Bundling and Competition for Slots*

Doh-Shin Jeon† and Domenico Menicucci‡

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Abstract

We consider competition between sellers selling multiple distinct products to a buyer having k slots. Under independent pricing, a pure strategy equilibrium often does not exist and equilibrium in mixed strategy is never efficient. When bundling is allowed, each seller has an incentive to bundle his products and an efficient "technology-renting" equilibrium always exists. Furthermore, in the case of digital goods or when sales below marginal cost are banned, all equilibria are efficient. Comparing the mixed strategy equilibrium with the technology-renting equilibrium reveals that bundling often increases the buyer's surplus. Finally, we derive clear-cut policy implications.

Key words: Bundling, Block Booking, Pure Bundling, Slots (or Shelf Space), Slotting Contracts, Foreclosure, Digital goods

JEL Code: D4, K21, L13, L41, L82

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†Toulouse School of Economics (GREMAQ, IDEI). dohshin.jeon@gmail.com

‡Università degli Studi di Firenze, Italy. domenico.menicucci@dmd.unifi.it

1 Introduction

Sellers with different portfolios of products often compete for limited slots (or shelf space) of a buyer who wants to build up her[1] own portfolio. In these situations, sellers may employ bundling as a strategy to win the competition for slots. Even though bundling has been a major antitrust issue and a subject of intensive research, no paper has studied how bundling affects portfolios' competition for slots. We provide a new perspective on bundling by addressing this issue.

Examples of the situations we described above are abundant. For instance, in the entertainment industry, movies compete for slots on a cineplex screen and television networks compete for slots on the cable lineup. Indeed, allocation of slots in movie theaters was one of the main issues raised in the movie industry during the last presidential election in France.[2] Furthermore, bundling in the movie industry (known as block booking[3]) was declared illegal in two Supreme Court decisions in the U.S.: U.S. v. *Paramount Pictures* (1948) and U.S. v. *Loew's* (1962).[4]

In retailing, manufacturers may practice bundling (often called full-line forcing) and/or exclusive dealing to win competition for retail shelf space.[5] For instance, the French Competition Authority fined Société des Caves de Roquefort for using selectivity or exclusivity contracts with supermarket chains.[6] In addition, slotting fees (the payments by manu-

[1]We use "he" for each seller and "she" for the buyer.

[2]Cahiers du Cinema (April, 2007) proposed to limit the number of copies per movie since certain movies, by saturating screens, restrict other movies' access to screens and asked each presidential candidate's opinion about the policy proposal.

[3]Block booking is "the practice of licensing, or offering for license, one feature or group of features on the condition that the exhibitor will also license another feature or group of features released by distributors during a given period" (Unites States v. Paramount Pictures, Inc., 334 U.S. 131, 156 (1948)).

[4]More recently, in *MCA Television Ltd. v. Public Interest Corp.* (11th Circuit, April 1999), the Court of Appeals reaffirmed the *per se* illegal status of block booking.

[5]For instance, Procter and Gamble uses 'golden-store' arrangement such that to be considerd a golden store, a retailer must agree to carry 40 or so P&G items displayed together. See "P&G has big plans for the shelves of tiny stores in emgering nations", Wall Street Journal, July 17, 2007.

[6]Société des Caves de Roquefort's market share in the Roquefort cheese market was 70% but, through the contract, could occupy eight among all nine brands that Carrefour, a supermarket chain, sold.

facturers for retail shelf space) have been the subject of recent antitrust litigation[7] and the focus of Federal Trade Commission studies.

In our model, we assume away any private information of the buyer, which allows us to depart from the existing literature on bundling that usually embraces a framework of second-degree price discrimination and to identify what can be a first-order effect of bundling. In fact, the second-degree price discrimination explanation of bundling is inconsistent with the facts of *Paramount* and *Loew's* since the prices of the blocks varied a great deal across markets (Kenney and Klein, 1983). Furthermore, recent advances in information technologies and the internet allow firms to offer personalized prices to each customer (an individual or a firm).[8]

We consider a simultaneous pricing game between two sellers (or firms) A and B, who offer their products to a buyer having $k(\geq 1)$ slots. Each seller has a fixed number of distinct products. We suppose that the prototype of each product is already made and thus the fixed cost of production has already been incurred. We call a product a *digital good* if the marginal cost of producing an extra unit is zero. The buyer has a unit demand for each product but a product needs to occupy a slot to generate value. We assume free disposal. In this setup, we study how the outcome of competition depends on the nature of products (digital goods or not) and on the different bilateral contractual arrangements between each seller and the buyer.

Our model fits well United States v. Loew's (1962), where block booking was practiced by six major distributors of pre-1948 copyrighted motion picture feature films for television exhibition. As in our model, the movies had already been produced, and each distributor could bundle a large number of movies and charge a personalized price to

[7] One case is R.J. Reynolds Tobacco Co. v. Philip Morris, Inc., 199 F. Supp. 2d 363 (M.D.N.C. 2002), in which Reynolds Tobacco accused Philip Morris of using a "Retail Leaders" contract that provides discounts to retailers on its popular Marlboro brand in exchange for the most advantageous display and signage space in retail establishments. See also American Booksellers Ass'n, Inc. v. Barnes & Noble, Inc., 135 F. Supp. 2d 1031 (N.D. Cal. 2001); Intimate Bookshop, Inc. v. Barnes & Noble, Inc., 88 F. Supp. 2d 133 (S.D.N.Y. 2000); FTC v. H.J. Heinz Co., 116 F. Supp. 2d 190 (D.C.C. 2000), rev'd, 246 F.3d 708 (D.C. Cir. 2001).

[8] A number of papers (Chen and Iyer, 2002, Choudhary et al., 2005, Ghose and Huand, 2009, Shaffer and Zheng, 2002, Thisse and Vives, 1988) model personalized pricing as perfect price discrimination as we do in our paper.

each different TV station. For instance, Loew's exacted from KWTV a contract for the entire Loew's library of 723 movies, involving payments of $314,725.20. In addition, our free disposal assumption seems to be satisfied since the policies of a distributor (C&C Super Corp.) "resulted in at least one station having to take a package in which 'certain of the films were unplayable, since they had a foreign language sound track.'"(U.S. v. Loew's). Of course, in our analysis, we regard movies as digital goods even though the notion of digital goods did not exist at the time of U.S. v. Loew's.

A reference pricing strategy is "independent pricing", which means that a seller chooses a price for each product and the price of each set of products is the sum of the individual prices of the products in the set. We define "bundling" as any contract that specifies a price for every subset of a seller's portfolio, but is different from independent pricing. Two particular forms of bundling are of interest: "technology-renting" and "pure bundling". First, technology-renting means that a seller rents his technology at a fixed fee such that upon paying the fee, the buyer can buy any subset of the seller's portfolio at the marginal cost of producing the subset. Hence, technology-renting generalizes marginal cost pricing to a situation in which a seller sells multiple distinct products. Second, pure bundling means that a seller puts all his products into one bundle and offers only that bundle. Note, however, that we assume free disposal (except in Section 6 on slotting contracts). Hence, even though the buyer buys the bundle of all products of a firm, she may then not use all of them. In the case of digital goods, pure bundling and technology-renting are equivalent under the assumption of free disposal.

Our main results are the following. First, under independent pricing, the fact that a multi-product firm faces competition among his own products can make equilibrium in pure strategies fail to exist; this non-existence is generic in the case of digital goods.[9] Furthermore, any mixed strategy equilibrium involves an inefficient allocation of slots and we characterize a mixed-strategy equilibrium for the case of $k = 2$. Second, each firm has an incentive to practice bundling since bundling eliminates competition among one's own products and thereby reduces damages from rival products. Third, bundling restores the existence of an equilibrium in pure strategies without causing inefficiency. When

[9] In the same setting, Jeon and Menicucci (2009) consider sequential pricing and find that an equilibrium in pure strategies always exists but that it often leads to an inefficient allocation of slots.

bundling is allowed, there always exists an efficient equilibrium in which each firm uses a technology-renting strategy, as is known from Bernheim and Whinston (1986, 1998) and O'Brien and Shaffer (1997). Under a mild condition related to decreasing marginal social values of products, called "weak m-submodularity", all equilibria are efficient if either all products are digital goods or sales below marginal cost are prohibited; furthermore, under the condition, each player's payoff is uniquely determined.[10] Finally, when we compare the mixed-strategy equilibrium to the technology-renting equilibrium, we find that the firms' total profits are often higher in the former than in the latter, implying that bundling often increases the buyer's surplus.[11] We illustrate these results through a simple example in Section 2.

To see the incentive to practice bundling, consider a simple setting in which products are digital goods and have independent values. Firm A offers two products of value 5 each, firm B offers one product of value 2, and the buyer has two slots. Suppose that A wants to sell both products. Then, under independent pricing, each product of A faces competition from B's product and A realizes a profit of $5 - 2 = 3$ from each and hence a total profit of 6. Let now A offer only the bundle of the two products. Then, without buying the bundle, the buyer can fill the slots with only B's product. If instead she buys the bundle, she can replace B's product with A's and thus A realizes a profit of $10 - 2 = 8$. Basically, under independent pricing the buyer has the option to buy only one product from A and to fill the second slot with B's product. Bundling eliminates this option and thereby prevents A's own products from competing with each other.

The intuition for why all equilibria under bundling are efficient is simple for digital goods with independent values. Imagine a situation in which a product occupying a slot is inferior to a product that is not occupying any slot. Then the seller owning the latter can include it in his bundle or, if not currently selling any bundle to the buyer, can provide this product on its own. Since the product is superior, it can be profitably sold as long as the production cost is below the product's incremental value, which always occurs for digital goods.

[10] We also identify another condition, which we call "unilateral improvement", under which all equilibria are efficient, but different equilibria may yield different payoffs to the players.

[11] In addition, under certain conditions, firms face a prisoner's dilemma since each firm has a weak incentive to practice bundling but is weakly better off with independent pricing (see Proposition 8).

In Section 6, we extend our results by allowing for slotting contracts. When a seller offers a bundle with a slotting contract, he can specify the minimum number of slots that the products in the bundle should occupy; therefore, free disposal does not hold under slotting contracts. The previous results still hold, except that slotting contracts can create inefficient equilibria even for digital goods and even when sales below marginal cost are banned.

Our paper generates clear-cut policy implications. In the case of digital goods, technology-renting, which is a sort of marginal-cost pricing, is equivalent to pure bundling (or block booking). Hence any seller can find a best response among pure bundling strategies (see Lemma 1) and competition among pure bundles leads to an efficient outcome in which each seller obtains a profit equal to the marginal social contribution of his portfolio. In addition, under the "weak m–submodularity" condition, pure bundling allows each seller to obtain this profit independently of the rivals' strategies. This suggests that pure bundling of digital goods (and hence block booking of movies) is socially desirable in terms of allocation of slots and does not generate any concern in terms of foreclosure. For non-digital goods, banning sales below marginal cost is socially desirable for similar reasons. However, such a ban prevents the use of pure bundling.

According to the leverage theory of tying, on which the Supreme Court's decisions to prohibit block booking were based, tying allows a distributor to extend its monopoly power on a desirable movie to an undesirable one. This theory was criticized by the Chicago School (see e.g. Bowman 1957, Stigler 1963, Posner 1976, Bork 1978) since the distributor is better off by selling only the desirable movie at a higher price. As an alternative, Stigler (1963) proposed a theory based on price discrimination which became a dominant strand in the literature (Schmalensee, 1984, McAfee et al. 1989, Shaffer, 1991, Salinger 1995 and Armstrong 1996), at least until Whinston (1990) resuscitated the leverage theory with its first formal treatment (for later work in this line, see Choi and Stefanadis 2001, Carlton and Waldman 2002, and Nalebuff 2004). Basically, in Whinston (1990), precommitment to pure bundling induces an incumbent to be aggressive, which discourages entry if there is a fixed cost of entry. We contribute to the literature by showing that pure bundling of digital goods is socially desirable in terms of allocation of slots and does not generate any concern in terms of foreclosure, absent slotting contracts.

Since each firm can bundle any number of products in our paper, we also contribute to the literature on bundling a large number of products. Armstrong (1999) and Bakos and Brynjolfsson (1999) show that bundling allows a monopolist to extract more surplus since it reduces the variance of average valuations by the law of large numbers. In our paper, the law of large number plays no role due to the assumption of complete information. Jeon and Menicucci (2006) consider a framework similar to the one in the current paper to study competition among publishers selling academic journals to a library facing a budget constraint (instead of a slot constraint). While both papers find that each firm has an incentive to bundle its products, Jeon and Menicucci (2006) show that bundling reduces social welfare since if large publishers extract more surplus with bundling, there is less (possibly zero) budget left for small publishers.

Our game when bundling is allowed is similar to the menu-auction game (Bernheim and Whinston, 1986) and the common agency game. For instance, Bernheim and Whinston (1998) and O'Brien and Shaffer (1997) consider competition between two sellers facing a common buyer. They identify a "sellout" equilibrium that maximizes the joint profit of all three players and Pareto-dominates any other equilibrium in terms of the sellers' payoffs. Although our model is a bit different from theirs,[12] their result applies to our setting as well. Our contribution mainly lies in analyzing independent pricing and the incentive to use bundling, identifying two sufficient conditions that make all equilibria efficient under bundling, and comparing the outcome under independent pricing with that under bundling.

The paper proceeds as follows. Section 2 illustrates the key results with a simple example. Section 3 presents the model. Sections 4 and 5 analyze independent pricing and bundling, respectively. Section 6 studies bundling with slotting contracts. Section 7 concludes. The Appendix contains some of the proofs, but not all of them for the sake of brevity.[13]

[12] Precisely, they assume that each seller sells a homogeneous product and thus each seller chooses a price schedule that depends only on quantity. Our setting is more general: since each seller sells heterogenous objects, he specifies a price for each subset of objects which depends not only on the number of products, but also on their identities. In addition, they do not consider the slot constraint.

[13] In particular, we do not provide the proofs for Lemma 1, Proposition 5 (these results have been previously discovered by Bernheim and Whinston (1986, 1998) and O'Brien and Shaffer (1997)), and for

2 Illustration with a simple example

In this section we give a simple example to illustrate some of our main results. There are two sellers, A and B. A has two products of values $(u_A^1, u_A^2) = (4, 3)$, while B has one product of value $u_B^1 = 2$. u_i^j denotes the value that the buyer, C, obtains from the j-th best product among seller i's products. The values are independent but each product needs to occupy a slot to generate a value, and C has only two slots. We consider digital goods, which means that the production cost is zero for each product. Thus efficiency requires that the two slots be occupied by A's two products.

2.1 Independent pricing

Consider a simultaneous pricing game without bundling: seller i $(= A, B)$ simultaneously chooses a price $p_i^j \geq 0$ for product j $(= 1, 2)$. We assume as a tie-breaking rule that if C is indifferent among several products, she buys the products with the highest values.[14]

Non-existence of equilibrium in pure strategies Here we prove that this game has no equilibrium in pure strategies. First, there is no equilibrium in which A sells only his best product. Indeed, in this case A can make a profit of 4 by setting $p_A^1 = 4$ and $p_A^2 > 3$,[15] so that C buys only A's best product from A. The best response of B is $p_B^1 = 2$. However, a profitable deviation exists for A: by setting $p_A^1 = 3.9$ and $p_A^2 = 2.9$, A succeeds in selling both products and earns $6.8 > 4$.

Now we prove that there is no equilibrium in which A sells his two products. In order to sell both products, A needs to set $p_A^1 \leq 2$ and $p_A^2 \leq 1$, otherwise B can profitably sell his own product by charging $p_B^1 > 0$ such that $2 - p_B^1 > \min\{4 - p_A^1, 3 - p_A^2\}$. Therefore the profit of A when he sells both products is not larger than 3. This is inconsistent with an equilibrium, since we know that A can earn 4 by selling only his best product.

In summary, a pure strategy equilibrium does not exist for the following reasons. On the one hand, if A occupies both slots, each of A's products faces competition from B's

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Propositions 2 and 3. These proofs can be obtained from the authors upon request.

[14] This tie-breaking rule is standard. See footnote 20 for more details.

[15] Actually, A can earn 4 also by playing $p_A^1 = 4$ and $p_A^2 = 3$. But then we cannot have an equilibrium since there is no best reply for B.

product such that A's total profit is lower than the profit he makes from selling only his best product. On the other hand, if A occupies only one slot, he can extract the full surplus of C from his best product. But then, B's best response is to do the same with his own product, which triggers A's deviation to occupy both slots.

Mixed-strategy equilibrium In this game, there exists a mixed-strategy equilibrium[16] in which (i) A chooses $p_A^1 = p_A^2 + 1$ and selects p_A^2 randomly in the interval $[\frac{3}{2}, 3]$ with a cumulative distribution function

$$F_A(p_A^2) = \begin{cases} \frac{2p_A^2-3}{2p_A^2-2} & \text{if } p_A^2 \in [\frac{3}{2}, 3), \\ 1 & \text{if } p_A^2 = 3; \end{cases}$$

(ii) B chooses p_B^1 randomly in the interval $[\frac{1}{2}, 2]$ according to the cumulative distribution function $F_B(p_B^1) = \frac{2p_B^1-1}{1+p_B^1}$. In this equilibrium, A's best product is always sold while A's second best product and B's product are sold with probabilities 0.46 and 0.54 respectively. A's profit is 4, B's profit is 1/2 and the buyer's payoff is 1.96.

2.2 Bundling

Suppose now that A offers only a bundle of his two products at a price $P_A \geq 0$. Let $P_B \geq 0$ denote the price that B charges for his product. In this game, the unique equilibrium is $P_A = 5$, $P_B = 0$ and C buys A's bundle; hence the outcome is efficient. It is easy to see why this is an equilibrium: A has no incentive to charge $P_A > 5$, as then C prefers buying B's product instead of A's bundle. Given that B's profit is zero for any $P_B \geq 0$, it follows that $P_B = 0$ is a best response.

Although this example is simple, it generates useful insights. First, it shows that a firm may have a strict incentive to use bundling since it prevents the firm's own products from competing with each other and thereby reduces the damage caused from the rival's product(s). If A wants to occupy both slots, under independent pricing, he needs to charge an aggressive price for the second product (i.e. $p_A^2 = 1$) and this forces him to reduce also p_A^1 to 2 because C has an option to buy only one product from A and to fill the second slot with B's product. Bundling eliminates this option and thereby prevents A's own products from competing with each other.

[16] See Section 4.2 for a general analysis of a mixed strategy equilibrium in the case of two slots.

What happens here can also be interpreted from the point of view of cooperative game theory.[17] If A attempts to sell both of his products, then the price for each product is limited by the added value of the product with respect to the product of B, that is $p_A^1 \leq 4 - 2 = 2$ and $p_A^2 \leq 3 - 2 = 1$. On the other hand, when A's products are sold as a bundle, the price of the bundle is limited by the added value of the bundle with respect to the product of B, i.e. $P_A \leq 4 + 3 - 2 = 5$, and the value of B's product is subtracted only once. In the terms of cooperative game theory, the coalition composed by the bundle of A's products has an added value of 5, which is higher than the sum of the added values $(2 + 1)$ of each individual product of A.

Second, bundling restores equilibrium in pure strategies without causing inefficiency. The intuition for efficiency is simple in the case of digital goods with independent values. In this case, efficiency requires the best k products to occupy the k slots. Suppose that one of the k best products does not occupy any slot. Then, its seller can increase his profit by including it in his bundle or, if not currently selling any bundle to the buyer, by providing the product on its own.

Finally, the buyer's surplus can be larger under bundling than under independent pricing. Indeed, the buyer's payoff is 2 under bundling, while she obtains 1.96 in the mixed-strategy equilibrium under independent pricing. This occurs because the mixed-strategy equilibrium does not implement the efficient allocation of slots with probability one, and therefore a lower social surplus is generated. As well, under independent pricing, competition is softer in the mixed-strategy equilibrium than in the candidate pure-strategy equilibrium (see Proposition 1) since A randomizes between selling both products and only the best product, and B responds to this by being less aggressive.

3 The Model

3.1 The setting

We consider two competing sellers (or firms), denoted by A and B; the extension to competition among more than two sellers can be done similarly. There is a single buyer,

[17] We thank an anonymous referee for suggesting the interpretation.

denoted by C. We use "he" for each seller i (firm i) with $i = A, B$ and "she" for the buyer. Each firm i has a portfolio $\mathcal{B}_i$ of $n_i \geq 1$ distinct products and we use b_i to represent a generic product in $\mathcal{B}_i$. Let $\mathcal{B} \equiv \mathcal{B}_A \cup \mathcal{B}_B$. We assume that the prototypes of all the products in $\mathcal{B}$ have already been produced before the firms engage in price competition. In other words, any fixed cost related to producing the first unit of each product has already been incurred. For any $B_i \subseteq \mathcal{B}_i$, let $C_i(B_i)$ represent the cost for firm i of producing one unit of every product in B_i, with $C_i(\phi) = 0$. We assume that for any product the marginal cost is non-negative: given $b_i \in \mathcal{B}_i$ and $B_i \subset \mathcal{B}_i$ such that $b_i \notin B_i$, we have $C_i(B_i \cup \{b_i\}) \geq C_i(B_i)$. Given any $B \subseteq \mathcal{B}$, let $B_A \subseteq \mathcal{B}_A$ and $B_B \subseteq \mathcal{B}_B$ be such that $B = B_A \cup B_B$ and define $C(B)$ as $C_A(B_A) + C_B(B_B)$. We say that the products are *digital goods* if $C(B) = 0$ for any $B \subseteq \mathcal{B}$.

The buyer has a unit demand for each product. For any set of products $B \subseteq \mathcal{B}$, let $U(B)$ represent the gross value that C obtains from using B. We assume $U(\phi) = 0$ and that for any product in $\mathcal{B}$, the marginal gross value is non-negative. Let $V(B) = U(B) - C(B)$ denote the social welfare for the economy composed of the buyer and the two firms when the firms produce $B \subseteq \mathcal{B}$ and C uses every product in B.

Since we are mainly interested in studying the effect of the buyer's slot constraint on the competition between the two firms, we assume that C has $k \geq 1$ slots and a product needs to occupy any slot to generate any value.[18] Therefore, if C buys $B \subseteq \mathcal{B}$ and $\#(B) \leq k$ then her gross payoff is given by $U(B)$; if instead $\#(B) > k$, then her gross payoff is given by $\max U(\hat{B})$ subject to $\hat{B} \subset B$ and $\#(\hat{B}) \leq k$. The buyer's net payoff from buying B is given by her gross payoff as described above minus the prices paid.

We have in mind a situation in which the two multi-product firms A and B compete in several separate markets. In each market, there is a single buyer and the firms charge different prices in each different market. Therefore, without loss of generality we can consider only one market (and only one buyer).

A special case of this setting is the case of independent products, which we now in-

[18] By assuming unit demand, we assume for simplicity that a product can occupy at most one slot in that the value generated from occupying a second unit of slot is zero. This assumption can be relaxed without changing the main results.

troduce. For any product $b_i \in \mathcal{B}_i$, let $u(b_i) = U(\{b_i\})$ represent the gross value generated by this product when C does not buy anything else. We say that *values are independent* if the following property holds:

$$U(B_A \cup B_B) = \sum_{b_A \in B_A} u(b_A) + \sum_{b_B \in B_B} u(b_B) \qquad \text{for} \quad \text{any} \quad B_A \subseteq \mathcal{B}_A \quad \text{and} \quad B_B \subseteq \mathcal{B}_B.$$

For any product $b_i \in \mathcal{B}_i$, let $c_i(b_i) = C_i(\{b_i\})$ represent the cost for firm i of producing one unit of the product when firm i does not produce anything else. We say that *costs are independent* if the following property holds:

$$C_i(B_i) = \sum_{b_i \in B_i} c_i(b_i) \qquad \text{for} \quad \text{any} \quad B_i \subseteq \mathcal{B}_i \quad \text{and} \quad \text{for} \quad i = A, B.$$

The *case of independent products* is such that both values and costs are independent. For this setting, let $v(b_i) \equiv u(b_i) - c_i(b_i)$ denote the *net value* of product b_i.

We study how the firms' pricing strategies (in particular, bundling or not) affect the set of products occupying the buyer's slots. Specifically, we are interested in knowing when the slots are occupied by the products that maximize social welfare, defined by

$$B^* \equiv \arg\max_{B \subseteq \mathcal{B}} V(B) \qquad \text{subject to } \#(B) \leq k. \tag{1}$$

We assume that B^* is unique and we let $V^* \equiv U(B^*) - C(B^*)$ and $B_i^* \equiv B^* \cap \mathcal{B}_i$. We say that an equilibrium (of the games we consider below) is (socially) *efficient* if the firms produce the set B^*, or a set B such that $B^* \subset B$ and $C(B) = C(B^*)$, and each product in B^* occupies a slot.

An important role is played by the products that maximize social welfare when C is restricted to buying from a single firm $i = A, B$, defined by

$$B_i^{*S} \equiv \arg\max_{B_i \subseteq \mathcal{B}_i} V(B_i) \qquad \text{subject to } \#(B_i) \leq k. \tag{2}$$

We assume that B_i^{*S} is unique and we let $V_i^{*S} \equiv U(B_i^{*S}) - C_i(B_i^{*S})$.

We make the following assumption.

Assumption A1.

$$B_A^* \neq \emptyset, \qquad V_B^{*S} > 0, \qquad V_A^{*S} + V_B^{*S} - V^* \geq 0.$$

The condition $B_A^* \neq \emptyset$ means that the efficient allocation includes some products of firm A; this is without loss of generality. The inequality $V_B^{*S} > 0$ means that if C is restricted to dealing only with firm B, then there exist mutually profitable trades; if $V_B^{*S} \leq 0$, then A does not face any competition from B. Finally, the assumption $V_A^{*S} + V_B^{*S} - V^* \geq 0$ means that the products of the two firms are substitutes in a weak sense and guarantees that there exists an efficient equilibrium (see Section 5.2).

3.2 Contracts and games

In this section, we first describe the bilateral contracts that each seller can propose to the buyer in our model and then introduce the timing of the games that we study.

3.2.1 Menu of bundles (without slotting contracts)

In the absence of slotting contracts, defined in Section 6, the most general contract between seller i and the buyer is that i offers a *menu of bundles* with a price $P_i(B_i) \geq 0$ for each bundle $B_i \subseteq \mathcal{B}_i$, such that $P_i(\phi) = 0$.[19] Then, if C buys bundle B_A from firm A and bundle B_B from firm B (some of these sets may be empty), then she pays $P_A(B_A) + P_B(B_B)$. Let $s_i = \{P_i(B_i)\}_{B_i \subseteq \mathcal{B}_i}$ denote a generic strategy of firm i.

- Technology-renting

A particular menu of bundles is what we call *technology-renting*, which Bernheim and Whinston (1998) refer to as a *sellout* contract. A technology-renting strategy for firm i is characterized by a fee $F_i \geq 0$ and is such that $P_i(B_i) = F_i + C_i(B_i)$ for any non-empty $B_i \subseteq \mathcal{B}_i$. In words, if the buyer wants to buy at least one product from firm i, then she must first pay F_i for the right to buy, and in addition she pays the production cost of the products that she selects to buy. In a sense, firm i rents his production technology to C by levying a fixed rental fee in addition to a term for cost reimbursement. Let tr_i denote a generic technology-renting strategy of firm i.

[19] Our definition of menu of bundles generalizes the notion of mixed bundling used in the context of two goods (see McAfee, McMillan and Whinston (1989) for example). In this case, mixed bundling means that the seller charges a price for each good and a third price for the bundle of both goods.

Note that if i plays a technology-renting strategy with fee F_i, then i's profit is F_i if C buys at least one of i's products and zero otherwise. Furthermore, if both firms play a technology-renting strategy and C rents both technologies, then she becomes the residual claimant of social welfare, which induces her to buy the efficient set B^*.

- Pure bundling

A pure bundling strategy is such that $P_i(B_i) = F_i$ for some $F_i \geq 0$ for any non-empty $B_i \subseteq \mathcal{B}_i$. Therefore each subset of $\mathcal{B}_i$ is offered by firm i at the same price F_i, which in a sense makes pure bundling an all-or-nothing deal. Pure bundling is equivalent to technology-renting in the case of digital goods.

- Independent pricing

An independent pricing strategy is such that firm i chooses an individual price $p_i(b_i)$ for each b_i in B_i and the price for any non-empty $B_i \subseteq \mathcal{B}_i$ is $P_i(B_i) = \sum_{b_i \in B_i} p_i(b_i)$.

In what follows, we use the word "bundling" for any pricing strategy that is different from independent pricing. When bundling is prohibited, each firm is constrained to using independent pricing. When bundling is allowed, each firm can use any menu of bundles including independent pricing.

3.2.2 Timing

We consider a two-stage pricing game in which

- at stage one, each firm simultaneously makes a contract offer;
- at stage two, C chooses the products (or bundles) to buy and allocates the slots.

At stage two, we assume that in case C is indifferent among different combinations of products, she chooses the combination that maximizes social welfare.[20]

[20] This tie-breaking rule is standard in that it is basically equivalent to the following rule applied in a Bertrand setting when two firms produce a homogenous good with different marginal costs: when the two firms charge the same price, the tie is broken by assuming that all consumers buy from the firm with the lower marginal cost (which generates a higher social welfare).

4 Independent pricing

In this section, we assume that firms are restricted to using independent pricing and focus on the case of independent products when there are more than k products with positive net value.[21] Then B^* is the set of the k products with the highest net values. We order the products in $\mathcal{B}_A$ and $\mathcal{B}_B$ such that $v(b_A^1) \geq v(b_A^2) \geq ... \geq v(b_A^{n_A})$ and $v(b_B^1) \geq v(b_B^2) \geq ... \geq v(b_B^{n_B})$. Firms A and B select the prices $\{p_A(b_A)\}_{b_A \in \mathcal{B}_A}$ and $\{p_B(b_B)\}_{b_B \in \mathcal{B}_B}$. Let $w_i(b_i) \equiv u(b_i) - p_i(b_i)$ represent the *net surplus* that C obtains from buying product b_i. Therefore she buys the k products with the highest net surpluses, provided that these surpluses are non-negative.

4.1 Equilibrium in pure strategies

We first study equilibrium in pure strategies. Given that $B_A^* \neq \emptyset$, we distinguish the case of $B_B^* \neq \emptyset$ from the case of $B_B^* = \emptyset$ (which implies $B^* = B_A^*$).

When $B_B^* \neq \emptyset$, an important role is played by the product with the highest net value among the products in $\mathcal{B} \backslash B^*$ (that is, the product $\bar{b}$ such that $v(\bar{b}) \geq v(b_i)$ for any $b_i \in \mathcal{B} \backslash B^*$). We suppose that such a product is unique and that it belongs to $\mathcal{B}_A$ (since $B_i^* \neq \emptyset$ for $i = A, B$, this is without loss of generality). Let $\bar{b}_A$ denote this best product in $\mathcal{B} \backslash B^*$ and note that $v(\bar{b}_A) > 0$ since there are more than k products with positive net value.

For the case of $B_B^* = \emptyset$, we define $\lambda(b_B) = \min\{u(b_B), v(b_B^1)\}$ for $b_B = b_B^2, ..., b_B^{n_B}$. Thus, for each $b_B \in B_B \backslash \{b_B^1\}$, $\lambda(b_B)$ is the minimum between the gross value of b_B and the net value of b_B^1 (we explain later in this section why $\lambda(b_B)$ plays a role when $B_B^* = \emptyset$.)[22] In order to simplify the exposition, we suppose that $u(b_B^2) \geq u(b_B^3) \geq ... \geq u(b_B^{n_B})$, which implies that $\lambda(b_B^2) \geq \lambda(b_B^3) \geq ... \geq \lambda(b_B^{n_B})$. In fact, this assumption is without loss of generality since the order of net values in $B_B \backslash \{b_B^1\}$ is irrelevant when $B_B^* = \emptyset$.

[21] If there are $\leq k$ products with positive net value, then each firm i can extract the full surplus from the buyer by charging $p_i(b_i) = \max\{u(b_i), c_i(b_i)\}$ for any $b_i \in \mathcal{B}_i$. The assumption that more than k products have positive net value creates competition among the products.

[22] We use this definition of $\lambda(b_B)$ becuase we allow for prices below marginal cost. If conversely we impose $p_B(b_B) \geq c_B(b_B)$ for any $b_B \in \mathcal{B}_B$, then we need to replace $\lambda(b_B)$ with $v(b_B)$ for $b_B = b_B^2, ..., b_B^{n_B}$.

Proposition 1 *[independent pricing: pure strategies] Suppose that products are independent, that there are more than k products with positive net value and that each firm uses independent pricing.*

(i) If an equilibrium in pure strategies exists, then it is efficient and there exists a number w^* such that $w_i(b_i) = w^*$ for any $b_i \in B^*$; moreover, $w^* = \max\{0, \max_{b_i \in (\mathcal{B} \setminus B^*)}\{w_i(b_i)\}\}$.

(ii) When $B_B^* \neq \emptyset$, if an equilibrium in pure strategies exists, then $w^* \geq v(\bar{b}_A)$. In addition, no pure-strategy equilibrium exists if

$$v(\bar{b}_A) > u(b_B) \qquad \text{for} \quad \text{any} \quad b_B \in \mathcal{B}_B \setminus B_B^*. \tag{3}$$

(iii) When $B_B^* = \emptyset$, there exists a pure-strategy equilibrium if and only if

$$\sum_{b_A \in B_A^*} [v(b_A) - v(b_B^1)] \geq \sum_{b_A \in \{b_A^1, b_A^2, ..., b_A^j\}} [v(b_A) - \lambda(b_B^{k-j+1})] \qquad \text{for} \quad j = 1, ..., k-1. \tag{4}$$

Proposition 1(i) describes some properties of pure-strategy Nash equilibrium (NE) under independent pricing. In any NE (if it exists), the slots are occupied by the products in B^*, and the net surpluses of these products are all equal to a common value w^*, which is the highest net surplus (if positive) among the products in $\mathcal{B} \setminus B^*$.

When $B_B^* \neq \emptyset$, the first part of Proposition 1(ii) reveals that a multi-product firm suffers from *internal competition* (i.e., competition among its own products) under independent pricing, in that A needs to leave C a net surplus at least equal to $v(\bar{b}_A)$ on any product he sells, even though A himself owns $\bar{b}_A$. In other words, in any NE each product in B_A^* must give the buyer at least as much surplus as it would give when the best product in $\mathcal{B} \setminus B^*$ is offered at cost by another firm. This occurs because if w^* in Proposition 1(i) is smaller than $v(\bar{b}_A)$, then B prices each product b_B in B_B^* such that $w_B(b_B) = w^* < v(\bar{b}_A)$, and this in turn allows A to find a profitable deviation that induces C to replace a product in B_B^* with $\bar{b}_A$ by setting $p_A(\bar{b}_A)$ between $c_A(\bar{b}_A)$ and $u(\bar{b}_A) - w^*$.[23] For this reason, no NE exists if (3) holds. In order to see why, it is useful to consider the case of digital goods. Then, $v(\bar{b}_A) = u(\bar{b}_A) > w_A(b_A)$ for each $b_A \in (\mathcal{B}_A \setminus B_A^*)$, $b_A \neq \bar{b}_A$ and (3) implies that $v(\bar{b}_A) = u(\bar{b}_A) > w_B(b_B)$ for each $b_B \in (\mathcal{B}_B \setminus B_B^*)$. Since $w^* \geq v(\bar{b}_A) > 0$ and $w^* = \max_{b_i \in (\mathcal{B} \setminus B^*)}\{w_i(b_i)\}$ from Proposition 1(i), $p_A(\bar{b}_A) = 0$ and $w^* = v(\bar{b}_A)$ must hold. However, A has an incentive to relax the internal competition by slightly increasing

[23] Simultaneously, A needs to decrease slightly the prices of his products in B_A^*.

$p_A(\bar{b}_A)$ above zero, in order to extract more surplus from each product in B_A^*. This leads to $w^* = w(\bar{b}_A) < v(\bar{b}_A)$, and we have seen that $w^* < v(\bar{b}_A)$ cannot hold in NE. It is worth noting that in the case of digital goods, condition (3) is satisfied as long as products have heterogeneous values. Therefore, NE non-existence generically holds for digital goods when $B_B^* \neq \emptyset$.[24] The next example illustrates this result.

Example 1 *Consider a setting of digital goods with independent values, $\mathcal{B}_A = \{b_A^1, b_A^2\}$, $\mathcal{B}_B = \{b_B^1\}$, $k = 2$ and $u(b_A^1) = 4$, $u(b_A^2) = 2$, $u(b_B^1) = 3$; thus $B^* = \{b_A^1, b_B^1\}$, $\bar{b}_A = b_A^2$ and (3) holds (trivially, since $\mathcal{B}_B \backslash B_B^* = \emptyset$). On the one hand, there exists no NE in which C buys $\{b_A^1, b_A^2\}$ since B can profitably induce C to replace b_A^2 with b_B^1. On the other hand, there exists no NE in which C buys $\{b_A^1, b_B^1\}$ because then $4 - p_A^1 = 3 - p_B^1 = w^*$ (from Proposition 1(i)) and (i) if $w^* < 2$, then A can profitably induce C to replace b_B^1 with b_A^2 by choosing $p_A^2 = 2 - w^* - \varepsilon > 0$; (ii) if $w^* \geq 2$, then A can profitably deviate by charging $p_A^1 = 4$ and $p_A^2 \geq 2$.*

When $B_B^* = \emptyset$, every product in B_A^* must give C a net surplus w^* such that $w^* \geq v(b_B^1)$; otherwise a profitable deviation exists for B. However, A might have an incentive to deviate by occupying less than k slots. Actually, A's optimization problem for the number of slots to occupy given the prices charged by B is similar to a standard monopoly pricing problem under a downward sloping demand: the fewer slots A occupies, the higher is the surplus per slot he extracts. To provide an intuition, consider digital goods and assume that B charges zero price for all his products (we explain below in this paragraph why this minimizes A's incentive to deviate). Then, A's deviation to occupy $k-1$ slots is profitable if $(k-1)(u(b_B^1) - u(b_B^2)) > u(b_A^k) - u(b_B^1)$ holds: A extracts more surplus, by $u(b_B^1) - u(b_B^2)$, from each of $k-1$ products sold but loses the profit $u(b_A^k) - u(b_B^1)$ from the k-th (unsold) product. More generally, minimizing A's incentive to deviate requires (i) minimizing the value of w^*, which implies $w^* = v(b_B^1)$ and $\pi_A = \sum_{b_A \in B_A^*}[v(b_A) - v(b_B^1)]$; and (ii) maximizing the net surpluses of the products $b_B^2, ..., b_B^{n_B}$ subject to the constraint $w_B(b_B) \leq v(b_B^1)$ for $b_B = b_B^2, ..., b_B^{n_B}$. This leads to $\lambda(b_B)$ as

[24] Likewise, if we impose $p_i(b_i) \geq c_i(b_i)$ for any $b_i \in \mathcal{B}$, then the condition $v(\bar{b}_A) > w_B(b_B)$ for each $b_B \in (\mathcal{B}_B \backslash B_B^*)$ holds as long as net values are heterogeneous. Thus, NE non-existence generically holds if prices cannot be below costs.

defined above for $b_B = b_B^2, ..., b_B^{n_B}$, and if A wants to sell the bundle $\{b_A^1, b_A^2, ..., b_A^j\}$ for some $j \leq k-1$, then each product sold must give C a net surplus of $\lambda(b_B^{k-j+1})$, yielding a profit of $\sum_{b_A \in \{b_A^1, b_A^2, ..., b_A^j\}} [v(b_A) - \lambda(b_B^{k-j+1})]$. We obtain (4) by comparing this deviation profit with π_A.

From Proposition 1, we obtain the following corollary for the case of digital goods.

Corollary 1 *In the case of digital goods with independent values, under independent pricing, there is no pure-strategy equilibrium if either of the two following conditions holds:*
(i) $B_B^* \neq \emptyset$ *and products have heterogenous values;*
(ii) $B_B^* = \emptyset$ *and* $\sum_{b_A \in B_A^*} [u(b_A) - u(b_B^1)] < \sum_{b_A \in \{b_A^1, b_A^2, ..., b_A^j\}} [u(b_A) - u(b_B^{k-j+1})]$ *holds for at least one* j *between* 1 *and* $k-1$.

4.2 Mixed strategies

In this section, we study equilibrium in mixed strategies in the simple setting of digital goods with $k = 2$, and then we provide a general result on the inefficiency of mixed strategy NE.

When $k = 2$, without loss of generality we consider $n_A = n_B = 2$. For notational simplicity we let $x = u(b_A^1)$, $y = u(b_A^2)$, $z = u(b_B^1)$, $t = u(b_B^2)$, and with some abuse of terminology we refer to product x, y, z, t instead of $b_A^1, b_A^2, b_B^1, b_B^2$. Firm A charges prices (p_x, p_y) and B charges (p_z, p_t). Assume without loss of generality that t is the lowest valued product. If values are heterogeneous, then there are two possible cases:

- case 1: $z > x > y > t$ or $x > z > y > t$,
- case 2: $x > y > z > t$.

In case 1, there exists no pure-strategy NE by Corollary 1(i), while in case 2 Proposition 1(iii) reveals that a pure-strategy NE exists if and only if $y \geq 2z - t$. If the inequality holds, the NE is $p_x = x - z$, $p_y = y - z$, $p_z = p_t = 0$. Therefore, we assume $y < 2z - t$ for case 2; this inequality is automatically satisfied in case 1. The next proposition describes a mixed-strategy NE:

Proposition 2 *[independent pricing: mixed strategies] Consider the setting, described just above, of digital goods with $k = 2$ and $y < 2z - t$. Suppose that each firm uses independent pricing.*

(i) There exists a mixed-strategy NE in which A sets $p_x = x - \alpha$, $p_y = y - \alpha$ and randomly chooses α in the interval $[t, \frac{1}{2}(y+t)]$ according to the c.d.f. $G_A(\alpha) = \frac{2z-t-y}{2z-2\alpha}$ (note that G_A has an atom at $\alpha = t$) and B sets $p_z = z - \beta$, $p_t = 0$ and randomly chooses β in the interval $[t, \frac{1}{2}(y+t)]$ according to the c.d.f. $G_B(\beta) = \frac{\beta-t}{y-\beta}$.

(ii) In this mixed-strategy NE, the payoffs of A and B are $x-t$ and $z-\frac{1}{2}t-\frac{1}{2}y$ respectively. The buyer buys either the combination $\{x,y\}$ or $\{x,z\}$.[25] For $y \leq z + (z-t)/3$, G_B first order stochastically dominates G_A, that is on average B leaves C more surplus than A does.

The strategies described in Proposition 2(i) can be easily interpreted as follows. A randomly selects a net surplus α for each of his products (which means that $p_x = x - \alpha$, $p_y = y - \alpha$) in $[\underline{\alpha}, \overline{\alpha}] = [t, \frac{1}{2}(y+t)]$ according to the c.d.f. $G_A(\alpha) = \frac{2z-t-y}{2z-2\alpha}$; B chooses $p_t = 0$ and randomly selects a net surplus β for his best product (that is $p_z = z - \beta$) in the interval $\left[\underline{\beta}, \overline{\beta}\right] = [t, \frac{1}{2}(y+t)]$ according to the c.d.f. $G_B(\beta) = \frac{\beta-t}{y-\beta}$. The outcome of this NE is such that the buyer always buys product x, never buys product t and buys either product y or z depending on the realization of the mixed strategies.

In order to get an intuition for this mixed strategy NE, consider case 1 and note that $B^* = \{x,z\}$ and $\bar{b}_A = y$. If a pure-strategy NE existed, then $x - p_x = z - p_z = y$ [as we argued when providing the intuition for Proposition 1(ii)], and $\pi_A = x - y$, $\pi_B = z - y$; this corresponds to $\alpha = y$, $\beta = y$. However, firm A has an incentive to increase p_y above zero in order to reduce the internal competition generated by product y and to sell good x at a higher price. In fact, A can always earn $x - t$ $(> x - y)$ by setting $p_x = x - t$, $p_y = y - t$; this corresponds to $\alpha = t$. Then B also gains from reducing β to t since he earns $z - t$ instead of $z - y$. However, at $\alpha = \beta = t$ firm A can increase his profit by slightly increasing α above t to sell both products and earn about $x + y - 2t > x - t$,

[25] The probability that C buys $\{x,y\}$ turns out to be $\frac{y-t}{2(y-z)^2}[y - z + (2z-t-y)\ln\frac{2z-t-y}{z-t}]$. For the example in Section 2, the parameters are $x = 4, y = 3, z = 2, t = 0$. Then the support for α and β is $[0, \frac{3}{2}]$ and $G_A(\alpha) = \frac{1}{4-2\alpha}$, $G_B(\beta) = \frac{\beta}{3-\beta}$. From $4 - p_x = 3 - p_y = \alpha$ and $p_z = z - \beta$ we obtain F_A and F_B described in Section 2.1. The probability that C buys $\{x,y\}$ is $\frac{3}{2}(1 + \ln\frac{1}{2}) \simeq 0.46$.

only to trigger an analogous reaction from B. Therefore, it is not surprising that we find a mixed strategy NE in which $\alpha \geq t$ and $\beta \geq t$ with probability one. However, the upper bound for α, $\overline{\alpha}$, coincides with the upper bound for β, $\overline{\beta}$. It is not equal to y but is such that A earns $x-t$ by selling both products at the lowest prices $p_x = x-\overline{\alpha}$ and $p_y = y-\overline{\alpha}$, i.e. $x + y - 2\overline{\alpha} = x - t$ or $\overline{\alpha} = \frac{1}{2}(y + t)$. Hence A chooses α between t and $\frac{1}{2}(y + t)$, which means that he is less aggressive than when playing $\alpha = y$ as in the candidate pure-strategy NE. That is, A reduces the competition that product y exerts on product x, and also on product z. As a consequence, this benefits B, who earns $z - \frac{1}{2}t - \frac{1}{2}y$ instead of $z - y$.

It is also interesting to note that when $y \leq z + (z - t)/3$, we find that B is more aggressive than A in the sense that G_B first order stochastically dominates G_A, and thus B leaves on average more surplus than A does. For this reason, for instance, when y is close to z, C buys product y (respectively, z) with probability about 1/4 (respectively, 3/4). The intuition for this result is that leaving a high surplus on product y reduces the profit that A makes from x, which C certainly buys.

The mixed-strategy NE described by Proposition 2 is inefficient, as C buys B^* with probability smaller than 1. More generally, we can prove that a similar property holds when the conditions for non-existence of a pure-strategy NE in Proposition 1 are satisfied: in such a case, no mixed-strategy NE is efficient, independently of k and of whether or not the products are digital goods.

Proposition 3 *Suppose that $B_B^* \neq \emptyset$ and (3) is satisfied or that $B_B^* \neq \emptyset$ and (4) is violated. Then, no pure-strategy NE exists and any mixed-strategy NE is inefficient.*

If no pure-strategy NE exists, then one (or more) mixed-strategy NE may exist. But each mixed-strategy NE is inefficient according to Proposition 3. This is consistent with Proposition 2 (and footnote 25) and implies that under a broad set of circumstances, neither efficient pure-strategy NE nor efficient mixed-strategy NE exists under independent pricing.

The logic behind Proposition 3 is quite simple. If an efficient mixed-strategy NE exists, then it is necessary that C buy all products in B^* with probability one, which requires that $w_i(b_i) \geq w_j(b_j)$ with probability one for each $b_i \in B^*$ and $b_j \in \mathcal{B}\backslash B^*$. But then for each product $b_i \in B^*$, it is profitable for i to choose a deterministic $p_i(b_i)$ such that $w_i(b_i)$ is equal to the lowest value that, with probability one, is weakly larger than $w_j(b_j)$ for each $b_j \in \mathcal{B} \backslash B^*$. In a sense, this brings us back to the pure-strategy setting and allows to use the arguments of Proposition 1 to show that an efficient mixed-strategy NE does not exist.

5 Bundling

In this section, we study competition among sellers when bundling is allowed. We show that each seller has an incentive to bundle his products (Section 5.1), that an efficient equilibrium always exists (Section 5.2), and we provide sufficient conditions for all equilibria to be efficient (Section 5.3). Finally, we compare the case of independent pricing with that of bundling in terms of social welfare and the buyer's surplus (Section 5.4).

We would like to emphasize that even though we focus on the slot constraint, all the principles underlying our results hold independently of this constraint. Thus, we can think of the role of the slot constraint as creating competition among products even when they have independent values and costs.

5.1 Incentive to bundle

We first describe an important property of the technology-renting strategies in the following lemma.

Lemma 1 *[Bernheim and Whinston, 1986]*[26] *For any profile of strategies (s_A, s_B), let π_i denote the profit of firm i given (s_A, s_B). Then firm i can also make profit π_i by playing a technology-renting strategy, instead of s_i, in which the fixed fee F_i is equal to π_i.*

Lemma 1 says that no firm i loses anything by restricting attention to technology-renting strategies *regardless of the strategies used by the other firm.* We will often use this result in our proofs. In particular, the lemma implies that each firm has at least a

[26]Technology-renting strategies are analogous to "truthful" strategies in Bernheim and Whinston (1986) and Lemma 1 in our paper is analogous to Theorem 1 in their paper.

weak incentive to practice bundling. Furthermore, the example in Section 2 illustrates a case in which a firm has a strict incentive to do so. Therefore, we have:

Proposition 4 *[incentive to bundle] Each firm has at least a weak incentive, and sometimes a strict incentive, to practice bundling.*

5.2 An efficient equilibrium

In this section we describe an efficient equilibrium. In this NE each firm i uses a technology-renting strategy and hence we can consider the strategy space for each i as given by $[0, +\infty)$, the set of possible values of F_i. The equilibrium fixed fees are

$$F_A^* = V^* - V_B^{*S}, \qquad F_B^* = V^* - V_A^{*S}. \tag{5}$$

The intuition for these values is simple. If C rents only j's technology, then she chooses $B_j \subseteq \mathcal{B}_j$ to maximize $U(B_j) - F_j - C_j(B_j)$ subject to $\#(B_j) \leq k$ and obtains payoff $V_j^{*S} - F_j$: see (2). If she rents i's technology as well, then she becomes the residual claimant over social welfare, thus chooses B^* and obtains a payoff equal to $V^* - F_j - F_i$. Therefore, the F_i that makes C indifferent between renting i's technology (in addition to renting j's technology) and not renting the technology is equal to $V^* - V_j^{*S}$.[27] Actually, $V^* - V_j^{*S}$ represents the incremental contribution to social welfare made by the products in $\mathcal{B}_i$ or, in the terms of Brandenburger and Stuart (1996), the added value of firm i.[28] Note that if $B_B^* = \emptyset$, then $V^* = V_A^{*S}$ and therefore $F_B^* = 0$.

Let tr_i^* denote the technology-renting strategy of firm i in which the fixed fee is F_i^*. As discovered by Bernheim and Whinston (1998) and O'Brien and Shaffer (1997) (for a somewhat different model: see footnote 12), the profile (tr_A^*, tr_B^*) is a NE, which we call the *technology-renting equilibrium*.

[27] This intuition may appear incomplete as it assumes that C has already rented the technology of j, but actually it is simple to see that there exists no NE in which C rents no technology. To be precise, a profitable deviation for any firm i is such that i plays a technology-renting strategy with a small positive F_i, given that $V_i^{*S} > 0$.

[28] Brandenburger and Stuart (1996) apply a similar idea to define the added value of a firm in a vertical chain as the value created by all agents in the chain minus the value created by the same agents without the firm in question.

Proposition 5 *[Bernheim and Whinston, 1998 and/or O'Brien and Shaffer 1997, technology-renting equilibrium and efficiency when $B_B^* = \emptyset$] Suppose that bundling is allowed.*
(i) There exists an efficient NE in which each firm i uses the technology-renting strategy tr_i^*. In this NE, the firms' profits are F_A^*, F_B^* in (5) and the buyer's payoff is $\pi_C^* \equiv V_A^{*S} + V_B^{*S} - V^* \geq 0$.
(ii) In any NE, the profit of each firm i is not larger than F_i^*; hence, the technology-renting equilibrium Pareto dominates any other equilibrium in terms of sellers' payoffs.
(iii) All equilibria are efficient if $B_B^* = \emptyset$.

The logic for Proposition 5(i) is straightforward. If the firms play (tr_A^*, tr_B^*), then C obtains the same payoff from renting only A's technology, or only B's technology, or both technologies. The tie-breaking rule selects the alternative that maximizes social welfare and thus C rents both technologies and selects the products in B^*, meaning that the NE is efficient.[29] Note that the payoff of C, π_C^*, is non-negative by assumption A1. Moreover, if firm A (for instance) increases F_A above F_A^*, then C strictly prefers renting only B's technology to renting only A's technology or both technologies. The intuition for Proposition 5(ii) is that if firm i attempts to make a profit larger than the incremental contribution of his portfolio $\mathcal{B}_i$ to social welfare, then j can find a profitable deviation by inducing C to buy only B_j^{*S}.

Furthermore, according to Proposition 5(iii), all NE are efficient if $B_B^* = \emptyset$. In words, if the efficient allocation requires C to use only products of firm A, then any NE induces the efficient allocation. The result is from O'Brien and Shaffer (1997) and its idea is simple. If C purchases an inefficient bundle, then an inefficiently low social surplus $\bar{V}(< V^*)$ is generated whereas A and C can jointly generate a surplus equal to V^*. Therefore if A plays a technology-renting strategy with fee equal to π_A (his profit before the deviation), C can increase her payoff by $V^* - \bar{V}$ by trading only with A; hence A can charge a fee slightly higher than π_A and still induce C to rent A's technology.

However, O'Brien and Shaffer (1997) also show that non-linear pricing (i.e., bundling in our setting) can generate inefficient NE when $B_B^* \neq \emptyset$. Similarly, the following example

[29] The technology-renting equilibrium generalizes the marginal cost pricing result in the literature on competition in non-linear pricing (Armstrong and Vickers, 2001, 2010 and Rochet and Stole 2002) to a situation in which each firm can produce any number of distinct products.

shows that pure bundling generates an inefficient NE.

Example 2 *(pure bundling and inefficiency) Consider a setting with independent products and* $k = 2$*. Firm* A *has two products:* $\mathcal{B}_A = \{b_A^1, b_A^2\}$ *such that* $u(b_A^1) = 12$*,* $u(b_A^2) = 8$*. Firm* B *has two products:* $\mathcal{B}_B = \{b_B^1, b_B^2\}$ *such that* $u(b_B^1) = 10$*,* $u(b_B^2) = 5$*. The marginal production cost for each product is* 3*. Then there exists a NE in which firm* A *plays pure bundling by charging* $F_A = 11$ *for each subset of* $\mathcal{B}_A$*, and* B *charges* $P_B(b_B^1) = 5$*,* $P_B(b_B^2) = 3$*,* $P_B(b_B^1, b_B^2) = 6$*. Then* C *buys* $\mathcal{B}_A$ *even though* $B^* = \{b_A^1, b_B^1\}$ *includes a product of* B*.*

In the NE of Example 2, firm B is unable to sell his superior product b_B^1 for the two following reasons. First, from the all-or-nothing deal of pure bundling, if C buys b_A^1, she also gets b_A^2 at the same total price. Second, if firm B tries to find a profitable deviation, in order not to make a loss he must charge a price for b_B^1 at least equal to the marginal cost 3, whereas the gain of C from replacing b_A^2 with b_B^1 is $10 - 8 = 2$.

This argument suggests that the inefficient NE would disappear if the marginal cost of b_B^1 were smaller than 2. In the next subsection we prove that this is indeed the case and provide general sufficient conditions to make all NE efficient.

5.3 Conditions under which all equilibria are efficient

In this subsection, we provide two (different) mild conditions guaranteeing that all NEs are efficient regardless of whether $B_B^* \neq \emptyset$ holds; note that both conditions are satisfied in the case of independent products. More precisely, when either of the two conditions holds, all NEs are efficient if the products are digital goods, or if sales below marginal cost are banned. Furthermore, under the first condition, each seller i can earn F_i^* regardless of the rival's strategy and in each NE each player has the same payoff as in the technology-renting NE.

5.3.1 Weak m-submodularity

In this section we consider a class of environments which satisfy a condition related to submodularity. Given any $B \subseteq \mathcal{B}$, we define $V^m(B)$ as follows:

$$V^m(B) \equiv \max_{\hat{B} \subseteq B} V(\hat{B}) \qquad \text{subject to} \quad \#(\hat{B}) \leq k,$$

where the superscript m comes from the max operator. In the next proposition, we assume that V^m satisfies the following property:

$$V^m(\mathcal{B}_j \cup \mathcal{B}_i) - V^m(\mathcal{B}_j) \leq V^m(B_j \cup \mathcal{B}_i) - V^m(B_j) \text{ for any } B_j \subset \mathcal{B}_j \text{ and } i \neq j = A, B. \quad (6)$$

In order to understand (6), it is useful to recall the standard property of decreasing marginal values:

$$V^m(B \cup \{b\}) - V^m(B) \leq V^m(B' \cup \{b\}) - V^m(B') \text{ for any } B' \subset B \subset \mathcal{B} \text{ and } b \in \mathcal{B} \backslash B, \quad (7)$$

which is well known to be equivalent to submodularity of V^m (see Moulin (1995)). Condition (6) is implied by (7) and has a related interpretation: the incremental social contribution of $\mathcal{B}_i$ given $\mathcal{B}_j$ is weakly smaller than the one given B_j, for any $B_j \subset \mathcal{B}_j$. But in fact, (6) is substantially weaker than (7) and for this reason we call condition (6) *weak m-submodularity.*[30]

In this class of environments, a strong result is obtained for digital goods, or when each firm is prohibited from setting the marginal price of any product below its marginal cost. Precisely, we consider the following restriction on firm i's strategies:

$$\begin{gathered} \text{for any } b_i \in \mathcal{B}_i \text{ and any } B_i \subset \mathcal{B}_i \text{ such that } b_i \notin B_i, \\ P_i(B_i \cup \{b_i\}) - P_i(B_i) \geq C_i(B_i \cup \{b_i\}) - C_i(B_i) \quad \text{for } i = A, B. \end{gathered} \quad (8)$$

The meaning of (8) is that as the number of products in a bundle of firm i increases, the price of the bundle needs to increase at least by the cost of the additional products in the bundle. In short, the marginal price of each product is not smaller than its marginal cost. In particular, this implies that $P_i(B_i) \geq C_i(B_i)$ for each $B_i \subseteq \mathcal{B}_i$, or pricing above total cost. In addition, if seller i is interested in selling a particular bundle B_i for a certain price $P^* \geq 0$, condition (8) forces him to offer *each subset* of B_i at a price (weakly)

[30] Notice that weak m-submodularity implies that the last inequality in A1 is satisfied.

smaller than P^* such that the buyer can save at least $C_i(B_i) - C_i(B_i \backslash \{b_i\})$ by cancelling a product b_i in B_i. In particular, (8) makes it impossible for a firm to use a pure bundling strategy except in the case of digital goods, and indeed the firms' strategies (the strategy of firm A, in particular) in the NE of Example 2 violate (8). On the other hand, every technology-renting strategy satisfies (8). Therefore, from Lemma 1 we know that for any s_j, firm $i(\neq j)$ can find a best response to s_j in the set of strategies satisfying (8).

Proposition 6 *[uniqueness of equilibrium outcome under weak m-submodularity] Suppose that bundling is allowed and that weak m-submodularity condition (6) holds. Consider either the setting of digital goods or assume that sales below marginal cost are banned (i.e., (8) must be satisfied). Then*
(i) each firm i can earn F_i^* by using the technology-renting strategy with fee F_i^*, independently of the pricing strategy chosen by firm j;
(ii) all NEs are outcome-equivalent, i.e. in each NE the allocation of slots is efficient and each player obtains the same payoff: F_A^*, F_B^* for the sellers and π_C^* for the buyer.

Proposition 6(i) says that in the case of digital goods or when sales below marginal cost are prohibited, any firm can guarantee a payoff equal to that of the technology-renting equilibrium F_i^* *independently of* the pricing strategy played by the rival firm. In other words, from Proposition 5(ii) we know that the upper bound of i's equilibrium profit is F_i^*, whereas Proposition 6(i) provides sufficient conditions to make this upper bound a lower bound. The intuition for this result is especially simple to convey for the case of digital goods. Consider firm A and note first that $F_A^* = V^* - V_B^{*S} = V^m(\mathcal{B}_A \cup \mathcal{B}_B) - V^m(\mathcal{B}_B)$, i.e. F_A^* is the marginal value of $\mathcal{B}_A$ for the buyer when she already owns $\mathcal{B}_B$. Weak m-submodularity implies that F_A^* is smaller than the marginal value of $\mathcal{B}_A$ when C already owns any other bundle B_B in $\mathcal{B}_B$. Suppose now that A plays the technology-renting strategy with fee F_A^*. Then A fails to earn F_A^* only if C refuses to rent A's technology, and in such a case C will buy some bundle $\bar{B}_B$ (possibly $\bar{B}_B = \emptyset$) from seller B. However, for any $\bar{B}_B$, it is at least weakly profitable for C to rent A's technology at the price of F_A^*, since the marginal value of $\mathcal{B}_A$ is at least F_A^*. Therefore no strategy played by B can prevent A from earning F_A^*.

Propositions 5(ii) and 6(i) imply that each firm i earns exactly F_i^* in any NE. Then, we can prove that C's payoff is π_C^* in any NE, as in the technology-renting NE. If C earns less than π_C^*, then any seller i can make a profitable deviation by inducing C to buy only $\mathcal{B}_i$, since trading only with seller i generates a surplus of $V_i^{*S} = F_i^* + \pi_C^*$. Finally, since the sum $F_A^* + F_B^* + \pi_C^*$ is equal to V^*, we conclude that any NE is efficient.

In the case of independent products, weak m-submodularity is satisfied, thus from Propositions 5 and 6 we obtain the following corollary.

Corollary 2 *Consider the setting with independent products when bundling is allowed. Then*
(i) (tr_A^*, tr_B^*) is an efficient NE, all equilibria are efficient if $B_B^* = \emptyset$, and in any NE the profit of each firm i is not larger than F_i^*;
(ii) if sales below marginal cost are banned or products are digital goods, then each firm i can earn F_i^* independently of the strategy of firm j, and all NEs are outcome-equivalent.

5.3.2 Unilateral improvement

Define $U^m(B)$ as $\max_{\hat{B}\subseteq B} U(\hat{B})$ subject to $\#(\hat{B}) \leq k$. The second class of environments we consider is characterized by a condition which we call *unilateral improvement* and is described as follows:[31]

$$\begin{gathered}\text{for every set } B \text{ that is inefficient, there exists } B' \text{ such that} \\ \#B' \leq k,\ B_i' \subseteq B_i \text{ for some firm } i \text{ and } U(B') - C(B') > U^m(B) - C(B).\end{gathered} \tag{9}$$

The interpretation of the condition is that starting from any inefficient bundle $B(= B_i \cup B_j)$, there exists at least a seller j who can propose B_j' such that a higher social welfare is achieved if C combines B_j' with a suitable subset of B_i. In this sense, there is a way for seller j to unilaterally improve social welfare upon set B; this eventually allows him to achieve a higher profit. The condition is satisfied by independent products.

In order to facilitate the understanding of condition (9), we below provide two examples.

Example 3 *Consider a setting with digital goods, $\mathcal{B}_A = \{b_A^1\}$, $\mathcal{B}_B = \{b_B^1, b_B^2\}$, $k = 2$*[32]

[31] We thank an anonymous referee for suggesting this condition.

[32] Since $k = 2$, the value of $U(b_A^1, b_B^1, b_B^2)$ does not matter. A similar remark applies also to Example 4.

and

$$\begin{aligned} U(b_A^1) &= U(b_B^1) = 6, \quad U(b_B^2) = 5, \\ U(b_A^1, b_B^1) &= 10, \quad U(b_A^1, b_B^2) = U(b_B^1, b_B^2) = 7. \end{aligned}$$

In this environment $B^* = \{b_A^1, b_B^1\}$. In order to see that (9) is satisfied, consider for instance $B = \{b_B^1, b_B^2\}$. Then $B' = \{b_A^1, b_B^1\}$ satisfies (9) and in particular $B'_B = \{b_B^1\}$ is a subset of $B_B = \{b_B^1, b_B^2\}$; an analogous argument applies to $B = \{b_A^1, b_B^2\}$. When $B = \{b_A^1\}$ or $B = \{b_B^1\}$, we can pick again $B' = \{b_A^1, b_B^1\}$; and when $B = \{b_B^2\}$ we can pick $B' = \{b_A^1, b_B^2\}$.

Example 4 *Consider a setting with digital goods, $\mathcal{B}_A = \{b_A^1, b_A^2\}$, $\mathcal{B}_B = \{b_B^1, b_B^2\}$, $k = 2$ and*

$$U(b_A^1) = U(b_B^1) = 5, \quad U(b_A^2) = U(b_B^2) = 6,$$
$$U(b_A^1, b_B^1) = 10, \quad U(b_A^1, b_A^2) = U(b_A^1, b_B^2) = U(b_B^1, b_A^2) = U(b_B^1, b_B^2) = 8, \quad U(b_A^2, b_B^2) = 12.$$

In order to see that in this environment (9) is violated, consider $B = \{b_A^1, b_B^1\}$. In order to satisfy $U(B') - C(B') > U^m(B) - C(B)$, B' needs to include $\{b_A^2, b_B^2\}$, but then B'_A includes b_A^2 and thus is not a subset of $B_A = \{b_A^1\}$. Likewise, B'_B includes b_B^2 and thus is not a subset of $B_B = \{b_B^1\}$. In words, increasing social welfare from set B requires new products from both firms and therefore (9) fails to hold since no seller can unilaterally improve upon set B.

We now state an efficiency result which relies on (9).

Proposition 7 *[efficiency under unilateral improvement] Suppose that bundling is allowed and that unilateral improvement condition (9) holds. Consider either the setting of digital goods, or assume that sales below–marginal cost are banned (i.e., (8) must be satisfied). Then all equilibria are efficient.*

In order to provide an intuition for this result, suppose that initially the firms play strategies such that C buys set B with $B \neq B^*$ and let B' satisfy (9) with $B'_B \subseteq B_B$, for instance. Then consider now that A deviates with a technology-renting strategy specifying $F_A = \pi_A + \varepsilon$, where π_A is A's profit before the deviation and $\varepsilon > 0$ is close to zero. If C purchases only products from seller B, then her payoff cannot be higher than before A's deviation, whereas if she buys B'_A, B'_B, we can prove that her payoff is higher than before the deviation. This implies that $B'_A \neq \emptyset$ and that C buys some product(s) of A, even though she may not buy the bundle B'_A, B'_B. Hence, A's deviation is profitable.

Precisely, if C buys B'_A and B'_B, then (i) social welfare increases by $V(B')-V(B) > 0$; (ii) the profit of A increases by ε; (iii) the profit of seller B does not increase, since (8) implies that $P_B(B'_B) - C_B(B'_B) \leq P_B(B_B) - C_B(B_B)$. As a consequence, the payoff of C increases at least by $V(B') - V(B) - \varepsilon > 0$, where ε could be any number smaller than $V(B') - V(B)$. The key of this argument is that cancelling the products in $B_B \backslash B'_B$ allows C to save at least the marginal cost of these products and therefore the increase in social welfare is split only between A and C.

We note that condition (9) delivers a weaker result than condition (6) does since Proposition 6 determines how the social surplus V^* is split among the firms and C, but Proposition 7 does not. Nevertheless, there are no relationships between these conditions in the sense that (9) does not imply (6),[33] and that (6) does not imply (9) either.[34]

Propositions 6(ii) and 7 generate the following policy implications when a mild condition (either weak m-submodularity or unilateral improvement) is satisfied. First, in the case of digital goods, when bundling (in the sense of a menu of bundles) is allowed, the efficient allocation is always achieved. Furthermore, pure bundling is equivalent to technology-renting, and competition between pure bundles leads to the efficient outcome, implying that pure bundling of digital goods is socially desirable. Second, in the case of non-digital goods, on the contrary, pure bundling of non-digital goods can generate inefficient equilibria; for this reason, banning sales below marginal cost is socially desirable since this makes all equilibria efficient when bundling (i.e. a menu of bundles) is allowed. Such a ban prevents the use of pure bundling.

Propositions 5(ii) and 6(i) generate a policy implication on foreclosure even though we

[33] We know that (9) is satisfied in Example 3, but (6) fails to hold since $V^m(\mathcal{B}_B \cup \mathcal{B}_A) - V^m(\mathcal{B}_B) = 10 - 7 = 3 > V^m(b_B^2 \cup \mathcal{B}_A) - V^m(b_B^2) = 7 - 5 = 2$. Furthermore, $F_A^* = 3$ but the strategies $p_A(b_A^1) = 2.8$ and $p_B(b_B^1) = 4$, $p_B(b_B^2) = 1.8$, $p_B(b_B^1, b_B^2) = 5.8$ constitute a NE in which C buys $B^* = \{b_A^1, b_B^1\}$ and firm A's profit is $2.8 < F_A^*$.

[34] We know that (9) is violated in Example 4, but it is simple to see that (6) is satisfied using $V^m(b_A^1, b_B^1, b_A^2) = V^m(b_A^1, b_B^1, b_B^2) = 10$, $V^m(b_A^1, b_A^2, b_B^2) = V^m(b_B^1, b_A^2, b_B^2) = V^m(b_A^1, b_B^1, b_A^2, b_B^2) = 12$.

have not formally investigated this dynamic issue. According to Proposition 6(i), when bundling is allowed, in the case of digital goods or when sales below marginal cost are banned, each firm can obtain at least a profit equal to the marginal social contribution of his portfolio *independently of* the strategy of the rival firm. Furthermore, according to Proposition 5(ii), this marginal contribution is the upper bound of any equilibrium profit under bundling. The results suggest that any incumbent's attempt to reduce a rival firm's profit in order to induce the latter's exit will fail. Therefore, pure bundling of digital goods cannot be an instrument of foreclosure. In contrast, in the case of non-digital goods, prohibiting sales below marginal cost is socially desirable in order to prevent foreclosure.[35]

5.4 Comparison

In this section, we compare the case of independent pricing with the case of bundling in terms of social welfare, sellers' profits, and the buyer's payoff. We focus on the setting of digital goods with independent values and $k = 2$ described in Section 4.2. In the case of independent pricing, we consider the pure-strategy NE when it exists (that is, when $y \geq 2z - t$) and the mixed-strategy NE described in Proposition 2 when no pure-strategy NE exists (that is, when $y < 2z - t$). In the case of bundling, Proposition 6 implies that there is a unique equilibrium outcome, and thus we consider the outcome of the technology-renting NE. Then, we have:

Proposition 8 *Consider the setting of digital goods with independent values and $k = 2$ described in Section 4.2.*
(i) Social welfare is weakly smaller under independent pricing than under bundling, and strictly smaller when no pure-strategy equilibrium exists under independent pricing.
(ii) If $B_B^* \neq \emptyset$, or if $B_B^* = \emptyset$ and $z + (z - t)/3 > y$, then no pure-strategy equilibrium exists under independent pricing and the sellers' total profits are higher under independent pricing than under bundling. Therefore the buyer's payoff is lower under independent pricing than under bundling. Conversely, when a pure-strategy equilibrium exists under

[35] In the practice of competition policy, below cost pricing has been discussed in the context of predation and Areeda and Turner (1975) were the first to propose to use below cost pricing to identify predation.

independent pricing, the sellers' total profits are lower (and the buyer's payoff is higher) under independent pricing than under bundling.

Since bundling allows a firm to eliminate competition among his own products, bundling could be expected to increase a firm's profit compared to independent pricing. This is true if a pure strategy NE exists under independent pricing, as then each product sold faces competition from the best product in $\mathcal{B}\backslash B^*$ (see Proposition 1(ii)). But in the interesting case characterized by $B_B^* \neq \emptyset$ (case 1 in Section 4.2), for instance, such a NE does not exist and, surprisingly, Proposition 8(ii) shows that profits are higher in the mixed-strategy NE under independent pricing than under bundling. The reason is that, as we explained in Section 4.2, both firms are less aggressive in the mixed strategy NE than in the hypothetical pure strategy NE, which increases their aggregate profits above the profits under bundling. Then, the buyer's payoff is necessarily lower under the mixed strategy NE than under bundling since an inefficient allocation of slots arises and thus social welfare is reduced in the mixed strategy NE.[36]

6 Extension: Bundling with slotting contracts

In this section, we study what happens if we allow for slotting contracts. If no seller uses a slotting contract, the buyer has full freedom in allocating the slots among all products she purchased. If seller i offers a bundle B_i with a slotting contract, the contract specifies the minimum number of slots $m_i(B_i)$ that must be occupied by the products in B_i and a price $P_i^s(B_i, m_i(B_i))$ for the bundle. Therefore, when slotting contracts are allowed, firm i offers a menu of bundles with prices $\{P_i(B_i), P_i^s(B_i, m_i(B_i))\}_{B_i \subseteq \mathcal{B}_i}$, where $P_i(B_i)$ is the price of B_i without slotting contract. A slotting contract specifying $m_i(B_i) = k$ is an exclusive dealing contract.

Under independent pricing, slotting contracts are redundant since the buyer will not pay a positive price for a product that will not occupy any slot. Under bundling we find that Lemma 1 and Proposition 5 apply to the case in which slotting contracts are

[36] In fact, when $B_B^* \neq \emptyset$, the sellers face a weak version of prisoner's dilemma in the sense that bundling is a weakly dominant strategy but each seller realizes a weakly smaller profit when bundling is allowed than when bundling is prohibited.

allowed. For instance, even though the other firm uses slotting contracts, a firm can find a best response among technology-renting strategies without specifying any slotting contract. However, slotting contracts can render efficient reallocation of slots difficult and thereby create inefficient equilibria even for digital goods, and even when condition (8) is imposed, as it occurs in the following example.

Example 5 *(slotting contracts and inefficiency) Consider a setting of digital goods with independent values,* $k = 3$, $\mathcal{B}_A = \{b_A^1, b_A^2, b_A^3\}$, $\mathcal{B}_B = \{b_B^1, b_B^2, b_B^3\}$ *such that*

$$\begin{aligned}(u(b_A^1), u(b_A^2), u(b_A^3)) &= (10, 7, 6);\\ (u(b_B^1), u(b_B^2), u(b_B^3)) &= (9, 8, 1).\end{aligned}$$

Here $B^* = \{b_A^1, b_B^1, b_B^2\}$*. However, there exists an inefficient NE in which each firm* i *chooses* $m_i(B_i) = 3$, $P_i^s(B_i, m_i(B_i)) = P_i^s(\mathcal{B}_i, 3)$ *for each* $B_i \subset \mathcal{B}_i$ *and* $P_i(B_i)$ *high enough for each* $B_i \subseteq \mathcal{B}_i$*; this essentially means that each firm offers pure bundling with exclusivity. Then, Bertrand competition between* $\mathcal{B}_A$ *and* $\mathcal{B}_B$ *determines the equilibrium prices* $P_A^s(\mathcal{B}_A, 3) = 5$, $P_B^s(\mathcal{B}_B, 3) = 0$*. The buyer buys* $\mathcal{B}_A$*, and the products of firm A occupy the three slots.*

The result in the example is similar to the result obtained by O'Brien and Shaffer (1997) that an inefficient NE can be supported by exclusive dealing. In the example, firm B with a product in B^* is unable to induce the buyer to replace an inferior product of firm A with B's superior product since A uses a slotting contract.[37] This also implies that Proposition 6(i) does not hold when slotting contracts are allowed. Actually, slotting contracts can be used as a direct instrument of foreclosure since a dominant firm can simply buy all slots and thereby foreclose rival firms if this is in its interest.

[37] We need to note, however, that the NE in Example 5 has a few unappealing features that may make it implausible. For instance, it is Pareto dominated for the sellers by the technology renting equilibrium and seller B plays a weakly dominated strategy. Furthermore, given that seller B does not make any profit, he does not lose anything by proposing a menu that includes renting his technology without slotting contract at a fee $F_B = 1$ (for instance); but this would destroy the equilibrium as A would find it profitable to play a technology-renting strategy with $F_A = 9$. We thank an anonymous referee for this comment.

7 Conclusion

We studied in a general setup how bundling affects competition between sellers selling multiple distinct products to a buyer having a slot constraint. In particular, in the case of digital goods, we obtained a number of clear-cut results. When bundling is prohibited, equilibrium in pure strategies generically fails to exist because of the internal competition among the products belonging to a same seller, and any mixed-strategy equilibrium involves inefficient allocation of slots. When bundling is allowed, each seller has an incentive to practice bundling in order to eliminate the internal competition and, under the mild condition of weak m-submodularity, there is a unique equilibrium outcome that is efficient. Each seller can find a best response among pure bundling strategies and competition among pure bundles leads to the unique equilibrium outcome mentioned above. Furthermore, pure bundling allows each seller to obtain a profit equal to the social marginal contribution of his portfolio independently of the rival's strategy. This suggests that pure bundling of digital goods (and hence block booking of movies) is socially desirable for efficient allocation of slots and that it cannot be used as an instrument of foreclosure.

Even though we focused on the slot constraint, all the principles underlying our results hold independently of such a constraint. As challenging issues for future studies, it would be interesting to explore a dynamic setting in which we make the portfolio of each firm endogenous. It would be also interesting to explicitly model a buyer as a downstream firm and study the interaction between bundling at upstream level and bundling at downstream level, which is very relevant for cable TV.[38]

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Appendix

Throughout this appendix, ε denotes a number which is positive and close to zero.

Proof of Proposition 1

(i) The proof is organized in three steps.

Step 1 In any NE, the buyer buys k products.

Suppose that C buys less than k products and thus leaves some slots empty; this requires that less than k products have positive or zero net surplus. Let b_i denote a product in B^* which C does not buy: then necessarily $p_i(b_i) > u(b_i)$. Hence, a profitable deviation for firm i exists: set $p_i(b_i)$ such that $c_i(b_i) < p_i(b_i) < u(b_i)$ [note that $u(b_i) > c_i(b_i)$ since $b_i \in B^*$]. Now one more product gives a positive net surplus and C buys b_i as well as the products she was buying previously. The profit of firm i increases by $p_i(b_i) - c_i(b_i) > 0$.

Step 2 Let B denote the set of products purchased by the buyer in an arbitrary NE; then $w_i(b_i) = w^*$ for any $b_i \in B$, for some $w^* \geq 0$.

If b_i and b_j are in B and $w_i(b_i) > w_j(b_j)$ (not necessarily $j \neq i$), then for firm i it is profitable to increase $p_i(b_i)$ slightly because then C will still buy the products in B and therefore firm i will make a higher profit. Hence, $w_i(b_i) = w_j(b_j)$ for any b_i and b_j in B and we let w^* denote this common value.

Step 3 In any NE the slots are occupied by the efficient products, that is the buyer buys B^*; furthermore, $w^* = \max\{0, \max_{b_i \in \mathcal{B} \backslash B^*}\{w_i(b_i)\}\}$.

Suppose that C buys a set B which consists of k products (by Step 1) but there is some product b_i in B^* which is not in B. We need to distinguish the case in which there exists a product $\tilde{b}_i \in B_i \backslash B_i^*$ from the case in which $B_i \subset B_i^*$.

In the first case we have $u(\tilde{b}_i) - p_i(\tilde{b}_i) = w^*$ by Step 2. Consider the deviation of i such that $p_i(\tilde{b}_i)$ is set very high and $p_i(b_i)$ is set equal to $u(b_i) - w^* - \varepsilon$. Then C buys from i the set of products $B_i \cup \{b_i\} \backslash \{\tilde{b}_i\}$, that is product $\tilde{b}_i$ is replaced by b_i. As a consequence, the profit of i changes by $u(b_i) - w^* - \varepsilon - c_i(b_i) - [u(\tilde{b}_i) - w^* - c_i(\tilde{b}_i)] = v(b_i) - v(\tilde{b}_i) - \varepsilon$, and this expression is positive since $b_i \in B^*$, $\tilde{b}_i \notin B^*$.

In the second case there exists a product b_j of firm $j \neq i$ in $B_j \backslash B_j^*$. By step 2, $u(b_j) - p_j(b_j) = w^*$ and $p_j(b_j) - c_j(b_j) = u(b_j) - w^* - c_j(b_j)$ is the profit of firm j from the sale of b_j; thus $p_j(b_j) - c_j(b_j) \geq 0$. Then firm i can increase his profit by setting $p_i(b_i) = u(b_i) - w^* - \varepsilon$ and reducing the price of each other product in B_i by ε; in this way C continues to buy all the products in B_i and in addition she buys b_i as well. The profit of firm i varies by

$u(b_i) - w^* - \varepsilon - c_i(b_i) - \#(B_i)\varepsilon = u(b_i) - c_i(b_i) - [u(b_j) - p_j(b_j)] - [\#(B_i)+1]\varepsilon$ and this term is positive since $p_j(b_j) \geq c_j(b_j)$ and $b_i \in B^*$, $b_j \notin B^*$.

In order to see that $w^* = \max\{0, \max_{b_i \in \mathcal{B}\backslash B^*}\{w_i(b_i)\}\}$, note that if w^* is smaller than $\max\{0, \max_{b_i \in \mathcal{B}\backslash B^*}\{w_i(b_i)\}\}$, then C does not buy all the products in B^*. If $w^* > \max\{0, \max_{b_i \in \mathcal{B}\backslash B^*}\{w_i(b_i)\}\}$, then firm i with $B_i^* \neq \emptyset$ can increase his profit by slightly increasing the prices of his products in B_i^* since then C will still purchase them.

(ii) **Step 1** $w^* \geq v(\bar{b}_A) = u(\bar{b}_A) - c_A(\bar{b}_A)$.

If $w^* < v(\bar{b}_A)$, then we exhibit a profitable deviation for A: set $p_A(\bar{b}_A) = c_A(\bar{b}_A) + \varepsilon$, and reduce by $\frac{\varepsilon}{k}$ the price of each of his products in B_A^* – they are at most $k-1$. As a consequence, the net surplus of product $\bar{b}_A$ is $v(\bar{b}_A) - \varepsilon > w^*$ and each product in B_A^* has net surplus equal to $w^* + \frac{\varepsilon}{k}$. Thus C buys $B_A^* \cup \{\bar{b}_A\}$ from A and the profit of A increases by at least $\varepsilon - (k-1)\frac{\varepsilon}{k} > 0$.

Step 2 If (3) is satisfied, then no NE exists.

When (3) holds we have that $v(\bar{b}_A) > w_B(b_B)$ for any $b_B \in (\mathcal{B}_B \backslash B_B^*)$. From Step 1 in the proof of (ii) we infer that $w^* \geq v(\bar{b}_A) > 0$ and Step 3 in the proof of (i) implies that $w^* = \max_{b_i \in \mathcal{B}\backslash B^*}\{w_i(b_i)\} = w_A(\hat{b}_A)$ for some $\hat{b}_A \in (\mathcal{B}_A \backslash B_A^*)$ (possibly with $\hat{b}_A = \bar{b}_A$). What is the most relevant is that $w^* > w_B(b_B)$ for any $b_B \in (\mathcal{B}_B \backslash B_B^*)$. We end the proof by showing that a profitable deviation exists for A. Let A set $p_A(b_A)$ very high for any $b_A \in (\mathcal{B}_A \backslash (B_A^* \cup \{\hat{b}_A\}))$, increase $p_A(b_A)$ by ε for any $b_A \in (B_A^* \cup \{\hat{b}_A\})$ such that $w_A(\hat{b}_A)$ remains larger than $w_B(b_B)$ for any $b_B \in (\mathcal{B}_B \backslash B_B^*)$. Then C still buys all products in B^* and A's profit is increased because $p_A(b_A)$ is increased for any $b_A \in B_A^*$.

(iii) When $B_B^* = \emptyset$, we know from (i) that in any NE, C buys B_A^* and nothing else. Hence $\pi_B = 0$, $w_A(b_A) = w^*$ for any $b_A \in B_A^*$ and $w^* \geq v(b_B^1)$, since if $w^* < v(b_B^1)$ then setting $p_B(b_B^1) = c_B(b_B^1) + \varepsilon$ is profitable deviation for seller B since it induces C to buy b_B^1 at a price above $c_B(b_B^1)$. Regarding A's deviations, the most favorable case for the existence of NE is when w^* is small as this maximizes the profit of A, $\pi_A = \sum_{b_A \in B_A^*}[v(b_A) - w^*]$. Thus we consider $w^* = v(b_B^1)$ and $\pi_A = \sum_{b_A \in B_A^*}[v(b_A) - v(b_B^1)]$. The only potentially profitable deviations for A are such that he sells fewer than k products to C. The lower are the prices $p_B(b_B^2), ..., p_B(b_B^{n_B})$, the lower is A's profit from such deviations. However, these prices must satisfy the inequalities $w_B(b_B) \leq w_B(b_B^1) = v(b_B^1)$ for $b_B = b_B^2, ..., b_B^{n_B}$; for this reason we need $\lambda(b_B) = \min\{u(b_B), v(b_B^1)\}$ for $b_B =$

$b_B^2, ..., b_B^{n_B}$. Then if A wants to sell only the products in $\{b_A^1, b_A^2, ..., b_A^j\}$ instead of B_A^*, for $j = 1, ..., k-1$, he needs to leave C a net surplus of $\lambda(b_B^{k-j+1})$ for each product in $\{b_A^1, b_A^2, ..., b_A^j\}$, earning $\sum_{b_A \in \{b_A^1, b_A^2, ..., b_A^j\}} [v(b_A) - \lambda(b_B^{k-j+1})]$. Comparing this profit with π_A yields (4).

Proof of Proposition 2

(i) We need to verify that the mixed strategies described in Proposition 2(i) constitute a NE. We start with firm A, and the following lemma is a useful step.

Lemma 2 *Given an arbitrary (pure or mixed) strategy played by firm B, if A plays a pure strategy (p_x, p_y) such that $x - p_x < y - p_y$, then there exists (p_x', p_y') such that $x - p_x' > y - p_y'$ and A's profit with (p_x', p_y') is at least as much as with (p_x, p_y).*

Proof. Given (p_x, p_y) such that $x - p_x < y - p_y$, let δ denote the probability that C buys both products x and y and let η denote the probability that C buys only y among the products of A. The expected profit of A is then $\delta(p_x + p_y) + \eta p_y$. Now consider (p_x', p_y') such that $x - p_x' = y - p_y$ and $y - p_y' = x - p_x$, so that $x - p_x' > y - p_y'$. Then the expected profit of A is $\delta(p_x' + p_y') + \eta p_x' = \delta(p_x + p_y) + \eta(x - y + p_y)$ and this is weakly higher than the expected profit of A with (p_x, p_y). ■

Now we verify that A's strategy in Proposition 2(i) is a best reply to B's strategy. As it is well known, it suffices to consider pure strategies of A, and Lemma 2 allows to restrict to pure strategies satisfying $x - p_x \geq y - p_y$. Consider first $p_y \in [\frac{1}{2}(y-t), y-t]$. Then $x - p_x \geq y - p_y \geq t$ holds and thus C certainly purchases product x; hence it is suboptimal for A to play $p_x < x - y + p_y$. Given $p_x = x - y + p_y$, we find

$$\begin{aligned} \pi_A &= x - y + p_y + p_y \Pr\{y - p_y > \beta\} \\ &= x - y + p_y + p_y G_B(y - p_y) = x - t \end{aligned} \tag{10}$$

for any $p_y \in [\frac{1}{2}(y-t), y-t]$.[39] Playing $p_y < \frac{1}{2}(y-t)$ is not a profitable deviation for A since then $y - p_y > \frac{1}{2}(y+t) \geq \beta$ with probability one and $x - p_x \geq y - p_y$ implies that C buys both x and y; therefore the profit of A is smaller than $x - \frac{1}{2}(y+t) + \frac{1}{2}(y-t) = x - t$.

[39] The strict inequality in (10) is correct in case 1, while in case 2 we should write $\Pr\{y - p_y \geq \beta\}$. But in fact there are no practical differences between the two cases, as G_B has no atoms.

Playing $p_y > y - t$ is not a profitable deviation since then $y - p_y < t$ – thus C certainly does not buy y – and in order to induce C to buy product x, A cannot set p_x larger than $x - t$ given that $z - p_z \geq t$ with probability one and $p_t = 0$.

Now we verify that B's strategy in Proposition 2(i) is a best reply to A's strategy. Since $x - p_x = y - p_y \geq t$, C does not buy product t and B only needs to choose p_z. When he plays $p_z \in [z - \frac{1}{2}y - \frac{1}{2}t, z - t)$, his payoff is

$$\pi_B = p_z \Pr\{z - p_z \geq \alpha\} = p_z G_A(z - p_z) = z - \frac{1}{2}t - \frac{1}{2}y \tag{11}$$

for any $p_z \in [z - \frac{1}{2}y - \frac{1}{2}t, z - t)$.[40] Playing $p_z = z - t$ is not a profitable deviation since then C does not buy product z in case 2 and buys it with probability $G_A(t) = \frac{2z-t-y}{2z-2t}$ in case 1 (then, the profit of B is $\frac{2z-y-t}{2z-2t}(z-t) = z - \frac{1}{2}(y+t)$). Playing $p_z > z - t$ is not a profitable deviation as it implies $z - p_z < t$ and certainly C does not buy product z. Finally, playing $p_z < z - \frac{1}{2}t - \frac{1}{y}y$ is not a profitable deviation since C buys product z but B's profit is smaller than $z - \frac{1}{2}y - \frac{1}{2}t$.

(ii) The payoffs of the two firms are obtained in (10) and (11). Let a denote the probability that C buys y given the mixed strategies; then

$$\begin{aligned} a &= \int_t^{\frac{1}{2}(y+t)} G_B(\alpha)G_A'(\alpha)d\alpha = \int_t^{\frac{1}{2}(y+t)} \frac{\alpha - t}{y - \alpha}\frac{2z - t - y}{2(z-\alpha)^2}d\alpha \\ &= \frac{y-t}{2(y-z)^2}[y - z + (2z - t - y)\ln\frac{2z-t-y}{z-t}]. \end{aligned}$$

For the example in Section 2, with $(x, y, z, t) = (4, 3, 2, 0)$, we find $a = \frac{3}{2}(1 + \ln\frac{1}{2}) \simeq 0.46$. Suppose that $y \leq z + (z-t)/3$. The inequality $G_A(\alpha) > G_B(\alpha)$ is equivalent to $h(\alpha) \equiv 2\alpha^2 - (4z - y + t)\alpha + 2zy - ty + 2tz - y^2 > 0$, and $y \leq z + (z-t)/3$ implies that h is strictly decreasing with respect to $\alpha \in [t, \frac{1}{2}(y+t)]$, with $h(\frac{1}{2}(y+t)) = 0$.

Proof of Proposition 3

Step 1 If $B_B^* = \varnothing$ and (4) is violated, then no pure-strategy NE exists and no efficient mixed-strategy NE exists.

[40]The weak inequality in (11) is correct in case 1, while in case 2 we should write $\Pr\{z - p_z > \alpha\}$. But in fact there are no practical differences between the two cases, as $z - p_z$ belongs to $(t, \frac{1}{2}(y+t)]$ from $p_z \in [z - \frac{1}{2}(y+t), z - t)$ and G_A has no atoms in $(t, \frac{1}{2}(y+t)]$.

When $B_B^* = \varnothing$ and (4) is violated, we know from Proposition 1(iii) that no pure-strategy NE exists. We now show that no efficient mixed-strategy NE exists.

Since $B_B^* = \varnothing$, efficiency requires that C buy all the products in B_A^* with probability one, and nothing else; this implies $\pi_B = 0$. Therefore $\Pr\{w_A(b_A) \geq v_B(b_B^1)\} = 1$ for each $b_A \in B_A^*$, otherwise firm B could play $p_B(b_B^1) = c_B(b_B^1) + \varepsilon$ and induce C to buy b_B^1 with positive probability, which would imply $\pi_B > 0$. From $\Pr\{w_A(b_A) \geq v_B(b_B^1)\} = 1$ for each $b_A \in B_A^*$ we infer that π_A is not larger than $\sum_{b_A \in B_A^*}[v(b_A) - v(b_B^1)]$. Since there exists no pure-strategy NE, we know from Proposition 1(iii) and (4) that there is some $\hat{\jmath}$ between 1 and $k-1$ such that $\sum_{b_A \in B_A^*}[v(b_A) - v(b_B^1)] < \sum_{b_A \in \{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}}[v(b_A) - \lambda(b_B^{k-\hat{\jmath}+1})]$. Note however that if $\lambda(b_B^{k-\hat{\jmath}+1}) = v(b_B^1)$, then selling $\{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}$ is definitely not a profitable deviation for A since the surplus he needs to leave to C for each of the $\hat{\jmath}$ products in $\{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}$ is the same as he needs to leave on each of the k products in B_A^*. Thus, given that selling $\{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}$ is a profitable deviation for A, it is necessary that $\lambda(b_B^{k-\hat{\jmath}+1}) = u(b_B^{k-\hat{\jmath}+1}) < v(b_B^1)$. Hence

$$\sum_{b_A \in B_A^*}[v(b_A) - v(b_B^1)] < \sum_{b_A \in \{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}}[v(b_A) - u(b_B^{k-\hat{\jmath}+1})] \tag{12}$$

holds for a certain $\hat{\jmath} < k$. We now exhibit a profitable deviation for firm A: set $p_A(b_A) = u(b_A) - u(b_B^{k-\hat{\jmath}+1})$ for $b_A \in \{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}$, and $p_A(b_A)$ large for $b_A \in \mathcal{B}_A \backslash \{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}$. In this way each product in $\{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}$ leaves C a net surplus equal to $u(b_B^{k-\hat{\jmath}+1})$, and at most $k - \hat{\jmath}$ products of firm B leave C a higher net surplus; therefore C buys all the products in $\{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}$. Firm A earns then $u(b_A) - u(b_B^{k-\hat{\jmath}+1}) - c_A(b_A) = v(b_A) - u(b_B^{k-\hat{\jmath}+1})$ on each $b_A \in \{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}$, and his total profit is $\sum_{b_A \in \{b_A^1, b_A^2, ..., b_A^{\hat{\jmath}}\}}[v(b_A) - u(b_B^{k-\hat{\jmath}+1})]$, which is larger than his (supposed) equilibrium profit π_A, given (12).

Step 2 If $B_B^* \neq \varnothing$ and (3) is satisfied, then no pure-strategy NE exists and no efficient mixed-strategy NE exists.

When $B_B^* \neq \varnothing$ and (3) holds, we know from Proposition 1(ii) that no pure-strategy NE exists. We now show that no efficient mixed-strategy NE exists.

We assume that an efficient mixed-strategy NE exists and we derive a contradiction. Efficiency requires that C buy all products in B_A^* and B_B^* with probability one. Therefore it is necessary that

$$\begin{array}{lcl} \Pr\{w_A(b_A) \geq \max\{w_A(\tilde{b}_A), w_B(\tilde{b}_B)\}\} = 1 & \text{for each} & b_A \in B_A^*, \quad b_B \in B_B^* \quad \text{and} \\ \Pr\{w_B(b_B) \geq \max\{w_A(\tilde{b}_A), w_B(\tilde{b}_B)\}\} = 1 & & \tilde{b}_A \in \mathcal{B}_A \backslash B_A^*, \quad \tilde{b}_B \in \mathcal{B}_B \backslash B_B^* \end{array} \tag{13}$$

If we define $w_A^M \equiv \inf\{w : \Pr\{w_A(\tilde{b}_A) \leq w \text{ for each } \tilde{b}_A \in \mathcal{B}_A \backslash B_A^*\} = 1\}$ and $w_B^M \equiv \inf\{w : \Pr\{w_B(\tilde{b}_B) \leq w \text{ for each } \tilde{b}_B \in \mathcal{B}_B \backslash B_B^*\} = 1\}$, we can write (13) as

$$\begin{array}{lcl} \Pr\{w_A(b_A) \geq \max\{w_A^M, w_B^M\}\} = 1 & \text{for each} & b_A \in B_A^*,\, b_B \in B_B^*. \\ \Pr\{w_B(b_B) \geq \max\{w_A^M, w_B^M\}\} = 1 & & \end{array}$$

This suggests that unless $w_A(b_A) = \max\{w_A^M, w_B^M\}$ with probability one for each $b_A \in B_A^*$, a profitable deviation exists for firm A which consists in lowering $w_A(b_A)$. The same argument suggests that $w_B(b_B)$ needs to be equal to $\max\{w_A^M, w_B^M\}$ with probability one for each $b_B \in B_B^*$. Given that $w_A(b_A) = w_B(b_B) = \max\{w_A^M, w_B^M\}$ for each $b_A \in B_A^*$, $b_B \in B_B^*$, it follows that $w_A^M = w_B^M \equiv w^M$. Precisely, if $w_A^M > w_B^M$ then A can profitably reduce both w_A^M and $w_A(b_A)$ for each $b_A \in B_A^*$; if $w_A^M < w_B^M$, then a symmetric argument applies.

Finally, since $\bar{b}_A \in \mathcal{B}_A \backslash B_A^*$ is the product in $\mathcal{B} \backslash B^*$ with the highest net value, we exhibit a profitable deviation for A: set $w_A(\bar{b}_A) = w^M + \varepsilon$ and $w_A(b_A) = w^M + \varepsilon$ for each $b_A \in B_A^*$ with probability one. Then C buys all these products of A and A's profit changes by $\Delta\pi_A > -\varepsilon k + p(\bar{b}_A) - c_A(\bar{b}_A) = v(\bar{b}_A) - w^M - \varepsilon(k+1)$. We find that $\Delta\pi_A > 0$ since $w^M = w_B^M \leq u(b_B)$ for some $b_B \in (\mathcal{B}_B \backslash B_B^*)$, and from (3) we know that $v(\bar{b}_A) > u(b_B)$ for any $b_B \in (\mathcal{B}_B \backslash B_B^*)$.

Proof of Lemma 1

Consider any arbitrary profile of strategies (s_i, s_j) and let π_i be the profit of firm i given (s_i, s_j). We show that i can achieve the same profit π_i by playing the technology-renting strategy such that $F_i = \pi_i$. This fact is obvious if $\pi_i = 0$ and therefore we consider the case of $\pi_i > 0$. In order to prove this result, it suffices to show that the buyer buys at least one product from i when i plays tr_i. We find that, (i) given (tr_i, s_j), the buyer can make the same payoff that she makes with (s_i, s_j) since she can buy the same products, with the same outlay; (ii) given (tr_i, s_j), the buyer cannot realize a higher payoff than with (s_i, s_j) without buying at least one product of firm i, because otherwise she would not buy anything from i given (s_i, s_j), and this contradicts $\pi_i > 0$.

Proof of Proposition 5

(i) First we note that if the firms play (tr_A^*, tr_B^*), the buyer rents both A's technology and B's technology. Precisely, the buyer's payoff is (i) $V^* - F_A^* - F_B^* = V_A^{*S} + V_B^{*S} - V^* \geq 0$ if she rents both technologies; (ii) $V_A^{*S} - F_A^* = V_A^{*S} + V_B^{*S} - V^*$ if she rents only A's technology; (iii) $V_B^{*S} - F_B^* = V_A^{*S} + V_B^{*S} - V^*$ if she rents only B's technology. Since she maximizes the social surplus when she is indifferent among two or more alternatives, she rents both technologies and thus buys the products in B^*.

Now we prove that when firm B plays tr_B^*, firm A cannot make a profit larger than F_A^* (the same argument applies to firm B). From Lemma 1, it is enough to consider firm A's deviations in the set of technology-renting strategies. Obviously, firm A has no incentive to decrease F_A below F_A^*. If instead A chooses $F_A = F_A^* + \varepsilon$ with $\varepsilon > 0$, then the buyer rents only B's technology since she earns a payoff $V_A^{*S} + V_B^{*S} - V^* - \varepsilon$ by renting both technologies or by renting only A's technology while she earns $V_A^{*S} + V_B^{*S} - V^*$ by renting only B's technology.

(ii) The proof is by contradiction. Suppose there exists a NE such that A makes a profit higher than $F_A^* = V^* - V_B^{*S}$. Precisely, suppose that in the NE, the players' payoffs are $\pi_A (> F_A^*)$, π_B and π_C. Given that $\pi_A + \pi_B + \pi_C \leq V^*$ necessarily holds, we infer that $\pi_A > F_A^*$ implies $\pi_B + \pi_C < V_B^{*S}$. Then, let firm B deviate by using tr_B such that $F_B = \pi_B + \varepsilon$ with $\varepsilon > 0$ and such that $\pi_C < V_B^{*S} - \pi_B - \varepsilon$. This deviation of B is profitable if and only if the buyer buys at least one product from firm B. In order to prove that this is the case, we note first that if the buyer does not buy anything from B, then she buys only bundles offered by A and they cannot yield C a payoff larger than π_C; otherwise we obtain a contradiction with the fact that the initial strategies generating payoffs (π_A, π_B, π_C) constitute a NE. We end the proof by observing that if the buyer pays F_B and trades only with firm B, a social surplus of V_B^{*S} is generated and the buyer obtains $V_B^{*S} - \pi_B - \varepsilon$, which is larger than π_C.

(iii) Suppose that C purchases an inefficient set of products. Then an inefficiently low social surplus $\bar{V} (< V^*)$ is generated, with payoffs $\pi_A \geq 0, \pi_B \geq 0, \pi_C \geq 0$ such that $\pi_A + \pi_B + \pi_C = \bar{V}$. Now suppose that A plays a technology-renting strategy with $F_A = \pi_A + \varepsilon$. By renting A's technology and buying $B^*(= B_A^*)$, C earns $V^* - \pi_A - \varepsilon$ and $V^* - \pi_A - \varepsilon > \pi_C = \bar{V} - \pi_A - \pi_B$.

Proof of Proposition 6

(i) Without loss of generality, we prove the result for firm A. It is useful to start by defining $U^m(B)$ as $\max_{B' \subseteq B} U(B')$ subject to $\#(B') \leq k$.
Let firm A play the technology-renting strategy with fee $F_A^* = V^m(\mathcal{B}_A \cup \mathcal{B}_B) - V^m(\mathcal{B}_B)$. We prove that, for any strategy of firm B, the buyer rents A's technology, and thus A earns F_A^*. First notice that if C does not rent A's technology, then she buys a bundle $\bar{B}_B$ which solves $\max\left[U^m(B_B) - P_B(B_B)\right]$ with respect to $B_B \subseteq \mathcal{B}_B$. We show that by renting A's technology, the buyer achieves a payoff higher than $U^m(\bar{B}_B) - P_B(\bar{B}_B)$.

Step 1 The case of digital goods.
The payoff of C from buying $\mathcal{B}_A \cup \bar{B}_B$ is $U^m(\mathcal{B}_A \cup \bar{B}_B) - F_A^* - P_B(\bar{B}_B)$. Since costs are zero, $V^m(B) = U^m(B)$ for every $B \subseteq \mathcal{B}$; thus $F_A^* = U^m(\mathcal{B}_A \cup \mathcal{B}_B) - U^m(\mathcal{B}_B)$ and (6) holds for U^m. Hence $F_A^* \leq U^m(\mathcal{B}_A \cup \bar{B}_B) - U^m(\bar{B}_B)$ since $\bar{B}_B \subseteq \mathcal{B}_B$, and thus $U^m(\mathcal{B}_A \cup \bar{B}_B) - F_A^* - P_B(\bar{B}_B) \geq U^m(\bar{B}_B) - P_B(\bar{B}_B)$. Thus the buyer obtains a weakly higher payoff by renting A's technology than by not renting it; in case of equality, the tie-breaking rule favors renting A's technology because the social surplus is higher.

Step 2 The case in which (8) needs to be satisfied.
Consider $V^m(\mathcal{B}_A \cup \bar{B}_B)$ and let $\hat{B}_A \subseteq \mathcal{B}_A$ and $\hat{B}_B \subseteq \bar{B}_B$ be such that $\#(\hat{B}_A \cup \hat{B}_B) \leq k$ and $V(\hat{B}_A \cup \hat{B}_B) = V^m(\mathcal{B}_A \cup \bar{B}_B)$. The payoff of the buyer if she buys $\hat{B}_A \cup \hat{B}_B$ is

$$U(\hat{B}_A \cup \hat{B}_B) - C_A(\hat{B}_A) - F_A^* - P_B(\hat{B}_B) + P_B(\bar{B}_B) - P_B(\bar{B}_B) \tag{14}$$

From (6), we obtain $F_A^* \leq V^m(\mathcal{B}_A \cup \bar{B}_B) - V^m(\bar{B}_B)$ (since $\bar{B}_B \subseteq \mathcal{B}_B$) and from (8), we obtain $P_B(\bar{B}_B) - P_B(\hat{B}_B) \geq C_B(\bar{B}_B) - C_B(\hat{B}_B)$. Therefore (14) is at least as large as $U(\hat{B}_A \cup \hat{B}_B) - C_A(\hat{B}_A) - V^m(\mathcal{B}_A \cup \bar{B}_B) + V^m(\bar{B}_B) + C_B(\bar{B}_B) - C_B(\hat{B}_B) - P_B(\bar{B}_B)$, which is equal to $V^m(\bar{B}_B) + C_B(\bar{B}_B) - P_B(\bar{B}_B)$. From the definitions of V^m and U^m it follows that $V^m(\bar{B}_B) + C_B(\bar{B}_B) \geq U^m(\bar{B}_B)$, and in case of equality, the tie-breaking rule implies that the buyer rents A's technology.

(ii) **Step 1** In any NE the payoff of the buyer is equal to π_C^*, the buyer's payoff in the technology-renting NE.

Given an arbitrary NE, let π_C denote the buyer's payoff in that NE. Before showing that $\pi_C = \pi_C^*$, we prove that $\pi_C = \max_{B_B \subseteq \mathcal{B}_B} [U^m(B_B) - P_B(B_B)] \equiv \pi_C^B$ and $\pi_C = \max_{B_A \subseteq \mathcal{B}_A} [U^m(B_A) - P_A(B_A)] \equiv \pi_C^A$. Consider an arbitrary NE and denote with $B_A \cup B_B$ the bundle purchased by C in this NE, and with $\hat{B}_A \cup \hat{B}_B \subseteq B_A \cup B_B$ the bundle of objects she uses. We prove that $\pi_C = \pi_C^B$, as $\pi_C = \pi_C^A$ is proved analogously. First notice that the inequality $\pi_C < \pi_C^B$ cannot hold (if $\pi_C < \pi_C^B$, then π_C is not the equilibrium payoff of C). Thus we show that if $\pi_C > \pi_C^B$, then there exists a profitable deviation for A which consists of playing the technology-renting strategy such that $F_A = F_A^* + \varepsilon$ (recall that A's equilibrium profit is F_A^*). If C refuses to rent A's technology, then her payoff is π_C^B; if instead she buys $B_A \cup B_B$ and uses $\hat{B}_A \cup \hat{B}_B$, just like before the deviation of A, then her payoff is $\pi_C - \varepsilon$, the same as before minus ε. Since $\pi_C > \pi_C^B$, it follows that $\pi_C - \varepsilon > \pi_C^B$. Now we show that $\pi_C = \pi_C^*$. First notice that the inequality $\pi_C > \pi_C^*$ cannot hold since then the sum of payoffs is $F_A^* + F_B^* + \pi_C > V^*$, which is impossible. Thus we prove that in case $\pi_C < \pi_C^*$, there exists a profitable deviation for A which consists of playing the technology-renting strategy with $F_A = F_A^* + \varepsilon$. If C does not rent A's technology, then her payoff is $\pi_C = \pi_C^B$ as we have seen above. But C can make a higher payoff by renting A's technology and buying and using only B_A^{*S}: in this way her payoff is $V_A^{*S} - F_A^* - \varepsilon = \pi_C^* - \varepsilon > \pi_C$.

Step 2 Each NE is efficient.

From Propositions 5(ii) and 6(i), and from Step 1, we know that in any NE the profits of the firms are F_A^*, F_B^* and the payoff of the buyer is π_C^*. Since $F_A^* + F_B^* + \pi_C^* = V^*$, it follows that any NE generates a social surplus equal to V^*, and thus it is efficient.

Proof of Proposition 7

Step 1 The case in which (8) needs to be satisfied.

Suppose that in equilibrium C buys a set B which is inefficient. Then, from (9) it follows that there exists $B' = (B'_A, B'_B)$ such that $\#B' \leq k$, $U(B') - C_A(B'_A) - C_B(B'_B) > U^m(B) - C_A(B_A) - C_B(B_B)$ and $B'_i \subseteq B_i$ for (at least) one firm i. A profitable deviation for firm j is the technology-renting strategy with $F_j = \pi_j + \varepsilon$. Precisely, if after this deviation C buys B'_i and B'_j, then her payoff is $U(B') - P_i(B'_i) - \pi_j - \varepsilon - C_j(B'_j)$, which now we prove to be larger than her payoff before the deviation, $U^m(B) - P_i(B_i) - \pi_j - C_j(B_j)$.

Precisely, we need to show that

$$P_i(B_i) - P_i(B_i') > \varepsilon + U^m(B) - U(B') + C_j(B_j') - C_j(B_j). \tag{15}$$

From (8) we obtain $P_i(B_i) - P_i(B_i') \geq C_i(B_i) - C_i(B_i')$, and thus a sufficient condition for (15) to hold is

$$C_i(B_i) - C_i(B_i') > \varepsilon + U^m(B) - U(B') + C_j(B_j') - C_j(B_j),$$

which is equivalent to $U(B') - C_A(B_A') - C_B(B_B') - \varepsilon > U^m(B) - C_i(B_i) - C_j(B_j)$.

Step 2 The case of digital goods.

For the case of digital goods, consider again the deviation of firm j such that he plays the technology-renting strategy with $F_j = \pi_j + \varepsilon$. After this deviation, let C buy B_i and B_j' and use $B_i' \subseteq B_i$ and B_j'. Then C's payoff is $U(B') - P_i(B_i) - \pi_j - \varepsilon$, which is larger than C's payoff before the deviation, $U^m(B) - P_i(B_i) - \pi_j$, since $U(B') > U^m(B)$.

Proof of Proposition 8

The result is immediate from Propositions 1, 2 and 6.