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***Giuseppe Marco Attanasi, Christian Gollier, Aldo Montesano and Noémie Pace***

**Institut d'Economie Industrielle (IDEI)** – Manufacture des Tabacs
Aile Jean-Jacques Laffont – 21, allée de Brienne – 31000 TOULOUSE – FRANCE
Tél. + 33(0)5 61 12 85 89 – Fax + 33(0)5 61 12 86 37 – **www.idei.fr** – contact@cict.fr

INSTITUT D'ECONOMIE INDUSTRIELLE

# Eliciting ambiguity aversion in unknown and in compound lotteries: A *KMM* experimental approach*

Giuseppe Attanasi
University of Strasbourg

Christian Gollier
Toulouse School of Economics

Aldo Montesano
Bocconi University, Milan

Noemi Pace
University Ca' Foscari of Venice

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We define coherent-ambiguity aversion within the Klibanoff, Marinacci and Mukerji (2005) smooth ambiguity model (henceforth *KMM*) as the combination of choice-ambiguity aversion and value-ambiguity aversion. We analyze theoretically five ambiguous decision tasks, where a subject faces two-stage lotteries with binomial, uniform or unknown second-order probabilities. We check our theoretical predictions through a 10-task laboratory experiment. In (unambiguous) tasks 1-5, we elicit risk aversion both through a portfolio choice method and through a *BDM* mechanism. In (ambiguous) tasks 6-10, we elicit choice-ambiguity aversion through the portfolio choice method and value-ambiguity aversion through the *BDM* mechanism. We find that more than 75% of classified subjects behave according to the *KMM* model in all tasks 6-10, independent of their degree of risk aversion. Further, the percentage of coherently-ambiguity-averse subjects is lower in the binomial than in the uniform and in the unknown treatment, with only the latter difference being significant. Finally, highly-risk-averse subjects are more prone to coherent-ambiguity.

*JEL* classification: D81, D83, C91.

*Keywords*: coherent-ambiguity aversion, value-ambiguity aversion, choice-ambiguity aversion, smooth ambiguity model, binomial distribution, uniform distribution, unknown urn.

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# 1 Introduction

In this paper we propose a series of ten experimental decision tasks involving two-outcome lottery choices. Five of these tasks are aimed at eliciting a subject's attitude towards risk and the other five are designed to study her attitude towards ambiguity. We derive specific theoretical predictions about a subject's behavior in the latter decision tasks, by relying on the Klibanoff, Marinacci and Mukerji (2005) smooth ambiguity model (henceforth *KMM*). The paper has three main goals.

The first objective is to propose a simple experimental method able to make *KMM* operational in individual decision tasks. This is why the experimental environment is explicitly designed in order to match *KMM* intuition of modelling ambiguity through two-stage lotteries. In such an environment, we are able to provide two different operational definitions of ambiguity aversion. The first one, namely *value-ambiguity attitude*, is based on Becker and Brownson (1964) idea that "individuals are willing to pay money to avoid actions involving ambiguity" (p. 5).[1] A value-ambiguity-averse subject values an ambiguous lottery less than its unambiguous equivalent with the same mean probabilities. In the *KMM* model, this is true if the subject's $\phi$ function is concave. The second definition, namely *choice-ambiguity attitude*, relies on Gollier (2012) intuition that more ambiguity-averse subjects should have a smaller demand for a risky asset whose distribution of return is ambiguous. Notice that a portfolio containing a larger share invested in the risky asset may be seen as a two-stage lottery where second-order objective probabilities are more dispersed. In the *KMM* framework, Gollier (2012) has shown that an ambiguity-averse subject could have a larger demand for the risky asset than another ambiguity-neutral subject with the same risk aversion, thereby stating that a choice-ambiguity-averse subject is not necessarily value-ambiguity-averse. On the other hand, Gollier (2012) provides sufficient conditions on the structure of the two-stage uncertainty to re-establish the link between the concavity of $\phi$ and ambiguity aversion. Given that one of these conditions is satisfied in our experimental decision tasks, we expect to find an equivalence between value-ambiguity attitude and choice-ambiguity attitude, that we define as *coherent-ambiguity attitude* within the *KMM* framework.

The second objective of the paper is to test the reliability of *KMM* in the five decision tasks aimed at studying a subject's attitude towards ambiguity. In all ambiguous decision tasks the subject faces always the same two (second-stage) lottery-outcomes. Thus,

[1] After Becker and Brownson (1964), the idea that information which reduces ambiguity has a positive value for ambiguity-averse subjects has been clearly stated within different decision-theoretic models: e.g., Quiggin (2007), using Machina (2004) concept of almost-objective acts; Attanasi and Montesano (2012), relying on the Choquet expected utility model. Moreover, focusing on a specific adaptation of *KMM*, Snow (2010) has proved that the value of information that resolves ambiguity increases with greater ambiguity and with greater ambiguity aversion. Attanasi and Montesano (2012) have found similar results within the Choquet model.

within the same treatment, each ambiguous task differs from the next one only because of the level of ambiguity of the decision setting and/or because of a first-degree stochastic improvement in the distribution of second-order probabilities. In particular, these tasks are designed such that, once a subject has been classified as coherent-ambiguity-averse, coherent-ambiguity-neutral or coherent-ambiguity-loving, the sign of the variation of her certainty equivalent from one task to the next one should depend only on this classification. Therefore, this sign should be predicted directly by the "sign" of her attitude towards ambiguity as determined within *KMM*. This means that, by construction, the verification of our main theoretical predictions in these tasks should be independent of the subject's degree of risk aversion as elicited in the five unambiguous tasks. Finding an effect of risk attitude over the behavioral verification of our theoretical hints would raise some doubts on the use of *KMM* as reference model for the tasks proposed in our experiment. The elicitation of risk attitude is also important in order to empirically state whether it influences the "sign" of the ambiguity attitude, i.e. which one of the three ambiguity attitudes (aversion, neutrality, or proneness) the subject could show. Our design is also aimed at stating whether this "sign" may depend on the riskiness of the second-stage lottery, i.e. on the spread of the difference of its two lottery-outcomes. In order to be consistent in the elicitation of risk attitude and of ambiguity attitude, we use the same pair of instruments for both attitudes. In particular, we elicit risk attitude both through a portfolio choice method and through a Becker–DeGroot–Marschak (1964) mechanism (henceforth, BDM). Correspondingly, we state choice-ambiguity aversion through the first method and value-ambiguity aversion through the second one. The combination of the two instruments has a twofold role. For risk attitude, it allows to check that both instruments lead to similar subjects' orderings. For ambiguity attitude, it enables to elicit separately the two features of (coherent)-ambiguity attitude introduced above within *KMM*. Concerning risk attitude, once verified the correlation between the two risk-aversion orderings, we rely on the results of the portfolio choice method: this has the advantage of imposing some theoretically derived constraints which allow to check whether the subject's selected portfolio is compatible with a constant absolute and/or a constant relative risk aversion specification. Concerning ambiguity attitude, throughout the article we consider as "classified subjects" only those who provide coherent answers under the two instruments. This provides a rationale for the term "coherent" to identify the kind of ambiguity attitude studied in this paper.

The third objective of the paper is to analyze how subjects' decisions under ambiguity react to different distributions of second-order probabilities. The experiment consists of three treatments, according to a between-subject design. The five unambiguous tasks do not vary among treatments, while the ambiguous tasks are different for each treatment according to the way in which uncertainty over the composition of the urns used to perform them is generated. More precisely, the first of these tasks relies on a 10-ball small urn with inside white and orange balls whose composition is not told to subjects. In all treatments this composition is generated through a random draw from a big urn, introduced in order to mimic *KMM* two-stage lottery approach. In treatment 1, the composition of the 10-ball small urn is determined through a Bernoullian process over a 50-white-50-orange balls big urn, thereby leading to a binomial distribution of second-order probabilities. In treatment 2, subjects are shown that second-order probabilities over the composition of the 10-ball small urn are uniformly distributed. In treatment 3, subjects have no information about the composition of the 10-ball small urn, although – to make it comparable to treatment 1 – ambiguity is generated through a two-stage lottery procedure similar to the one of the binomial treatment, but without giving any information about the composition of the big urn. The uniform distribution of the second-order probabilities in treatment 2 is clearly a mean-preserving spread of the binomial distribution obtained in treatment 1. Treatment 3 is intrinsically more ambiguous than treatment 1. Therefore, under ambiguity aversion, we expect that in the first ambiguous task of both the uniform and the unknown treatment the subject assigns a lower value to the ambiguous lottery than in the corresponding task of the binomial treatment. This should happen also in the remaining ambiguous tasks, given that, once the 10-ball small urn is generated, the way its composition is "modified" in order to vary the level of ambiguity and the distribution of second-order probabilities is the same in each treatment. Although our design is not within-subject, we can check the above stated predictions by comparing the distribution of subjects' decisions in the ambiguous tasks of the three treatments. This treatment comparison would hold only under the assumption that the distribution of subjects' degree of risk aversion does not differ among the three treatments. This is an additional motivation for eliciting risk attitude before looking at subjects' decisions in the ambiguous tasks.

Since Ellsberg (1961), several papers have empirically investigated the descriptive and predictive power of theories of decision making under ambiguity.[2] Some of them have investigated ambiguity attitude by explicitly excluding two-stage probability models.[3] Some others have produced experimental designs aimed at comparing the performance of *KMM* to that of non-expected utility models. Within this second group of studies, Halevy (2007) is surely the closest to our paper in terms of experimental design: we use a similar BDM mechanism to elicit risk attitude and value-ambiguity attitude. However, there are two main differences. On the one hand, compared to Halevy (2007) our experimental design allows to study the variation of the subject's certainty equivalent for a larger number of ambiguity levels and of distribution of second-order probabilities. This enables to formulate a richer set of theoretical relations that a subject's decisions have to satisfy in order

[2] Early literature is surveyed in Camerer and Weber (1992) and Camerer (1995).

[3] See for example Hey, Lotito and Maffioletti (2010), and Hey and Pace (2011): both experimental designs are aimed at testing only non-two stage probability models.

for *KMM* to pass the test. On the other hand, we propose the three treatments "binomial", "uniform" and "unknown" between subjects: Halevy (2007) proposes only the last two treatments, and, more importantly, within subjects. This enables him to examine the relation between attitude towards ambiguity and attitude towards reduction of compound (objective) lotteries, an issue that is outside the goals of our paper. Halevy (2007) finds that there is no unique theoretical decision model that captures all subjects' behavior. However, 15%-20% of his subjects are ambiguity neutral and able to reduce compound lotteries.[4] Another 35% of subjects exhibit ambiguity aversion (proneness) together with aversion (proneness) to mean preserving spreads in the second-order distribution. Both these categories of subjects are consistent with *KMM*.

Also Conte and Hey (2012) compare the performance of different theoretical decision models – expected utility, *KMM*, rank dependent expected utility, and Alpha model of Ghirardato, Maccheroni and Marinacci (2004) – with an experimental design quite different from Halevy (2007) and so from ours. They find results in favor of *KMM* both through individual estimates (56% of subjects have behavior consistent with *KMM*) and by classifying subjects through posterior probabilities of each of them being coherent with one over four types of preferences (50% for *KMM*). Their results clearly suggest that *KMM* performs the best among the four tested models.

Not all experimental studies find support for *KMM*. Ahn et al. (2011) find a result opposite to the one of Conte and Hey (2012) by performing a portfolio choice experiment aimed at investigating rank-dependent theories versus smooth ambiguity *à la KMM*. Their tests of significance suggest that the majority of subjects are well described by the subjective expected utility model. Moreover, among the remaining subjects, *KMM* is not able to explain the behavior of those subjects showing ambiguity aversion. Close to our paper, Ahn et al. (2011) implement an experimental design where subjects are asked to choose between different lotteries that duplicate the return of a portfolio containing a safe asset and an ambiguous asset. However, differently from their study, in each task of our experiment the asset contained in the portfolio is either safe or ambiguous. Moreover, we simplify the choice problem by limiting the choice set to only four possible portfolios, and by considering an uncertain environment with only two states of nature.

There are not so many experiments explicitly designed to test the *KMM* model only: Chakravarty and Roy (2009) is one of them. As in our paper, they try to separate attitude towards risk from that towards ambiguity, although using an experimental instrument – the multiple price list method – different from the two used in this paper. Their main objective is different from ours: investigating potential differences in subject's behavior under uncertainty over gains versus uncertainty over losses. For what concerns the domain

[4] In a similar experimental environment, Abdellaoui, Klibanoff and Placido (2011) find evidence that do not support the equivalence between ambiguity neutrality and reduction of compound lotteries.

of gains (the only one that can be compared with our design), they find a positive correlation between risk attitude and ambiguity attitude (although in the aggregate subjects are risk-averse and ambiguity-neutral). Their result is not isolated. Among several experimental studies about a possible relation between risk attitude and ambiguity attitude, only few papers find no correlation (Cohen, Jaffray and Said, 1987; Cohen, Tallon and Vergnaud, 2011), while many studies find a positive correlation (e.g., Lauriola and Levin, 2001; Di Mauro and Maffioletti, 2004; Bossaerts et al., 2010; Ahn et al., 2011).

The idea that attitudes toward risk and ambiguity can be qualitatively different in a subject has been stated by Andersen et al. (2009) within an experimental design that directly refers to *KMM*'s second-order acts. They estimate attitudes toward ambiguity, attitudes toward risk, and subjective probabilities in a rigorous manner within *KMM*, by making some parametric assumptions about the form of the distribution of the priors and the uncertain process. They find subjects who are risk-averse, and yet at the same time ambiguity-loving. To the best of our knowledge, this is the only experimental work that clearly states that attitudes toward risk and ambiguity can be opposite.

Our experimental findings are in line with Conte and Hey (2012) about the performance of *KMM*: almost 90% of subjects can be classified as averse, neutral or loving according to our operational definition of coherent-ambiguity-attitude. Moreover, more than 75% of classified subjects comply with our theoretical predictions in all ambiguous tasks of the experiment, independent of their degree of risk aversion. This percentage decreases if we consider only coherent-ambiguity-loving subjects or only the uniform treatment. Coherent-ambiguity-neutral subjects are those who have the highest percentage of compliance with *KMM* predicted behavior. Recalling that for these subjects *KMM* reduces to the expected utility model, we can relate this result to the one found by Halevy (2007) about the higher ability of ambiguity-neutral subjects to reduce compound lotteries.

Further, we find that highly-risk-averse subjects are more prone to coherent-ambiguity. This may lead to think to a sort of negative correlation between risk attitude and ambiguity attitude, as the one found by Andersen et al. (2009). This point requires a more thorough discussion. Notice that our experimental design only allows to state the "sign" of the subject's (coherent)-ambiguity attitude, i.e. whether she is (coherent)-ambiguity-averse, neutral or loving. Separating all classified subjects in three groups according to this "sign", we find significant differences in the distributions of the degree of risk aversion among the three groups. In particular, (coherent)-ambiguity-averse and (coherent)-ambiguity neutral subjects have on average a low degree of risk aversion, while the vast majority of (coherent)-ambiguity-loving subjects is highly-risk-loving. A careful analysis of the experimental data clarifies that this result is linked to the riskiness of the second-stage lottery the subject is assigned in the ambiguous tasks. The higher the degree of risk aversion, the riskier the chosen lottery in an ambiguous task, the higher the reduction of the value of this lottery when the distribution of outcomes becomes ambiguous.

Finally, we find that the percentage of coherently-ambiguity-averse subjects is lower in the binomial than in the uniform and in the unknown treatment. However, only the difference between the binomial and the unknown treatment is statistically significant. This result is partially in line with the findings of Abdellaoui, Klibanoff and Placido (2011), for the part where they state that attitude towards ambiguity and attitude towards compound risks are related but distinct, with this relationship being quite sensitive to the type of compound risks considered. They define as compound risk those decision tasks where the second-order probability distribution over the one-stage lotteries is objective: both our binomial and our uniform treatment belong to this category. Therefore, also in our case the relation between what they call ambiguity (our unknown treatment) and what they call compound risk depends on the type of compound risk considered, e.g. binomial or uniform. However, there are two crucial differences between our experimental design and that of Abdellaoui, Klibanoff and Placido (2011). First of all, although analyzing our uniform case, they do not analyze our binomial case, focusing instead on the hypergeometric case. And they actually find that it is the latter case the one having the strongest relationship with ambiguity attitude. Second, and more importantly, as Halevy (2007), they have a within-subject design, while in our experiment each subject only participates in one treatment, hence facing only one of the three second-order probability distributions that we generate in order to implement ambiguity: binomial, uniform or unknown.

The rest of the article is structured as follows. Section 2 describes our experimental design, by highlighting the motivations behind the ten decision tasks. Section 3 analyzes the five decision tasks under ambiguity and presents the main theoretical results. Section 4 presents the results of our experiment. Section 5 concludes.

# 2 Experimental Design

Experimental subjects were graduate students in Economics of the Toulouse School of Economics (TSE). Computerized sessions where conducted at the Laboratory of Experimental Economics of TSE.

A total of 105 experimental subjects (42 women, 63 men, average age = 23.7) participated in our experiment, with each subject participating only once. Average earnings were approximately € 20.50 per subject, including a € 5.00 show-up fee. The experiment was programmed using the z-Tree software (Fischbacher, 2007) and subjects were seated in isolated cubicles in front of computer terminals. Three treatments were run through a between subjects design, with the same number of subjects ($N = 35$) participating in each treatment. The number of subjects in each session varied from a minimum of 9 to a maximum of 18.[5]

The experiment consists of ten decision tasks per treatment. At the beginning of the experiment, participants were told how many tasks there are. However, instructions of the new task were given and read aloud prior to that task. After instructions were read aloud, the decision task appeared on the screen and participants had three minutes to answer the task. The average duration of the experiment was 65 minutes, including construction of the "unknown" small urns (only for treatment 1 and 3), performance of one over the ten tasks and participants' final payment. The final payment of each participant depended only on the choice made by this participant in the ten decision tasks and on some random draws which we explain in detail below. Only one of the ten decision tasks was randomly selected at the end of the experiment to determine participants' final earnings.

The ten tasks of our experimental design differ in terms of the elicitation method applied and/or of the scope of that elicitation (see Table 1). Tasks 1-5 do not vary among treatments, while tasks 6-10 are different for each treatment according to the way in which uncertainty over the composition of the urns used to perform these tasks is generated.

<table>
<tr><td></td><td>All Treatments</td><td>Treatment 1</td><td>Treatment 2</td><td>Treatment 3</td></tr>
<tr><td>Task</td><td>Elicitation Method</td><td colspan="3">Features of the lotteries</td></tr>
<tr><td>1-4</td><td>Portfolio Choice</td><td colspan="3" rowspan="2">Simple Lottery</td></tr>
<tr><td>5</td><td rowspan="2">BDM mechanism</td></tr>
<tr><td>6-9</td><td rowspan="2">Binomial Compound Lottery</td><td rowspan="2">Uniform Compound Lottery</td><td rowspan="2">Unknown (Compound) Lottery</td></tr>
<tr><td>10</td><td>Portfolio Choice</td></tr>
</table>

Table 1. Main features of the ten decision tasks.

In each task from 1 to 4 the experimental subject is shown the same small urn with 5 white balls and 5 orange balls inside. She is asked to choose among four simple lotteries of the type $l_t^j = (\overline{x}_t^j, 0.5; \underline{x}_t^j, 0.5)$, with $\overline{x}_t^j, \underline{x}_t^j \in \mathbb{R}_+$, $\overline{x}_t^j > \underline{x}_t^j$ for each $j$ and $t$, where $j = A, B, C, D$ indicates the four lotteries in each task and $t = 1, 2, 3, 4$ indicates the task (see Table 2). All $l_t^j$ in the four tasks rely on the same 5-5 balls small urn, with white balls assigned to the highest of the two outcomes, $\overline{x}_t^j$. Each $l_t^j$ differs from the other fifteen lotteries proposed in the four tasks in terms of both expected value and standard deviation. In particular, in each of the four portfolio choices, the higher the index of the lottery, the higher both its expected value and its standard deviation (see Table B in the Appendix). Let $j_t \in \{A, B, C, D\}$ be the index of the lottery chosen by the subject in

[5] For treatment 1, we run two sessions, respectively with 17 and 18 students. For treatment 2, we run three sessions, respectively with 16, 10 and 9 students. For treatment 3, we run three sessions, respectively with 12, 10 and 13 subjects.

task $t \in \{1,2,3,4\}$. If a task $t$ between 1 and 4 is selected to be paid at the end of the experiment, the subject plays for the pair of outcomes she has chosen in that task, namely $\overline{x}_t^{j_t}$ and $\underline{x}_t^{j_t}$. She is paid $\overline{x}_t^{j_t}$ if a white ball is randomly drawn from the 5-5 balls small urn and she is paid $\underline{x}_t^{j_t}$ otherwise.

| | *Task t=1* | | *Task t=2* | | *Task t=3* | | *Task t=4* | |
|---|---|---|---|---|---|---|---|---|
| | $\overline{x}_1^j$ | $\underline{x}_1^j$ | $\overline{x}_2^j$ | $\underline{x}_2^j$ | $\overline{x}_3^j$ | $\underline{x}_3^j$ | $\overline{x}_4^j$ | $\underline{x}_4^j$ |
| lottery $j = A$ | 12 | 6 | 11 | 6 | 20 | 14 | 19 | 14 |
| lottery $j = B$ | 16 | 4 | 14 | 4 | 24 | 12 | 22 | 12 |
| lottery $j = C$ | 20 | 2 | 17 | 2 | 28 | 10 | 28 | 8 |
| lottery $j = D$ | 24 | 0 | 20 | 0 | 32 | 8 | 34 | 4 |

Table 2. Portfolio Choice in Tasks 1-4: pair of lottery-outcomes.

Tasks 1 to 4 are called "portfolio choices" because the random outcome parallels the outcome of a portfolio with one risk-free asset and one risky asset. Indeed, we have that the outcome $l_t^j$ of choice $j$ in task $t$ can be written as $(w_t - \alpha_t^j)(1 + r_f) + \alpha_t^j(1 + \widetilde{y}_t)$, where $w_t$ can be interpreted as initial wealth in task $t$, and $\alpha_t^j$ is the euro investment in the risky asset: $r_f$ is the risk-free rate that is always normalized to 0, and $\widetilde{y}_t$ is the return of the risky asset in task $t$. The return of the risky asset can take two possible values $\overline{y}_t$ and $\underline{y}_t$ with equal probabilities. In Table 3, we reinterpret the portfolio contexts and portfolio choices in the four tasks.

| | $w_t$ | $\overline{y}_t$ | $\underline{y}_t$ | $\alpha_t^{j=A}$ | $\alpha_t^{j=B}$ | $\alpha_t^{j=C}$ | $\alpha_t^{j=D}$ |
|---|---|---|---|---|---|---|---|
| *Task t=1* | 8 | 4 | -2 | 1 | 2 | 3 | 4 |
| *Task t=2* | 8 | 3 | -2 | 1 | 2 | 3 | 4 |
| *Task t=3* | 16 | 4 | -2 | 1 | 2 | 3 | 4 |
| *Task t=4* | 16 | 3 | -2 | 1 | 2 | 4 | 6 |

Table 3. Reinterpretation of the lottery choices into portfolio choices for tasks 1 to 4.

In task 5 we propose to the subject the same pair of lottery-outcomes she has chosen in task 4, namely $\overline{x}_4^{j_4}$ and $\underline{x}_4^{j_4}$. We use again the same 5-5 balls small urn of tasks 1-4, with white balls again assigned to $\overline{x}_4^{j_4}$, in order to build the lottery $l_4^{j_4} = (\overline{x}_4^{j_4}, 0.5; \underline{x}_4^{j_4}, 0.5)$. Therefore, the subject's "initial endowment" in task 5 is her preferred lottery in task 4. In task 5 the subject has the possibility to sell $l_4^{j_4}$ through a BDM mechanism.[6] She is asked

[6] Given that the subject has to set the price at which to sell a random "initial endowment", we assign to her a lottery that she has just declared to prefer among four possible lotteries (task 4). Therefore, her "initial endowment" in task 5 (and, as will we see, in tasks 6-9) depends on the choice made in task 4,

to state the minimal price at which she is willing to sell $l_4^{j_4}$, by setting a price between $\underline{x}_4^{j_4}$ and $\overline{x}_4^{j_4}$. This reservation price should provide an approximation to the subject's certainty equivalent of $l_4^{j_4}$. Our BDM mechanism is very close to the one implemented by Halevy (2007). In contrast to Halevy (2007) however, we have four different lotteries $l_4^j$ for which a subject may state her reservation price and the set of possible "buying/selling prices" is discrete.[7]

In each of the tasks 6-9 we propose to the subject the same pair of lottery-outcomes that she has chosen in task 4, with white balls again assigned to $\overline{x}_4^{j_4}$, and we give her the possibility to sell the respective lottery through the same BDM mechanism of task 5.[8] However, the 10-ball small urn used to determine the likelihood of $\overline{x}_4^{j_4}$ and of $\underline{x}_4^{j_4}$ is not the same as in tasks 1-5.

In particular, the three treatments differ according to the way in which the composition of the 10-ball small urn used to perform task 6 is determined. More specifically:[9]

- **Treatment 1: Binomial**. The 10-ball small urn used to perform task 6 is generated from a *transparent big urn* containing 50 white balls and 50 orange balls. At the beginning of task 6, 10 balls are randomly drawn (one after the other, with replacement) from the big urn. The colors of these 10 balls determine the composition of the 10-ball small urn. The outcomes of the 10 random draws are not shown to the subjects. Therefore, at the moment when the subject states her reservation price in task 6, the composition of the *unknown small urn* is a *binomial* random variable taking 11 possible values.

although the subject does not know this in task 4.

[7]As we will see below, in our experiment none of the random draws from any urn is computerized. In the same spirit of concreteness, also our BDM mechanism is implemented through real tools. There are four different envelops, labeled respectively with letter $A$, $B$, $C$ and $D$, i.e. one for each lottery available in task 4. Each of these envelops contains eleven different numbered tickets. The distance between each numbers on the tickets in an envelop is the same, so to have the same number of tickets in each envelop, with the lowest numbered ticket being equal to $\underline{x}_4^j$ and the highest being equal to $\overline{x}_4^j$. In particular, the eleven tickets inside envelop $A$ are $14, 14.5, ..., 18.5, 19$; those inside envelop $B$ are $12, 13, ..., 21, 22$; those inside envelop $C$ are $8, 10, ..., 26, 28$; those inside envelop $D$ are $4, 7, ..., 31, 34$. The eleven tickets in envelop $j$ represent the set of possible prices of lottery $l_4^{j_4}$, with $j = A, B, C, D$. A ticket is randomly drawn from each envelop. The ticket drawn from envelop $j$ determines the random "buying price" for lottery $j$. Then, without knowing this price, the subject states her minimal selling price (reservation price) for her lottery $l_4^{j_4}$, by choosing one among the eleven possible prices for lottery $j$. In case task 5 is selected for payment at the end of the experiment, the following happens: if, for the lottery the subject owned in task 5, the subject's minimal selling price is lower than the respective random "buying price", the subject sells her lottery and is paid the latter price. Otherwise, she has to play her lottery, and her payoff ($\overline{x}_4^{j_4}$ or $\underline{x}_4^{j_4}$) depends on the ball randomly drawn from the small urn.

[8]In particular, the subject is told that in each task from 6 to 9 the "buying prices" for the four lotteries are respectively the same four numbered tickets randomly drawn at the beginning of task 5. Therefore, although unknown to the subject, the reference "buying price" for the assigned lottery is the same as in tasks 5-9. Furthermore, also the set of possible selling prices for each lottery is maintained constant among tasks 5-9.

[9]See Figure A in the Appendix.

- **Treatment 2: Uniform**. At the beginning of task 6, we show to the subject a *transparent construction urn*[10] which contains 11 transparent small urns of 10 balls each. Each of the 11 small urns has a different composition in terms of white and orange balls. One of the 11 small urns is randomly drawn from the construction urn. Therefore, at the moment when the subject states her reservation price, the composition of the *unknown small urn* is a (discrete) *uniform* random variable taking 11 possible states.

- **Treatment 3: Unknown**. The 10-ball small urn used to perform task 6 is generated from an *opaque big urn* containing 100 white and orange balls with *unknown* composition. As in treatment 1, at the beginning of task 6, we draw from the big urn (one after the other, with replacement) 10 balls whose color determines the composition of the 10-ball small urn. The outcomes of the 10 random draws are not shown to the subject. Therefore, the subject states her reservation price in task 6 without having any information about the composition of the *unknown small urn.* The reason why ambiguity is generated through a two-stage lottery is to make this treatment comparable to treatment 1.[11]

Tasks 7-9 involve the elimination of some possible compositions of the 10-ball *unknown small urn* used to perform task 6. At the beginning of task 7, the subject is told that if this task would be performed at the end of the experiment, the number of white balls in the *unknown small urn* will be between 3 and 7 (and so the number of orange balls). This would be implemented in the following way. In treatment 1 and treatment 3, 6 balls will be taken out from the unknown small urn constructed at the beginning of task 6 and replaced with 3 white balls and 3 orange balls. In treatment 2, 6 transparent small urns (the three with less than 3 white balls and the three with less than 3 orange balls) will be taken out from *transparent construction urn.* Task 8 (9) differs from task 7 only for the fact that in the *unknown small urn* the number of white (orange) balls will be between 3 and 10.

In each of the tasks 6-9 the subject, besides stating her reservation price for the lottery resulting from the corresponding unknown small urn, is also asked to guess the number of white balls in that urn. In case a task from 6 to 9 is randomly selected to be performed at the end of the experiment, the subject is paid additional € 5.00 if her guess of the number of white balls in the unknown small urn of that task was right.

[10] The term "construction urn" is borrowed from Klibanoff, Marinacci and Mukerji (2012). Epstein (2010) calls this the "second-order urn".

[11] In treatment 3, at the beginning of the experiment, after the unknown small urn has been constructed, a random draw is made from a 2-ball urn containing 1 white ball and 1 orange ball. The color of the randomly drawn ball is assigned to the highest of the two outcomes in each lottery in all the ten tasks of the experiment. This additional random draw has been inserted in the design of treatment 3 in order to make the subject aware that no manipulation from the experimenter is possible about the composition of the unknown small urn used for tasks 6-10.

Finally, task 10 is the same as task 4 in terms of elicitation method (portfolio choice) and in the set of possible pair of outcomes among which the subject has to pick one pair. However, the 10-ball small urn used to determine the likelihood of the chosen pair, namely $\overline{x}_{10}^{j_{10}}$ and $\underline{x}_{10}^{j_{10}}$, is the same as in task 6.

Notice that the subject in each task has no feedback about any random draw performed in any of the previous tasks. This is because only one of the tasks is selected and actually performed and only at the end of the experiment.[12] Therefore, in our experimental design the subject cannot make any updating neither about the actual composition of the unknown small urns nor about the random "buying prices" in the BDM mechanism. Notice also that in each session all the urns are real urns (not computerized) and all the random draws in the experiment (construction of the small urns, random "buying prices" in the BDM mechanism, selection of the task determining the subject's final earnings, performance of this task) are executed by one of the subjects (indicated in the experimental instructions as the "drawer"). This subject is randomly chosen before the beginning of the experiment among the subjects showing up for the experimental session. She does not participate in the experiment and is paid a fix amount of money ($ 20.00) independent of her random draws. The reason why we opted for a random human "drawer" instead of computerized random draws is to make participants in the experiment aware that no manipulation from the experimenter is possible in any of the random draws characterizing the experimental setting.

# 3 Theoretical Predictions

We rely on Klibanoff, Marinacci and Mukerji (2005) smooth ambiguity model (henceforth *KMM*). Therefore, we assume that the subject's preferences are represented by the von Neumann - Morgenstern Expected Utility (henceforth *EU*) function for simple lotteries and we relax reduction between first and second-order probabilities in two-stage lotteries in order to account for multiplicity/uncertainty of the possible compositions of the second-stage lottery.

Let us first present our predictions about subject's behavior in the first half of the experimental design, i.e., in the five tasks aimed at estimating her degree of risk aversion.[13]

[12] For tasks 1-4 and 10, performing the task means playing the chosen lottery (random draw of one ball from the 10-ball small urn). For tasks 5-9, it means playing the assigned lottery only if the subject's selling price is not lower than the random "buying price" for that lottery.

[13] Our ten decision tasks are shown to the subject always in the same order. The reason why we propose tasks 1-5 (which rely on the 5-5 balls small urn) always before tasks 6-10 is because we want to elicit subject's risk-aversion before introducing unknown/multiple small urns. Further, Halevy (2007) has shown that the (usually) higher reservation price for the 5-5 balls small urn (our task 5) is not a consequence of this urn being proposed before the unknown/multiple ones (our tasks 6-9). Finally, our theoretical results for the subject's reservation price in tasks $t = 6, ..., 9$ do not suggest that this price should be

These five tasks involve only simple lotteries.

Tasks 1-4 rest on the well-know result in expected utility theory (e.g., Pratt 1964) that the value of a simple lottery decreases if subject's risk aversion increases. The value of a simple lottery $l$ with possible returns $X$ is measured by its certainty equivalent $CE(l)$, which is defined by the following condition:

$$u(CE(l)) = EU(X),$$

where we assume that the utility function $u$ is increasing and that it is concave for risk-averse subjects and convex for risk-loving ones. From the previous relation, we have that $CE(l)$ decreases if we increase the concavity of $u$ in the sense of Arrow-Pratt. This implies that, for any task 1-4, an increase in risk aversion will never induce subjects to select a less risky lottery (in our case, a lottery with less exposure to the risky asset). Given the fact that lotteries $A, B, C, D$ correspond to different portfolios with an increasing exposure to the risky asset, we also know from Arrow (1964) that preferences are unimodal in $(A, B, C, D)$. Thus, if for example $C$ is preferred to $B$, it is also the case that it is preferred to $A$. If one limits the analysis to a set of utility functions that can be ordered by a single risk aversion parameter, this allows us to compute for each task three critical degrees of risk aversion, one for indifference between the least risky lottery $A$ and the riskier lottery $B$, one for indifference between lotteries $B$ and $C$, and one for indifference between lotteries $C$ and $D$.

Suppose first that the subject has $C$onstant $A$bsolute $R$isk $A$version (henceforth $CARA$), so that $u(c) = 1 - \exp(-ARA\ c)$ for all $c$. Under this specification, one can compute for task 1 the critical $ARA_1^{AB}$ that yields indifference between lotteries $A$ and $B$:

$$\frac{1}{2}\exp(-ARA_1^{AB}\overline{x}_1^A) + \frac{1}{2}\exp(-ARA_1^{AB}\underline{x}_1^A) = \frac{1}{2}\exp(-ARA_1^{AB}\overline{x}_1^B) + \frac{1}{2}\exp(-ARA_1^{AB}\underline{x}_1^B)$$

We obtain $ARA_1^{AB} = 0.077$. We can proceed in a similar fashion for the other pairs of lotteries $(B, C)$ and $(C, D)$, and for the other tasks 2, 3 and 4. Under $CARA$, it is well known (e.g., Gollier 2001) that the optimal portfolio composition is independent of initial wealth. From Table 3, we know that tasks 1 and 3 correspond to the same portfolio problem, but with different initial wealth levels respectively equal to $w_1 = 8$ and $w_3 = 16$. This implies that $ARA_1^{j,j+1} = ARA_3^{j,j+1}$ for all pairs of lotteries $(j, j+1)$. In other words, a $CARA$ subject should answer in exactly the same way for these two tasks. A similar observation can be made for tasks 2 and 4. The interpretation of Table 4 is the following: if the subject's $ARA$ is inside the interval $(0.054, 0.077]$, then she should pick the pattern

always increasing or always decreasing with $t$. Rather, it should depend on the subject's attitude towards ambiguity (e.g., see (5) and (7) below).

$(l_1^{j_1}, l_2^{j_2}, l_3^{j_3}, l_4^{j_4}) = (B, A, B, A)$ in the four portfolio choice problems and here $CARA$ index is 8. Notice that the *higher* the subject's degree of risk aversion, the *higher* her $CARA$ index, the *less* risky is the pattern she chooses. Table 4 shows that in our experiment more than 1/2 of subjects select lotteries in tasks 1 to 4 in a way that is compatible with $CARA$.[14] Results of the elicitation are provided disentangled by treatments in order to show possible differences in the distribution of the $CARA$ ordering among the three subject pools. Indeed, although the percentage of explained patterns is higher for subjects participating in treatment 2, we do not find any significant difference in the distribution of $CARA$ ordering in the three treatments (see section 4.1).

| Predicted pattern under $CARA$ | | | Experimental Data | | | | |
|---|---|---|---|---|---|---|---|
| Intervals of $ARA$ | Pattern $(l_1^{j_1}, l_2^{j_2}, l_3^{j_3}, l_4^{j_4})$ | Index $CARA$ | Tr. 1 | Tr. 2 | Tr. 3 | All | % TOT |
| $0.077 < ARA < +\infty$ | $(A, A, A, A)$ | 9 | 1 | 3 | 0 | 4 | 7.41% |
| $0.054 < ARA \leq 0.077$ | $(B, A, B, A)$ | 8 | 0 | 3 | 2 | 5 | 9.26% |
| $0.046 < ARA \leq 0.054$ | $(B, B, B, B)$ | 7 | 2 | 3 | 4 | 9 | 16.67% |
| $0.033 < ARA \leq 0.046$ | $(C, B, C, B)$ | 6 | 1 | 3 | 1 | 5 | 9.26% |
| $0.032 < ARA \leq 0.033$ | $(D, B, D, B)$ | 5 | 2 | 0 | 1 | 3 | 5.56% |
| $0.027 < ARA \leq 0.032$ | $(D, C, D, B)$ | 4 | 0 | 2 | 0 | 2 | 3.70% |
| $0.023 < ARA \leq 0.027$ | $(D, C, D, C)$ | 3 | 2 | 0 | 1 | 3 | 5.56% |
| $0.016 < ARA \leq 0.023$ | $(D, D, D, C)$ | 2 | 7 | 1 | 2 | 10 | 18.52% |
| $-\infty < ARA \leq 0.016$ | $(D, D, D, D)$ | 1 | 3 | 6 | 4 | 13 | 24.07% |
| No. of Observations | | | 18 | 21 | 15 | 54 | |
| % Explained | | | 51% | 60% | 43% | 51% | |

Table 4. Optimal answers for Tasks 1-4 under $CARA$.

Now, suppose that the subject has $C$onstant $R$elative $R$isk $A$verse (henceforth $CRRA$), so that $u(c) = c^{1-RRA}/(1-RRA)$ for all $c$. Under this specification, one can compute for task 1 the critical $RRA_1^{AB}$ that yields indifference between lotteries $A$ and $B$:

$$\frac{1}{2}\left(\overline{x}_1^A\right)^{1-RRA_1^{AB}} + \frac{1}{2}\left(\underline{x}_1^A\right)^{1-RRA_1^{AB}} = \frac{1}{2}\left(\overline{x}_1^B\right)^{1-RRA_1^{AB}} + \frac{1}{2}\left(\underline{x}_1^B\right)^{1-RRA_1^{AB}}$$

We obtain $RRA_1^{AB} = 1.320$. We proceed in a similar fashion for the other pairs of lotteries $(B, C)$ and $(C, D)$, and for the other tasks 2, 3 and 4. We order $C$onstant $R$elative $R$isk

[14] When checking if a behavioral pattern in tasks 1-4 is compatible with $CARA$, we allow up to only one possible deviation of at most one lottery $l_t^{j_t}$ from each of the theoretical patterns. For example, we assign a $CARA$ index to pattern $(B, C, B, B)$, namely index 7, but we do not to assign any index to $(B, D, B, B)$ or to $(C, C, B, B)$.

*Averse* (henceforth *CRRA*) subjects according to their lottery choices in tasks 1-4, as in Table 5. The interpretation of Table 5 is the same as in Table 4, with $RRA$ in place of $ARA$. Again, the *higher* the subject's degree of risk aversion, the *higher* her $CRRA$ index, the *less* risky is the pattern she chooses. Table 5 shows that in our experiment almost 3/4 of subjects have a quadruplet of choices that is compatible with $CRRA$. Although the percentage of explained patterns is higher for subjects participating in treatment 2, we do not find any significant difference in the distribution of $CARA$ ordering in the three treatments (see section 4.1)[15]

| Predicted pattern under $CRRA$ | | | Experimental Data | | | | |
|---|---|---|---|---|---|---|---|
| Intervals of $RRA$ | Pattern $(l_1^{j_1}, l_2^{j_2}, l_3^{j_3}, l_4^{j_4})$ | Index $CRRA$ | Tr. 1 | Tr. 2 | Tr. 3 | All | % TOT |
| $1.320 < RRA < +\infty$ | $(A, A, A, A)$ | 12 | 1 | 3 | 0 | 4 | 5.26% |
| $0.890 < RRA \leq 1.320$ | $(A, A, B, A)$ | 11 | 1 | 0 | 3 | 4 | 5.26% |
| $0.805 < RRA \leq 0.890$ | $(A, A, B, B)$ | 10 | 1 | 1 | 2 | 4 | 5.26% |
| $0.670 < RRA \leq 0.805$ | $(A, A, C, B)$ | 9 | 0 | 0 | 1 | 1 | 1.32% |
| $0.575 < RRA \leq 0.670$ | $(B, A, C, B)$ | 8 | 3 | 5 | 4 | 12 | 15.79% |
| $0.440 < RRA \leq 0.575$ | $(B, A, D, B)$ | 7 | 3 | 2 | 2 | 7 | 9.21% |
| $0.439 < RRA \leq 0.440$ | $(B, A, D, C)$ | 6 | 0 | 0 | 2 | 2 | 2.63% |
| $0.382 < RRA \leq 0.439$ | $(B, B, D, C)$ | 5 | 5 | 2 | 3 | 10 | 13.16% |
| $0.244 < RRA \leq 0.382$ | $(C, B, D, C)$ | 4 | 3 | 1 | 2 | 6 | 7.89% |
| $0.197 < RRA \leq 0.244$ | $(C, C, D, D)$ | 3 | 2 | 1 | 2 | 5 | 6.58% |
| $0.123 < RRA \leq 0.197$ | $(D, C, D, D)$ | 2 | 0 | 0 | 1 | 1 | 1.32% |
| $-\infty < RRA \leq 0.123$ | $(D, D, D, D)$ | 1 | 9 | 6 | 5 | 20 | 26.32% |
| No. of Observations | | | 28 | 21 | 27 | 76 | |
| % Explained | | | 80% | 60% | 77% | 72% | |

Table 5. Optimal answers for Tasks 1-4 under *CRRA*.

Notice that tasks 1-4 have been designed such that both a $CARA$ subject and a $CRRA$ subject, in order to show that she is not risk-averse (respectively, $RRA \leq 0$ and $ARA \leq 0$), should pick the riskiest pattern $(l_1^{j_1}, l_2^{j_2}, l_3^{j_3}, l_4^{j_4}) = (D, D, D, D)$, thereby being assigned ($CARA$ or $CRRA$) index 1. That is why, independently from the assumption of $CARA$ or $CRRA$, if the number of explained patterns is the same under the two specifications, we should find by construction the same percentage of non-risk-averse subjects. Indeed,

[15] As for Table 4, when checking if a behavioral pattern in tasks 1-4 is compatible with $CARA$, we allow up to only one possible deviation of at most one lottery $l_t^{j_t}$ from each of the theoretical patterns. For example, we assign a $CRRA$ index to pattern $(B, B, C, B)$, namely index 8, but we do not assign any index to $(B, C, C, B)$ or to $(C, B, C, B)$.

we find that this percentage is the same under the two specifications, although $CRRA$ captures a higher number of patterns than $CARA$: around 1/4 of the explained patterns are compatible with risk neutrality or risk loving. This percentage is close to the one found in other experimental studies on risk-aversion elicitation in simple lotteries.[16]

Through the BDM mechanism proposed in task 5, a risk-averse (-loving) subject should declare a certainty equivalent for $l_4^{j_4}$ – the simple lottery she has been assigned in task 5 – lower (higher) than its expected value, i.e.[17]

$$CE(l_4^{j_4}) < (>) EV(l_4^{j_4}) \ .$$

Given that in task 5 the lottery assigned to the subject is the same she has chosen in task 4, $l_4^{j_4}$, our portfolio choice problem provides a theoretical prediction on $CE(l_4^{j_4})$ in task 5 both under $CARA$ and under $CRRA$ specification. Suppose that the subject's pattern in tasks 1-4 is compatible with $CARA$. Then, given her $CARA$ index $h = 1, 2, ..., 9$, her $ARA$ belongs to the interval $(\underline{ARA}_h, \overline{ARA}_h]$ for each $h$. Hence, given $l_4^{j_4} = (\overline{x}_4^{j_4}, 0.5; \underline{x}_4^{j_4}, 0.5)$, $\underline{ARA}_h$, $\overline{ARA}_h$, it is

$$CE(l_4^{j_4}; ARA_h) = -\frac{1}{ARA_h} \ln \left( \frac{1}{2} \exp(-ARA_h \overline{x}_4^{j_4}) + \frac{1}{2} \exp(-ARA_h \underline{x}_4^{j_4}) \right) \qquad (1)$$

with $ARA_h = \underline{ARA}_h, \overline{ARA}_h$. Then, it should be $CE(l_4^{j_4}) \in (CE(l_4^{j_4}; \overline{ARA}_h), CE(l_4^{j_4}; \underline{ARA}_h)]$. If the subject's pattern in tasks 1-4 is compatible with $CRRA$, then, given $l_4^{j_4}$ and her $CRRA$ index $k = 1, 2, ..., 12$, her $RRA$ belongs to the interval $(\underline{RRA}_k, \overline{RRA}_k]$ for each $k$. Hence, given $l_4^{j_4} = (\overline{x}_4^{j_4}, 0.5; \underline{x}_4^{j_4}, 0.5)$, $\underline{RRA}_k$, $\overline{RRA}_k$, it is

$$CE(l_4^{j_4}; RRA_k) = \left( \frac{1}{2} (\overline{x}_4^{j_4})^{1-RRA} + \frac{1}{2} (\underline{x}_4^{j_4})^{1-RRA} \right)^{\frac{1}{1-RRA}} \qquad (2)$$

with $RRA_k = \underline{RRA}_k, \overline{RRA}_k$. Then, it should be $CE(l_4^{j_4}) \in (CE(l_4^{j_4}; \underline{RRA}_k), CE(l_4^{j_4}; \overline{RRA}_k)]$.

Let us now analyze subject's optimal behavior in tasks 6-10.

Consider a two-stage lottery $L$ where the second stage is represented by a set of $n+1$ lotteries $\widetilde{l}_\theta \sim (x_1, p_{1\theta}; ...; x_S, p_{S\theta})$, with possible payoffs $x_1 > ... > x_S$, $\theta \in \{0, ..., n\}$, $p_{s\theta} \geq 0$ and $\Sigma_s p_{s\theta} = 1$. The first stage is represented by the lottery $L$ having as possible outcomes the second-stage lotteries $\widetilde{l}_\theta$ with probabilities $(q_1, ..., q_n)$, with $q_\theta \geq 0$ and $\Sigma_{\theta=0}^n q_\theta = 1$.

[16] For example, Holt and Laury (2002) finds 34% of subjects with $RRA \in (-\infty, 0.150)$ in the "low real" payoffs task and 19% of subjects with $RRA \in (-\infty, 0.150)$ in the "20x real" payoffs task. Our tasks 1-4 contain lotteries whose expected payoffs are between Holt and Laury's "low real" and "20x real" lotteries expected payoffs.

[17] Karni and Safra (1987) has shown that the "certainty equivalent" of a lottery elicited using the BDM mechanism respects the preference ordering if and only if preferences satisfy the independence axiom. This is assumed in our theoretical analysis, given that we rely on $KMM$ and so on $EU$.

These are the second-order probabilities over the plausible probability distributions for $\widetilde{l_\theta}$.

In all treatments of our experiment, we characterize the impact of information on the value of lotteries in the *KMM* framework. Following *KMM*, it is assumed that the subject's ex ante utility is measured by:

$$u(CE(L)) = \phi^{-1}\left(\sum_{\theta=0}^{n} q_\theta \phi(EU(\widetilde{l_\theta}))\right) \tag{3}$$

with

$$EU(\widetilde{l_\theta}) = \sum_{s=1}^{S} p_{s\theta} u(x_s).$$

Function $u$ is a von Neumann-Morgenstern utility function, and $\phi$ captures subject's smooth ambiguity attitude. In fact, $\phi$ is a von Neumann-Morgenstern index function accounting for the attitude toward mean preserving spreads in the induced distribution of the expected utility of the one-stage lottery conditional to $\theta$, namely $EU(\widetilde{l_\theta})$. *KMM* define "smooth ambiguity aversion" and shows that it is equivalent to $\phi$ being concave. Therefore, it is equivalent to aversion to mean preserving spreads of the expected utility values induced by the second-order subjective probability and lottery $\widetilde{l_\theta}$. Then, defining function $v$ as $v = \phi \circ u$, the certainty equivalent of the two-stage lottery is

$$CE(L) = v^{-1}\left(\sum_{\theta=0}^{n} q_\theta v(CE(\widetilde{l_\theta}))\right), \tag{4}$$

where $CE(\widetilde{l_\theta})$ is the certainty equivalent of the one-stage lottery conditional to $\theta$. Function $v$ is a von Neumann-Morgenstern index function accounting for the attitude toward mean preserving spreads in certainty equivalents of the one-stage lottery conditional to $\theta$, namely $CE(\widetilde{l_\theta})$.

Recall that in each task of our experiment, there are only two possible payoffs, namely $\overline{x}, \underline{x} \in \mathbb{R}_+$, $\overline{x} > \underline{x}$. Therefore, the small urn is represented by the 10-ball one-stage lottery $\widetilde{l_\theta} \sim (\overline{x}, p_\theta; \underline{x}, 1 - p_\theta)$, where $p_\theta = \frac{\theta}{10}$ is the objective probability given by the ratio of the number of white balls $\theta \in \{0, 1, ..., 10\}$ over 10. The second-order probabilities on the possible compositions of the small urn depends upon the treatment under consideration. In tasks 6-10 of treatments 1 and 2, the probability distribution $(q_0, ..., q_{10})$ over the one-stage lotteries is objective. It is binomial in treatment 1 and uniform in treatment 2. Therefore, in treatment 1, given a task from 6 to 10, the second-order objective probabilities are always less dispersed than in the corresponding task in treatment 2.[18]

[18] In particular, in treatment 1 the objective second-order probabilities are as follows: In tasks 6 and 10, $q_{10} = q_0 = 1/1024 \simeq 0.1\%$, $q_9 = q_1 = 10/1024 \simeq 1\%$, $q_8 = q_2 = 45/1024 \simeq 4.4\%$, $q_7 = q_3 = 120/1024 \simeq 11.7\%$, $q_6 = q_4 = 210/1024 \simeq 20.5\%$, and $q_5 = 252/1024 \simeq 24.6\%$; in task 7, $q_7 = q_3 = 1/16 = 6.25\%$,

Recall that in our experiment second-stage lotteries in tasks 5-9 have the same pair of outcomes, so that their variety depends only on first-order probabilities. Notice that the simple lottery in task 5, $l_4^{j_4}$, is analogous to a two-stage lottery with all second-stage lotteries $\widetilde{l}_{\theta=5}$ being $l_4^{j_4}$, namely $L_5 := (q_1, l_4^{j_4}; q_2, l_4^{j_4}; .., q_n; l_4^{j_4})$. We trivially assume that $L_5 \sim l_4^{j_4}$. In order to identify whether a subject shows aversion, neutrality or proneness to ambiguity, we can rely on comparing the subject's answer to tasks 5 and 6. Recall that in task 5, the subject is asked to value the unambiguous lottery that he/she selected in task 4. In task 6, the subject is asked to do the same thing for an ambiguous urn with the same expected probability for the two outcomes. This suggests the following operational Definition 1.

**Definition 1** ***(value-ambiguity attitude)*** *Call $CE(L_t)$ the subject's reservation price for the two-stage lottery assigned in task $t \in \{5, 6\}$. It can be interpreted as the certainty equivalent of the two-stage lottery in task $t$. Then, a subject is value-ambiguity-averse if $CE(L_6) \leq CE(L_5)$. She is value-ambiguity-neutral if $CE(L_6) = CE(L_5)$. She is value-ambiguity-loving if $CE(L_6) \geq CE(L_5)$.*

In short, a value-ambiguity-averse subject values an ambiguous lottery less than its unambiguous equivalent with the same mean probabilities. In the *KMM* model, this is true if the subject's $\phi$ function is concave.

Our experimental design offers us an alternative to test ambiguity-aversion by comparing the subject's answers to tasks 4 and 10. Remember that the two possible outcomes in lotteries $\{A, B, C, D\}$ are the same in the two tasks. The difference lies in the fact that probabilities are unambiguously 1/2 in task 4, whereas there are ambiguous in task 10, with mean 1/2. Define a dispersion order $\succ$ on set $\{A, B, C, D\}$, such that $D \succ C \succ B \succ A$. A more dispersed lottery is equivalent to a portfolio containing a larger share invested in the risky asset.

**Definition 2** ***(choice-ambiguity attitude)*** *Call $j_t \in \{A, B, C, D\}$ the index of the lottery chosen by a subject in task $t \in \{4, 10\}$. Then, a subject is choice-ambiguity-averse if $j_{10} \preceq j_4$, i.e., if lottery $j_{10}$ is not more dispersed than lottery $j_4$. She is choice-ambiguity-neutral if $j_{10} = j_4$. She is choice-ambiguity-loving if $j_{10} \succeq j_4$, i.e., lottery $j_{10}$ is not less dispersed than $j_4$.*

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$q_6 = q_4 = 4/16 = 25\%$, and $q_5 = 6/16 = 37.5\%$; in task 8, $q_{10} = q_3 = 1/128 \simeq 0.8\%$, $q_9 = q_4 = 7/128 \simeq 5.5\%$, $q_8 = q_5 = 21/128 \simeq 16.4\%$, $q_7 = q_6 = 35/128 \simeq 27.3\%$; in task 9, $q_7 = q_0 = 1/128 \simeq 0.8\%$, $q_6 = q_1 = 7/128 \simeq 5.5\%$, $q_5 = q_2 = 21/128 \simeq 16.4\%$, $q_4 = q_3 = 35/128 \simeq 27.3\%$. All other $q_\theta$ are zero. In treatment 2 the objective second-order probabilities are: In tasks 6 and 10, $q_\theta = 1/11 \simeq 9.1\%$ for every $\theta = 0, 1, ..., 10$; in task 7, $q_\theta = 1/5$ for every $\theta = 3, 4, ..., 7$; in task 8, $q_\theta = 1/8$ for every $\theta = 3, 4, ..., 10$; in task 9, $q_\theta = 1/8$ for every $\theta = 0, 1, ..., 7$. All other $q_\theta$ are zero.

Equivalently, a choice-ambiguity-averse subject always reduces her demand for the risky asset when the distribution of outcomes becomes ambiguous. In the *KMM* smooth ambiguity aversion framework, Gollier (2012) has shown that it is not true in general that the concavity of the $\phi$ function implies the choice-ambiguity-aversion of the subject. In other words, a smooth ambiguity-averse subject could have a larger demand for the ambiguity asset than another ambiguity-neutral subject with the same risk aversion. However, Gollier (2012) provides sufficient conditions on the structure of the two-stage uncertainty to re-establish the link between the concavity of $\phi$ and ambiguity aversion. One of these sufficient conditions is that the different second-stage distributions of the risky asset can be ordered by the Monotone Likelihood Ratio stochastic order. Because the set of distributions $\{(3, p_\theta; -2, 1-p_\theta) \,|\theta = 0, ..., 10\}$ can always be ordered by Monotone Likelihood Ratio, we conclude that, in the *KMM* framework, the two definitions of value-ambiguity aversion and choice-ambiguity aversion are equivalent, and are satisfied if $\phi$ is concave. This justifies the following definition.

**Definition 3** ***(coherent-ambiguity attitude)*** *A subject is coherently-ambiguity-averse if $CE(L_6) \leq CE(L_5)$ and $j_{10} \preceq j_4$, with at least one of the two relations holding strictly. She is coherently-ambiguity-neutral if $CE(L_6) = CE(L_5)$ and $j_{10} = j_4$. She is coherently-ambiguity-loving if $CE(L_6) \geq CE(L_5)$ and $j_{10} \succeq j_4$, with at least one of the two inequalities holding strictly.*

Our operational definition of coherent-ambiguity attitude is based on a double-check: we compare subject's behavior in task 5 versus task 6 and in task 4 versus task 10. The first comparison tells us whether, given the two second-stage lottery-outcomes, she prefers to know first-order probability $p_\theta$ than facing a mean-preserving spread of second-order probabilities over the all possible $p_\theta$. The second comparison tells us whether she prefers a less risky lottery (a less dispersed performance of the portfolio in Table 3) where this mean-preserving spread takes place.

Let us now analyze how the certainty equivalent of the two-stage lottery varies when moving from task 6 to tasks 7, 8 or 9 and whether this variation depends on the fact the subject is ambiguity-averse. Let us first compare $CE(L_7)$ to $CE(L_6)$. Remember that, in each of our three treatments, the two-stage lottery in task 7 is obtained from task 6 by symmetrically eliminating the plausibility of the extreme urns $\theta = 0, 1, 2, 8, 9, 10$. This implies that we must objectively have $q_\theta = 0$ in task 7 for these $\theta$. Compared to task 6, the subject's subjective second-order probabilities must be symmetrically transferred from the extreme urns to the less dispersed urns $\theta = 3, ..., 7$. This yields a mean-preserving contraction in the distribution of $\widetilde{U} \sim (EU(\widetilde{l}_0), q_0; ...; EU(\widetilde{l}_{10}), q_{10})$, as we show now. In the remainder of this section, let us normalize $u$ in such a way that $u(\underline{x}_4^{j_4}) = 0$ and $u(\overline{x}_4^{j_4}) = 1$, so that $EU(\widetilde{l}_\theta) = p_\theta$.

**Lemma 4** *Consider a symmetric random variable* $\widetilde{p} \sim (p_0, q_0; ...; p_n, q_n)$*, with* $p_\theta = \theta/n$*,* $q_\theta = q_{n-\theta}$ *for all* $\theta$*, and* $n > 2$*. Consider another symmetric random variable* $\widetilde{p}' \sim (p_0, q_0'; ...; p_n, q_n')$ *on the same support, but with* $q_0' = q_n' = 0$ *and* $q_\theta' = q_{n-\theta}' \geq q_\theta = q_{n-\theta}$ *for all* $\theta \in \{1, ..., n-1\}$*. It implies that* $E\phi(\widetilde{p}') \geq E\phi(\widetilde{p})$ *for all concave functions* $\phi$*, i.e., that* $\widetilde{p}'$ *is a Rothschild-Stiglitz mean-preserving contraction of* $\widetilde{p}$*.*

**Proof.** Proof: Observe that, by symmetry, we have that

$$E\widetilde{p} = \sum_{\theta=0}^{n} q_\theta \frac{\theta}{n} = \sum_{\theta=0}^{n/2} q_\theta \left( \frac{\theta}{n} + \frac{n-\theta}{n} \right) = \sum_{\theta=0}^{n/2} q_\theta = \frac{1}{2}.$$

Because the same observation can be made for $\widetilde{p}'$, we have that $E\widetilde{p} = E\widetilde{p}' = 1/2$. Because $\widetilde{p}'$ is obtained from $\widetilde{p}$ by a transfer of probability mass from the extreme states to the center of the distribution, we conclude that $\widetilde{p}$ is a mean-preserving spread of $\widetilde{p}'$. This concludes the proof. ■

Repeating this lemma three times, we obtain that $CE(L_7)$ must be larger than $CE(L_6)$ under smooth ambiguity aversion. Because $L_7$ is still ambiguous, we also have that $CE(L_7)$ is smaller than $CE(L_5)$. Thus we have that $CE(L_5) \geq CE(L_7) \geq CE(L_6)$. The opposite result would hold under smooth ambiguity-loving. Observe that a crucial assumption for the lemma is the symmetry of the second-order probability distributions. In treatments 1 and 2, the second-order probability distribution on the composition of the small urn is either binomial or uniform, which are clearly symmetric. In treatment 3, the symmetry of the second-order distribution will depend upon the subject's beliefs on the composition of the big urn from which the small urn is built. However, the principle of insufficient reason suggests that the subject has symmetric beliefs on the composition of the big urn, and therefore on the composition of the small urn generated by the Bernoullian process. Under this principle, we can write the following proposition.

**Proposition 5** *If the subject is ambiguity-averse, then* $CE(L_5) \geq CE(L_7) \geq CE(L_6)$*. If she is ambiguity-loving, then* $CE(L_5) \leq CE(L_7) \leq CE(L_6)$*. If she is ambiguity-neutral, then* $CE(L_5) = CE(L_7) = CE(L_6)$*.*

Let us now compare tasks 8 and 9 to task 6. Task 8 is similar to task 6 except that the worst urns have been eliminated. Proposition 6 shows that the certainty equivalent of the two-stage lottery proposed in that task 8 is greater than the one of the two-stage lottery proposed in task 6, whatever the degree of ambiguity of the subject, i.e. independently of the fact that she is ambiguity-averse, neutral or loving. The opposite result prevails for task 9, in which the best urns have been removed. Therefore, comparison between task 6, 8 and 9 always leads to $CE(L_8) \geq CE(L_6) \geq CE(L_9)$, whatever the subject's attitude toward ambiguity.

**Proposition 6** *Suppose that new information implies that the worst (best) urns become implausible, without reducing the probability $q_\theta$ of any of the other urns. This new information raises (reduces) the certainty equivalent of the lottery independent of the degree of ambiguity aversion.*

**Proof.** Because $\phi$ is increasing and concave, it is obvious that any first-degree or second-degree stochastic dominance improving shift in the distribution of $(q_0, EU(\widetilde{l}_0); ...; q_n, EU(\widetilde{l}_n))$. Because $p_\theta = \theta/n$ is increasing in $\theta$, so is $EU(\widetilde{l}_\theta) = p_\theta u(\overline{x}_4^{j_4}) + (1 - p_\theta)u(\underline{x}_4^{j_4})$. Suppose that a new information makes the worst lotteries $(\widetilde{l}_0, \widetilde{l}_1, ..., \widetilde{l}_m)$, $m < n$, totally implausible. This implies that the new second-order probabilities take the form $(\widehat{q}_0, \widehat{q}_1, ..., \widehat{q}_n)$, with $\widehat{q}_1 = \widehat{q}_2 = ... = \widehat{q}_m = 0$. This yields a first-degree stochastic improvement if $\widehat{q}_i \geq q_i$ for all $i \in \{m+1, ..., n\}$. Therefore, under the assumption that both $u$ and $\phi$ are strictly monotone, this new information raises the certainty equivalent of the lottery independent of the degree of ambiguity aversion. Of course, the symmetric case also holds: Suppose that new information implies that the best scenarii become implausible, without reducing the probability $q_\theta$ of any of the other scenarii. This new information reduces the certainty equivalent of the lottery independent of the degree of ambiguity aversion. ■

This result also applies to the comparison between task 8 (task 9) and task 7: the certainty equivalent of the two-stage lottery proposed in the former task must be greater (smaller) than the one of the two-stage lottery proposed in task 6, whatever the degree of ambiguity of the subject. Task 7 may be seen as a modification of Task 9 through new information implying that the worse scenarii become implausible, without reducing the probability $q_\theta$ of any of the other scenarii in task 9. Task 7 may be also seen as a modification of Task 8 through new information implying that the best scenarii become implausible, without reducing the probability $q_\theta$ of any of the other scenarii in task 8. Therefore, we must have $CE(L_8) \geq CE(L_7) \geq CE(L_9)$ independent of the shape of $\phi$.

Let us now try to establish the complete ranking of the values of tasks 5 to 9 under smooth ambiguity aversion. We have seen earlier that smooth ambiguity aversion implies that $CE(L_5) \geq CE(L_7) \geq CE(L_6)$. Combining these three sequences of inequalities implies that, under smooth ambiguity aversion, we have that

$$\left.\begin{array}{c} CE(L_5) \\ CE(L_8) \end{array}\right\} \geq CE(L_7) \geq CE(L_6) \geq CE(L_9), \tag{5}$$

independent of subject's attitude toward risk. The only degree of freedom under smooth ambiguity aversion is thus about the relative values of task 5 (no ambiguity: $q_5 = 1$) and task 8 (ambiguity with worst urns eliminated: $q_0 = q_1 = q_2 = 0$). If ambiguity aversion is small enough, i.e., if the concavity of $\phi$ is small, then the large expected probability of the high outcome enjoyed in task 8 will dominate the ambiguity aversion effect to yield $CE(L_8) \geq CE(L_5)$, otherwise $CE(L_8) < CE(L_5)$. The following result is a direct consequence of Gollier (2001, Section 6.3.2).

**Proposition 7** *Suppose that a subject prefers the unambiguous lottery $L_5$ to the ambiguous lottery $L_8$ (this is possible only under ambiguity aversion). Then, an increase in ambiguity aversion in the* KMM *model can never reverse this ranking.*

This implies that, assuming similar attitudes toward risk, any subject with $CE(L_5) < CE(L_8)$ has a smaller degree of smooth ambiguity aversion than any subject with $CE(L_5) \geq CE(L_8)$. Thus, comparing the values of task 5 and 8 for ambiguity-averse subjects allows us to get some information about their degree of ambiguity aversion.

Of course, in the limit case of smooth ambiguity-neutrality, we must have that

$$CE(L_8) > CE(L_5) = CE(L_6) = CE(L_7) > CE(L_9), \tag{6}$$

independent of subject's attitude toward risk. Finally, for an ambiguity-loving subject,we obtain that

$$CE(L_8) \geq CE(L_6) \geq CE(L_7) \geq \begin{cases} CE(L_5) \\ CE(L_9) \end{cases}, \tag{7}$$

independent of her attitude toward risk. If the degree of ambiguity proneness is small enough, i.e., if the convexity of $\phi$ is small, then the low expected probability of the high outcome faced in task 9 will dominate the attractiveness of this ambiguous lottery for ambiguity-loving subjects, so that $CE(L_9) < CE(L_5)$, otherwise $CE(L_9) \geq CE(L_5)$.

**Proposition 8** *Suppose that a subject prefers the unambiguous lottery $L_5$ to the ambiguous lottery $L_9$ (this is possible also under ambiguity proneness). Then, a concave transformation of the $\phi$ function in the* KMM *model can never reverse this ranking.*

Thus, comparing the values of task 5 and 9 for ambiguity-loving subjects allows us to get some information about their degree of ambiguity proneness.

The next corollary shows the difference among certainty equivalents of two-stage lotteries in the same task of different treatments. The comparison of treatments 1 and 2 is the easiest. The uniform distribution of the second-order probabilities in treatment 2 is clearly a mean-preserving spread of the binomial distribution obtained in treatment 1. Comparing the certainty equivalents for treatments 1 and 3 is more difficult. In both treatments, a Bernoullian process is applied to build the small urn, but the parameter $p$ of the Bernoulli distribution is 1/2 in treatment 1, whereas it is unknown in treatment 3. If one accepts the principle of insufficient reason, then one may assume that the third-order probabilities on parameter $p$ yields $Ep = 1/2$. Under this assumption, treatment 3 always yields a mean-preserving spread of the second-order probability distribution $(q_0, ..., q_n)$. Under ambiguity aversion, this yields a reduction of the certainty equivalents. This yields the following result.

**Corollary 9** *If the subject is ambiguity-averse (-loving), then $CE_t$ is greater (smaller) in treatment 1 than in treatments 2 and 3 for every $t = 6, 7, 8, 9$.*

By combining Proposition 7, Proposition 8 and Corollary 9, we obtain an interesting behavioral prediction about possible treatment differences. Although our design is not within-subject, we have seen from Table 4 and Table 5 that the distribution of the degree of risk aversion does not differ among the three treatments, both if we use a $CARA$ and if we use a $CRRA$ specification. Then, if we assume that the distribution of the degree of ambiguity aversion is the same among the three treatments, then for similar degrees of risk aversion we should find that the percentage of ambiguity-averse subjects with $CE(L_5) \geq CE(L_8)$ is lower in treatment 1 than in treatments 2 and 3. By combining Proposition 8 and Corollary 9 for ambiguity-loving subjects: if the distribution of the degree of ambiguity proneness is the same among the three treatments, then the percentage of ambiguity-loving subjects with $CE(L_9) \geq CE(L_5)$ is lower in treatment 1 than in treatment 2 and 3 inside the same class of risk aversion.

# 4 Experimental Results

In this section we present our experimental results. First of all, in section 4.1 we briefly analyze the results of the elicitation of subject's risk aversion through the portfolio choice method of tasks 1-4. Then, in section 4.2, we classify subjects according to their ambiguity attitude relying on the operational definition introduced in section 3. In section 4.3 we test the main theoretical predictions derived in section 3. In section 4.4 we analyze treatment effects on the distribution of subjects' beliefs over the second-order probabilities and on the two-stage lotteries certainty equivalents given the task.

## 4.1 Risk aversion elicitation

The portfolio choice method used at the beginning of the experiment enables all subjects to face the same set of lotteries in tasks 1-4: this allows to build a risk-attitude ordering of subjects independent of $l_4^{j_4}$, the lottery chosen in task 4. This is the first reason why we prefer to rely on it rather than on the certainty equivalent elicited in task 5, which instead depends on $l_4^{j_4}$. Further, our portfolio choice method has the advantage of imposing some theoretically derived constraints which allow to check whether the subject's selected pattern is compatible with a $CARA$ and/or a $CRRA$ specification. This provides an empirical verification of what in many experimental studies on risk aversion is generally assumed.

First, we check whether the portfolio choice method elicitation (tasks 1-4) leads to the same ordering in terms of $CE(L_5)$ as the (more standard) BDM mechanism proposed in task 5. Indeed, we find a positive correlation (coeff. = 0.46) and highly significant (*P-value* = 0.000) between $CE(L_5)$ as predicted by the $CARA$ ordering derived from the selected pattern in tasks 1-4 (see Table 4) and the one elicited through the BDM mechanism in task 5. If we use the $CRRA$ in place of the $CARA$ ordering, the former correlation is slightly lower (coeff. = 0.36) and again statistically significant (*P-value* = 0.006).[19]

Second, we check whether there is any significant difference among the three treatments in the distribution of *CARA* indexes or in the distribution of *CRRA* indexes. Although the percentage of explained patterns under each specification is different in the *Uniform* treatment (see respectively Table 4 and Table 5), we do not find any significant difference in the distribution of risk-aversion ordering in the three treatments. This is what Figure C in the Appendix seems to suggest, both if we rely on the $CARA$ and the *CRRA* specification. To provide support to the graphical representation, we have tested the differences in distribution of $CARA$ ordering and *CRRA* ordering in the three treatments with two different test: a Kruskal-Wallis test[20] and a Kolmogorov-Smirnov equality-of-distributions test[21] with a pairwise comparison between treatments.[22]

Therefore, both orderings are correlated with the certainty equivalent of task 5 and lead to similar distributions of risk attitude among treatments. Without assuming whether subjects are $CARA$ or $CRRA$, we use both specifications when analyzing possible relations between the subject's degree of risk aversion and her behavior in tasks 6-10.

It is true that all the theoretical predictions derived in section 3 within the *KMM* framework should hold whatever the subject's risk aversion. Nevertheless we look for possible correlations between risk attitude and ambiguity attitude. Further, we want to check that risk attitude does not play any role when testing our main theoretical predictions, which we have shown to hold independently of the subject's risk attitude. Finally, including risk aversion as an explanatory variable in our econometric analysis may be useful in order to provide an experimental answer to some open theoretical questions as the sign

[19] More precisely, as $CE(L_5)$ predicted by the $CARA$ ordering we consider the average between $CE(l_4^{j_4};\overline{ARA}_h)$ and $CE(l_4^{j_4};\underline{ARA}_h)$ in (1), for $h = 1, 2, ..., 9$. Similarly, for the $CE(L_5)$ predicted by the $CRRA$ ordering we consider the average between $CE(l_4^{j_4};\overline{RRA}_k)$ and $CE(l_4^{j_4};\underline{RRA}_k)$ in (2), for $k = 1, 2, ..., 12$.

[20] The Kruskal-Wallis equality-of-populations rank test (non-parametric) tests the hypothesis that several samples are from the same population.

[21] The Kolmogorov-Smirnov test (non-parametric) compares two observed distributions f(x) and g(x). The procedure involved forming the cumulative frequency distributions F(x) and G(x) and finding the size of the largest difference between these. The hypothesis that has been tested is whether the two observed distributions are equal (we perform a pairwise comparison between treatment 1-treatment 2, treatment 1-treatment 3 and treatment 2-treatment 3).

[22] According to the Kruskal-Wallis test we cannot reject the null hypothesis of equality of distributions (*P-value* = 0.401 for *CARA* and *P-value* = 0.357 for *CRRA*). The Kolmogorov-Smirnov test confirms this result.

of $CE(L_8) - CE(L_5)$ for ambiguity-averse subjects or the sign of $CE(L_9) - CE(L_5)$ for ambiguity-loving ones. Notice that for ambiguity-neutral subjects it is always $CE(L_8) > CE(L_5)$ and $CE(L_9) < CE(L_5)$.

## 4.2 Ambiguity aversion elicitation

In Table 6, we classify subjects being ambiguity-averse, ambiguity-neutral and ambiguity-loving in each treatment according to Definition 3 (coherent-ambiguity-attitude). Notice that almost 1/2 of the classified subjects are ambiguity-averse, while less than 1/5 are ambiguity-loving. Only 13 subjects (less than 12% of the sample) participating in our experiment cannot be classified according to Definition 3: around half of them are ambiguity-averse according to Definition 1 (value-ambiguity attitude) and ambiguity-loving according to Definition 2 (choice-ambiguity attitude). The other half of them are value-ambiguity-loving and choice-ambiguity-averse.[23] Given the small percentage of unclassified subjects, we can conclude that concavity of the $\phi$ function implies choice-ambiguity-aversion in our experimental tasks. This was exactly our theoretical prediction, given that the different second-stage distributions of the risky asset have been set such that they can be ordered according to the Monotone Likelihood Ratio stochastic order (see Gollier, 2012). Indeed, the correlation between strong value-ambiguity-aversion ($CE(L_6) < CE(L_5)$) and strong choice-ambiguity-aversion ($j_{10} \prec j_4$) is positive, not very high (coeff. $= 0.18$), but statistically significant (*P-value* $= 0.074$). We will further analyze this last result at the end of section 4.3, by showing that in our sample subjects with strong choice-ambiguity-aversion are usually non-strongly choice-ambiguity-averse.

| | Binomial | Uniform | Unknown | TOTAL |
|---|---|---|---|---|
| *coherent-AA* | 13 | 15 | 17 | 45 |
| *coherent-AN* | 16 | 5 | 8 | 29 |
| *coherent-AL* | 5 | 9 | 4 | 18 |
| Total Classified | **34** | **29** | **29** | **92** |
| *value-AA* & *choice-AL* | 1 | 4 | 2 | 7 |
| *value-AL* & *choice-AA* | 0 | 2 | 4 | 6 |
| Total Unclassified | 1 | 6 | 6 | 13 |

Table 6. Classification of (coherent) ambiguity attitude according to Definition 3.

[23] Although the number of unclassified subjects is lower in the *Binomial* than in the other two treatments, unclassified subjects are not statistically different from classified ones neither with respect to *CARA* or *CRRA* ordering nor with respect to the lottery chosen in task 4.

From Table 6, one can also see that the percentage of classified subjects being ambiguity-averse is lower in the *Binomial* than in the *Uniform* treatment and in the *Unknown* treatment. Further, the percentage of classified subjects being ambiguity-neutral is higher in the *Binomial* than in the other two treatments.

Let us define the "*sign*" of the ambiguity attitude as being negative if the subject is ambiguity-averse, null if she is ambiguity-neutral and positive if she is ambiguity-loving. Looking at the multinomial logistic regression of the sign of the ambiguity attitude over the treatment, we find that the relative risk ratio for being ambiguity-neutral versus being ambiguity-averse is 0.27 (*P-value* = 0.040) when switching from the *Binomial* to the *Uniform* treatment and 0.38 (*P-value* = 0.091) when switching from the *Binomial* to the *Unknown* treatment. In other words, the expected probability of being ambiguity-neutral seems to be higher for subjects who participate in the *Binomial* treatment. Table 6 also shows that the percentage of subjects being ambiguity-loving is lower in the *Binomial* than in the *Uniform* treatment, but not in the *Unknown* treatment. However, a multinomial logistic regression of the ambiguity attitude over the treatment shows that the relative risk ratio for being ambiguity-loving versus being ambiguity-averse is 1.56 (not statistically significant: *P-value* = 0.510) when switching from the *Binomial* to the *Uniform* treatment and 0.62 (not statistically significant: *P-value* = 0.521) when switching from the *Binomial* to the *Unknown* treatment.[24]

A possible explanation of this result relies on Corollary 9. Given the degree of ambiguity attitude, $|CE(L_6) - CE(L_5)|$ is lower in the *Binomial* than in the *Uniform* treatment. This is due to the fact that the distribution of second-order probabilities is less dispersed in the *Binomial* than in the *Uniform* treatment. Moreover, recall that the set of possible certainty equivalent values that a subject may select is discrete. Therefore, if a subject is slightly-ambiguity-averse or slightly-ambiguity-loving, it is more likely for her to choose $CE(L_6) = CE(L_5)$ in the *Binomial* than in the *Uniform* treatment.[25]. The intuition based on Corollary 9 applies also to the comparison between the *Binomial* and the *Unknown* treatment. Indeed, the percentage of ambiguity-averse (loving) subjects in the *Unknown* treatment is higher (lower) with respect to the other two treatments, although this difference is not significant (Kruskal-Wallis, *P-value* = 0.258). However, doing a pair-wise comparison between treatments about the percentage of ambiguity-averse subjects, we find that there is not statistically significant difference between the *Binomial* and the *Uni-*

[24] These results are not shown but are available upon request.

[25] This intuition is reinforced by the fact the correlation between (strong) value-ambiguity-aversion and (strong) choice-ambiguity-aversion found above in all the sample of classified subjects is higher (coeff. 0.45) and significant (*P-value* 0.007) only if we restrict the analysis at the *Binomial* treatment. In this treatment, it is plausible that only highly-ambiguity-averse subjects show at the same time $CE(L_6) < CE(L_5)$ and $j_{10} \prec j_4$.

*form* treatment (t-test, *P-value* = 0.290)[26], and between the *Uniform* and the *Unknown* treatment (*P-value* = 0.605), while the difference between the *Binomial* and the *Unknown* treatment is almost significant (*P-value* = 0.109). This distortion confirms our intuition about the interpretation of the *Unknown* treatment. Both in the *Binomial* and in the *Unknown* treatment the 10-ball small urn in task 6 has been generated through the same Bernoullian process. However, the latter treatment is intrinsically more ambiguous, given that there is no information about the composition of the big urn from which the small unknown urn is generated. According to *KMM*, this generates smaller $CE(L_6)$ through (4) and/or lower $j_{10}$ through (3) in the *Unknown* than in the *Binomial* treatment, thereby significantly increasing the percentage of subjects for which it is $CE(L_6) \leq CE(L_5)$ and $j_{10} \preceq j_4$. Notice that if we disentangle value-ambiguity aversion and choice-ambiguity aversion, we still find a higher percentage of subjects in the *Unknown* than in the *Binomial* treatment, although this difference is no more statistically significant.

Let us conclude this paragraph by analyzing the relation between the sign of the ambiguity attitude (as defined in the following sentence) and the degree of risk aversion. Figure 1 shows that both the distribution of *CARA* indexes and the distribution of *CRRA* indexes differ according to the sign of the ambiguity attitude. Indeed, under both specifications, the modal risk-aversion index for ambiguity-averse and for ambiguity-neutral is 1 (non-risk-averse subjects), while the modal index for ambiguity-loving subjects is 8 (highly-risk-averse subjects). We do find that differences in the distributions of our risk-aversion ordering among different signs of the ambiguity attitude are significant.[27] This supposed negative correlation between the sign of the ambiguity attitude and the index of risk aversion is confirmed by rank correlation tests for ambiguity-neutral subjects under *CARA* (coeff. $= -0.28$, *P-value* = 0.063) and for ambiguity-loving subjects under both specifications (under *CARA*: coeff. $= -0.39$, *P-value* = 0.007; under *CRRA,* coeff. $= -0.30$, *P-value* = 0.013). We do not find that this negative correlation is significant for ambiguity-averse subjects. However, if we disentangle ambiguity-averse subjects according to Definition 1 and Definition 2, we find that the negative correlation between the fact of being choice-ambiguity-averse and the index of risk aversion is significant (under *CARA*: coeff. $= -0.30$, *P-value* = 0.029; under *CRRA,* coeff. $= -0.31$, *P-value* = 0.007).

[26] The t-test is any statistical hypothesis test (parametric) in which the test statistic follows a Student's t distribution if the null hypothesis is supported. Here we run a two-sample t-tests for a difference in mean (the null hypothesis is that the two samples have the same mean).

[27] In order to test the equality in the distribution of the risk-aversion indexes among different signs of the ambiguity attitude, we have performed two different tests: Kruskal-Wallis equality-of-populations rank test and Kolmogorov-Smirnov equality-of-distributions test. We find that both under *CARA* and under *CRRA*, we can reject the null hypothesis of equality in distributions according to the Kruskal-Wallis equality-of-populations rank test (respectively for *CARA* and *CRRA*: *P-value* = 0.0100, *P-value* = 0.0100). By perfoming the Kolmogorov-Smirnov equality-of-distributions test with a pairwise comparison between different signs of the ambiguity attitude, we find that the results are consistent with the Kruskal-Wallis test.

We can empirically observe the relation between the $CARA$ (or $CRRA$) ordering and the sign of the ambiguity attitude also by eliciting choice-ambiguity-aversion (tasks 4 and 10), which relies on the same method used to elicit risk aversion (tasks 1-4). Figure 1 seems to confirm our intuition: very few ambiguity-averse subjects (less than 5%) choose the least risky lottery in task 4, while the vast majority of ambiguity-loving ones (61%) choose this lottery in task 4; moreover, none of the latter chooses the riskiest lottery in task 4. We do find that differences in the distributions of $l_4^{j_4}$ among the three groups of subjects averse, neutral and prone to ambiguity are significant.[28]

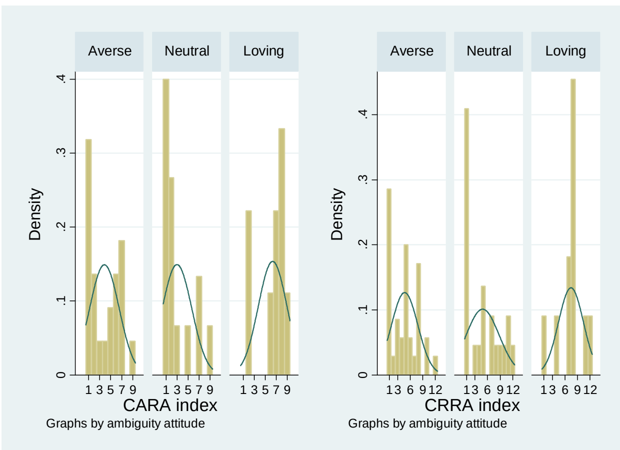


Figure 1. Distribution of $CARA$ index and $CRRA$ index by ambiguity attitude.

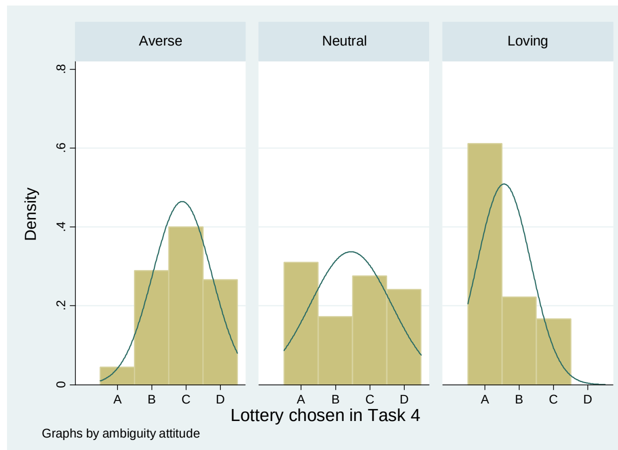


Figure 2. Distribution of lottery chosen in task 4 by ambiguity attitude.

This point can be further clarified by introducing in the analysis the subject's guess on the number of balls linked to the highest of the two lottery-outcomes (henceforth, "winning balls") in task 6, that we could interpret as the subject's modal belief on the composition of the small unknown urn. This guess may have influenced the certainty equivalent in task 6 and so the sign of value-ambiguity-aversion.[29] By looking at the upper-left part of Figure D, one can notice that also the distribution of subjects' guess in task 6 is significantly different among different signs of the ambiguity attitude.[30]

Indeed, with a multivariate regression analysis we find that – controlling or not for treatment effect – both $l_4^{j_4}$ and the guess in task 6 are highly significant (respectively, *P-*

[28] Again we have performed a Kruskal-Wallis test to check whether the distributions of $l_4^{j_4}$ are different by treatment. According to this test, we can reject the null hypothesis of equality in distribution (*P-value* = 0.000). We have also perfomed a Kolmogorov-Smirnov test with a pairwise comparison between different signs of the ambiguity attitude and the results are consistent with the Kruskal-Wallis test.

[29] The guess in task 6 is positively correlated (coeff. = 0.20, *P-value* = 0.045) with the "normalized" $CE(L_6)$. In section 4.4 we explain what we intend with "normalized" certainty equivalent.

[30] Through the Kruskal-Wallis test, we can reject the null hypothesis of equality in distribution (*P-value* = 0.000). The Kolmogorov-Smirnov test confirms this result.

*value* = 0.000 and *P-value* = 0.001): the former has a positive effect on being ambiguity-averse, while the latter has a negative one.

About the positive effect of $l_4^{j_4}$, we may think that subjects have made some kind of hedging between the higher risk they accepted through the choice of a risky lottery in task 4 and the ambiguous second-order probabilities over this (second-stage) lottery (task 10). Indeed, if we disentangle value-ambiguity aversion and choice-ambiguity aversion, we find a positive and highly significant correlation between $l_4^{j_4}$ and each measure of ambiguity attitude: respectively, coeff. = 0.24 (*P-value* = 0.023) for value-ambiguity aversion and coeff. = 0.42 (*P-value* = 0.000) for choice-ambiguity aversion.

About the negative effect of the guess of the number of winning balls in task 6, this plays a role through a decrease of $CE(L_6)$ and of $j_{10}$. We will see in section 4.4 that the distribution of subjects' guess in task 6 does not depend on the treatment where the subjects have participated, i.e. on the different distributions of second-order probabilities that we have generated through our experimental design.

## 4.3 Test of the main theoretical results

In this paragraph, we report and comment about the percentage of classified subjects who satisfy the theoretical predictions stated in section 3. We disentangle classified subjects according to the sign of their ambiguity attitude (averse, neutral and loving) and according to the treatment where they participated. All the theoretical predictions tested in this paragraph should hold independent of the treatment. A first set of theoretical predictions (Proposition 5) state different conditions for ambiguity-averse, ambiguity-neutral and ambiguity-loving subjects. A second set of theoretical predictions (Proposition 6) should hold whatever the sign of the ambiguity attitude.

Figure 3 summarizes the percentage of subjects classified according to Definition 3 whose behavior in tasks 5-7 complies with Proposition 5. In particular, for 78% (35/45) of ambiguity-averse subjects it is $CE(L_5) \geq CE(L_7) \geq CE(L_6)$; for 86% (25/29) of ambiguity-neutral subjects it is $CE(L_5) = CE(L_7) = CE(L_6)$; for about 78% (14/18) of ambiguity-loving subjects it is $CE(L_5) \leq CE(L_7) \leq CE(L_6)$. Although the percentage of subjects fulfilling predictions of Proposition 5 is higher among the ambiguity-neutral ones, validity of these predictions does not depend on the sign of the ambiguity attitude. On the other hand, there is some dependences on the treatment. Through a t-test, we get that the percentage of subjects complying with Proposition 5 is almost significantly higher in the *Binomial* (*P-value* = 0.124) and in the *Unknown* (*P-value* = 0.120) than in the *Uniform* treatment. Finally, this compliance is uncorrelated with subject's *CARA* or

$CRRA$ ordering, as predicted by Proposition 5.[31]

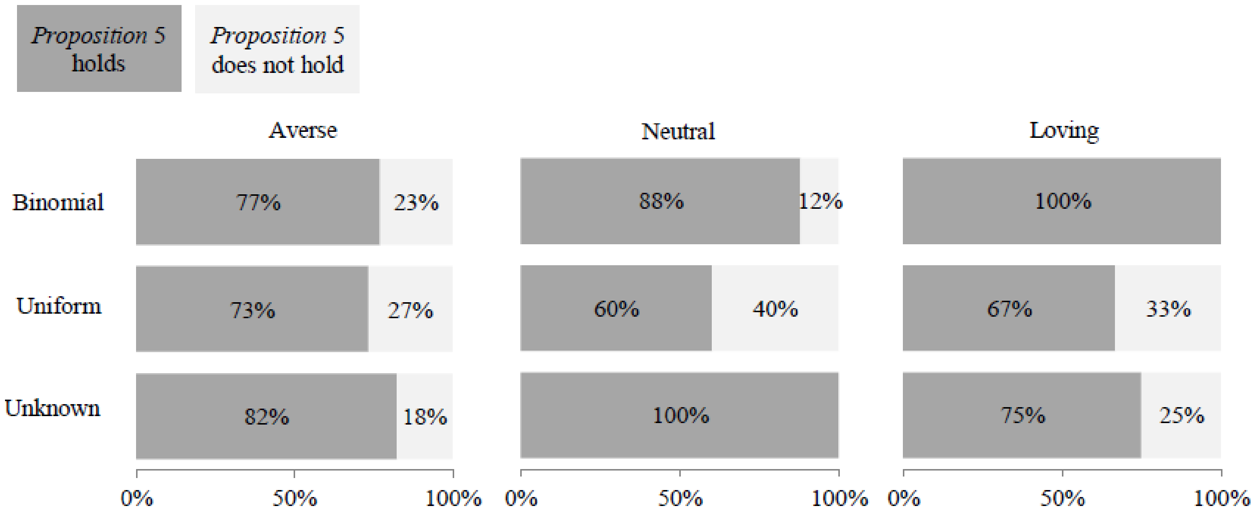


Figure 3. Percentage of subjects satisfying Proposition 5, by treatment and ambiguity attitude.

Figure 4 reports the percentage of subjects whose behavior in tasks 6-9 complies with Proposition 6 respectively w.r.t to $CE(L_6)$ (Figure 4.a) and w.r.t. $CE(L_7)$ (Figure 4.b). Recall that Proposition 6 provides the same prediction for all subjects being classified according to Definition 3, whatever the sign of their ambiguity attitude and their degree of ambiguity aversion. It is easy to notice that the percentage of subjects fulfilling Proposition 6 is even higher than for Proposition 5 and that it does not depend on which of the two reference certainty equivalents we applied it, $CE(L_6)$ or $CE(L_7)$. In both cases, more than 90% of ambiguity-averse subjects (42/45 for $CE(L_6)$, 41/45 for $CE(L_7)$), all ambiguity-neutral subjects (29/29) and more than 70% ambiguity-loving subjects (13/18) comply with Proposition 6. As further proof of "rational" behavior of this huge pool of subjects, notice that there are only 4/84 subjects satisfying Proposition 6 w.r.t $CE(L_6)$ and not satisfying it also w.r.t to $CE(L_7)$; there are only 3/83 subjects who fulfills the predictions w.r.t $CE(L_7)$ and not w.r.t to $CE(L_6)$.

There are not significant differences by treatment if we take as reference $CE(L_6)$. If instead we take as reference $CE(L_7)$, the percentage of subjects being consistent with Proposition 6 is slightly higher in the *Binomial* than in the *Uniform* treatment, with this difference being significant ($P\text{-}value = 0.055$). On the other hand, whatever the reference certainty equivalent, $CE(L_6)$ or $CE(L_7)$, we find a significant difference by the sign of the ambiguity attitude. Indeed, the percentage of ambiguity-loving subjects complying with Proposition 6 is significantly lower than the proportion of ambiguity-averse ones[32]

[31] $P\text{-}value = 0.546$ for $CARA$, $P\text{-}value = 0.841$ for $CRRA$.
[32] $P\text{-}value = 0.023$ for $CE(L_6)$, $P\text{-}value = 0.054$ for $CE(L_7)$.

and the proportion of ambiguity-neutral ones[33]. Again, compliance with the prediction of Proposition 6 is uncorrelated with subject's *CARA* or *CRRA* ordering.[34]

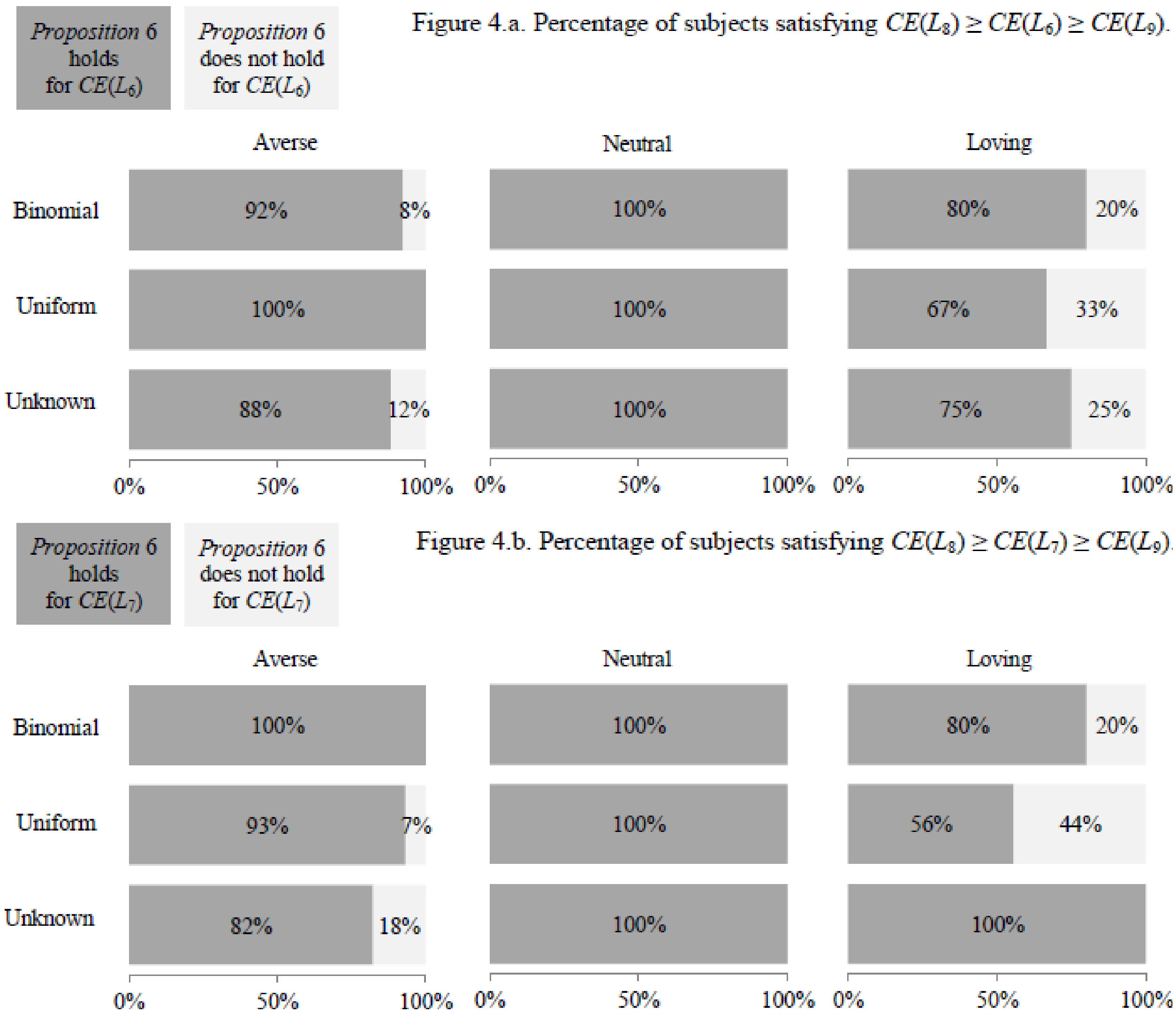


Figure 4. Percentage of subjects satisfying Proposition 6, by treatment and ambiguity attitude.

Figure 5 shows the percentage of classified subjects whose behavior in tasks 5-9 satisfies at the same time both Proposition 5 and Proposition 6. As we have seen in section 3, the two propositions taken together lead to relation (5) for ambiguity-averse, relation (6) for ambiguity-neutral, and relation (7) for ambiguity-loving subjects. Indeed, more than 75% of classified subjects (70/92) states their certainty equivalents in tasks 5-9 in a way that

[33] *P-value* $= 0.002$ for both $CE(L_6)$ and $CE(L_7)$.

[34] If we take as reference $CE(L_6)$, it is *P-value* $= 0.202$ for *CARA*, *P-value* $= 0.278$ for *CRRA*. If we take as reference $CE(L_7)$, it is *P-value* $= 0.216$ for *CARA*, *P-value* $= 0.333$ for *CRRA*.

all the "rationality" constraints imposed by the *KMM* model are satisfied. Notice that between Proposition 5 and Proposition 6 the former cuts much more observations, given that percentages of verification in Figure 5 are only slightly lower than those in Figure 3. In addition, ambiguity-loving subjects have a lower ratio of fulfillment of propositions 5 and 6 taken together (61%, 11/18) than ambiguity-averse subjects (76%, 34/45) and ambiguity-neutral ones (86%, 25/29), although this difference is not significant, maybe because of the low number of ambiguity-loving subjects in our sample. Notice that all ambiguity-neutral subjects fulfilling Proposition 5 also fulfill Proposition 6.

Furthermore, we find significant differences both by sign of the ambiguity attitude and by treatment. About the former, we find that the percentage of ambiguity-loving subjects complying with both propositions is again significantly lower than the proportion of ambiguity-neutral ones (*P-value* = 0.038). About the latter, we have that (as for Proposition 5) the percentage of subjects complying with both propositions is significantly higher in the *Binomial* (*P-value* = 0.038) and in the *Unknown* (*P-value* = 0.018) than in the *Uniform* treatment. It is particularly striking that in the *Unknown* treatment all subjects satisfying Proposition 5 satisfy also Proposition 6, both w.r.t to $CE(L_6)$ and w.r.t. $CE(L_7)$: in fact, percentages of compliance with the theoretical predictions are the same in Figure 3 and in Figure 5, whatever the sign of the ambiguity attitude. One more time, compliance with the whole set of our theoretical predictions is uncorrelated with subject's $CARA$ or $CRRA$ index.[35]

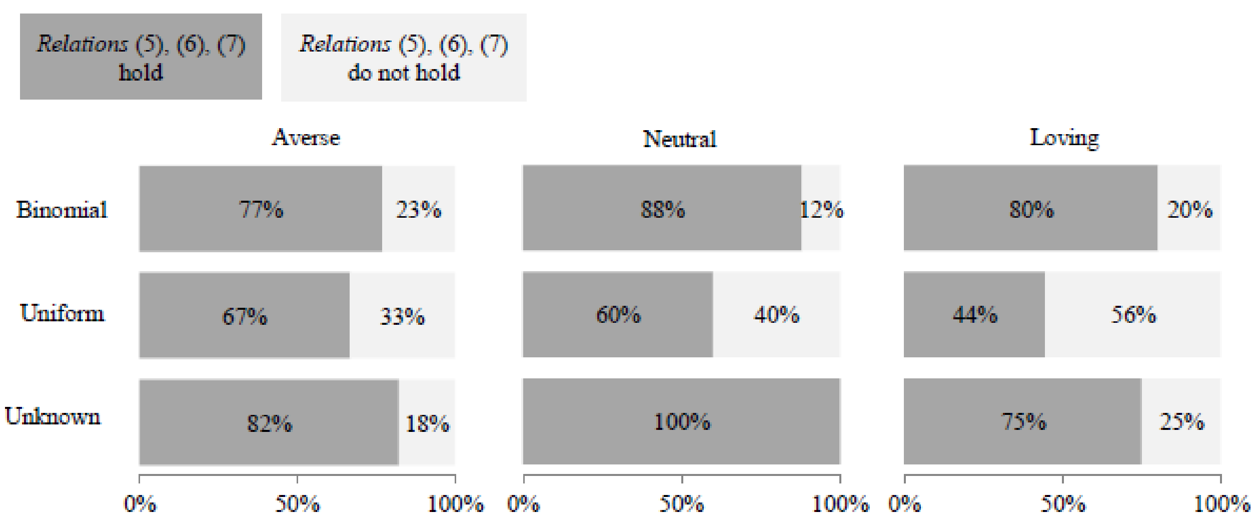


Figure 5. Percentage of subjects satisfying (5), (6) and (7), by treatment and ambiguity attitude. [Note: Relations (5), (6) and (7) refer respectively to ambiguity-averse, ambiguity-neutral and ambiguity-loving subjects.]

[35] *P-value* = 0.838 for $CARA$, *P-value* = 0.765 for $CRRA$.

Finally, let us analyze subject's behavior in the only two tasks for which we do not have a sharp theoretical prediction. We refer here to tasks 5 and 8 for ambiguity-averse subjects and to tasks 5 and 9 for ambiguity-loving ones. From relation (6) we know that in the limit case of smooth ambiguity-neutrality, we must have $CE(L_8) > CE(L_5)$ and $CE(L_5) > CE(L_9)$. Therefore, in Figure 6 we classify as "*Low*" the ambiguity attitude for those ambiguity-averse subjects and for those ambiguity-loving ones who behave as the ambiguity-neutral ones, respectively in tasks 5 and 8 and in tasks 5 and 9. "*Medium*" and "*High*" ambiguity attitude are classified accordingly, that is a subject is highly-ambiguity-averse if $CE(L_8) < CE(L_5)$ and highly-ambiguity-loving if $CE(L_5) < CE(L_9)$.

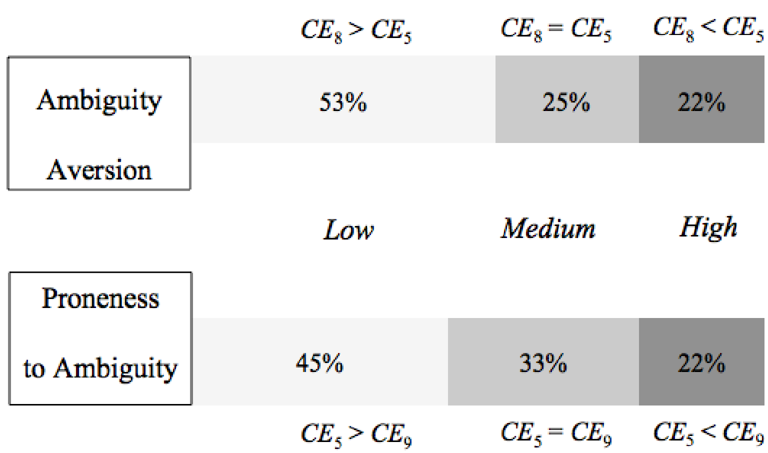


Figure 6. Disentangle of Low, Medium and High ambiguity aversion w.r.t. $CE(L_8) \gtreqless CE(L_5)$ and ambiguity proneness w.r.t. $CE(L_5) \gtreqless CE(L_9)$.

Surprisingly enough, we find the same percentage of highly-ambiguity-averse (10/45) and of highly-ambiguity-loving (4/18) subjects in our sample. Subjects in the former group prefer to know with certainty the composition of the small urn (task 5) rather than knowing only that the worst scenarii are implausible (task 8). Specularly, subjects in the latter group prefer to know only that the best scenarii are implausible (task 9) rather than knowing with certainty the composition of the small urn (task 5). Notice that neither the sign of $CE(L_8) - CE(L_5)$ for ambiguity-averse subjects nor the sign of $CE(L_5) - CE(L_9)$ for ambiguity-loving ones is correlated with any of the explanatory variables introduced above (treatment, $CARA$ ordering, $CRRA$ ordering, lottery chosen in task 4, guess on the winning balls respectively in task 8 and in task 9).

The effects of all these variables are not significant at all, neither in the univariate analysis considering each control singularly, nor in the multivariate regressions. The only significant result that we find is actually quite counterintuitive. We find that $CE(L_8) - CE(L_5) >$

0 depends positively on strong choice-ambiguity-aversion (i.e. $j_{10} \prec j_4$). This would lead to conclude that strongly-choice-ambiguity-averse subjects are not so strongly-value-ambiguity-averse ($CE(L_6) < CE(L_5)$), thereby explaining why the above found correlation between strong choice-ambiguity-attitude and strong value-ambiguity-attitude, although positive and significant, is not so high.

As anticipated at the end of section 3, the combination of Proposition 7 and Corollary 9 suggests a possible treatment effect on the percentage of ambiguity-averse subjects showing $CE(L_5) \geq CE(L_8)$ and on the percentage of ambiguity-loving subjects showing $CE(L_9) \geq CE(L_5)$. This prediction relies on the assumption of a similar distribution of risk attitude among the three treatments, that we have shown is satisfied in section 4.1. Indeed, our prediction on the treatment effect over the size of the ambiguity attitude is verified. The percentage of medium and highly-ambiguity-averse subjects is lower in the *Binomial* (31%, 4/13) than in the *Uniform* (60%, 9/15) and in the *Unknown* treatment (47%, 8/17), with the difference being almost significant between the *Binomial* and the *Uniform* (*P-value* = 0.131). Specularly, the percentage of medium- and highly-ambiguity-loving subjects is lower in the *Binomial* (40%, 2/5) than in the *Uniform* (67%, 6/9) and in the *Unknown* treatment (50%, 2/4).

## 4.4 Treatment Effects over beliefs and certainty equivalents

Let us conclude our analysis of the experimental results through a quick look at possible treatment effects over the certainty equivalents and over the guess of winning balls in tasks 6-9.

Corollary 9 states that $CE(L_t)$ for $t = 6, ..., 9$ should be higher in the *Binomial* than in the *Uniform* and in the *Unknown* treatment. If the subject's guess on winning balls in task $t$ would be correlated with $CE(L_t)$ for $t = 6, ..., 9$, then we should find that also this guess should be higher in the *Binomial* treatment. Now, it is true that our experimental design is between-subject, hence we cannot state whether and how a subject changes her certainty equivalent (and her guess) according to the way in which at the beginning of task 6 the unknown small urn is generated. However, under the assumption that the distribution of risk attitudes and of ambiguity attitudes is not too different among treatments – that is what we have shown respectively in section 4.1 and in section 4.2 – we can look at possible differences among treatments in the distribution of certainty equivalents and of guesses in tasks 6-9.

In Figure 7 we report the distribution of subjects' guess on the number of winning balls in tasks 6-9 disentangled by treatment. The graphs by treatment seem to suggest that the distribution of guesses in the *Unknown* treatment is close to the one in the *Binomial* treatment and both are quite different from the one in the *Uniform* treatment. This result is in line with the principle of insufficient reason, that should lead a subject in the *Binomial* and in the *Unknown* treatment to provide a guess equal to 5 in tasks 6 and 7, between 6 and 7 in task 8, and between 3 and 4 in task 9. Conversely, in the *Uniform* treatment, all guesses should be equivalent: any guess over a scenario that is plausible in a specific task is justifiable.

However, we find that the distribution of subject's guess in task $t$ is not significantly different among treatments, for $t = 6, 8, 9$.[36] Our intuition is instead right in task 7. According to the Kruskal-Wallis test on the equality in distribution of guesses by treatment, we can reject the null hypothesis at a 10% level (*P-value* = 0.082). More precisely, according to the Kolmogorov-Smirnov equality-of-distributions test, there is not equality in the distribution of guesses between the *Binomial* and the *Uniform* treatment (*P-value* = 0.003) and between the *Unknown* and the *Uniform* treatment (*P-value* = 0.016). In this specific task it seems that subjects in the *Unknown* treatment state similar guesses to those of subjects in the *Binomial* one.

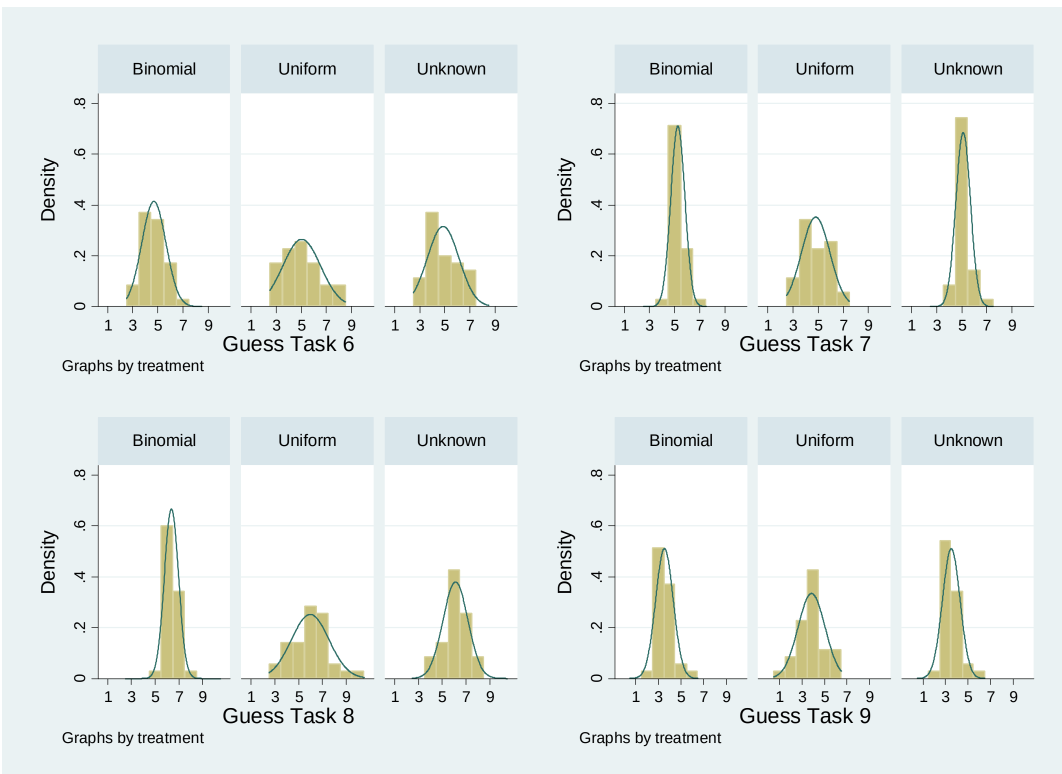


Figure 7. Distribution of subjects' guess on the number of winning balls in tasks 6-9 by treatment.

[36] According to the Kruskal-Wallis test on the equality in distribution of guesses by treatment, we cannot reject the null hypothesis (respectively for task 6, 8, 9, *P-value*: 0.739, 0.375, 0.175). The Kolmogorov-Smirnov equality-of-distributions test with a pairwise comparison between treatments confirms this result.

Let us now focus on the relation between certainty equivalents and subject's guesses. We find that, overall, the former are never correlated with the latter in tasks 6-9 within the same task.[37]

In general, certainty equivalents in task $t$ ($CE(L_t)$) are negatively correlated with the $CARA$ and/or with the $CRRA$ ordering (recall that the ordering index increases with risk aversion). This is reasonable: as shown above, the subject's certainty equivalent elicited in task 5 (and in the following four tasks) rely on the lottery chosen in task 4. This choice also depends on the $CARA$ or the $CRRA$ ordering elicited in tasks 1-4 (we have seen in section 4.2 that it influences also the sign of the ambiguity attitude). Moreover, in order to further investigate the link between certainty equivalent and risk aversion, we consider the lottery chosen in task 4 ($l_4^{j_4}$) and its relation with the "normalized" values of the certainty equivalents, i.e. their index in tasks 6-9, with $CE(L_t) = \underline{x}_4^{j_4}$ being assigned index 1 and $CE(L_t) = \overline{x}_4^{j_4}$ being assigned index 11.[38] We report in Figure E in the Appendix the "normalized" values of $CE(L_t)$ for $t = 6, ..., 9$.

Once we normalize $CE(L_t)$, we find that they are still correlated with the risk-attitude index. However, while the relation between risk aversion and $CE(L_6), CE(L_7)$ and $CE(L_8)$ is negative, $CE(L_9)$ shows instead a positive relation with risk aversion. The last result can be referred to the positive correlation between risk aversion and ambiguity proneness highlighted in section 4.2.

Further, as long as the distribution of the second-order probabilities is symmetric around $\theta = 5$ (tasks 6 and 7), the normalized certainty equivalents are correlated with the guesses in the same task. We do not find such a correlation in tasks 8 and 9.

A strong regularity is that by regressing either $CE(L_t)$ or its normalized version over the guess in the same task, the index of risk attitude and the treatment, we do not find significant treatment effects, for any $t = 6, 7, 8, 9$.[39]

As one can notice from Figure E, the differences among treatments in the distribution of normalized $CE(L_t)$ are not statistically significant for every $t = 6, ..., 9$. In addition, the sign of all differences $CE(L_{t+1}) - CE(L_t)$ for $t = 5, 6, 7$ and $CE(L_{t+2}) - CE(L_t)$ for $t = 6, 7$ do not depend on the treatment.[40]

[37] However, we find that $CE(L_6)$ is positively correlated (coeff. $= 0.35$) with the guess in task 6 only in the *Uniform* treatment (*P-value* $= 0.037$). We also find that $CE(L_9)$ is positively correlated (coeff. $= 0.36$) with the guess in task 9 only in the *Binomial* treatment (*P-value* $= 0.035$).

[38] In task 5-9 we allow each subject to choose always among 11 possible selling prices. Therefore, for every $t = 5, 6, ..., 9$ , we can always assign index 1 to $CE(L_t) = \underline{x}_4^{j_4}$, index 11 to $CE(L_t) = \overline{x}_4^{j_4}$ and indexing the internal $CE(L_t)$ accordingly. See footnote 7.

[39] A relevant exception is again represented by the (normalized) certainty equivalent in task 7. Controlling for $CARA$ and treatment and taking the *Binomial* as reference treatment, we find that the *Unknown* treatment has a positive and significant effect (*P-value* $= 0.059$) over $CE(L_7)$. Controlling for $CRRA$ and treatment, we find that also the *Uniform* treatment has a positive and significant effect (*P-value* $= 0.041$) over $CE(L_7)$.

[40] To be more precise, only in the regressions for $CE(L_7) - CE(L_6) > 0$ ($CE$ normalized) we find that

All this would lead to conclude that, despite some differences in the distribution of guesses among treatments, we do not find any treatment effect on the distribution of certainty equivalents in the ambiguous tasks. Therefore, the only significant difference among treatments is the one shown at the beginning of section 4.2 about the distribution of $(CE(L_6)-CE(L_5))$ if combined with the distribution of $(j_{10}-j_4)$, i.e. the two conditions leading to state a subject's coherent-ambiguity attitude.

# Conclusion

Our work shows how to identify two features of smooth ambiguity attitude *à la KMM*: value-ambiguity attitude and choice-ambiguity attitude. A value-ambiguity-averse subject values an ambiguous lottery less than its unambiguous equivalent with the same mean probabilities. A choice-ambiguity-averse subject always reduces her demand for the risky asset when the distribution of outcomes becomes ambiguous.

We elicit these two attitudes in a series of experimental decision tasks designed in order to match the main insights of *KMM* model. Our decision tasks are parameterized so that a value-ambiguity averse (loving) subject should not necessarily behave as a choice-ambiguity averse (loving) one, i.e. showing coherent-ambiguity aversion. Indeed, we find that 88% of our subjects (92/105) show a coherent-ambiguity attitude, independent of the treatment, i.e. of the distribution of second-order probabilities (binomial, uniform or unknown). This result clearly indicates an equivalence between value-ambiguity aversion and choice-ambiguity aversion in subjects participating in our experiment. However, we do not find the same equivalence between strong value-ambiguity aversion and strong choice-ambiguity aversion.

We believe that the most important contribution of the paper concerns the theoretical analysis of those ambiguous tasks designed with the aim of testing *KMM* predictive power. In section 3 we provide two kinds of theoretical predictions: those holding independent of subject's (coherent)-ambiguity attitude and those stating specific behavior in correspondence of a specific attitude to (coherent)-ambiguity. We find that the former are satisfied by more than 90% of our classified subjects (84/92), while the latter comply with behavior of more than 80% of classified subjects (74/92). Overall, a large number of classified subjects (76%, 70/92) satisfy all our theoretically derived constraints. This extremely high compliance of subjects' behavior to *KMM* indirectly provides support both to our operational definition of (coherent)-ambiguity attitude and to the fact that our experimental design may be a correct representation of the main features of *KMM* in the laboratory.

the *Uniform* treatment increases significantly the probability that $CE(L_7) - CE(L_6) > 0$ with respect to the *Binomial* treatment (*P-value* $= 0.040$). This result holds also when controlling for the difference in the guesses about the number of winning balls in task 7 and task 6.

We do not find any significant correlation between compliance to *KMM* and gender, age, level of education and degree of risk attitude.

A secondary contribution of the paper concerns the analysis of possible relations between risk attitude and ambiguity attitude. We elicit risk attitude through the same two methods – portfolio choice and *BDM* – used to elicit the two features – respectively, choice and value – of coherent-ambiguity attitude. Risk-aversion orderings provided by the two methods are correlated. Relying on the two risk-aversion orderings built through the first method, we find that more than 1/2 of subjects may be classified as constant-absolute-risk-averse and almost 3/4 may be constant-relative-risk-averse. Under both specifications of risk aversion, we find a negative correlation between the degree of risk aversion elicited in the unambiguous tasks and the fact of showing coherent-ambiguity aversion in the ambiguous tasks. This correlation is significant if we consider only choice-ambiguity-averse subjects. More specifically, in our sample many coherent-ambiguity-averse subjects have a low degree of risk aversion, while the most part of coherent-ambiguity-loving subjects have a high degree of risk aversion (77% of the coherent-ambiguity-loving subjects are in the last two quintiles of the *CARA* index distribution and 73% are in the last two quintiles of the *CRRA* index distribution). We have an explanation for this apparently surprising result. In our design a subject is elicited her (coherent)-ambiguity attitude in correspondence to a lottery that she has previously chosen among a set of lotteries having different levels of riskiness. We find that the riskier the lottery chosen by the subject when there was no uncertainty about first-order probabilities, the greater the decrease in her value for this lottery when first-order probabilities become ambiguous. The positive correlation between the riskiness of the lottery assigned in the most ambiguous task and both value-ambiguity aversion and choice-ambiguity aversion that we find in the data supports this explanation. However, we found this result through a between-subject design: we ask a subject to state her certainty equivalent under different levels of ambiguity of the setting, but always facing the same pair of (second-stage) lottery-outcomes. One may test whether the effect that we find between-subjects holds also when the same subject is asked to state her attitude towards ambiguity for different pairs of lottery-outcomes. We leave this for further research.

The third contribution of the paper concerns the analysis of subjects' attitudes and decisions in ambiguous tasks with specific distributions of second-order probabilities. We find that the percentage of coherently-ambiguity-averse subjects is lower (though not significantly) in the binomial than in the uniform treatment. This was easily predictable, given that the latter distribution of the second-order probabilities is a mean-preserving spread of the former. We also verify the prediction of a significantly lower percentage of coherent-ambiguity-averse subjects in the binomial than in the unknown treatment (the latter is intrinsically more ambiguous than the former). What is more surprising is the absence of a significant difference in the percentage of coherent-ambiguity averse subjects between the uniform and the ambiguous treatment. This would lead to validate the assumption that subjective second-order probabilities may be thought as uniformly distributed when the subject is not given any information about the composition of the unknown urn. This conclusion is even stronger if one thinks at the fact that in our experiment ambiguity in the unknown treatment has been generated through a process similar to the one used in the binomial treatment. It seems that this process has influenced subjects' guess about the number of "winning" balls in the unknown urn: the distribution of this guess in the unknown treatment is closer to the one in the binomial than in the uniform treatment in one ambiguous task, with no significant differences in the remaining ambiguous tasks. Despite that, it seems that the level of a subject's confidence about her guess is completely different when an objective probability distribution is given with respect to the case where no information is provided about objective second-order probabilities. This may represent a further justification to the significant difference found between the percentage of ambiguity-averse subjects in the binomial and in the unknown treatment.

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# Appendix

## Appendix A

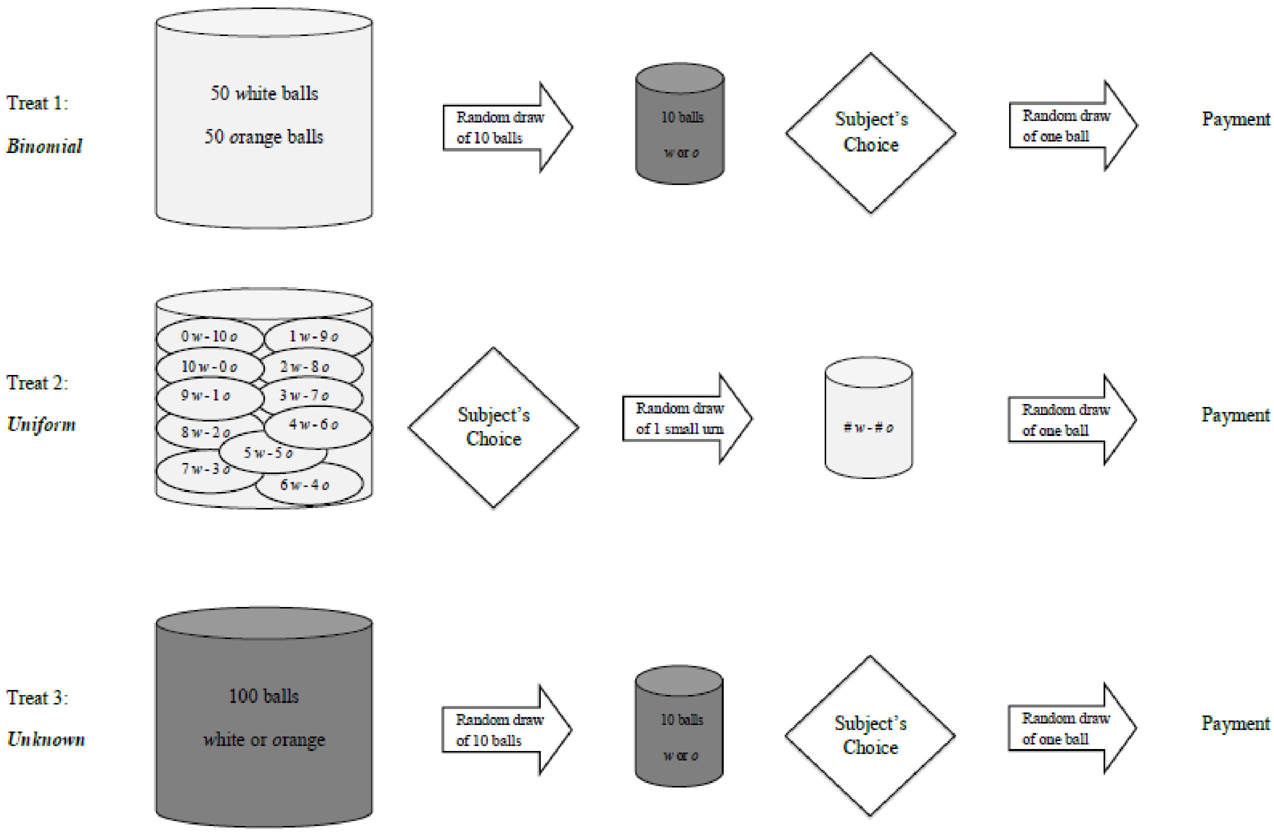


Figure A. Description of the procedure followed to generate of the composition of the 10-ball small urn used to perform task 6.

## Appendix B

For each lottery $l_t^j = (\overline{x}_t^j, 0.5; \underline{x}_t^j, 0.5)$ in Table 2, we have expected value and standard deviation respectively equal to $EV = 0.5(\overline{x} + \underline{x})$ and $\sigma = 0.5(\overline{x} - \underline{x})$. The two lottery-outcomes can be expressed in terms of the two moments, i.e. $\overline{x} = EV + \sigma$ and $\underline{x} = EV - \sigma$. In Table B, we classify the set of lotteries in tasks 1-4 in terms of the triple $\left(EV, \sigma, \frac{d\sigma}{dEV}\right)$, where the ratio $\frac{d\sigma}{dEV}$ is the same for all lotteries in the same task. In particular, it is $\frac{d\sigma}{dEV} = 3$ in tasks 1 and 3 and $\frac{d\sigma}{dEV} = 5$ in tasks 2 and 4.

| | Task $t=1$ | | | Task $t=2$ | | | Task $t=3$ | | | Task $t=4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Lottery | $EV_1^j$ | $\sigma_1^j$ | $\frac{d\sigma_1^j}{dEV_1^j}$ | $EV_2^j$ | $\sigma_2^j$ | $\frac{d\sigma_2^j}{dEV_2^j}$ | $EV_3^j$ | $\sigma_3^j$ | $\frac{d\sigma_3^j}{dEV_3^j}$ | $EV_4^j$ | $\sigma_4^j$ | $\frac{d\sigma_4^j}{dEV_4^j}$ |
| $j = A$ | 9.0 | 3.0 | 3.0 | 8.5 | 2.5 | 5.0 | 17.0 | 3.0 | 3.0 | 16.5 | 2.5 | 5.0 |
| $j = B$ | 10.0 | 6.0 | 3.0 | 9.0 | 5.0 | 5.0 | 18.0 | 6.0 | 3.0 | 17.0 | 5.0 | 5.0 |
| $j = C$ | 11.0 | 9.0 | 3.0 | 9.5 | 7.5 | 5.0 | 19.0 | 9.0 | 3.0 | 18.0 | 10.0 | 5.0 |
| $j = D$ | 12.0 | 12.0 | 3.0 | 10.0 | 10.0 | 5.0 | 20.0 | 12.0 | 3.0 | 19.0 | 15.0 | 5.0 |

Table B. Risk Attitude Elicitation method: reinterpretation in terms of $EV$, $\sigma$, and $(d\sigma/dEV)$.

# Appendix C

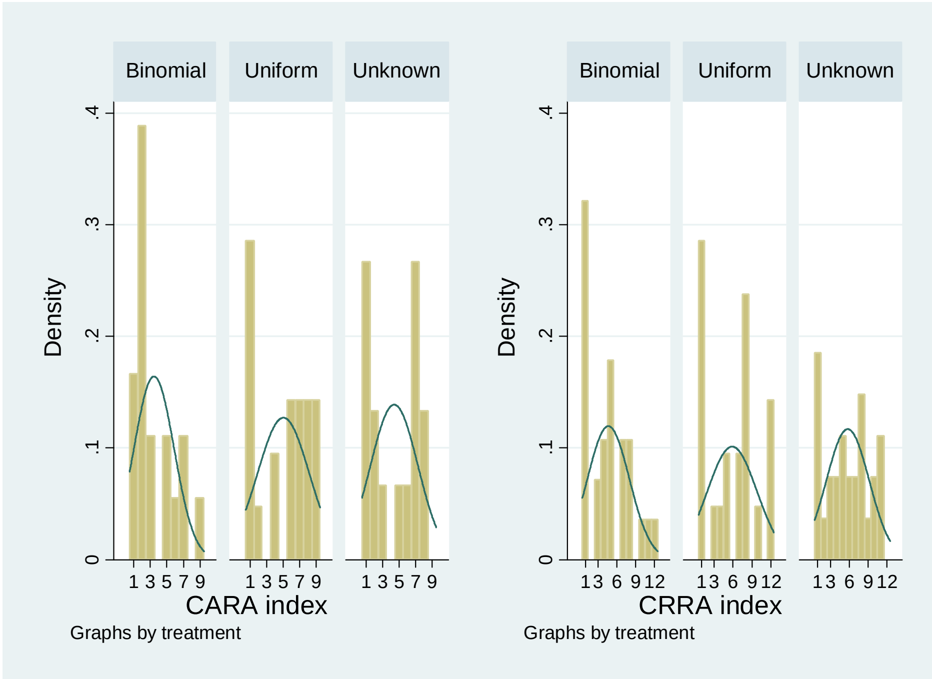


Figure C. Distribution of *CARA* index and *CRRA* index by treatment.

# Appendix D

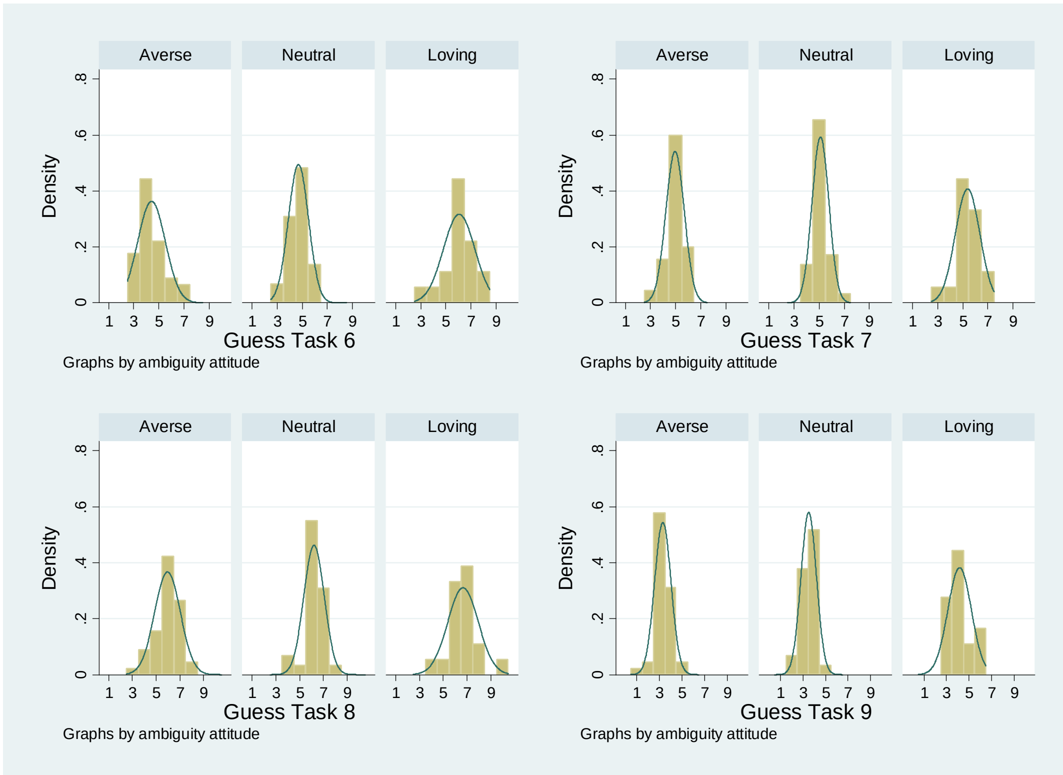


Figure D. Distribution of subjects' guess on the number of winning balls in tasks 6-9 by ambiguity attitude.

# Appendix E

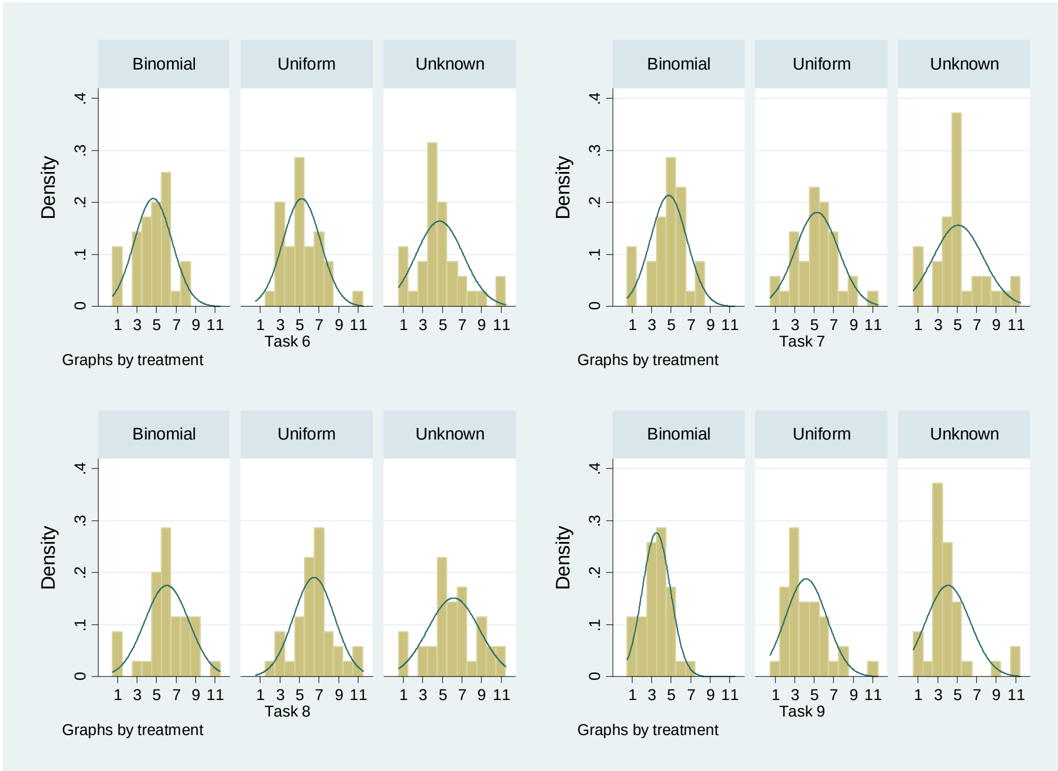


Figure E. Distribution of “normalized” $CE(L_t)$ in tasks 6-9 by treatment.