« Information, Tranching and Liquidity »

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Abstract

The paper revisits and qualifies existing insights on security design. A rich literature argues that tranching creates debt-like instruments that are robust to adverse selection or discourage wasteful information acquisition. Yet, for a given information structure, while tranching confines and liquefies the safe part of a cash flow (the insulation effect), bundling makes the risky part more liquid (the trading adjuvant effect). Moreover, tranching always has adverse welfare effects on information acquisition: It encourages (discourages) information acquisition when it should be deterred (encouraged). The paper provides conditions under which tranching reduces welfare even when the insulation effect dominates the trading adjuvant effect. The paper's second contribution is to analyze the velocity of assets that are repeatedly traded. The dynamic model can be nested into the static one and insights are shown to be closely related to those on tranching. The central insight is that liquidity is self-fulfilling: A perception of future illiquidity creates current illiquidity.

Keywords: Liquidity, velocity, security design, tranching, information acquisition.

JEL numbers: D82, E51, G12, G14.

1 Introduction

Financial institutions and corporations need to store value to meet cash shortages and take advantage of acquisition and investment opportunities. The attractiveness of an asset as a store of value hinges on its liquidity—its owner's ability to rapidly part with the asset at a fair price. Liquidity in turn requires buyers not to cherry pick high-quality assets and sellers not to trade only their lemons. As the recent crisis and other episodes suggest, suspicions about asset quality may have serious consequences for the functioning of the economy.

Starting with Akerlof (1970), economists have investigated the impact of information held by either sellers or buyers on the volume of trade and efficiency. It has been established that informed sellers have an incentive to engage in limited securitization/resale in order to signal asset quality (Leland-Pyle 1977, Myers-Majluf 1984) and that security design prior to seller information acquisition may reduce signaling costs (DeMarzo-Duffie 1999, DeMarzo 2005, Plantin 2009). Sellers' ability to part with their assets at fair prices is also hindered when buyers have private information, the focus of much market micro-structure economics; in this spirit, Gorton and Pennacchi (1990) recommend the use of tranching to create low-information-intensity (debt-like) securities that protect sellers from the "seller's curse", namely the risk of selling only high-quality assets when trading with an informed buyer.

An important recent strand of this literature, initiated by Dang-Gorton-Holmström (2011), notes that information structures are endogenous and so argues that securities, to be liquid, should be designed with an eye on their impact on information acquisition.[1] Dang et al. show that debt contracts optimally deter buyer information acquisition and may thereby maximize seller welfare. Two central and recurring insights of the literature are:

- *Tranching is optimal*. The creation of debt-like securities alleviates buyer concerns about the seller's ability to foist a lemon, and seller concerns about the seller's curse. It further minimizes incentives for information acquisition. Tranching thus boosts liquidity, the value of assets and welfare.

- *Ignorance is bliss*. The acquisition of information by the potentially informed party is to be deterred or at least limited.

This paper's first contribution is to revisit and qualify these conclusions. Section 2 develops the canonical model. An asset has value $s\delta + S$ to the seller and $b\delta + B$ to the buyer, where $\delta = 1$ (high-quality asset) or 0 (lemon) and $S \geq B$ and $b \geq s$ (gains from trade). The seller and the buyer can secretly learn δ at a cost (that may differ between the two parties). They then bargain over the sale of the asset. Uncertainty about the quality of the asset is sufficient to create a suboptimal volume of trade when one of the parties is informed.

Section 4 shows how equilibrium liquidity relates to the buyer's information acquisition cost and to the parties' bargaining powers. Tranching impacts both liquidity (the

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[1]See Yang (2011) for a neat extension of the Dang et al framework, and Cremer-Khalil (1992) for an early paper on optimal mechanism design with endogenous information acquisition by the agent.

likelihood of trading value) for a given information structure, and information acquisition. We first take the information structure as given (like in much of the literature) and unveil two conflicting effects of tranching. On the one hand, tranching insulates the debt-like/safe component from the risk of illiquidity. This "insulation" effect has attracted much attention in the literature (since Gorton-Pennacchi 1990). On the other hand, the absence of bundled debt in the equity trade reduces the cost of not trading the equity part and thereby may make the trading of the equity part less likely: the bundling of the safe part is a "trading adjuvant" for the transfer of the risky part. The global impact of these "*insulation*" and "*trading adjuvant*" effects depends on the liquidity of the bundle: if the bundle is liquid, then tranching can only hinder the transfer of value between parties. By contrast, if the bundle is illiquid, tranching increases liquidity by ensuring that the debt component is transferred.

Endogenizing the information structure, the paper argues that spinning off debt-like securities actually increases the incentive to acquire information when information is to be deterred (because one party finds it too expensive to acquire information anyway) and reduces this incentive when information acquisition is to be encouraged (so to re-establish the symmetry of information). The paper thereby identifies an important cost of tranching.

To highlight the adverse social impact of tranching on information acquisition, we then restrict attention to parameter values such that the insulation effect dominates the trading adjuvant effect, and so tranching is a superior alternative when parties are asymmetrically informed (tranching is obviously liquidity-neutral under symmetric information). Even in this most favorable case for tranching, tranching may become undesirable once information acquisition is endogenized.

The thrust of the argument goes as follows: Consider a situation in which parties remain ignorant and so the asset is liquid. One party's temptation to become informed is associated with the option not to trade. The seller would like to identify a gem so as to keep it and sell the asset only if it has mediocre quality; likewise, the buyer aspires at identifying a lemon so as to refrain from acquiring it. Deterring information acquisition thus requires making the absence of trade costly. Spinning off a safe tranche reduces the cost of not trading the risky (information intensive) part; it thereby encourages information acquisition and may reduce the overall liquidity.[2] By contrast "bundling" (selling the

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[2]This mechanism is reminiscent of Whinston (1990)'s argument that bundling a monopoly good with a competitive one makes it more costly for a seller not to sell the competitive one and so increases his volume of sales in the competitive market. Whinston's application and focus - entry deterrence- are rather unrelated with the current paper, though.

entire asset untranched) reduces information collection.[3]

By contrast when one party is informed, the incentive of the other party to become informed is associated with an increase in the probability of trading as symmetric information delivers efficient trade. Because information acquisition then helps create efficient trade, it is important to increase the cost of not trading; this is exactly what bundling does. The literature has mostly considered situations in which at most one party to the potential trade is informed. When both sides can acquire information, ignorance may no longer be desirable. Indeed, liquidity stems not from the lack of information, but more generally from the commonality of information. And so if one party is likely to be informed, it may well be optimal to encourage information acquisition by the other party so as to reestablish the symmetry of information and thereby recreate asset liquidity.

While the literature cited above is primarily concerned with a one-shot trade, the paper's second contribution is to embody these considerations in a dynamic framework in which an asset changes hands repeatedly. The counterpart of asset liquidity is then asset velocity. Section 3 constructs a dynamic-trading model in which the asset's unknown dividends are i.i.d. and parties can learn the realization of the dividend one period ahead; it shows how this dynamic framework can be nested in the canonical model of Section 2, making it possible to apply the latter's general results. The dynamic framework conveniently turns out to be a special case of our canonical model, in which the endogenous resale values are part of the asset's return stream.

Section 5 accordingly applies the results obtained in Section 4 to the dynamic setting. Its central insight is that liquidity is self-fulfilling: A perception of future illiquidity creates current illiquidity. The intuition for this result is closely related to the insights obtained for the analysis of tranching. A high liquidity in the future makes the asset a desirable store of value. The high price thus fetched in the future technically resembles a "safe component" and is therefore a "trading adjuvant" in current negotiations. The expectation of future liquidity makes it more costly not to trade today and thereby boosts current liquidity.

Section 6 concludes with some alleys for future research.

Related literature. The literature on security design has focused on a single trade and therefore not investigated the velocity and self-fulfilling liquidity issues studied in the repeated trading part of the paper. Within the one-shot trade paradigm, the literature has

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[3]Subrahmanyam (1991) and Gorton-Pennacchi (1993) point out that pooling different assets, each with its own underlying information, reduces asymmetries of information (by the law of large numbers) and therefore boosts liquidity. Their insight is unrelated to the one just developed, as we consider a single asset, which can be tranched or sold as is.

argued that low-information intensity (LII)/debt-like securities mitigate adverse selection (held by an informed seller in DeMarzo-Duffie 1999 and an informed buyer in Gorton-Pennacchi 1990). Biais and Mariotti (2005) show that debt protects not only against adverse selection, but also against monopsony power. In their paper, the issuer receives a signal for the underlying asset's final payoff realization after the security design stage, but before trading takes place. The buyer offers a schedule specifying a transfer $T(q)$ for arbitrary fractions $q \in [0, 1]$ of the security. For example, when the signal is the future payoff realization, a debt claim has the same value for all realizations beyond the nominal debt claim; this creates an elastic demand curve and limits the buyer's market power.

Dang-Gorton-Holmström (2011) and its extension by Yang (2011) argue that carving out and marketing only LII securities further discourages the emergence of adverse selection by minimizing the buyer's incentive to acquire information. So the overall picture is that debt claims perform best against adverse selection, market power and information acquisition.

A recent literature emphasizes the role of informational asymmetries in an economy with a shortage of liquid assets. Lester-Postlewaite-Wright (forthcoming) considers an interesting and highly tractable environment embodying extreme adverse selection in an economy with search frictions: Agents are either fully knowledgeable about the asset that can be used to trade goods; or they cannot even recognize a counterfeit (and their counterparty knows that). Then trade can only occur between cognoscenti. "Money" then refers to assets with which many agents in the economy are familiar. A key contribution of Lester et al, which emphasizes themes rather different from ours, is to embody the lemons idea into a general equilibrium in which monetary assets are scarce and compete with each other. When information is endogenized, multiple equilibria may co-exist. Gorton-Ordonez (2012) develops a dynamic version of Dang et al in which the quality of a collateralized asset moves over time and mean-reverts. Borrowers scale down the use of collateral in bad macroeconomic times in order to prevent information acquisition, leading to an output decline. Like Lester et al, the focus is on macroeconomic behavior rather than on the micro-economic themes emphasized in our paper.

Our paper is also related to the literature on bargaining under endogenously asymmetric information. Like Dang et al and Yang, this literature is preoccupied mainly with information acquisition deterrence, albeit in simpler games of bargaining with take-it-or-leave-it offers for the bundle (there is no security design). Shavell (1994) studies the voluntary and mandatory disclosure of hard, private-value information in a trading relationship between a seller and a buyer; he shows for instance that the seller has excessive incentives to acquire information about the buyer's valuation whether this informa-

tion is socially useful or not, and that disclosure should be mandated. Dang (2008), in a common-value environment, points out that no trade and no information acquisition may simultaneously arise in equilibrium, and also shows that, in contrast with conventional wisdom, a party receiving the offer may obtain up to the full social surplus of the transaction as the offer is tailored to discourage him from acquiring information. Besides some modeling differences (in particular, we assume that parties decide whether to acquire their information before bargaining, so offers cannot by themselves deter information acquisition), these papers again have a different focus and do not address the two main themes of our paper, tranching and repeated trading.

2   Static Model

Consider a meeting between a seller and a buyer. The seller is endowed with an asset. He can sell this asset to the buyer. The surplus from owning the asset for the seller and the buyer are given by respectively $s\delta + S$ and $b\delta + B$, where $\delta = 1$ with probability ρ and $\delta = 0$ with probability $1 - \rho$. Both the buyer and the seller are risk neutral. We make the following assumption, which ensures that there are gains from trade.

Assumption 1. *(Gains from Trade).* $b \geq s$ *and* $B \geq S$.

We assume that δ is initially unknown to both the buyer and the seller. The seller and the buyer can learn δ at a cost c^S and c^B respectively. One party cannot observe whether the other party is informed or not.

Bargaining takes place as follows. With probability α^S, the seller makes a take-it-or-leave-it offer, and with the complementary probability $\alpha^B = 1 - \alpha^S$, the buyer makes a take-it-or-leave-it-offer. Hence we can take α^S (α^B) to represent the bargaining power of the seller (buyer).

The timing is as follows. First, parties decide whether or not to become informed. Then nature determines who gets to make a take-it-or-leave-it offer. Finally the offer is made and is either accepted or rejected, and payoffs are realized.

Equilibrium concept. Our equilibrium concept is that of Perfect Bayesian Equilibrium (PBE). We will often be confronted with a situation of bargaining under asymmetric information—both on the equilibrium path, and also off the equilibrium path when we consider the incentives of parties to acquire information. When an uninformed party makes an offer to an informed party, the uninformed party just sets its monopoly price, and the informed party either accepts or rejects the offer. Things are more complex when an informed party makes an offer to an uninformed party. The reason is that the offer
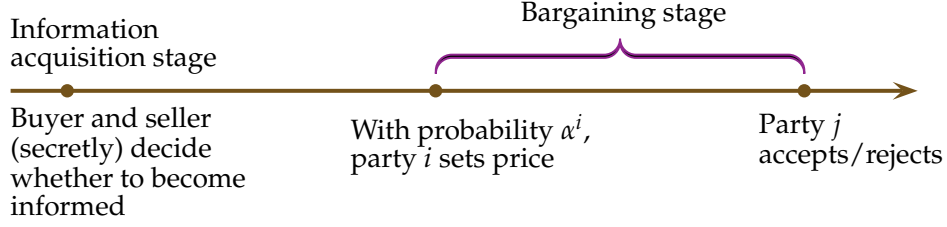
Figure 1: Timing.

potentially conveys information and acts as a signaling mechanism. This typically leads to multiple PBEs. We will always select the trade maximizing equilibrium. Imagine for example that the informed party is the seller, and the uninformed party the buyer. Then in the trade maximizing equilibrium, the seller sets a price and sells the asset only if δ is below a cutoff (0 or 1). The price is such that the buyer is indifferent between buying and not buying the asset, knowing that the asset is offered to him only if δ is below the cutoff. We can always construct beliefs such that this outcome is a PBE. It is the PBE that maximizes both the probability of trade and the welfare of the party making the offer.

In the paper, we will use the following wording convention. With a slight abuse, we will say that no offer is made when an offer is made but this offer is rejected with probability 1. This convention obviously has no material impact on our analysis.

Inefficient trade with asymmetric information. Unless otherwise stated, we make two additional assumptions:

Assumption 2. (Inefficient Trade when Only S is Informed). $s + S > b\rho + B$.

Assumption 3. (Inefficient Trade when Only B is Informed). $s\rho + S > B$.

The left-hand side of Assumption 2 represents an informed seller's reservation value when $\delta = 1$. The right-hand side represents the most that an uninformed buyer is willing to pay. Similarly, the left-hand side of Assumption 3 represents the reservation value of an uninformed seller. The right-hand side represents an informed buyer's willingness to pay when $\delta = 0$.

To understand the role of these assumptions, it is necessary to anticipate the nature of the several types of equilibria that can arise in our model. There are equilibria with asymmetric information where only one party is informed. When only the seller is informed, we refer to the equilibrium as an (Only S) equilibrium. Similarly, when only the buyer is informed, we call the equilibrium (Only B). There are also equilibria with symmetric information. When both parties are informed, we call the equilibrium (I). When no party is informed, we call the equilibrium (NI).

Together with Assumption 1, Assumptions 2 and 3 are sufficient for the equilibria with asymmetric information (Only S) and (Only B) to feature less than full trade—this is to be compared with equilibria with no or full information, where trade always occurs.[4] They are also necessary and sufficient so that in (NI), a party who secretly becomes informed will trade less. They imply that in (I), a party who secretly becomes uninformed sets a monopoly price that features less than full trade. Similarly, they imply that in an equilibrium where only one party is informed (Only S or Only B), if the uninformed party makes the offer, it sets a monopoly price that features less than full trade. Finally, the conditions are also necessary and sufficient so that in an equilibrium where only one party is informed (Only S or Only B), if the informed party decides to secretly become uninformed, then there is no trade.

2.1 Four types of equilibrium information structures

We investigate the potential pure-strategy equilibria of the game. We derive conditions on the cost of acquiring information (c^S, c^B) for these equilibria to exist. For each candidate equilibrium, we describe the equilibrium strategies.

2.1.1 Non-Informed equilibrium (NI)

Equilibrium. We first characterize equilibria where neither the seller nor the buyer are informed. If the seller makes the offer, he sets a price equal to $b\rho + B$. If the buyer makes the offer, he sets a price equal to $s\rho + S$. Trade always occurs. The party making the offer appropriates all the trade surplus $(b\rho + B) - (s\rho + S)$. The payoffs to the seller and the buyer are

$$
\begin{aligned}
v^S &= \alpha^S [b\rho + B] + \alpha^B [s\rho + S], \\
v^B &= \alpha^S [0] + \alpha^B [(b\rho + B) - (s\rho + S)].
\end{aligned}
$$

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[4]Introduce the following two weaker versions of Assumptions 2 and 3 (we sometimes refer to them as such).

Assumption 4. *(Weak Version of Assumption 2).* $b\rho + B - (s + S) < (1 - \rho)(B - S)$.

Assumption 5. *(Weak Version of Assumption 3).* $B < \rho(b + B) + (1 - \rho)S$.

Then a necessary and sufficient condition for (Only S) to feature less than full trade if the seller makes the offer is Assumption 2; a necessary and sufficient condition for (Only S) to feature less than full trade if the buyer makes the offer is Assumption 4. Similarly, a necessary and sufficient condition for (Only B) to feature less than full trade if the seller makes the offer is Assumption 5; a necessary and sufficient condition for (Only B) to feature less than full trade if the buyer makes the offer is Assumption 3.

Incentives to acquire information. Let us now turn to the incentives to acquire information. Suppose that the seller becomes informed. If the seller makes the offer, then he does not sell the asset if $\delta = 1$, and sells the asset at price $b\rho + B$ if $\delta = 0$. The fact that the seller prefers not to sell at price $b\rho + B$ if $\delta = 1$ is a direct consequence of Assumption 2.[5] Suppose now that the buyer makes the offer. The seller accepts the offer and sells the asset if $\delta = 0$ and keeps the asset if $\delta = 1$. As a result, the condition that the seller does not acquire information is

$$c^S \geq \alpha^B \rho \left[(s + S) - (s\rho + S) \right] + \alpha^S \rho \left[(s + S) - (b\rho + B) \right].$$

Similarly, using Assumption 3 the condition that the buyer does not acquire information is

$$c^B \geq \alpha^B (1 - \rho) \left[s\rho + S - B \right] + \alpha^S (1 - \rho) \left[b\rho + B - B \right].$$

Define

$$
\begin{aligned}
\underline{c}^S &= \alpha^B \rho (1 - \rho) s + \alpha^S \rho \left[(s + S) - (b\rho + B) \right], \\
\underline{c}^B &= \alpha^B (1 - \rho) \left[s\rho + S - B \right] + \alpha^S \rho (1 - \rho) b.
\end{aligned}
$$

We then have the following proposition.

Proposition 1. (NI). *The Non-Informed (NI) equilibrium exists if and only if $c^S \geq \underline{c}^S$ and $c^B \geq \underline{c}^B$.*

The incentives to become informed in the (NI) equilibrium derive solely from the possibility of refusing disadvantageous trades, whether or not the party who becomes informed makes the offer. Importantly, the incentives to become informed do not arise from an ability to charge different prices when trade does occur. For example, if a seller becomes informed, he chooses not to sell when $\delta = 1$ whether or not he makes the offer.

2.1.2 Informed equilibrium (I)

Equilibrium. Here we characterize the equilibrium where both the seller and the buyer are informed. If the seller makes the offer, he sets a price equal to $b\delta + B$. If the buyer makes the offer, he sets a price equal to $s\delta + S$. Trade always occurs. The payoffs to the

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[5]To check that this outcome maximizes trade given the information structure, suppose that the seller makes an offer different from $b\rho + B$, generating off-the-equilibrium path beliefs $\hat{\rho} > \rho$ (if $\hat{\rho} \leq \rho$, the seller doesn't benefit from offering an unexpected price). The seller trades the high-quality asset only if $b\hat{\rho} + B \geq s + S$, but then the value to the buyer is only $b\rho + B$, leading to a rejection.

seller and the buyer are the same as in (NI) minus the information acquisition costs:

$$v^S = \alpha^S [b\rho + B] + \alpha^B [s\rho + S] - c^S,$$
$$v^B = \alpha^S [0] + \alpha^B [(b\rho + B) - (s\rho + S)] - c^B.$$

Incentives to acquire information. Let us now turn to the incentives to acquire information. Suppose that the seller decides not to become informed. Suppose first that the seller makes the offer. Provided that $\rho(b + B) + (1 - \rho)S \geq B$, which is implied by Assumption 3, the seller sets a price equal to $b + B$ (rather than B), so that the buyer buys the asset if $\delta = 1$ and not if $\delta = 0$. Suppose now that the buyer makes the offer. Then the buyer's offer reveals δ, and the seller always accepts the buyer's offer. Hence the condition that the seller becomes informed can be written as

$$c^S \leq \alpha^S [(b\rho + B) - \rho(b + B) - (1 - \rho)S].$$

Similarly, provided that $(1 - \rho)[B - S] \geq b\rho + B - (s + S)$, which is implied by Assumption 2, the condition that the buyer becomes informed can be written as

$$c^B \leq \alpha^B \rho [(b + B) - (s + S)].$$

Define

$$\bar{c}^S = \alpha^S (1 - \rho)[B - S],$$
$$\bar{c}^B = \alpha^B \rho [(b + B) - (s + S)].$$

We then have the following proposition.

Proposition 2. *(I). The Informed (I) equilibrium exists if and only if $c^S \leq \bar{c}^S$ and $c^B \leq \bar{c}^B$.*

The incentives to become informed for a party in the (I) equilibrium come from the possibility to perfectly price discriminate the other party when making the offer. If instead the party under consideration does not become informed, he has to revert in this case to imperfect price discrimination. He thus extracts less surplus and trades less if it does not become informed.

2.1.3   Only S Informed equilibrium (Only S)

Equilibrium. Here we characterize the equilibrium where the seller is informed but the buyer is not. If the seller makes the offer, then he sets a price equal to B and sells the asset

only if $\delta = 0$. The buyer accepts the seller's offer if it is made. The other candidate price, $b\rho + B$, can be ruled out using Assumption 2 which guarantees that if $\delta = 1$, the seller does not want to sell at this price. If the buyer makes the offer, then he sets a price equal to S, and the seller accepts the offer only if $\delta = 0$. Provided that $(b\rho + B) - (s + S) \leq (1 - \rho)[B - S]$, which is implied by Assumption 2, the other candidate price, $s + S$, is an inferior strategy for the buyer. The payoffs to the seller and the buyer are

$$
\begin{aligned}
v^S &= \alpha^S \left[\rho (s + S) + (1 - \rho) B \right] + \alpha^B \left[s\rho + S \right] - c^S, \\
v^B &= \alpha^S [0] + \alpha^B (1 - \rho) [B - S].
\end{aligned}
$$

Incentives to acquire information. Let us now turn to the incentives to acquire information. We analyze the incentives of the seller and of the buyer in turn. Suppose that the seller decides not to become informed. If the seller makes the offer, then the seller does not want to sell—a strategy preferred to that of selling under Assumption 3. Suppose now that the buyer makes the offer. Then the seller does not sell. Hence the condition that the seller becomes informed can be written as

$$
c^S \leq \alpha^S \left[\rho (s + S) + (1 - \rho) B - (s\rho + S) \right].
$$

We now analyze the incentives of the buyer. Suppose that the buyer decides to become informed. If the seller makes the offer, then the buyer still accepts the offer of the seller if it is made, i.e. if $\delta = 0$. Suppose now that the buyer makes the offer. Then he sets a price equal to $s\delta + S$, and the seller always accepts the offer. Hence the condition that the buyer does not become informed can be written as

$$
c^B \geq \alpha^B \left[\rho \left[(b + B) - (s + S) \right] + (1 - \rho) [B - S] - (1 - \rho) [B - S] \right].
$$

We then have the following proposition.

Proposition 3. (Only S). *The (Only S) equilibrium exists if and only if $c^S \leq \bar{c}^S$ and $c^B \geq \bar{c}^B$.*

The incentives for the seller to become informed in the (Only S) equilibrium are that if the seller makes the offer, he can sell at a low price if $\delta = 0$ and extract all the surplus of the buyer. By contrast, if the seller does not become informed, he finds it best not to sell at all, extracting less surplus and trading less. The incentives for the buyer to become informed in the (Only S) equilibrium come from the possibility to perfectly price discriminate the seller when the buyer makes the offer. If instead the buyer does not become informed, he has to revert in this case to imperfect price discrimination by charging a monopoly price.

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The buyer thus extracts less surplus if it does not become informed, and also trades less.

2.1.4   Only B informed equilibrium (Only B)

Equilibrium. Here we characterize the equilibrium where the buyer is informed but the seller is not. If the seller makes the offer, then he sets a price equal to $b + B$, and the buyer accepts the offer only if $\delta = 1$. Provided that $B \leq \rho(b + B) + (1 - \rho)S$, which is implied by Assumption 3, the other candidate price, B, is an inferior strategy for the seller. If the buyer makes the offer, then he sets a price equal to $s + S$ and buys the asset only if $\delta = 1$. The seller accepts the offer of the buyer if it is made. The other candidate price, $s\rho + S$, can be ruled out using Assumption 3, which guarantees that if $\delta = 0$, the buyer does not want to buy at this price. The payoffs to the seller and the buyer are

$$
\begin{aligned}
v^S &= \alpha^S \left[\rho(b + B) + (1 - \rho)S \right] + \alpha^B \left[s\rho + S \right], \\
v^B &= \alpha^S \left[0 \right] + \alpha^B \rho \left[(b + B) - (s + S) \right] - c^B.
\end{aligned}
$$

Incentives to acquire information. Let us now turn to the incentives to acquire information. The analysis is similar to that of the (Only S) case.

Proposition 4. (Only B). *The (Only B) equilibrium exists if and only if $c^S \geq \bar{c}^S$ and $c^B \leq \bar{c}^B$.*

The incentives to become informed in the (Only B) equilibrium are the mirror image of the corresponding incentives in the (Only S) equilibrium. We do not discuss them in detail for brevity.

2.2   Equilibrium regions with endogenous information acquisition

It is useful to note that the incentives to acquire information are the same for both parties in (I), (Only S) and (Only B). That is, the increase in expected payoff from becoming informed for a party who is uninformed in equilibrium, or the loss in expected payoff from becoming uninformed for a party who is informed in equilibrium, are the same in equilibria (I), (Only S) and (Only B).

We depict equilibrium regions in the (c^S, c^B) space. The configuration of the equilibrium regions is different depending on whether $\underline{c}^B$ ($\underline{c}^S$) is lower or greater than $\bar{c}^B$ ($\bar{c}^S$), i.e. depending on wether the incentives to become informed in (NI) are lower than in (I), (Only S) and (Only B) or vice versa.

Note that the condition for $\underline{c}^B \leq \bar{c}^B$ holds for some (α^B, α^S) if and only if

$$
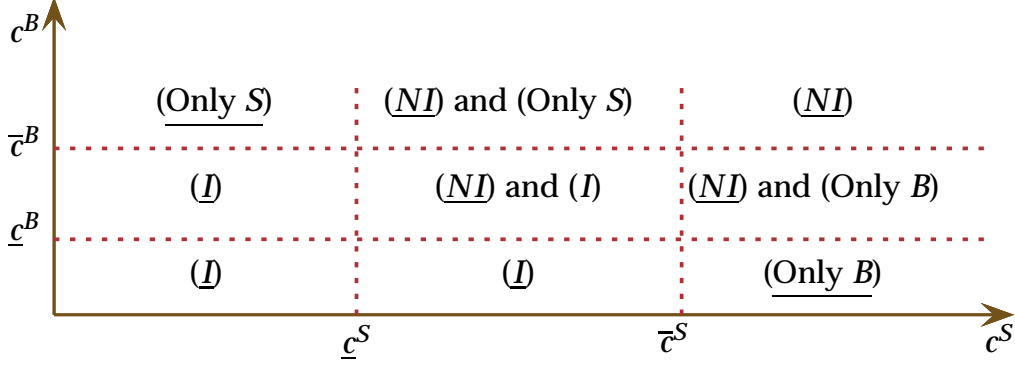(b\rho + B) - (s\rho + S) - \rho(1 - \rho)s \geq 0, \tag{1}
$$

Figure 2: Equilibrium regions (the Pareto dominant equilibrium is underlined).

i.e. if the gains from trade are large enough and the dispersion of δ is low enough. Similarly the condition for $\underline{c}^S \leq \bar{c}^S$ holds for some $\left(\alpha^B, \alpha^S\right)$ if and only if

$$(b\rho + B) - (s\rho + S) - \rho\,(1 - \rho)\,b \geq 0. \tag{2}$$

The two conditions $\underline{c}^B \leq \bar{c}^B$ and $\underline{c}^S \leq \bar{c}^S$ can simultaneously hold for some $\left(\alpha^B, \alpha^S\right)$ if and only if

$$(b\rho + B) - (s\rho + S) \geq \rho\,(1 - \rho)\,b$$

and

$$\frac{b\rho\,(1 - \rho)}{(b\rho + B) - (s\rho + S) - \rho\,(1 - \rho)\,s}\frac{\rho\,(1 - \rho)\,s}{(b\rho + B) - (s\rho + S) - \rho\,(1 - \rho)\,b} \leq 1. \tag{3}$$

In this case, the two conditions $\underline{c}^B \leq \bar{c}^B$ and $\underline{c}^S \leq \bar{c}^S$ hold simultaneously when (α^B, α^S) is in some non-empty convex set.

2.3 Ranking of equilibria

Note that when $\underline{c}^B \leq \bar{c}^B$ (respectively, $\underline{c}^S \leq \bar{c}^S$), there are complementarities between the beliefs that the buyer (respectively, the seller) is informed, and the incentives of the buyer (respectively, the seller) to become informed—as illustrated by the comparison of the incentives of the buyer (respectively, the seller) to acquire information in (NI) versus (Only B) (respectively (NI) versus (Only S)).[6] These complementarities lead to the existence of regions where multiple equilibria coexist: (NI) and either (I), (Only S) or (Only B).

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[6]There are also complementarities between the seller's and the buyer's decisions to acquire information—as illustrated by the comparison of the incentives of the buyer (resp. seller) to acquire information in (NI) versus (Only S) (resp. (NI) versus (Only B))—although in this case, the buyer (resp. seller) actually makes the same decision not to acquire information in both equilibria, even though his incentives to acquire information are higher in the latter.

The equilibria can be partially Pareto-ranked under the conditions leading to their coexistence. The equilibrium (NI) can potentially coexist with (I), (Only S) and (Only B). The equilibrium (I) can never coexist with (Only S) and (Only B), except in the knife-edge case where $c^B = \bar{c}^B$ and $c^S = \bar{c}^S$. The same is true of the equilibria (Only S) and (Only B).

The following proposition formalizes these observations.

Proposition 5. *(Pareto-Ranking of Equilibria). The equilibrium payoffs are ranked as follows:*

$$v^S(NI) \geq \max\{v^S(I), v^S(Only\,S), v^S(Only\,B)\},$$
$$v^B(NI) \geq \max\{v^B(I), v^B(Only\,S), v^B(Only\,B)\}.$$

Therefore (NI) always Pareto dominates (I), (Only S), and (Only B) whenever they coexist.

The equilibrium with no information (NI) always dominates the equilibria with asymmetric information (Only S) and (Only B) since it is more liquid. It also dominates the equally liquid equilibrium with symmetric information (I) since it economizes on information acquisition costs.

3   Dynamic Model

In this section, we construct a related infinite horizon dynamic-trading model a la Woodford (1990). Agents are risk neutral and discount the future at rate $\beta \in (0, 1)$. They are of one of two types: type 1 and type 2. Type-1 agents have investment opportunities in odd periods. In those periods, they have access to a linear technology that yields an instantaneous unit return of $R > 1$. Type-2 agents have identical investment opportunities but in even periods. Agents of type 1 (2) also have a large endowment of goods in even (odd) periods.

Agents cannot borrow: when they have investment opportunities, they can only invest up to their net worth. Together with the alternating investment opportunity structure, this generates a demand for liquidity (stores of value). We assume that the supply of liquidity is given by a unit mass of assets. These assets are Lucas trees with dividends δ_t in period t. They provide liquidity, as they can be bought by type-1 agents at the end of an even period, and sold to type-2 agents at the end of an odd period. We therefore refer to a type-1 (type-2) agent in an odd period as a seller (buyer), and vice-versa in even periods. The dividends take value 1 with probability ρ and 0 with probability $1 - \rho$, are i.i.d. across

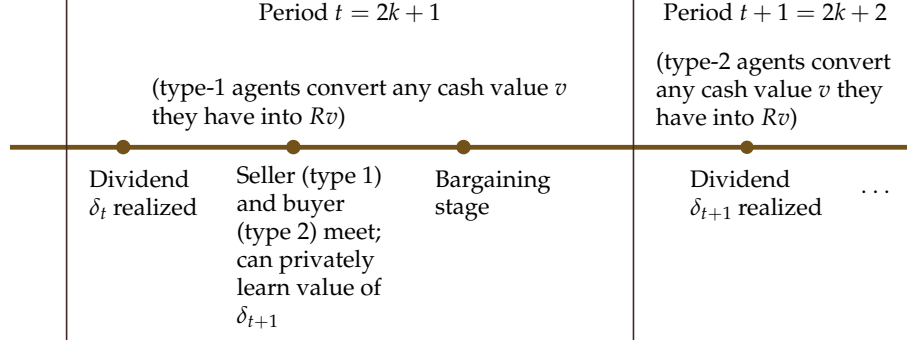| Period $t = 2k+1$			Period $t+1 = 2k+2$	
	(type-1 agents convert any cash value v they have into Rv)		(type-2 agents convert any cash value v they have into Rv)	
Dividend δ_t realized	Seller (type 1) and buyer (type 2) meet; can privately learn value of δ_{t+1}	Bargaining stage	Dividend δ_{t+1} realized	. . .

Figure 3: Timing of the dynamic model.

assets and over time but the value of δ_{t+1} can be learned one period in advance at t at a cost c^B (c^S) for type-1 agents and c^S (c^B) for type-2 agents if t is even (odd).

We assume that asset sales and purchases are made asset by asset, in a decentralized way. In every period, an asset held by an agent of a given type generates exactly one opportunity to trade with an agent of the opposite type (two different assets owned by a given agent of a given type generate meetings with different agents of the opposite type). We detail the timing within each period. At the beginning of period t, the dividend δ_t is realized. Each asset then generates a trading opportunity with an agent of the opposite type for its owner. Agents can then decide whether or not to become informed about this asset's next period dividend δ_{t+1} by incurring an information cost. Then bargaining takes place: the seller makes a take-it-or-leave-it offer with probability α^S, and the buyer makes a take-it-or-leave-it offer with probability $\alpha^B = 1 - \alpha^S$.

We focus on stationary equilibria. We show that this dynamic framework can be nested in the canonical model developed in the previous section. This makes it possible to apply the corresponding general results.

3.1 Stationary equilibria

Just like in the static model developed in Section 2, stationary equilibria are of four kinds: (NI), (I), (Only S) and (Only B). The reader can get a grasp for the methodology by looking at the (NI) case, and skip the other configurations, which employ very similar reasoning.

Non-Informed equilibrium (NI). In the (NI) equilibrium, agents do not acquire information. Every asset changes hands in a given period t for all values of δ_{t+1}. Let $\overline{p}$ and $\underline{p}$ denote the prices when the seller and the buyer make offers, respectively. Let $p \equiv \alpha^S \overline{p} + \alpha^B \underline{p}$. We denote by V^S the ex-dividend value of an asset for a seller.

A buyer's net valuation for the asset is

$$\beta[R\delta_{t+1} + V^S] - p_t$$

and a seller's net valuation for the asset is

$$Rp_t - \beta[\delta_{t+1} + \beta[R\rho + V^S]]$$

where we have made use of the no-trade theorem, namely that if at date t, a seller does not sell to the buyer (bargaining breakdown), this seller has buyer preferences at date $(t+1)$ and so keeps the asset until date $(t+2)$.

The prices p, $\underline{p}$ and $\bar{p}$ and the ex-dividend value of an asset for a seller are determined by the following equations. First, $\bar{p}$ is equal to reservation value of the buyer:

$$\bar{p} = \beta(\rho R + V^S).$$

Second, $\underline{p}$ is equal to the reservation value of the seller.

$$\underline{p} = (\beta/R)[\rho + \beta(\rho R + V^S)].$$

Third, the average price p is the weighted average of $\underline{p}$ and $\bar{p}$, the weights being given by the probabilities α^S and α^B:

$$p \equiv \alpha^S \bar{p} + \alpha^B \underline{p}.$$

Finally, the ex-dividend value of the asset for a seller is simply given by the product of the rate of return R and the average price p of the asset:

$$V^S = Rp.$$

This forms a system of four equations and four unknowns. The prices p, $\underline{p}$ and $\bar{p}$ can be directly computed using these equations once V^S is known. In turn, those equations can be manipulated to show that V^S is the solution of the fixed-point equation

$$V^S = T^{NI}(V^S)$$

where the fixed-point operator T^{NI} is defined by

$$T^{NI}(V^S) \equiv R\alpha^S \beta(\rho R + V^S) + R\alpha^B(\beta/R)[\rho + \beta(\rho R + V^S)].$$

The solution can be found in closed form:[7]

$$p = \frac{\alpha^S R + \alpha^B (1 + \beta R)/R}{1 - \alpha^S \beta R - \alpha^B \beta^2} \beta \rho,$$

$$V^S = \frac{\alpha^S R + \alpha^B (1 + \beta R)/R}{1 - \alpha^S \beta R - \alpha^B \beta^2} \beta R \rho.$$

Informed equilibrium (I). In the (I) equilibrium, agents acquire information. Every asset changes hands in a given period t for all values of δ_{t+1}. We denote by $\underline{p}$, $\bar{p}$ and p the average prices at which trades occur (averaged over the realizations of δ_{t+1}). We denote by V^S the ex-dividend value of an asset for a seller.

We now have the following equations:

$$\bar{p} = \beta(\rho R + V^S - c^S),$$

$$\underline{p} = \frac{\beta}{R}[\rho + \beta(\rho R + V^S - c^S)],$$

$$p = \alpha^S \bar{p} + \alpha^B \underline{p},$$

$$V^S = Rp.$$

This forms a system of four equations and four unknowns. The prices p, $\underline{p}$ and $\bar{p}$ can be directly computed using these equations once V^S is known. In turn, those equations can be manipulated to show that V^S is the solution of the fixed-point equation

$$V^S = T^I(V^S)$$

where the fixed-point operator T^I is defined by

$$T^I(V^S) \equiv R\alpha^S \beta(\rho R + V^S - c^S) + R\alpha^B \frac{\beta}{R}[\rho + \beta(\rho R + V^S - c^S)].$$

---

[7]In these formulas, and the corresponding formulas for the other equilibria, we assume that the denominators are strictly positive. A necessary and sufficient condition for all denominators to be positive is $1 - \alpha^S \beta R - \alpha^B \beta^2 > 0$.

The solution can be found in closed form:

$$p = \frac{\alpha^S \beta(\rho R - c^S) + \alpha^B \frac{\beta}{R}[\rho + \beta(\rho R - c^S)]}{1 - \alpha^S \beta R - \alpha^B \beta^2},$$

$$V^S = \frac{\alpha^S \beta R(\rho R - c^S) + \alpha^B \beta[\rho + \beta(\rho R - c^S)]}{1 - \alpha^S \beta R - \alpha^B \beta^2}.$$

(Only S) equilibrium. In the (Only S) equilibrium, in a given period, only the seller acquires information. The asset changes hands in a given period t if and only if $\delta_{t+1} = 0$. We denote by $\underline{p}$, $\bar{p}$ and p the prices at which trades occur if they occur (that is if $\delta_{t+1} = 0$). We denote by V^S the ex-dividend value of an asset for a seller.

We have the following equations:

$$\bar{p} = \beta(V^S - c^S),$$

$$\underline{p} = \frac{\beta}{R}[\beta(\rho R + V^S - c^S)],$$

$$p = \alpha^S \bar{p} + \alpha^B \underline{p},$$

$$V^S = \rho \left[\beta + \beta^2(\rho R + V^s - c^S) \right] + (1 - \rho)Rp.$$

This forms a system of four equations and four unknowns. The prices p, $\underline{p}$ and $\bar{p}$ can be directly computed using these equations once V^S is known. In turn, those equations can be manipulated to show that V^S is the solution of the fixed-point equation

$$V^S = T^{OnlyS}(V^S)$$

where the fixed-point operator T^{OnlyS} is defined by

$$T^{OnlyS}(V^S) \equiv \rho \left[\beta + \beta^2(\rho R + V^s - c^S) \right] + (1 - \rho)R \left[\alpha^S \beta(V^S - c^S) + \alpha^B \frac{\beta}{R}[\beta(\rho R + V^S - c^S)] \right].$$

The solution can be found in closed form:

$$p = \frac{\alpha^S \left[\rho \beta^2 (1 + \rho \beta R) - \beta c^S \right] + \alpha^B \left[\rho \beta^2 \left(\frac{\beta}{R} + 1 \right) - \frac{\beta^2}{R} c^S \right]}{1 - \rho \beta^2 - \beta(1 - \rho)(\alpha^S R + \alpha^B \beta)},$$

$$V^S = \frac{\rho \left[\beta + \beta^2(\rho R - c^S)\right] + (1-\rho)R \left[-\alpha^S \beta c^S + \alpha^B \frac{\beta^2}{R}(\rho R - c^S)\right]}{1 - \rho\beta^2 - (1-\rho)\beta \left(\alpha^S R + \alpha^B \beta\right)}.$$

(Only B) equilibrium. In the (Only B) equilibrium, in a given period, only the buyer acquires information. The asset changes hands in a given period t if and only if $\delta_{t+1} = 1$. We denote by $\underline{p}$, $\bar{p}$ and p the prices at which trades occur if they occur (that is if $\delta_{t+1} = 1$). We denote by V^S the ex-dividend value of an asset for a seller.

We have the following equations:

$$\bar{p} = \beta(R + V^S),$$

$$\underline{p} = \frac{\beta}{R}[1 + \beta(\rho R + V^S)],$$

$$p = \alpha^S \bar{p} + \alpha^B \underline{p},$$

$$V^S = \rho R p + (1-\rho)\beta^2(\rho R + V^S).$$

This forms a system of four equations and four unknowns. The prices p, $\underline{p}$ and $\bar{p}$ can be directly computed using these equations once V^S is known. In turn, those equations can be manipulated to show that V^S is the solution of the fixed-point equation

$$V^S = T^{OnlyB}(V^S)$$

where the fixed-point operator T^{OnlyB} is defined by

$$T^{OnlyB}(V^S) \equiv \rho R \left[\alpha^S \beta(R + V^S) + \alpha^B \frac{\beta}{R}[1 + \beta(\rho R + V^S)]\right] + (1-\rho)\beta^2(\rho R + V^S).$$

The solution can be found in closed form:

$$p = \beta \frac{\alpha^S[R[1 - (1-\rho)\beta^2] + (1-\rho)\rho\beta^2 R] + \alpha^B[\frac{1-(1-\rho)\beta^2}{R} + \beta\rho]}{1 - (1-\rho)\beta^2 - \beta\rho(\alpha^S R + \alpha^B \beta)},$$

$$V^S = \rho R \frac{\alpha^S \beta R + \alpha^B \frac{\beta}{R}(1 + \beta\rho R) + (1-\rho)\beta^2}{1 - (1-\rho)\beta^2 - \beta\rho(\alpha^S R + \alpha^B \beta)}.$$

3.2 Mapping the dynamic model to the static model

In all these candidate stationary equilibria, the payoffs from buying and selling, acquiring information or not are the same as in the static model considered in Section 2, for particular values of b, s, B and S. In other words, we can find a simple mapping from V^S to b, s, B and S. This mapping is the same across all stationary equilibria, up to information acquisition costs in the case of I and $OnlyS$. To represent this dependence, we introduce an indicator variable $1_{\{I,OnlyS\}}$ that takes the value 1 for the equilibria (I) and (Only S) and 0 otherwise.

Proposition 6. *(Mapping the Dynamic Model). For every stationary equilibrium (NI), (I), (Only S) and (Only B), the dynamic model can be mapped to the static model. The associated values of b, s, B, and S are given by:*

$$b = \beta R,$$
$$s = \frac{\beta}{R},$$
$$B = \beta \left(V^S - c^S 1_{\{I,OnlyS\}} \right),$$
$$S = \frac{\beta^2}{R}(\rho R + V^S - c^S 1_{\{I,OnlyS\}}).$$

It is important to note that the coefficients b and s are independent of the equilibrium. By contrast, B and S depend on the equilibrium. More precisely, B and S depend on the continuation equilibrium. We will make heavy use of this important observation in Section 5. A sufficient statistic for the dependence of B and S on the continuation equilibrium for $t \geq 1$ is given by $V^S - c^S 1_{\{I,OnlyS\}}$, the ex-dividend value of the asset for a seller net of the information cost for the seller if the seller acquires information in equilibrium. Moreover B, S and $B - S$ are increasing in $V^S - c^S 1_{\{I,OnlyS\}}$. Note that only the information cost for the seller c^S but not the information cost for the buyer c^B may enter in these formula. This is because the relevant buyer's information decisions are sunk whenever a buyer enters a negotiation with a seller (only his future decision to become informed when he turns into a seller in the next period is relevant to his valuation of the asset).

Proposition 7. *(Satisfying the Assumptions of the Associated Static Model). Assumption 1 is always satisfied as long as $R \geq 1$. There exists $\bar{R} > 1$ such that for all $R \in [1, \bar{R})$, Assumptions 2 and 3 of the static model associated with the stationary equilibria (NI), (I), (Only S) and (Only B) are satisfied whenever these equilibria exist.*

For R low enough, the gains from trade are small enough that there are inefficiencies

when parties are asymmetrically informed—which is the content of Assumptions 2 and 3.

Given these values of B and S, we can compute the associated values of $\underline{c}^B, \bar{c}^B, \underline{c}^S$ and $\bar{c}^S$ for the different stationary equilibria that we have considered:

$$
\begin{aligned}
\underline{c}^S &= \alpha^B \rho \left(1 - \rho\right) s + \alpha^S \rho \left[(s + S) - (b\rho + B)\right], \\
\underline{c}^B &= \alpha^B \left(1 - \rho\right) \left[s\rho + S - B\right] + \alpha^S \rho \left(1 - \rho\right) b, \\
\bar{c}^S &= \alpha^S \left(1 - \rho\right) \left[B - S\right], \\
\bar{c}^B &= \alpha^B \rho \left[(b + B) - (s + S)\right].
\end{aligned}
$$

Once again, in interpreting these information thresholds, it is important to keep in mind that these thresholds now depend on the continuation equilibrium for $t \geq 1$. We sometimes emphasize the dependence of V^S, B, S, $\underline{c}^B, \bar{c}^B, \underline{c}^S$ and $\bar{c}^S$, by explicitly writing these thresholds as functions. For example, we sometimes write $V^S(NI)$ or $\bar{c}^B(NI)$ to denote the values of V^S and $\bar{c}^B$ when the continuation equilibrium for $t \geq 1$ is (NI).

The existence conditions for the different stationary equilibria can then be derived exactly as in the static model. A detailed derivation is provided in Propositions 16 and 17 in Appendix A.2. This characterization is valid under the additional conditions that $R \in [1, \bar{\bar{R}})$, $\beta \in (\bar{\bar{\beta}}, 1)$, (α^B, α^S) are such that $\alpha^B \beta > 1 - \rho$, $\alpha^S \beta > 1 - \rho$, and $2\alpha^S < 3 + 2\rho(1 - \rho) - \sqrt{(3 + 2\rho(1 - \rho))^2 - 8\rho}$, where $\bar{\bar{R}} \in (1, \bar{R})$, and $\bar{\bar{\beta}} \in (0, 1)$. In the rest of the paper, unless stated otherwise, we always assume that these conditions are verified when we consider the dynamic version of the model.

3.3 Prices and liquidity across equilibria

It is useful to compare the value of V^S across equilibria.

Proposition 8. (Dynamic Liquidity and Asset Prices). *The more information is common, and the less information is acquired, the higher the ex-dividend value V^S of an asset for the seller. More precisely, whenever (NI) and another equilibrium $(E) \in \{I, Only\,S, Only\,B\}$ coexist, we have*

$$
V^S(NI) \geq \max\{V^S(E)\}.
$$

Moreover whenever (I) and (Only S) coexist, we have

$$
V^S(I) \geq V^S(Only\,S).
$$

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That $V^S(NI) \geq V^S(I)$ reflects the fact that the (NI) equilibrium economizes on information costs. These costs are capitalized and reflected in the value of the asset so that $p(NI) \geq p(I)$: an asset is more valuable today if it can be sold without needing to acquire information in the future. In other words, decreasing information acquisition while preserving commonality of information increases asset prices and welfare.

That $V^S(NI)$ is greater than $V^S(Only\ S)$ and $V^S(Only\ B)$ is the result of three effects: first, (NI) is more liquid today, and hence leads to more efficient trading; second, (NI) is more liquid in the future, which enhances the value of the asset today and creates greater gains from trade today; third, (NI) economizes on sellers' information costs. That $V^S(I)$ is greater than $V^S(Only\ S)$ is also the result of the same first two effects ((Only S) economizes on buyers' information costs, but those are not capitalized in V^S). The comparison between $V^S(Only\ B)$ and $V^S(I)$ (or $V^S(Only\ S)$) is ambiguous and depends on the value of the information acquisition cost c^S. This is because the three aforementioned effects depend on c^S.

Note that (I) and (Only S), and similarly (I) and (Only S) can now coexist in the dynamic model, which was not the case in the static model. This is because the corresponding information thresholds are endogenous to the continuation equilibrium through the values of B and S, which were instead exogenous in the static model.

4   Tranching

Suppose that the payoff of the asset is $\Delta + \delta$ where Δ is safe, $\delta = 0$ with probability $1 - \rho$ and $\delta = 1$ with probability ρ. To apply the analysis of Section 2, we renormalize B and S as $b\Delta$ and $s\Delta$, and so we can look at what happens when the asset is tranched into a pure debt tranche and a pure equity tranche.

The timing is as follows. First, the asset is tranched. Then, parties decide to acquire information. Finally, bargaining takes place. We assume that the payoff for holding debt for the seller and the buyer are $S = s\Delta$ and $B = b\Delta$. The payoffs to trading equity are $s\delta$ and $b\delta$ with $\delta \in \{0,1\}$.

Tranching has a direct effect on trading. It also modifies the incentives to acquire information. We examine these two effects in turn.

4.1   Tranching with exogenous information

We first analyze the properties of tranching when information is exogenous. In our propositions we focus for simplicity on the case where c^B and c^S are either 0 or ∞.

Observe that tranching is completely neutral if information is symmetric. Both the debt and the equity part are traded with probability 1 and the game has the exact same equilibrium payoffs for all parties.

Proposition 9. (Neutrality of Tranching with Common Information). *Consider the (I) equilibrium ($c^B = 0$ and $c^S = 0$) or the (NI) equilibrium ($c^B = \infty$ and $c^S = \infty$). Under bundling, trade occurs with probability* 1*. Under tranching, trade of both the safe and the risky tranche occurs with probability* 1*. Bundling and tranching are Pareto-equivalent.*

When there is commonality of information, there is no illiquidity and both the insulation and the trading adjuvant effects have no bite.

By contrast, tranching is not neutral when information is asymmetric and trade only occurs with some probability. Indeed, when information is asymmetric, the effect of tranching on liquidity is ambiguous. Tranching isolates a safe debt part that is completely liquid (traded with probability one). Spinning off this safe tranche insulates it against the distrust generated by the risky tranche. This tends to increase overall liquidity.

But tranching also makes the residual equity part riskier and hence less liquid. The safe tranche no longer serves as an adjuvant in negotiations over the risk tranche. Another way to say it is that spinning off the safe tranche lowers the cost of not trading the risky tranche—the safe tranche no longer serves as a form of "mutual hostage". This effect is similar to Whinston (1990)'s observation that bundling leads to more competitive pricing. This tends to reduce overall liquidity.

Thus the liquidity benefit of tranching is an *insulation effect* while the liquidity benefit of bundling is a *trading adjuvant* effect. We illustrate these effects with two propositions, starting with the insulation effect.

Proposition 10. (Insulation Effect of Tranching).*We have the following.*

    i. *Consider the (Only S) equilibrium ($c^B = \infty$ and $c^S = 0$). Under bundling, trade occurs only if $\delta = 0$. Under tranching, the safe tranche is traded with probability* 1 *but the equity tranche is traded only if $\delta = 0$. Tranching Pareto-dominates bundling.*

    ii. *Consider the (Only B) equilibrium ($c^B = 0$ and $c^S = \infty$). Under bundling, trade occurs only if $\delta = 1$. Under tranching, the safe tranche is traded with probability* 1 *but the equity tranche is traded only if $\delta = 1$. Tranching Pareto-dominates bundling.*

Proof. Note that under tranching, the safe tranche is always traded. The rest of the proposition follows from the fact that the equivalent of Assumptions 2 and 3 for the risky tranche, namely $s > b\rho$ and $s\rho > 0$ are implied by Assumptions 2 and 3. □

We continue with the trading adjuvant effect. This effect requires suspending Assumptions 2 and 3.

Proposition 11. *(Trading Adjuvant Effect of Bundling). We have the following.*

i. *Assume that the weak version of Assumption 2 (Assumption 4) is violated so that $(b\rho + B) - (s + S) > (1 - \rho)(B - S)$. In addition, assume that $s > b\rho$. Consider the (Only S) equilibrium ($c^B = \infty$ and $c^S = 0$). Under bundling, trade occurs with probability 1. Under tranching, the safe tranche is traded with probability 1 but the equity tranche is traded only if $\delta = 0$. The seller is better off under bundling than under tranching, and the buyer is worse off under bundling than under tranching.*

ii. *Assume that the weak version of Assumption 3 (Assumption 5) is violated so that $B > \rho(b + B) + (1 - \rho)S$. Consider the (Only B) equilibrium ($c^B = 0$ and $c^S = \infty$). Under bundling, trade occurs with probability 1. Under tranching, the safe tranche is traded with probability 1 but the equity tranche is traded only if $\delta = 1$. The buyer is better off under bundling than under tranching, and the seller is worse off under bundling than under tranching.*

Proof. We treat the (Only S) equilibrium. The analysis for the (Only B) equilibrium is similar.

Consider first the case of bundling. The trade maximizing equilibrium can be described as follows. If the seller makes the offer, then he sets a price equal to $b\rho + B$ and sells the asset with probability 1. The seller is better off selling at that price even if $\delta = 1$ because $b\rho + B > s + S$. If the buyer makes the offer, then he sets a price equal to $s + S$ and buys the asset with probability 1. The other candidate price, S, leads to a lower payoff for the buyer since $(1 - \rho)(B - S) < (b\rho + B) - (s + S)$. The payoffs are

$$v^S = \alpha^S [b\rho + B] + \alpha^B [s + S],$$
$$v^B = \alpha^S [0] + \alpha^B [(b\rho + B) - (s + S)].$$

Consider now the case of tranching. Then the safe tranche is traded with probability 1, so we focus on the risky tranche. If the seller makes the offer, then he sets a price equal to 0 and sells the risky tranche only if $\delta = 0$ (in which case the risky tranche is worth 0 to both the buyer and the seller). The other candidate price, $b\rho$, can be ruled out since $s > b\rho$ which guarantees that if $\delta = 1$, the seller does not want to sell at this price. If the buyer makes the offer, then he sets a price equal to 0, and the seller accepts the offer only if $\delta = 0$ (in which case the risky tranche is worth 0 to both the buyer and the seller). Since

$s > b\rho$, the other candidate price, s, is an inferior strategy for the buyer. The payoffs are

$$
\begin{aligned}
v^S &= \alpha^S \left[s\rho + B \right] + \alpha^B \left[s\rho + S \right], \\
v^B &= \alpha^S \left[0 \right] + \alpha^B \left[B - S \right].
\end{aligned}
$$

It is apparent that the seller is better off under bundling and that the buyer is better off under tranching. □

Under the hypotheses of Proposition 11, bundling always increases liquidity when only one party is informed so that information is asymmetric. As a result, there are more gains from trade. However, these additional gains from trade are entirely captured by the informed party. This emphasizes that bundling has both a trading adjuvant effect and also a *tilting of bargaining power effect* that always favors the informed party.

Actually, while bundling makes the informed party better off, it also makes the uninformed party worse off. Intuitively, when the uninformed party makes the offer, it prefers to propose a very attractive offer in order to trade the asset with probability 1 under bundling, whereas under tranching it can make a less attractive offer, trade the equity tranche only with some probability but trade the debt tranche with probability 1. In other words, under bundling, the informed party extracts some surplus even when the uninformed party makes the offer.

4.2 Tranching with endogenous information acquisition

When information acquisition is endogenous, tranching modifies the incentives to acquire information. This plays out differently in different cases. Indeed, starting at some bundling equilibrium, tranching can either increase or decrease the incentives to acquire information. Whether this *information effect* of tranching enhances or hinders liquidity depends on whether parties were both informed, asymmetrically informed, or both uninformed at the original bundling equilibrium.

For conciseness, we consider only the buyer's incentives to acquire information, and focus on the case where the seller is either uninformed ($c^S = \infty$) or informed ($c^S = 0$). We make Assumptions 2 and 3 and so the insulation effect dominates the trading adjuvant effect. However the liquidity effect of tranching is still present. Hence any eventual adverse effect of tranching must be due to the information effect.

Recall that

$$\bar{c}^B = \alpha^B \rho \left[(b + B) - (s + S) \right],$$
$$\underline{c}^B = \alpha^B (1 - \rho) \left[s\rho + S - B \right] + \alpha^S \rho (1 - \rho) b.$$

As long as $\alpha^B > 0$, $\underline{c}^B$ is decreasing in $B - S$ and that $\bar{c}^B$ is increasing in $B - S$, implying that $\underline{c}^B$ increases with tranching, and that $\bar{c}^B$ decreases with tranching. We find it convenient to indicate the dependence of these information thresholds on tranching with a (T), for Tranching and on bundling with an (NT), for No Tranching. We have established the following proposition.

Lemma 1. *(Information Effect of Tranching). As long as $\alpha^B > 0$:*

i. when $c^S = \infty$, tranching increases the incentives of the uninformed buyer to become informed in equilibrium (NI), $\underline{c}^B(T) > \underline{c}^B(NT)$;

ii. when $c^S = \infty$, tranching decreases the incentives of the uninformed buyer to become informed in equilibrium (Only B), $\bar{c}^B(T) < \bar{c}^B(NT)$;

iii. when $c^S = 0$, tranching reduces the incentives of the buyer to acquire information in equilibrium (I) and (Only S), $\bar{c}^B(T) < \bar{c}^B(NT)$.

We start with (i). That $\underline{c}^B(T) > \underline{c}^B(NT)$ shows that at the equilibrium where both parties are uninformed (NI), tranching increases the incentives of the buyer to acquire information. This is because at the (NI) equilibrium, the benefit of becoming informed for the buyer hinges on refusing some trades. Under bundling, refusing trades for the risky tranche comes with the *collateral damage* of not trading the safe tranche. This collateral damage is absent under tranching—another implication of the trading adjuvant effect of bundling which disappears under tranching. Therefore, refusing trades is less costly under tranching than under bundling. This enhances the incentives of the buyer to become informed.

Indeed, when the seller makes an offer (probability α^S) and the buyer identifies the asset as a lemon $\delta = 0$ (probability $1 - \rho$), the latter can simply turn down the trade. This increases his payoff compared to the case where he does not acquire information by $\alpha^S (1 - \rho)[b\rho + B - B]$. Similarly, when the buyer makes an offer (probability α^B), and identifies the asset as a lemon $\delta = 0$ (probability $1 - \rho$), the buyer can simply not make an offer. This increases his payoff compared to the case where he does not acquire information by $\alpha^B (1 - \rho)[(s\rho + S) - B]$. In the first case, the gain of the buyer from becoming informed is independent of tranching. In the second case, under tranching, the buyer's

gain from becoming informed is increased from $\alpha^B(1 - \rho)[(s\rho + S) - B]$ to $\alpha^B(1 - \rho)s\rho$: the buyer can still purchase the safe tranche and not make an offer on the risky tranche.

We continue with (iii), which we discuss in the context of the (I) equilibrium (we could have alternatively chosen the (Only S) equilibrium, or the (Only B) equilibrium). That $\bar{c}^B(T) < \bar{c}^B(NT)$ shows that at the equilibrium where both parties are informed (I), tranching reduces the incentives of the buyer to acquire information. This is because at the (I) equilibrium, the benefit of becoming informed for the buyer hinges on the possibility of making some trades. Under bundling, making trades for the risky tranche comes with the *collateral benefit* of making trades for the safe tranche. This collateral benefit of bundling disappears under tranching. Tranching therefore reduces the incentives of the buyer to acquire information.

Indeed when the buyer makes an offer (probability α^B), and the buyer identifies that $\delta = 1$ (probability ρ), he can offer $s + S$ and get the seller to sell him the asset yielding a benefit $(b + B) - (s + S)$. If the buyer is uninformed, he prefers not to generate that trade because he fears being sold a lemon $\delta = 0$. An uninformed buyer offers to pay S and the informed seller accepts if $\delta = 0$. Hence acquiring information increases the payoff of the buyer by $\alpha^B\rho[(b + B) - (s + S)]$. Under tranching, this gain is reduced to $\alpha^B\rho(b - s)$ because an uninformed buyer can still buy the safe tranche at price S when confronted with an informed seller who observes $\delta = 1$.

4.2.1   Uninformed seller ($c^S = \infty$)

We start by analyzing the case where the seller is uninformed but the buyer can decide to acquire information. This is the case considered by Dang-Gorton-Holmström (2011) and Yang (2011). Note also that the case analyzed by Gorton-Penacchi (1990) is a particular case where in addition $c^B = 0$ so that the buyer is informed.

We now translate Lemma 1 into equilibrium predictions and show that, despite the fact that making Assumptions 2 and 3 stacks the deck in favor of tranching, bundling may dominate tranching once information is endogeneized.

Proposition 12. (Tranching with Uninformed Seller). *Assume that $c^S = \infty$, $\alpha^B > 0$ and $\underline{c}^B(T) < \bar{c}^B(T)$ (and so from Proposition 1, we have $\underline{c}^B(NT) < \bar{c}^B(NT)$ as well). Then:*

i. for $c^B \in [\underline{c}^B(NT), \underline{c}^B(T))$, bundling Pareto-dominates tranching;

ii. for $c^B \in [0, \underline{c}^B(NT))$, tranching Pareto-dominates bundling.

Proof. Under both bundling and tranching, for $c^B < \underline{c}^B$ the only equilibrium is (Only B), for $\underline{c}^B \leq c^B \leq \bar{c}^B$, there are two possible equilibria (Only B) and (NI), and for $\bar{c}^B < c^B$, the

only equilibrium is (NI). When $\underline{c}^B \leq c^B \leq \bar{c}^B$, we select the Pareto-dominant equilibrium (NI). Hence the equilibrium is (Only B) for $c^B < \underline{c}^B$ and (NI) for $\underline{c}^B \leq c^B$.

Lemma 1 shows that as long as $\alpha^B > 0$, we have $\underline{c}^B(T) > \underline{c}^B(NT)$, so that (NI) is more likely to be the equilibrium under bundling than under tranching. Indeed for $c^B \in [\underline{c}^B(NT), \underline{c}^B(T))$, the equilibrium is (NI) under bundling and (Only B) under tranching. Both parties are then better off under bundling than under tranching. This illustrates the adverse information effect of tranching, which reduces overall liquidity by increasing the incentives of the buyer to acquire information.

By contrast, when $c^B \in [0, \underline{c}^B(NT))$, then the equilibrium is (Only B) under both tranching and bundling. Both parties are then better off under tranching. This is a manifestation of the benefits of the insulation effect: tranching allows to trade the safe tranche with probability 1. $\qquad\square$

4.2.2 Informed seller ($c^S = 0$)

We now deal with the case where the seller is informed and the buyer can decide whether to acquire information. Myers-Majluf (1984) and DeMarzo-Duffie (1999) can be considered as special cases where in addition $c^B = \infty$ so that the buyer is uninformed.

Proposition 13. *(Tranching with Informed Seller).* *Assume that $c^S = 0$ and $\alpha^B > 0$. Then:*

i. *for $c^B \in (\bar{c}^B(T), \bar{c}^B(NT)]$, the buyer is worse off and the seller is better off under bundling than under tranching;*

ii. *for $c^B \in (\bar{c}^B(T), \min\{\bar{c}^B(NT), \frac{1}{\alpha^B}\bar{c}^B(T)\}]$, bundling increases total welfare $v^S + v^B$ and for $c^B \in (\min\{\bar{c}^B(NT), \frac{1}{\alpha^B}\bar{c}^B(T)\}, \bar{c}^B(NT)]$, bundling decreases total welfare $v^S + v^B$.*

iii. *for $c^B \in (\bar{c}^B(NT), \infty)$, tranching Pareto-dominates bundling.*

Proof. Under both bundling and tranching, for $c^B < \bar{c}^B$ the only equilibrium is (I), for $\bar{c}^B < c^B$, the only equilibrium is (Only S).

Lemma 1 shows that as long as $\alpha^B > 0$, we have $\bar{c}^B(T) < \bar{c}^B(NT)$, so that (I) is more likely to be the equilibrium under bundling than under tranching. Indeed for $c^B \in (\bar{c}^B(T), \bar{c}^B(NT)]$, the equilibrium is (I) under bundling and (Only S) under tranching. Using Lemma 2 in Appendix A.4, we know that under tranching, we have $v^B(I) \geq v^B(OnlyS)$ if and only if $c^B \leq \bar{c}^B(T)$. Using the fact that $v^B(I)$ is the same under tranching and bundling, we conclude that the buyer is worse off under bundling. By contrast, the seller is obviously better off. Lemma 2 in Appendix A.4 also shows that bundling

increases total welfare $v^S + v^B$ for $c^B \in (\bar{c}^B(T), \min\{\bar{c}^B(NT), \frac{1}{\alpha^B}\bar{c}^B(T)\}]$ and decreases it for $c^B \in (\min\{\bar{c}^B(NT), \frac{1}{\alpha^B}\bar{c}^B(T)\}, \bar{c}^B(NT)]$.

By contrast, when $c^B \in (\bar{c}^B(NT), \infty)$, then the equilibrium is (Only S) under both tranching and bundling. Both parties are then better off under tranching. This is a manifestation of the benefits of the insulation effect: tranching allows to trade the safe tranche with probability 1. $\qquad\square$

Case (i) in Proposition 12, and cases (i) and (ii) in Proposition 13 illustrate the adverse information effect of tranching. When the seller is uninformed, tranching *increases* the incentives of the buyer to acquire information. When the seller is informed, tranching *decreases* the incentives of the buyer to acquire information. In both cases, tranching works against commonality and information and towards asymmetric information, to the detriment of liquidity and welfare.

4.2.3 Tranching and commonality of information

We now provide a more general argument solidifying the intuition, provided in the introduction, that tranching encourages information acquisition when it should be deterred and discourages it when it should be promoted. We thereby also shed further light on Lemma 1, and Propositions 12 and 13. The analysis relies on a simple convexity argument, allows arbitrary tranching and does not require Assumptions 2 and 3.

Suppose that the asset is split into I tranches ($i = 1, ..., I$); each tranche i is composed of a fraction x_i of equity (cash-flow right on δ) and of a fraction y_i of debt, such that $\Sigma_i x_i = \Sigma_i y_i = 1$. The seller and the buyer bargain over the entire bundle under bundling, and enter piecemeal negotiations for each tranche under tranching.

Proposition 14. (Tranching Works Against Commonality of Information).

 i. *If (NI) is an equilibrium under tranching, then (NI) is a fortiori an equilibrium under bundling;*

 ii. *if (I) is an equilibrium under tranching, then (I) is a fortiori an equilibrium under bundling.*

Proof. We start with (i). Suppose that (NI) is an equilibrium. Let us compute the buyer's gain from information acquisition under tranching ($G_T^B(NI)$) and under bundling ($G_{NT}^B(NI)$) (the reasoning is symmetrical for the seller). Under tranching, when the seller makes the offer, the seller offers price $y_i B + x_i \rho b$ for tranche i. The buyer's gain from being informed is then $(1 - \rho)x_i(\rho b)$, so the total gain over all tranches is $\Sigma_i(1 - \rho)x_i(\rho b) = (1 - \rho)\rho b$, and

so is the same as under bundling. Suppose now that the buyer makes the offer. When un-informed, the buyer offers $y_i S + x_i \rho s$ for tranche i and this offer is accepted. The buyer's gain on tranche i from being informed is $\max\{a_i, 0\}$, where

$$a_i \equiv (1 - \rho) \left[y_i S + x_i \rho s - y_i B \right].$$

Under bundling the gain from being informed is $\max\{a, 0\}$, where

$$a \equiv (1 - \rho) \left[S + \rho s - B \right] = \Sigma_i a_i.$$

And so,
$$G_T^B(NI) \equiv \alpha^B \Sigma_i \max\{a_i, 0\} \geq G_{NT}^B(NI) \equiv \alpha^B \max\{\Sigma_i a_i, 0\}.$$

We now deal with (ii). Suppose that (I) is an equilibrium and let us compute the losses $L_T^B(I)$ and $L_{NT}^B(I)$ for the buyer from not being informed (again the reasoning is symmetrical for the seller).

With probability α^S, the seller offers price B in the bad state and price $B + b$ in the good state. The buyer has no surplus and therefore there is no loss from being uninformed (besides, the offer reveals the state of nature). So suppose that the buyer makes the offer. The loss for tranche i from not being informed is

$$a_i \equiv (1 - \rho)y_i(B - S) + \rho \left[y_i(B - S) + x_i(b - s) \right] - y_i(1 - \rho)(B - S)$$
$$\text{if} \quad y_i \rho(B - S) \leq x_i(s - \rho b)$$

and

$$b_i \equiv (1 - \rho)y_i(B - S) + \rho \left[y_i(B - S) + x_i(b - s) \right] - \left[y_i(B - S) + x_i(\rho b - s) \right]$$
$$\text{if} \quad y_i \rho(B - S) \geq x_i(s - \rho b).$$

And so
$$L_T^B(I) \equiv \alpha^B \Sigma_i \min\{a_i, b_i\} \leq L_{NT}^B(I) \equiv \alpha^B \min\{\Sigma_i a_i, \Sigma_i b_i\}.$$

$\square$

While it is instructive to compute actual gains and losses (in particular, we see that both are associated with the possibility of making an offer), the reasoning in the proof does not hinge on the exact expressions. The key insight is that the gain from becoming

informed in a non-informed equilibrium is linked to the possibility of refusing a disadvantageous trade (buying a lemon, selling a high quality asset). In this respect, tranching offers more flexibility in the trade pattern and therefore a higher gain from deviating from the non-informed equilibrium. Similarly, the loss associated with not being informed in an informed equilibrium is associated with the possibility of either not trading or not capturing the other side's surplus. Minimizing this loss piecewise is easier than minimizing it globally, and so the incentive to deviate from an informed equilibrium is greater under tranching.

Finally, we note that we have taken the view that opportunities for trade are unaffected by security design: The buyer, say, can buy the same overall security under tranching and bundling. For instance, an informed buyer can under tranching acquire in several negotiations the various pieces of the whole bundle that he can acquire in a single negotiation under bundling. In making this assumption, we implicitly follow the literatures on market microstructure and on mechanism design. An opposite view would be that one should cut securities in an arbitrary number of different tranches that would be traded by different groups of agents; in such a world, it would be difficult to see how asymmetric information would ever emerge, since the cost of acquiring information could never be recouped through purchasing a tiny piece of the overall cake. In our view, the reason why the formalism adopted in this section is more relevant is that there is in practice, a second kind of information acquisition. Economic agents who trade an asset must have a minimum amount of familiarity with the properties of this asset (as in Lester et al, forthcoming). Thus, cutting into small pieces would create illiquidity rather than enhance liquidity. We hope that future research will develop and clarify this line of thought, that seems crucial for market microstructure and security design.

4.3 Discussion of the related literature

The security design literature is more general in some respects, and less general in some others. Outcomes can usually take a continuum of values instead of being binary. And optimal tranching involves mixing the safe part with as much equity as is consistent with keeping the former liquid. On the other hand, the literature usually considers special cases, as we do in this section (for example in DeMarzo-Duffie (1999), $c^S = 0$ and $c^B = \infty$). More importantly, the literature makes two key assumptions: (a) the seller has full bargaining power ($\alpha^S = 1$); and (b) the seller can commit to sell some tranche and keep the rest. Concerning this commitment assumption, note that the seller benefits from selling the equity tranche after disposing of the safe one. Whether the seller is likely to be able

to abide to such a commitment to forego beneficial trades is context-dependent, and we find both cases to be of interest.

Despite these differences, we can compare our results with those of the literature. DeMarzo and Duffie consider the case of an exogenously informed seller. Our Assumption 2 corresponds to their assumption that the bundle leads to wasteful under-trade; and Proposition 10 (i) is broadly consistent with their identification of the insulation effect.

Dang, Gorton and Holmström (2011) study the case of an uninformed seller ($c^S = \infty$) and endogenous information acquisition by the buyer ($0 < c^B < \infty$). They find that, in contrast with the analysis of this section, tranching always optimally deters information acquisition by the buyer. The difference with Proposition 12 can be grasped by returning to assumptions (a) and (b) stated above. To understand the role of the commitment assumption, suppose that $\alpha^S = 1$. While tranching deters information acquisition by the buyer in Dang et al, it is neutral with respect to information acquisition in our model: Recall that the buyer's incentive to acquire information in the (NI) equilibrium is the ability to refuse trading when $\delta = 0$. Regardless of how many tranches one constructs out of the bundle and of how these tranches are structured, the total overcharging in the bad state of nature is equal to ρb. And so the buyer's incentive to acquire information is independent of financial engineering. This is not so when the seller can commit not to trade (risky) tranches; the buyer's incentive to acquire information is then reduced. Second, when $\alpha^B > 0$, tranching is no longer neutral as shown by Proposition 12 (i). Tranching enables the buyer to make a finer use of his information, i.e. to pick and choose, and thereby encourages information acquisition.

4.4 Applying the results to the dynamic model

We can directly apply our results to the dynamic model using the mapping to the static model. The interpretation is the following. We are considering the impact of tranching at date t, once the period-t dividend δ_t has been paid. The asset is tranched into a claim on δ_{t+1} and a separate claim on future dividends δ_{t+2}, δ_{t+3},... No further tranching is allowed. All the results derived in this section for the static model (Lemma 1 and Propositions 12 and 13) can then be applied to the dynamic model for the form of tranching that we have just detailed.

We can also derive results for a different form of tranching, where in every period t (as opposed to only in one period), the asset is tranched into a claim to δ_{t+1} and a separate claim to future dividends δ_{t+2}, δ_{t+3},...The results for this model are almost identical. The only difference is that part (ii) of Proposition 13 needs to be modified as follows: for

$c^B \in (\bar{c}^B(T), \min\{\bar{c}^B(NT), \hat{c}^B\}]$, bundling increases total welfare $v^S + v^B$ and for $c^B \in (\min\{\bar{c}^B(NT), \hat{c}^B\}, \bar{c}^B(NT)]$, bundling decreases total welfare $v^S + v^B$, where a different threshold $\hat{c}^B$ takes the place of $\frac{1}{\alpha^B}\bar{c}^B(T)$ in these expressions (this threshold $\hat{c}^B$ is defined by the condition that total welfare $v^S + v^B$ is the same under tranching and under no tranching).[8] The proofs of these results are omitted for brevity.

5   Dynamic Self-Fulfilling Liquidity

In this section, we focus on the dynamic aspects of liquidity. In particular, we are interested in the impact of expectations regarding future liquidity on contemporaneous liquidity.

One way to approach this question is to consider simple equilibria of the dynamic game where the continuation equilibrium is independent of the date (stationary). More precisely, we assume that independently of the actions taken at $t = 0$, the equilibrium from $t = 1$ on is the same stationary equilibrium, either (NI), (I), (Only S) or (Only B). We can then analyze how the set of equilibria at $t = 0$ changes as we vary the continuation equilibrium for $t \geq 1$. For example, we can ask how the conditions of existence of (NI) at $t = 0$ change depending on whether (NI) or (Only B) is played for $t \geq 1$.

Given a continuation equilibrium, the set of equilibria is identical to the one of the static game we analyzed in Section 2, for some specific values of b, s, B and S. The key is that B and S now depend on the continuation equilibrium. This dependence turns out to be exactly the one outlined above in Section 3.2. We can then simply analyze the dependence of the information thresholds $\underline{c}^B$, $\bar{c}^B$, $\underline{c}^S$ and $\bar{c}^S$ of the static game on B and S. There are many equilibrium thresholds. For simplicity and in the interest of space, we focus on the case where $c^S \in \{0, \infty\}$ or $c^B \in \{0, \infty\}$.

Proposition 15. (Dynamic Self-Fulfilling Liquidity). *The dependence of the information thresholds at $t = 0$ on the continuation equilibrium (NI), (I), (Only S) and (Only B) for $t \geq 1$ is as follows, as long as the corresponding continuation equilibria exist for $t \geq 1$:*

*   *i. ($c^S = \infty$) $\underline{c}^B(NI) \leq \underline{c}^B(Only\ B)$ with a strict inequality if and only if $0 < \alpha^B < 1$, so that the (NI) equilibrium is more likely to exist at $t = 0$ if the continuation equilibrium for $t \geq 1$ is (NI) than if it is (Only B);*

*   *ii. ($c^B = \infty$) $\underline{c}^S(NI) \leq \underline{c}^S(OnlyS)$ with strict inequality if and only if $\alpha^B < 1$, so that the (NI) equilibrium is more likely to exist at $t = 0$ if the continuation equilibrium for $t \geq 1$ is (NI) than if it is (Only S).;*

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[8] One can show that $\hat{c}^B \leq \frac{1}{\alpha^B}\bar{c}^B(T)$.

iii. ($c^S = 0$) $\bar{c}^B(OnlyS) \le \bar{c}^B(I)$ with a strict inequality if and only if $\alpha^B > 0$, so that (I) is more likely to exist at $t = 0$ if the continuation equilibrium for $t \ge 1$ is (I) than if it is (Only S);

iv. ($c^B = 0$) $\bar{c}^S(OnlyB) \le \bar{c}^S(I)$ with a strict inequality if and only if $\alpha^S > 0$, so that (I) is more likely to exist at $t = 0$ if the continuation equilibrium for $t \ge 1$ is (I) than if it is (Only B).

Consider first the case of an uninformed seller ($c^S = \infty$). If (NI) is the continuation equilibrium for $t \ge 1$, then the condition for the existence of the equilibrium (NI) at $t = 0$ is $c^B \ge \underline{c}^B(NI)$. Similarly, if (Only B) is the continuation equilibrium for $t \ge 1$, then the condition for the existence of (NI) at $t = 0$ is $c^B \ge \underline{c}^B(OnlyB)$. Part (i) of Proposition 15 states that $\underline{c}^B(NI) \le \underline{c}^B(OnlyB)$. This means that the (NI) equilibrium is more likely to exist at $t = 0$ if the continuation equilibrium for $t \ge 1$ is (NI) than if it is (Only B). This is because the (ex-dividend) value of the asset V^s is higher under (NI) than under (Only B). Intuitively, the asset is expected to be more liquid under (NI) than under (Only B). This in turn increases the gains from trading the asset at $t = 0$. But the only reason for a buyer to acquire information at $t = 0$ is to use this information to turn down disadvantageous trades. The gains from trade are higher under (NI) than under (Only B). Hence the incentives for a buyer to acquire information under (NI) are lower than under (Only B). In other words, when the seller is not informed, the buyer is less likely to acquire information at $t = 0$ if the continuation equilibrium for $t \ge 1$ is (NI) than if it is (Only B).

Consider now the case of an informed seller ($c^S = 0$). If (I) is the continuation equilibrium for $t \ge 1$, then the condition for the existence of (I) is $c^B \le \bar{c}^B(I)$. If (Only S) is the continuation equilibrium for $t \ge 1$, then the condition for the existence of (I) is $c^B \le \bar{c}^B(OnlyS)$. Part (iii) of Proposition 15 shows that $\bar{c}^B(OnlyS) \le \bar{c}^B(I)$, establishing that (I) is more likely to exist at $t = 0$ if the continuation equilibrium for $t \ge 1$ is (I) than if it is (Only S).

Parts (ii) and (iv) of Proposition 15 can be used to establish similar dynamic self-fulfilling liquidity results in the case of an uninformed buyer ($c^B = \infty$) and the case of an informed buyer ($c^B = 0$). In the case of an uninformed buyer, (NI) is more likely to exist at $t = 0$ if the continuation equilibrium for $t \ge 1$ is (NI) than if it is (Only S). In the case of an informed buyer, (I) is more likely to exist at $t = 0$, if the continuation equilibrium for $t \ge 1$ is (I) than if it is (Only B).

These results illustrate an important dynamic aspect of liquidity in our model. Through an information channel, liquidity is dynamically self-fulfilling. The expectation of liquid-

ity tomorrow increases liquidity today.

6   Conclusion

The paper aims at revisiting and developing new insights on security design. After pitting the insulation effect (tranching confines and liquefies the safe part of a cash flow) against the trading adjuvant effect (bundling makes the risky part more liquid), the paper's first substantive contribution was to show that tranching always has adverse welfare effects on information acquisition: Tranching reduces a party's cost of not trading and therefore works against commonality of information and the concomitant liquidity of the asset. Tranching thereby encourages (discourages) information acquisition when it should be deterred (encouraged). The paper provides conditions under which tranching reduces welfare even when the insulation effect dominates the trading adjuvant effect.

The security design literature has focused on a single transaction between a seller and a buyer. Yet, the essence of a "liquid asset" is that it serves as collateral or simple means of exchange in a series of economic transactions (repos, acquisitions...). The faster it circulates, the more useful it is as a store of value. The paper's second contribution was to analyze the velocity of assets that are repeatedly traded. The dynamic model can be nested into the static one, enabling us to make use of existing results. The central insight is that liquidity is self-fulfilling: A perception of future illiquidity creates current illiquidity. Insights on velocity are shown to be closely related to those on tranching.

The focus of this paper leaves many alleys open to future research. One of the most challenging, but also potentially most rewarding ones is to embody these considerations in a general equilibrium framework with a shortage of stores of value. Endogenously varying demand for liquidity impacts the velocity of existing stores of value and therefore the supply of liquidity. Another extension would look at security design once the veil of ignorance is lifted. The issuer then would use security design to signal underlying security values, as in Nachman-Noe (1994). Yet another worthwhile line of investigation would try to find conditions under which the standard assumption of "learning by holding" (made e.g. in Plantin 2009 and papers assuming that the seller is superiorly informed) is likely to hold; while seller's superiority of information is natural in primary markets, it is arguably less so in secondary markets. A fourth extension would allow for a larger set of information acquisition strategies. We have followed much of the literature in assuming that the parties can acquire a piece of information about the value of the asset; this was natural in our binary-state environment. Had we considered a continuum of outcomes, say, we could have allowed, in the spirit of Yang (2011) and the rational inat-

tention literature, parties to focus their attention on specific regions of the outcome space; the impact of tranching on focused attention is an interesting alley for research.

Finally, related ideas could be developed outside the realm of security design. The issue of lump-sum vs. piecemeal negotiations is relevant in many areas of the social sciences. For example, opinions diverge as to whether climate change negotiations should be conducted at the sectoral level or globally. Piecemeal negotiations (the equivalent of tranching) affect incentives for information acquisition as well as incentives for lobbies to build up resistance to an agreement. Clearly substantial additional modeling effort will be required to investigate these issues, but some of the intuitions gleaned in this paper might be useful in this area. Another perhaps unexpected extension would bring an additional consideration to the literature on the separation between investment and retail banking (Glass-Steagall Act, Volker and Vickers rules).[9] Such separation is akin to the tranching of a universal bank into a relatively low-information-intensity entity (the retail bank) and a high-information-intensity entity (the investment bank). "Buyers" of claims on the bank (retail depositors- i.e. the banking regulator/taxpayers-, wholesale depositors, etc.) face different incentives to collect information in the two arrangements, an issue that has been overlooked in the policy discussions on the matter. We hope that these and other topics related to this paper will be investigated in future research.

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A   Appendix

We will make repeated use of the following observation. If a given continuation equilibrium for $t \geq 1$ exists, then the corresponding value for the continuation equilibrium satisfies $V^S \geq \frac{\rho\beta(1+\beta R)}{1-\beta^2}$, where the RHS is the ex-dividend value that a seller would obtain if he kept the asset forever.

A.1   Proof of Proposition 7

Checking Assumptions 1, 2, and 3.   We have just described how every stationary equilibrium can be associated with a particular parametrization of the static model. We now proceed to derive conditions on the primitives of the dynamic model such that Assumptions 1, 2, and 3 in the associated static model are verified.

We start with Assumptions 1: $b \geq s$ and $B \geq S$. Note that for all equilibria, we have $b > s$. The condition that $B > S$ amounts to

$$V^S - c^S 1_{\{I,OnlyS\}} > \frac{\rho\beta R}{R-\beta}.$$

We can use the lower bound $V^S - c^S 1\{I, OnlyS\} \geq \frac{\rho\beta(1+\beta R)}{1-\beta^2}$ (which holds across all equilibria), to conclude that this inequality is always verified as long as $R > 1$, so that we have $B > S$.

We move on to Assumptions 2 and 3: $S + s > B + \rho b$ and $\rho s > B - S$. These assumptions are equivalent to

$$\frac{\rho\beta R + 1 - \rho R^2}{R-\beta} > V^S - c^S 1_{\{I,OnlyS\}}$$

and

$$\rho\frac{\beta R + 1}{R-\beta} > V^S - c^S 1_{\{I,OnlyS\}}.$$

In light of the results below, namely that $V^S(NI) \geq \max\{V^S(OnlyB), V^S(OnlyS), V^S(I)\}$, it suffices to check that these assumptions are verified for (NI). We get the following condition for Assumption 2:

$$\rho\beta R + 1 - \rho R^2 > \alpha^S\beta R + \alpha^B\beta\left[\rho R + (1-\rho)\beta\right]. \tag{4}$$

Similarly, we get the following condition for Assumption 3:

$$\beta R + 1 > \alpha^S\beta R\left(R^2 + 1\right) + \alpha^B\beta R(1 + \beta R). \tag{5}$$

In order for Assumption 2 to be verified, ρ should not be too large and R should not be too large. In order for Assumption 3 to be verified, R should not be too large (this condition is independent of ρ).

Note also that for $R = 1$, these conditions become respectively

$$1 + \beta > \alpha^S\beta.$$

and

$$1 > \alpha^S\beta + \alpha^B\beta^2,$$

so that they are automatically verified. Hence Assumptions 1, 2 and 3 are guaranteed to hold as long as R is close enough to 1.

A.2 Equilibrium regions for the dynamic model

The conditions for the existence of (NI) are $c^B \geq \underline{c}^B(NI)$ and $c^S \geq \underline{c}^S(NI)$, and the conditions for the existence of (Only B) are $c^B \leq \bar{c}^B(Only\ B)$ and $c^S \geq \bar{c}^S(Only\ B)$. However we encounter the following complication for equilibria (I and Only S): for those equilibria, B, S and hence the corresponding information thresholds $\bar{c}^B(I)$, $\bar{c}^B(Only\ S)$, $\bar{c}^S(I)$, $\bar{c}^S(Only\ S)$ depend on c^S. This poses no particular problem, and the existence conditions for the equilibria can be expressed exactly as before. However, it is useful to solve out these conditions further. No change is required for the buyer's information thresholds $\bar{c}^B(I)$, $\bar{c}^B(Only\ S)$, one should simply bear in mind that they now depend on c^S. For the seller's information thresholds, we find it more useful to derive two thresholds $\hat{c}^S(I)$ and

$\hat{c}^S(Only\ S)$ that are independent of c^S.[10] Hence $c^S \leq \bar{c}^S(I)$ if and only if $c^S \leq \hat{c}^S(I)$ and $c^S \leq \bar{c}^S(OnlyS)$ if and only if $c^S \leq \hat{c}^S(OnlyS)$. The conditions for the existence of (I) are then $c^B \leq \bar{c}^B(I)$ and $c^S \leq \hat{c}^S(I)$, and the conditions for the existence of (Only S) are $c^B \geq \bar{c}^B(Only\ S)$ and $c^S \leq \hat{c}^S(Only\ S)$.

The number of relevant regions in the information cost space $(c^S, c^B) \in \mathbb{R}^{+2}$ is higher than in the static case because the information thresholds now depend on the equilibrium. For completeness, we list them all in Proposition 17 below. This involves describing the boundaries of a large number of regions of the information cost space. For ease of exposition, we focus here on the case $c^S = 0$ (informed seller) and $c^S = \infty$ (uninformed seller) in Proposition 16 below, and refer the reader to Proposition 17 for the complete treatment and the proofs of these propositions.

Proposition 16. (Equilibrium Regions for the Dynamic Model). *There exists $\bar{\bar{R}} \in (1, \bar{R})$, and $\bar{\bar{\beta}} \in (0,1)$ such that for all $R \in (1, \bar{\bar{R}})$ and $\beta \in (\bar{\bar{\beta}}, 1)$, and all (α^B, α^S) such that $\alpha^B \beta > 1 - \rho$ and $\alpha^S \beta > 1 - \rho$, we have $\underline{c}^B(NI) \leq \bar{c}^B(Only\ B)$ and:*

i. *When $c^S = 0$ (informed seller), we have $\bar{c}^B(Only\ S) < \bar{c}^B(I)$ and*

    (a) *for $c^B \in [0, \bar{c}^B(Only\ S))$, (I) is the only stationary equilibrium;*

    (b) *for $c^B \in [\bar{c}^B(Only\ S), \bar{c}^B(I)]$, (I) and (Only S) are the only stationary equilibria;*

    (c) *for $c^B \in (\bar{c}^B(I), \infty)$, (Only S) is the only stationary equilibrium;*

ii. *When $c^S = \infty$ (uninformed seller), we have*

    (a) *for $c^B \in [0, \underline{c}^B(NI))$ (Only B) is the only stationary equilibrium;*

    (b) *for $c^B \in [\underline{c}^B(NI), \bar{c}^B(Only\ B)]$ (Only B) and (NI) are the only stationary equilibria;*

    (c) *for $c^B \in (\bar{c}^B(Only\ B), \infty)$ (NI) is the only stationary equilibrium;*

We now provide a generalization of Proposition 16 without restricting ourselves to the cases $c^S = 0$ and $c^S = \infty$ and its proof.

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[10]These information thresholds can be computed as follows:

$$\hat{c}^S(I) = \frac{\alpha^S \rho(1-\rho)\beta^2(R-\beta)\left[\alpha^S((R-\beta)^2 - 1) + \beta(1 - \frac{\beta}{R})\right]}{R\left[1 - \beta(\alpha^B\beta + \alpha^S(R - \beta(1-\rho)(R-\beta)))\right]},$$

$$\hat{c}^S(Only\ S) = \frac{\beta^2\rho(1-\rho)\alpha^S\beta(R^2 - 1)}{R\left[1 - \rho\beta^2 - (1-\rho)\beta(\alpha^S R + \alpha^B\beta) + \alpha^S(1-\rho)\beta(1 - \frac{\beta}{R})\right]}.$$

Proposition 17. *(Equilibrium Regions for the Dynamic Model). There exists $\bar{\bar{R}} \in (1, \bar{R})$, and $\bar{\bar{\beta}} \in (0, 1)$ such that for all $R \in (1, \bar{\bar{R}})$ and $\beta \in (\bar{\bar{\beta}}, 1)$, and all (α^B, α^S) such that $\alpha^B \beta > 1 - \rho$, $\alpha^S \beta > 1 - \rho$, and $2\alpha^S < 3 + 2\rho(1 - \rho) - \sqrt{(3 + 2\rho(1 - \rho))^2 - 8\rho}$, we have $\hat{c}^S(\text{Only } S) < \bar{c}^S(\text{Only } B) < \hat{c}^S(I)$ and $\underline{c}^B(NI) \le \bar{c}^B(\text{Only } B)$ and:*

   i. For $c^S \in [0, \underline{c}^S(NI))$, $\bar{c}^B(\text{Only } S) < \bar{c}^B(I)$ and

      (a) for $c^B \in [0, \bar{c}^B(\text{Only } S))$, (I) is the only stationary equilibrium;

      (b) for $c^B \in [\bar{c}^B(\text{Only } S), \bar{c}^B(I)]$, (I) and (Only S) are the only stationary equilibria;

      (c) for $c^B \in (\bar{c}^B(I), \infty)$, (Only S) is the only stationary equilibrium;

   ii. For $c^S \in [\underline{c}^S(NI), \hat{c}^S(\text{Only } S)]$, $\bar{c}^B(\text{Only } S) < \bar{c}^B(I)$ and

      (a) for $c^B \in [0, \underline{c}^B(NI))$, (I) is the only stationary equilibrium;

      (b) for $c^B \in [\underline{c}^B(NI), \bar{c}^B(\text{Only } S))$, (NI) and (I) are the only stationary equilibria;

      (c) for $c^B \in [\bar{c}^B(\text{Only } S), \bar{c}^B(I)]$, (NI), (I) and (Only S) are the only stationary equilibria;

      (d) for $c^B \in (\bar{c}^B(I), \infty)$, (NI) and (Only S) are the only stationary equilibria;

   iii. For $c^S \in (\hat{c}^S(\text{Only } S), \bar{c}^S(\text{Only } B))$,

      (a) for $c^B \in [0, \underline{c}^B(NI))$, (I) is the only stationary equilibrium;

      (b) for $c^B \in [\underline{c}^B(NI), \bar{c}^B(I)]$, (I) and (NI) are the only stationary equilibria;

      (c) for $c^B \in (\bar{c}^B(I), \infty)$, (NI) is the only stationary equilibrium;

   iv. For $c^S \in [\bar{c}^S(\text{Only } B), \hat{c}^S(I))$, $\bar{c}^B(\text{Only } B) < \bar{c}^B(I)$ and

      (a) for $c^B \in [0, \underline{c}^B(NI))$ (I) and (Only B) are the only stationary equilibria;

      (b) for $c^B \in [\underline{c}^B(NI), \bar{c}^B(\text{Only } B)]$ (I), (Only B) and (NI) are the only stationary equilibria;

      (c) for $c^B \in (\bar{c}^B(\text{Only } B), \bar{c}^B(I)]$ (I) and (NI) are the only stationary equilibria;

      (d) for $c^B \in (\bar{c}^B(I), \infty)$ (NI) is the only stationary equilibrium;

   v. For $c^S \in [\hat{c}^S(I), \infty)$,

      (a) for $c^B \in [0, \underline{c}^B(NI))$ (Only B) is the only stationary equilibrium;

      (b) for $c^B \in [\underline{c}^B(NI), \bar{c}^B(\text{Only } B)]$ (Only B) and (NI) are the only stationary equilibria;

(c) for $c^B \in (\bar{c}^B(Only\ B), \infty)$ (NI) is the only stationary equilibrium;

We derive a number of conditions for the ranking of the information thresholds corresponding to different stationary equilibria. The proposition follows using the existence conditions associated with these information thresholds.

Conditions for

$$\underline{c}^S(NI) \leq \min\{\bar{c}^S(Only\ S),\ \bar{c}^S(Only\ B), \bar{c}^S(I)\}$$

and

$$\underline{c}^B(NI) \leq \min\{\bar{c}^B(Only\ S),\ \bar{c}^B(Only\ B), \bar{c}^B(I)\}.$$

We use the following two functions of $(B - S)$

$$\phi_{\bar{c}^S}(B - S) = \alpha^S(1 - \rho)(B - S),$$

$$\phi_{\underline{c}^S}(B - S) = \rho \left[(1 - \rho)s - \alpha^S \left(B - S - \rho(b - s)\right) \right].$$

Using the fact that b and s are independent across equilibria,

$$b = \beta R,$$

$$s = \frac{\beta}{R},$$

we see that $\phi_{\bar{c}^S}$ is increasing, $\phi_{\underline{c}^S}$ decreasing, and the two functions cross at $B - S = X = \frac{\rho(1-\rho)\beta}{\alpha^S R} - \rho^2 \beta \left(R - \frac{1}{R}\right)$. Now we have

$$B - S = \beta \left(1 - \frac{\beta}{R}\right)\left(V^S - c^S 1_{\{I,Only S\}}\right) - \beta^2 \rho.$$

In Appendix A.3, we show that $V^S(NI)$ is greater than $V^S(I)$, $V^S(Only\ S)$ and $V^S(Only\ B)$. Hence a necessary condition for $\underline{c}^S(NI) \leq \min\{\bar{c}^S(Only\ S),\ \bar{c}^S(Only\ B), \bar{c}^S(I)\}$ is that at (NI), $B - S$ be greater than X. This is equivalent to

$$\beta \left(1 - \frac{\beta}{R}\right) \frac{\alpha^S R^2 + \alpha^B(1 + \beta R)}{1 - \alpha^S \beta R - \alpha^B \beta^2} > \frac{1 - \rho}{\alpha^S R} - \rho \left(R - \frac{1}{R} - 1\right). \tag{6}$$

Once this condition is verified, a sufficient condition for $\underline{c}^S(NI) \leq \min\{\bar{c}^S(Only\ S),\ \bar{c}^S(Only\ B), \bar{c}^S(I)\}$ is that for (I), (Only S), and (Only B), $B - S$ be greater than X. Using the lower bound

$V^S - c^S 1\{I, OnlyS\} \geq \frac{\rho\beta(1+\beta R)}{1-\beta^2}$, we find that a sufficient condition is

$$\beta\left(1 - \frac{\beta}{R}\right)\frac{\rho\beta(1+\beta R)}{1-\beta^2} - \beta^2\rho > \frac{1-\rho}{\alpha^S R} - \rho\left(R - \frac{1}{R} - 1\right), \tag{7}$$

which implies (6).

Similarly, a sufficient condition for $\underline{c}^B(NI) \leq \min\{\bar{c}^B(Only\ S), \bar{c}^B(Only\ B), \bar{c}^B(I)\}$ is

$$\beta\left(1 - \frac{\beta}{R}\right)\frac{\rho\beta(1+\beta R)}{1-\beta^2} - \beta^2\rho > \rho\beta\left[R\left(\frac{1-\rho}{\alpha^B} + \rho - 2\right) + \frac{2-\rho}{R}\right] \tag{8}$$

Hence (7) is a sufficient condition for $\underline{c}^S(NI) \leq \min\{\bar{c}^S(Only\ S), \bar{c}^S(Only\ B), \bar{c}^S(I)\}$ and (8) is a sufficient condition for $\underline{c}^B(NI) \leq \min\{\bar{c}^B(Only\ S), \bar{c}^B(Only\ B), \bar{c}^B(I)\}$.

When $R = 1$, these sufficient conditions become respectively

$$\alpha^S\beta > 1 - \rho,$$

$$\alpha^B\beta > 1 - \rho.$$

Since Assumptions 1, 2 and 3 are automatically verified when $R = 1$, we conclude that for R close enough to 1, we can have at the same time Assumptions 1, 2, 3 and $\underline{c}^S(NI) \leq \min\{\bar{c}^S(Only\ S), \bar{c}^S(Only\ B), \bar{c}^S(I)\}$, $\underline{c}^B(NI) \leq \min\{\bar{c}^B(Only\ S), \bar{c}^B(Only\ B), \bar{c}^B(I)\}$.

Conditions for $\bar{c}^S(OnlyB) \leq c^{S*}$. We define c^{S*} to be the value of c^S for which $V^S(I) = V^S(OnlyS)$: $c^{S*} = \frac{(R^2-1)[1-(1-\rho)(\alpha^B\beta+\alpha^S R)]}{R-\beta}$. We can check that $V^S(I) > V^S(OnlyS)$ if and only if $c^S < c^{S*}$.

The condition for $\bar{c}^S(OnlyB) \leq c^{S*}$ is

$$\frac{\alpha^S\rho(1-\rho)\beta^2(R^2-1)(\alpha^S R + \alpha^B\beta)}{R(1-(1-\rho)\beta^2 - \beta\rho(\alpha^S R + \alpha^B\beta))} \leq \frac{(R^2-1)(1-(1-\rho)\beta(\alpha^B\beta + \alpha^S R))}{R-\beta}.$$

Let $X = \alpha^B\beta + \alpha^S R$. We can rewrite this condition as

$$\rho(1-\rho)\beta^2\alpha^S RX - \rho(1-\rho)\beta^3\alpha^S X$$
$$\leq R\left[1 - (1-\rho)\beta^2 - \beta\rho X - (1-\rho)\beta X + (1-\rho)^2\beta^3 X + \rho(1-\rho)\beta^2 X^2\right],$$

which becomes when $R \to 1$,

$$\rho(1-\rho)\beta^2 \alpha^S X(1-\beta) \leq 1 - (1-\rho)\beta^2 - \beta\rho X - (1-\rho)\beta X + (1-\rho)^2\beta^3 X + \rho(1-\rho)\beta^2 X^2.$$
(9)

The LHS and the RHS of equation (9) are equal for $\beta = 1$. We can now derive a condition for the derivative of the LHS with respect to β for $\beta = 1$ to be greater than the derivative of the RHS for $\beta = 1$:

$$(\alpha^S)^2 - \alpha^S(3 + 2\rho(1-\rho)) + 2\rho > 0.$$

Hence we have $\bar{c}^S(OnlyB) \leq c^{S*}$ for R and β close enough to 1 as long as

$$2\alpha^S < 3 + 2\rho(1-\rho) - \sqrt{(3 + 2\rho(1-\rho))^2 - 8\rho}.$$

Conditions for $\bar{c}^S(OnlyB) \leq \hat{c}^S(I)$. It is easy to see that when $R = 1$, we have $\bar{c}^S(OnlyB) = 0$ while $\hat{c}^S(I) > 0$. This shows that $\bar{c}^S(OnlyB) \leq \hat{c}^S(I)$ for R close enough to 1.

Conditions for $\hat{c}^S(OnlyS) \leq \bar{c}^S(OnlyB)$. Using $X = \alpha^B\beta + \alpha^S R$, we find that $\hat{c}^S(OnlyS) \leq \bar{c}^S(OnlyB)$ if and only if

$$X + (1-\rho)\beta(\beta^2 - X^2) + \alpha^S(1-\rho)\beta(1 - \frac{\beta}{R})X \geq \beta,$$

which becomes when $R \to 1$,

$$\alpha^S(1-\beta) + (1-\rho)(\beta^3 - \beta^4 - (\alpha^S)^2\beta^2(1-\beta)^2 - 2\alpha^s\beta^3(1-\beta))$$
$$+ \alpha^S(1-\rho)\beta(1-\beta)(\beta + \alpha^S(1-\beta)) \geq 0.$$

The LHS of this equation is equal to 0 for $\beta = 1$. We can check that the derivative of the LHS with respect to β for $\beta = 1$ is strictly negative.

Hence we have $\hat{c}^S(OnlyS) \leq \bar{c}^S(OnlyB)$ for R and β close enough to 1.

A.3 Proof of Propositions 8 and 15

We prove Proposition 15 and in the process, we also prove Proposition 8.

Self-fulfilling liquidity with $c^S = \infty$ (NI and Only B) and proof that $V^S(NI) > V^S(OnlyB)$.
We start by proving that when (NI) and (Only B) are possible continuation equilibria for

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$t \geq 1$, we have $\underline{c}^B(NI) < \underline{c}^B(OnlyB)$. To perform this comparison we note that this will occur if and only if $V^S(NI) > V^S(OnlyB)$. To investigate this inequality, we look at the corresponding operators T^{NI} and T^{OnlyB}:

$$T^{NI}(V^S) = \alpha^S \rho \beta R^2 + \alpha^B \rho \beta [1 + \beta R] + [\alpha^S \beta R + \alpha^B \beta^2] V^S.$$

$$T^{OnlyB}(V^S) = \alpha^S \rho \beta R^2 + \alpha^B \rho \beta [1 + \beta \rho R] + \rho(1-\rho)\beta^2 R + \left[\rho \left(\alpha^S \beta R + \alpha^B \beta^2\right) + (1-\rho)\beta^2\right]] V^S.$$

Note that T^{NI} is steeper. The two functions cross at $V^S = \frac{\rho \beta R}{R - \beta}$. We can use the lower bound $V^S \geq \frac{\rho \beta(1+\beta R)}{1-\beta^2}$, which holds as long as long as the two continuation equilibria (NI) and (Only B) exist, to check that V^S will always be to the right of this crossing point. Hence, we can rank the two fixed point operators $T^{NI} > T^{OnlyB}$ over the relevant region except when $\alpha^B = 1$, in which case $T^{NI} = T^{OnlyB}$. We conclude that $\underline{c}^B(NI) < \underline{c}^B(OnlyB)$ (illiquidity tomorrow leads to illiquidity today) as long as $\alpha^B < 1$ (otherwise the two thresholds are equal).

Self-fulfilling liquidity with $c^B = \infty$ (NI and Only S) and proof that $V^S(NI) > V^S(OnlyS)$. We now turn to the case $c^B = \infty$. Now we need to check a different condition:

$$-\beta^2 \rho + \frac{\beta}{R} V^S(NI)(R - \beta) > -c^S \left(1 - \frac{\beta^2}{R}\right) - \beta^2 \rho + \frac{\beta}{R} V^S(OnlyS)(R - \beta)$$

A sufficient (but not necessary) condition for that is that $V^S(NI) > V^S(OnlyS)$. To investigate this inequality, we look at the corresponding operators T^{NI} and T^{OnlyS}:

$$T^{NI}(V^S) = \alpha^S \rho \beta R^2 + \alpha^B \rho \beta [1 + \beta R] + [\alpha^S \beta R + \alpha^B \beta^2] V^S.$$

$$T^{OnlyS}(V^S) = \rho \beta \left(1 + \rho \beta R\right) + (1-\rho)R \left[\alpha^B \beta^2 \rho\right] + \left[\rho \beta^2 + (1-\rho)\left(\alpha^S \beta R + \alpha^B \beta^2\right)\right](V^S - c^S).$$

Note that T^{NI} is steeper. The two functions cross at

$$V^S = \frac{1 + \rho \beta R - R^2}{R - \beta} + \frac{\left[\rho \beta^2 + (1-\rho)\left(\alpha^S \beta R + \alpha^B \beta^2\right)\right]}{\rho[\alpha^S \beta R + \alpha^B \beta^2 - \beta^2]}(-c^S)$$

We can use the lower bound $V^S \geq \frac{\rho\beta(1+\beta R)}{1-\beta^2}$, which holds as long as long as the two continuation equilibria (NI) and (Only S) exist, to check that V^S will always be to the right of the crossing point above. Hence, we can rank the two fixed point operators over the relevant region.

Self-fulfilling liquidity with $c^s = 0$ (I and Only S) and proof that $V^S(I) > V^S(OnlyS)$. We now turn to the case $c^S = 0$. We can check that $V^S(I) > V^S(OnlyS)$ if and only if $c^S < c^{S*}$, where $c^{S*} = \frac{(R^2-1)[1-(1-\rho)(\alpha^B\beta+\alpha^S R)]}{R-\beta}$, which is automatically verified if the continuation equilibrium (Only S) exists. This implies that as long as (I) and (Only S) exist, we have $\bar{c}^B(OnlyS) \leq \bar{c}^B(I)$.

Self-fulfilling liquidity with $c^B = 0$ (I and Only B). We now turn to the case $c^B = 0$. We have already established that $\bar{c}^S(OnlyB) \leq \hat{c}^S(I)$ which is equivalent to $\bar{c}^S(OnlyB) \leq \bar{c}^S(I)$. This implies that as long as the continuation equilibria (I) and (Only B) exist, we have $\bar{c}^S(OnlyB) \leq \bar{c}^S(I)$.

Proof that $V^S(NI) > V^S(I)$. The proof is immediate using

$$T^{NI}(V^S) = \alpha^S \rho\beta R^2 + \alpha^B \rho\beta[1 + \beta R] + [\alpha^S \beta R + \alpha^B \beta^2]V^S,$$

and

$$T^I(V^S) = \alpha^S \rho\beta R^2 + \alpha^B \beta\rho[1 + \beta R] + [\alpha^S \beta R + \alpha^B \beta^2]\left(V^S - c^S\right).$$

A.4   Lemma for Section 4

In Section 4, we make use of some comparison of payoffs across equilibria even when they might not coexist. For example, we establish that $v^S(I) > v^S(Only\ B)$ when $c^S < \bar{c}^S$ and $v^S(I) < v^S(Only\ B)$ when $c^S > \bar{c}^S$. As long as $c^B < \underline{c}^B$, the equilibrium is (I) or (Only B). It is (I) when $v^S(I) > v^S(Only\ B)$ and (Only B) when $v^S(I) < v^S(Only\ B)$.

Lemma 2. *(Further Comparison of Payoffs). The payoffs are ranked as follows:*

  i. *for the seller: $v^S(NI) \geq v^S(I) \geq v^S(Only\ S)$, and $v^S(NI) \geq v^S(Only\ B)$; furthermore $v^S(I) \geq v^S(Only\ B)$ if and only if $c^S \leq \bar{c}^S$;*

  ii. *for the buyer: $v^B(NI) \geq v^B(I) \geq v^B(Only\ B)$, and $v^B(NI) \geq v^B(Only\ S)$; furthermore $v^B(I) \geq v^B(Only\ S)$ if and only if $c^B \leq \bar{c}^B$.*

iii. *for total welfare $v = v^S + v^B$: $v(NI) \geq \max\{v(I), v(Only\ S), v(Only\ B)\}$; furthermore $v(I) \geq v(Only\ B)$ if and only if $c^S \leq \frac{\bar{c}^S}{\alpha^S}$, and $v(I) \geq v(Only\ S)$ if and only if $c^B \leq \frac{\bar{c}^B}{\alpha^B}$.*