Information and the equity premium[*]

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Abstract

We determine how better information affects the average equity premium in a standard representative-agent exchange economy. Perfect information obviously eliminates the equity premium, and a particular kind of information about the *level* of future consumption always lowers the average equity premium. Surprisingly, information sometimes raises the average equity premium, no matter what the preferences of the representative agent. Information purely about the *volatility* either of consumption or the marginal utility of consumption raises the equity premium for a wide class of preferences. Moreover, information can raise the average equity premium by an arbitrarily large percentage (while still matching important magnitudes, such as average growth and the risk-free rate). We consider two different economies: a two-period economy with arbitrary preferences for the representative agent; and an infinite horizon economy, in which we restrict both preferences and the endowment distribution.

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1 Introduction

Informative signals move asset prices.[1] But how does *better* information affect asset prices, returns and the equity premium? Personal computers and the internet have made investment information cheaper, through news services, databases and online financial advice.[2] As investors learn more, will the equity premium on average rise or fall? We answer this question using a standard representative-agent exchange economy. Intuitively, information should lower the equity premium, since on average it reduces risk.[3] Indeed, in the limiting case of perfect information, risk–and with it the equity premium–vanishes altogether, so the equity premium falls if information is close to perfect.

The equity premium is the expected return on equity divided by the risk-free rate (minus 1). Following Gollier (2001), we show that information lowers the average equity premium if the expected return on equity and the forward price of equity are *comonotonic*: they move in the same direction as beliefs change (Lemma 1). If they move in the opposite direction, then information raises the average equity premium. The lemma is an important first step, but it does not tell us what conditions on primitives–preferences, endowments and beliefs–determines the effect of information, and it does not give much intuition. Our main task is to provide these conditions and explain their role.

In a representative agent exchange economy, uncertainty is about future aggregate consumption.

[1] See, e.g. Shiller (1981), Mitchell and Mulherin (1994), Fair (2002) and Boyd, Hu and Jagannathan (2005), who relate aggregate stock returns to news about world events, monetary policy and employment announcements.

[2] For a few illustrations of the falling cost of information, see Shapiro and Varian (1999, Chapter 2). Guiso, Haliassos and Jappelli (2003) give evidence suggesting that computer literacy, financial literacy, and transparency of financial institutions–all three are correlated with improved information about the return distribution of stocks–help explain why stock ownership has both increased through time and still varies so widely across developed countries.

[3] The dual version of this conjecture is that information raises the average demand for equity. A recent paper by Van Nieuwerburgh and Veldkamp (2005) confirms this property in a mean-variance model.

We find that information lowers the average equity premium if information only reveals something about the *level* of future consumption (Theorem 1), no matter what preferences of the representative agent. If however information is purely about *volatility* of consumption, then it sometimes raises the average equity premium, no matter what the preferences of the representative agent (Proposition 4); and information about volatility *raises* the average equity premium for a wide class of preferences (Theorems 2 and 3).

To clarify terminology, an *information structure*–in short *information*–is a mapping between the state of the world, which determines future consumption, and a probability distribution of the observed signal. We use *news* to mean a particular signal realization. A published estimate that annual growth next year will be 3% is news. How the forecaster comes up with that number, the research method used, corresponds to an information structure. If a forecaster publishes just a point forecast, then information is about the level of consumption growth. If the forecaster publishes an interval forecast, its width reveals something about the volatility of growth, as would the spread of different point forecasts, or of course a direct forecast of volatility (the size of the business cycle, say). If the forecaster's research method improves, or the forecasts reach more ears more often, then the signal becomes more informative.

Besides explaining how information affects the equity premium, our results might also explain part of the equity premium puzzle. Suppose an economist calculates the equity premium implied by a model and does not use some information that agents use. The difference between the average equity premium with and without information gives the bias from ignoring private information. If the average equity premium rises with information, then the economist will calculate an equity premium that is too low for the model economy. In the seminal work of Mehra and Prescott (1985) and their followers, the only information available to consumers in the model and to the economist is past and present consumption growth. If agents use other information, and information raises the equity premium, then the economist would overestimate how risk averse agents must be to generate the observed equity premium.

The usual view is that the economist is less informed than agents: "The events that make the price of IBM stock change by a dollar, like the events that make the price of tomatoes change by

10 cents, are inherently unobservable to economists or would-be social planners" (Cochrane, 2001, p. 132).[4] Another view is that the economist is better informed than agents–or least uses more information than is reflected in consumption decisions. If consumption is costly to adjust, then it sometimes does not change as news arrives.[5] On this interpretation, the correct equity premium is the one calculated with no information and the economist's calculation is one with additional information: now the economist will calculate an equity premium that is too low if information *lowers* the average equity premium.

Our work is related to the recent literature on the effect of the predictability of asset returns on portfolio management.[6] Asset returns are predictable if their distribution depends upon past observables. This literature focuses on the hedging demand for assets whose returns are correlated with these observables. If predictability reduces the demand for equity, the equilibrium price of equity is reduced and the equity premium is increased by information. In the special case with constant relative risk aversion, the sign of the hedging demand for the risky asset depends upon whether relative risk aversion is smaller or larger than 1, as shown by Gollier (2004) in a general setting.

With their emphasis on the impact of learning on asset prices, the closest papers to ours are Veronesi (2000), Weitzman (2007), and Ai (2007). Weitzman (2007) considers a Bayesian decision maker who learns from past experience about the variance of growth of log consumption and shows

[4] This assertion echoes Hayek's (1945) earlier complaint that the economist cannot observe most of the "knowledge of the particular circumstances of time and place" that guides economic action.

[5] The consumption-inertia hypothesis has been explored by Grossman and Laroque (1991), Lynch (1996) and Gabaix and Laibson (2002). A recent trend of the literature initiated by Carrillo and Mariotti (2000) and Bénabou and Tirole (2002) justifies the hypothesis that consumers disregard information to solve their dynamic inconsistency problem.

[6] See for example Detemple (1986), Kim and Omberg (1996), Brennan (1998), Campbell and Viceira (1999, 2001), and Gollier (2004).

that the equity premium can be arbitrarily large for moderate levels of risk aversion. Veronesi (2000) considers a continuous-time exchange economy in which the representative agent with CES utility learns about the unknown growth of log consumption. He finds that information about expected growth rate of consumption raises the instantaneous equity premium when relative risk aversion is larger than one (and lowers it otherwise). Ai (2007) finds the *opposite* result in an otherwise identical economy with production and Kreps-Porteus preferences: information about expected growth rate of consumption *lowers* the instantaneous equity premium when relative risk aversion is larger than one (and raises it otherwise). These last two papers assume that the observed signals are continuous-time versions of "signal equals the state plus normally-distributed noise." This informational assumption fits our Theorem 1 (information purely about the level of consumption growth) where we show that such information *always* lowers the average equity premium in our discrete-time exchange economy, no matter what the preferences of the representative agent.[7]

Since we restrict preferences and the information structure, our work is related to the literature that extends Blackwell's (1953) information order by restricting the class of decision problems. Lehman (1988) and Athey and Levin (2000) consider "monotone decision problems," ones for which the posterior beliefs generated by an information structure can be ordered so that the decision maker's optimal action is increasing in the signal. Lehman considers posterior beliefs that stand in the monotone likelihood ratio order; Athey and Levin extend Lehman (1988) to other orders, including first- and second-order stochastic dominance. In each case, they find changes in the information structure that make all agents facing a class of monotone decision problems better off. Our work differs from this literature on three counts. First, although we have a representative agent we consider the effect of information on equilibrium in a market, not a decision problem. Second, we determine how information affects the equity premium, not welfare.[8] Third, we only

[7] Veronesi (2000) and Ai (2007) consider signals that are bounded away from perfect information. If information were perfect–agents knew *exactly* the entire path of future consumption–then the equity premium would obviously be zero, exactly as in our model.

[8] Indeed, a representative agent is always indifferent about information in an exchange economy, since that agent always consumes the aggregate endowment. We eschew any welfare comparisons in a representative agent model: Even if it is a good positive model, it can lead to incorrect welfare conclusions about information (Schlee, 2001).

consider Blackwell improvements in information. Despite these differences, all our results are for monotone decision problems: To prove comonotonicity of the price of equity and the expected rate of return we show each is monotone. Since the price of equity increases with the demand for it, we can exploit the large literature on portfolio comparative statics in the standard one-safe, one-risky asset problem.

We begin by writing down a two-period version of the Lucas (1978) exchange economy, the standard model to examine asset pricing.[9] The two-period assumption allows us to consider general preferences for the representative agent. This generality allows us to identify exactly what preference properties are responsible for the results. At the end of the paper we drop the two-period assumption, but to make headway we are forced to specialize the endowment distribution and preferences. But under the restrictions we impose, we are able extend *all* our major results to an infinite horizon model.

2 The equity premium in a two-period model

There are two dates, 0 and 1, and a single consumption good. The representative agent maximizes the expectation of the function $u(c_0)+v(c_1)$ where c_t is consumption at date t.[10] We assume that u and v are three-times continuously differentiable, with positive first and negative second derivatives on $\mathbb{R}_{++}$. The agent is endowed with $z_0 > 0$ units of the good at date 0. There are two assets, one risky, one safe, and the agent is endowed with one unit of the risky asset and zero units of the safe asset. A unit of the risky asset is a claim on the random date-1 endowment $\widetilde{z}$ (assumed positive in each state). The price of the risky asset at date 0 is P_e. The safe asset has a price of one at date 0

[9] It is used by Prescott and Mehra (1985), Barsky (1989), Weil (1989), Franke, Stapleton and Subrahmanyam (1998), Weitzman (2007) and many others.

[10] Additively separability is just to reduce notation. All the two-period results hold for any increasing, concave function $U(c_0, c_1)$: simply replace $v(z)$ by $U(z_0, z)$ in each of the results in Section 4.

and has a gross payoff of R_f at date 1.

In the economy with additional information, the agent at date zero observes a simple random variable $\widetilde{\eta}$ correlated with $\widetilde{z}$. Since $\widetilde{\eta}$ and $\widetilde{z}$ are correlated, the agent's posterior beliefs about the aggregate consumption, and so equilibrium prices, depend on η. After observing η at date 0 the representative agent buys b units of the risk-free asset and buys a share a of the future random endowment. Normalize the price of date-0 consumption to one, and redefine the units of the risk-free asset so that its date-0 price is one. The optimal portfolio solves

$$\max_{(a,b)\in\mathbb{R}^2} u\left(z_0 + (1-a)P_e - b\right) + E\left[v\left(a\widetilde{z} + bR_f\right)|\eta\right], \tag{1}$$

and satisfies the first order conditions

$$\begin{aligned} u'\left(z_0 + (1-a)P_e - b\right)P_e &= E\left[v'\left(a\widetilde{z} + bR_f\right)\widetilde{z}|\eta\right] && (2)\\ u'\left(z_0 + (1-a)P_e - b\right) &= E\left[v'\left(a\tilde{z} + bR_f\right)|\eta\right]R_f. && (3) \end{aligned}$$

After imposing the equilibrium condition $(a,b) = (1,0)$, and using the normalization $u'(z_0) = 1$, equation (2) gives us the equilibrium price of equity $P_e(\eta) = E\left[v'\left(\widetilde{z}\right)\widetilde{z}|\eta\right]$ and equation (3) gives us the risk-free rate

$$R_f(\eta) = \frac{1}{E\left[v'(\widetilde{z}) \mid \eta\right]}. \tag{4}$$

The realized return on equity is $z/P_e(\eta)$, so the expected return on equity is $R_e(\eta) = E\left[\widetilde{z}|\eta\right]/P_e(\eta)$. The equity premium (plus 1) is

$$\phi(\eta) = \frac{R_e(\eta)}{R_f(\eta)} = \frac{E\left[\widetilde{z} \mid \eta\right]\ E\left[v'(\widetilde{z}) \mid \eta\right]}{E\left[\widetilde{z}v'(\widetilde{z}) \mid \eta\right]}. \tag{5}$$

Combine (2) and (3) to get

$$P_e(\eta)R_f(\eta) = \frac{E\left[\widetilde{z}v'\left(\widetilde{z}\right)|\eta\right]}{E\left[v'\left(\tilde{z}\right)|\eta\right]}. \tag{6}$$

The product $P_e(\eta)R_f(\eta)$ is the *forward price of equity*, the price if the payment is made at date 1 instead of date 0.

From an ex ante viewpoint, the average equity premium is $E[\phi(\widetilde{\eta})]$. In an economy with no additional information, the equilibrium risk-free rate is

$$R_f^0 = \frac{1}{E\left[v'(\widetilde{z})\right]}, \tag{7}$$

and the equity premium (plus 1) is

$$\phi^0 = \frac{E\left[\widetilde{z}\right]\ E\left[v'(\widetilde{z})\right]}{E\left[\widetilde{z}v'(\widetilde{z})\right]}. \tag{8}$$

The equity premium is of course nonnegative ($\phi(\eta) \geq 1$, $\phi^0 \geq 1$) if the agent is risk averse, and it increases as the agent becomes more risk averse.

We compare the equity premium in the economy without additional information to the average equity premium in an economy with additional information. As already mentioned, we interpret our results three ways.

- The first and most fundamental is the standard one: the agent gets better information about consumption growth and we want to know how the equity premium changes in response. In the infinite horizon version of the model, it is natural to think that information improves through time, perhaps as a result of technological progress. Our results would then tell us what happens to the average equity premium through time.
- The second interpretation is that information is private to the agent and not used by the economist. In this situation, the economist calculates an equity premium of ϕ^0. The actual value is $\phi(\eta)$ when the agent observes η, so the bias from private information is $\phi^0 - E[\phi(\widetilde{\eta})]$.
- The third interpretation uses the consumption inertia hypothesis: the economist uses public information not reflected in consumption choices. Here the actual and estimated values are reversed from the second interpretation. For these last two intrepretations, we want to know when there is a bias and what its direction is.

In the second and third interpretations the economist has an unbiased view of the agent's beliefs over the date-1 endowment: we assume that the economist knows the agent's prior belief; and the average posterior belief with information always equals the prior. Since the mean growth rate $E[\widetilde{z}]$ is linear in beliefs, there is no bias in the economist's calculation of average growth: by the iterated law of expectations, $E_{\widetilde{\eta}}\left[E_{\widetilde{z}}[\widetilde{z}|\widetilde{\eta}]\right] = E[\widetilde{z}]$. Similarly, since the price of equity is linear in beliefs the bias in asset prices is zero: $E_{\widetilde{\eta}}[P_e(\eta)] = P_e^0$. But since the equity premium is not linear in beliefs, we will *not* in general have $E_{\widetilde{\eta}}[\phi(\widetilde{\eta})] = \phi^0$.

We describe an information structure by the distribution of posterior beliefs. A more common approach specifies a prior belief p for date-1 consumption z, and, for each z in the support of p, a conditional distribution $\pi(\cdot|z)$ over signal realizations, η. From these primitives, the prior distribution of signals is $\lambda(\eta) = \sum_z \pi(\eta|z)p(z)$, and the posterior probability that date-1 consumption is z after observing signal η is $p(z|\eta) = p(z)\pi(\eta|z)/\lambda(\eta)$. We instead specify $p(\cdot|\eta)$ for each η; these conditional beliefs, together with a prior signal distribution $\lambda(\cdot)$, determine the prior belief and the conditional signal densities, π. Of course the two approaches are equivalent, but ours is more convenient here since the important property of information is how signals order the posterior beliefs $p(\cdot|\eta)$.

3 How information affects the equity premium: a general sufficient condition

The most surprising result is that information can raise the average equity premium. To explain informally, note first that information raises the average ex ante equity premium if the equity premium ϕ is a *convex* function of beliefs.[11] More formally, let F_η be the probability distribution of

[11] Indeed one version of Blackwell's Theorem is that experiment A is more informative than B if and only if the expectation of any convex function of beliefs is higher with A than B (Blackwell, 1953, Theorem 1, p. 266). Intuitively, with no information, posterior beliefs must be equal to the prior. With information, posterior beliefs are random, with mean equal to the prior. And the more informative the signal, the more spread out the distribution of posteriors, which raises the expectation of any convex function of beliefs.

$\widetilde{z} \mid \eta$, and let $\varphi(F_\eta) = \phi(\eta)$ be the equity premium as a function of the this probability distribution. Then, information reduces the average equity premium if $E\varphi(F_{\widetilde{\eta}}) \leq \varphi(EF_{\widetilde{\eta}})$, and the inequality holds for every information structure if and only if φ is concave. If φ is not concave, information must sometimes raise the average equity premium, and if it is convex, information always raises it.[12] Thus, the technical question is whether the equity premium is concave (or convex) in beliefs about date-1 consumption.

To illustrate what determines how information affects the equity premium, suppose for a moment that the signals do not affect the price of equity: $P_e(\eta) = E[zv'(z)|\eta]$ is constant in η. We can interpret this supposition either as a restriction on information, or on preferences. (With log utility, for example, the price of equity is *always* equal to 1.) In this case, information lowers the average equity premium if

$$E_{\widetilde{\eta}}\Big[E[z|\widetilde{\eta}] \;\; E[v'(z)|\widetilde{\eta}]\Big] \leq E_{\widetilde{\eta}}\Big[E[z|\widetilde{\eta}]\Big] \;\; E_{\widetilde{\eta}}\Big[E[v'(z)|\widetilde{\eta}]\Big], \tag{9}$$

which just says that the covariance between expected consumption and the expected marginal utility of consumption is negative. Inequality (9) holds if expected date-1 consumption and the risk-free rate ($= 1/E[v'|\eta]$) are comonotonic (as the signal varies). Since the price of equity is constant, the last condition is the same as comonotonicity between the rate of return on equity, $E[z]/E[zv'(z)]$, and the *forward price of equity*, $E[zv'(z)]/E[v'(z)]$. The next result, an application of Lemma 11 in Gollier (2001, page 415), shows that this conclusion still holds if the price of equity depends on the signal. Let D denote a set of cumulative distribution functions for $\widetilde{z}$ with bounded support in $\mathbb{R}_{++}$. We show in the appendix that this Lemma follows as a simple corollary from the familiar Covariance Inequality (Hardy, et. al., Theorem 43).[13]

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[12] The risk-free rate $R_f(F_\eta) = 1/E\left[v'(\widetilde{z}) \mid \eta\right]$ is clearly convex in F_η, so information always raises the average risk-free rate – even for the case in which the risk-free rate in the informed economy does not depend on the realized signals.

[13] Hardy, et. al. attribute it to Tchebychef.

Lemma 1 *Let v be a differentiable function on $\mathbb{R}_{++}$ with $v' > 0$. Define functionals φ, V_1 and V_2 on D by* [14]

$$\begin{aligned}
\varphi(F) &= \frac{\int z dF(z) \int v'(z) dF(z)}{\int z v'(z) dF(z)}, \\
V_1(F) &= \frac{\int z dF(z)}{\int z v'(z) dF(z)} \quad \text{and} \quad V_2(F) = \frac{\int z v'(z) dF(z)}{\int v'(z) dF(z)}.
\end{aligned}$$

Consider any $(F_1, F_2) \in D^2$. The following two conditions are equivalent:

(i) For all $\lambda \in [0,1]$, we have $\lambda\varphi(F_1) + (1-\lambda)\varphi(F_2) \leq \varphi(\lambda F_1 + (1-\lambda)F_2)$.

(ii) $[V_1(F_1) - V_1(F_2)]\,[V_2(F_1) - V_2(F_2)] \geq 0$.

Proof: Appendix.

The functional V_1 is the expected return on equity and V_2 is the forward price of equity. If condition (i) in Lemma 1 holds for all pairs of c.d.f.'s in D, then φ is concave on D. Condition (ii) says that V_1 and V_2 rank the two distributions the same way; if (ii) holds for all pairs of possible realizations of the posterior distribution for $\widetilde{z}$ such that the signal leading to F_2 is higher than the signal leading to F_1, then V_1 and V_2 are comonotonic in the signal η. By Lemma 1, concavity of φ requires V_1 and V_2 to rank *all* distributions the same way.

Notice that the equality $V_2(F) = p$ is the first-order condition for the maximization problem with objective $Ev(a\widetilde{z} + (1-a)p)$, which is the static two-asset portfolio problem. We will exploit the large literature on that problem to determine when $p(\eta) = P_e(\eta)R_f(\eta)$ is monotone in η.

[14] We are abusing notation by taking the domain of φ to be D rather than the set of signal realizations.

4 The CRRA/log-normal case

To illustrate some possibilities, we specialize for now to the familiar case of constant relative risk aversion (CRRA) preferences and log-normal endowment distributions: for every signal realization η, $\ln \widetilde{z} \mid \eta$ is normally distributed with mean $\mu(\eta)$ and variance $\sigma^2(\eta)$; and $v'(z) = z^{-\gamma}$, where $\gamma > 0$ is the constant degree of relative risk aversion. In this case, we have closed form solutions for the relevant expectations:[15]

$$E\left[\widetilde{z} \mid \eta\right] = \exp(\mu + \tfrac{1}{2}\sigma^2) \tag{10}$$

$$E\left[v'(\widetilde{z}) \mid \eta\right] = \exp -\gamma(\mu - \tfrac{1}{2}\gamma\sigma^2) \tag{11}$$

$$E\left[\widetilde{z}v'(\widetilde{z}) \mid \eta\right] = \exp(1-\gamma)(\mu + \tfrac{1}{2}(1-\gamma)\sigma^2). \tag{12}$$

Insert these three calculations into (5) to find that the conditional equity premium is

$$\phi(\eta) = \exp(\gamma\sigma^2(\eta)). \tag{13}$$

The mean equity premium in the informed economy is $E\phi(\widetilde{\eta}) = E\exp(\gamma\widetilde{\sigma}^2)$, where $\widetilde{\sigma}^2 = \sigma^2(\widetilde{\eta})$. The equity premium in the uninformed economy is (here $\widetilde{\mu} = \mu(\widetilde{\eta})$)

$$\phi^0 = \frac{E\left[\exp(\widetilde{\mu} + \frac{1}{2}\widetilde{\sigma}^2)\right] E\left[\exp(-\gamma(\widetilde{\mu} - \frac{1}{2}\gamma\widetilde{\sigma}^2))\right]}{E\left[\exp((1-\gamma)(\widetilde{\mu} + \frac{1}{2}(1-\gamma)\widetilde{\sigma}^2))\right]}. \tag{14}$$

Information lowers the average equity premium if and only if $E\phi(\widetilde{\eta}) \leq \phi^0$. Define $x_i(\eta) = \ln V_i(F_\eta)$, and use equations (10), (11) and (12), to find

$$x_1(\eta) = \gamma\mu(\eta) + \tfrac{1}{2}\gamma(2-\gamma)\sigma^2(\eta)$$

[15] These equalities follow since the moment generating function for the normal distribution is $exp(t\mu + \frac{1}{2}t^2\sigma^2)$. Equivalently, the Arrow-Pratt approximation is exact in the normal-CARA case: when $\widetilde{x}$ is normal we have

$$E\exp k\widetilde{x} = \exp(k(E\widetilde{x} + \tfrac{1}{2}kVar(\widetilde{x}))).$$

$$x_2(\eta) = \mu(\eta) + \tfrac{1}{2}(1 - 2\gamma)\sigma^2(\eta).$$

The following proposition is a direct consequence of Lemma 1.

Proposition 1 *Suppose that* $\ln \widetilde{z} \mid \eta$ *is normally distributed with mean* $\mu(\eta)$ *and standard deviation* $\sigma(\eta)$*, for all* η*, and that that relative risk aversion equals a constant* γ*. Information lowers (raises) the average equity premium if the functions* $x_1(\cdot)$ *and* $x_2(\cdot)$ *are (anti-)comonotonic.*

By Proposition 1, information can either raise or lower the equity premium even in the CRRA/log-normal case, whatever the value of γ. We consider 4 examples. The first one generalizes the case of perfect information ($\sigma(\eta) = 0$ for all η).

Example 1 ***(Learning about the mean of log consumption only,*** $\sigma(\eta)$ ***constant)*** *Suppose that* σ^2 *does not depend on the signal. Clearly,* $x_1(\cdot)$ *and* $x_2(\cdot)$ *are comonotonic in that case. By Proposition 1, information purely about the mean of log consumption lowers the average equity premium. Indeed, since the conditional equity premium* $\phi(\eta)$ *does not depend on the signal, the equity premium falls for each signal realization. A special case is when* $\widetilde{\mu}$ *itself is normally distributed with mean* μ_0 *and variance* σ_0^2*. In that case, the prior distribution of* $\ln \widetilde{z}$ *is normal with mean* μ_0 *and variance* $\sigma^2 + \sigma_0^2$*, so that* $\phi^0 = \exp \gamma(\sigma^2 + \sigma_0^2)$*, which is larger than* $\phi(\eta) = E\phi(\widetilde{\eta}) = \exp(\gamma\sigma^2)$*. Information about the mean of log consumption lowers the variance of log consumption from* $\sigma^2 + \sigma_0^2$ *to* σ^2*. The (instantaneous) equity premium is* $\gamma(\sigma^2 + \sigma_0^2)$ *without information, and only* $\gamma\sigma^2$ *with information. The fall in the equity premium is thus proportional to the quality of information (measured by* $\sigma_0^2/(\sigma^2 + \sigma_0^2)$*). With perfect information (*$\sigma^2 = 0$*) the equity premium goes to zero.*

Example 2 ***(Learning about the variance of log consumption only,*** $\mu(\eta)$ ***constant)****. Suppose now that* μ *does not depend on the signal, as in Weitzman (2007). Then* $x_1(\cdot)$ *and* $x_2(\cdot)$ *are comonotonic precisely when* $2 - \gamma$ *and* $1 - 2\gamma$ *have the same sign, that is, when* γ *lies outside the interval* $[1/2, 2]$*. When* γ *belongs to this interval (log utility for example), the expectation of learning about the variance of log consumption always* ***raises*** *the average equity premium!*[16]

[16] In the log case, it is easy to check that information increases the average equity premium by an amount that is

Example 3 ***(Learning about the variance of consumption only, mean consumption constant)*** *In Example 2 we kept the mean of* $\ln \widetilde{z}$ (μ) *constant across the different possible signals. But since* $E\widetilde{z} = \mu_z = \exp(\mu + \sigma^2(\eta)/2)$*, the signal affects the mean of consumption itself. Now suppose that mean consumption* $E\widetilde{z} = \mu_z$ *does not depend on the signal. In this case,* $\mu(\eta) = \ln \mu_z - \sigma^2(\eta)/2$ *for all* η*. Rewrite* x_1 *and* x_2 *as a function of* (μ_z, σ) *rather than of* (μ, σ) *so that* $x_1(\eta) = \gamma \ln \mu_z + \gamma(1-\gamma)\sigma^2(\eta)/2$ *and* $x_2(\eta) = \ln \mu_z - \gamma\sigma^2(\eta)$ *to conclude that* x_1 *and* x_2 *are comonotonic, and information lowers the average equity premium if and only if* $\gamma > 1$*. When relative risk aversion is less than one, information raises the average equity premium.*

Example 4 ***(Learning about both the mean and variance of log consumption, risk-free rate constant)****. Suppose that* $E\left[v'(\widetilde{z}) \mid \eta\right] = \exp\left(-\gamma(\mu - \gamma\sigma^2/2)\right) = R_f^{-1}$ *does not depend on the signal, equivalently, the signal does not affect the risk-free rate in the economy with information. A constant risk-free rate implies that* $\mu(\eta) = \gamma^{-1}\ln R_f + \gamma\sigma^2(\eta)/2$*, so that the signal affects both the mean and variance of log consumption; in particular they are positively correlated. Rewrite* $x_1(\eta) = \ln R_f + \gamma\sigma^2(\eta)$ *and* $x_2(\eta) = \gamma^{-1}\ln R_f + (1-\gamma)\sigma^2(\eta)/2$ *and to see that* x_1 *and* x_2 *are anti-comonotonic and information* ***raises*** *the average equity premium precisely when* $\boldsymbol{\gamma}$ ***is larger than one****.*

We summarize the examples in Table 1. Information lowers the equity premium when signals affect mean log consumption only; or when they affect volatility of log consumption only and relative risk aversion is less than $\frac{1}{2}$ or greater than 2. Information has the opposite effect on the average equity premium in Examples 3 and 4. Examples 1, 2, and 4 are special cases of results in Section 5, and the last column of Table 1 indicates which one.

Example 4 is the most surprising. Given CRRA, the preference restriction $\gamma > 1$ just says that the agent is moderately risk-averse. Although the risk-free rate certainly does move in response to news, it is likely to move much less than the price of equity or the expected return on equity, and Example 4 is just the limiting case in which the risk-free rate is constant. *Yet these two seemingly*

just equal to $Var(\exp(\widetilde{\sigma}^2/2))$.

value of $\gamma \rightarrow$		1/2		1		2		Special case of (Section 5) $\downarrow$
Ex 1: $\sigma(\eta) = \text{constant}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	Theorem 1
Ex 2: $\mu(\eta) = \text{constant}$	$-$	0	$+$	$+$	$+$	0	$-$	
Ex 3: $\ln \mu_z(\eta) = \text{constant}$	$+$	$+$	$+$	0	$-$	$-$	$-$	Theorem 2
Ex 4: $\ln R_f(\eta) = \text{constant}$	$-$	$-$	$-$	0	$+$	$+$	$+$	Theorem 3

Table 1: **The effect of information on the average equity premium in the CRRA/log-normal case.** The first column gives what variable the realized signal *does not* affect, the last the corresponding result in Section 5. (μ = mean log consumption, σ = standard deviation of log consumption, μ_z = mean consumption, R_f = risk-free rate, and γ = relative risk aversion.)

innocuous assumptions imply that information raises the average equity premium.

To help understand Example 4, recall that information always raises the average equity premium if the equity premium is a convex function of beliefs. With the risk-free rate constant, the equity premium is just $E\left[\widetilde{z}|\eta\right]/P_e(\eta)R_f$. Suppose (wlog) that higher signals η lead to both higher means and variances of log consumption, so $E\left[\widetilde{z}|\eta\right]$ rises with η. If relative risk aversion is greater than 1, there are two competing effects of higher signals on the price of equity. The increased variance of date-1 log consumption tends to increase saving (for precautionary reasons), but the increased mean of date-1 consumption tends to decrease saving (from wealth effects); the first raises and the second lowers the demand for equity. If the risk-free rate is constant ($\mu(\eta) = \gamma^{-1}\ln R_f + \gamma\sigma^2(\eta)/2$) and $\gamma > 1$, then the mean rises "fast enough" as the variance rises, which lowers the demand for equity, hence its price. So a higher signal leads both to higher mean date-1 consumption and lower date-0 price of equity, and the combination ensures that the equity premium rises at an increasing rate as beliefs about the mean of log date-1 consumption rise.[17]

Example 4 suggests a conjecture for more general economies: if the risk-free rate doesn't vary "too much" in response to news and the representative agent is at least moderately risk averse then information raises the equity premium. We return to this conjecture in Section 5.4.

Quantitative Importance: How Much Can Information Raise the Equity Pre-

[17] One can also directly verify that the ratio f/g of two positive linear functions with f increasing and g decreasing is convex. ($E[z]$ and P_e are both linear in beliefs.)

MIUM? Examples 2-4 show that information about volatility can raise the average equity premium. The following calibration argument shows it can be quantitatively important in a model that matches some important historical averages. We consider a realistic calibration of the model and use it to calculate the effect of information on the equity premium in the setting of Example 4. Using annual data from 1889 to 1978, Kocherlakota (1996) calculates that $-1 + E\widetilde{z}/z_0 = 1.8\%$ and $Var(\widetilde{z}/z_0) = (3.6\%)^2$. If we assume $\widetilde{z}$ is log-normal, this calculation is consistent with $\sigma = 3.54\%$ and $\mu = 1.72\%$. We suppose that information affects the conditional variance of log consumption. There are two possible signals. Signal η_1 yields a low variance $\sigma_1^2(t) = (3.54\%)^2(1-t)$, whereas signal η_2 yields a higher variance $\sigma_2^2(t) = (3.54\%)^2(1+kt)$,with k positive. In order to preserve the average variance to its historical mean, we set the probability of the low variance signal to be $\pi = k/(1+k)$. We assume that the signals do not affect the risk-free rate, so that $\mu(\eta_i) = \gamma^{-1}\ln R_f + \gamma\sigma^2(\eta_i)/2$.

Assume that $t = 0.75$, $k = 99$, and $\gamma = 5$. We describe the information structure for these parameter values in Table 2. There is a 99% chance that the low volatility state occurs, in which case the volatility equals half of its historical mean. There is a 1% chance that the volatility be 8.67 times its historical mean. In Figure 1, we have drawn the prior density of $\widetilde{z}$. The dotted curve is the log-normal distribution with the same mean and variance (μ, σ), but obviously thiner tails.

	Probability	$\mu(\eta)$	$\sigma(\eta)$	$\ln R_f$	$\phi(\eta) - 1$
η_1	99%	1.49%	1.77%	7.03%	0.16%
η_2	1%	24.98%	30.71%	7.03%	60.24%
Mean	-	1.72%	2.06%	7.03%	0.76%

Table 2: The information structure with $t = 0.75$, $k = 99$, and $\gamma = 5$.

In the absence of information, we obtain that the equity premium ϕ^0 equals 0.66%,[18] whereas

[18] An external observer could have used the average volatility $E\widetilde{\sigma} = 2.06\%$ to estimate the equity premium around $-1 + \exp\gamma(E\widetilde{\sigma}) = 0.21\%$. The existence of fat tails in this calibration explains why ϕ^0 is three times larger than this estimation. As claimed by Abel (2002) for example, fat tails can solve the equity premium puzzle.

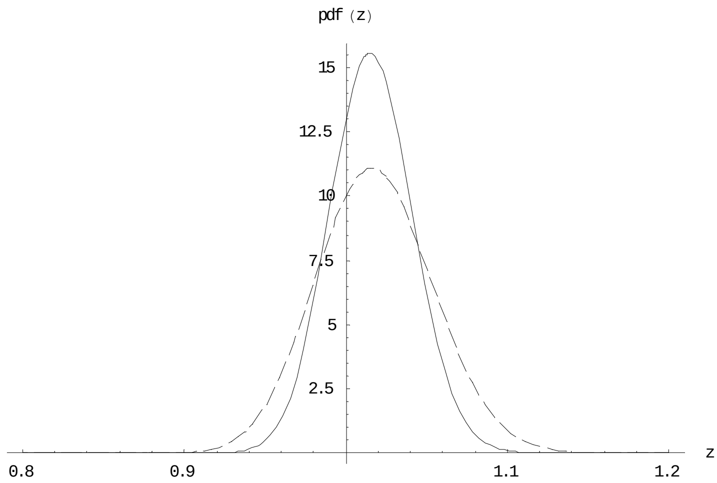

Figure 1: The unconditional distribution of aggregate consumption. The dotted curve is the log-normal distribution with the same first two moments.

the average equity premium when information is available to investors equals 0.76%. Information raises the equity premium by 16%. In Figure 2, we have drawn the average equity premium with information (plain curve) and without information (dashed curve), as a function of the representative agent's risk aversion.

More generally we can use Example 4 to show that, if $\gamma > 1$, information can raise the equity premium by *any* percentage while matching any risk-free rate and growth rate. Suppose as in Example 4 that the signals do not affect the risk-free rate. Substitute $\mu(\eta) = \gamma^{-1} \ln R_f + \gamma\sigma^2(\eta)/2$ into (14) to find the equity premium in the uninformed economy is

$$\phi^0 = \frac{E[\exp\left((1+\gamma)\frac{1}{2}\sigma^2(\widetilde{\eta})\right)]}{E[\exp\left((1-\gamma)\frac{1}{2}\sigma^2(\widetilde{\eta})\right)]}. \tag{15}$$

The average equity premium in the informed economy is still $E[\exp(\gamma\sigma^2(\widetilde{\eta}))]$. Now fix any risk-free rate $\overline{R}_f$ and any growth rate $\overline{z}$. *If $\gamma > 1$, then there is a distribution of the variance $\sigma^2(\widetilde{\eta})$ such that the ratio $E[\phi(\widetilde{\eta})]/\phi^0$ can be arbitrarily large, and the risk-free rate in the informed economy is $\overline{R}_f$ and the ex ante average growth rate, $E_\eta\left[E_z[\widetilde{z}|\widetilde{\eta}]\right]$, equals $\overline{z}$.*

There are several ways to prove this fact. One uses the property that for $\gamma > 1$, both the

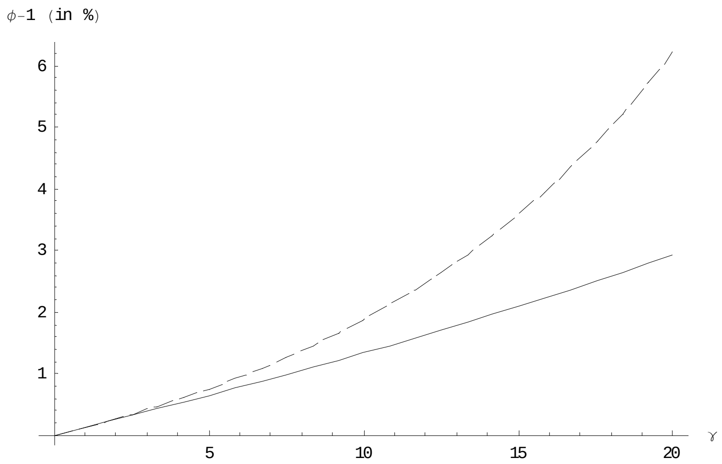

Figure 2: The average equity premium with (plain) and without (dashed) information as a function of risk aversion.

equilibrium price of equity in the informed economy, $\exp\big((1-\gamma)\sigma^2/2\big)$, and the equity premium in the informed economy, $\exp(\gamma\sigma^2)$, are *more convex* than expected consumption in the informed economy, $\exp\big((1+\gamma)\sigma^2/2\big)$.[19] Now continually make the distribution of expected consumption, $\exp\big((1+\gamma)\sigma^2(\widetilde{\eta})/2\big)$, riskier, while preserving its mean at $\overline{z}\overline{R}_f^{-1/\gamma}$. The average equity premium in the informed economy, $E[\exp(\gamma\sigma^2(\widetilde{\eta}))]$, increases without bound, while the equity premium in the uniformed economy falls (the numerator in (15) stays the same and the denominator rises).[20] And the informed economy grows on average at $\overline{z}$: with the risk-free rate constant at $\overline{R}_f$, we have $E_\eta\left[E_z[\widetilde{z}|\widetilde{\eta}]\right] = \overline{R}_f^{1/\gamma} E\left[\exp\big((\gamma+1)\sigma^2(\widetilde{\eta})/2\big)\right] = \overline{z}$.

Remark 1 *Weitzman (2007) shows that the equity premium can be arbitrarily large for any level of risk aversion in his model of learning about volatility of consumption from past consumption*

[19]For two functions f and g defined on a real interval I, g is *more convex* than f if there is a convex function T on the range of f such that $g(z) = T(f(z))$ for all z in I. For example, we have $\exp(\gamma\sigma^2) = T(\exp\big((1+\gamma)\sigma^2/2\big))$ for the function $T(y) = y^{2\gamma/(\gamma+1)}$, which is convex whenever $\gamma \geq 1$.

[20]These facts follow from Theorem 3 in Diamond and Stiglitz (1974) on mean-utility-preserving increases in risk. Suppose that a change in the distribution of z causes a mean-preserving increase in risk in the distribution of a function f of z. Their Theorem 3 implies that if g is more convex than f, the distribution change raises the mean of g. Their theorem is a simple extension of the Arrow-Pratt theorem (found for example in Gollier (2002), pp. 20-1).

realizations. That information can raise the equity premium arbitrarily for any level of risk aversion. Weitzman goes further than we do by assuming an inverse-χ^2 distribution for $\widetilde{\sigma}$, which implies a Student–t distribution for $\widetilde{z}$. It implies $\phi^0 = +\infty$.

Remark 2 *We can substantially generalize Example 4. As just emphasized, the assumption that the risk-free rate does not depend on the signal imposes a restriction on the relationship between the mean and variance of log consumption across signals. For any two signals η' and η'' let $\Delta\mu = \mu(\eta'') - \mu(\eta')$, $\Delta\sigma^2 = \sigma^2(\eta'') - \sigma^2(\eta')$, and $\Delta x_i = x_i(\eta'') - x_i(\eta')$. Use the definitions of $x_i(\cdot)$ to find that*

$$\frac{\Delta x_1 \Delta x_2}{\gamma} = (\Delta\mu)^2 - \tfrac{3}{2}\Delta\mu\Delta\sigma^2(\gamma - 1) + \tfrac{1}{4}(\Delta\sigma^2)^2(2\gamma^2 - 5\gamma + 2). \tag{16}$$

Suppose (wlog) that $\Delta\sigma^2 \geq 0$. It is easy to confirm from equation (16) that $\Delta x_1 \Delta x_2 < 0$ provided that $(\gamma/2 - 1)\Delta\sigma^2 \leq \Delta\mu \leq (\gamma - (1/2))\Delta\sigma^2$. Setting $\Delta\mu = \gamma\Delta\sigma^2/2$ gives us Example 4. The other three examples can also be verified directly from (16).

5 The effect of information on the equity premium

We now drop the assumptions of log-normal signals and constant relative risk aversion and extend all the results from the last section. By considering more general preferences and distributions, we are able to see the forces behind these results more clearly.

5.1 Information sometimes raises the equity premium, no matter what the preferences of the representative agent

The following result in Gollier (2001, Proposition 99) shows that the equity premium cannot be (globally) concave, so information sometimes raises the average equity premium, no matter what the representative agent's preferences.

Proposition 2 *For any strictly increasing, strictly concave date-1 utility function v for the repre-*

sentative agent, there is an information structure that raises the average equity premium.

Proof: Appendix.

Since perfect information lowers the equity premium, we can show that information always lowers (or always raises) the average equity premium only by restricting the information structure. We do so by restricting the distribution of posterior beliefs over date-1 consumption. We suppose that the set of posterior beliefs that an information structure generates can be ordered by one of two stochastic dominance relations: first-order stochastic dominance (FSD); and riskiness. Let G and F be two c.d.f.'s with bounded support D in $\mathbb{R}_+$.

Definition 1 *G **first order stochastically dominates** F if $\int f(y)dG(y) \geq \int f(y)dF(y)$ for all continuous increasing functions f on $\mathbb{R}_+$, with the inequality strict for at least one such function.*

Definition 2 *G is **riskier than** F if $\int f(y)dG(y) \leq \int f(y)dF(y)$ for all concave functions f on $\mathbb{R}_+$, with the inequality strict for at least one such function.*

These two cases are natural ones to consider because they are easy to interpret. Roughly, if the set of posterior beliefs over date-1 consumption can be ordered by FSD, then the information is about the *level* of consumption, as in Example 1; if the posterior beliefs over date-1 consumption can be ordered by riskiness, then information is purely about the *volatility* of consumption, as in Example 3. We also consider information that combines both elements, as in Example 4.

5.2 Information about the level of consumption: posterior beliefs ordered by FSD

As just mentioned, if the posterior beliefs are ordered by first-order stochastic dominance for a given prior about $\widetilde{z}$, then information is about the level of date-1 consumption. If we consider *all* prior belief/information structure combinations that order the posteriors by FSD, then relative risk aversion must be *constant* and *equal to one*, implying that date-1 utility is log.[21]

Proposition 3 *Suppose that the representative agent's date-1 utility satisfies the Inada condition* $v'(\infty) = 0$*. Then the average equity premium falls with information for every prior belief/information structure combination that orders the posteriors by FSD if and only if* $RRA(z) = 1$ *for all* $z > 0$*, i.e. the representative agent's date-1 utility is log.*

Proof: Appendix.

To explain intuitively, suppose that the Inada condition $v'(\infty) = 0$ holds. Recall that $(zv'(z))' = v'(z)(1 - RRA(z))$ so the size of relative risk aversion determines whether zv' is increasing or not. If relative risk aversion exceeds 1 on some interval, then clearly the expected return on equity V_1 rises with an FSD improvement if the change is confined to that interval (since the price of equity falls and the expected date-1 consumption rises, with such an FSD improvement). Unfortunately, we know that an FSD improvement sometimes lowers the demand for a risky asset in a two-asset problem (and therefore reduces the equilibrium forward price of equity V_2) if relative risk aversion is somewhere larger than 1 (Fishburn and Porter, 1976). By Lemma 1, one can therefore find an information structure with posteriors that can be ordered according to FSD that raises the average equity premium. On the other hand, if relative risk aversion is somewhere less than 1, we know from Fishburn and Porter (1976) that the demand for a risky asset rises with an FSD improvement if the change is confined to that interval. Therefore, this improvement raises the forward price of equity V_2. Unfortunately, the expected return on equity V_1 sometimes falls with an FSD improvement if $RRA < 1$. Hence V_1 and V_2 cannot always be comonotonic when the posterior beliefs are ordered by FSD unless $RRA = 1$ globally.

Proposition 3 shows that we must restrict the information structure further if we want to use anything other than log utility.

[21] Athey and Levin (2000) consider posteriors ordered by FSD for a fixed prior. To prove the necessity of (b), we must allow the prior to vary.

Definition 3 *F_2 dominates F_1 in the* ***monotone likelihood ratio order*** *(MLR) if there is an increasing function ℓ on $\mathbb{R}_{++}$ such that $F_2(x) = \int_0^x \ell(\xi) dF_1(\xi)$ for all $x \leq \inf\{y | F_1(y) = 1\}$.*

If the cdf's are differentiable then the definition collapses to the ratio of the densities, F_1'/F_0', being increasing. Milgrom (1981) defines signal η_2 to be *better news* than η_1 if the posterior given η_2, $F(\cdot|\eta_2)$, first-order stochastically dominates $F(\cdot|\eta_1)$ for *every* prior belief. He proves that η_2 is *better news* than η_1 if and only if $F(\cdot|\eta_2)$ MLR dominates $F(\cdot|\eta_1)$.[22] For example, suppose that, conditional on z, the signal equals z "plus noise": $\tilde{\eta}|z = z + \tilde{\epsilon}$, where $\tilde{\epsilon}$ is a random variable with mean zero and independent of z. If the distribution of $\tilde{\epsilon}$ is given by a density whose log is concave (e.g. the Normal distribution), then the posterior beliefs will be ordered by MLR.

Theorem 1 ***(MLR)*** *If the signals order the posterior c.d.f.'s by MLR, then information lowers the average equity premium.*

Proof: Appendix.

For this natural restriction of information to be about the level of consumption, information lowers the average equity premium, as casual intuition suggests it should. We stress that this conclusion holds for all strictly increasing, strictly concave utilities and all prior beliefs, the only such result we present.

Remark 3 *In Example 1 the signals only affect the mean of log consumption and information lowers the average equity premium. This example is a special case of Theorem 1 since an increase in the mean of a log-normal random variable is an MLR improvement for the random variable itself: letting $f(\cdot|\mu)$ be the density of z when $\ln z$ is normal with mean μ, it is easy to show that, for any $\mu' > \mu$, $f(z|\mu')/f(z|\mu)$ is increasing in z, so that $f(\cdot|\mu')$ MLR dominates $f(\cdot|\mu)$.*

[22] Since Proposition 5 requires all *combinations* of prior beliefs and information structures to order the posteriors by FSD, that result considers a wider set of information structures than does Theorem 1.

5.3 Information about the volatility of consumption: posterior beliefs ordered by riskiness

Information about volatility has dramatically different effects on the equity premium than information about levels. To model information purely about volatility we assume that the posteriors are ordered by riskiness, implying that the mean date-1 endowment is known (and so does not depend on the signal). For example, if, conditional on η, the distribution of date-1 consumption is equal to $\tilde{x} + \eta\tilde{\varepsilon}$, where $E[\tilde{\varepsilon}|x] = 0$ for all x, then higher realizations of η lead to riskier posterior beliefs about date-1 consumption. The assumption that the posteriors are ordered by riskiness excludes perfect information: it might be possible to learn the state of nature, but it is impossible to learn the state with probability 1.

Proposition 4 *If an information structure orders the posteriors by riskiness and if date-1 utility v satisfies $zv''' + 2v'' = 0$ for all $z > 0$ (equivalently, zv' is affine), then information has no effect on the average equity premium. If $zv''' + 2v'' \neq 0$ for some $z > 0$, then there is a prior belief and an information structure that orders the posteriors by riskiness such that information raises the average equity premium.*

Proof: Appendix.

Recall that *prudence* is $P(z) = -v'''(z)/v''(z)$ and *relative prudence* is $zP(z)$. The condition in the first sentence of Proposition 4 says that $zP(z) = 2$ for all $z > 0$, equivalently that date-1 utility takes the form $v(z) = az + \log z$ for $a \geq 0$. Proposition 4 implies that if information is purely about the riskiness of consumption, then there is *no* utility function such that information always lowers the average equity premium.

Indeed, information always raises the equity premium for a class of date-1 utility functions if the posteriors are ordered by riskiness of consumption. Consider the expected rate of return on equity, $V_1 = \int zdF / \int zv'dF$. An increase in risk does not affect the expected return on equity (the numerator), and it decreases the price of equity (the denominator) if zv' is strictly concave, or

$zv''' + 2v'' < 0$. Now consider the forward price of equity, $V_2 = \int zv'dF / \int v'dF$.[23] The denominator increases (or is constant) with an increase in risk if v' is convex, or $v''' \geq 0$. And the numerator decreases if zv' is strictly concave in z. So if v' is convex and zv' is strictly concave – more compactly, if $0 \leq zP(z) < 2$ – then V_1 and *minus* V_2 are comonotonic, implying by Lemma 1 that information always raises the average equity premium if the posteriors are ordered by riskiness.

Theorem 2 ***(Riskiness of*** z***)*** *Consider any information structure that orders the posterior beliefs by riskiness. The equity premium increases with such information if* v' *is convex and* zv' *is strictly concave; or equivalently if* $0 \leq zP(z) < 2$ *for all* z*.*

To interpret the preference restriction, consider the static two-asset portfolio problem with one safe and one risky asset. The restriction on relative prudence is exactly what ensures that the demand for the risky asset falls as its return distribution becomes riskier (Hadar and Seo, 1990, Theorem 2).[24] One way to interpret Theorem 2 is that, if demand for a risky asset always falls as its risk increases, then information about the volatility of equity must raise the average equity premium.

Why is it that information about the volatility of equity increases the average equity premium for this class of preferences? The average equity premium rises with information if the equity premium increases at an increasing rate as the date-1 price of equity gets riskier. The assumption that $0 \leq zP(z) < 2$ insures that the demand for equity falls as it becomes riskier; this fall in demand lowers the date-0 price of equity, $P_e(F) = \int zv'dF$. Since the mean endowment is not affected by information, the lower price for equity raises its expected rate of return. And since $v''' > 0$, the agent is "prudent"–the agent saves more in response to an increase in future income risk (Leland, 1968). The higher supply of saving pushes the risk-free rate down. Each effect on its own ensures that the equity premium rises as equity gets riskier. Together, they ensure that the

[23] Abel (2002) shows that V_2 is decreasing in the riskiness of $\tilde{z}$ in the case of small risks. It can easily be shown that this is generally not true for large risks.

[24] More precisely, let $v' > 0, v'' < 0$, and $v''' \geq 0$. Hadar and Seo prove that an increase in risk always lowers the demand for the risky asset for every wealth level if and only if zv' is concave.

equity premium rises at an increasing rate as as equity becomes riskier, so the equity premium is convex in beliefs.

What can be said about the size of relative prudence? Under decreasing absolute risk aversion (DARA), relative risk aversion is bounded above by relative prudence. Under DARA, relative prudence less than 2 implies that relative risk aversion is also less than 2. Indeed, for CRRA utility ($v'(z) = z^{-\gamma}$), relative prudence is equal to $\gamma - 1$, so Example 2 (learning only about the variance of consumption) is a special case of Proposition 2. Although there is no agreement about what is a "reasonable" number for relative risk aversion in an aggregate model, opinion seems to cluster in the interval $(1, 3)$ (see, e.g. the survey by Kocherlakota (1996)). So 2 might seem a low number for relative prudence. On the other hand, studies which try to estimate relative prudence directly with household level data often report estimates below 2, even below 1.[25]

5.4 Constant risk-free rate in the informed economy

Return now to Example 4, the CRRA/log-normal specification in which the signals do not affect the risk-free rate. It suggests that if risk aversion is at least moderately high ($\gamma > 1$) and the signals do not affect the risk free rate (much), then information raises the average equity premium. Does this conclusion generalize to other preferences and distributions? What aspects of the CRRA/log-normal case is responsible for the result?

The risk-free rate in that case is $1/E[v'|\eta] = \exp(\gamma(\mu(\eta) - \gamma\sigma^2(\eta)/2))$. To keep the risk-free rate constant, the mean and variance of log consumption must move in the same direction as news arrives. Recall that information raises the average equity premium if the rate of return of equity and the forward price of equity move in opposite directions as news arrives (Lemma 1). The striking conclusion from Example 4 now follows from Lemma 1 and two facts: if $\gamma > 1$, the function $zv'(z) = z^{1-\gamma}$ is *more concave* than $v'(z) = z^{-\gamma}$; and when the variance and mean of log

[25] See e.g. Dynan (1993) and her references.

consumption rise in such a way to keep $E[v'|\eta]$ constant, *the distribution of* v' *becomes riskier.*[26] Since zv' is more concave than v', $E[zv']$ falls whenever the distribution of v' undergoes a (mean-preserving) increase in risk. And since $v'^{-\gamma}$ is convex ***and decreasing***, the identity function is more ***convex*** than v', so $E[z]$ ***rises*** with an increase in the riskiness of v'.[27]

We conjecture that information raises the average equity premium if (i) the information structure orders the posteriors by riskiness of v'; (ii) $v''' > 0$ (to ensure that z is more ***convex*** than v'); and (iii) zv' is more concave than v'. Condition (iii) it turns out is equivalent to the condition that risk aversion is less than twice prudence.

Lemma 2 *For an increasing, concave* C^3 *utility* v *on a real open interval* I*, the function* zv' *is more concave than* v' *if and only if* $P(z) \leq 2A(z)$ *for all* $z \in I$*.*

Proof: Appendix.

Lemmata 1 and 2 give us

Theorem 3 *(**Riskiness of** v') Suppose that (i) the information structure orders the posterior beliefs by riskiness of* v' *(so that the risk-free rate is not affected by the signal), and (ii) prudence is positive and uniformly less than twice risk aversion (*$0 < P(z) < 2A(z)$ *for all* $z \geq 0$*). Then information raises the average equity premium.*

For CRRA utility prudence is less than twice risk aversion precisely when relative risk aversion (γ) is greater than 1. *But this relationship between risk aversion and the inequality* $P(z) < 2A(z)$ *is*

[26] That the distribution of v' becomes riskier is easy to verify formally from the density function for v' and the relationship $\mu(\eta) - \gamma\sigma^2(\eta)/2 =$ constant. Also $z^{1-\gamma} = T(z^{-\gamma})$ for the function $T(y) = y^{(\gamma-1)/\gamma}$, which is concave whenever $\gamma \geq 1$.

[27] $z = T(z^{-\gamma})$ for $T(y) = y^{-1/\gamma}$, which is convex. Note that a linear function can be more convex than a *decreasing* convex function.

an artifact of CRRA preferences. If absolute risk aversion is constant, prudence and risk aversion are always the same, so clearly $P < 2A$ no matter what the degree of risk aversion. And for the function $v(z) = \ln z - z$ on $[0, 1]$, prudence is *greater than* twice risk aversion; but that function is *more* risk averse than $\ln z$ (for which $P = 2A$). Despite what the CRRA case suggests, the level of risk aversion in general has nothing to do with whether information raises the equity premium when the risk-free rate is constant. Theorem 3 once again illustrates the danger of using single-parameter utility functions, such as CRRA, to understand asset pricing.

How do we interpret the inequality $P < 2A$? It has appeared before, especially in the literature on whether independent risky assets are complements or substitutes (Gollier, 2001, chapter 10). We now argue that $P < 2A$ means that *equity and total saving are substitutes*, in the sense that allowing an agent to invest in equity lowers total saving. Suppose that an agent (with additively separable and concave utility) lives two periods, has a nonrandom consumption endowment of date-1 consumption, and can save by investing in a safe asset. Now a risky asset becomes available and the agent optimally invests a positive amount in it. The question is: Does optimal saving go up or down? It clearly goes down if the marginal utility of saving in the one-asset problem is greater than the expected marginal utility of saving in the two-asset problem (evaluated at the solution for the two-asset problem). We know already that adding a risky asset raises the marginal utility of wealth in a one-period problem if and only if $P < 2A$ globally (Gollier and Kimball, 1995).[28] Replace "wealth" with "saving" to conclude that adding a risky asset always lowers total saving if and only if $P < 2A$. Intuitively, adding a risky asset does two things: it raises expected date 2 consumption; and it introduces uncertainty about date-1 consumption. Raising date-1 consumption lowers the marginal utility of saving since v' is decreasing, and what governs how much v' falls when consumption increases is absolute risk aversion. Increasing uncertainty about date-1 consumption tends to raise it if v' is convex, and what governs how much v' increases with an increase in uncertainty is absolute prudence. The condition $P < 2A$ ensures that prudence isn't "too large," so marginal utility of saving falls with the addition of a risky asset.

[28] See Gollier (2001, pp. 146-7). That the introduction of a risk asset lowers the marginal utility of wealth when $P < 2A$ follows almost immediately from Lemma 2 and the Arrow-Pratt theorem on comparative risk aversion.

Whether equity and saving are substitutes is an empirical question, one made challenging by the definition of substitutability: saving falls when a consumer previously constrained to hold no equity is allowed to hold equity. The fraction of consumers who hold no equity has fallen steeply in the U.S. over the past few decades. Perhaps one way to answer the question is to determine how total saving changes after a consumer begins to hold equity.

Finally, the quantitative argument at the end of Section 4–that information can raise the equity premium by any percentage while matching mean growth and the risk-free rate–holds in the general case if $P < 2A$: as the distribution of v' becomes riskier, the average equity premium in the informed economy grows without bound and the equity premium in the uninformed economy falls.

6 Information and the equity premium in an infinite horizon model

We can interpret our two-period results as applying to "one-period ahead" equity which pays off the random endowment one period hence, then expires. Although the two-period model allows us to use arbitrary preferences for the representative agent, one may wonder whether our results are an artifact of the two-period assumption. We can extend our results to the infinite horizon case if we restrict both preferences and the endowment. In what follows we assume that the vN-M utility takes the additively separable form

$$v(z_0) + \sum_{t=1}^{\infty} \beta^{t-1} v(z_t)$$

where $\beta \in (0, 1)$. In this setting "equity" denotes a claim on *all* future state-contingent consumption. Proceeding along of the lines of the argument in Section 2, the equilibrium date-t price of equity is

$$P_e(t) = \frac{E_t[\sum_{j=1}^{\infty} \ \beta^j v'(\widetilde{z}_{t+j})\widetilde{z}_{t+j}]}{v'(z_t)}, \tag{17}$$

where E_t is the expectations operator conditional on date-t information. The date-t return on equity is

$$R(t) = \frac{z_{t+1} + P_e(t+1)}{P_e(t)} \tag{18}$$

and the risk-free rate between dates t and t+1 is

$$R_f(t+1) = \frac{v'(z_t)}{\beta E_t[v'(\widetilde{z}_{t+j})]}. \tag{19}$$

The date-t equity premium, $\phi(t)$, is

$$\frac{E_t[R(t)]}{R_f(t+1)} = \frac{\beta E_t\left[v'(\widetilde{z}_{t+1})\right]}{E_t\left[\sum_{j=1}^{\infty}\beta^j v'(\widetilde{z}_{t+j})\widetilde{z}_{t+j}\right]}\left(E_t\left[\widetilde{z}_{t+1}\right] + E_t\left[\frac{\sum_{j=1}^{\infty}\beta^j v'(\widetilde{z}_{t+1+j})\widetilde{z}_{t+1+j}}{v'(\widetilde{z}_{t+1})}\right]\right). \tag{20}$$

The capital gains component, $P_e(t+1)$, in the return to equity prevents us from applying Lemma 1 directly to determine the effect of information. In the case of log utility at each date, however, $P_e(t) = z_t\beta/(1-\beta)$ for all t, in which case the date-t equity premium collapses to

$$\phi(t) = \; E_t\left[\widetilde{z}_{t+1}\right]\; E_t\left[\frac{1}{\widetilde{z}_{t+1}}\right] \qquad \left(= E_t[\widetilde{z}_{t+1}]\, E_t[v'(\widetilde{z}_{t+1})]\right),$$

exactly as in the two-period model: *every* result in the two-period model for log utility continues to hold in the infinite horizon model, no matter what the endowment distribution. To move beyond log utility, we restrict the endowment process and possibly preferences.

6.1 Independent growth rates, isoelastic period utility

For $t = 0, 1, 2...$, now let $\widetilde{z}_{t+1} = \widetilde{y}_{t+1} z_t$, for some random variable $\widetilde{y}_{t+1}$ with support in a bounded interval $[a, b]$ with $b > a > 0$, with the $\widetilde{y}_{t+1}$ independently (but *not* necessarily identically) distributed. And now for $t = 0, 1, 2...$, let period utility take the form $v(z) = (1-\gamma)^{-1} z^{1-\gamma}$ for some $0 \neq \gamma < 1$; or $v(z) = \ln z$. Now find the date-t return is

$$R(t) = \frac{y_{t+1}\left(1 + E_t[\widetilde{\Omega}_{t+1}]\right)}{\beta E_t[\widetilde{y}_t^{1-\gamma}](1 + E_t[\widetilde{\Omega}_{t+1}])} \tag{21}$$

where $\Omega_{t+1} = \sum_{j=1}^{\infty} \beta^j \left(\Pi_{i=1}^{j} y_{t+1+i}^{1-\gamma} \right)$, and the expected return simplifies to

$$E_t[R(t)] = \frac{E_t[\widetilde{y}_{t+1}]}{\beta E_t[\widetilde{y}_{t+1}^{1-\gamma}]}. \tag{22}$$

The date-t equity premium is just

$$\phi(t) = \frac{E_t[\widetilde{y}_{t+1}^{-\gamma}] E_t[\widetilde{y}_{t+1}]}{E_t[\widetilde{y}_{t+1}^{1-\gamma}]}, \tag{23}$$

exactly as in the two-period model. We consider two different comparative statics. Suppose that the $\widetilde{y}_t$ are identically distributed (say all equal to $\tilde{y}$), so the date t equity premium is $\phi = E[\widetilde{y}^{-\gamma}]E[\widetilde{y}]/E_t[\widetilde{y}^{1-\gamma}]$ for all t. We can then ask what would happen to the average equity premium if the agent receives information *at date zero* about the distribution of $\widetilde{y}$. Here the information is received only once, but is long-lived. Or we can assume that the agent receives a signal at each date, but concerning *only* next period's growth rate. Information is received every period, but is short-lived. In either case we can apply Lemma 1 and Proposition 1 to $\phi(t)$ or to ϕ to determine how information affects the equity premium. We summarize the results for the infinite-horizon model in Table 2. The first column restricts the information structure.

posteriors ordered by ↓	$\gamma < 1$	$\gamma = 1$	$\gamma > 1$	Corollary of ↓
FSD	$+/-$	$-$	$+/-$	Proposition 5
MLR	$-$	$-$	$-$	Theorem 1
Riskiness of z	$+$	0	$-$	Theorem 2
Riskiness of v'	$-$	0	$+$	Theorem 3

Table 2: **Effect of information on the average equity premium in the infinite-horizon model: period marginal utility $v'(z) = z^{-\gamma}$ and independent growth rates.** The first column restricts the information structure by how the signals order the posteriors; the first row specifies the elasticity of substitution γ.

6.2 I.i.d. endowment, arbitrary period utility

With independent growth rates and isoelastic period utility all the results from the two-period model hold. But as we pointed out in Section 5.4, the assumption of isoelastic period utility can obscure the conditions which determine how information affects the equity premium. We can prove results for arbitrary period utility–provided we assuming that the endowment is independently *and* identically distributed across time. The iid assumption unfortunately implies that there is no growth, but this subsection should help convince the reader that the two-period results are at least somewhat robust.

Use the assumption that the $\widetilde{z}_t$ are identically and independently distributed in equation (20) to find the equity premium to be

$$\phi = (1-\beta)\frac{E[\widetilde{z}]\,E[v'(\widetilde{z})]}{E[v'(\widetilde{z})\widetilde{z}]} + \beta E\left[\frac{1}{v'(\widetilde{z})}\right] E[v'(\widetilde{z})]. \tag{24}$$

Since we assume that the endowment is i.i.d. to obtain (24), we have to restrict information to date 0 and it affects beliefs about all future consumption (long-lived information obtained once). Our Theorems about the one-period equity premium determine whether the first term is concave or convex on a set of beliefs. The second term is the product of two linear functions of beliefs. It is concave whenever $E[1/v']$ and $E[v']$ are anti-comonotonic, and convex whenever they are co-monotonic. For any FOSD change, $E[1/v']$ and $E[v']$ are anti-comonotonic, so the Theorem 1 (MLR) goes through for this case without modification. If the posteriors are ordered by riskiness of v', then the second term is linear in beliefs, so our Theorem 3 (Riskiness of v') goes through without modification as well. If posteriors are ordered by riskiness of z, then $E[1/v']$ and $E[v']$ are comonotonic if $1/v'$ is convex or Prudence is at least twice Risk Aversion. (Of course the last assumption is merely sufficient; the equity premium can be convex overall if $P < 2A$.) To sum up: In the infinite horizon economy with i.i.d. endowment, with C^3, strictly increasing and strictly concave (but otherwise arbitrary) period utility v and discount factor $0 < \beta < 1$, the conclusions of Theorems 1 (MLR) and 3 (Riskiness of v') hold; if, in addition, Prudence is at least twice Risk Aversion, the conclusion of Theorem 2 (Riskiness of z) holds. Note that as $\beta \to 0$, the date-t equity premium converges to the static counterpart. As $\beta \to 1$ it converges to the "capital gains" part of the equity premium, $E[1/v'(\widetilde{z})]E[v'(\widetilde{z})]$.

7 Concluding Remarks

Information can either raise or lower the average equity premium. When information is about the level of future consumption in the sense that the signals order the posterior beliefs by MLR, information lowers the average equity premium–as the intuition from perfect information suggests it should. Information not purely about the level of consumption, however, can easily raise the average equity premium, especially if the signals do not affect the risk-free rate much.

These results extend to an infinite horizon model for the special case of independent growth rates and standard time-separable, isoelastic preferences (or i.i.d. endowment with arbitrary period utility). As mentioned, one natural interpretation of that extension comes from supposing that information improves through time (perhaps because of faster and cheaper personal computers leading to cheaper information). Since information is apt to be about both the level and the volatility of equity, our results do not predict what should happen to the equity premium over time, but they do remove some surprise at the possibility that it could rise through time. More broadly, the important condition to check is the one given in Lemma (ii), which says that the expected rate of return on equity and the forward price of equity are comonotonic, a condition which could in principle be tested.

We have assumed that people have the same information. If different people have different information (and this asymmetric information is not revealed by equilibrium prices), then the effect of better information on the equity premium is harder to determine. Clearly, if public information is close enough to perfect, then it must lower the equity premium: individuals not only have more information, but informational asymmetry falls with better public information. Determining how information affects the average equity premium when information is asymmetric would be both challenging and interesting.[29]

As a final observation, return to the interpretation that information is private to investors

[29] For a survey of information aggregation in markets, see Vives (2008).

but unobserved by the economist. If information is purely about volatility (of consumption, log consumption or the marginal utility of consumption) then the average equity premium could be higher than one calculated by an economist who ignores private information. As we already mentioned, Weitzman (2007) considers a Bayesian model in which agents learn about consumption variance from past experience. He shows that the equity premium which takes into account the "sampling error" from such learning can be dramatically higher than the one calculated assuming that the variance is known and equal to the sample variance from historical data. In his model, as in the private information interpretation of ours, at each date t agents and the economist have different beliefs about future consumption. Since our agent's beliefs are an updated version of the economist's, our agent cannot believe an event possible that the economist thinks impossible. In principle, Weitzman's Bayesian agent could believe that some outcomes are possible that an economist who just uses historical frequencies thinks are impossible. In that sense our model imposes added discipline on the relationship between beliefs of the economist and agents.

Appendix: Proofs

We use the following Lemma in several proofs. Although it follows from known results (see Machina, 1982, and Wang, 1993, for example), we include the simple proof to make our presentation self-contained.

Lemma 3 *For $i = 1, 2$ we have*

$$V_i(G) - V_i(F) = \int_0^1 \left(\int U_i(z; \lambda G + (1-\lambda)F) d(G - F) \right) d\lambda, \tag{A.1}$$

where

$$U_1(z; F) = \frac{z - zv'(z)V_1(F)}{\int \xi v'(\xi) dF(\xi)}, \tag{A.2}$$

and

$$U_2(z; F) = \frac{zv'(z) - v'(z)V_2(F)}{\int v'(\xi) dF(\xi)}. \tag{A.3}$$

Thus if $U_i(\cdot, H)$ is increasing for every c.d.f. H, then $V_i(G) \geq V_i(F)$ whenever G FSD dominates F; if $U_i(\cdot, H)$ is convave for every c.d.f. H, then $V_i(G) \geq V_i(F)$ whenever F is riskier than G.

Proof of Lemma 3: First note that, by the Fundamental Theorem of Calculus, we have

$$V_i(G) - V_i(F) = \int \left(\frac{d}{d\alpha} V_i(\alpha G + (1-\alpha)F)|_{\alpha=\lambda} \right) d\lambda. \tag{A.4}$$

Now

$$\frac{d}{d\alpha} V_1(\alpha G + (1-\alpha)F) = \frac{\int z d(G-F)}{\int z v'(z) d(\alpha G + (1-\alpha)F)} - \frac{V_1(\alpha G + (1-\alpha)F) \int z v'(z) d(G-F)}{\int z v'(z) d(\alpha G + (1-\alpha)F)}.$$

Rearrange and use (A.2) to find

$$\frac{d}{d\alpha} V_1(\alpha G + (1-\alpha)F)|_{\alpha=\lambda} = \int U_1(z; \alpha G + (1-\alpha)F) d(G-F),$$

and substitute this expression into (A.4) for $i = 1$ to find that

$$V_1(G) - V_1(F) = \int_0^1 \left(\int U_1(z; \lambda G + (1-\lambda)F) d(G-F) \right) d\lambda.$$

For $i = 2$, we have

$$\frac{d}{d\alpha} V_2(\alpha G + (1-\alpha)F) = \frac{\int z v'(z) d(G-F)}{\int v'(z) d(\alpha G + (1-\alpha)F)} - \frac{V_2(\alpha G + (1-\alpha)F) \int v'(z) d(G-F)}{\int v'(z) d(\alpha G + (1-\alpha)F)}.$$

Use (A.3) and substitute into (A.4) to get (A.1) for $i = 2$. The rest of the Lemma follows from standard stochastic dominance arguments (e.g. Gollier, 2001, chapter 3). ■

Proof of Lemma 1: We now show that Lemma 1 is a consequence of the following extension of the covariance inequality (Hardy, et. al., Theorem 43), namely, for a finite set S and real-valued positive functions f, g on S, we have $\sum_{s \in S} f(s) g(s) \pi(s) \leq \sum_{s \in S} f(s) \pi(s) \sum_{s \in S} g(s) \pi(s)$ for all positive π with $\sum \pi(s) = 1$ if and only if g and $-f$ are comonotonic. We write $E_\pi[f] = \sum_{s \in S} f(s) \pi(s)$.

Lemma 4 *For a finite set S and real-valued positive functions f, g and h and π on S, we have*

$$E_\pi\left[\frac{fg}{h}\right] \leq \frac{E_\pi[f]E_\pi[g]}{E_\pi[h]}$$

for all π with $\sum \pi(s) = 1$ if and only if g/h and $-f/h$ are comonotonic.

To prove Lemma 4, define

$$\pi^*(s) = \frac{h(s)\pi(s)}{E_\pi[h]}$$

for all $s \in S$ and note that

$$E_\pi\left[\frac{fg}{h}\right] = E_\pi[h]E_{\pi*}\left[\frac{f}{h}\frac{g}{h}\right]. \tag{A.5}$$

By the Covariance Inequality, we have

$$E_{\pi*}\left[\frac{f}{h}\frac{g}{h}\right] \leq E_{\pi*}\left[\frac{f}{h}\right]E_{\pi*}\left[\frac{g}{h}\right] \tag{A.6}$$

for all probability distributions π^* (equivalently, all probability distributions π) if and only if f/h and $-g/h$ are comonotonic. Combining (A.5) and (A.6) we have

$$E_\pi\left[\frac{fg}{h}\right] \leq E_\pi[h]E_{\pi*}\left[\frac{f}{h}\right]E_{\pi*}\left[\frac{g}{h}\right] \leq \frac{E_\pi[f]E_\pi[g]}{E_\pi[h]}, \tag{A.7}$$

for all probability distributions π if and only if f/h and $-g/h$ are comonotonic, which proves Lemma 4. To prove Lemma 1, take $S = \{F_1, F_2\}$, $f(F_i) = \int z dF_i$, $g(F_i) = \int v'(z)dF_i$, and $h(F_i) = \int zv'(z)dF_i$, for $i = 1, 2$ and note that f, g and h are linear (e.g. $f(\lambda F_2 + (1-\lambda)F_1) = \lambda f(F_2) + (1-\lambda)f(F_1)$), so the last term in (A.7) is the equity premium with no information. ■

Proof of Proposition 2: By Proposition 4, a necessary condition for the average equity premium never to increase with information is that the utility function takes the form $v(z) = az + \log z$ for $a \geq 0$. The rate of return on equity in this case is

$$V_1(F) = \frac{\int z dF(Z)}{\int az dF(z) + 1}$$

and the forward price of equity is

$$V_2(F) = \frac{\int azdF(z) + 1}{a + \int (1/z)dF(z)}.$$

Let G and F be nondegenerate c.d.f.'s in D with G riskier than F, so that $V_2(G) < V_2(F)$ and $V_1(G) = V_1(F)$. Since the V_i's are continuous on D (and F does not maximize V_1 on D), there is an H in D such that $V_2(H) < V_2(F)$ and $V_1(H) > V_1(F)$. ■

Proof of Proposition 3: Proposition 3 is a direct consequence of

Lemma 5 *The following are equivalent:*

1. *The average equity premium falls for every prior belief/information structure combination that orders the posteriors by FSD.*
2. *For all* $z > 0$, $1 - \frac{v'(\infty)}{v'(z)} \leq RRA(z) \leq 1$.

Proof of Lemma 5: To prove that (b) implies (a), suppose that $1-(v'(\infty)/v'(z)) \leq RRA(z) \leq 1$ for all $z > 0$ and let F_2 FSD dominate F_1. If $v'(\infty) = 0$, then the result is immediate by Lemma 1, since $RRA(z) \equiv 1$. So suppose that $v'(\infty) > 0$. Since $R \leq 1$, we have that $U_2(\cdot; F)$ defined by (A.3) is increasing for every F, so that $V_2(F_2) \geq V_2(F_1)$. We now show that $V_1(F_2) \geq V_1(F_1)$. We have for any c.d.f. H

$$\begin{aligned}
\left(\int \xi v'(\xi)dH(\xi)\right) U_1'(z;H) &= 1 - \big(v'(z) + zv''(z)\big)\, V_1(H) \\
&= 1 - v'(z)\,(1 - RRA(z))\, V_1(H) \\
&\geq 1 - v'(z)\frac{v'(\infty)}{v'(z)} V_1(H) \\
&= 1 - \frac{\int \xi dH(\xi)}{\int \xi v'(\xi)dH(\xi)} v'(\infty) \\
&\geq 1 - \frac{\int \xi dH(\xi)}{v'(\infty)\int \xi dH(\xi)} v'(\infty) = 0.
\end{aligned}$$

Since $\int zv'(z)dH(z) > 0$, these inequalities imply that $U_1(\cdot, H)$ is increasing for any H. By Lemma 3, $V_1(F_2) \geq V_1(F_1)$.

To prove that (a) implies (b), suppose first that $R(y) = -yv''(y)/v'(y) > 1$ for some y, so that (b) fails. By continuity of $R(\cdot)$, there is some positive open interval I containing y such that $-zv''(z)/v'(z) > 1$ for all z in I. Hence, for any c.d.f H, we have $U_1'(z; H) > 0$ on I. Let G_1 and G_2 be any c.d.f.'s which are equal off of I but G_2 FSD dominates G_1. Define $F_i = p\delta_{1-p} + (1-p)G_i$ for $i = 1, 2$ where $1 - p > 0$ is less than any point in I.[30] Then F_2 FSD dominates F_1 and $F_2 = F_1$ off of I, so that $V_1(F_2) - V_1(F_1) > 0$ for any such p.

Now

$$\begin{aligned}
&\left[\int v' d(\lambda F_2 + (1-\lambda)F_1)\right] \int U_2(z; \lambda F_2 + (1-\lambda)F_1) d(F_2 - F_1) \\
= & \int zv'(z)d(G_2 - G_1) + V_2(\lambda F_2 + (1-\lambda)F_1) \int v'(z)d(G_2 - G_1)
\end{aligned}$$

We have $\int zv'(z)d(G_2 - G_1) < 0$, since zv' is strictly decreasing on I. Moreover,

$$V_2(\lambda F_2 + (1-\lambda)F_1) = \frac{(1-p)\int zv' d(\lambda G_2 + (1-\lambda)G_1) + p(1-p)v'(1-p)}{(1-p)\int v' d(\lambda G_2 + (1-\lambda)G_1) + pv'(1-p)}$$

As p tends to 1, $V_2(\lambda F_2 + (1-\lambda)F_1)$ tends to 0. Thus for p close enough to 1, $\int U_2(z; \lambda F_2 + (1-\lambda)F_1)d(F_2 - F_1) < 0$ and by Lemma A.1, $V_2(F_2) - V_2(F_1) < 0$.

Now suppose that $1 - (v'(\infty)/v'(y)) > R(y)$ for some $y > 0$. By continuity of $R(\cdot)$ and of v', there is some open interval N of y such that $1 - (v'(\infty)/v'(y)) > RRA(z)$ for all z in N. Hence $U_2'(z; H) > 0$ for any c.d.f. H and any z in N. Suppose first that $v'(\infty) > 0$. Since $RRA(z) < 1$ on

[30] The notation δ_p stands for the c.d.f. that puts probability 1 on the point p. Since there are at least three states of the world, it is always possible to construct F_i's and G_i's with the indicated properties.

N and v' is decreasing, we have, for any z in N and any F

$$\begin{aligned} \left[\int \xi v'(\xi)dF(\xi)\right] U_1'(z,F) &= 1-\big(v'(z)+zv''(z)\big)\frac{\int \xi dF(\xi)}{\int \xi v'(\xi)dF(\xi)} \\ &\leq 1-\big(v'(z)+zv''(z)\big)\frac{1}{v'(\infty)} \\ &= 1-(1-RRA(z))\frac{v'(z)}{v'(\infty)}<0. \end{aligned}$$

Thus, if F_2 first-order dominates F_1 and the two distributions are equal off of the interval N, we have $V_2(F_2)-V_2(F_1)>0$ and $V_1(F_2)-V_1(F_1)<0$.

If $v'(\infty)=0$, let G_1 and G_2 be any c.d.f.'s which are equal off of N but G_2 FSD dominates G_1. Define $F_i=p\delta_{(1-p)^{-1}}+(1-p)G_i$ for $i=1,2$ where $(1-p)^{-1}$ exceeds any point in N. Then F_2 FSD dominates F_1 and $F_2=F_1$ off of N, so that $V_2(F_2)-V_2(F_1)>0$ for any such p.

Now

$$\begin{aligned} &\left[\int yv'(y)d(\lambda F_2+(1-\lambda)F_1)\right] U_1'(z,\lambda F_2+(1-\lambda)F_1) \\ =\ & 1-\big(v'(z)+zv''(z)\big)V_1(\lambda F_2+(1-\lambda)F_1), \end{aligned}$$

where

$$\begin{aligned} V_1(\lambda F_2+(1-\lambda)F_1) &= \frac{(1-p)\int yd(\lambda G_2+(1-\lambda)G_1)+p(1-p)^{-1}}{(1-p)\int yv'(y)d(\lambda G_2+(1-\lambda)G_1)+p(1-p)^{-1}v'((1-p)^{-1})} \\ &= \frac{(1-p)^2\int yd(\lambda G_2+(1-\lambda)G_1)+p}{(1-p)^2\int yv'(y)d(\lambda G_2+(1-\lambda)G_1)+pv'((1-p)^{-1})}. \end{aligned}$$

As p tends to 1, V_1 tends to ∞. Thus, for p close enough to 1, $U_1'(z,\lambda F_2+(1-\lambda)F_1)<0$ on N and hence $V_1(F_2)-V_1(F_1)<0$. ■

Proof of Theorem 1: V_2 rises with MLR by Milgrom (1981, Section 3). We will show that V_1 rises with MLR as well. Suppose that F_2 MLR dominates F_1. Letting

$$k=\frac{\int zdF_1}{\int zv'(z)dF_1},$$

we need to show that

$$\frac{\int z dF_2}{\int zv'(z)dF_2} \geq k$$

or equivalently $\int (z - kzv'(z))\, dF_2 \geq 0$. Since v is strictly concave, $j(z) = z\,(1 - kv'(z))$ crosses zero on $\mathbb{R}_{++}$ at most once from below, when $1 = kv'(z)$. Hence j satisfies the single-crossing property in z and therefore $\int (z - kzv'(z))\, dF_1 = 0$ implies that $\int (z - kzv'(z))\, dF_2 \geq 0$ (see e.g. Gollier, 2001, p. 102, Proposition 16). ∎

Proof of Proposition 4: We have that

$$\left(\int \xi v'(\xi) dF(\xi)\right) U_1''(z;F) = -\left(2v'' + zv'''\right) V_1 \tag{A.8}$$

and

$$\begin{aligned}\left(\int v'(\xi) dF(\xi)\right) U_2''(z;F) &= 2v'' + zv''' - v'''V_2 &\text{(A.9)}\\ &= \frac{-U_1''(z;F)}{V_1} - v'''V_2. &\text{(A.10)}\end{aligned}$$

Observe that if $2v'' + zv''' \equiv 0$, then $U_1''(z;H) = 0$ for every H and so $V_1(F_2) = V_1(F_1)$ whenever F_2 is riskier than F_1. Hence the equity premium is unchanged by information when the posteriors are ordered by riskiness.

Suppose now that $2v''(y) + yv'''(y) \neq 0$ for some $y > 0$. Since v'' and v''' are continuous, there is a positive number ε and an interval (a,b) with $a > 0$ such that $|2v''(z) + zv'''(z)| > \varepsilon$ on (a,b). Let G_1 and G_2 be any c.d.f.'s with support in $[0,2b]$ which are equal outside (a,b) with G_2 riskier than G_1. Define $F_i^p = p\delta_{1-p} + (1-p)G_i$ for $i = 1,2$ where $a > 1-p > 0$. Clearly F_2 is riskier than F_1. Letting $G^\lambda = \lambda G_2 + (1-\lambda)G_1$ for $\lambda \in [0,1]$, we have

$$V_2(\lambda F_2^p + (1-\lambda)F_1^p) = \frac{p(1-p)v'(1-p) + (1-p)\int zv'dG^\lambda}{pv'(1-p) + (1-p)\int v'dG^\lambda} = (1-p)\frac{p + \int zv'dG^\lambda}{p + \frac{1-p}{v'(1-p)}\int v'dG^\lambda}$$

which tends to zero uniformly in λ as p tends to 1.

And since v''' is continuous, it is bounded on $[0, 2b]$, so the product $v'''(z)V_2(\lambda F_2^p + (1-\lambda)F_1^p)$ tends to zero uniformly in λ, z as p tends to 1. Moreover $V_1(\lambda F_2^p + (1-\lambda)F_1^p)$ is bounded as a function of (p, λ). So for some p close enough to 1, $U_2''(z; \lambda F_2 + (1-\lambda)F_1)$ and $U_1''(z; \lambda F_2 + (1-\lambda)F_1)$ will be of opposite signs for every λ in $[0, 1]$ and $z \in [0, 2b]$. By Lemma 3,

$$\Big(V_1(F_2^p) - V_1(F_1^p)\Big)\Big(V_2(F_2^p) - V_2^p(F_1)\Big) < 0,$$

and by Lemma 1 information raises the average equity premium. ■

Proof of Lemma 2: Since v' is strictly decreasing, there is a function T defined on the range of v' such that $zv'(z) = T(v'(z))$ for all $z \in I$. Differentiate to find that

$$zv''(z) + v'(z) = T'(v'(z))v''(z), \tag{A.11}$$

so

$$T'(v'(z)) = z - \frac{1}{A(z)}. \tag{A.12}$$

Differentiate (A.11) and use (A.12) to conclude

$$T''(v'(z))v''(z) = 2 - \frac{P(z)}{A(z)}, \tag{A.13}$$

so T is concave if and only if $P/A < 2$ on I. ■

REFERENCES

Abel, Andrew, (2002), "An Exploration of the Effects of Pessimism and Doubt on Asset Returns, *Journal of Economic Dynamics and Control*, 26, 1075-1092.

Ai, Hengjie (2007), "Information Quality and Long-run Risk: Asset Pricing and Welfare Implications," Duke University Working Paper, forthcoming in the *Journal of Finance*.

Athey, Susan and Jonathan Levin, (2000), "The Value of Information in Monotone Decision Problems," Stanford University working paper.

Barsky, Robert, (1989), "Why Don't the Prices of Stocks and Bonds Move Together?," *American Economic Review*, 79, 1132-1145.

Bénabou, Roland, and Jean Tirole, (2002), "Self-confidence and Personal Motivation," *Quarterly Journal of Economics*, 117, 871-915.

Blackwell, David, (1953), "Equivalent Comparisons of Experiments," *Annals of Mathematical Statistics*, 24, 265-272.

Boyd, John, Jian Hu and Ravi Jagannathan, (2005), "The Stock Market's Reaction to Unemployment News: Why Bad News is Usually Good for Stocks," *Journal of Finance*, 60, 649-72.

Brennan, Michael, (1998), "The Role of Learning in Dynamic Portfolio Decisions," *European Finance Review*, 1, 295-306.

Campbell, John, and Luis Viciera, (1999), "Consumption and Portfolio Decisions When Expected Returns are Time Varying," *Quarterly Journal of Economics*, 114, 433-95.

Campbell, John, and Luis Viciera, (2001), "Who Should Buy Long-Term Bonds?," *American Economic Review*, 91, 99-127

Carrillo, Juan, and Thomas Mariotti, (2000), "Strategic Ignorance as a Self Disciplining Device," *Review of Economic Studies*, 67, 529-544.

Cochrane, John, (2001), *Asset Pricing*, Princeton University Press.

Detemple, Jerome, (1986), "Asset Pricing in an Economy with Incomplete Information," *Journal of Finance*, 61, 383-392.

Diamond, Peter and Stiglitz, Joseph, (1974), "Increases in Risk and in Risk Aversion," *Journal of Economic Theory* 8, 337-360.

Dynan, Karen, (1993), "How Prudent are Consumers?," *Journal of Political Economy*, 101, 1104-13.

Fair, Ray, (2002), "Events that Shook the Market," *Journal of Business*, 75, 713-31.

Fishburn, Peter, and R. Burr Porter, (1976), "Optimal Portfolios with One Safe and One Risky Asset: Effects of Changes in Rate of Return and Risk," *Management Science*, 22, 1064-73.

Franke, Günter, Richard Stapleton, and Marti Subrahmanyam, (1998), "Who Buys and Who Sells Options: The Role of Options in an Economy with Background Risk," *Journal of Economic Theory*, 82, 89-109.

Gabaix, Xavier and David Laibson, (2002), "The 6D bias and the Equity Premium Puzzle," Ben Bernanke and Ken Rogoff ed., *NBER Macroeconomics Annual*, 16, 257-312.

Gollier, Christian, (2001), *The Economics of Risk and Time*, MIT Press, Cambridge.

Gollier, Christian, (2004), "Optimal Portfolio Risk with First-Order and Second -Order Predictability," *Contributions to Theoretical Economics*, Vol. 4, Issue 1, Article 4.

Grossman, Sanford and Guy Laroque, (1990), "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Assets," *Econometrica*, 58, 25-52.

Guiso, Luigi, Michael Haliassos, and Tullio Jappelli, (2003), "Household Stockholding in Europe: Where Do We Stand and Where Do We Go?," *Economic Policy*, 36, 123-170.

Hadar, Josef and Tae Kun Seo., (1990), "The Effect of Shifts in a Return Distribution on Optimal Portfolios," *International Economic Review*, 31, 721-736.

Hardy, Godfrey, John Littlewood, and George Polya, (1952) *Inequalities*, 2nd edition, Cambridge University Press.

Hayek, Frederick, (1945), "The Use of Knowledge in Society," *American Economic Review*, 35, 519-530.

Ingersoll, Jonathan, and Stephen Ross, (1992), "Waiting to Invest: Investment and Uncertainty," *Journal of Business*, 65, 1-29.

Kim, T.S., and E. Omberg, (1996), "Dynamic Nonmyopic Portfolio Behavior," *Review of Financial Studies*, 9, 141-61.

Kocherlakota, Narayana, (1996), "The Equity Premium: It's Still a Puzzle," *Journal of Economic Literature*, 34, 42-71.

Lehmann, Erich., (1988), "Comparing Location Experiments," *The Annals of Statistics*, 16, 521-533.

Leland, Hayne, (1968), "Savings and Uncertainty: the Precautionary Demand for Saving," *Quarterly Journal of Economics*, 82, 465-473.

Lucas, Robert, (1978), "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429-46.

Lynch, Antony, (1996), "Decision Frequency and Synchronization Across Agents: Implications for Aggregate Consumption and Equity Returns," *Journal of Finance,* 51, 1479-97.

Machina, Mark, (1982) ""Expected Utility" Analysis without the Independence Axiom," *Econometrica*, 50, 277-323.

Mehra, Rajnish and Edward Prescott, (1985), "The Equity Premium: a Puzzle," *Journal of Monetary Economics*, 10, 335-339.

Milgrom, Paul, (1981), "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics,* 12, 380-391.

Mitchell, Mark and John Mulherin, (1994), "The Impact of Public Information on the Stock Market," *Journal of Finance*, 49, 923-50.

Rothschild, Michael and Joseph Stiglitz, (1971), "Increasing risk II: Its Economic Consequences," *Journal of Economic Theory* 3, 66-84.

Schlee, Edward, (2001), “The Value of Information in Efficient Risk-Sharing Arrangements,” *American Economic Review*, 91, 509-524.

Shapiro, Carl and Hal Varian, (1999), *Information rules: a strategic guide to the network economy*, Harvard University Press, Boston.

Shiller, Robert, (1981), “Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?,” *American Economic Review*, 71, 421–436.

Van Nieuwerburgh, Stijn, and Laura Veldkamp, (2005), “Information Acquisition and Portfolio Under-Diversification,” Discussion Paper, New York University.

Veronesi, Pietro, (2000), “How Does Information Quality Affect Stock Returns?,” *Journal of Finance*, 55, 807-837.

Vives, Xavier, (2008), *Information and Learning in Markets: The Impact of Market Microstructure*, Princeton University Press.

Wang, Tan, (1993), “L_p-Frechet Differentiable Preferences and “Local Utility” Analysis,” *Journal of Economic Theory*, 61, 139-159.

Weil, Phillipe, (1992), “Equilibrium Asset Prices with Undiversifiable Labor Income Risk,” *Journal of Economic Dynamics and Control*, 16, 769-90.

Weitzman, Martin, (2007), “Subjective Expectations and Asset-Return Puzzles,” *American Economic Review*, 97, 1102-1130.