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Abstract

We study how the presence of a news aggregator affects quality choices of newspapers competing on the Internet. To provide a microfoundation for the role of the aggregator, we build a model of multiple issues where each newspaper chooses quality on each issue. This model captures the "business-stealing effect" and the "readership expansion effect" of the aggregator. We find that the presence of the aggregator leads newspapers to specialize in news coverage, changes quality choices from strategic substitutes to strategic complements and is likely to increase the quality of newspapers and social welfare, with an ambiguous effect on newspapers' profits.

JEL Classification: D21, D43, L13, L82

Key words: Newspapers, News Aggregator, Internet, Quality, Strategic Substitutes, Strategic Complements, Advertising, Business-stealing, Readership expansion, Opting Out.

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1 Introduction

The traditional ad-based business model of newspapers has been in crisis because of declining revenues from newspaper advertising and increasing competition from new online media, such as web-only news, blogs and news aggregators. There are serious concerns that this crisis may lead to a decrease in the quality of journalism.

Newspapers' revenues from advertising have fallen approximately 50% since 2000. According to The State of the News Media (2013), print advertising accounted for $48.7 billion in revenue for newspapers in 2000, $19.1 billion in 2012 (See Figure 1).[1] In particular, entry of online classified-ad competitors such as Craigslist substantially reduced newspapsers' revneue (Kroft and Pope, 2012, Seamans and Zhu, 2012). The above numbers become more meaningful if we know that for most newspapers, about 80% of revenues came from advertising and 20% came from sales (FTC, 2010). In France, not a single national newspaper is profitable despite around €1.2 billion in direct and indirect government subsidies.[2]

Newspapers are in stiff competition with new online media. The online media is the only one among all media which has audience growth in the past decade, as Figure 2 confirms. Among online media, news aggregators are the most important. According to Outsell report (2009), 57 percent of users now go to digital sources and they are also likelier to turn to an aggregator (31 percent) than to a newspaper site (8 percent) or other site (18 percent). Indeed, Pew Research Center (2012) shows that the aggregators (Yahoo! News, Google News, MSN, AOL News and Huffington Post) attract more than half of the traffic of online news in U.S. (see Table 1). In particular, in South Korea, the share of traffics originated from the top two domestic search engines reaches 85% of the total traffics to newspapers; NAVER, the number one search engine, alone accounts for 70% [3].

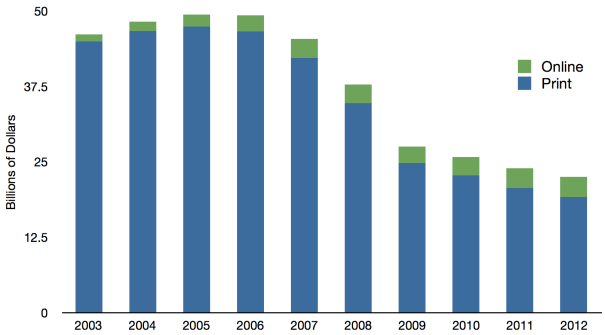
The success of news aggregators raises a hot debate about the effect of news aggregators on newspapers. At the heart of the debate is the effect on newspapers' incentive to produce high quality content. The debate has already attracted the attention of governments and regulatory bodies. For instance, during 2009 to 2010 the FTC hosted three workshops on the Future of Journalism and has published a controversial "discussion draft" that hints of copyright reform and protection of newspapers from aggregators. In Europe, the German government recently adopted a project to introduce a law on Google "Lex Google" of which the purpose is to make Google pay for indexing the content of German news sites[4]. A similar law has been proposed in

[1] In addition, the report of Pew Research Center (2013) shows that after the recent financial crisis, the advertising revenue bounced back for all media except for newspapers.

[2] "Taxing times", The Economist, 10 Nov. 2012

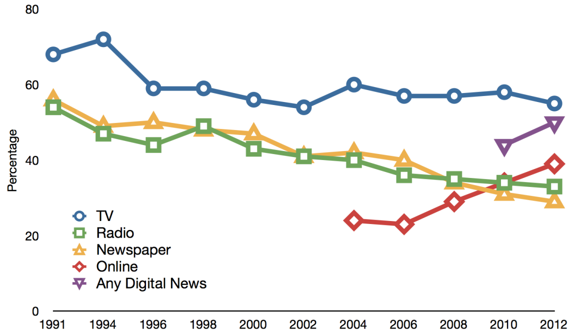
[3] See http://www.rankey.com/blog/blog.php?type=column&sub_type=all&writer= &no=327&page=9&search_type=subject&search_wd=

[4] "Polémique sur la 'Lex Google' en Allemagne", Le Monde, 30 Aug. 2012



Source: The State of the News Media, 2013

Figure 1: Advertising Revenue



Source: The State of the News Media, 2013

Figure 2: Where People Got News Yesterday

Italy[5] and French newspapers want the same[6]. Recently, France's President François Hollande unveiled a settlement that Google made with French newspapers according to which Google would create a €60 million fund to help them develop their Internet presence.[7]

There are two opposite arguments in the debate. On the one hand, content producers argue that news aggregators make money by stealing high quality content. Since this money is pull out of the content producers' pocket, they have less incentive to produce high quality content. For instance, according to Rupert Murdoch (2009), chairman of News Corp.,

"When this work is misappropriated without regard to the investment made, it destroys the economics of producing high quality content. The truth is that the "aggregators" need news organizations. Without content to transmit, all our flat-screen TVs, computers, cell phones, iPhones and blackberries, would be blank slates. (p.13)."

On the other hand, news aggregators argue that news aggregators conduct huge traffics to news sites that they can make money out of them. Google (2010), for instance, in a response to the FTC report (2010), claims that they send more than four billion clicks each month to news publishers via Google Search, Google News, and other products. Google believes each click – each visit – provides publishers with an opportunity to show users ads, register users, charge users for access to content, and so forth.

In this paper, we study how the presence of a news aggregator affects competition between newspapers and their quality choice. Our multi-issue model provides a microfoundation for the role of the aggregator. In addition, the model creates rich strategic interactions

[5] "Taxing times", The Economist, 10 Nov. 2012

[6] Le Figaro, les Echos and le Nouvel Observateur are in favor of Google tax:
"Taxe Google : Le Figaro, les Echos et le Nouvel Obs veulent être payés", ZDNet, 11 Sep. 2012

[7] "Google Settles Dispute with French Newspapers", Wall Street Journal, 1 Feb. 2013

Table 1: Where Do People Get News Online?

Online News Sources	%
Yahoo/Yahoo News	26
Google/Google News	17
CNN	14
Local news sources	13
MSN	11
FOX	9
MSNBC	6
New York Times	5
AOL	5
Huffington Post	4
Facebook	3
ABC/ABC News	3
Wall Street Journal	3
BBC	2
USA Today	2
Internet service providers	2
ESPN	2
Washington Post	2
The Drudge Report	2

Source: Pew Research Center, 2012

between newspapers by allowing each newspaper to choose quality for each among many issues. Hence, each newspaper's strategy has both a vertical dimension (through quality choice) and a horizontal dimension (through choice of issues to cover in depth). Finally, we embed this feature of multiple issues into the classic Hotelling model, which serves to capture product differentiation of newspapers, which can be interperted as ideological differentiation as in the article of Mullainathan and Shleifer (2005) on media bias. In contrast to Mullainathan and Shleifer (2005), we consider the locations of the newspapers as given and instead focus on how the presence of the aggregator affects their choice of quality and covergae.

We have in mind a sequential reading process in which a reader first reads a homepage (i.e. an index page) and then click on the issues that she or he wants to read more about. More precisely, we assum that a reader spends a unit of attention on any given issue at the homepape by reading the title and the abstract and spends extra $\delta > 0$ unit of attention on the original article by clicking on the link to the article if the issue is covered with high quality. Each newspaper's homepage provides links to its own articles only. We model the aggregator to capture Google News: the aggregator has no original article and its homepage provides a link to the highest quality article on each issue. Therefore, when a reader switches from her preferred newspaper to the aggregator, there is a clear benefit in terms of quality and a clear loss in terms of preference mismatch (or ideological mismatch). In addition, this microfoundation of the aggregator allows us to capture two opposite effects, which are at the core of the debate on news aggregators, namely the business-stealing effect and the readership expansion effect. The business-stealing effect arises in our model in terms of homepage consumption as long as some readers switch from a newspaper to the aggregator. However, there also exists the readership expansion effect in terms of consumption of original articles: since the aggregator improves match between each reader's attention and high quality content, it expands the readership for high quality articles.

In the baseline model, we consider two symmetric newspapers. We find that the presence of an aggregator would lead each newspaper to specialize in a different set of issue (i.e. maximum differentiation) when the advertising revenue increases substantially with quality increase (i.e. δ high) and would lead both newspapers to invest in the same issues (i.e. minimum differentiation) otherwise. When both newspapers use the maximum differentiation strategy, the presence of the aggregator changes the strategic interactions of quality choices from strategic substitutes to strategic complements. As a consequence, its presence increases the average quality of newspapers compared to case without aggregator, which in turn implies that the presence increases consumer surplus and social welfare. However, the effect on the newspapers' profits is ambiguous.

The intuition for the change in the strategic interactions is the following. In the absence of the aggregator, if newspaper 2, say, chooses a higher quality, this decreases the market share of newspaper 1 and hence reduces 1's marginal revenue from increase in quality. On the contrary, when both newspapers use the maximum differentiation strategy in the presence of the aggregator, if newspaper 2 increases its quality, this expands the market share of the aggregator. This in turn implies that the high quality content of newspaper 1 can reach a larger number of readers since it can reach both its loyal readers and the readers who use the aggregator. Therefore, an increase in 2's quality increases 1's marginal revenue from quality increase.

When the presence of the aggregator induces minimum differentiation, the aggregator has zero market share and we find that there is a continuum of symmetric equilibria such that the maximum quality (respectively, the minimum quality) is higher (lower) than the quality in the absence of the aggregator. However, when we allow each newspaper to choose to opt out (i.e. to break the hyper link to the aggregator's site), only the equilibrium quality in the absence of the aggregator survives. Therefore, introducing opting out possibility leads to a sharp prediction: the presence of the aggregator either leads to no change or to the maximum differentiation equilibrium.

In section 7, we make an extension to capture asymmetry among newspapers. In reality, there are many small news sites which would receive very negligible traffics in the absence of the aggregator. Therefore, these sites have strong incentives to use "the maximum differentiation and opt-in strategy" in order to attract traffics from the aggregator. In order to capture this scenario, we introduce one important modification into our model: by using the aggregator, consumers can get additional utility, u_T, where T means third party content. When u_T is important enough, we find that it is a dominant strategy for each newspaper to adopt the maximum differentiation strategy and hence quality choices become strategic complements. There is a unique symmetric equilibrium of which the quality increases with δ. To obtain a lower bound on δ, we rely on the findings from Athey and Mobius (2012) and Chiou and Tucker (2012) that an increase in the content indexed by Google News boosts traffics to the news sites that have been indexed by Google News. Then, we find that the presence of the aggregator increases the quality of the newspapers and that no newspaper has an incentive to opt out.

There are a few theoretical papers on news aggregators. Dellarocas, Katona, and Rand (2012) consider a single-issue model with focus on interactions between quality choice and link decisions (i.e. every newspaper can provide a link to a rival's content). The aggregator benefits consumers by providing links to the highest quality content. They show that the presence of an aggregator might decrease (increase) competition among content providers if content providers can (can not) link to each other. In George and Hogendorn (2012), news aggregators increase multi-homing viewers, which reduces multi-homing advertisers. They consider switching of a given mass of viewers from single-homing to multi-homing and find that it is likely to reduce (increase) a news outlet's advertising revenue if the outlet initially has a high (small) share of exclusive viewers. The major difference of our paper with respect to these papers is that we consider a model of multiple issues with endogenous quality and coverage, which allows us to provide a microfoundation to the role of the aggregator and clearly captures the business-stealing effect and the readership expansion effect. Furthermore, our result that the presence of the aggregator changes strategic interactions of quality choices from strategic substitutes to strategic complements does not exist in their papers.

There are two empirical papers on news aggregators (Chiou and Tucker, 2012, and Athey and Mobius, 2012). They provide evidence for the dominance of the readership expansion effect over the business-stealing effect. Chiou and Tucker (2012) study a natural experiment where Google News had a dispute with the Associated Press and hence did not show Associated Press content for some period. They find that after the removal of Associated Press content, users of Google News subsequently visited other news sites less often than users of Yahoo! News which did not remove the content. They conclude that users of aggregators are more likely to seek additional sources and read further rather than merely being satisfied with the summary. Athey and Mobius (2012) study a case where Google News added local content to its homepage for those users who chose to enter their location. By comparing the consumers who use this feature with controlled users, they find that users who adopt the feature increase their usage of Google News, which in turn leads to additional consumption of local news. They conclude that their results support the view that news aggregators are complement for local news outlets. This occurs in our paper if readership expansion effect dominates business-stealing effect.

Our work relates to the literature on interconnection among online sites. In particular, Jeon and Menicucci (2011) studies interconnection among academic journal websites either through a multilateral platform (such as CrossRef) or through bilateral arrangements. News Aggregators can be considered a multilateral platform of interconnection. However, this paper is different from Jeon and Menicucci (2011) in the sense that the strategic variables are completely different. The former studies how the presence of a multilateral platform affects newspapers' choice of content (when content is free) whereas the latter studies how interconnections interact with pricing of academic journals for given content.

Our paper builds on the literature on two-sided markets (Rochet and Tirole, 2003, 2006, Caillaud and Jullien, 2001, 2003, Anderson and Coate, 2005, Armstrong, 2006, Hagiu, 2006, Weyl, 2010). Two-sided markets can be roughly defined as industries where platforms provide intermediation services between two (or several) kinds of users. Typical examples include dating agencies, payment cards (Rochet and Tirole, 2002), media (Anderson and Coate, 2005), operating systems (Parker and Van Alstyne, 2005), video games (Hagiu, 2006), academic journals (Rochet and Jeon, 2010) etc. In such industries, it is vital for platforms to find a price structure that attracts sufficient numbers of users on each side of the market. In the application to media (Anderson and Coate, 2005), the two sides refer to readers and advertisers. For instance, Athey, Calvano, and Gans (2012) study how applying consumer tracking technology to advertising affects competition between online news media in a two-sided market framework. Instead of explicitly modeling the competition in the market for advertising, we describe this market with a reduced-form in order to focus on rich strategic interactions in the newspaper content market. In addition, with the reduced-form approach, we intend to capture the fact that newspapers compete with other media and non-media firms such as Craigslist in the advertising market.

The rest of the paper is organized as follows. We present the model in Section 2. In Section 3, we study newspapers competition in the absence of aggregator as a benchmark. Section 4 studies how the presence of an aggregator affects newspaper competition. Section 5 introduces opting out possibility and refines equilibria obtained in Section 4. Section 6 compares the outcome without the aggregator with the one with the aggregator in terms of quality, consumer surplus, profit and social welfare. Section 7 is about the extension with content from third party sites. Section 8 concludes the paper. All the proofs except for the short proof of Lemma 2 are gathered in the Appendix.

2 Model

We consider two newspapers and one aggregator for simplicity. To provide a microfoundation for the role of the aggregator, we introduce some novel features into the classic Hotelling model (Hotelling, 1929, Tirole, 1990). The Hotelling model is used to capture horizontal product differentiation between the newspapers and has been used by Mullainathan and Shliefer (2005) to represent ideological differentiation. The novel features we introduce are multiple issues and endogenous choice of quality and coverage, as is explained below.

2.1 Newspapers and Consumers

Throughout the paper, we assume that consumers single-home[8], which means that a consumer consumes only one of the two newspapers in the absence of the aggregator. In the presence of the aggregator, a consumer chooses one among newspaper 1, newspaper 2 and the aggregator.

2.1.1 Product Differentiation

The two newspapers are located at the extreme points of a line of length 1;[9] newspaper 1 on the left extreme point and newspaper 2 on the right extreme point. Mass 1 of consumers are uniformly distributed on the line. A location in the line can represent the ideological view of a consumer or a newspaper (Mullainathan and Shliefer, 2005, and Gentzkow and Shapiro, 2011). If a consumer located at x consumes an article of a newspaper located at y, the consumer incurs a transportation cost of $t\,|x-y|$ with $t>0$. The transportation cost represents utility losses due to imperfect preference matching.

2.1.2 Multiple Issues and Choice of Quality and Coverage

We assume that there is a continuum of issues which each newspaper covers. Let S be the set of issues. On each given issue, a newspaper can provide either high or low quality content. So the strategy of newspaper i, with $i \in \{1,2\}$, is a subset of issues $s_i \in S$ which it covers with high quality content; for the rest of issues $S - s_i$, the quality of content is low. Let $\mu(s)$ represent the measure of any set $s \in S$. Without loss of generality, assume $\mu(S) = 1$. Then, $\mu(s_i)$ represents the average quality of newspaper i. Therefore, the strategy s_i has a vertical dimension in terms of average quality: from now on, we simply call $\mu(s_i)$ quality of newspaper i. Furthermore, even when both newspapers choose the same quality, the strategy has a horizontal dimension since each newspaper can cover, with high quality content, a different subset of issues or the same subset. Given $0 < \mu(s_1), \mu(s_2) \leq 1/2$, for newspaper $i \in \{1,2\}$, if i chooses s_i such that $s_i \cap s_j = \emptyset$, we say that i uses *the maximum differentiation strategy* (equivalently, the specialization strategy). If i chooses s_i such that $\mu(s_1 \cap s_2) = \min(\mu(s_1), \mu(s_2))$, then we say that i uses *the minimum differentiation strategy* (equivalently, the no-specialization strategy).

[8] Basically, the aggregator's technology allows consumers to have access to content from all newspapers. Given that we consider only two newspapers, this technological difference between the aggregator and newspapers can be captured by the single homing assumption. However, if we consider a large number of newspapers, we can allow consumers to read two or three newspapers without using the aggregator and still capture the technological difference.

[9] We here follow the maximum differentation result in the Hotelling model. Actually, Mullainathan and Shliefer (2001) rediscover the maxium diffentiation result in the context of media bias. Anyway, our results would hold for any symmetric locations of the newspapers.

2.1.3 Consumer Preferences

We have in mind a sequential reading process in which a reader first reads a homepage (i.e. a index page) and then click on the issues that she wants to read more about. For each article, a homepage provides the title, a summary and a link to the original content. After reading the title and the summary, each reader decides to click on the link to read the original article. We assume that each reader clicks on the link to an original article only if its quality is high.[10] Therefore, a reader spends one unit of attention per article at a homepage and $\delta > 0$ extra unit of attention at the original article only if it is of high quality. A simplifying assumption we make is that each consumer is interested in all issues.[11]

Let u_0 represent a consumer's gross utility from reading the homepage of a newspaper when all its content is of low quality. We assume $u_0 > t$, which implies that even when all content is of low quality, each consumer ends up consuming one of the newspapers. This is a standard full participation assumption in the Hotelling model. Let $\Delta u > 0$ represent the utility increase that a consumer experiences when a low quality article is replaced by a high quality one. Then, the utility that a consumer located at x obtaints from consuming newspaper 1 (or 2) is given by

$$U^1(x) = u_0 + \mu(s_1)\Delta u - xt; \tag{1}$$

$$U^2(x) = u_0 + \mu(s_2)\Delta u - (1-x)t. \tag{2}$$

Define β as $\beta \equiv \Delta u/t$. We can interpret β as *the measure of disloyality*, in the sense that the smaller β is, the more loyal are consumers to the newspapers. Small β means that ideological characteristic of newspapers matters more than their quality for consumers. To make sure that each newspaper has a positive market share in the presence of the aggregator, we make the following assumption:[12]

A1: $\beta < 1$ (i.e. consumers are loyal enough to the newspapers).

2.1.4 Advertising Revenues and Content Production Technology

We consider a business model based on advertising in which newspapers' content on the Internet is free. Each unit of attention brings an advertising revenue of $\varpi > 0$.

[10] Alternatively, after clicking on the link to a low quality article, a reader immediately stops reading it.

[11] Alternatively, we can assume that each consumer is interested in a constant fraction of (randomly selected) issues. Enriching heterogeneity among the readers is a direction for future research.

[12] In the absence of the aggregator, it is sufficient to have $\beta < 2$ to discard cornering equilibrium.

For tractability, we model the cost of investing in news quality by a quadratic function. Furthermore, we are intrested in a situation in which each newspaper can take the two key stragetic decisions in a separate way: the choice of (average) quality, on the one hand, and the choice of differentiation in terms of issues covered with high quality, on the other hand. Therefore, we assume that the cost of investing in a subset s_i of measure $\mu(s_i)$ for newspaper $i \in \{1, 2\}$ is given by

A2:

$$C(\mu(s_i)) = \begin{cases} \infty & \mu(s_i) > \frac{1}{2} \\ c\mu(s_i)^2 & \mu(s_i) \leq \frac{1}{2} \end{cases}$$

where $c > 0$ is a positive constant. In A2, the cost of investing in a subset of measure greater than $\frac{1}{2}$ is infinity. Limting i's choice to $\mu(s_i) \leq \frac{1}{2}$ mainly serves the purpose of allowing each newspaper to make the two decisions separately. Without this assumption, the two choices cannot be made independetly: for instance, when $\mu(s_1 \cup s_2) = 1$, increasing i's quality implies an increase in $\mu(s_1 \cap s_2)$. In general, when there is no upper bound on the quality of an article on an issue, each newspaper is able to make the two decisions separately. We introduce the restriction $\mu(s_i) \leq \frac{1}{2}$ to capture this situation in our simple model with an exogenous upper bound on the quality of an article.[13]

Thus, the profit of newspaper $i \in \{1, 2\}$ in the absence of the aggregator is

$$\pi_i(s_i) = \varpi\alpha_i [1 + \mu(s_i)\delta] - C(\mu(s_i)), \tag{3}$$

where α_i is the market share of newspaper i.

In what follows, without loss of generality, we normalize ϖ at one since what matters is only c/ϖ. However, the interpretation of our results will be done in terms of c/ϖ (see the end of Section 6).

2.2 Aggregator

2.2.1 Benefit of Using the Aggregator

The value added of an aggregator consists in recognizing high quality content ex post. In the real world, some aggregators, like Huffington Post, use editorial staff, while others, like Google News,

[13] However, a model of continuous quality choice with no upper bound would be much less tractable while it does not deliver much new insight. Actually, even our simple model becomes technically involved because providing the microfoundation for the utility that a consumer obtains from the aggregator creates some technical challenges: as is shown in Lemma 3, in the presence of the aggregator, the denominator in the expression for a given newspaper's market share is a function of the strategies $(\mu(s_1), \mu(s_2))$, which makes the analysis complex. This is why we consider a quadratic cost function.

use an algorithm to find high quality content. After finding high quality articles, each aggregator posts them on its site. However, there are different ways. Someone, like Yahoo! News, posts the whole article in its site, without putting any link to the original content. Usually, this is because the aggregator pays the newspaper for that content and hence has the right to publish it. In 2006, Yahoo! signed an agreement with Newspaper Consortium[14] to use their content. Others, like Google News, show the title and a short summary of an article and provide a link to the original article. The first pages and sample articles of Yahoo! News and Google News can be seen in Figures 7, 8, 9, and 10. Indeed, these two types of aggregators bring revenue to newspapers in different ways, the first one by buying the license and the second by sending traffic to newspaper sites.

We model the aggregator mostly in the form of Google News and relegate the licensing issue for future work. Hence, the aggregator in our model provides only a homepage without having its own original articles. It generates benefit to consumers by improving the match between their attention and high quality content. More precisely, for each given issue, the aggregator finds the highest quality article and publishes its title and summary with a link to the original article. A consumer who goes to the aggregator's homepage spends one unit of attention per article regardless of its quality. After that, if the quality is high, the consumer clicks on the link to read the original article. By doing this she spends δ unit of attention on the newspaper site to which she or he is directed. If the quality is low, the consumer only reads the title and the abstract, and does not click on the link.[15]

While a consumer who directly chooses a newspaper spends $1 + \delta$ unit of attention per high quality article and one unit per low quality one, a consumer who is directed to a newspaper through the aggregator spends δ unit of attention only for high quality articles. *The business-stealing effect* captures this loss of the traffics to the homepages of the newspapers. However, there is also *a readership expansion effect* since high quality articles of a given newspaper can reach not only its loyal readers but also those using the aggregator. The latter includes consumers who would read the rival newspaper if there were no news aggregator.

2.2.2 Cost of Using the Aggregator

We capture the cost of using the aggregator by assuming that if both newspapers or none of them produce high quality articles on a given issue, the aggregator will provide a link only to one of them with an equal probability. So for a given consumer, using the aggregator involves

[14] http://www.npconsortium.com/
"Is Yahoo a Better Friend to Newspapers Than Google?", New York Times, 8 Apr. 2009

[15] Alternatively, she might click the link but quickly stop reading the article upon realizing that the quality is low.

a higher cost of preference mismatch than using her preferred newspaper. Actually, this is the way the aggregators work. For instance, Google News provides one link per issue for all topics except for the top story for which it shows multiple links (see Figure 10).

In summary, for any given consumer, using the aggregator, in comparison to reading her preferred newspaper, allows her to enjoy more high quality content at a higher cost of preference mismatch.

2.3 Timing

In what follows, we analyze the following two-stage game.

- Stage 1: each newspaper i simultaneously chooses s_i.
- Stage 2: each consumer chooses one between the two newspapers if there is no aggregator (otherwise, one among the two newspapers and the aggregator).

When there is an aggregator, we also study a two-stage game in which each newspaper is allowed to opt out in stage 1 where opting out means that a newspaper breaks the link with the aggregator. Then, stage 1 is replaced by

- Stage 1': each newspaper i simultaneously decides whether to opt out or not and chooses s_i.

3 No Aggregator

In this section, we analyze the two-stage game in the absence of the aggregator. As usual we use backward induction and start from Stage 2. In this section, what matters is only $\mu(s_i) = \mu_i$ for $i = 1, 2$, given our single-homing assumption.

Let x denote the location of the consumer who is indifferent between 1 and 2, which is determined by:

$$\mu_1 \Delta u - tx = \mu_2 \Delta u - t(1-x).$$

Equivalently, we have

$$x = \frac{1}{2} + \frac{\beta}{2}\left(\mu_1 - \mu_2\right).$$

From A1, we have $0 < x < 1$. Therefore, each newspaper's market share is positive: $0 < \alpha_i < 1$ for $i = 1, 2$.

Newspaper i's profit is given by

$$\pi_i = \left[\frac{1}{2} + \frac{\beta}{2}\left(\mu_i - \mu_j\right)\right]\left[1 + \mu_i \delta\right] - c\mu_i^2 \text{ for } \left(\mu_i, \mu_j\right) \in [0, 1/2]^2.$$

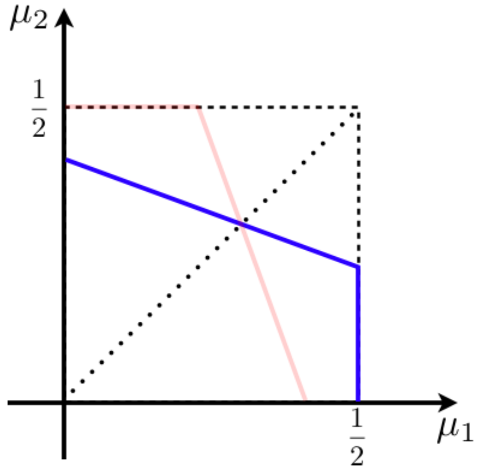


Figure 3: Best reply function of newspaper 1 when there is no aggregator

If $c \leq \beta\delta/2$, the profit function is convex. As $\pi_i'(0) = \beta + \delta - \beta\delta\mu_j > 0$ for any $\mu_j \in [0, 1/2]$, newspaper i's best response is $1/2$ for any $\mu_j \in [0, 1/2]$. If $c > \beta\delta/2$, the profit function is strictly concave. The best reply function of i is given by

$$BR_i^N(\mu_j) = \begin{cases} \frac{1}{2} & \text{if } \mu_j \leq 1 - \frac{2c-(\beta+\delta)}{\beta\delta}, \\ \frac{\beta+\delta-\beta\delta\mu_j}{4c-2\beta\delta} & \text{if } \mu_j > 1 - \frac{2c-(\beta+\delta)}{\beta\delta}; \end{cases}$$

where the superscript N means no aggregator. In this case, the sign of best reply function's slope is zero or $-\beta\delta/(4c - 2\beta\delta)$. Therefore, we can conclude:

Lemma 1. *In the absence of the aggregator, newspapers' quality choices* (μ_1, μ_2) *are strategic substitutes.*

In the absence of the aggregator, if newspaper j increases its quality, this reduces newspaper i's market share and thereby i's marginal revenue from an increase in quality. This is why quality choices are strategic substitutes.[16] Figure 3 describes newspaper 1's best reply function when $c > \beta\delta/2$.

Let (μ_1^*, μ_2^*) denote the equilibrium quality in the absence of the aggregator. The next proposition shows that there is a unique equilibrium.

Proposition 1. *Under A1 and A2, in the absence of the aggregator, there is a unique equilibrium,*

[16] This is similar to why quantities are strategic substitutes in Cournot competition: an increase in firm j's quantity reduces the price of firm i's good and hence the latter's marginal revenue from production. The intuition also shows that the result holds even if we allow newspapers to charge for subscriptions: for any given prices, quality choices are strategic substitutes.

which is symmetric. In the equilibrium,
(i) the average quality of each newspaper is

$\mu^* = \mu_1^* = \mu_2^* = \frac{1}{2}$ *if* $0 \leq c \leq \frac{\delta\beta}{4} + \frac{\delta}{2} + \frac{\beta}{2}$,

$\mu^* = \mu_1^* = \mu_2^* = \frac{\delta+\beta}{4c-\delta\beta}$ *if* $c > \frac{\delta\beta}{4} + \frac{\delta}{2} + \frac{\beta}{2}$;

(ii) the profit of each newspaper is $\pi^* = -c{\mu^*}^2 + \frac{\delta}{2}\mu^* + \frac{1}{2}$.

One can easily check that μ^* and π^* are increasing in δ and decreasing in c. μ^* is increasing in β but π^* is decreasing in β. It means that newspapers like customer loyalty but their quality decreases with loyalty.

From now on, we assume that the equilibrium quality in the absence of the aggregator is interior (i.e. $\mu^* \in (0, 1/2)$):

A3: $c > \frac{\delta\beta}{4} + \frac{\delta}{2} + \frac{\beta}{2}$.

If A3 does not hold, each newspaper i's best reply is $\mu_i = \frac{1}{2}$ for any $\mu_j \in [0, 1/2]$, which is not interesting.

4 Aggregator

In this section, the two newspapers compete in the presence of an aggregator.

4.1 Market shares for given qualities

Given (s_1, s_2), the utility that a consumer with location x obtains from using the aggregator is given by:

$$\begin{aligned} U^{Agg}(x) &= u_0 + \mu(s_1 - s_2)(\Delta u - xt) + \mu(s_2 - s_1)(\Delta u - (1-x)t) \\ &\quad + \mu(s_1 \cap s_2)\left(\Delta u - \frac{1}{2}xt - \frac{1}{2}(1-x)t\right) + (1 - \mu(s_1 \cup s_2))\left(-\frac{1}{2}xt - \frac{1}{2}(1-x)t\right) \end{aligned} \tag{4}$$

where $s_1 - s_2$ means $s_1 \cap s_2^c$. Given an issue, when both newspapers provide the same quality content on it, the aggregator displays one of them with equal probability and therefore the consumer's expected transportation cost is $\frac{1}{2}xt + \frac{1}{2}(1-x)t$.

Using $\mu(s_1 \cup s_2) = \mu(s_1) + \mu(s_2) - \mu(s_1 \cap s_2)$ and $\mu(s_i - s_j) = \mu(s_i) - \mu(s_1 \cap s_2)$, we can rewrite $U^{Agg}(x)$, $U^1(x)$ and $U^2(x)$ as follows:

$$\begin{aligned}
U^{Agg}(x) &= u_0 - \frac{t}{2} + \mu(s_1 \cup s_2)\Delta u + t(x - \frac{1}{2})(\mu(s_2) - \mu(s_1));\\
U^1(x) &= u_0 - \frac{t}{2} + \mu(s_1)\Delta u + t(\frac{1}{2} - x);\\
U^2(x) &= u_0 - \frac{t}{2} + \mu(s_2)\Delta u + t(x - \frac{1}{2}).
\end{aligned}$$

Hence, it is clear that a consumer located at $x = 1/2$ loses nothing by choosing the aggregator; $U^{Agg}(1/2) \geq \max\left\{U^1(1/2), U^2(1/2)\right\}$. Consider now a consumer with location $x < \frac{1}{2}$. We have

$$U^{Agg}(x) - U^1(x) = \underbrace{(\mu(s_1 \cup s_2) - \mu(s_1))\,\triangle u}_{\text{Benefit from higher quality}} - \underbrace{t(\frac{1}{2} - x)\,(1 + \mu(s_2) - \mu(s_1))}_{\text{Cost from higher preference mismatch}}. \tag{5}$$

The benefit of using the aggregator instead of newspaper 1 is captured by the term $(\mu(s_1 \cup s_2) - \mu(s_1))\,\triangle u$, which means that the consumer consumes more high quality content. This benefit comes with the cost of more preference mismatch since, for a consumer with location $x < \frac{1}{2}$, the favorite newspaper is 1. More precisely, the last term in (5) has always a negative sign for $x < \frac{1}{2}$ and represents the cost of using the aggregator.

More generally, we have the following lemma which shows that newspapers are not directly in competition with each other.

Lemma 2. *Newspapers are not directly in competition with each other: For any given* (s_1, s_2)*, there exists no* $x \in [0,1]$ *such that* $\min\left\{U^1(x), U^2(x)\right\} > U^{Agg}(x)$.

Proof. To prove the lemma we consider two cases.

1) If $x < \frac{1}{2}$, then $U^{Agg}(x) > U^2(x)$ since $\mu(s_2) - \mu(s_1) < \frac{1}{2}$.

2) If $x > \frac{1}{2}$, then $U^{Agg}(x) > U^1(x)$ since $\mu(s_1) - \mu(s_2) < \frac{1}{2}$. □

Let x_i denote the location of the consumer who is indifferent between newspaper i ($i = 1, 2$) and the aggregator. Then, for any $x < x_1$, we have $U^1(x) > U^{Agg}(x)$. This, together with Lemma 2, implies $U^1(x) > U^2(x)$ for any $x < x_1$. Therefore, 1's market share is given by x_1. For similar reason, 2's market share is given by $1 - x_2$. Furthermore, $U^{Agg}(1/2) \geq \max\left\{U^1(1/2), U^2(1/2)\right\}$ means that $x_1 \leq 1/2 \leq x_2$. Therefore, in general, we have $x_1 \in [0, 1/2]$ and $x_2 \in [1/2, 1]$ and the aggregator's market share is $x_2 - x_1$. The next lemma shows that each newspaper has a positive market share under A1.

Lemma 3. *Under A1, for any given* (s_1, s_2) *satisfying* $\mu(s_i) \leq 1/2$ *for* $i = 1, 2$*, the market shares of* 1 *and* 2 *are*

$$0 < \alpha_1 = \frac{1}{2} - \beta \frac{\mu(s_2) - \mu(s_1 \cap s_2)}{1 - \mu(s_1) + \mu(s_2)} \leq \frac{1}{2}; \tag{6}$$

$$0 < \alpha_2 = \frac{1}{2} - \beta \frac{\mu(s_1) - \mu(s_1 \cap s_2)}{1 + \mu(s_1) - \mu(s_2)} \leq \frac{1}{2}. \tag{7}$$

One of the effects of the aggregator is to decrease the market share of the newspapers. In lemma 3, we have shown that for any given (s_1, s_2) satisfying $\mu(s_i) \leq 1/2$, the market share of a newspaper cannot be larger than $\frac{1}{2}$, whereas it is possible for a newspaper to have a market share larger than a half (although not in equilibrium) when there is no aggregator. This result holds even when the quality of newspaper 1, say, is the maximum possible, i.e. $\frac{1}{2}$, and the quality of 2 is zero because the consumers located at $x \in (1/2, 1]$ prefer the aggregator to newspaper 1. By using the aggregator, they consume all the high quality content from 1 whereas they can still consume low quality content from newspaper 2 half of the time.

The market share of each newspaper decreases in β, which means that the more loyal consumers are, the more market shares the newspapers have. Keeping $(\mu(s_1), \mu(s_2))$ constant, increasing $s_1 \cap s_2$ reduces high quality content available at the aggregator and thereby increases the market share of both newspapers. In the extreme case of $s_1 = s_2$, there is no room for the aggregator and each newspaper shares the whole market equally.

From Lemma 4, we can see the effect of the quality of $\mu(s_i)$ with $i = 1, 2$ on the market share of newspaper 1, say:

- α_1 increases, if i $(= 1, 2)$ increases the quality, $\mu(s_i)$, by investing on the issues which are covered by $j(\neq i)$ too, i.e. by increasing $\mu(s_1 \cap s_2)$.
- α_1 decreases, if i $(= 1, 2)$ increases the quality, $\mu(s_i)$, by investing on the issues which are not covered by $j(\neq i)$, i.e. by increasing $\mu(s_i - s_j)$.

For subsequent analysis, it is important to understand the above rather non-standard effects of changes in quality $\mu(s_i)$ on the market share of 1. This requires to examine how a quality change affects the choice of the marginal consumer who is indifferent between newspaper 1 and the aggregator. For instance, if newspaper 1 increases its quality by investing on the issues not covered by 2, this reduces 1's market share by strengthening the aggregator: the marginal consumer enjoys this quality increase regardless of whether she or he chooses newspaper 1 or the aggregator and in addition the quality increase reduces the preference mismatch from using the aggregator. In contrast, if newspaper 1 increases its quality by investing on the issues covered by 2, this increases 1's market share by weakening the aggregator.

4.2 Business-stealing vs. readership expansion for given qualities

Given (s_1, s_2), newspaper i's profit is given by:

$$\pi_i(s_i) = \alpha_i \left[1 + \mu(s_i)\delta\right] + \delta(1 - \alpha_i - \alpha_j) \left(\mu(s_i - s_j) + \frac{1}{2}\mu(s_i \cap s_j) \right) - c\mu(s_i)^2; \tag{8}$$

where $j \in \{1, 2\}$, $j \neq i$.

The following proposition states that there exists no equilibrium in which the common issues covered by 1 and 2, $s_1 \cap s_2$, is neither the maximum nor the minimum.

Proposition 2. *Given $\mu(s_i)$ satisfying $0 < \mu(s_i) \leq 1/2$ for newspaper $i \in \{1, 2\}$, choosing s_i such that $0 < \mu(s_1 \cap s_2) < \min(\mu(s_1), \mu(s_2))$ is strictly dominated by choosing s_i such that $\mu(s_1 \cap s_2) = 0$ or $\mu(s_1 \cap s_2) = \min(\mu(s_1), \mu(s_2))$. In other words, each newspaper is always better off to choose maximum or minimum differentiation.*

The proof of Proposition 2 shows that newspaper i's profit is convex with respect to $\mu(s_1 \cap s_2)$. So the profit is maximized at the corners. This result is so robust that it does not depend on A2. Indeed, it holds for any arbitrary cost function. From Lemma 4 and the discussion following the lemma, we know that the aggregator's market share is minimized under minimum differentiation and maximized under maximum differentiation. Hence, Proposition 2 implies that newspaper i finds it optimal either to "accommodate" the aggregator by maximum differentiation or to "fight" it by minimum differentiation.

Consider a given symmetric quality $\mu(s_1) = \mu(s_2) = \mu \in (0, 1/2) \leq 1/2$. Then, if newspaper i uses the minimum differentiation strategy, the aggregator gets zero market share and hence each newspaper's profit is not affected by the presence of the aggregator. If i uses instead the maximum differentiation strategy, each newspaper has the same market share ($\alpha_1 = \alpha_2 = \alpha = 1/2 - \beta\mu$) and obtains the same profit equal to is $\alpha\left[1 + \mu\delta\right] + \delta(1 - 2\alpha)\mu$. Therefore, the difference between a newspaper's profit under maximum differentiation and its profit under minimum differentiation (i.e. which is equal to the profit without the aggregator) is given by:

$$\pi_i(\mu, \mu \mid \max) - \pi_i(\mu, \mu)|_{\text{no aggregator}} = \underbrace{-\beta\mu * 1}_{\text{Business-stealing effect}} + \underbrace{\beta\mu * \delta\mu}_{\text{readership expansion effect}} = \beta\mu\left(\delta\mu - 1\right). \tag{9}$$

The first term in the R.H.S. of the above equation shows **the business-stealing effect** of the aggregator; the aggregator reduces the total attention spent on the homepages of each newspaper by $\beta\mu*1$. The second term in the R.H.S. of the above equation shows **the readership expansion effect** of the aggregator. Namely, the aggregator improves the match between attention and high quality content and thereby allows each newspaper i's high quality content to reach more customers which include some customers who would read only the rival newspaper j without

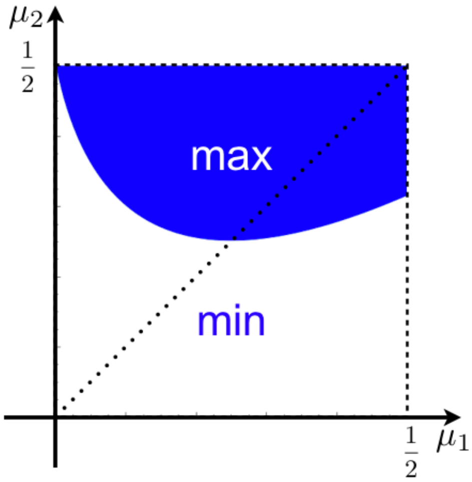


Figure 4: Best reply function of newspaper 1 given μ_1, and μ_2

the aggregator. $\beta\mu * \delta\mu$ measures this increase in attention from the readership expansion effect. From the previous discussion, we have:

Lemma 4. *Consider any symmetric equilibrium candidate $\mu(s_1) = \mu(s_2) = \mu$ satisfying $0 < \mu \leq 1/2$. Then, in the candidate, the newspapers use the maximum differentiation strategy (respectively, the minimum differentiation strategy) if $\delta\mu > 1$ (respectively, if $\delta\mu < 1$).*

Although we considered here symmetric quality, this trade-off between the business-stealing effect and the readership expansion effect is quite general. All other things being equal, as μ_j increases, the aggregator has a larger market share under maximal differentiation and hence the readership expansion effect is more likely to dominate the business-stealing effect from i's point of view. As δ increases, the profit from high quality content is more important relative to the profit from low quality content, which also makes the readership expansion effect more likely to dominate the business-stealing effect. More generally, Figure 4 describes, given $(\mu_1, \mu_2) \in (0, 1/2]^2$, when minimum differentiation (respectively, maximum differentiation) is optimal for newspaper 1.

Remark: The previous discussion shows that the presence of the aggregator can never decrease each newspaper's profit for given symmetric quality since each newspaper can reduce the aggregator's market share to zero by using the minimum differentiation strategy and thereby obtain the profit in the absence of the aggregator. However, this is a consequence of the fact that we consider only two newspapers. On the contrary, if there are many newspapers and some of them use maximum differentiation, a single newspaper cannot reduce the market share of the aggregator to zero. Then, it is possible for the business-stealing effect to dominate the readership

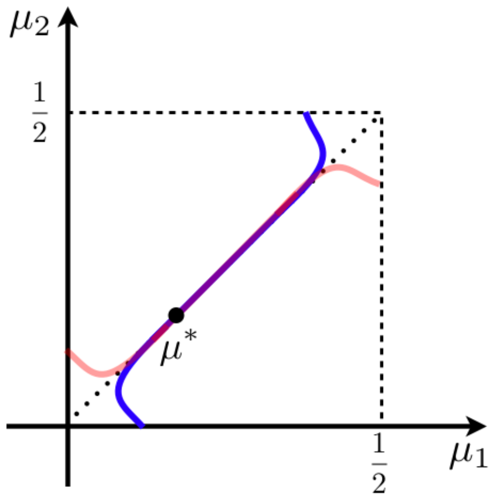


Figure 5: Best reply function of newspaper 1 given min differentiation

expansion effect regardless of whether a given newspaper adopts the minimum differentiation or the maximum differentiation strategy. After completely characterizing the outcomes for two newspapers, we make an extension to the case in which the aggregator provides content from a third-party different from the two newspapers (see Section 7).

As a consequence of Proposition 2, there are two equilibrium candidates, one with minimum differentiation and the other with maximum differentiation. We go through them in the two next subsections.

4.3 Minimum differentiation (no specialization) equilibrium

In this section, we study the existence of the equilibrium in which the newspapers choose the minimum differentiation, or equivalently $s_1 = s_2$. Let (μ_1^m, μ_2^m) denote the equilibrium qualities under the minimum differentiation strategy. We have:

Proposition 3. *Under A1-A3, there are* $\left(\underline{\delta}^m, \bar{\delta}^m\right)$ *satisfying* $0 < \underline{\delta}^m \leq \bar{\delta}^m$ *such that for any* $\delta > \bar{\delta}^m$ *there exists no symmetric equilibrium in which newspapers invest on the same set of issues: for any* $\delta \leq \underline{\delta}^m$ *there exist multiple symmetric equilibria in which newspapers invest on the same set of issues:*

i) $\mu_1^m = \mu_2^m = \mu^m \in \left[\frac{\delta}{4c-\delta\beta}, \frac{1}{2}\right]$ *if* $\frac{\delta}{2} + \frac{\delta\beta}{4} + \frac{\beta}{2} < c \leq \frac{\delta}{2} + \frac{\delta\beta}{4} + \beta$;

ii) $\mu_1^m = \mu_2^m = \mu^m \in \left[\frac{\delta}{4c-\delta\beta}, \frac{\delta+2\beta}{4c-\delta\beta}\right]$ *if* $c > \frac{\delta}{2} + \frac{\delta\beta}{4} + \beta$.

Corollary 1. $\mu^m = \mu^*$ *(the quality in the equilibrium without the aggregator) is one of the minimum differentiation equilibria.*

The intuition behind this result is simple. If the revenue from high quality content is high enough, each newspaper has an incentive to use the maximum differentiation strategy since the readership expansion effect dominates the business-stealing effect. On the contrary, when the revenue from high quality content is low enough, the business-stealing effect dominates the readership expansion effect and each newspaper uses the minimum differentiation strategy. Since any equilibrium quality μ_1^m is a best response to μ_2^m, for the interval of equilibrium qualities described in Proposition 3, the best reply curve has a slope of 45 degree (see also Figure 5). Hence, quality choices are strategic complements for this interval. The reason is that given $\mu(s_2) = \mu_2^m$, newspaper 1 finds it optimal to "fight" against the aggregator by choosing $s_1 = s_2$, which leaves zero market share to the aggregator. More precisely, conditional on using the minimum differentiation strategy, newspaper 1's profit increases when μ_1 increases to μ_2^m and decreases when when μ_1 increases from μ_2^m. Figure 5 also shows that the equilibrium quality without the aggregator μ^* belongs to the interval of equilibrium quality under the minimum differentiation strategy.

4.4 Maximum differentiation (specialization) equilibrium

In this section, we study the equilibrium candidate with maximum differentiation. The profit of newspaper $i \in \{1, 2\}$ conditional on maximum differentiation is given by:

$$\pi_i(s_i \mid \max) = \frac{1}{2} + \frac{\delta}{2}\mu_i - \beta\frac{\mu_j}{1+\mu_j-\mu_i} + \delta\beta\frac{\mu_i^2}{1+\mu_i-\mu_j} - c\mu_i^2.$$

Let (μ_1^M, μ_2^M) denote the equilibrium qualities under the maximum differentiation strategy. Figure 6(a) shows the best reply conditional on that both newspapers use the maximum differentiation strategy. It shows that the curve crosses the 45 degree line only once and has a positive slope when (and after) the curve crosses it. More precisely, we have

$$\frac{\partial \pi_i}{\partial \mu_i \partial \mu_j} = -\beta\frac{1-\mu_i-\mu_j}{(1-\mu_i+\mu_j)^2} + 2\delta\beta\frac{\mu_i(1-\mu_j)}{(1+\mu_i-\mu_j)^3}$$

which is positive for $\delta\mu_i \geq 1/2$. Since $\delta\mu^M > 1$ holds from Lemma 4, quality choices are strategic complements for quality above μ^M and for quality just below (and close to) μ^M. Therefore, we have:

Lemma 5. *In the presence of the aggregator, suppose that a symmetric equilibrium with maximum differentiation strategy $\mu_1^M = \mu_2^M = \mu^M$ exists. Then, there exists some μ' satisfying $\mu' < \mu^M$ such that conditional on that newspaper i uses the maximum differentiation strategy, an increase in μ_j induces an increase in μ_i for any $\mu_j \geq \mu'$: newspapers' quality choices (μ_1, μ_2)*

are strategic complements for $\mu_j \geq \mu'$.

When newspaper 1 uses the maximum differentiation strategy, an increase in μ_2 expands the market share of the aggregator and hence increases the readership expansion effect. This increased readership expansion effect in turn increases the marginal revenue from an increase in μ_1, which makes quality choices strategic complements. Figure 6(b) shows that this property holds true even when a newspaper is not restricted to the maximum differentiation strategy since it is optimal for i to use this strategy for μ_j larger than a threshold (see Figure 4).

We have:

Proposition 4. *Under A1-A3, there exists a threshold* $\bar{\delta}^M > 0$ *such that for any* $\delta \geq \bar{\delta}^M$ *there is a unique symmetric equilibrium,* $\mu_1^M = \mu_2^M = \mu^M$*, in which newspapers invest in disjoint sets of issues;* μ^M *is*

$$\begin{array}{ll} i)\frac{1}{2} & if \quad c \leq \frac{\delta}{2} - \frac{\beta}{2} + \frac{3}{4}\delta\beta; \\ ii)\frac{(-\beta+2\delta\beta-2c)+\sqrt{(-\beta+2\delta\beta-2c)^2+2\delta^2\beta}}{2\delta\beta} & if \quad c > \frac{\delta}{2} - \frac{\beta}{2} + \frac{3}{4}\delta\beta \end{array}.$$

Moreover, there exist another threshold $\underline{\delta}^M \in \left(0, \bar{\delta}^M\right)$ *such that for any* $\delta < \underline{\delta}^M$*, there exist no equilibrium with maximum differentiation.*

Note that from Lemma 4, a necessary condition to have an equilibrium with maximum differentiation is $\delta > 2$. One can check that μ^M is increasing in δ: as the revenue from high quality content increases, the newspapers have more incentive to invest in quality. Moreover, if consumers are less loyal (i.e. as β increases), the competition becomes tougher and hence the newspapers invest more in quality. Moreover, one can check that $\lim_{\beta\to 0}\mu^M = \frac{\delta}{4c} = \lim_{\beta\to 0}\mu^*$, where $\frac{\delta}{4c}$ is the monopoly quality. It means if consumers are too much loyal, the presence of aggregator has no effect on the quality.

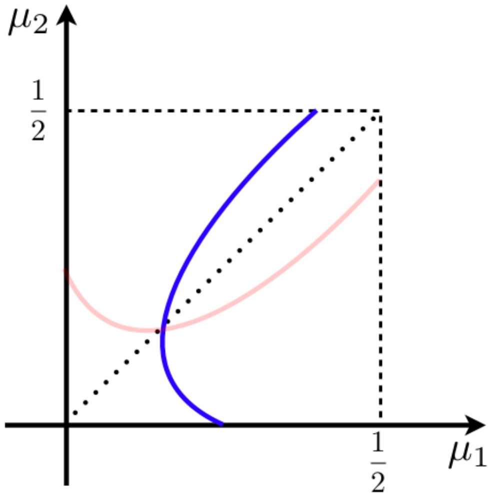
5 Opting out possibility

In this section, we analyse the following two-stage game.

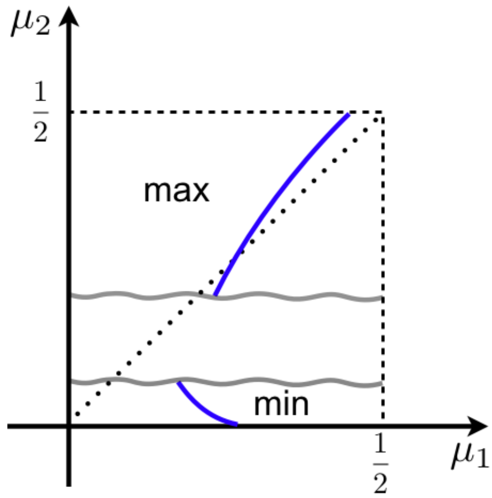
- Stage 1: each newspaper i simultaneously decides whether to opt out or not and chooses s_i.
- Stage 2: each consumer chooses one among the two newspapers and the aggregator.

Note that if newspaper i opts out, the aggregator has content only from j and in this case we break the tie by assuming that consumers prefer using newspaper j to the aggregator.[17] Then,

[17] This tie-breaking makese sense since the navigation between the aggregator's site and the newspaper j's site is less seamless than the navigation within the newspaper j's site.



(a) Given max differentiation



(b) Actual best reply functions of Newspaper 1 given $\delta > \bar{\delta}^M$

Figure 6: Best reply functions

we always have an equilibrium in which all newspapers opt out. In this opting out equilibrium, each newspaper chooses the quality μ^*. This equilibrium trivially exists regardless of the number of competing newspapers.

We now check how opting out possibility affects the equilibria under minimum differentiation. Given $\mu(s_j) = \mu^m$, does the opting out possibility induce newspaper i to deviate from choosing $s_i = s_j$? The answer is yes for any μ^m is different from μ^*. Note first that in the minimum differentiation equilibrium candidate, each newspaper gets the profit it obtains in the absence of the aggregator from symmetric quality μ^m. Therefore, as long as μ^m is different from $BR_i^N(\mu^m)$, i.e. newspaper i's best response to $\mu(s_j) = \mu^m$ in the absence of the aggregator, newspaper i has an incentive to opt out and to choose $BR_i^N(\mu^m)$. Since we have a unique equilibrium without the aggregator, $\mu^m = BR_i^N(\mu^m)$ holds if and only if $\mu^m = \mu^*$. This implies that only $\mu^m = \mu^*$ survives the opting out possibility.

In the case of the maximum differentiation equilibrium, things are different since for given $\mu(s_1) = \mu(s_2) = \mu^M$, from (9), each firm gets a higher profit in the equilibrium than in the absence of the aggregator. If i opts out for given $\mu(s_j) = \mu^M$ satisfying $\delta\mu^M > 1$, its best response is $BR_i^N(\mu^M)$. It is possible that this deviation profit is lower than the equilibrium profit. To see this, note that in the absence of the aggregator, an increase in μ_j reduces the marginal profit of i and that $\mu^M > \mu^*$ (see Proposition 6).

Therefore, introducing opting out possibility leads to a sharp prediction: the presence of the aggregator either leads to no change or to the specialization equilibrium. Summarising, we have:

Proposition 5. *When newspapers can opt out,*
(i) There always exists an equilibrium in which every newspaper opts out and chooses the equilibrium quality without the aggregator ($\mu_1 = \mu_2 = \mu^*$).
(ii) Only the equilibrium quality without the aggregator survives opting out possibility among all equilibria with minimum differentiation.
(iii) The maximum differentiation equilibrium survives opting out possibility if the deviation to "opting out and choosing $\mu_i = BR_i^N(\mu^M)$" is not profitable.

6 Comparison: quality, consumer surplus, profit and welfare

In this section, we study how the aggregator affects quality, consumer surplus, profit and welfare. From Proposition 5, we compare the equilibrium without the aggregator with the specialization equilibrium.

The next proposition reports the effect of the news aggregator on the quality of newspapers:

Proposition 6. *Under A1-A3, the quality of newspapers is higher in the maximum differentiation equilibrium than in the equilibrium without the aggregator, i.e. $\mu^M > \mu^*$.*

Note that the existence of the maximum differentiation equilibrium requires δ large enough (i.e. $\delta\mu^M > 1$). In the presence of the aggregator, for δ large enough, $\mu_1 = \mu_2 = \mu^*$ is not an equilibrium. Then, the readership expansion effect dominates the business-stealing effect and hence each newspaper finds it optimal to respond by increasing quality above μ^* and using maximum differentiation. Furthermore, quality choices are strategic complements. Therefore, they end up choosing $\mu_1 = \mu_2 = \mu^M > \mu^*$.

We now study how the aggregator affects consumer surplus and the profits of newspapers. The consumer surplus and the profit of each newspaper when there is no aggregator are given by:

$$CS^* = \int_0^{\frac{1}{2}} (\mu^* \triangle u + u_0 - xt)\, dx + \int_{\frac{1}{2}}^{1} (\mu^* \triangle u + u_0 - (1-x)t)\, dx = \mu^* \triangle u + u_0 - \frac{t}{4}; \tag{10}$$

$$\pi^* = -c\mu^{*2} + \frac{\delta}{2}\mu^* + \frac{1}{2}. \tag{11}$$

Since the aggregator induces each newspaper to choose a higher quality, this increases every consumer's surplus. Even if a consumer continues to use her preferred newspaper, she benefits from quality increase. In addition, she has the option of using the aggregator.

The profit of each newspaper in the specialization equilibrium is $\pi^M = \alpha^M[1 + \delta\mu^M] + (1 - 2\alpha^M)\delta\mu^M - c{\mu^M}^2$, where α^M is the share of each newspaper and is equal to $\frac{1}{2} - \beta\mu^M$ due to (6) and (7). Thus, the profit is

$$\pi^M = {\mu^M}^2(\delta\beta - c) + \mu^M(-\beta + \frac{\delta}{2}) + \frac{1}{2}. \tag{12}$$

The presence of the aggregator increases each newspaper's profit if and only if

$${\mu^M}^2(\delta\beta - c) + \mu^M(-\beta + \frac{\delta}{2}) + \frac{1}{2} \geq \frac{1}{2} + \frac{\delta}{2}\mu^* - c{\mu^*}^2;$$

equivalently

$${\mu^M}^2(\delta\beta - c) + \mu^M\left(\frac{\delta}{2} - \beta\right) - \frac{\delta}{2}\mu^* + c{\mu^*}^2 \geq 0. \tag{13}$$

We have:

Proposition 7. *Suppose that the presence of the aggregator leads to the maximum differentiation equilibrium. Then:*
i) Every consumer gets a higher surplus;
ii)The profits of newspapers increases if the cost is low enough, and decreases otherwise. More precisely, there exists $\hat{c}$ such that $\pi^M < \pi^*$ for all $c > \hat{c}$ and $\pi^M > \pi^*$ for all $c < \hat{c}$.
iii) Social welfare is higher.

The profits of the newspapers can be lower in the specialization equilibrium than in the equilibrium without the aggregator. More generally, Proposition 7 shows that whether the profits increase or decrease depends on the level of cost c. As we noted in Section 4.2, for given quality, the aggregator cannot decrease each newspaper's profit. Furthermore, from (9), the profit in the maximum differentiation equilibrium (gross of the investment cost) strictly increases with μ^M. This implies that the aggregator increases each newspaper's profit if the investment cost does not increase much (i.e. if c is low enough).

Actually, the relevant cost is c/ϖ where ϖ is advertising revenue per unit of attention, which was normalised at one. If the Internet creates advertising congestion (Anderson, Foros, Kind, and Peitz, 2012) by expanding massively advertising possibilities and thereby reduces ϖ, this increases c/ϖ, suggesting that the presence of the aggregator would decrease profits of newspapers. This may explain why the current debate on news aggregators is so heated.

Finally, we show that the presence of the aggregator increases social welfare. We proceed in two steps. First, for given symmetric quality (for instance, μ^*), the presence of the aggregator increases social welfare. This is because consumer surplus increases and the total traffics to the newspapers and the aggregator increase. The latter increases since the total traffics to the homepages are constant whereas the traffics to high quality articles increase thanks to the aggregator. Second, we can show that in the presence of the aggregator, the newspapers choose too low quality from a social point of view, which implies that the increase in quality from μ^* to μ^M (in the presence of the aggregator) is welfare-improving. To see this, consider a marginal change in μ_1 for any given μ_2. We have

$$\frac{\partial SW}{\partial \mu_1} = \frac{\partial \pi_1}{\partial \mu_1} + \frac{\partial \pi_2}{\partial \mu_1} + \frac{\partial \pi_A}{\partial \mu_1} + \frac{\partial CS}{\partial \mu_1}$$

where π_A is the profit of the aggregator. $\frac{\partial CS}{\partial \mu_1} > 0$ is obvious and we can show

$$\frac{\partial \pi_2}{\partial \mu_1} + \frac{\partial \pi_A}{\partial \mu_1} > 0.$$

From Lemma 4, as μ_1 increases, newspaper 1's market share decreases under maximum differentiation. This implies that as μ_1 increases, the total traffics to the homepages of newspaper 2 and the aggregator increases. Furthermore, it also implies that the traffics to the high quality articles of 2 increase. Therefore, an increase in μ_1 generates positive externalities on the joint profit of newspaper 2 and the aggregator and on consumers. Hence, if $\frac{\partial \pi_1}{\partial \mu_1} = 0$, then $\frac{\partial SW}{\partial \mu_1} > 0$.

7 Extension: content from third-party providers

We believe what is happening in the online world can be represented by the specialization equilibrium. However, one may argue that the model does not reflect the real world since each newspaper has a huge market power such that it can unilaterally eliminate the aggregator by opting out. In the real world, each newspaper has very little effect on the aggregator since the aggregator contains content from many news outlets.[18] In particular, there are many small news sites which would receive very negligible traffics in the absence of the aggregator. Therefore, these sites have strong incentives to use "the maximum differentiation and opt-in strategy" in order to attract traffics from the aggregator. In order to capture this heterogeneity among news sites in our model and to show the robustness of our main results, we introduce one important modification into our model: by using the aggregator, consumers can get utility u_T generated by the aggregation of the content from numerous small third party providers.[19] Therefore, even if

[18] In case of Google News, there are about 25000 news outlets.

[19] Although u_T can depend on a consumer's ideological taste, we abstract from this dimension for simplicity.

the two newspapers opt out, a consumer can get a utility equal to u_T from using the aggregator. However, in the absence of the aggregator, it is impossible for a consumer to obtain u_T from numerous small third party providers. This implies that the introduction of u_T does not affect the analysis of the case without aggregator.

In the presence of the aggregator, the utility that a consumer located at x obtains from the aggregator is given by:

$$U^{Agg}(x) = \mu(s_1 \cup s_2)\Delta u + u_0 - tx\mu(s_1) - t(1-x)\mu(s_2) - \frac{t}{2}(1 - \mu(s_1) - \mu(s_2)) + u_T. \quad (14)$$

The market shares of the newspapers are given as follows:

$$\alpha_1 = \frac{1}{2} - \frac{1}{t}\frac{(\mu(s_2) - \mu(s_1 \cap s_2))\,\Delta u + u_T}{1 - \mu(s_1) + \mu(s_2)}; \quad (15)$$

$$\alpha_2 = \frac{1}{2} - \frac{1}{t}\frac{(\mu(s_1) - \mu(s_1 \cap s_2))\,\Delta u + u_T}{1 - \mu(s_2) + \mu(s_1)}. \quad (16)$$

We focus on the case in which the utility form third party content is important enough:
A4: $u_T \geq \frac{\Delta u}{2}\max\{1, \frac{3}{\delta}\}$.

To avoid corner solutions under A4 (i.e. to guarantee a positive market share for each newspaper), we should modify A1 as follows.

A1': $4u_T < t$

A1's puts an upper bound on u_T. Hence, under A1' and A4, depending on the parameter values, the equilibrium market share of the aggregator can vary from (close to) zero to (close to) one. We have:

Proposition 8. *Suppose that the utility from third party content is high enough (i.e. A4 holds). Under A1', A2, A3;*
i) For any $(\mu(s_1), \mu(s_2)) \in [0, 1/2]^2$, the maximum differentiation, $\mu(s_1 \cap s_2) = 0$, is a dominant strategy for each newspaper.
ii) For any $\delta > 0$, newspapers' quality choices $(\mu(s_1), \mu(s_2))$ are strategic complements.
iii) For any $\delta > 0$, there is a unique symmetric equilibrium, $\mu(s_1) = \mu(s_2) = \mu^T$, where newspapers invest on disjoint set of issues, $\mu(s_1 \cap s_2) = 0$. There are two thresholds of δ such that $\mu^T = 0$ for all $\delta \leq \underline{\delta}^T$ and $\mu^T = \frac{1}{2}$ for all $\delta \geq \bar{\delta}^T (> \underline{\delta}^T)$. For $\delta \in \left[\underline{\delta}^T, \bar{\delta}^T\right]$, μ^T strictly increases with δ.

When the utility from third party content is high enough, the aggregator already has a non-negligible market share independently of what a single newspaper does. Therefore, this induces each newspaper to accommodate the aggregator by adopting the maximum differentiation strategy such that the minimum differentiation equilibrium does not exist whereas the maximum differentiation equilibrium exists for all $\delta > 0$. For the same reasons, newspapers' quality choices $(\mu(s_1), \mu(s_2))$ are strategic complements for all $\delta > 0$. Furthermore, the equilibrium quality is increasing in δ such that the presence of the aggregator can increase or decrease the quality with respect to the case without the aggregator. If δ is small (respectively, large), the business-stealing effect is large (respectively, small) relative to the readership expansion effect.

Therefore, the value of δ is the key parameter determining whether the presence of the aggregator increases or reduces quality of newspapers. Even if there has not been any empirical study directly estimating δ, we think that the studies by Athey and Mobius (2012) and Chiou and Tucker (2012) allow us to pin down a lower bound of δ. For instance, Athey and Mobius (2012) study a natural experiment in which Google News introduces news from local outlets for readers who entered their zip code. They find that after adding content from new local outlets to Google News, traffics increases not only to these new outlets but also to the old (local and non-local) ones that have been indexed by Google News. Chiou and Tucker (2012) exploit a contract dispute which led Google News to remove the content from Associated Press (AP). They show that the presence of the AP content on Google News would have increased traffics to the news sites indexed by Google News, which are not necessarily members of the AP network. Therefore, we can infer from these papers that an increase in the third party content u_T would increase traffics to the two newspapers for given equilibrium quality of the newspapers, implying

$$\frac{\partial \pi^T}{\partial u_T} \mid_{\mu^T=cst} > 0 \Leftrightarrow \delta\mu^T > 1$$

where π^T is the profit of each newspaper and is given by:

$$\pi^T = \frac{1}{2} + \frac{\delta}{2}\mu^T + \frac{1}{t}(\mu^T \Delta u + u_T)(\delta\mu^T - 1) - c{\mu^T}^2$$

This means that the readership expansion effect is larger than business-stealing effect at the equilibrium:

$$\begin{aligned} \pi_i^T(\mu, \mu \mid \max) - \pi_i(\mu, \mu)|_{\text{no aggregator}} &= \underbrace{-(\beta\mu + \frac{u_T}{t}) * 1}_{\text{Business-stealing effect}} + \underbrace{(\beta\mu + \frac{u_T}{t}) * \delta\mu}_{\text{Readership expansion effect}} \\ &= (\beta\mu + \frac{u_T}{t})\,(\delta\mu - 1) > 0, \end{aligned}$$

where $\beta\mu + \frac{u_T}{t}$ is the market share loss of each newspaper to the aggregator. For $\delta\mu^T > 1$, we can show that the presence of the aggregator increases the quality and that each newspaper has no incentive to opt out in the equilibrium.

Proposition 9. *Under A1', A2-A4, if* $\delta\mu^T > 1$:
i) the presence of the aggregator increases the quality of newspapers, $\mu^T \geq \mu^*$*;*
ii) when the aggregator is present, each newspaper has no incentive to opt out;
iii) the presence of the aggregator increases consumer surplus and social welfare.

The result that each newspaper has no incentive to opt out is proved in the Appendix. To obtain the result on consumer surplus and social welfare, we can apply the same logic that is used in Section 6.

8 Conclusion

In this paper, we studied the impact of news aggregator on the quality choices of newspapers by considering two scenarios: symmetric newspapers and asymmetric newspapers. Regardless of the scenarios, we find that the presence of the news aggregator induces each newspaper to get specialized in order to increase the traffics from the aggregator and this in turn changes the strategic interactions of quality choices from strategic substitutes to strategic complements. In addition, when newspapers are symmetric, the presence of the aggregator induces them to choose higher quality, which increases consumer surplus and social welfare. When newspapers are asymmetric such that small newspapers prefer their content indexed by the aggregator, its presence can increase or decrease the quality chosen by large newspapers depending on how sensitively time spent on news sites responds to quality increase. However, if an increase in the content indexed by the aggregator increases the traffics to the newspapers that have been indexed by the aggregator (as in the empirical findings of Athey and Mobius (2012) and Chiou and Tucker (2012)), then its presence increases the quality (and thereby consumer surplus and welfare).

Our simple model providing a microfoundation for news aggregators can be regarded as a first step and can be enriched for future research. For instance, we find that the impact of the news aggregator on the profits of newspapers is ambiguous. Actually, if we take into account advertising congestion (Anderson, Foros, Kind, and Peitz, 2012), the presence of the aggregator is likely to reduce their profits, which worsens their financial shapes. Therefore, it would be interesting to study alternative business models for newspapers such as strengthening IP protection of content, versioning,[20] Google tax etc. One can also study the impact of the

[20] Calzada and Ordóñez (2012) study a newspaper's reaction to the aggregator in terms of versioning in the framework of a monopolist's second-degree price discrimination.

aggregators on news slanting by making each newspaper's position endogenous as in Mullainathan and Shleifer (2005).

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Appendix A

8.1 Proof Lemma 3

Proof. We prove it for newspaper 1. $U^1(x_1) = U^{Agg}(x_1)$ is equivalent to

$$x_1 = \frac{1}{2} - \beta\frac{\mu(s_1 \cup s_2) - \mu(s_1)}{1 - \mu(s_1) + \mu(s_2)}.$$

Using $\mu(s_1 \cup s_2) = \mu(s_1) + \mu(s_2) - \mu(s_1 \cap s_2)$, we get

$$x_1 = \frac{1}{2} - \beta\frac{\mu(s_2) - \mu(s_1 \cap s_2)}{1 - \mu(s_1) + \mu(s_2)}.$$

We now show $0 < x_1 \leq 1/2$, which is equivalent to

$$\frac{1}{2} > \beta\frac{\mu(s_2) - \mu(s_1 \cap s_2)}{1 - \mu(s_1) + \mu(s_2)} \geq 0.$$

The second inequality is obvious. The first comes from

$$\beta\frac{\mu(s_2) - \mu(s_1 \cap s_2)}{1 - \mu(s_1) + \mu(s_2)} < \frac{\mu(s_2)}{1 - \mu(s_1) + \mu(s_2)} \leq \frac{\mu(s_2)}{1/2 + \mu(s_2)} < \frac{1}{2}.$$

□

8.2 Proof Proposition 1

Proof. There are four equilibrium candidates.

i) $(\frac{1}{2}, \frac{1}{2})$: This is an equilibrium if and only if $\frac{1}{2} \leq 1 - \frac{2c-\delta-\beta}{\delta\beta}$, or equivalently $c \leq \frac{\delta\beta}{4} + \frac{\delta}{2} + \frac{\beta}{2}$.

ii & iii) $\left(\frac{1}{2}, \frac{\delta+\beta-\frac{\delta\beta}{2}}{4c-2\delta\beta}\right)$ and $\left(\frac{\delta+\beta-\frac{\delta\beta}{2}}{4c-2\delta\beta}, \frac{1}{2}\right)$: To have one of them as an equilibrium we should have $\frac{\delta+\beta-\frac{\delta\beta}{2}}{4c-2\delta\beta} \leq 1 - \frac{2c-\delta-\beta}{\delta\beta}$, and $1 - \frac{2c-\delta-\beta}{\delta\beta} < \frac{1}{2}$. By rearranging the inequalities, one get $-8\left(c - \frac{\delta}{2} - \frac{\beta}{2} - \frac{\delta\beta}{4}\right)\left(c - \frac{3}{4}\delta\beta\right) \geq 0$, and $c > \frac{\delta}{2} + \frac{\beta}{2} + \frac{\delta\beta}{4}$ which are totally inconsistent.

iv) $\left(\frac{\delta+\beta}{4c-\delta\beta}, \frac{\delta+\beta}{4c-\delta\beta}\right)$: This is an equilibrium if and only if $\frac{1}{2} > \frac{\delta+\beta}{4c-\delta\beta} > 1 - \frac{2c-\delta-\beta}{\delta\beta}$, or equivalently $c > \frac{\delta\beta}{4} + \frac{\delta}{2} + \frac{\beta}{2}$.

□

8.3 Proof Proposition 2

Proof. We prove the proposition for $i = 1$; for $i = 2$ is the same. To prove the result, we decompose the profit of the newspaper 1, (8), using (6), (7), $\mu(s_1 \cup s_2) = \mu(s_1) + \mu(s_2) - \mu(s_1 \cap s_2)$,

$\mu(s_1 - s_2) = \mu(s_1) - \mu(s_1 \cap s_2)$ and $\mu(s_2 - s_1) = \mu(s_2) - \mu(s_1 \cap s_2)$. So we get

$$\begin{aligned} \pi_1(s_1) &= \delta\alpha_1\mu(s_1) + \alpha_1 + \delta(1-\alpha_1-\alpha_2)\left(\mu(s_1) - \frac{1}{2}\mu(s_1 \cap s_2)\right) - c\mu(s_1)^2 \\ &= h(\mu(s_1), \mu(s_2)) + \frac{\delta\beta\mu(s_1 \cap s_2)}{1-(\mu(s_1)-\mu(s_2))^2}\left[\mu(s_1 \cap s_2) - g(\mu(s_1), \mu(s_2))\right] \qquad (17) \end{aligned}$$

, where

$$h(\mu(s_1), \mu(s_2)) = \frac{1}{2} + \frac{\delta}{2}\mu(s_1) - \beta\frac{\mu(s_2)}{1+\mu(s_2)-\mu(s_1)} + \delta\beta\frac{\mu(s_1)^2}{1+\mu(s_1)-\mu(s_2)} - c\mu(s_1)^2 \qquad (18)$$

$$g(\mu(s_1), \mu(s_2)) = -\frac{3}{2}\mu(s_1)^2 + \mu(s_1)\left(2\mu(s_2) - \frac{1}{\delta} + \frac{3}{2}\right) + (1-\mu(s_2))\left(\frac{1}{2}\mu(s_2) - \frac{1}{\delta}\right) \qquad (19)$$

There are two cases:

1)$\min(\mu(s_1), \mu(s_2)) < g(\mu(s_1), \mu(s_2))$: In this case, $\mu(s_1 \cap s_2) < g(\mu(s_1), \mu(s_2))$. Therefore, the second term of (17) is negative, if 1 chooses $0 < \mu(s_1 \cap s_2)$. So any s_1 and s_2 such that $0 < \mu(s_1 \cap s_2)$ is strictly dominated by $\mu(s_1 \cap s_2) = 0$. It worths to mention that it is always possible for 1 to choose s_1 such that $\mu(s_1 \cap s_2) = 0$, thanks to A1.

2)$\min(\mu(s_1), \mu(s_2)) \geq g(\mu(s_1), \mu(s_2))$: 1 is always better off to choose $\mu(s_1 \cap s_2) = \min(\mu(s_1), \mu(s_2))$ rather than $\mu(s_1 \cap s_2) < \min(\mu(s_1), \mu(s_2))$.

In other words, the profit function of 1, (17), is convex with respect to $\mu(s_1 \cap s_2)$. So the maximum is achieved at the corners. □

8.4 Proof Proposition 3

Proof. We can rewrite (8) as

$$\pi_1(s_1 \mid \min) = \begin{cases} \frac{\delta}{2}\mu_1 + \frac{1}{2} + \delta\beta\frac{(\mu_1-\mu_2)(\mu_1-\frac{1}{2}\mu_2)}{1+\mu_1-\mu_2} - c\mu_1^2 & \mu_1 > \mu_2 \\ \frac{\delta}{2}\mu_1 + \frac{1}{2} - \frac{\delta\beta}{2}\frac{(\mu_2-\mu_1)\mu_1}{1+\mu_2-\mu_1} - \beta\frac{(\mu_2-\mu_1)}{1+\mu_2-\mu_1} - c\mu_1^2 & \mu_1 \leq \mu_2 \end{cases}$$

, where $\pi_1(s_1 \mid \min)$ is the profit of 1 given $\mu(s_1 \cap s_2) = \min(\mu(s_1), \mu(s_2))$, which is in fact the maximum intersection. And its first, second and third derivatives are

$$\pi_1'(s_1 \mid \min) = \begin{cases} \frac{\delta}{2} + \delta\beta\frac{(\mu_1-\mu_2)}{1+\mu_1-\mu_2} + \delta\beta\frac{(\mu_1-\frac{1}{2}\mu_2)}{(1+\mu_1-\mu_2)^2} - 2c\mu_1 & \mu_1 > \mu_2 \\ \frac{\delta}{2} - \frac{\delta\beta}{2}\frac{(\mu_2-\mu_1)}{1+\mu_2-\mu_1} + \frac{\delta\beta}{2}\frac{\mu_1}{(1+\mu_2-\mu_1)^2} + \frac{\beta}{(1+\mu_2-\mu_1)^2} - 2c\mu_1 & \mu_1 < \mu_2 \end{cases}$$

$$\pi_1''(s_1 \mid \min) = \begin{cases} \frac{\delta\beta}{(1+\mu_1-\mu_2)^2} + \delta\beta\frac{(1-\mu_1)}{(1+\mu_1-\mu_2)^3} - 2c & \mu_1 > \mu_2 \\ \frac{\delta\beta}{2(1+\mu_2-\mu_1)^2} + \frac{\delta\beta}{2}\frac{1+\mu_1+\mu_2}{(1+\mu_2-\mu_1)^3} + \frac{2\beta}{(1+\mu_2-\mu_1)^3} - 2c & \mu_1 < \mu_2 \end{cases}$$

$$\pi_1'''(s_1 \mid \min) = \begin{cases} -\delta\beta\left(\frac{2}{(1+\mu_1-\mu_2)^3} + \frac{4-2\mu_1-\mu_2)}{(1+\mu_1-\mu_2)^4}\right) & \mu_1 > \mu_2 \\ \frac{\delta\beta}{(1+\mu_2-\mu_1)^3} + \delta\beta\frac{2+\mu_1+2\mu_2}{(1+\mu_2-\mu_1)^4} + \frac{6\beta}{(1+\mu_2-\mu_1)^4} & \mu_1 < \mu_2 \end{cases}$$

We consider two cases:

i)$\frac{\delta}{2} + \frac{\beta}{2} + \frac{\delta\beta}{4} < c \leq \frac{\delta}{2} + \frac{\delta\beta}{4} + \beta$: Any equilibrium candidate,(μ_1, μ_2), can be seen in two sub-cases:

a)$\mu_1, \mu_2 < \frac{\delta}{4c-\delta\beta}$: In this case, always there is a deviation, and so there is not any equilibrium in this form. To show that, suppose $\mu_2 \leq \mu_1 < \frac{\delta}{4c-\delta\beta}$. We will show there is a deviation for 1.

$$\begin{aligned} \pi_1'(s_1 \mid \min)^+ &= \frac{\delta}{2} + \delta\beta\frac{(\mu_1-\mu_2)}{1+\mu_1-\mu_2} + \delta\beta\frac{(\mu_1-\frac{1}{2}\mu_2)}{(1+\mu_1-\mu_2)^2} - 2c\mu_1 \\ &> \frac{\delta}{2} + \delta\beta\frac{(\mu_1-\mu_2)}{1+\mu_1-\mu_2} + \delta\beta\frac{(\mu_1-\frac{1}{2}\mu_2)}{(1+\mu_1-\mu_2)^2} - \frac{\delta}{2} - \frac{\delta\beta}{2}\mu_1 \\ &= \frac{\delta\beta}{(1+\mu_1-\mu_2)^2}\left((\mu_1-\mu_2)(1+\mu_1-\mu_2) + (\mu_1-\frac{1}{2}\mu_2) - \frac{\mu_1}{2}(1+\mu_1-\mu_2)^2\right) \\ &= \frac{\delta\beta(\mu_1-\mu_2)}{(1+\mu_1-\mu_2)^2}\left(\frac{3}{2} - \mu_1 + (\mu_1-\mu_2)\left(1-\frac{\mu_1}{2}\right)\right) \geq 0 \end{aligned}$$

[21]

Thus, 1 benefits from investing more on quality.

b)$\exists j \in \{1,2\} \mid \mu_j \geq \frac{\delta}{4c-\delta\beta}$: We will show if $\mu_j \geq \frac{\delta}{4c-\delta\beta}$ the best response of the other newspaper, i, should be equal to the average quality of j, $\mu_j = \mu_i$. First, we show any $\mu_i > \mu_j$ is strictly dominated by $\mu_i = \mu_j$.

$$\begin{aligned} \pi_i(s_i \mid \min, \mu_i \geq \mu_j) &= \frac{\delta}{2}\mu_i + \frac{1}{2} + \delta\beta\frac{(\mu_i-\mu_j)\left(\mu_i-\frac{1}{2}\mu_j\right)}{1+\mu_i-\mu_j} - c\mu_i^2 \\ &= \frac{1}{2} + \frac{\delta}{2}\mu_j - c\mu_j^2 + (\mu_i-\mu_j)k(\mu_i,\mu_j) \end{aligned}$$

, where

[21] The first inequality is obtained from the fact that $\mu_1 < \frac{\delta}{4c-\delta\beta}$, and so $2c\mu_1 < \frac{\delta}{2} + \frac{\delta\beta}{2}\mu_1$

$$
\begin{aligned}
k(\mu_i,\mu_j) &= \frac{\delta}{2} + \delta\beta\frac{(\mu_i - \frac{1}{2}\mu_j)}{1+\mu_i-\mu_j} - c(\mu_i+\mu_j) \\
&= \frac{\delta}{2} + \delta\beta\frac{(\mu_i - \frac{1}{2}\mu_j)}{1+\mu_i-\mu_j} - c(\mu_i-\mu_j) - 2c\mu_j \\
&\leq \frac{\delta}{2} + \delta\beta\frac{(\mu_i - \frac{1}{2}\mu_j)}{1+\mu_i-\mu_j} - c(\mu_i-\mu_j) - \frac{\delta}{2} - \frac{\delta\beta}{2}\mu_j \\
&= (\mu_i-\mu_j)\left(\delta\beta\frac{(1-\frac{1}{2}\mu_j)}{1+\mu_i-\mu_j} - c\right) \\
&\leq (\mu_i-\mu_j)\left(\delta\beta(1-\frac{1}{2}\mu_j) - c\right) \\
&\leq (\mu_i-\mu_j)\left(\delta\beta(1-\frac{1}{2}\frac{\delta}{4c-\delta\beta}) - c\right) \\
&\leq \frac{(\mu_i-\mu_j)}{4c-\delta\beta}\left(-4c^2 + 5\delta\beta c - \delta\beta\left(\delta\beta + \frac{\delta}{2}\right)\right) \\
&\leq \frac{(\mu_i-\mu_j)}{4c-\delta\beta}\left(-4c^2 + 5\delta\beta c - \delta\beta\left(\delta\beta + \frac{\delta\beta}{2}\right)\right) \\
&= \frac{(\mu_i-\mu_j)}{4c-\delta\beta}(-4)\left(c - \frac{3}{4}\delta\beta\right)\left(c - \frac{\delta\beta}{2}\right) < 0
\end{aligned}
$$

Therefore, this part of the proof completes since $\pi_i\left(s_i \mid \min, \mu_i > \mu_j\right) < \pi_i\left(s_i \mid \min, \mu_i = \mu_j\right)$. Now, we will proof that any $\mu_i < \mu_j$ is also strictly dominated by $\mu_i = \mu_j$.

$$
\begin{aligned}
\pi_i\left(s_i \mid \min, \mu_i \leq \mu_j\right) &= \frac{\delta}{2}\mu_i + \frac{1}{2} - \frac{\delta\beta}{2}\frac{(\mu_j-\mu_i)\mu_i}{1+\mu_j-\mu_i} - \beta\frac{(\mu_j-\mu_i)}{1+\mu_j-\mu_i} - c\mu_i^2 \\
&= \frac{1}{2} + \frac{\delta}{2}\mu_j - c\mu_j^2 + (\mu_i-\mu_j)z(\mu_i,\mu_j)
\end{aligned}
$$

, where

$$
\begin{aligned}
z(\mu_i,\mu_j) &= \frac{\delta}{2}+\frac{\delta\beta}{2}\frac{\mu_i}{1+\mu_j-\mu_i}+\frac{\beta}{1+\mu_j-\mu_i}-c(\mu_i+\mu_j)\\
&\geq \frac{\delta}{2}+\frac{\delta\beta}{2}\frac{\mu_i}{1+\mu_j-\mu_i}+\frac{\beta}{1+\mu_j-\mu_i}-\left(\frac{\delta}{2}+\frac{\delta\beta}{4}+\beta\right)(\mu_i+\mu_j)\\
&= \frac{\delta}{2}(1-\mu_i-\mu_j)+\frac{\delta\beta}{2}\frac{\mu_i}{1+\mu_j-\mu_i}-\frac{\delta\beta}{4}(\mu_i+\mu_j)+\frac{\beta}{1+\mu_j-\mu_i}\left(\mu_i^2-\mu_i+1-\mu_j-\mu_j^2\right)\\
&\geq \frac{\delta\beta}{2}(1-\mu_i-\mu_j)+\frac{\delta\beta}{2}\frac{\mu_i}{1+\mu_j-\mu_i}-\frac{\delta\beta}{4}(\mu_i+\mu_j)+\frac{\beta}{1+\mu_j-\mu_i}(1-2\mu_j)\\
&\geq \frac{\delta\beta}{4}\frac{1}{1+\mu_j-\mu_i}\left(3\mu_i^2-3\mu_i+2-\mu_j-3\mu_j^2\right)\\
&\geq \frac{\delta\beta}{4}\frac{1}{1+\mu_j-\mu_i}(2-4\mu_j)>0
\end{aligned}
$$

As a result, $\pi_i(s_i \mid \min,\mu_i<\mu_j) < \pi_i(s_i \mid \min,\mu_i=\mu_j)$. Therefore, the proof completes. The equilibrium candidates in this case are (μ_1,μ_2) such that $\mu_1=\mu_2\in\left[\frac{\delta}{4c-\delta\beta},\frac{1}{2}\right]$.

ii)$\frac{\delta}{2}+\frac{\delta\beta}{4}+\beta<c$: We consider four cases:

a)$\mu_1,\mu_2<\frac{\delta}{4c-\delta\beta}$: There can't be an equilibrium satisfying this condition. For proof, see part (a) of 1st case.

b)$\mu_1,\mu_2>\frac{\delta+2\beta}{4c-\delta\beta}$: We will show there is always a deviation. Suppose $\mu_1\leq\mu_2$.

$$
\begin{aligned}
\pi_1'(s_1 \mid \min)^- &= \frac{\delta}{2}-\frac{\delta\beta}{2}\frac{(\mu_2-\mu_1)}{1+\mu_2-\mu_1}+\frac{\delta\beta}{2}\frac{\mu_1}{(1+\mu_2-\mu_1)^2}+\frac{\beta}{(1+\mu_2-\mu_1)^2}-2c\mu_1\\
&< \frac{\delta}{2}-\frac{\delta\beta}{2}\frac{(\mu_2-\mu_1)}{1+\mu_2-\mu_1}+\frac{\delta\beta}{2}\frac{\mu_1}{(1+\mu_2-\mu_1)^2}+\frac{\beta}{(1+\mu_2-\mu_1)^2}-\frac{\delta}{2}-\frac{\delta\beta}{2}\mu_1-\beta\\
&= -\frac{\delta\beta}{2}\frac{(\mu_2-\mu_1)}{1+\mu_2-\mu_1}+\left(\frac{\delta\beta}{2}+\beta\right)\left(\frac{1}{(1+\mu_2-\mu_1)^2}-1\right)<0
\end{aligned}
$$

[22]

Therefore, 1 benefits from reducing its investment on quality. As a consequence, there is no equilibrium in this form.

c)$\exists j\in\{1,2\} \mid \frac{\delta}{4c-\delta\beta}\leq\mu_j\leq\frac{\delta+2\beta}{4c-\delta\beta}$: We show that any $\mu_i\neq\mu_j$ is strictly dominated by $\mu_i=\mu_j$. We know from part (ii) of 2nd case that any $\mu_i>\mu_j$ is strictly dominated. If we compute the right and left derivative of π_i at $\mu_i=\mu_j$ we get

[22] The first inequality is obtained from the fact that $\mu_1>\frac{\delta+2\beta}{4c-\delta\beta}$, and so $2c\mu_1>\frac{\delta}{2}+\frac{\delta\beta}{2}\mu_1+\beta$

$$\pi_i'(s_j \mid \min, \mu_i = \mu_j)^+ = \frac{\delta}{2} + \frac{\delta\beta}{2}\mu_j - 2c\mu_j$$

$$\pi_i'(s_j \mid \min, \mu_i = \mu_j)^- = \frac{\delta}{2} + \frac{\delta\beta}{2}\mu_j - 2c\mu_j + \beta$$

Therefore, $\pi_i'(s_j \mid \min, \mu_i = \mu_j)^+ \leq 0 \leq \pi_i'(s_j \mid \min, \mu_i = \mu_j)^-$. If we do the same computation for the second derivative of π_i we get

$$\pi_i''(s_j \mid \min, \mu_i = \mu_j)^- = \frac{\delta\beta}{2}(2 + \mu_i + \mu_j) + 2\beta - 2c$$

$\pi_i''(s_j \mid \min, \mu_i = \mu_j) < 0$ thanks to $c > \frac{\delta}{2} + \frac{\delta\beta}{4} + \beta$. And $\pi_i''(s_j \mid \min, \mu_i \leq \mu_j) < 0$ since $\pi_i'''(s_j \mid \min)^- > 0$. Therefore, we have $\pi_i'(s_j \mid \min, \mu_i \leq \mu_j) > 0$, which means any $\mu_i < \mu_j$ is strictly dominated. As a result, the equilibrium candidates in this case are (μ_1, μ_2) that $\mu_1 = \mu_2 \in \left[\frac{\delta}{4c-\delta\beta}, \frac{\delta+2\beta}{4c-\delta\beta}\right]$.

So far, we pin down all symmetric equilibrium candidates - which means there is no deviation given $\mu(s_1 \cap s_2) = \min(\mu(s_1), \mu(s_2))$. However, we should check for any deviations which decreases $\mu(s_1 \cap s_2)$. When $s_1 = s_2$ with $\mu_1 = \mu_2 = \mu \in \left[\frac{\delta}{4c-\delta\beta}, \frac{\delta+2\beta}{4c-\delta\beta}\right]$, the most profitable deviation for newspaper $i \in \{1, 2\}$ consists in going from minimum differentiation, $s_i = s_j$, to maximum differentiation, $s_i \cap s_j = \emptyset$ according to proposition 2. To rule out this type of deviation we should have

$$\forall \mu_i \in [0, \frac{1}{2}] \qquad \frac{\delta}{2}\mu + \frac{1}{2} - c\mu^2 \geq \delta x_i \mu_i + x_i + \delta(1 - x_i - x_j)\mu_i - c\mu_i^2 \tag{20}$$

, where the left hand side is the profit of i when $s_i = s_j$ and $\mu(s_i) = \mu$, while the right hand side is the profit of i where $s_i \cap s_j = \emptyset$, $\mu(s_i) = \mu_i$, and $\mu(s_j) = \mu$. By rearranging (20), we get:

$$\begin{aligned} d(\mu_i, \mu, , \delta, \beta, c) &= c\mu_i^4 - \left(\frac{\delta}{2} + \delta\beta + 2c\mu\right)\mu_i^3 + \left(\frac{3\delta}{2}\mu + \delta\beta(1+\mu) - c\right)\mu_i^2 \\ &\quad + \left(\frac{\delta}{2} - \frac{3\delta}{2}\mu^2 - \beta\mu + 2c\mu^3\right)\mu_i - \frac{\delta}{2}\mu + \frac{\delta}{2}\mu^3 - \beta\mu + \beta\mu^2 + c\mu^2 - c\mu^4 \\ &\leq 0 \end{aligned}$$

First, we compute the $\lim_{\delta \to 0} d(\mu_i, \mu, , \delta, \beta, c)$.

$\lim_{\delta \to 0} d(\mu_i, \mu, , \delta, \beta, c) = c\mu_i^4 - (2c\mu)\mu_i^3 + (-c)\mu_i^2 + (-\beta\mu + 2c\mu^3)\mu_i - \beta\mu + \beta\mu^2 + c\mu^2 - c\mu^4$ As $\delta \to 0$, two cases can happen depending on the value of c[23]:

[23] We assume that $, \beta > 0$, otherwise the result of the game is trivial.

1)$c \le \beta$: Any $\mu \in \left[0, \frac{1}{2}\right]$ can be an equilibrium.

$$\begin{aligned}
\lim_{\delta \to 0} d(\mu_i, \mu, , \delta, \beta, c) &= c\mu_i^4 - (2c\mu)\,\mu_i^3 + (-c)\,\mu_i^2 + \left(-\beta\mu + 2c\mu^3\right)\mu_i - \beta\mu + \beta\mu^2 + c\mu^2 - c\mu^4 \\
&\le c\mu_i^2(\mu_i^2 - 1) - (2c\mu)\,\mu_i^3 + \left(-\beta\mu + 2\beta\mu^3\right)\mu_i - \beta\mu(1 - 2\mu + \mu^3) < 0
\end{aligned}$$

2)$\beta < c$: In this case, any $\mu \in \left[0, \frac{\beta}{2c}\right]$ could be an equilibrium.

$$\begin{aligned}
\lim_{\delta \to 0} d(\mu_i, \mu, , \delta, \beta, c) &= c\mu_i^2(\mu_i^2 - 1) - (2c\mu)\,\mu_i^3 + \left(-\beta\mu + 2c\mu^3\right)\mu_i - \beta\mu + \beta\mu^2 + c\mu^2 - c\mu^4 \\
&\le c\mu_i^2(\mu_i^2 - 1) - (2c\mu)\,\mu_i^3 + \left(-\beta\mu + \beta\mu^2\right)\mu_i - \beta\mu(1 - \mu - \frac{1}{2} + \frac{\mu^2}{2}) < 0
\end{aligned}$$

Thus, we have shown that $\lim_{\delta \to 0} d(\mu_i, \mu, , \delta, \beta, c) < 0$. This implies that there exist a $\underline{\delta}^m > 0$ such that $\forall \mu_i \in \left[0, \frac{1}{2}\right], \forall \delta \le \underline{\delta}^m \mid d(\mu_i, \mu, , \delta, \beta, c) < 0$ due to continuity of d; which means $\mu_1 = \mu_2 = \mu$ is an equilibrium.

We can also find a large enough δ in which no symmetric equilibrium with minimum differentiation can be sustained any more. To have an equilibrium, we should have $\forall \mu_i \in \left[0, \frac{1}{2}\right] \mid d(\mu_i, \mu, , \delta, \beta, c) < 0$. Therefore, if $d(\mu_i = \mu, \mu, , \delta, \beta, c) > 0$ holds, no equilibrium can be sustained.

$$\begin{aligned}
d(\mu_i = \mu, \mu, , \delta, \beta, c) &= \delta\beta\mu^2 - \beta\mu > 0 \\
&\Leftrightarrow \delta\mu > 1
\end{aligned}$$

For any $c, 0 <, 0 < \beta < 1$ we can find $\hat{\delta}$ such that $c < \frac{\delta}{2} + \frac{\delta\beta}{4}$; which means $\mu = \frac{1}{2}$. Therefore, $\forall \delta > \bar{\delta}^m = \max(2, \hat{\delta}) \mid d(\mu_i = \mu, \mu, , \delta, \beta, c) > 0$, which means μ can't be sustained as an equilibrium.

□

8.5 Proof Proposition 4

Proof. In this case, we can rewrite the profit of $i \in 1, 2$ as

$$\pi_i(s_i \mid \max) = \frac{1}{2} + \frac{\delta}{2}\mu_i - \beta\frac{\mu_j}{1 + \mu_j - \mu_i} + \delta\beta\frac{\mu_i^2}{1 + \mu_i - \mu_j} - c\mu_i^2$$

The derivatives are

$$\pi_i'(s_i \mid \max) = \frac{\delta}{2} - \beta\frac{\mu_j}{(1 + \mu_j - \mu_i)^2} + 2\delta\beta\frac{\mu_i}{1 + \mu_i - \mu_j} - \delta\beta\frac{\mu_i^2}{(1 + \mu_i - \mu_j)^2} - 2c\mu_i \tag{21}$$

$$\pi_i''(s_i \mid \max) = -2\beta \frac{\mu_j}{(1+\mu_j-\mu_i)^3} + \frac{2\delta\beta(1-\mu_j)^2}{(1+\mu_i-\mu_j)^3} - 2c$$

$$\pi_i'''(s_i \mid \max) = -6\beta \frac{\mu_j}{(1+\mu_j-\mu_i)^4} - \frac{6\delta\beta(1-\mu_j)^2}{(1+\mu_i-\mu_j)^4}$$

At the end of this proof we will show that $\delta > 2$ which is a necessary condition to have a maximum differentiation equilibrium. For now, we use this condition.

$$\begin{aligned} \pi_i'(s_i \mid \max, \mu_i = 0) &= \frac{\delta}{2} - \beta \frac{\mu_j}{(1+\mu_j)^2} \\ &\geq \frac{\delta}{2} - \beta \frac{2}{9} > 0 \end{aligned}$$

This and the negativity of π_i''' imply that the solution of $\pi_i'(s_i \mid \max) = 0$ is a global maximum of $\left[0, \frac{1}{2}\right]$, given the solution is in $\left[0, \frac{1}{2}\right]$; and if the solution is out of it the global maximum is reached at $\frac{1}{2}$. Therefore, the best response of i is either $\frac{1}{2}$ or the solution of $\pi_i'(s_i \mid \max) = 0$. As we are looking for symmetric equilibriums, there are not more than two possibilities, $\mu_1 = \mu_2 = \frac{1}{2}$, and $\mu_1 = \mu_2 = \hat{\mu}$ where $\hat{\mu}$ is the solution of

$$Q(\hat{\mu}) = \hat{\mu}^2(-\delta\beta) + \hat{\mu}(-\beta + 2\delta\beta - 2c) + \frac{\delta}{2} = 0 \tag{22}$$

which is obtained from putting $\mu_i = \mu_j = \hat{\mu}$ in (21).

i)To have $(\frac{1}{2}, \frac{1}{2})$ as an equilibrium we should have $\pi_i'(s_l \mid \max, \mu_i = \mu_j = \frac{1}{2}) > 0$ for $i, j \in \{1, 2\}$. This is equivalent to $c \leq \frac{\delta}{2} - \frac{\beta}{2} + \frac{3}{4}\delta\beta$.

ii) It is simple to check $\frac{\delta}{2} - \frac{\beta}{2} + \frac{3}{4}\delta\beta < c$ implies $\hat{\mu} < \frac{1}{2}$. As a result, given $\mu_j = \hat{\mu}$, $\mu_i = \hat{\mu}$ is the best response of i as it is discussed before.

To show the existence of the equilibrium, we should prove there is no deviation. So far we have shown that there is no deviation given the maximum differentiation. However, there is another possible deviation to check. The only possible deviation is increasing the $s_i \cap s_j$. According to the proposition 2, the most profitable deviation is choosing the maximum intersection.

Suppose (μ, μ) is the equilibrium candidate. We consider two cases:

a)$\mu_i \leq \mu$: To rule out profitable deviation, we should have

$$\frac{1}{2} + \frac{\delta}{2}\mu - \beta\mu + \delta\beta\mu^2 - c\mu^2 \geq \frac{1}{2} + \frac{\delta}{2}\mu_i - \frac{\delta\beta}{2}\frac{(\mu-\mu_i)\mu_i}{1+\mu-\mu_i} - \beta\frac{\mu-\mu_i}{1+\mu-\mu_i} - c\mu_i^2$$

, where the left hand side represent the profit in equilibrium, (μ, μ), and the right hand side shows the profit of i when she deviates from equilibrium. This inequality is equivalent to

$$(\mu-\mu_i)\left(\delta\left(-\frac{}{2}-\frac{\beta}{2}\frac{\mu_i}{1+\mu-\mu_i}\right)+c(\mu+\mu_i)-\beta\frac{1}{1+\mu-\mu_i}\right)+\beta\mu-\delta\beta\mu^2\leq 0$$

This should hold $\forall\mu_i\leq\mu$. For particular case $\mu_i=\mu$, this inequality is equivalent to $\delta\mu\geq 1$. Therefore, $\delta\geq 2$ is a necessary condition to have an equilibrium with maximum differentiation.

As the coefficient of δ in the inequality is negative, there exist a $\hat{\delta}>0$ such that $\forall\delta>\hat{\delta}$ the left term takes negative values. The negativity of the right term is a necessary condition, $\delta>\frac{1}{\mu}$.

b)$\mu_i\geq\mu$: In this case $\mu(s_i\cap s_j)=\mu$. This deviation is profitable if $\min(\mu_i,\mu)>g(\mu_i,\mu)$. If it is not the case i can increase its profit by reducing the measure of intersection with j to zero, but we know there is no profitable deviation if the sets are disjoint, since i is choosing the best reply given the empty intersection.

From (19), we know $\frac{\partial g(\mu_i,\mu)}{\partial\mu_i}=3(\frac{1}{2}-\mu_i)+2\mu-\frac{1}{\delta}>0$, if $\mu>\frac{1}{\delta}$ (which is a necessary condition for case (a)). As $g(\mu_i=\mu,\mu)=2\mu-\frac{1}{\delta}>\mu$, it means $\forall\mu_i\geq\mu,\quad\mu<g(\mu_i,\mu)$. This means this case does not matter as long as there is no deviation in case (i).

There exists a $\hat{\hat{\delta}}$ such that $\forall\delta>\hat{\hat{\delta}}\mid c<\frac{\delta}{2}-\frac{\beta}{2}+\frac{3}{4}\delta\beta$ which implies $\mu=\frac{1}{2}$. Hence, there is a $\bar{\delta}^M=\max\left(2,\hat{\delta},\hat{\hat{\delta}},\right)$ such that $\forall\delta>\bar{\delta}^M$ there exists an equilibrium in which newspapers invest on different sets of issues.

Moreover, we can set $\underline{\delta}^M=2$ which implies $\forall\delta<\underline{\delta}^M$ there exists no equilibrium in which newspapers invest on different sets of issues. This is due to the fact that the necessary condition, $\delta\geq\frac{1}{\mu}$, is violated. □

8.6 Proof Proposition 6

Proof. In terms of c, we have two cases:

1)$c>\frac{\delta}{2}-\frac{\beta}{2}+\frac{3}{4}\delta\beta$:

First of all, to have a specialization equilibrium, we have shown in the proof of proposition 4 it is necessary $\mu^M>\frac{1}{\delta}$. From (22), we have

$$Q(\mu^M)=\mu^{M^2}(-\delta\beta)+\mu^M(-\beta+2\delta\beta-2c)+\frac{\delta}{2}=0$$

In other hand, from proposition 1 we know, $\frac{\delta}{2}=2c\mu^*-\frac{\beta}{2}-\frac{\delta\beta}{2}\mu^*$. By substituting this in the $Q(\mu^M)$ we get

$$\begin{aligned}2(\mu^M-\mu^*)(c-\frac{\delta\beta}{4}) &= \mu^{M^2}(-\delta\beta)+(\frac{3\delta\beta}{2}-\beta)\mu^M-\frac{\beta}{2}\\ &= \delta\beta\mu^M(\frac{1}{2}-\mu^M)+\beta\mu^M(\frac{\delta}{2}-1)+\frac{\beta}{2}(\mu^M\delta-1)\\ &\geq 0\end{aligned}$$

Hence, $\mu^M \geq \mu^*$ since $c \geq \frac{\delta\beta}{4}$.

2)$c \leq \frac{\delta}{2}-\frac{\beta}{2}+\frac{3}{4}\delta\beta$: In this case, $\mu^M=\frac{1}{2}$ which for sure is greater than μ^*.

□

8.7 Proof Proposition 7

Proof. ii) First, we show $\pi^M-\pi^*$ is decreasing with c. We consider two cases:
a)$c \geq \frac{\delta}{2}-\frac{\beta}{2}+\frac{3}{4}\delta\beta$: In this case, $\mu^* < \mu^M < \frac{1}{2}$. From (13), we have

$$\pi^M-\pi^*=h(c)=(\mu^M-\mu^*)\left(-c(\mu^M+\mu^*)+\frac{\delta}{2}\right)+\beta\mu^M(\delta\mu^M-1)$$

We want to show $\frac{\partial(\pi^M-\pi^*)}{\partial c}<0$. We can write $\frac{\partial(\pi^M-\pi^*)}{\partial c}$ as

$$h'(c)=\mu^{M'}\left(-2c\mu^M+2\delta\beta\mu^M+\frac{\delta}{2}-\beta\right)+\mu^{*'}\left(2c\mu^*-\frac{\delta}{2}\right)-\left(\mu^{M^2}-\mu^{*^2}\right)$$

From proposition 1, we know $c\mu^*=\frac{\delta}{4}+\frac{\beta}{4}+\frac{\delta\beta}{4}\mu^*$. Moreover, the derivation of (22) gives us

$$\mu^{M'}\left(-2c\mu^M+2\delta\beta\mu^M\right)=\mu^{M'}\left(\beta\mu^M+2\delta\beta\mu^{M^2}\right)+2\mu^{M^2} \tag{23}$$

By substituting in $\frac{\partial(\pi^M-\pi^*)}{\partial c}$, we get

$$\mu^{M'}\left(2\delta\beta\mu^M+\beta\mu^M+\frac{\delta}{2}-\beta\right)+\mu^{*'}\left(\frac{\beta}{2}+\frac{\delta\beta}{2}\mu^*\right)+\mu^{M^2}+\mu^{*^2}$$

Since $\mu^{M'},\mu^{*'}<0$, it is sufficient to show $\mu^{M'}\left(\delta\beta\mu^M+\frac{\delta}{2}\right)+\mu^{*'}\left(\frac{\beta}{2}+\frac{\delta\beta}{2}\mu^*\right)+\mu^{M^2}+\mu^{*^2}<0$.
By substituting $\mu^{M'}=\frac{-2\mu^M}{2\delta\beta\mu^M+\beta+2c-2\delta\beta}$, and $\mu^{*\prime}=\frac{-4\mu^*}{4c-\delta\beta}$, we get

$$\begin{aligned}
\mu^{M\prime}\left(\delta\beta\mu^M+\frac{\delta}{2}\right)+\mu^{M^2}+\mu^{*\prime}\left(\frac{\beta}{2}+\frac{\delta\beta}{2}\mu^*\right)+\mu^{*^2} &= \mu^M\frac{2\delta\beta\mu^{M^2}+\mu^M(\beta+2c-2\delta\beta)-\delta-2\delta\beta\mu^M}{2\delta\beta\mu^M+\beta+2c-2\delta\beta} \\
&+ \mu^*\frac{-2\beta-2\delta\beta\mu^*+\delta+\beta}{4c-\delta\beta} \\
&= \frac{-\mu^M(\beta+2c-2\delta\beta)-2\delta\beta\mu^M}{2\delta\beta\mu^M+\beta+2c-2\delta\beta} \\
&+ \mu^*\frac{-2\delta\beta\mu^*+\delta-\beta}{4c-\delta\beta} \\
&= \frac{-\mu^M\beta-2c\mu^M}{2\delta\beta\mu^M+\beta+2c-2\delta\beta}+\mu^*\frac{-2\delta\beta\mu^*+\delta-\beta}{4c-\delta\beta} \\
&< \frac{-2\mu^M\beta-4c\mu^M}{4c-\delta\beta}+\mu^*\frac{-2\delta\beta\mu^*+\delta-\beta}{4c-\delta\beta} \\
&< \frac{-2\mu^M\beta-(2\delta+3\delta\beta-2\beta)\mu^M}{4c-\delta\beta} \\
&+ \mu^*\frac{-2\delta\beta\mu^*+\delta-\beta}{4c-\delta\beta} \\
&= \frac{-2\delta\mu^M+\delta\mu^*-3\delta\beta\mu^M-2\delta\beta\mu^{*^2}-\beta\mu^*}{4c-\delta\beta} \\
&< 0
\end{aligned}$$

b)$\frac{\delta}{2}+\frac{\beta}{2}+\frac{\delta\beta}{4}\leq c<\frac{\delta}{2}-\frac{\beta}{2}+\frac{3}{4}\delta\beta$: In this case, $\frac{\delta}{4c-\delta\beta}=\mu^*<\mu^M=\frac{1}{2}$. (13) can be written as:

$$h(c)=\frac{1}{4}(\delta\beta-c)+\frac{1}{2}\left(\frac{\delta}{2}-\beta\right)-\frac{\delta}{2}\mu^*+c\mu^{*^2}$$

Hence,

$$\begin{aligned}
h'(c)=-\frac{1}{4}-\frac{\delta}{2}\mu^{*\prime}+2c\mu^*\mu^{*\prime}+\mu^{*^2} &= -\frac{1}{4}+\mu^{*^2}+\mu^{*\prime}\left(-\frac{\delta}{2}+2c\mu^*\right) \\
&= -\frac{1}{4}+\mu^{*^2}+\mu^{*\prime}\left(\frac{\beta}{2}+\frac{\delta\beta}{2}\mu^*\right)<0
\end{aligned}$$

We show $\pi^M-\pi^*$ is strictly decreasing. To prove (ii) it is sufficient to show $\pi^M-\pi^*$ gets both positive and negative values for some values of c. For $c=\frac{\delta}{2}+\frac{\beta}{2}+\frac{\delta\beta}{4}$, $\mu^M=\mu^*=\frac{1}{2}$. Thus, $\pi^M-\pi^*=\frac{\beta}{2}(\frac{\delta}{2}-1)>0$. We also know, for $c=\frac{\delta^2}{4}+\delta\beta-\beta$, $\mu^*<\mu^M=\frac{1}{\delta}$. Substituting in 13 we get

$$\begin{aligned}\pi^M - \pi^* &= (\mu^M - \mu^*)\left(-c(\mu^M + \mu^*) + \frac{\delta}{2}\right)\\ &= \frac{1}{2}(\mu^M - \mu^*)\left(\delta\beta\mu^{M^2} + \beta\mu^M - 2\delta\beta\mu^M - \frac{\beta}{2} - \frac{\delta\beta}{2}\mu^*\right) < 0\end{aligned}$$

□

8.8 Proof Proposition 8

Proof. i) First, we prove the newspaper are always better to invest on disjoint set of issues. By introducing u_T we should modify (17):

$$\begin{aligned}\pi_1(s_1) &= \delta\alpha_1\mu(s_1) + \alpha_1 + \delta(1-\alpha_1-\alpha_2)\left(\mu(s_1) - \frac{1}{2}\mu(s_1\cap s_2)\right) - c\mu(s_1)^2\\ &= h(\mu(s_1),\mu(s_2)) + \frac{\delta\beta\mu(s_1\cap s_2)}{1-(\mu(s_1)-\mu(s_2))^2}\left[\mu(s_1\cap s_2) - g(\mu(s_1),\mu(s_2))\right] \quad (24)\end{aligned}$$

, where

$$h(\mu(s_1),\mu(s_2)) = \frac{1}{2} + \frac{\delta}{2}\mu(s_1) - \frac{1}{t}\frac{\mu(s_2)\Delta u + u_T}{1+\mu(s_2)-\mu(s_1)} + \frac{\delta\mu(s_1)}{t}\frac{\mu(s_1)\Delta u + u_T}{1+\mu(s_1)-\mu(s_2)} - c\mu(s_1)^2 \quad (25)$$

$$g(\mu(s_1),\mu(s_2)) = -\frac{3}{2}\mu(s_1)^2 + \mu(s_1)\left(2\mu(s_2) - \frac{1}{\delta} + \frac{3}{2}\right) + (1-\mu(s_2))\left(\frac{1}{2}\mu(s_2) - \frac{1}{\delta}\right) + \frac{u_T}{\Delta u} \quad (26)$$

There are two cases:
a) $\mu_1 \leq \mu_2$: in this case max differentiation is profitable if and only if $\mu_1 \leq g(\mu_1,\mu_2)$ or:

$$-\frac{3}{2}\mu_1^2 + \mu_1\left(2\mu_2 - \frac{1}{\delta} + \frac{1}{2}\right) + (1-\mu_2)\left(\frac{1}{2}\mu_2 - \frac{1}{\delta}\right) + \frac{u_T}{\Delta u} \geq 0 \quad (27)$$

Now, we show this inequality holds for any μ_1, and μ_2, as long as $u_T \geq \frac{\Delta u}{\delta}$. First,

$$(1-\mu_2)\left(\frac{1}{2}\mu_2 - \frac{1}{\delta}\right) + \frac{u_T}{\Delta u} \quad (28)$$

is positive for any $0 \leq \mu_2 \leq \frac{1}{2}$. Therefore, it sufficient to show (27) is positive if $\mu_1 = \mu_2$. This holds if $\mu^T - \frac{1}{\delta} + \frac{u_T}{\Delta u} > 0$ which is consistent with $u_T \geq \frac{\Delta u}{\delta}$.

b)$\mu_1 \geq \mu_2$: in this case max differentiation is profitable if and only if $\mu_2 \leq g(\mu_1, \mu_2)$. From (a), we know if $\mu_2 = \mu_1$, $\mu_2 \leq g(\mu_1, \mu_2)$ holds as long as $\frac{\Delta u}{8} + u_T \geq \frac{\Delta u}{2\delta}$.

Now, we should prove $\mu_2 \leq g(\frac{1}{2}, \mu_2)$. This is equivalent to

$$-\frac{1}{2}\mu_2^2 + \mu_2(\frac{1}{2} + \frac{1}{\delta}) + \frac{3}{8} - \frac{3}{2\delta} + \frac{u_T}{\Delta u} \geq 0 \tag{29}$$

$\frac{3\Delta u}{8} + u_T \geq \frac{3\Delta u}{2\delta}$ is sufficient to satisfy the above inequality. And in total, if we have $u_T \geq \frac{3\Delta u}{2\delta}$ then the maximum differentiation is always profitable for any μ_1, and μ_2.

ii) We will show $\frac{\partial^2 \pi_i}{\partial \mu_i \partial \mu_j} > 0$.

$$\frac{\partial^2 \pi_i}{\partial \mu_i \partial \mu_j} = \frac{1}{t}\left[\frac{-(1-\mu_i-\mu_j)\Delta u + 2u_T}{(1-\mu_i+\mu_j)^2} + \delta\frac{u_T(1-\mu_j) + 2\Delta u \mu_i - u_T\mu_i - 2\mu_i^2\Delta u}{(1+\mu_i-\mu_j)^3}\right] \tag{30}$$

The right expression is always positive. The left one is also positive, since $2u_T > \Delta u$.

iii) First, we show there is always a unique response.

$$\pi_i'(s_i) = \frac{\delta}{2} - \frac{1}{t}\frac{\mu_j\Delta u + u_T}{(1+\mu_j-\mu_i)^2} + 2\delta\beta\frac{\mu_i}{1+\mu_i-\mu_j} - \frac{\delta}{t}\frac{\mu_i^2\Delta u - u_T(1-\mu_j)}{(1+\mu_i-\mu_j)^2} - 2c\mu_i \tag{31}$$

$$\pi_i''(s_i) = -\frac{2}{t}\frac{\mu_j\Delta u + u_T}{(1+\mu_j-\mu_i)^3} + \frac{2\delta(1-\mu_j)}{t}\frac{(1-\mu_i-\mu_j)\Delta u - u_T}{(1+\mu_i-\mu_j)^3} - 2c \tag{32}$$

$$\pi_i'''(s_i) = -\frac{6}{t}\frac{\mu_j\Delta u + u_T}{(1+\mu_j-\mu_i)^4} - \frac{6\delta(1-\mu_j)}{t}\frac{(1-\mu_i-\mu_j)\Delta u - u_T}{(1+\mu_i-\mu_j)^4} \tag{33}$$

There are two cases:
a)$u_T \geq \Delta u$: in this case, the profit function is concave no matter of what μ_j.
b)$u_T < \Delta u$: they may exist some μ_j which the profit function is convex. If this is the case, the third derivative would be negative. On the other hand, we know $u_T \geq \frac{3\Delta u}{2\delta}$. Therefore, $u_T < \Delta u$ implies $\delta \geq \frac{3}{2}$. $\pi'(\mu_i = 0, \mu_j) > 0$ is sufficient to prove there exists a unique best response. This

is equivalent to:

$$\begin{aligned}
\pi'(\mu_i = 0, \mu_j) &= \frac{\delta}{2} - \frac{1}{t}\frac{\mu_j \Delta u + u_T}{(1+\mu_j)^2} + \frac{\delta}{t}\frac{u_T}{1-\mu_j} \\
&> -\frac{1}{t}\frac{\mu_j \Delta u + u_T}{(1+\mu_j)^2} + \frac{1}{t}\frac{u_T}{1-\mu_j} \\
&= \frac{1}{t(1+\mu_j)^2(1-\mu_j)}\left[u_T(1+\mu_j)^2 - u_T(1-\mu_j) - \mu_j \Delta u(1-\mu_j)\right] \\
&= \frac{1}{t(1+\mu_j)^2(1-\mu_j)}\left[u_T(\mu_j + \mu_j^2) + \mu_j(2u_T - \Delta u(1-\mu_j))\right] \\
&> 0
\end{aligned}$$

Depending on the value of δ, the best response could take three values, 0, $\frac{1}{2}$ or the solution of $\pi'(\mu_i, \mu_j) = 0$.
1)To have $(0, 0)$ as an equilibrium we should have $\pi_i'(\mu_i = \mu_j = 0) < 0$ for $i, j \in \{1, 2\}$. This is equivalent to $\delta < \underline{\delta}^T = \frac{u_T/t}{u_T/t+1/2}$.

2) For $\underline{\delta}^T \leq \delta \leq \bar{\delta}^T$, the best response is the solution of $\pi'(\mu_i, \mu_j) = 0$. Therefore the equilibrium quality is the solution of Q :

$$Q(\hat{\mu}) = \hat{\mu}^2(-\delta\beta) + \hat{\mu}(-\beta + 2\delta\beta - 2c - \frac{u_T\delta}{t}) + \frac{\delta}{2} + \frac{u_T}{t}(\delta - 1) \tag{34}$$

.

3)To have $(\frac{1}{2}, \frac{1}{2})$ as an equilibrium we should have $\pi_i'(\mu_i = \mu_j = \frac{1}{2}) > 0$ for $i, j \in \{1, 2\}$. This is equivalent to $\delta > \bar{\delta}^T = \frac{c+\Delta u/2t+u_T/t}{u_T/2t+1/2+3\Delta u/4t}$.

Now, we prove that μ^T is increasing with δ. For $\delta < \underline{\delta}^T$, μ^T is zero, and for $\delta < \bar{\delta}^T$, μ^T is 1/2. So it is sufficient to prove μ^T is increasing when $\delta \in [\underline{\delta}^T, \bar{\delta}^T]$. When $\delta \in [\underline{\delta}^T, \bar{\delta}^T]$, we have

$$\mu^{T^2}(-\delta\beta) + \mu^T(-\beta + 2\delta\beta - 2c - \frac{u_T\delta}{t}) + \frac{\delta}{2} + \frac{u_T}{t}(\delta - 1) = 0$$

Hence, the derivative would be

$$\mu^{T'}[-2\mu^T\delta\beta - \beta + 2\delta\beta - 2c - u_T\delta/t] - \beta\mu^{T^2} + \mu^T(2\beta - u_T/t) + 1/2 + u_T/t = 0$$

As $-2\mu^T\delta\beta - \beta + 2\delta\beta - 2c - u_T\delta/t < 0$, and $-\beta\mu^{T^2} + \mu^T(2\beta - u_T/t) + 1/2 + u_T/t > 0$, $\mu^{T'}$ is positive. $\square$

8.9 Proof Proposition 9

Proof. i) From the proof of proposition 6, we can write

$$\begin{aligned}
2(\mu^T - \mu^*)(c - \frac{\delta\beta}{4}) &= {\mu^T}^2(-\delta\beta) + (\frac{3\delta\beta}{2} - \beta - \frac{u_T\delta}{t})\mu^T - \frac{\beta}{2} + \frac{u_T}{t}(\delta - 1) \\
&= \delta\beta\mu^T(\frac{1}{2} - \mu^T) + \beta\mu^T(\frac{\delta}{2} - 1) + \frac{\beta}{2}(\mu^T\delta - 1) + \frac{u_T}{t}(\delta - 1 - \delta\mu_T) \\
&\geq 0
\end{aligned}$$

since $\delta > \frac{1}{\mu^T} > 2$.

ii) Please see the online appendix.

iii) The proof is the same as proposition 7 (iii). □

Appendix B

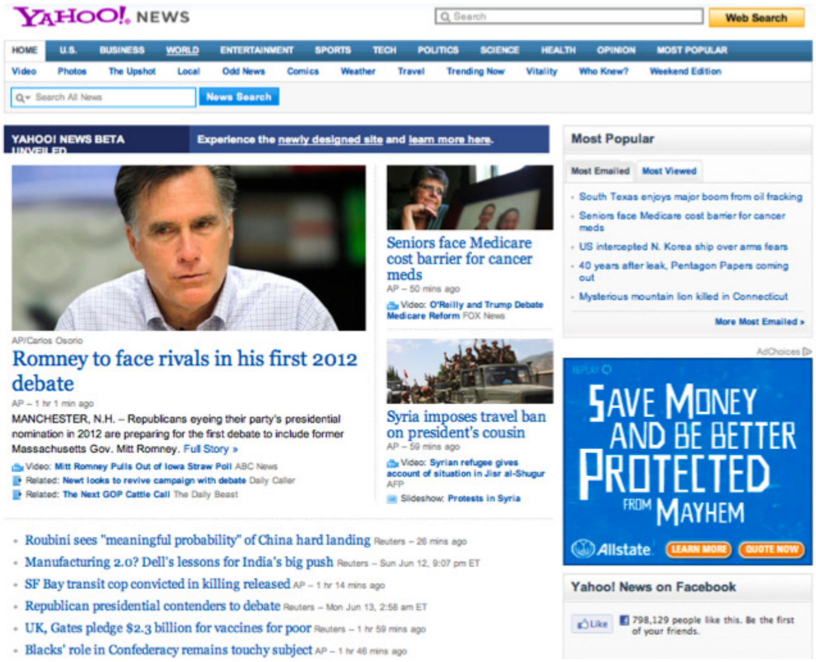

Figure 7: The Yahoo! News

Figure 8: An article in the Yahoo! News. As you can see there is no link to the original article.

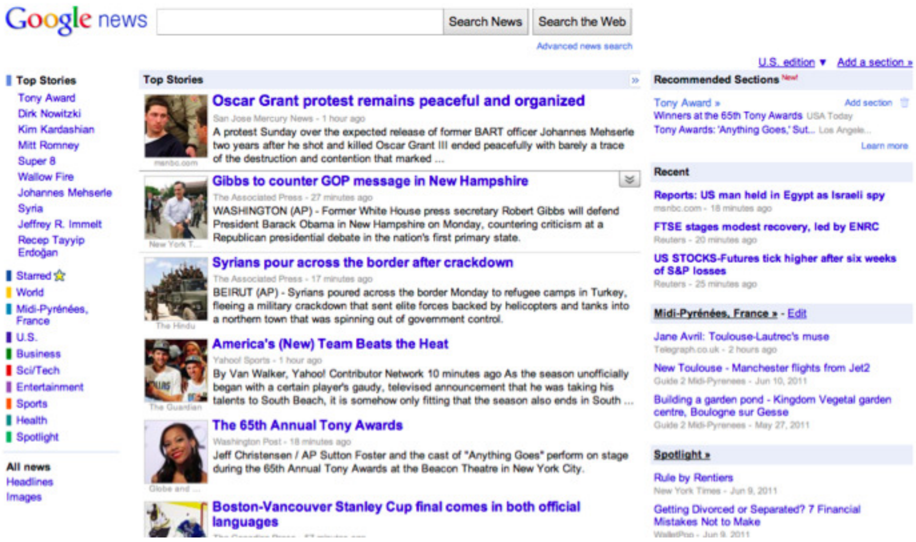

Figure 9: The Google News

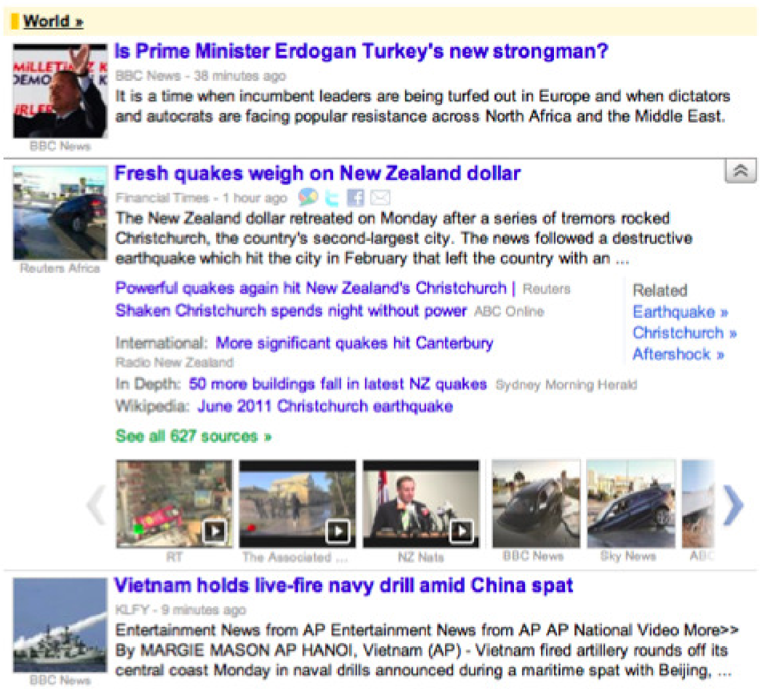

Figure 10: An article from Financial Times in the Google News. There is a short abstract of the article in the two or three lines, and a link to the original article.

For Online Publication

8.10 Proof Proposition 9 (ii)

Proof. If newspaper i opts out, its market share would be $\alpha_i' = \frac{1}{2} - \frac{(\mu^T - \mu_i)\Delta u + u_T}{2t}$. So we can write $\alpha_i' - \alpha_i = \frac{1}{2t}(u_T + \Delta u(\mu^T + \mu_i))$, where α_i is the equilibrium market share. We also know the best deviation quality is $\mu_i = \frac{\delta+\beta-\frac{\delta(\mu^T\Delta u + u_T)}{t}}{4c-2\delta\beta}$. Therefore, the gain from deviation is

$$
\begin{aligned}
\alpha_i'(1+\delta\mu_i) - c\mu_i^2 - \alpha_i(1+\delta\mu^T) - 2\delta(\frac{u_T}{t} + \beta\mu^T)\mu^T + c{\mu^T}^2 &= \\
\frac{1}{2t}(\mu^T\Delta u + u_T + \mu_i\Delta u) + \frac{1}{2t}(\delta\mu_i\mu^T\Delta u) - \frac{3}{2t}\delta\mu^T(\mu^T\Delta u + u_T) &- \\
(\mu^T - \mu_i)\left[-c(\mu_i + \mu^T) + \delta/2 - \frac{\delta(\mu^T - \mu_i)\Delta u}{2t} - \frac{\delta u_T}{2t}\right]&
\end{aligned}
$$

By using (34) and $\mu_i = \frac{\delta+\beta-\frac{\delta(\mu^T\Delta u + u_T)}{t}}{4c-2\delta\beta}$, we get:

$$
\begin{aligned}
(\mu^T\Delta u + u_T + \mu_i\Delta u) + \delta\mu_i\mu^T\Delta u - 3\delta\mu^T(\mu^T\Delta u + u_T) &- \\
(\mu^T - \mu_i)[{\mu^T}^2\delta\Delta u + \mu^T(\Delta u + \delta u_T) - \frac{5}{2}\mu^T\delta\Delta u - \frac{3}{2}u_T\delta + u_T - \Delta u/2]&
\end{aligned}
$$

We can rearrange it to

$$
\begin{aligned}
u_T + 2\mu^T\Delta u - 2\delta{\mu^T}^2\Delta u - 3\delta\mu^T u_T &- \\
(\mu^T - \mu_i)[{\mu^T}^2\delta\Delta u + \mu^T(\Delta u + \delta u_T - \frac{3}{2}\delta\Delta u) - \frac{3}{2}u_T\delta + u_T + \Delta u/2]&
\end{aligned}
$$

Using (34), we can write:

$$
(\mu_i - \mu^T)(4c - 2\delta\beta) = \frac{1}{t}\left[2{\mu^T}^2\delta\Delta u + 2\mu^T(\Delta u - \frac{3}{2}\delta\Delta u + u_T\delta) - 3u_T\delta + 2u_T + \Delta u\right] \tag{35}
$$

Hence, the gain is

$$
u_T + 2\mu^T\Delta u - 2\delta{\mu^T}^2\Delta u - 3\delta\mu^T u_T + \frac{t}{2}(4c - 2\delta\beta)(\mu^T - \mu_i)^2
$$

or equivalently

$$
{\mu^T}^2\left(\frac{t}{2}(4c - 2\delta\beta) - 2\delta\Delta u\right) + \mu^T\left(2\Delta u - 3\delta u_T - t\mu_i(4c - 2\delta\beta)\right) + u_T + \frac{t}{2}(4c - 2\delta\beta)\mu_i^2
$$

We first want to show the gain is decreasing in μ^T, and then show it is negative for $\mu^T = \mu^*$. This implies opting out is not profitable for $\mu^T \geq \mu^*$. The derivative is

$$t(4c - 2\delta\beta)(\mu^T - \mu_i) - 4\delta\Delta u\mu^T + 2\Delta u - 3\delta u_T - t\mu^T(4c - 2\delta\beta)\mu_i' + t\mu_i(4c - 2\delta\beta)\mu_i'$$

We can replace $(4c - 2\delta\beta)\mu_i'$ by $-\delta\beta$ and $t(4c - 2\delta\beta)(\mu^T - \mu_i)$ from (35). Hence, we have

$$-2{\mu^T}^2\delta\Delta u - 2\mu^T\Delta u + 3\delta\Delta u\mu^T - 2\delta u_T\mu^T + 3u_T\delta - 2u_T - \Delta u - 3\delta\Delta u\mu^T + 2\Delta u - 3\delta u_T - \delta\Delta u\mu_i$$

or equivalently

$$-2{\mu^T}^2\delta\Delta u - 2\mu^T\Delta u - 2\delta u_T\mu^T - 2u_T + \Delta u - \delta\Delta u\mu_i$$

which is negative since $2u_T \geq \Delta u$.

We also know

$$\begin{aligned}
\delta\mu^T > 1 \Rightarrow Q(\frac{1}{\delta}) > 0 \quad &\Rightarrow \quad c < \frac{\delta^2}{4} + \frac{\delta^2 u_T}{2t} + \frac{\delta}{t}(\Delta u - u_T) - \beta \\
&\Rightarrow \quad c < \frac{\delta^2}{4} + \delta\beta - \beta + \frac{\delta^2 u_T}{2t} - \frac{\delta u_T}{t} \\
&\Rightarrow \quad c < \frac{3\delta^2}{8} + \frac{5\delta\beta}{8} \Rightarrow \delta\mu^* > \frac{2}{3}
\end{aligned}$$

If $\mu^T = \mu^*$ then we have $t(4c - 2\delta\beta)(\mu^* - \mu_i) = \delta u_T$. Therefore, the gain from opting out when $\mu^T = \mu^*$ is

$$-\delta\mu^*(u_T + 2\mu^*\Delta u + \frac{3}{2}u_T) + u_T + 2\mu^*\Delta u - \frac{\delta u_T}{2}\mu_i$$

which is less than

$$-\frac{2}{3}u_T - \frac{4}{3}\mu^*\Delta u - u_T + u_T + 2\mu^*\Delta u - \frac{\delta u_T}{2}\mu_i = -\frac{2}{3}(u_T - \mu^*\Delta u) - \frac{\delta u_T}{2}\mu_i < 0$$

Therefore, the gain from opting out is negative for all $\mu^T \geq \mu^*$. And since $\delta\mu^T > 1$ implies $\mu^T > \mu^*$ opting out is not beneficial if $\delta\mu^T > 1$.

$\square$