_0$, he is charged $\hat{p}(0, L)$ that correctly reveals that the product is of low quality. As expected utility of quality is the same in both situations, it remains to check whether the expected price under costly revelation can be higher than $\hat{p}(c, q_0)$. We finally obtain the following result.

Claim 3 *Suppose $q_0 \geq \hat{q}_0$ and $c \leq \bar{c}$. With Mussa-Rosen preferences, the pooling price $\hat{p}(c, q_0)$ is cheaper than the expected price under revelation provided that $c < \mu_0 \sqrt{H\Delta}$. Hence, ex ante, for all $\mu_0 \in (\frac{c}{\sqrt{H\Delta}}, 1)$, the pooling equilibrium price $\hat{p}(c, q_0)$ is preferred by both firms and consumers to revelation.*

Proof: See Appendix C. ∎

4 The vertically decentralized structure: the case of delegation

We now consider a similar set-up but where the manufacturer M delegates the task of distributing and pricing the product on the market to an independent retailer R. Hence, it is now the retailer R that signals the product quality by choosing its price. Nevertheless, the decision of the retailer will be influenced via the procurement contract signed with the manufacturer.

We consider in the following that the set of possible contracts is limited to the set of two-part tariffs. More precisely, the good is exchanged between M and R at a per-unit price w, while R is paying a franchise F to M. Furthermore, we make two simplifying assumptions: i) M has all the bargaining power and hence proposes a take-it-or-leave-it offer (w, F) to R, ii) consumers know that the contract set is the set of two-part tariffs but do not observe the terms of the contract that remain private information shared by M and R. Hence, consumers only observe the final price set up by R. Finally, the rest of the notations and assumptions made

in the vertically integrated structure case also hold in the vertically decentralized structure.

We investigate a four-period signaling game which proceeds as follows. In period one, Nature selects quality q from the set $\{H, L\}$ according to the commonly known probability distribution μ_0. In period two, both M and R learn the actual q and M makes a take-it-or-leave-it offer (w_q, F_q) to R which accepts or refuses. In period three, in case of acceptance, R charges p. In period four, consumers observe p but not w_q nor F_q, and update their prior beliefs μ_0 and hence their quality expectation q_0, thereby making their choice between products on the basis of this observation. Consumers' posterior quality expectation after observing p will also be denoted by $\tilde{q}(p) : \mathbb{R}^+ \to [0, 1]$.

In the spirit of subgame perfection, equilibrium is characterized by first looking for the consumer prices in the subgame $G(w, F)$ that starts in period three, $(w, F) = (w_H, F_H, w_L, F_L)$ being the pair of accepted franchise fees and prices. Lastly, the contract choices are determined.

The profits of the manufacturer and the retailer can be defined as functions of their price, given consumers' beliefs and the actual quality. The profits of M and R will be denoted respectively, for $q = H, L$:[7]

$$\Pi^M = (w_q - c_q)D(p, \tilde{q}) + F_q$$

and

$$\Pi^R = (p - w_q)D(p, \tilde{q}) - F_q \tag{10}$$
$$= \pi(p; w_q, \tilde{q}) - F_q$$

As before, from the viewpoint of both M and R, $\tilde{q} = L$ is the least favorable quality expectation that consumers can form, regardless of the actual quality. Indeed, as the demand is increasing in expected quality, so are the retailer's and manufacturer's profits. As long as demand $D(p, \tilde{q})$ is positive, the optimal price that maximizes Π^R with respect to p and considering $\tilde{q}$ fixed is $\hat{p}(w_q, \tilde{q})$ and the retailer's maximized profit is $\hat{\pi}(w_q, \tilde{q}) - F_q$.

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[7]To save on notations, we omit to condition the profit functions of firms of type q on the contract (w_q, F_q).

4.1 Full information

We record here the benchmark case of complete information. Not surprisingly, the manufacturer is able to extract the full surplus of the vertical structure using marginal cost pricing and charging a fee equal to the corresponding monopoly profit, which makes the retailer indifferent between accepting or not the contract. Indeed, it is well known that, in our context, delegating the pricing task to the retailer entails the emergence of double marginalization and that two-part tariffs are sufficient to eliminate this vertical externality.

To sum up, for $q = H, L$, the equilibrium wholesale price is equal to the marginal cost, i.e., $w_q = c_q$, the retailer charges the complete information price $\hat{p}(c_q, q)$ and gets zero profit while the manufacturer earns the optimal fee equal to the monopoly profit $\hat{\pi}(c_q, q)$.

4.2 Asymmetric information

Under asymmetric information, the quality of the product is supposed known to both the manufacturer and the retailer, but not to the consumers. We start by defining the equilibrium concept.

4.2.1 The equilibrium concept

A perfect Bayesian equilibrium of the whole game is a set of price strategies $\left\{ (w_q^*, F_q^*, p_q^*)_{q=H,L} \right\}$ and quality expectation $\tilde{q}^*(p)$ such that, at any period of the game, strategies must be optimal given that the following player, if any, responds optimally, and given consumers' expectations. Formally, the optimal condition for M requires that, for $q = H, L$,

$$
\begin{aligned}
w_q^*, F_q^* \quad &\in \quad \arg\max_{w_q, F_q} \Pi^M = (w_q - c_q) D\left(p_q^*, \tilde{q}^*(p_q^*)\right) + F_q \\
\text{s.t. } \Pi^R \quad &= \quad \pi(p_q^*; w_q, \tilde{q}^*(p_q^*)) - F_q \geq 0. \tag{FC$_q$}
\end{aligned}
$$

The feasibility constraint (FC$_q$) means that the retailer of type q should accept the contract.[8]

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[8] In equilibrium, we expect the manufacturer of any type to always propose a contract that meets the feasibility constraints. Suppose on the contrary that in equilibrium the contract offered by one type of M is not accepted; then, this type of M can always secure a positive profit with an acceptable contract simply by reducing her franchise without affecting the relationship between R and M of the other type.

The perfection condition for R is for $q = H, L$,

$$p_q^* \in \arg\max_p \Pi^R = \pi(p; w_q, \tilde{q}^*(p)) - F_q. \tag{11}$$

Bayes' consistency of beliefs demands that consumers form posterior expectations about q from observing prices only by using Bayes' rule out of equilibrium. It follows that

$$\text{if } p_H^* \neq p_L^*, \text{ then } \tilde{q}^*(p_H^*) = H \text{ and } \tilde{q}^*(p_L^*) = L, \tag{12}$$

$$\text{and if } p_H^* = p_L^*, \text{ then } \tilde{q}^*(p_H^*) = \tilde{q}^*(p_L^*) = q_0.$$

4.2.2 The set of separating outcomes in $G(w, F)$

In this section, we first characterize the set of separating equilibria if any, for a given pair of wholesale prices and franchise fees, assuming that contracts have been accepted by the corresponding type of retailer. Note that our search for separating equilibria in this subgame amounts to generalize what has been done in the integrated case, to the procurement cost configuration (w_L, w_H), provided that the franchises meet the feasibility constraints. In particular, this requires to explore separating possibilities when the procurement cost of the high-quality product is lower than the low-quality one.

Constraints for separating equilibria Consider a potential separating equilibrium (p_H^*, p_L^*) of the subgame $G(w, F)$. Without loss of generality, let consumers' beliefs be the least favorable ones out of equilibrium from the retailer's point of view, i. e., $\tilde{q}^*(p) = L$ for all $p \notin \{p_H^*, p_L^*\}$. Such beliefs will generate all of the possible perfect Bayesian equilibrium paths. Indeed, if the retailer of any type does not have an incentive to charge p when $\tilde{q}^*(p) \neq L$, then he will not have an incentive when $\tilde{q}^*(p) = L$, since his profit is lower.

As for the integrated case, we now introduce the two individual rationality constraints. A type H retailer who is clearly identified as a high-quality provider by consumers must find profitable to choose p_H^* instead of any other price that fool consumers, giving at best the variable profit $\hat{\pi}(w_H, L)$:

$$\pi(p_H^*; w_H, H) - F_H \geq \hat{\pi}(w_H, L) - F_H. \tag{IR$_H$}$$

18

Similarly, the individual rationality constraint for a type L retailer is:

$$\pi(p_L^*; w_L, L) - F_L \geq \hat{\pi}(w_L, L) - F_L \tag{IR$_L$}$$

which means that p_L^* must be an optimal strategy when consumers correctly identify the product as being a low-quality one.

We now turn to the incentive compatibility constraints. First, the type H retailer must not find profitable to mimic the equilibrium price chosen by the type L retailer, that is:

$$\pi(p_H^*; w_H, H) - F_H \geq \pi(p_L^*; w_H, L) - F_H. \tag{IC$_H$}$$

Second, the type L retailer must not find profitable to behave as if he were the type H retailer, that is:

$$\pi(p_L^*; w_L, L) - F_L \geq \pi(p_H^*; w_L, H) - F_L. \tag{IC$_L$}$$

Given that the contract has already been accepted, the franchise fees do not impact directly individual rationality nor incentive compatibility constraints: only the wholesale prices matter.

Analysis We first establish similar claims as in the integrated case and for the same reasons. Indeed, observe that the constraint (IR$_L$) reduces to

$$\pi(p_L^*; w_L, L) \geq \hat{\pi}(w_L, L)$$

and it is necessarily binding. Hence,

$$p_L^* = \hat{p}(w_L, L)$$

that is the complete information equilibrium price corresponding to a procurement cost w_L. We assume that $\hat{\pi}(w_L, L) > 0$ or equivalently that $w_L < p_L^{\max}$, so that $\hat{p}(w_L, L)$ is an interior maximum. As will be clear below, this condition is necessary for (IC$_L$) to hold.[9]

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[9]Recall that in the integrated case, we have similarly assumed that $\hat{\pi}(0, L) > 0$.

Also, as before, the constraint (IR_H) for a type H retailer implies constraint (IC_H). We are thus left with, first, the constraint (IR_H):

$$\pi(p_H^*; w_H, H) - F_H \geq \hat{\pi}(w_H, L) - F_H \tag{IR$_H$}$$

and second the constraint (IC_L):

$$\pi(p_L^*; w_L, L) - F_L \geq \pi(p_H^*; w_L, H) - F_L$$

together with the fact that $p_L^* = \hat{p}(w_L, L)$. We can rewrite (IC_L) as

$$\hat{\pi}(w_L, L) - F_L \geq \pi(p_H^*; w_L, H) - F_L. \tag{IC$_L$}$$

Simplifying these constraints, we are thus looking for the set Σ of price p_H^* such that the two following constraints hold:

$$\pi(p_H^*; w_H, H) \geq \hat{\pi}(w_H, L) \tag{13}$$

$$\hat{\pi}(w_L, L) \geq \pi(p_H^*; w_L, H). \tag{14}$$

Note that the assumption $\hat{\pi}(w_L, L) > 0$ is needed for (14) to hold.

The set of separating price equilibria is characterized in the following Proposition.

Proposition 4 *Consider a given pair of wholesale prices (w_H, w_L) such that $w_L < p_L^{\max}$. Any separating price equilibrium (p_H^*, p_L^*) of $G(w, F)$ is such that:*

$$p_L^* = \hat{p}(w_L, L)$$

$$p_H^* \in \Sigma$$

where the set Σ of high-quality separating prices is given by:

$$\Sigma = \begin{cases} \left[\max\left(\overline{p}(w_L, H), \underline{p}(w_H, H)\right), \overline{p}(w_H, H)\right] & \text{when } w_H > w_L \\ \left\{\underline{p}(w_L, H), \overline{p}(w_L, H)\right\} & \text{when } w_H = w_L \\ \left[\underline{p}(w_H, H), \min\left(\underline{p}(w_L, H), \overline{p}(w_H, H)\right)\right] & \text{when } w_H < w_L \end{cases}$$

Proof: See Appendix D. ∎

We also obtain straightforwardly the following Corollary.

Corollary 5 *The least cost separating price that signals quality H, whenever it exists, is $p_H^* \in \Sigma$ such that*

$$p_H^* = \begin{cases} \min\left(\underline{p}(w_L, H), \hat{p}(w_H, H)\right) & \text{if } w_H < w_L, \\ \overline{p}(w_L, H) \text{ if } w_H = w_L \\ \max\left(\overline{p}(w_L, H), \hat{p}(w_H, H)\right) & \text{if } w_L < w_H \end{cases}.$$

Note first, that in the limit case where $w_H = w_L$, both separating prices give the same gross revenue by definition,

$$\overline{p}(w_L, H) D(\overline{p}(w_L, H), H) = \underline{p}(w_L, H) D(\underline{p}(w_L, H), H)$$

but as $\overline{p}(w_L, H) > \underline{p}(w_L, H)$, the corresponding profits are ranked accordingly so that $\overline{p}(w_L, H)$ is the least cost separating price for high quality.

The insight from Corollary 5 is twofold. First, to signal high quality, the retailer may distort the consumer price relative to the full information situation, either downward ($p_H^* = \underline{p}(w_L, H)$) or upward ($p_H^* = \overline{p}(w_L, H)$) depending on whether the wholesale price for high quality falls short or not of the wholesale price for low quality. Second, to the extent that p_H^* is biased away from $\hat{p}(w_H, H)$, p_H^* becomes insensitive to changes in w_H as shown by the definitions of $\underline{p}(w_L, H)$ and $\overline{p}(w_L, H)$.

As the complete information price $\hat{p}(w_H, H)$ is increasing in w_H, there exists two threshold values for w_H, denoted $\underline{w}_H$ and $\overline{w}_H$ and defined by $\hat{p}(\underline{w}_H, H) = \underline{p}(w_L, H)$ and $\hat{p}(\overline{w}_H, H) = \overline{p}(w_L, H)$.[10] Figure 2 depicts p_H^* as a function of w_H and for a fixed w_L, in the special case of a linear demand $D(p, \tilde{q}) = 1 - p/\tilde{q}$ where the perfect information price $\hat{p}(w_H, H)$ is a linear increasing function of w_H.

The sense of the signaling distortion necessary to signal high quality to consumers, if any, is determined backward by the manufacturer. If the wholesale price discriminates on behalf of low (resp. high) quality, i.e., $w_L \leq w_H$, the manufacturer may induce the retailer to signal high quality with a price higher (resp. lower) than what would prevail under full information.

---

[10]More precisely, $\overline{w}_H$ is given by the solution in w_H of the equation $D(\overline{p}(w_L, H), H) + (\overline{p}(w_L, H) - w_H)\frac{\partial D(\overline{p}(w_L, H), H)}{\partial p} = 0$. That is, $\overline{w}_H = \overline{p}(w_L, H) + \frac{D(\overline{p}(w_L, H), H)}{\frac{\partial D(\overline{p}(w_L, H), H)}{\partial p}}$. Similarly, the other threshold is $\underline{w}_H = \underline{p}(w_L, H) + \frac{D(\underline{p}(w_L, H), H)}{\frac{\partial D(\underline{p}(w_L, H), H)}{\partial p}}$.
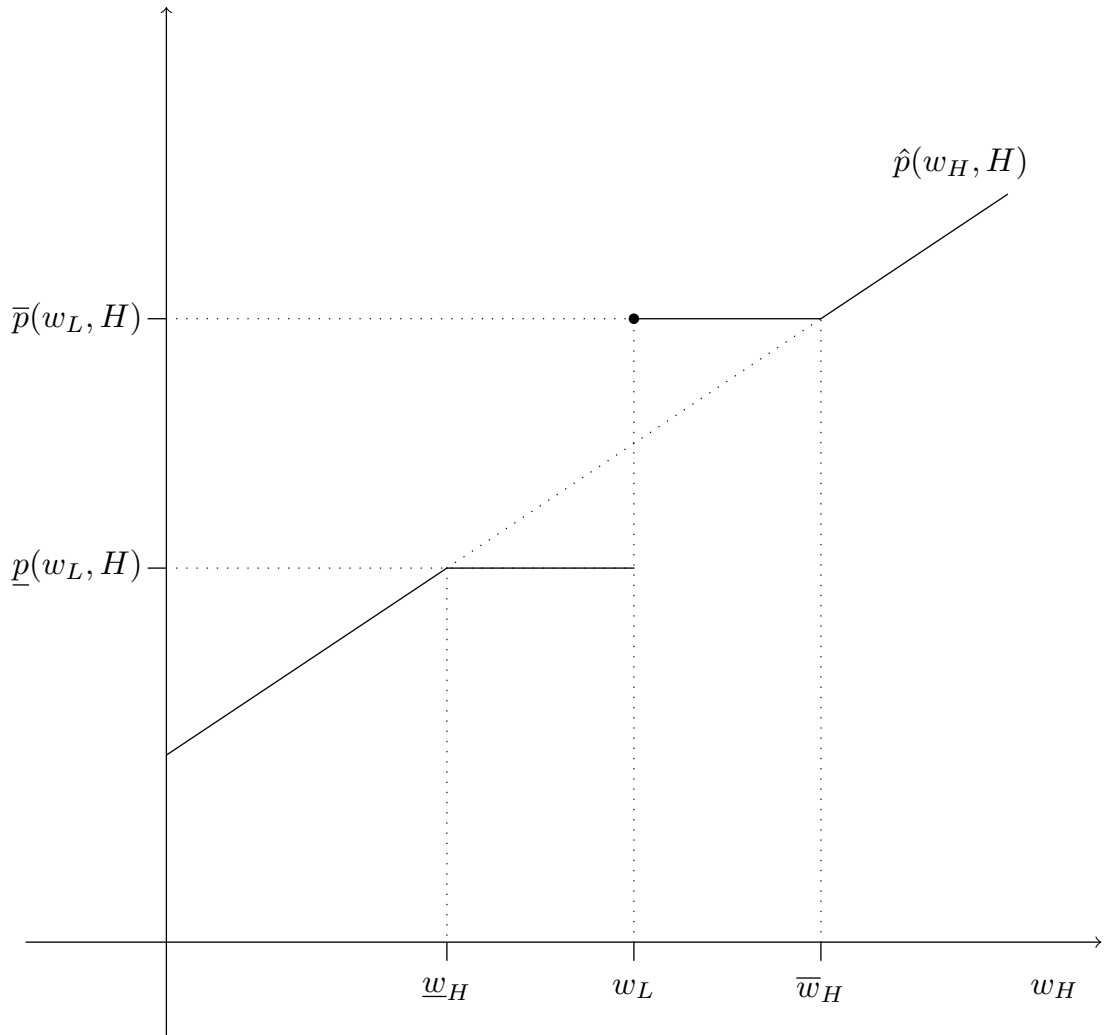
Figure 2: Least cost separating price p_H^* for high quality as a function of w_H

Setting a lower wholesale price to either quality amounts to reducing the retailer's cost of selling this quality. When the retailer considers changing consumer price away from the full information level, he has to take into account both a direct and an indirect effects on his profit, which play in opposite directions. First, the retailer directly modifies his gross revenue at the current sale. But second, the retailer changes the demand for his product, thereby modifying indirectly the profit from sales (net from the wholesale price). Thus, a downward deviation from the full information price both reduces the gross revenue and increases the net profit by boosting demand, whereas an upward deviation has the converse effects: it raises the gross revenue but entails a loss in profit due to lower demand. If the wholesale price for the high quality is lower than that for the low quality, the type H retailer has more incentive than the type L to distort downward the consumer price because increased demand for the retailer's product is more profitable to the high quality due to less expensive sales. Conversely, if the type H retailer must pay a wholesale price higher than that proposed to the type L retailer, distorting upward the consumer price can successfully signal high quality because the global effect of the foregone profit from adverse business switching on one hand, and the higher gain from increased price on the other hand, is less damaging to the type H retailer, but also his gain from increased price is higher. As shown by Figure 2, the cost of signaling is maximum when w_H and w_L are alike.

4.2.3 The set of pooling outcomes in $G(w, F)$

In this section, we characterize the set of pooling equilibria $p_H^* = p_L^* = p^*$ if any, for a given pair of wholesale prices w and franchise fees F. Consider a potential pooling price p^* of the subgame $G(w, F)$ such that $\tilde{q}^*(p^*) = q_0$. Without loss of generality, let consumers' beliefs be the least favorable ones out of equilibrium from the retailer's point of view, i. e., $\tilde{q}(p) = L$ for all $p \neq p^*$. As for the separating prices case, individual rationality constraints should hold

for a potential pooling price, that is:

$$\pi(p^*; w_H, q_0) - F_H \geq \hat{\pi}(w_H, L) - F_H \qquad \text{(IR}_H)$$

$$\pi(p^*; w_L, q_0) - F_L \geq \hat{\pi}(w_L, L) - F_L \qquad \text{(IR}_L)$$

The next Proposition characterizes the set P of pooling equilibrium.

Proposition 6 *In the subgame $G(w, F)$, there may exist a continuum of pooling equilibrium prices p^* that conceal information about quality. More precisely, the set $P(q_0)$ of pooling prices is given by:*

$$P(q_0) = \begin{cases} \left[\underline{p}(w_H, q_0), \overline{p}(w_L, q_0)\right] \neq \varnothing \ \text{iff} \ \overline{p}(w_L, q_0) > \underline{p}(w_H, q_0) \ \text{when} \ w_H \geq w_L \\ \left[\underline{p}(w_L, q_0), \overline{p}(w_H, q_0)\right] \neq \varnothing \ \text{iff} \ \underline{p}(w_L, q_0) < \overline{\overline{p}}(w_H, q_0) \ \text{otherwise.} \end{cases}$$

Proof: See Appendix E. ∎

4.3 The optimal contract

We now examine the contract chosen by the manufacturer. Given that the manufacturer has all the bargaining power, the feasibility constraints should bind, that is for a separating equilibrium:

$$\pi(p_H^*; w_H, H) - F_H = 0 \qquad (15)$$

$$\pi(p_L^*; w_L, L) - F_L = 0 \qquad (16)$$

with $p_L^* = \hat{p}(w_L, L)$ and $p_H^* \in \Sigma$ and for a pooling equilibrium

$$\pi(p^*; w_H, q_0) - F_H = 0$$

$$\pi(p^*; w_L, q_0) - F_L = 0$$

with $p^* \in P(q_0)$. Hence, for $q = L, H$, the profit of the type$-q$ manufacturer writes as

$$\begin{aligned} \Pi^M &= (w_q - c_q)D(p_q^*, q) + \pi(p_q^*; w_q, q) \\ &= \pi(p_q^*; c_q, q) \end{aligned}$$

for a separating equilibrium and

$$\Pi^M = (w_q - c_q)D(p^*, q_0) + \pi(p^*; w_q, q_0)$$

$$= \pi(p^*; c_q, q_0)$$

for a pooling equilibrium.

The role of the wholesale price is to select a particular final price and the franchise helps to recover any variable profit (or loss) made by the retailer.

Proposition 7 *Optimal contracts set up by the manufacturer make it possible for the retailer to commit on a unique final price for each quality level.*

(i) *If $q_0 \leq \hat{q}_0$, the manufacturer optimally sets up a contract that achieves separation at the least cost, i. e., $(p_L^*, p_H^*) = (p_L^I, p_H^I)$. This contract involves the following wholesale prices and franchises:*

$$w_L^* = 0, F_L^* = \hat{\pi}(0, L)$$

and

$$w_H^* = \begin{cases} \in [0, \bar{c}] & \text{when } c \leq \bar{c} \\ c & \text{otherwise.} \end{cases} , \quad F_H^* = \begin{cases} \pi(\bar{p}(0, H); w_H^*, H) & \text{when } c \leq \bar{c} \\ \hat{\pi}(c, H) & \text{otherwise.} \end{cases}$$

(ii) *If $q_0 > \hat{q}_0$, and $c \leq \bar{c}$, the manufacturer optimally sets up a contract that conceals information and leads to its best pooling price, namely $\hat{p}(0, q_0)$ for low quality, and $\hat{p}(c, q_0)$ for high quality. If $c > \bar{c}$ then the optimal contract implements separation without distortion, as when $q_0 \leq \hat{q}_0$.*

Proof: The proof proceeds by first assuming that the manufacturer seeks separation and by finding the optimal contract. Second, we assume that the manufacturer wants to conceal information and we characterize the optimal contract. Finally, we prove that separation is more profitable than pooling when $q_0 \leq \hat{q}_0$. See appendix F. ∎

25

When separation is not costly ($c > \bar{c}$), there is a unique contract that leads the retailer to charge the complete information price for high quality. This constitutes the optimal policy for the manufacturer whatever the prior q_0 of consumers when $c > \bar{c}$.

On the contrary, when separation is costly ($c \leq \bar{c}$), note that the manufacturer's profit is insensitive to the choice of wholesale price w_H provided it belongs to $[0, \bar{c}]$. As a consequence, there is an infinite number of contracts that all lead the high-quality retailer to choose the separating distorted price $\overline{p}(0, H)$. Implementing one of theses contracts would prove to be optimal only if $q_0 \leq \hat{q}_0$. In that event, the vertical contract selects the signaling prices involved in the Riley separating outcome. Not only is this the least costly way of disclosing full information about quality, but also it pays more to disclose full information than to conceal information.

By contrast, when $q_0 > \hat{q}_0$, and separation is costly ($c \leq \bar{c}$), the manufacturer prefers a contract that implements the best pooling price given its type, thereby concealing information. As previously seen, if we compare this result to the predictions obtained by using refinement criterions to select "reasonable" signaling equilibria in the vertically integrated structure, we have that both pooling prices $\hat{p}(0, q_0)$ and $\hat{p}(c, q_0)$ would be eliminated as "un-intuitive" by Cho and Kreps (1987), but would survive as being "undefeated" by Mailath et *alii* (1993). Interestingly enough, there is a continuity in pricing behaviors between contracts under complete and incomplete information. The pooling price $\hat{p}(c, q_0)$ when quality is high tends to $\hat{p}(c, H)$ as q_0 approaches H.

As shown in section 3 with the example of Mussa-Rosen preferences, there also exists circumstances where consumers prefer *ex ante* not to learn the true quality through price, that is, $q_0 \geq \hat{q}_0$ and $\mu_0 > c/\sqrt{H\Delta}$.

5 Conclusion

This paper addresses the issue of price signaling in a model of vertical relationship between a manufacturer and a retailer who share the same private information about quality, whereas

consumers do not observe this characteristic *a priori*. We show that delegating the price setting task to a retailer and controlling it through a vertical contract (actually a two-part tariff) implements a unique price signaling quality, thereby solving the multiplicity problem at the consumer level. The franchise paid by the retailer to the manufacturer allows the latter to restrict the former's choice of signaling prices to the most profitable outcome from the manufacturer's standpoint.

The outcome of a unique price charged to consumers emerges without invoking the consumer sophistication usually required by selection criterions. In this paper, the only assumption made on consumer behavior is that they revise their beliefs according to Bayes' rule when observing a price. The vertical contract turns to be the most efficient way for the vertical chain to tie its hands on a unique final price. This price may disclose or not information to consumers depending on their initial optimism about quality.

Using Mussa-Rosen preferences, we prove that there may be circumstances under which consumers prefer *ex ante* not to learn the true quality through price. The result of concealing information with the vertical contract is intriguing in that it proves socially efficient when consumers hold optimistic beliefs about quality before purchase. It raises the issue of what would be the optimal anti-trust policy in such a context. Further research is needed to examine this problem with more general consumer preferences.

Appendix

A Proof of Lemma 1

First note that $\hat{\pi}(x, L)$ is strictly positive and decreasing on $[0, p_L^{\max})$ and $\hat{\pi}(x, L) = 0$ on $[p_L^{\max}, p_H^{\max}]$. Also the strict concavity of profit function with respect to price implies that the function $f(.; x, \tilde{q})$ is also strictly concave. Moreover, we have $f(x; x, \tilde{q}) = f(p_{\tilde{q}}^{\max}; x, \tilde{q}) = -\hat{\pi}(x, L) \leq 0$ and also $\max_p f(p; x, \tilde{q}) = \max_p (p - x) D(p, \tilde{q}) - \max_p (p - x) D(p, L) > 0$ as long as $\tilde{q} > L$.

It follows that there exist two solutions in p to the equation $f(p; x, \tilde{q}) = 0$ and that $f(.; x, \tilde{q})$ is positive between the roots. If $\hat{\pi}(x, L) = 0$, the roots are equal to the bounds of the interval $[0, p_L^{\max})$. On the contrary, if $\hat{\pi}(x, L) > 0$, they are interior values.

Finally, if we denote a generic (interior) root $p(x, \tilde{q})$, for any x and $\tilde{q}$:

$$f(p(x, \tilde{q}); x, \tilde{q}) = 0$$

Differentiating with respect to x, we get:

$$\frac{\partial f}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial f}{\partial x} = 0 \tag{17}$$

and when $p(x) = \underline{p}(x, \tilde{q})$ (resp. $\overline{p}(x, \tilde{q})$), we have $\frac{\partial f}{\partial p} > 0$ (< 0). Moreover,

$$\frac{\partial f(p(x, \tilde{q}), x)}{\partial x} = -D(p(x, \tilde{q}), \tilde{q}) - \frac{\partial \hat{\pi}(x, L)}{\partial x}.$$

Recall that $\frac{\partial \hat{\pi}(x, L)}{\partial x} = -D(\hat{p}(x, L), L)$ and then we have

$$\frac{\partial f(p(x, \tilde{q}), x)}{\partial x} = D(\hat{p}(x, L), L) - D(p(x, \tilde{q}), \tilde{q})$$

From (17), it follows that $\frac{\partial p(x, \tilde{q})}{\partial x} > 0$ if and only if

$$D(\overline{p}(x, \tilde{q}), \tilde{q}) < D(\hat{p}(x, L), L) < D(\underline{p}(x, \tilde{q}), \tilde{q}). \tag{18}$$

But by definition, the roots $\overline{p}(x, \tilde{q})$ and $\underline{p}(x, \tilde{q})$ are such that:

$$
\begin{aligned}
(\underline{p}(x, \tilde{q}) - x) D(\underline{p}(x, \tilde{q}), \tilde{q}) &= (\hat{p}(x, L) - x) D(\hat{p}(x, L), L) \\
&= (\overline{p}(x, \tilde{q}) - x) D(\overline{p}(x, \tilde{q}), \tilde{q})
\end{aligned}
$$

28

and $\underline{p}(x,\tilde{q}) < \hat{p}(x,L) < \overline{p}(x,\tilde{q})$. This implies that (18) holds true and hence, the generic root is increasing in x, i.e. $\frac{\partial p(x,\tilde{q})}{\partial x} > 0$.

B   Proof of Proposition 1

Let us denote the set of prices that satisfy $f(p;c,H) \geq 0$ as P_H and the set of prices that satisfy $f(p;0,H) \leq 0$ as P_L. The set of separating prices is $P = P_H \cap P_L$. A separating price equilibrium exists whenever this set is non empty and entails some distortion compared to complete information whenever the optimal price under complete information $\hat{p}(c,H)$ does not belong to that set.

First of all, note that $\hat{\pi}(0,L) = \max_p pD(p,L)$ is strictly positive because $D(0,L) > 0$ by assumption. Hence, following Lemma 1, the two roots of $f(p;0,H)$, namely $\underline{p}(0,H)$ and $\overline{p}(0,H)$, are interiors and consequently the set P_L is $\left[0,\underline{p}(0,H)\right] \cup \left[\overline{p}(0,H),p_H^{\max}\right]$.

Now for P_H, we have two cases that we examine in turn.

(a) $\hat{\pi}(c,L) = 0$. This is equivalent to $c \geq p_L^{\max}$ so that a high-quality producer, identified as a low-quality one, cannot expect making any profits given the cost c. Therefore, the set P_H is simply $[c,p_H^{\max}]$. In other words, (IR$_H$) always hold for the range of prices. Also, it is easy to check that $\underline{p}(0,H) < p_L^{\max}$ and hence we have $\underline{p}(0,H) < c$.[11] Consequently, $P = \left[\max\left(c,\overline{p}(0,H)\right),p_H^{\max}\right]$.

(b) $\hat{\pi}(c,L) > 0$. This is equivalent to $c < p_L^{\max}$. This means that the set P_H is now $\left[\underline{p}(c,H),\overline{p}(c,H)\right] \subset [c,p_H^{\max}]$. Also, using Lemma 1, we have that $\underline{p}(0,H) < \underline{p}(c,H)$ and $\overline{p}(0,H) < \overline{p}(c,H)$. It follows that $P = \left[\max\left(\overline{p}(0,H),\underline{p}(c,H)\right),\overline{p}(c,H)\right]$.

Finally, note that in both cases, if $\hat{p}(c,H) \in P$, that is when $\overline{p}(0,H) < \hat{p}(c,H)$, then the least-cost separating price is $\hat{p}(c,H)$.[12] Otherwise, that is when $\overline{p}(0,H) > \hat{p}(c,H)$, then the least-cost separating price is $\overline{p}(0,H)$.

---

[11] Indeed, by construction, we have $\underline{p}(0,H) < \hat{p}(0,L)$ and also $\hat{p}(0,L) < p_L^{\max}$, thus $\underline{p}(0,H) < p_L^{\max}$.
[12] When $\max\left(\overline{p}(0,H),\underline{p}(c,H)\right) = \underline{p}(c,H)$, then obviously $\hat{p}(c,H) \in P$. The same holds when $\max(c,\overline{p}(0,H)) = c$.

C   Proof of Claim 3

Consider the parameter configuration such that, first, the separating price intended to signal high quality is the upward-distorted price $p_H^I = \overline{p}(0, H)$– which happens when $c < \overline{c}$ –and second, the pooling price $\hat{p}(c, q_0)$ is more profitable to the high-quality monopolist than $p_H^I = \overline{p}(0, H)$– which happens when $\hat{\pi}(c, \hat{q}_0) > \pi(\overline{p}(0, H); c, H)$. In this parameter configuration, consumers are *ex ante* better off with $\hat{p}(c, q_0)$ than with the expected price under costly revelation if and only if

$$\hat{p}(c, q_0) < \mu_0 \overline{p}(0, H) + (1 - \mu_0)\hat{p}(0, L)$$

With Mussa-Rosen preferences, we have $\hat{p}(0, L) = \arg\max_p p(1 - p/L) = L/2$, $\overline{p}(0, H) = \frac{H + \sqrt{H\Delta}}{2}$, $\overline{c} = \sqrt{H\Delta}$ and $\hat{p}(c, q_0) = \frac{q_0 + c}{2}$. Hence, the inequality above is equivalent to $c < \mu_0\sqrt{H\Delta}$, which is more binding than $c < \overline{c}$.

D   Proof of Proposition 4

We have already seen that $p_L^* = \hat{p}(w_L, L)$. It remains to characterize P. Denote P_H the set of p_H^* for which (13) holds. Similarly, denote P_L the set of p_H^* for which (14) holds. Hence, $P = P_H \cap P_L$.

Actually, the characterization of P is very similar to the integrated case, although w_H can be higher or lower than w_L. Indeed, (13) and (14) write respectively:

$$f(p_H^*; w_H, H) \geq 0$$
$$f(p_H^*; w_L, H) \leq 0.$$

Using Lemma 1, we have $P_L = \left[w_L, \underline{p}(w_L, H)\right] \cup \left[\overline{p}(w_L, H), p_H^{\max}\right]$. Also, when $\hat{\pi}(w_H, L) > 0$, or equivalently $w_H < p_L^{\max}$, then $P_H = \left[\underline{p}(w_H, H), \overline{p}(w_H, H)\right] \subset [w_H, p_H^{\max}]$. If on the contrary, $\hat{\pi}(w_H, L) = 0$, that is a high-quality producer identified as a low-quality one cannot survive on the market given the procurement cost w_H, then $P_H = [w_H, p_H^{\max}]$.

Consequently, the set P depends on whether the procurement cost are positively cor-
related with quality or not.

(a) Suppose first that $w_H > w_L$, and use Lemma 1:

- if $\hat{\pi}(w_H, L) > 0$, then $P = \left[\max\left(\bar{p}(w_L, H), \underline{p}(w_H, H)\right), \bar{p}(w_H, H)\right]$,

- and if $\hat{\pi}(w_H, L) = 0$, then $P = [\max\left(\bar{p}(w_L, H), w_H\right), p_H^{\max}]$, because $w_H > p_L^{\max} > \underline{p}(w_L, H)$.

(b) Suppose now that $w_H < w_L$. Because the condition $\hat{\pi}(w_L, L) > 0$ is necessary for an equilibrium to exist, this implies here that $\hat{\pi}(w_H, L)$ is also strictly positive as $\hat{\pi}(., L)$ is decreasing. Using Lemma 1, we have

$$\underline{p}(w_H, H) < \underline{p}(w_L, H) \text{ and } \bar{p}(w_H, H) < \bar{p}(w_L, H)$$

because the roots are increasing and hence, by using Lemma 1, we get $P = \left[\underline{p}(w_H, H), \min\left(\underline{p}(w_L, H), \bar{p}(w_H, H)\right)\right]$.

(c) Suppose finally that $w_L = w_H$, then this implies that P is the set of prices such that $f(p_H^*; w_H, H) = 0$. In others words, $P = \left\{\underline{p}(w_L, H), \bar{p}(w_L, H)\right\}$.

E   Proof of Proposition 6

The set of constraints can be rewritten as follows, for $q = H, L$,

$$\pi(p^*; w_q, q_0) \geq \hat{\pi}(w_q, L).$$

It follows from Lemma 1 that the set of pooling prices is

$$P = \left[\underline{p}(w_L, q_0), \bar{p}(w_L, q_0)\right] \cap \left[\underline{p}(w_H, q_0), \bar{p}(w_H, q_0)\right].$$

Hence, when $w_H > w_L$, $P = \left[\underline{p}(w_H, q_0), \bar{p}(w_L, q_0)\right]$ and when $w_H \leq w_L$, $P = \left[\underline{p}(w_L, q_0), \bar{p}(w_H, q_0)\right]$.

F   Proof of Proposition 7

The proof proceeds as follows. We first assume that the manufacturer seeks separation and find the optimal contract. Then, we assume that the manufacturer wants to conceal

information and we characterize the optimal contract. Finally, we prove that separation is better than pooling when $q_0 \leq \hat{q}_0$.

1. At a separating equilibrium, the low-quality retailer plays as under perfect information. The manufacturer has thus no incentives to deviate from the optimal contract under perfect information, i.e., $w_L^* = 0$ and thus $F_L^* = \hat{\pi}(0, L)$. The low-quality retailer charges p_L^I and pays the franchise $F_L^* = \hat{\pi}(0, L)$.

Now Corollary 5 with $w_L = 0$ gives us the least-cost separating price for the high quality:

$$p_H^* = \begin{cases} \min\left(\underline{p}(0, H), \hat{p}(w_H, H)\right) & \text{if } w_H < 0, \\ \overline{p}(0, H) \text{ if } w_H = 0 \\ \max\left(\overline{p}(0, H), \hat{p}(w_H, H)\right) & \text{if } 0 < w_H \end{cases}$$

.

- We first show that choosing $w_H < 0$ is dominated by choosing $w_H = 0$. In other words it does not pay to charge less than $w_H = 0$ for the high quality. Indeed, whenever w_H is such that $\min\left(\underline{p}(0, H), \hat{p}(w_H, H)\right) = \underline{p}(0, H)$ then the manufacturer earns $(\underline{p}(0, H) - c)D(\underline{p}(0, H), H)$ which is clearly less than $(\overline{p}(0, H) - c)D(\overline{p}(0, H), H)$ because

$$\underline{p}(0, H)D(\underline{p}(0, H), H) = \overline{p}(0, H)D(\overline{p}(0, H), H)$$

by definition and because demand is downward sloping together with $\overline{p}(0, H) > \underline{p}(0, H)$. Hence the manufacturer can earn more by choosing $w_H = 0$.

Also, whenever w_H is such that $\min\left(\underline{p}(0, H), \hat{p}(w_H, H)\right) = \hat{p}(w_H, H)$ then this gives a positive profit to the manufacturer only if $\hat{p}(w_H, H) > c$. Hence the relevant possible set for w_H is such that $\underline{p}(0, H) > \hat{p}(w_H, H) > c$. As $\underline{p}(0, H)$ belongs to the set of prices where the profit function $(p - c)D(p, H)$ is increasing in p (see Figure 1 [13]), it follows that choosing w_H that implements $\hat{p}(w_H, H)$ is

---

[13] And recall that $\underline{p}(0, H) < \hat{p}(0, H) < \hat{p}(c, H)$.

dominated by choosing w_H that implements $\underline{p}(0, H)$, which itself is dominated by choosing $w_H = 0$ as shown above.

- Now we know that the best price for maximizing the high-quality manufacturer profit $(p - c)D(p, H)$ is $\hat{p}(c, H)$ by definition. This price can be implemented through $w_H = c$ whenever $c \geq \bar{c}$. Indeed, in this situation we have

$$
\begin{aligned}
p_H^* &= \max\left(\overline{p}(0, H), \hat{p}(c, H)\right) \\
&= \hat{p}(c, H).
\end{aligned}
$$

On the contrary, when $c < \bar{c}$, we have $\max\left(\overline{p}(0, H), \hat{p}(c, H)\right) = \overline{p}(0, H)$. It is still possible to implement $\hat{p}(w_H, H)$ by choosing $w_H > \bar{c}$ but in that case this price is larger than $\overline{p}(0, H)$ and thus even more upward distorted than $\overline{p}(0, H)$ with respect to $\hat{p}(c, H)$, thereby giving less profit. Alternatively, by choosing any w_H lower than $\bar{c}$, the price $\overline{p}(0, H)$ is implemented.

- From this discussion, it follows that the optimum for the manufacturer can be implemented through the following contract

$$
w_H^* = \left\{ \begin{array}{l} \in [0, \bar{c}] \ \ when \ c \leq \bar{c} \\ c \ \ otherwise. \end{array} \right. \ , \ F_H^* = \left\{ \begin{array}{l} \pi(\overline{p}(0, H); w_H^*, H) \ \ when \ c \leq \bar{c} \\ \hat{\pi}(c, H) \ \ otherwise. \end{array} \right.
$$

2. We here characterize the optimal contract when the manufacturer wants to conceal information about quality. A high-quality manufacturer then earns $(p - c)D(p, q_0)$ for any price p and his best possible price is $\hat{p}(c, q_0)$. A low-quality manufacturer would earn $pD(p, q_0)$ and the best possible price is $\hat{p}(0, q_0)$. It is possible in principle to have both prices as part of the set of pooling prices $P(q_0)$. Indeed, it would be sufficient to have w_H and $w_L \leq w_H$ such that

$$
\underline{p}(w_H, q_0) \leq \hat{p}(0, q_0) < \hat{p}(c, q_0) \leq \overline{p}(w_L, q_0).
$$

That is w_L sufficiently large and w_H sufficiently low.[14]

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[14] Or the same condition with inverting w_H and w_L when $w_H \leq w_L$.

Note that compared to the vertical integrated case, here the set of pooling prices can be enlarged as we can choose w_L larger than 0 and w_H smaller than c. This makes it possible to reach $\hat{p}(0, q_0)$ or $\hat{p}(c, q_0)$ whereas this might not be possible under vertical integration if both prices do not belong to $\left[\underline{p}(c, q_0), \overline{p}(0, q_0)\right]$.

In general, for given quality q, if the manufacturer offers a wholesale price w_q the retailer may want to implement $\hat{p}(w_q, q_0)$ if this price belongs to the pooling set P_0. For this it is sufficient that the manufacturer asks for the franchise

$$
\begin{aligned}
F_q &= (\hat{p}(w_q, q_0) - w_q)D(\hat{p}(w_q, q_0), q_0) \\
&= \hat{\pi}(w_q, q_0).
\end{aligned}
$$

Given this franchise, the unique optimal choice of the retailer is to charge $\hat{p}(w_q, q_0)$. Choosing any other price provides the retailer with negative profits.

3. As soon as the best pooling price for a high-quality manufacturer, $\hat{p}(c, q_0)$, gives a larger profit than the one the manufacturer can get with $\overline{p}(0, H)$, a pooling price will be preferred. This happens when $q_0 \geq \hat{q}_0$ and when $c \leq \bar{c}$.

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