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On the Size of the Government Spending Multiplier in the Euro Area

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This article addresses the existence of a wide range of estimated government spending multipliers in a dynamic stochastic general equilibrium model of the euro area. Our estimation results and counterfactual exercises provide evidence that omitting the interactions of key ingredients at the estimation stage (such as Edgeworth complementarity/subtitutability between private consumption and government expenditures, endogenous government spending policy and general time nonseparable preferences) paves the way for potentially large biases. We argue that uncertainty on the quantitative assessments of fiscal programmes could partly originate from these biases.

JEL: *C32, E32, E62.*
Keywords: *Government spending multiplier, DSGE models, Estimation bias, Euro area.*

Stimulus packages facing the Great Recession and the ongoing consolidation programs in most EU Member States have put fiscal policy at the heart of current economic policy discussions. A particularly debated issue is the evaluation of government spending multipliers, i.e. the increase in output consecutive to an increase in government spending, on which fiscal policy choices are partly based. However, there is not a single figure behind this concept, and a large uncertainty is surrounding its measurement (Hall, 2009, Ramey, 2011). The value of the multiplier depends on many factors such as the econometric approach, the underlying model, the nature and duration of the fiscal change, or the state of the economy (see among others, Cogan *et al.*, 2010, Christiano *et al.*, 2011, Auerbach and Gorodnichenko, 2012, Coenen *et al.*, 2012, Blanchard and Leigh, 2013, or Erceg and Lindé, forthcoming), leaving the decision maker in trouble.

This paper contributes to this literature by providing an explanation for the wide range of estimated government spending multipliers within a given dynamic stochastic general equilibrium (DSGE) model of the euro area. We argue that omitting the interactions of key ingredients at the estimation stage, through cross-equation restrictions, paves the way for potentially large biases. This shows up in our medium-scale model by studying the simultaneous combination of three mechanisms: *(i)* Edgeworth complementarity/substitutability between private consumption and government spending, *(ii)* endogeneity of government spending and *(iii)* general habits in

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consumption.

These ingredients have already been considered in the relevant literature. First, the Edgeworth complementarity/substitutability has been used to account for the aggregate interaction between private consumption and public spending (see Bailey, 1971, Aschauer, 1985, Barro, 1981 and Christiano and Eichenbaum, 1992, among others).[1] While there exist many concrete examples for which private consumption and public expenditures are complements or substitutes (health care, education, etc.),[2] a proper assessment of their degree of complementarity/substitutability remains an important issue for any economy (in our case the euro area over the period 1985Q1-2007Q4). Second, the endogeneity of government spending has been well documented in the literature (see McGrattan, 1994, Jones, 2002, Curdia and Reiss, 2010, Leeper *et al.*, 2010, among others), and can be represented, for instance, by a simple feedback rule on public spending that accounts for automatic stabilizers. Third, habit persistence has proven to be a key ingredient for aggregate persistence. In most of estimated DSGE models, habit persistence is basically represented by a one lag model in consumption (see e.g. Smets and Wouters, 2007). However, the macro-finance literature has insisted on more general specifications (including habit stock and/or local durability, see Heaton, 1995) to more accurately replicate the joint behavior of consumption and asset prices, including the real interest rate. Our objective is to assess how the interplay between these three features impacts the estimated government spending multiplier.

The mechanisms underlying the existence of bias in the estimated multiplier are the following. First, Edgeworth complementarity/substitutability and counter-cyclicality of government expenditures work in opposite direction. Indeed, higher level of Edgeworth complementarity leads to increase the response of both private consumption and output to a government spending shock, whereas a countercyclical policy acts as an automatic stabilizer. If fiscal policy is wrongly assumed to be exogenous, an econometrician will then underestimate the true level of Edgeworth complementarity to match the same correlation pattern of the data (as, for example, the positive relationships between private consumption and public expenditures). This translates into lower estimates of the government spending multiplier. A wrong assessment of the effects of government spending shocks is then obtained when automatic stabilizers are not taking into account. Second, Edgeworth complementarity and habit persistence in consumption work also in opposite direction. A high degree of habit formation tends to reduce (not eliminate) the crowding-out effect of public spending on private consumption. A moderate level of Edgeworth complementarity is then needed to replicate the properties of the data. If an important dimension of habit formation (in our case, habit stock) is wrongly omitted at the estimation stage, the level of Edgeworth complementarity will be over-estimated to compensate the lack of habit formation, turning to over-estimate the multiplier. This means that the

[1] Public spending displays Edgeworth complementarity (*resp.* substitutability) with private consumption when it increases (*resp.* decreases) the marginal utility of consumption.

[2] As discussed in Fiorito and Kollintzas (2004), the complementarity may reveal relative inefficiency in the provision of public goods. Let us consider the case of education. One may observe the coexistence of public school and private tutors if the quality of public teachers is considered as being to low by the private agents. In addition, the complementarity may occur because public education allows a higher level of income and thus increases the demand for other goods. Similar arguments hold for health care.

specification of the utility function matters a lot for when estimating the government spending multiplier.

To address these two quantitative issues, we consider a medium-scale DSGE model *à la* Smets and Wouters (2007). The model combines a neoclassical growth core with several shocks and frictions. In addition to the three above mentioned features, it includes investment adjustment costs, variable capital utilization, monopolistic competition in goods and labor markets, and nominal price and wage rigidities. Our maximum likelihood estimation with euro area data reveals *(i)* a high level of Edgeworth complementarity between private consumption and public expenditures, *(ii)* a countercyclical endogenous component of government spending, and *(iii)* a sizeable degree of habit stock. This implies that the data clearly favor a model version including these three new features. At the same time, this framework allows us to explain why the interactions between the three mechanisms may alter in different ways the estimated government spending multiplier. The multiplier on impact is around 1.30 (*resp.* 1.81) when wrongly omitting the endogeneity of government policy (*resp.* habit formation), to be compared to 1.60 in the benchmark specification. So, the short-run effect of an increase in government spending displays a wide range of estimates, depending on which relevant mechanism is wrongly excluded from the empirical analysis. A similar result holds when it comes to the long-run multiplier which is underestimated (by 25%) or overestimated (by 15%) depending on the restrictive assumption relative to economic policy or preferences. It is clear that both downward and upward biases obtained here are not negligible numbers, especially if the model is used to evaluate fiscal programs in the euro area.

Our quantitative investigations consider Edgeworth complementarity/substitutability and general habit formation (together with automatic stabilizers) as a simple way to account for the transmission mechanism of public spending in the euro area. Other candidates could be considered: non-separable preferences between consumption and leisure (Bilbiie, 2009), externalities in labor supply (Fève *et al.*, 2011) and in production (Devereux *et al.*, 1996), deep habits (Ravn *et al.*, 2006) and rule of thumb consumers (Gali *et al.*, 2007). We do not consider these alternative mechanisms in the rest of the paper for at least two reasons. First, some of them (e.g. the models including externalities) are observationally equivalent to our specification when focusing on the steady-state multiplier, but they induce severe difficulties at the estimation stage. Second, our estimation results indicate that they do not improve the model's fit, when our three modeling features are maintained. This is the case especially when considering rule of thumb consumers. Indeed, their estimated share remains too small (less than 7%) such that rule of thumb consumers can not be viewed as a quantitatively relevant transmission mechanism for the effects of government spending in our setup.[3]

The rest of the paper is organized as follows. In the next section, we expound the DSGE model that is subsequently estimated. In Section 2, the quantitative analysis is conducted and a model comparison is done. In Section 3, we inspect the mechanisms at work. The last section contains the concluding remarks.

[3]Coenen and Straub (2005) have shown that the share of rule of thumb consumers must be sufficiently large to reproduce the dynamic effects of government spending, as suggested by the SVAR literature.

I. A medium–scale model for the euro area

The present section describes our structural model of the euro area economy, which is close to Christiano *et al.* (2005) and Smets and Wouters (2007). The model combines a neoclassical growth core with several shocks and frictions. It includes features such as general habit formation, Edgeworth complementarity between private consumption and government spending, investment adjustment costs, variable capital utilization, monopolistic competition in goods and labor markets, nominal price and wage rigidities, and countercyclical government expenditures. The economy is populated by five classes of agents: producers of a final good, intermediate goods producers, households, employment agencies and the public sector (government and monetary authorities).

A. Household sector

Employment agencies

Each household indexed by $j \in [0,1]$ is a monopolistic supplier of specialized labor $N_{j,t}$. At every point in time t, a large number of competitive "employment agencies" combine households' labor into a homogenous labor input N_t sold to intermediate firms, according to $N_t = \left[\int_0^1 N_{j,t}{}^{\frac{1}{\varepsilon_{w,t}}} \mathrm{d}j\right]^{\varepsilon_{w,t}}$. Profit maximization by the perfectly competitive employment agencies implies the labor demand function $N_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}} N_t$, where $W_{j,t}$ is the wage paid by the employment agencies to the household supplying labor variety j, while $W_t \equiv \left(\int_0^1 W_{j,t}{}^{\frac{1}{\varepsilon_{w,t}-1}} \mathrm{d}j\right)^{\varepsilon_{w,t}-1}$ is the wage paid by intermediate firms for the homogenous labor input sold to them by the agencies. The exogenous variable $\varepsilon_{w,t}$ measures the substitutability across labor varieties and its steady–state is the desired steady–state wage markup over the marginal rate of substitution between consumption and leisure.

Household's preferences

The preferences of the jth household display time non–separability. Following Heaton (1995), preferences are time additive over a good called 'services'. In addition, labor supply enters separably in the utility function given by

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{b,t+s} \left(\log\left(S_{t+s}\right) - \frac{N_{j,t+s}^{1+\nu}}{1+\nu} \right),$$

where E_t denotes the mathematical expectation operator conditional upon information available at t, $\beta \in (0,1)$ is the subjective discount factor and $\nu > 0$ is the inverse of the Frisch labor supply elasticity. $N_{j,t}$ is labor of type j, $\varepsilon_{b,t}$ is a 'discount factor' shock, and the services S_t are defined by

$$S_t = S_{d,t} - h\left(1 - \delta_h\right) S_{h,t},$$

where $S_{d,t}$ is the 'durability stock' governing the degree of substitutability in consumption decision and $S_{h,t}$ is the 'habit stock' governing the degree of intertemporal complementarity in consumption (Otrok, 2001). The parameter $h \in [0,1]$ denotes the degree of habit formation. The two stocks evolve according to

$$S_{d,t} = \sum_{\tau=0}^{\infty} \delta_d^{\tau} C_{t-\tau}^{*} \quad \text{and} \quad S_{h,t} = \sum_{\tau=0}^{\infty} \delta_h^{\tau} C_{t-\tau}^{*},$$

where δ_h and δ_h are comprised between 0 and 1 and the consumption bundle C_t^* is defined by

$$C_t^* = C_t + \alpha_g G_t.$$

C_t and G_t denote private consumption and public expenditures, respectively. The parameter α_g accounts for the complementarity/substitutability between private consumption and public expenditures.[4] If $\alpha_g > 0$, government spending substitutes for private consumption, with perfect substitution if $\alpha_g = 1$, as in Christiano and Eichenbaum (1992). In this case, a permanent increase in government spending has no effect on output and hours but reduces private consumption, through a perfect crowding-out effect. In the special case $\alpha_g = 0$, we recover the standard business cycle model, with government spending operating through a negative income effect on labor supply (see Aiyagari *et al.*, 1992, Baxter and King, 1993). When the parameter $\alpha_g < 0$, government spending complements private consumption. Then, it can be the case (depending on the labor supply elasticity) that private consumption will react positively to an unexpected increase in government spending.

Combining the two stocks implies that services at time t are given by[5]

$$S_t = \frac{1 - (\delta_h + h(1-\delta_h)) L}{(1-\delta_d L)(1-\delta_h L)} C_t^*.$$

As we explain below, households are subject to idiosyncratic shocks about whether they are able to re–optimize their wage. Hence, the above described problem makes the choices of wealth accumulation contingent upon a particular history of wage rate decisions, thus leading to households heterogeneity. Combining the assumption of separability between services and labor supply in the utility function with the assumption of a complete set of contingent claims market, all the households will make the same choices regarding consumption and will only differ by their wage rate and supply of labor. This is directly reflected in our notations.

Household j's period budget constraint is given by

[4] The specification adopted here follows Christiano and Eichenbaum (1992), McGrattan (1994), Finn (1998), among others. An alternative specification is a CES function between C_t and G_t (see McGrattan *et al.*, 1997, Bouakez and Rebei, 2007, Coenen *et al.*, 2013). Note that these two specifications yield exactly the same log–linearized equation for the marginal utility of consumption.

[5] This specification is flexible enough to encompass many popular cases such as time–separable utility ($h = \delta_d = 0$), one lag habit formation ($\delta_h = \delta_d = 0$), and pure durability ($h = 0$).

$$\begin{aligned} P_t\left(C_t+I_t\right)+T_t+B_t &\leq R_{t-1}B_{t-1}+A_{j,t}+D_t \\ &+W_{j,t}N_{j,t}+R_t^k u_t\bar{K}_{t-1}-P_t\vartheta\left(u_t\right)\bar{K}_{t-1}, \end{aligned}$$

where I_t is investment, T_t denotes nominal lump–sum taxes (transfers if negative), B_t is the one-period riskless bond, R_t is the nominal interest rate on bonds, $A_{j,t}$ is the net cash flow from household's j portfolio of state contingent securities, D_t is the equity payout received from the ownership of firms, and R_t^k is the rental rate of capital. The capital utilization rate u_t transforms physical capital $\bar{K}_t$ into the service flow of effective capital K_t according to $K_t = u_t\bar{K}_{t-1}$,and the effective capital is rented to intermediate firms at the nominal rental rate r_t^k. The costs of capital utilization per unit of capital is given by the convex function $\vartheta\left(u_t\right)$. We assume that $u=1$, $\vartheta\left(1\right)=0$, and we define[6] $\eta_u \equiv \left[\vartheta''\left(1\right)/\vartheta'\left(1\right)\right]/\left[1+\vartheta''\left(1\right)/\vartheta'\left(1\right)\right]$. The physical capital accumulates according to

$$\bar{K}_t=\left(1-\delta\right)\bar{K}_{t-1}+\varepsilon_{i,t}\left(1-\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right)I_t$$

where $\delta\in\left[0,1\right]$ is the depreciation rate of capital, and $\mathcal{S}\left(.\right)$ is an adjustment cost function which satisfies $\mathcal{S}\left(\gamma_z\right)=\mathcal{S}'\left(\gamma_z\right)=0$ and $\mathcal{S}''\left(\gamma_z\right)=\eta_k>0$, γ_z is the steady–state growth rate of technology, and $\varepsilon_{i,t}$ is an investment shock.

Households set nominal wages according to a staggering mechanism. In each period, a fraction θ_w of households cannot choose its wage optimally, but adjusts it to keep up with the increase in the general wage level in the previous period according to the indexation rule $W_{j,t}=\gamma_z\pi^{1-\gamma_w}\pi_{t-1}^{\gamma_w}W_{j,t-1}$, where $\pi_t\equiv P_t/P_{t-1}$ represents the gross inflation rate, π is steady–state (or trend) inflation and the coefficient $\gamma_w\in\left[0,1\right]$ is the degree of indexation to past wages. The remaining fraction of households chooses instead an optimal wage, subject to the labor demand function $N_{j,t}$.

B. *Business sector*

Final good producers

At every point in time t, a perfectly competitive sector produces a final good Y_t by combining a continuum of intermediate goods $Y_t\left(\varsigma\right)$, $\varsigma\in\left[0,1\right]$, according to the technology $Y_t=\left[\int_0^1 Y_{\varsigma,t}{}^{\frac{1}{\varepsilon_{p,t}}}\mathrm{d}\varsigma\right]^{\varepsilon_{p,t}}$. Final good producing firms take their output price, P_t, and their input prices, $P_{\varsigma,t}$, as given and beyond their control. Profit maximization implies $Y_{\varsigma,t}=\left(\frac{P_{\varsigma,t}}{P_t}\right)^{-\frac{\varepsilon_{p,t}}{\varepsilon_{p,t-1}}}Y_t$, from which we deduce the relationship between the final good and the prices of the intermediate goods $P_t\equiv\left[\int_0^1 P_{\varsigma,t}{}^{\frac{1}{\varepsilon_{p,t}-1}}\mathrm{d}\varsigma\right]^{\varepsilon_{p,t}-1}$. The exogenous variable $\varepsilon_{p,t}$ measures the substitutability across differentiated intermediate goods and its steady state is then the desired steady–state price markup

[6]Later, we estimate η_u rather than the elasticity $\vartheta''\left(1\right)/\vartheta'\left(1\right)$ to avoid convergence issues.

over the marginal cost of intermediate firms.

Intermediate-goods firms

Intermediate good ς is produced by a monopolist firm using the following production function

$$Y_{\varsigma,t} = K_{\varsigma,t}{}^{\alpha} \left[Z_t N_{\varsigma,t}\right]^{1-\alpha} - Z_t F,$$

where $\alpha \in (0,1)$ denotes the capital share, $K_{\varsigma,t}$ and $N_{\varsigma,t}$ denote the amounts of capital and effective labor used by firm ς, $F > 0$ is a fixed cost of production that ensures that profits are zero in steady state, and Z_t is an exogenous labor–augmenting productivity factor whose growth–rate, denoted by $\varepsilon_{z,t} \equiv Z_t/Z_{t-1}$. In addition, we assume that intermediate firms rent capital and labor in perfectly competitive factor markets.

Intermediate firms set prices according to a staggering mechanism. In each period, a fraction θ_p of firms cannot choose its price optimally, but adjusts it to keep up with the increase in the general price level in the previous period according to the indexation rule $P_{\varsigma,t} = \pi^{1-\gamma_p}\pi_{t-1}^{\gamma_p} P_{\varsigma,t-1}$, where the coefficient $\gamma_p \in [0,1]$ indicates the degree of indexation to past prices. The remaining fraction of firms chooses its price $P^{\star}_{\varsigma,t}$ optimally, by maximizing the present discounted value of future profits

$$\mathrm{E}_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left\{ \Pi^p_{t,t+s} P^{\star}_{\varsigma,t} Y_{\varsigma,t+s} - \left[W_{t+s} N_{\varsigma,t+s} + R^k_{t+s} K_{\varsigma,t+s} \right] \right\}$$

where

$$\Pi^p_{t,t+s} = \begin{cases} \prod_{\nu=1}^{s} \pi^{1-\gamma_p}\pi_{t+v-1}^{\gamma_p} & s > 0 \\ 1 & s = 0, \end{cases}$$

subject to the demand from final goods firms and the production function. Λ_{t+s} is the marginal utility of consumption for the representative household that owns the firm.

C. Public sector

The stationary component of government spending is given by

$$\frac{G_t}{Z_t} = g\tilde{G}_t \varepsilon_{g,t},$$

where g denotes the deterministic steady–state value of G_t/Z_t. $\varepsilon_{g,t}$ is a government spending shock. The endogenous component of the policy $\tilde{G}_t$ is assumed to follow the simple rule

$$\tilde{G}_t = \left(\frac{Y_t}{\gamma_z Y_{t-1}} \right)^{\varphi_g}.$$

The parameter φ_g is the policy rule parameter linking the stationary component of government policy to demeaned output growth. If $\varphi_g > 0$, the policy rule contains a procyclical component that triggers an increase in government expenditures whenever output growth is above its average value. In contrast, if $\varphi_g < 0$, the policy rule features a countercyclical component, and thus reflects automatic stabilizers (see e.g. Jones, 2002, Curdia and Reis, 2010, and Fève *et al.*, 2013). In both cases however, assessing the degree of pro- or counter-cyclicality of the overall level of government spending requires taking the stochastic trend in productivity into account. For example, assuming that $\varphi_g = 0$, the growth rate of government expenditures would still be positively correlated with total factor productivity growth.

The monetary authorities follow a generalized Taylor rule by gradually adjusting the nominal interest rate in response to inflation and output growth:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{\gamma_z Y_{t-1}}\right)^{\phi_y}\right]^{(1-\phi_r)} \varepsilon_{r,t},$$

where $\varepsilon_{r,t}$ is a monetary policy shock.

D. Market clearing and stochastic processes

Market clearing condition on final goods market is given by

$$\begin{aligned} Y_t &= C_t + I_t + G_t + \vartheta\left(u_t\right)\bar{K}_{t-1}. \\ \Delta_{p,t} Y_t &= \left(u_t \bar{K}_{t-1}\right)^{\alpha}\left[Z_t N_t\right]^{1-\alpha} - Z_t F \end{aligned}$$

where $\Delta_{p,t} = \displaystyle\int_0^1 \left(\frac{P_{\varsigma,t}}{P_t}\right)^{-\frac{\varepsilon_{p,t}}{\varepsilon_{p,t-1}}} \mathrm{d}\varsigma$ is a measure of the price dispersion.

Regarding the properties of the stochastic variables, productivity and monetary policy shocks evolve according to $\log\left(\varepsilon_{x,t}\right) = \zeta_{x,t}$,with $x \in \{z, r\}$. The remaining exogenous variables follow an AR(1) process $\log\left(\varepsilon_{x,t}\right) = \rho_x \log\left(\varepsilon_{x,t-1}\right) + \zeta_{x,t}$,with $x \in \{b, i, g\}$, except the substitutabilities across labour varieties and across differentiated intermediate goods which are assumed to follow ARMA(1,1) processes, $\log\left(\varepsilon_{x,t}\right) = (1-\rho_x)\log\left(\varepsilon_x\right) + \rho_\iota \log\left(\varepsilon_{x,t-1}\right) + \zeta_{x,t} - \varrho_x \zeta_{x,t-1}$, with $x \in \{w, p\}$, in order to capture the moving average, high frequency component of both wages and inflation. In all cases, $\zeta_{x,t} \sim i.i.d.N\left(0, \sigma_x^2\right)$.

II. Quantitative analysis

In this section, our formal econometric procedure is expounded. We then present the estimation results of an unconstrained (hereafter referred as *benchmark*) and a constrained (hereafter referred as *Smets–Wouters*) version of the model. The *Smets–Wouters* model corresponds to the case when our three ingredients are absent $\left(\varphi_g = \alpha_g = \delta_d = \delta_h = 0\right)$. Finally, we discuss the implications for the estimated government spending multipliers.

A. Data and econometric approach

The quarterly euro area data run from 1985Q1 to 2007Q4 and are extracted from the AWM database compiled by Fagan *et al.* (2005), except hours worked and the working age population. Inflation is measured by the first difference of the logarithm of GDP deflator (YED), the short–term nominal interest rate is a three month rate (STN), and real wage growth is the first difference of the logarithm of nominal wage (WRN) divided by GDP deflator. Private consumption growth is constructed by multiplying real private consumption (PCR) times the private consumption deflator (PCD), divided by GDP deflator and transformed into first difference of the logarithm; Private investment growth is defined as the aggregate euro area total economy gross investment minus general government investment, scaled by GDP deflator and transformed into first difference of the logarithm; government spending growth is defined as the sum of nominal general government final consumption expenditure (GCN) and nominal government investment (GIN), scaled by GDP deflator and transformed into first difference of the logarithm. Real variables are divided by the working age population, extracted from the OECD Economic Outlook. Ohanian and Raffo (2012) have build a new dataset of quarterly hours worked for 14 OECD countries. We have then made an average of their series of hours worked for France, Germany and Italy to obtain a series of total hours for the euro area. Interestingly, the series thus obtained is very close to that provided by the ECB on the common sample, *i.e* 1999Q1–2007Q4. The growth of total hours worked is the first difference of the logarithm of total hours worked.

We follow the Bayesian approach to estimate various versions of the model (see An and Schorfheide, 2007, for an overview). Letting $\boldsymbol{\theta}$ denote the vector of structural parameters to be estimated and $\mathbf{S}_T \equiv \{\mathbf{S}_t\}_{t=1}^{T}$ the data sample, we use the Kalman filter to calculate the likelihood $L(\boldsymbol{\theta}, \mathbf{S}_T)$, and then combine the likelihood function with a prior distribution of the parameters to be estimated, $\Gamma(\boldsymbol{\theta})$, to obtain the posterior distribution, $L(\boldsymbol{\theta}, \mathbf{S}_T)\Gamma(\boldsymbol{\theta})$. Given the specification of the model, the posterior distribution cannot be recovered analytically but may be computed numerically, using a Monte-Carlo Markov Chain (MCMC) sampling approach. More specifically, we rely on the Metropolis–Hastings algorithm to obtain a random draw of size 1,000,000 from the posterior distribution of the parameters.

We use growth rates for the non-stationary variables in our data set (private consumption, private investment, government spending and the real wage) and express gross inflation, gross interest rates and the first difference of the logarithm of hours worked in percentage deviations from their sample mean. We write the measurement equation of the Kalman filter to match the seven observable series with their model counterparts. Before taking the model to the data, we induce stationarity by getting ride of the stochastic trend component Z_t and we log–linearized the resulting system in the neighborhood of the deterministic steady state. Thus, the state–space form of the model is characterized by the state equation

$$\mathbf{X}_t = \mathbb{A}(\boldsymbol{\theta})\mathbf{X}_{t-1} + \mathbb{B}(\boldsymbol{\theta})\xi_t, \qquad \xi_t \sim i.i.d.N\left(0, \Sigma_\xi\right),$$

where $\mathbf{X}_t$ is a vector of endogenous variables, ξ_t is a vector of innovations to the seven structural shocks, and $\mathbb{A}(\boldsymbol{\theta})$ and $\mathbb{B}(\boldsymbol{\theta})$ are complicated functions of the model's parameters. The measurement equation is given by

$$\mathbf{M}_t = \mathbb{C}(\boldsymbol{\theta}) + \mathbb{D}\mathbf{X}_t,$$

where $\mathbf{M}_t$ is a vector of observable variables, that is,

$$\mathbf{M}_t = 100 \times [\Delta \log C_t, \Delta \log I_t, \Delta \log G_t, \Delta \log (W_t/P_t), \Delta \log N_t, \pi_t, R_t],$$

$\mathbb{D}$ is a selection matrix and $\mathbb{C}(\boldsymbol{\theta})$ is a vector that is function of the structural parameters.

The *benchmark* model contains twenty two structural parameters, excluding the parameters relative to the exogenous shocks. We calibrate seven of them: the discount factor β is set to 0.99, the inverse of the Frisch labor supply elasticity $\nu = 2$, the capital depreciation rate δ is equal to 0.025, the parameter α in the Cobb–Douglas production function is set to 0.30 to match the average capital share in net (of fixed costs) output (McAdam and Willman, 2013), the steady–state price and wage markups ε_p and ε_w are set to 1.20 and 1.35 respectively (Everaert and Schule, 2008), and the steady–state share of government spending in output is set to 0.20 (the average value over the sample period). The remaining fifteen parameters are estimated. The prior distribution is summarized in Table 1. Our choices are in line with the literature, especially with Smets and Wouters (2007), Sahuc and Smets (2008) and Justiniano *et al.* (2010). Importantly, we specify for φ_g and α_g uniform priors centered on 0 and -1, and with a standard deviation of 1.00 and 1.33, respectively, to reflect our agnostic view concerning these key parameters. Finally, the rate of depreciation δ_d and the weight parameter on habit stock δ_h are assumed to follow a Beta distribution centered on 0.50 with a standard error of 0.20.

B. *Estimation results*

The estimation results are reported in Table 1, together with the posterior mean and the 90% confidence interval. Several results are worth commenting on.

First, the two model versions display very similar estimated values of the common structural parameters. Neither the parameters related to real rigidities nor those related to nominal rigidities are affected by the presence of Edgeworth complementarity, endogenous government policy and general time non–separable preferences.[7]

Second, the parameter estimates are in line with previous results (Smets and Wouters, 2003, Sahuc and Smets, 2008, Coenen *et al.*, 2013). For example, the adjustment cost parameter is $\eta_\kappa \approx 4.30$ and the parameter related to capital utilization is $\eta_u \approx 0.60$. In addition, the wage indexation parameter is $\gamma_w \approx 0.40$ in the two model versions, higher than the price indexation parameter $\gamma_p \approx 0.20$. This reflects a now standard result that the euro area data do not require too high a degree of price indexation. The probability that firms are not allowed to re-optimize their price is $\theta_p \approx 0.82$, implying an average duration of price contracts of about 15 months. The probability of no wage change is $\theta_w \approx 0.80$, implying an average duration of wage contracts of about 20 months. All these numbers are consistent with the results reported in the survey done by Druant *et al.* (2012). The monetary policy parameters $(\phi_\pi, \phi_y) \approx (1.51, 0.16)$ and $\phi_r \approx 0.85$ indicate that the monetary authorities act very gradually with a large weight on inflation.

[7] Note also that the parameters related to monetary policy are almost insensitive.

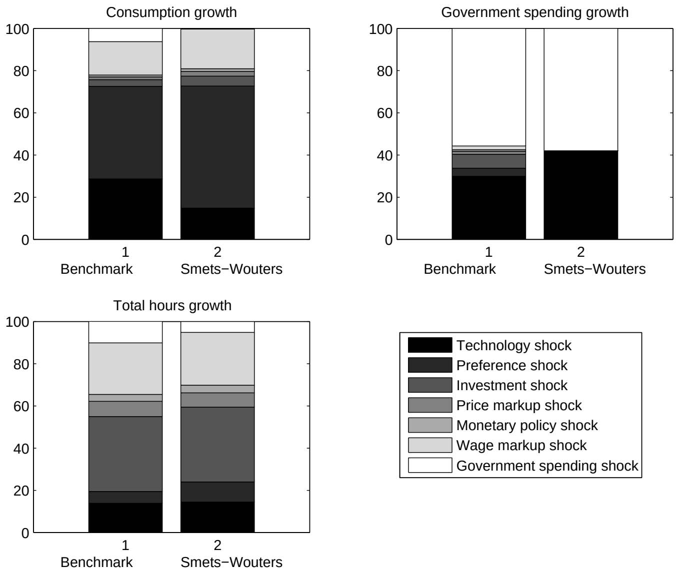
Table 1—Prior Densities and Posterior Estimates

Parameter	Prior	Posterior			
		Smets–Wouters Model		Benchmark Model	
h	$\mathcal{B}$[0.70,0.10]	0.852	[0.808,0.899]	0.813	[0.745,0.886]
δ_h	$\mathcal{B}$[0.50,0.20]	–	–	0.429	[0.181,0.671]
δ_d	$\mathcal{B}$[0.50,0.20]	–	–	0.256	[0.062,0.444]
η_u	$\mathcal{B}$[0.50,0.10]	0.586	[0.438,0.740]	0.600	[0.454,0.750]
η_k	$\mathcal{G}$[4.00,1.00]	4.379	[2.934,5.833]	4.302	[2.805,5.765]
α_g	$\mathcal{U}$[–3.30,1.30]	–	–	–0.647	[–0.986,–0.316]
φ_g	$\mathcal{U}$[–2.45,2.45]	–	–	–0.782	[–1.032,–0.536]
$\log(\gamma_z)$	$\mathcal{G}$[0.45,0.10]	0.306	[0.198,0.415]	0.329	[0.217,0.439]
θ_p	$\mathcal{B}$[0.66,0.10]	0.823	[0.761,0.884]	0.815	[0.746,0.885]
θ_w	$\mathcal{B}$[0.66,0.10]	0.808	[0.719,0.903]	0.765	[0.680,0.855]
γ_p	$\mathcal{B}$[0.50,0.15]	0.195	[0.068,0.318]	0.197	[0.068,0.324]
γ_w	$\mathcal{B}$[0.50,0.15]	0.458	[0.238,0.675]	0.389	[0.185,0.592]
ϕ_r	$\mathcal{B}$[0.75,0.15]	0.842	[0.809,0.874]	0.849	[0.815,0.883]
ϕ_π	$\mathcal{G}$[1.70,0.30]	1.509	[1.241,1.766]	1.562	[1.266,1.845]
ϕ_y	$\mathcal{G}$[0.125,0.05]	0.158	[0.069,0.245]	0.163	[0.067,0.255]
ρ_w	$\mathcal{B}$[0.60,0.15]	0.867	[0.788,0.945]	0.913	[0.863,0.961]
ρ_b	$\mathcal{B}$[0.60,0.15]	0.298	[0.132,0.449]	0.656	[0.478,0.833]
ρ_i	$\mathcal{B}$[0.60,0.15]	0.872	[0.785,0.961]	0.855	[0.756,0.955]
ρ_p	$\mathcal{B}$[0.60,0.15]	0.855	[0.722,0.965]	0.868	[0.752,0.989]
ρ_g	$\mathcal{B}$[0.60,0.15]	0.948	[0.914,0.982]	0.951	[0.911,0.991]
ϱ_w	$\mathcal{B}$[0.60,0.15]	0.670	[0.518,0.821]	0.725	[0.600,0.853]
ϱ_p	$\mathcal{B}$[0.60,0.15]	0.567	[0.388,0.756]	0.610	[0.432,0.795]
σ_w	$\mathcal{IG}$[0.25,2.00]	0.144	[0.113,0.172]	0.147	[0.117,0.176]
σ_b	$\mathcal{IG}$[2.00,2.00]	2.819	[1.916,3.685]	2.073	[1.439,2.694]
σ_i	$\mathcal{IG}$[0.25,2.00]	0.273	[0.212,0.333]	0.294	[0.226,0.357]
σ_p	$\mathcal{IG}$[0.25,2.00]	0.123	[0.096,0.150]	0.129	[0.102,0.158]
σ_z	$\mathcal{IG}$[0.25,2.00]	0.813	[0.713,0.910]	0.819	[0.714,0.920]
σ_g	$\mathcal{IG}$[0.25,2.00]	0.942	[0.825,1.059]	0.955	[0.824,1.088]
σ_r	$\mathcal{IG}$[0.25,2.00]	0.126	[0.113,0.172]	0.124	[0.107,0.141]
Marginal likelihood		–462.546		–438.369	
Posterior odds ratio		0.000		1.000	

Note: This table reports the prior distribution, the mean and the 90 percent confidence interval (within square brackets) of the estimated posterior distribution of the structural parameters.

Third, the three retained mechanisms are essential as they heavily increase the fit of the model, as shown by the improvement of the marginal likelihood in Table 1. The posterior odds ratios offer a complementary way of seeing this. Starting from a prior distribution on model versions with equal probability (1/2), we obtain that the *benchmark* model represents the whole probability mass. The estimated value for α_g is negative suggesting strong Edgeworth complementarity between private consumption and public expenditures. This result is in the line with those obtained in Coenen *et al.* (2013) for the euro area and Fève *et al.* (2013) for the United States. Moreover, the endogenous component of government expenditures is negatively related to output growth.[8] Finally, the parameters related to habit stock and durability are positive and significant. This implies that the habit parameter h is higher in the *Smets–Wouters* model since such a version omits habit stock in preferences.

Figure 1. Variance decomposition of a selection of variables



Note: This figure displays the contribution of each shock to the variance of observable variables in the benchmark model and in the Smets–Wouters model (at posterior estimates).

Finally, we compare the two model versions by computing the forecast error variance decomposition of consumption growth, government spending growth and total hours growth.[9] This decomposition is reported in Figure 1, where the forecast horizon is set to one.[10] In the *benchmark* model, the government spending shock explains 6% (*resp.* 20%) of the variance of consumption growth (*resp.* total hours growth),

[8] We also investigate various forms of the endogenous component of government spending and obtain that the specification adopted here is preferred by the data.

[9] We do not focus on the other variables as the effect of government spending shock is negligible and/or does not differ across the two model versions.

[10] The results do not differ too much when considering the contribution of the government spending shocks at other horizons.

whereas it is almost zero (*resp.* 10%) in the *Smets–Wouters* model. At the same time, the share of the variance of government expenditures is the same in the two model versions (around 56%), meaning that our key additional ingredients allow to reinforce the transmission mechanism of government policy.

C. Implications for government spending multipliers

We can now investigate the quantitative implications of these two model versions for the government spending multipliers. Two types of multipliers are considered.

First, the short run multiplier, defined as $\Delta y_{t+q}/\Delta g_t$ for $q = 0, 1, ..., 40$ (see Gali *et al.*, 2007), is obtained from the parameter estimates of the Table 1. An important issue when it comes to the evaluation and to the comparison of short–run multipliers between several structural models is the degree of persistence of the government policy shock, i.e. ρ_g in our notation (see Aiyagari *et al.*, 1992, and Campbell, 1995). In our case, this is not problematic because the autoregressive parameter is almost identical across model versions. In addition, as we have shown that the common structural parameters are very similar, we can suspect that the short–run multipliers are mainly driven by our three mechanisms.

Second, we consider the long–run multiplier, defined as the increase in steady–state output consecutive to an increase in steady–state government spending expenditures. Interestingly, this multiplier can be easily derived, even in a medium–scale DSGE model, since the steady state is independent from both real an nominal rigidities. From this definition and the structure of the *benchmark* model, the following proposition states key properties of the long–run multipliers.

Proposition 1. *Under the benchmark model:*

1) *The long–run multiplier is*

$$\frac{\Delta y}{\Delta g} = \frac{(1 + s_f)(1 - \alpha_g)}{1 + s_f - s_i + \nu(s_c + \alpha_g s_g)},$$

where s_c, s_i and s_g denotes the consumption to output ratio, investment to output ratio and government expenditures to output ratio, respectively. The parameter s_f is defined by F/y, where the fixed cost F is assumed to be constant.

2) *The long–run multiplier is a decreasing function of the Edgeworth complementarity parameter α_g.*

Proof.

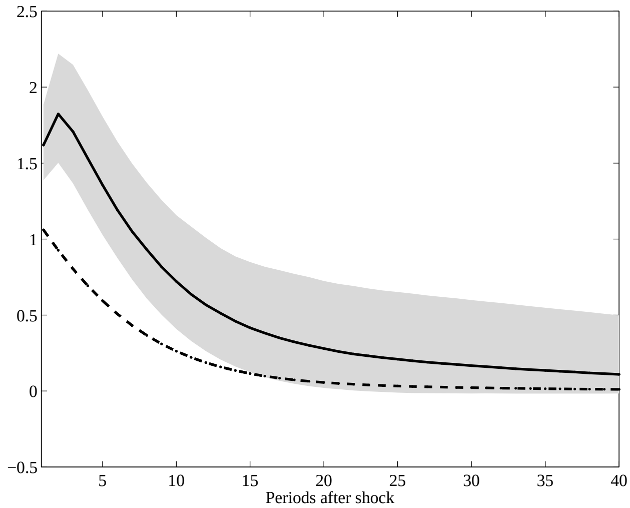
1) At the deterministic steady–state (see Appendix C), the log–linearized production function is $\widehat{\tilde{y}} = \alpha\hat{k} + (1 - \alpha)\hat{n}$, where $\widehat{\tilde{y}} = (y/(y + F))\hat{y}$. From the Euler equation on consumption, we get that $(y + F)/k$ and i/k are constant and independent of g, implying that $\widehat{\tilde{y}} = \hat{k} = \hat{\imath}$. Plugging this in the production function yields $\widehat{\tilde{y}} = \hat{n}$. Otherwise, from the real wage equation, it comes that $\hat{w} = \widehat{\tilde{y}} - \hat{n}$. Using the marginal rate of substitution between consumption and leisure, we deduce $\nu\hat{n} = \hat{w} - (c/(c + \alpha_g g))\hat{c} - \alpha_g(g/(c + \alpha_g g))\hat{g}$,

or equivalently $\nu\widehat{\tilde{y}} = -(c/(c+\alpha_g g))\,\hat{c} - \alpha_g\,(g/(c+\alpha_g g))\,\hat{g}$. Finally, from the aggregate resource constraint, we get $\hat{c} = ((\tilde{y}-i)/c)\,\widehat{\tilde{y}} - (g/c)\,\hat{g}$. We can now replacing $\hat{c}$ in the marginal rate of substitution equation to obtain $\hat{y} = [(1+F/y)(1-\alpha_g)(g/y)] / [1+F/y-i/y+\nu(c/y+\alpha_g(g/y))]\,\hat{g}$. Knowing that $\Delta y = y \times \hat{y}$ and $\Delta g = g \times \hat{g}$, we deduce the long–run multiplier formula.

2) Differentiating the multiplier with respect to α_g implies $\partial(\Delta y/\Delta g)/\partial\alpha_g = -[(1-s_f)(1+s_f-s_i+\nu(s_c+s_g))] / [1+s_f-s_i+\nu(s_c+\alpha_g s_g)]^2 < 0$. ■

This multiplier is obtained from the elasticities of non-stochastic steady–state employment and output with respect to public spending. The steady–state real interest rate is independent from all frictions (both real and nominal), the government endogenous policy and the exogenous process of public spending. It follows that the output to total hours ratio is constant and thus the real wage is unaffected. Importantly, the long–run multiplier directly depends on two structural parameters related to preferences (α_g and ν) as well as on the great ratios of the economy (so implicitly on β, δ, α). Given the fact that β, δ, α and ν are calibrated prior to estimation, the long–run multiplier *(i)* depends only on the estimated value of the Edgeworth complementarity parameter α_g, and *(ii)* unambiguously decreases with α_g. From this property and the estimation results of Table 1, we can directly expect a higher long–run multiplier in our *benchmark* model than in the *Smets–Wouters* model. We now quantify this discrepancy relative to the *benchmark* model.

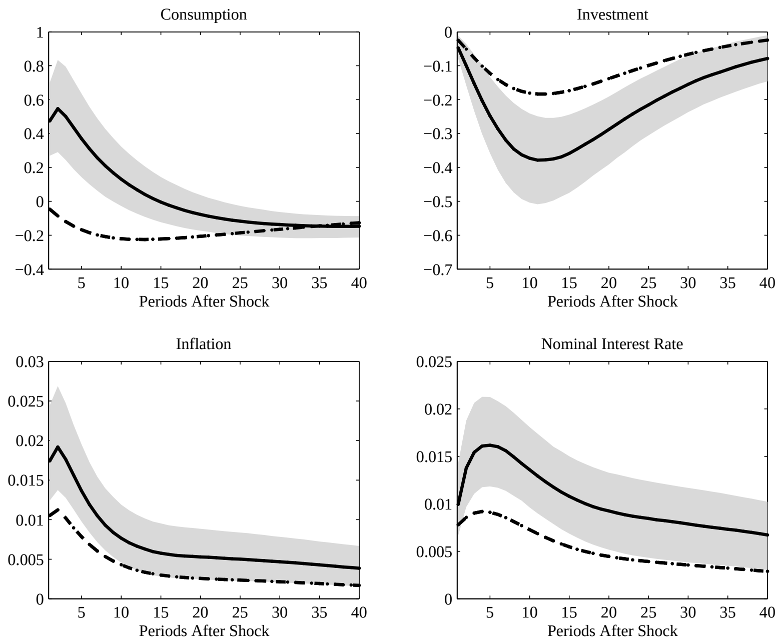
Figure 2. Short–run multiplier on output



Note: The solid line and dashed line corresponds to the mean of the short–run multiplier in the benchmark model and in the Smets–Wouters model, respectively. The grey area corresponds to the 90 percent confidence interval in the benchmark model.

Let us first consider the short–run dynamics. Figure 2 displays the short–run multiplier of the *benchmark* model together with its 90% confidence interval. For comparison purpose, the figure also includes the multiplier obtained from the *Smets–Wouters* model. On impact the multiplier is around 1.60, displays a hump pattern and then steadily goes back to zero. Notice that the short–run multiplier remains greater that one during almost two years. This result is in contrast with the *Smets–Wouters* model for which the multiplier is around one on impact but then quickly decreases (See Cogan *et al.,* 2010, for a similar number). In addition, this multiplier remains outside the confidence interval of the *benchmark* model during four years.

Figure 3. Dynamic responses

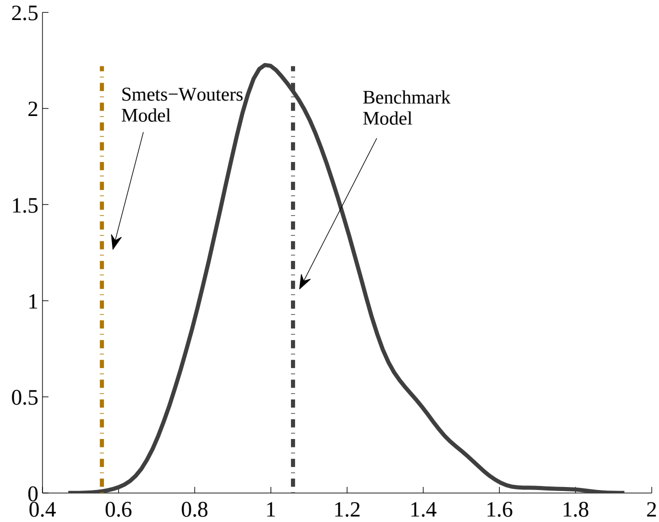


Note: The solid line and dashed line corresponds to the mean of the short–run multiplier in the benchmark model and in the Smets–Wouters model, respectively. The grey area corresponds to the 90 percent confidence interval in the benchmark model.

Figure 3 reports the responses of other aggregate variables. This figure shows huge differences between the two model versions. First, the *benchmark* model displays a persistent and hump–shaped response of private consumption after an increase in public spending. In contrast, the response of consumption is persistently negative in the *Smets–Wouters* model. Second, the decrease in investment is more pronounced in the *benchmark* model than in the *Smets–Wouters* one. The reason is that Edgeworth complementarity leads to reduce private saving, despite the increase in labor supply. Third, the effects on inflation and the nominal interest rate of the government policy shock are more pronounced. Such a demand shock triggers higher inflation pressures in the *benchmark* model than in the *Smets–Wouters* model, due to a larger increase in the real marginal cost. For the nominal interest rate this is a direct consequence of the Taylor rule. The dynamic responses obtained from the *Smets–Wouters* model appear outside the confidence interval for almost all the selected horizon (ten years).

We now investigate the effect of a permanent change in government spending. Figure 4 reports the empirical distribution of the long–run multiplier to output obtained from the *benchmark* model, together with the average long–run multiplier for the two model versions. This figure clearly shows that the two model versions yield very different estimates of the long–run multiplier. Indeed, the average multiplier is around 1.04 in the *benchmark* model whereas it is two times smaller (0.54) when we shut down together the three mechanisms that we put forward. Such a difference is clearly not neutral if the model is used to assess recovery plans or consolidation programmes in the euro area.

Figure 4. Empirical distribution of the long–run multiplier



Note: The vertical dotted line and the dashed line correspond to the mean of the long–run multiplier in the benchmark model and in the Smets–Wouters model, respectively.

III. Inspecting the mechanisms

Having shown that the *benchmark* model generates both short–run and long–run multipliers that exceed unity, we now inspect the key mechanisms at work after a shock to government spending. To do so, we conduct two types of counterfactual experiments. First, we shut down one or several transmission mechanisms by altering a parameter (or a combination thereof) and we re-estimate the model using Bayesian techniques (Counterfactual #1 in Table 2). Second, we perturb the same set of parameters while keeping all the other at the estimated values obtained in our *benchmark* case (Counterfactual #2 in Table 2). If the implied government spending multiplier is strongly affected in both types of counterfactual experiments, the isolated mechanism plays an essential role in the transmission of fiscal shocks. This simply means that other forces (or parameters) are not adjusted, meaning that altering the parameter reveals the mechanism at work. Conversely, if the other parameters adjust to fit the data, there exist potentially other forces that propagate the government spending shock. In this case, we may obtain very different multipliers in the two type of counterfactual experiments.

Table 2—Estimated effects of government spending shocks

		1stQ	5thQ	9thQ	LR			
Benchmark		1.59	1.38	0.88	1.04			
	Counterfactual #1				Counterfactual #2			
	1stQ	5thQ	9thQ	LR	1stQ	5thQ	9thQ	LR
No Edgeworth Complementarity	1.01	0.72	0.40	0.55	0.96	0.73	0.44	0.55
No feedback Rule	1.28	0.81	0.49	0.74	1.65	1.09	0.69	1.04
No Habit Stock	1.65	1.41	0.88	1.08	1.61	1.28	0.79	1.04
No Local Durability	1.61	1.40	0.90	1.05	1.64	1.40	0.88	1.04
One Lag Habit	1.81	1.46	0.82	1.16	1.66	1.30	0.80	1.04

Table 3—Parameter estimates under Counterfactual #1

	Benchmark	(1)	(2)	(3)	(4)	(5)
α_g	–0.650	0.000	–0.275	–0.696	–0.658	–0.776
φ_g	–0.745	–0.606	0.000	–0.759	–0.746	–0.807
h	0.812	0.842	0.829	0.886	0.768	0.799
δ_h	0.469	0.503	0.500	0.000	0.613	0.000
δ_d	0.225	0.234	0.223	0.530	0.000	0.000
η_u	0.613	0.580	0.566	0.618	0.614	0.625
η_k	3.962	4.156	3.874	3.955	3.985	4.000
θ_p	0.829	0.832	0.817	0.833	0.834	0.835
θ_w	0.784	0.776	0.760	0.794	0.788	0.826
γ_p	0.166	0.159	0.161	0.162	0.165	0.159
γ_w	0.366	0.389	0.383	0.376	0.369	0.419
ϕ_r	0.853	0.855	0.850	0.850	0.854	0.846
ϕ_π	1.558	1.553	1.588	1.544	1.564	1.507
ϕ_y	0.144	0.145	0.149	0.146	0.145	0.143

Note: (1) No Edgeworth complementarity, (2) No feedback rule, (3) No habit stock, (4) No local durability, and (5) One lag habit.

We concentrate our analysis on our three modeling features: *(i)* Edgeworth complementarity, *(ii)* endogenous component of government spending, and *(iii)* habit formation in consumption. All our experiments are reported in Tables 2 and 3. Table 2 reports the short–run multiplier $\Delta y_{t+q}/\Delta g_t$ on impact ($q = 0$), one year after impact ($q = 5$) and two years after impact ($q = 9$) and the long–run multiplier (i.e. after a permanent change in the level of government spending). Remind that this multiplier depends only on the estimated value of Edgeworth complementarity, α_g, since the other parameters in the formula remain constant (see Proposition 1). Table 3 reports the structural parameters estimates under Counterfactual #1.

A. Edgeworth complementarity

Let us first consider the role played by Edgeworth complementarity. We set $\alpha_g = 0$. The first line of Tables 2 reports the quantitative results. We see that both the short and long–run multipliers decrease heavily. This result applies for the two counterfactual experiments. So, no other parameters can adjust to compensate the role played by Edgeworth complementarity. This is confirmed by comparing the two first columns of Table 3 (benchmark and no Edgeworth complementarity). Indeed, the other model's parameters remain almost identical under the benchmark specification and the constrained version. The large decrease in the government spending thus results in the crowding out effect on consumption in the absence of Edgeworth complementarity. This mechanism thus appears essential for a proper transmission mechanism of government spending shocks.

B. Endogenous government spending

In this exercise, we assume that government spending is exogenous ($\varphi_g = 0$). In Counterfactual #1, the model's parameters remain unaffected, with the noticeable exception of α_g. When $\varphi_g = 0$, the estimated value of α_g decreases (it is divided by more than 2) and both the short–run and long–run multipliers decreases, i.e. by 0.30 points which is not a negligible figure. This results comes from the interplay of Edgeworth complementarity and countercyclicality of government spending. Indeed, these two mechanisms work in opposite directions in terms of generating a correlation pattern between output or consumption and government expenditures. Edgeworth complementarity tends to increase this correlation, because it induce that agents are willing to consume more. A countercyclical component in the government policy rule reduces this correlation, by construction. So, in order to replicate a given correlation pattern between output and government spending, a higher degree of Edgeworth complementarity is needed to compensate a highly countercyclical government policy. This mechanically translate to higher government spending multiplier. When the government policy is assumed to be exogenous, there is no need for high Edgeworth complementarity, thus yielding a smaller multiplier. Consequently, omitting the endogeneity of government spending may mask sizeable crowding in effects on private consumption. In Counterfactual #2, setting $\varphi_g = 0$ has very little effect. Only the multiplier on impact increases by 0.07 point. This is because, we shut down the automatic stabilizer effect of government spending. Notice that the long–run multiplier is the same since α_g is maintained to its benchmark value. The results show that omitting the endogenous policy rule at the estimation stage would lead an analyst to under–estimate the government spending multiplier at all horizons.

C. Habit formation on consumption

We consider three cases. In the first one, we eliminate the habit stock specification ($\delta_h = 0$). For the second case, we consider a pure habit model ($\delta_d = 0$). In the third case, we specify a one lag habit, as usual in the DSGE literature ($\delta_h = \delta_d = 0$). Overall, our results show that omitting habit formation can have potentially strong effects on the estimated government spending multiplier trough the estimation of α_g (Counterfactual# 1 in Table 2). For example, if we compare the results obtained in the one lag habit case with those of the benchmark case, we obtain a sizeable difference in the estimated multipliers. Both the short–run and the long–run multipliers are now overestimated, because the estimated value of α_g increases (in absolute value). The reason for this result is the following. Suppose that the econometrician will seek to estimate the Edgeworth complementarity, but she wrongly omits habit stock in consumption. Habit formation creates intertemporal complementarity in consumption decision and thus tends to limit the crowding out effect of public spending on private consumption. Because habit stock is (wrongly) absent in the estimated model, there is a need for a higher degree of Edgeworth complementarity to match the data, and thus this implies a higher multiplier. This is illustrated by the two columns "benchmark" and (5) in Table 3. For example, a typical version of the Smets and Wouters model considers only one lag in habit formation. This yields to estimate $\alpha_g = -0.776$ (to be compared to $\alpha_g = -0.650$) and then a multiplier on impact of 1.81, 0.22 point over the benchmark one. Notice that the effects of local durability remain small.

D. Summing up

The previous analysis shed light on two opposite forces affecting the estimation of the government spending multiplier. On the one hand, omitting the Edgeworth complementarity (together with endogenous government spending policy) leads to under–estimate the multiplier. On the other hand, ignoring habit stock in utility function tends to over–estimate the multiplier. The estimation of the *benchmark* model and the counterfactual exercises clearly show that *(i)* the first mechanism dominates and *(ii)* the multiplier is mainly driven by the complementarity between private consumption and public spending.

IV. Concluding remarks

This paper has assessed the main mechanisms at work when estimating government spending multipliers in a DSGE model of the euro area. First, we have shown that the level of Edgeworth complementarity between private and public consumption is essential to fit the data and thus lead to larger multiplier. Moreover, we have explored the consequences of misspecifying the government spending rule and habit formation on the estimated government spending multiplier. We have notably shown that omitting these last two features may exert a severe downward or upward biases on the estimated multipliers and thus can seriously affect the quantitative assessment of fiscal stimulus.

In our framework, we deliberately abstracted from relevant details in order to highlight, as transparently as possible, the empirical link between the three mechanisms we pushed forward. However, the recent literature insists on other modeling or policy issues that might potentially affect our results. We mention two of them. First, we assumed lump-sum taxes to finance government deficit but a more realistic case could consider distortionary taxes instead. In this latter case, we expect an even greater difference between the two model versions because taxes (labor income or VAT) would need to go up much more in the *Smets–Wouters* model for a given spending hike. Second, our paper focused on the size of the government spending multiplier in the euro area as a whole, abstracting from any form of heterogeneity (especially fiscal) among its members. Our framework could be extended to a model of a monetary union to account for cross-country spillovers.

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Model Appendix

A. Equilibrium conditions

This section reports the first–order conditions for the agents' optimizing problems and the other relationships that define the equilibrium of the *benchmark* model.

Effective capital:

$$K_t = u_t \bar{K}_{t-1}$$

Capital accumulation:

$$\bar{K}_t = (1-\delta)\,\bar{K}_{t-1} + \varepsilon_{i,t}\left(1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$$

Marginal utility of consumption:

$$\Lambda_t = \Lambda_{h,t} - \beta\,(\delta_h + h\,(1-\delta_h))\,E_t\left\{\Lambda_{h,t+1}\right\}$$

and

$$\Lambda_{h,t} = \frac{\varepsilon_{b,t}}{S_t} + \beta\,(\delta_h + \delta_d)\,\mathrm{E}_t\left\{\Lambda_{h,t+1}\right\} - \beta^2\,(\delta_h\delta_d)\,\mathrm{E}_t\left\{\Lambda_{h,t+2}\right\}$$

Consumption services:

$$S_t = (\delta_h + \delta_d)\,S_{t-1} - \delta_h\delta_d S_{t-2} + C_t^* - (\delta_h + h\,(1-\delta_h))\,C_{t-1}^*$$

Consumption bundle:

$$C_t^* = C_t + \alpha_g G_t$$

Consumption Euler equation:

$$\Lambda_t = \beta R_t \mathrm{E}_t\left\{\Lambda_{t+1}\frac{P_t}{P_{t+1}}\right\}$$

Investment equation:

$$1 = Q_t\varepsilon_{i,t}\left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}}\mathcal{S}'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta\mathrm{E}_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}\varepsilon_{i,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\mathcal{S}'\left(\frac{I_{t+1}}{I_t}\right)\right\}$$

Tobin's Q:

$$Q_t = \beta\mathrm{E}_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{R_{t+1}^k}{P_{t+1}}u_{t+1} - \vartheta\,(u_{t+1}) + (1-\delta)\,Q_{t+1}\right]\right\}$$

Capital utilization:

$$R_t^k = P_t\vartheta'\,(u_t)$$

Production function:

$$Y_{i,t} = K_{i,t}{}^{\alpha} \left[Z_t N_{i,t}\right]^{1-\alpha} - Z_t F$$

Labor demand:

$$W_t = (1-\alpha)\, Z_t \left(\frac{K_t}{Z_t N_t}\right)^{\alpha} MC_t$$

where MC_t is the nominal marginal cost.

Capital renting:

$$R_t^k = \alpha \left(\frac{K_t}{Z_t N_t}\right)^{\alpha-1} MC_t$$

Price setting:

$$\mathrm{E}_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} Y_{t,t+s}^{\star} \left[P_t^{\star} \Pi_{t,t+s}^{p} - \varepsilon_{p,t+s} MC_{t+s}\right] = 0$$

Aggregate price index:

$$P_t = \left[(1-\theta_p)\,(P_t^{\star})^{1/(\varepsilon_{p,t}-1)} + \theta_p \left(\pi^{1-\gamma_p} \pi_{t-1}^{\gamma_p} P_{t-1}\right)^{1/(\varepsilon_{p,t}-1)}\right]^{(\varepsilon_{p,t}-1)}$$

Wage setting:

$$\mathrm{E}_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \Lambda_{t+s} N_{t,t+s}^{\star} \left[W_t^{\star} \Pi_{t,t+s}^{w} - \varepsilon_{b,t} \varepsilon_{w,t+s} \frac{\left(N_{t,t+s}^{\star}\right)^{\nu}}{\Lambda_{t+s}}\right] = 0$$

Aggregate wage index:

$$W_t = \left[(1-\theta_w)\,(W_t^{\star})^{1/(\varepsilon_{w,t}-1)} + \theta_w \left(\gamma_z \pi^{1-\gamma_p} \pi_{t-1}^{\gamma_w} W_{t-1}\right)^{1/(\varepsilon_{w,t}-1)}\right]^{(\varepsilon_{w,t}-1)}$$

Government spending:

$$\frac{G_t}{Z_t} = g \times \left(\frac{Y_t}{\gamma_z Y_{t-1}}\right)^{\varphi_g} \varepsilon_{g,t}$$

Monetary policy rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{\gamma_z Y_{t-1}}\right)^{\phi_y}\right]^{(1-\phi_r)} \varepsilon_{r,t}$$

Resource constraint:

$$\begin{aligned} Y_t &= C_t + I_t + G_t + \vartheta(u_t)\,\bar{K}_{t-1} \\ \Delta_{p,t} Y_t &= \left(u_t \bar{K}_{t-1}\right)^{\alpha} \left[Z_t N_t\right]^{1-\alpha} - Z_t F \end{aligned}$$

B. Stationary equilibrium

To find the steady–state, we express the model in stationary form. Thus, for the non–stationary variables, let lower–case denote their value relative to the technology process Z_t :

$$\begin{array}{lllll} y_t \equiv Y_t/Z_t & k_t \equiv K_t/Z_t & \bar{k}_t \equiv \bar{K}_t/Z_t & i_t \equiv I_t/Z_t & c_t \equiv C_t/Z_t \\ g_t \equiv G_t/Z_t & c_t^* \equiv C_t^*/Z_t & s_t \equiv S_t/Z_t & w_t \equiv W_t/\left(Z_t P_t\right) & w_t^\star \equiv W_t^\star/\left(Z_t P_t\right) \\ \lambda_t \equiv \Lambda_t Z_t & \lambda_{h,t} \equiv \Lambda_{h,t} Z_t & & & \end{array}$$

where we note that the marginal utility of consumption Λ_t will shrink as the economy grows, and we express the wage in real terms. Also, we denote the real rental rate of capital and real marginal cost by

$$r_t^k \equiv R_t^k/P_t \text{ and } mc_t \equiv MC_t/P_t,$$

and the optimal relative price as

$$p_t^\star \equiv P_t^\star/P_t.$$

Then we can rewrite the model in terms of stationary variables as follows.

Effective capital:

$$k_t = \frac{u_t \bar{k}_{t-1}}{\varepsilon_{z,t}}$$

Capital accumulation:

$$\bar{k}_t = (1-\delta)\frac{\bar{k}_{t-1}}{\varepsilon_{z,t}} + \varepsilon_{i,t}\left(1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\varepsilon_{z,t}\right)\right) i_t$$

Marginal utility of consumption:

$$\lambda_t = \lambda_{h,t} - \beta\left(\delta_h + h\left(1-\delta_h\right)\right) \mathrm{E}_t \left\{\frac{\lambda_{h,t+1}}{\varepsilon_{z,t+1}}\right\}$$

and

$$\lambda_{h,t} = \frac{\varepsilon_{b,t}}{s_t} + \beta\left(\delta_h + \delta_d\right) \mathrm{E}_t \left\{\frac{\lambda_{h,t+1}}{\varepsilon_{z,t+1}}\right\} - \beta^2\left(\delta_h \delta_d\right) \mathrm{E}_t \left\{\frac{\lambda_{h,t+2}}{\varepsilon_{z,t+2}\varepsilon_{z,t+1}}\right\}$$

Consumption services:

$$s_t = \left(\delta_h + \delta_d\right)\frac{s_{t-1}}{\varepsilon_{z,t}} - \delta_h \delta_d \frac{s_{t-2}}{\varepsilon_{z,t}\varepsilon_{z,t-1}} + c_t^* - \left(\delta_h + h\left(1-\delta_h\right)\right)\frac{c_{t-1}^*}{\varepsilon_{z,t}}$$

Consumption bundle:

$$c_t^* = c_t + \alpha_g g_t$$

Consumption Euler equation:

$$\lambda_t = \beta R_t \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\varepsilon_{z,t+1}\pi_{t+1}} \right\}$$

Investment equation:

$$\begin{aligned} 1 \quad = \quad & q_t \varepsilon_{i,t} \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}} \varepsilon_{z,t} \right) - \frac{i_t}{i_{t-1}} \varepsilon_{z,t} \mathcal{S}'\left(\frac{i_t}{i_{t-1}} \varepsilon_{z,t} \right) \right] \\ & + \beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \varepsilon_{z,t+1}} q_{t+1} \varepsilon_{i,t+1} \left(\frac{i_{t+1}}{i_t} \varepsilon_{z,t+1} \right)^2 \mathcal{S}'\left(\frac{i_{t+1}}{i_t} \varepsilon_{z,t+1} \right) \right\} \end{aligned}$$

Tobin's Q:

$$q_t = \beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \varepsilon_{z,t+1}} \left[r_{t+1}^k u_{t+1} - \vartheta\left(u_{t+1}\right) + (1-\delta) q_{t+1} \right] \right\}$$

Capital utilization:

$$r_t^k = \vartheta'\left(u_t\right)$$

Production function:

$$y_{i,t} = k_{i,t}^{\alpha} N_{i,t}^{1-\alpha} - F$$

Labor demand:

$$w_t = (1-\alpha) \left(\frac{k_t}{N_t} \right)^{\alpha} mc_t$$

Capital renting:

$$r_t^k = \alpha \left(\frac{k_t}{N_t} \right)^{\alpha - 1} mc_t$$

Price setting:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta \theta_p\right)^s \frac{\lambda_{t+s}}{\lambda_t} y_{t,t+s}^{\star} \left[p_t^{\star} \Pi_{t,t+s}^p - \varepsilon_{p,t+s} mc_{t+s} \right] = 0$$

Aggregate price index:

$$1 = \left[\left(1-\theta_p\right) \left(p_t^{\star}\right)^{1/(\varepsilon_{p,t}-1)} + \theta_p \left(\pi^{1-\gamma_p} \pi_{t-1}^{\gamma_p} \frac{1}{\pi_t} \right)^{1/(\varepsilon_{p,t}-1)} \right]^{(\varepsilon_{p,t}-1)}$$

Wage setting:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta \theta_w\right)^s \lambda_{t+s} N_{t,t+s}^{\star} \left[w_t^{\star} \frac{P_t}{P_{t+s}} \frac{Z_t}{Z_{t+s}} \Pi_{t,t+s}^w - \varepsilon_{w,t+s} \frac{N_{t,t+s}^{\nu}}{\lambda_{t+s}} \right] = 0$$

Aggregate wage index:

$$w_t = \left[(1-\theta_w) \left(w_t^\star\right)^{1/(\varepsilon_{w,t}-1)} + \theta_w \left(\gamma_z \pi^{1-\gamma_p} \pi_{t-1}^{\gamma_w} \frac{w_{t-1}}{\pi_t \varepsilon_{z,t}} \right)^{1/(\varepsilon_{w,t}-1)} \right]^{(\varepsilon_{w,t}-1)}$$

Government spending:

$$g_t = g \times \left(\frac{\varepsilon_{z,t} y_t}{\gamma_z y_{t-1}} \right)^{\varphi_g} \varepsilon_{g,t}$$

Monetary policy rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\phi_r} \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{\varepsilon_{z,t} y_t}{\gamma_z y_{t-1}} \right)^{\phi_y} \right]^{(1-\phi_r)} \varepsilon_{r,t}$$

Resource constraint:

$$\begin{aligned} y_t &= c_t + i_t + g_t + \vartheta(u_t) \bar{k}_{t-1} / \varepsilon_{z,t} \\ \Delta_{p,t} y_t &= \left(u_t \bar{k}_{t-1} \right)^\alpha N_t^{1-\alpha} - F \end{aligned}$$

C. Steady state

We use the stationary version of the model to find the steady state, and we let variables without a time subscript denote steady-state values. First, we have that $R = \gamma_z \pi / \beta$ and the expression for Tobin's Q implies that the rental rate of capital is

$$r^k = \frac{\gamma_z}{\beta} - (1-\delta)$$

and the price-setting equation gives marginal cost as

$$mc = \frac{1}{\varepsilon_p}.$$

The capital/labor ratio can then be retrieved using the capital renting equation:

$$\frac{k}{N} = \left(\alpha \frac{mc}{r^k} \right)^{1/(1-\alpha)},$$

and the wage is given by the labor demand equation as

$$w = (1-\alpha) \, mc \left(\frac{k}{N} \right)^\alpha$$

The production function gives the output/labor ratio as

$$\frac{y}{N} = \left(\frac{k}{N} \right)^\alpha - \frac{F}{N},$$

and the fixed cost F is set to obtain zero profits at the steady state, implying

$$\frac{F}{N}=\left(\frac{k}{N}\right)^{\alpha}-w-r^{k}\frac{k}{N}.$$

The output/labor ratio is then given by

$$\begin{aligned}\frac{y}{N} &= w+r^{k}\frac{k}{N}\\ &= \frac{r^{k}}{\alpha}\frac{k}{N}.\end{aligned}$$

Finally, to determine the investment/output ratio, use the expressions for effective capital and physical capital accumulation to get

$$\frac{i}{k}=\left(1-\frac{1-\delta}{\gamma_z}\right)\gamma_z,$$

implying that

$$\begin{aligned}\frac{i}{y} &= \frac{i}{k}\frac{k}{N}\frac{N}{y}\\ &= \left(1-\frac{1-\delta}{\gamma_z}\right)\frac{\alpha\gamma_z}{r^{k}}.\end{aligned}$$

Given the government spending/output ratio g/y, the consumption/output ratio is then given by the resource constraint as

$$\frac{c}{y}=1-\frac{i}{y}-\frac{g}{y}.$$

In addition, we have:

$$\lambda=\frac{\gamma_z-\beta\left(\delta_h+h\left(1-\delta_h\right)\right)}{\gamma_z}\lambda_h,$$

$$\lambda_h=\frac{\gamma_z^2}{\gamma_z^2-\beta\left(\left(\delta_h+\delta_d\right)\gamma_z-\beta\delta_h\delta_d\right)}\frac{1}{s},$$

$$s=\frac{\gamma_z\left(\gamma_z-\beta\left(\delta_h+h\left(1-\delta_h\right)\right)\right)}{\gamma_z^2-\left(\delta_h+\delta_d\right)\gamma_z+\delta_h\delta_d}c^{*},$$

$$c^{*}=c+\alpha_g g,$$

D. Log–linearized version

We log-linearize the stationary model around the steady state. Let $\hat{\chi}_t$ denote the log deviation of the variable χ_t from its steady-state level χ: $\hat{\chi}_t\equiv\log\left(\frac{\chi_t}{\chi}\right)$. The log-linearized model is then given by the following system of equations for the endogenous variables.

Effective capital:

$$\hat{k}_t + \hat{\varepsilon}_{z,t} = \hat{u}_t + \widehat{\overline{k}}_{t-1}$$

Capital accumulation:

$$\widehat{\overline{k}}_t = \frac{1-\delta}{\gamma_z}\left(\widehat{\overline{k}}_{t-1} - \hat{\varepsilon}_{z,t}\right) + \left(1 - \frac{1-\delta}{\gamma_z}\right)\left(\hat{\imath}_t + \hat{\varepsilon}_{i,t}\right)$$

Marginal utility of consumption:

$$\begin{aligned}
\hat{\lambda}_t &= \frac{\gamma_z}{\gamma_z - \beta\left(\delta_h + h\left(1-\delta_h\right)\right)}\hat{\lambda}_{h,t} \\
&\quad -\beta\left(\delta_h + h\left(1-\delta_h\right)\right)\left(\frac{\gamma_z}{\gamma_z - \beta\left(\delta_h + h\left(1-\delta_h\right)\right)}\right)\left(\hat{\lambda}_{h,t+1} - \hat{\varepsilon}_{z,t+1}\right)
\end{aligned}$$

and

$$\begin{aligned}
\hat{\lambda}_{h,t} &= \frac{\gamma_z^2 - \beta\left(\left(\delta_h + \delta_d\right)\gamma_z - \beta\delta_h\delta_d\right)}{\gamma_z^2}\left(\hat{\varepsilon}_{b,t} - \hat{s}_t\right) + \frac{\beta\left(\delta_h + \delta_d\right)}{\gamma_z}\left(\hat{\lambda}_{h,t+1} - \hat{\varepsilon}_{z,t+1}\right) \\
&\quad -\frac{\beta^2\left(\delta_h\delta_d\right)}{\gamma_z^2}\left(\hat{\lambda}_{h,t+2} - \hat{\varepsilon}_{z,t+2} - \hat{\varepsilon}_{z,t+1}\right)
\end{aligned}$$

Consumption services:

$$\begin{aligned}
\hat{s}_t &= \frac{\delta_h + \delta_d}{\gamma_z}\left(\hat{s}_{t-1} - \hat{\varepsilon}_{z,t}\right) - \frac{\delta_h\delta_d}{\gamma_z^2}\left(\hat{s}_{t-2} - \hat{\varepsilon}_{z,t} - \hat{\varepsilon}_{z,t-1}\right) + \frac{\gamma_z^2 - \left(\delta_h + \delta_d\right)\gamma_z + \delta_h\delta_d}{\gamma_z\left(\gamma_z - \beta\left(\delta_h + h\left(1-\delta_h\right)\right)\right)}\hat{c}_t^* \\
&\quad -\frac{\left(\gamma_z^2 - \left(\delta_h + \delta_d\right)\gamma_z + \delta_h\delta_d\right)\left(\delta_h + h\left(1-\delta_h\right)\right)}{\gamma_z^2\left(\gamma_z - \beta\left(\delta_h + h\left(1-\delta_h\right)\right)\right)}\left(\hat{c}_{t-1}^* - \hat{\varepsilon}_{z,t}\right)
\end{aligned}$$

Consumption bundle:

$$\hat{c}_t^* = \frac{c}{c + \alpha_g g}\hat{c}_t + \frac{\alpha_g g}{c + \alpha_g g}\hat{g}_t$$

Consumption Euler equation:

$$\hat{\lambda}_t = \mathrm{E}_t\hat{\lambda}_{t+1} + \left(\hat{R}_t - \mathrm{E}_t\hat{\pi}_{t+1}\right) - \mathrm{E}_t\hat{\varepsilon}_{z,t+1}$$

Investment equation:

$$\hat{\imath}_t = \frac{1}{1+\beta}\left(\hat{\imath}_{t-1} - \hat{\varepsilon}_{z,t}\right) + \frac{\beta}{1+\beta}\mathrm{E}_t\left(\hat{\imath}_{t+1} + \hat{\varepsilon}_{z,t+1}\right) + \frac{1}{\eta_k\gamma_z^2\left(1+\beta\right)}\left(\hat{q}_t + \hat{\varepsilon}_{i,t}\right)$$

Tobin's Q:

$$\hat{q}_t = \frac{\beta\,(1-\delta)}{\gamma_z}\mathrm{E}_t\hat{q}_{t+1} + \left(1 - \frac{\beta\,(1-\delta)}{\gamma_z}\right)\mathrm{E}_t\hat{r}^k_{t+1} - (\hat{r}_t - \mathrm{E}_t\hat{\pi}_{t+1})$$

Capital utilization:

$$\hat{u}_t = \eta_u \hat{r}^k_t$$

Production function:

$$\hat{y}_t = \frac{y+F}{y}\left(\alpha\hat{k}_t + (1-\alpha)\,\hat{n}_t\right)$$

Labor demand:

$$\hat{w}_t = \widehat{mc}_t + \alpha\hat{k}_t - \alpha\hat{n}_t$$

Capital renting:

$$\hat{r}^k_t = \widehat{mc}_t - (1-\alpha)\,\hat{k}_t + (1-\alpha)\,\hat{n}_t$$

Phillips curve:

$$\hat{\pi}_t = \frac{\gamma_p}{1+\beta\gamma_p}\pi_{t-1} + \frac{\beta}{1+\beta\gamma_p}\mathrm{E}_t\pi_{t+1} + \frac{(1-\beta\theta_p)\,(1-\theta_p)}{\theta_p\left(1+\beta\gamma_p\right)}\left(\widehat{mc}_t + \hat{\varepsilon}_{p,t}\right)$$

Wage curve:

$$\begin{aligned}\hat{w}_t \;\; = \;\; & \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\mathrm{E}_t\hat{w}_{t+1} - \frac{(1-\beta\theta_w)\,(1-\theta_w)}{\theta_w\,(1+\beta)\left(1+\nu\frac{\varepsilon_w}{\varepsilon_w-1}\right)}\left(\widehat{mrs}_t + \hat{\varepsilon}_{w,t}\right) \\ & + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}\mathrm{E}_t\hat{\pi}_{t+1} - \frac{1}{1+\beta}\hat{\varepsilon}_{z,t} + \frac{\beta}{1+\beta}\mathrm{E}_t\hat{\varepsilon}_{z,t+1}\end{aligned}$$

Marginal rate of substitution:

$$\widehat{mrs}_t = \hat{w}_t - \left(\nu\hat{n}_t - \hat{\lambda}_t + \hat{\varepsilon}_{b,t}\right)$$

Government spending:

$$\hat{g}_t = \varphi_g\left(\hat{y}_t - \hat{y}_{t-1} + \hat{\varepsilon}_{z,t}\right) + \hat{\varepsilon}_{g,t}$$

Monetary policy rule:

$$\hat{R}_t = \phi_r R_{t-1}\,(1-\phi_r)\left[\phi_\pi\hat{\pi}_t + \phi_y\left(\hat{y}_t - \hat{y}_{t-1} + \hat{\varepsilon}_{z,t}\right)\right] + \hat{\varepsilon}_{r,t}$$

Resource constraint:

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{g}{y}\hat{g}_t + \frac{r^k k}{y}\hat{u}_t$$