Partial regulation in vertically differentiated industries[*]

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Abstract

This paper provides theoretical foundations for a price-and-quality cap regulation of recently liberalized utilities in which vertically differentiated services are provided by a regulated incumbent and an unregulated entrant competing in price and quality. The model may equally well represent competition across industries. We establish that optimal weights in the cap depend also on the market served by the entrant, despite the latter is not directly concerned by regulation. This calls for the possibility that regulators use information about the whole industries, rather than on the sole incumbents. However, in unit demand frameworks, regulation can be performed as if the incumbent were still a monopolist, provided its decisions have no impact on the quality of the unregulated firm's products. In all cases, the proposed regulatory mechanism is evidenced to be robust to small errors in the social value of quality.

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1 Introduction

In markets where not only price (a monetary dimension) but also quality (a non-monetary dimension) matters, pure price regulation does not yield overall desirable outcomes, in general (see, for instance, Armstrong and Sappington [4] and Sappington [32]).

Specifically, when firms are compelled to obey a price cap, they are induced to cut costs, which may translate into quality under-provision. This issue regards potentially all network industries in which price cap is adopted. With reference to telecommunications, Vogelsang [33] observes that concerns about quality deterioration are widespread and that, indeed, such services are subject to price cap in most OECD countries and in several others. In fact, Rovizzi and Thompson [31] report that noticeable quality reduction was registered in British Telecom's services as soon as the company was submitted to price cap, after privatization.[1] According to Crew and Parker [18], among the various quality aspects that might suffer from price ceiling, most penalized seems to be service reliability, which is a crucial part of service value to end users[2].

De Fraja and Iozzi [20] look for a way to use price cap in environments with relevant quality aspects, under the motivation that:

> "Price-cap regulation (...) strikes a very good compromise between the theoretically rigorous foundation of the theory of optimal regulation (...) and the practitioner's requirement of the simple, easy-to-understand, easy-to-apply rule."

They integrate quality dimensions into the "standard" price cap, restructuring the latter as a *price-and-quality cap*. Characterizing the ideal composition of this incentive scheme with regard to multiproduct monopolies, they find two essential results. The first result is that, in the index that enters the formula, the appropriate weights of the different prices are (proportional to) the optimal quantities of the products sold by the regulated firm. In other words, it does not differ from the

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[1]On the other hand, Ai and Sappington [1] find that, after the introduction of incentive regulation in US telecommunications sectors, service quality deteriorated in some aspects but improved in others, so that the overall impact is ambiguous.

[2]Service reliability introduces an element of heterogeneity even in electricity, a product that is otherwise perfectly homogeneous. Specifically, in power sectors, reaction lags and supply interruptions are relevant quality dimensions (Crampes and Moreaux [15]).

"standard" price cap, where quality is not an issue[3] (Brennan [13]; Laffont and Tirole [26]). The second result in De Fraja and Iozzi [20] is that quality weights in the "extended" cap should be equal to consumer marginal surplus evaluated at the optimal prices and qualities. Billette de Villemeur [9] obtains similar findings with reference to monopolistic airline industries, where relevant dimensions are price and *service frequency*[4].

The findings described above do not need to extend to vertically differentiated oligopolies in which regulation concerns one sole firm. With regard to non-monopolistic sectors, it is known that, if a regulated incumbent competes with an unregulated passive fringe, then total market quantities are the optimal weights in the pure price cap only if fringe profits are *not* included in social welfare. By contrast, if fringe profits are taken to contribute to social welfare, then appropriate weights relate to the optimal quantities of the sole regulated firm. These results are found in Brennan [13], who further acknowledges that, when competitors are *not* price-takers, a different recipe is required. The author does not provide such a recipe though, even just for pure price cap.

The goal of this article is to provide theoretical foundations to price-and-quality cap regulation of oligopolies where a regulated dominant firm (a Stackelberg leader) competes in price and quality with one (or few) strategic follower(s), which operate unregulated.

The market structure and the regulatory setting we consider, namely a Stackelberg oligopoly with regulation of the sole leader, closely reflect the most common evolution that formerly public utilities have recently undergone as a result of the liberalization process. Typically, in those sectors, the incumbent, *i.e.* the former monopolist, acquires the Stackelberg leadership of the market, whereas the few competitors that have entered after liberalization place themselves as followers. The incumbent goes subject to regulatory control. By contrast, competitors operate unregulated, despite exerting market power in the seek for highest attainable profits. Our model is thus meant to stylize concentrated and partially regulated industries of this sort. To fix ideas, one may think about competition between regulated telephone companies and unregulated cable voice-over-Internet-Protocol or wireless cellular companies in voice telephony. One may further consider competition between regulated and unregulated cable pay-televisions. Another instance is inter-modal competition between regulated train operators and deregulated air

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[3]Standard price cap may find specifications according to the context. For instance, Billette de Villemeur, Cremer, Roy and Toledano [10] characterize a price-cap scheme that fits postal sector features.

[4]Under monopoly (though not in other frameworks), service frequency is equivalent to a pure quality attribute.

carriers in European transport industries.

To capture the relevance of quality provision and motivate quality regulation, we represent a market where vertically differentiated services are supplied to consumers exhibiting heterogeneous quality valuations. Our choice to model vertical (rather than horizontal) differentiation follows from the observation that, in the industrial contexts we refer to, consumers tend to share the same quality ranking, e.g. they perceive the product provided by the incumbent as superior to the product(s) offered by the competitor(s). For instance, in voice telephony, the services provided by telephone companies are generally more reliable than those provided by cable companies. Similarly, most of the times, air transport is considered to be more comfortable and reliable than rail transport. In turn, regulated cable televisions typically broadcast higher quality programmes and propose less advertising than unregulated competitors.

Importantly, the possibility that quality be regulated follows from the circumstance that it is observable and verifiable. This is actually the case in nearly all network industries. Insisting on the sectors aforementioned, it is indeed possible to observe and collect data about connection interruptions in voice telephony, advertising frequency and programme content in cable TV services, travel time and departure/arrival delays in transport services[5]. This is why, as the literature has pointed out, in network industries, service quality tends to be regulated even heavily, whereas infrastructure quality, which is hardly observable and verifiable, remains unregulated in general (see Martimort and Sand-Zantman [29], for instance).

Furthermore, in the utilities recalled above, the quality of the provided goods and/or services (though not the inner quality of the network infrastructures) can be adjusted in the short run. To reflect this circumstance, in our model, we take quality to be as flexible as price[6].

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[5] To mention only a few examples, data about TV channels (types of broadcast, audience shares), advertising (spot duration, frequency) and programmes (duration, content) are largely available (see, for instance, Carat [14]). In Italy, the regulated rail company is currently compelled to disclose information about delays at arrival. Analogous information about airlines could be made available by collecting data from air traffic controllers. Furthermore, delays are being increasingly monitored by consumers' associations and other concerned institutions (see, for instance, the report by Legambiente [28], based on Censis data, about the situation of railways commuter transport in Italy).

[6] If quality were a longer-run decision variable, as compared to price, then it would represent a very strategic instrument. This is the situation represented, for instance, in Grilo [23] and Cremer, De Rycke and Grimaud [16]. These are both mixed oligopoly models in which, however, the public firm has *no* strategic advantage over the private firm. Competitors play a two-stage game, in which they set qualities anticipating the impact their choices will have on prices. In Grilo [23], it emerges that first best is viable in mixed duopolies, while it is not in private regulated duopolies, because public managers are better informed than regulators. First best entails also in Cremer, De Rycke and Grimaud [16] as long as the budget constraint of the public firm does not bind. Otherwise, a second-best outcome arises, which is still preferable to that a private duopoly would

We begin by characterizing the *optimal policy*, which is assumed to be the target of the regulator. In the presence of strategic rivals that remain unregulated, the relevant benchmark is the policy that arises when a welfare-maximizing firm acts as a Stackelberg leader with respect to one (or more) profit-maximizing rival(s), under the requirement that its profits be non-negative. From this perspective, our work is reminiscent of the mixed oligopoly models in which the public firm is taken to enjoy a strategic advantage *vis-à-vis* the private operator(s)[7].

We then demonstrate how the optimal policy can be decentralized by means of a price-and-quality cap targeted to the sole dominant firm. Decentralization requires that the weights in the cap be set by taking into account not only the market served by the incumbent, but also the market(s) covered by the unregulated competitor(s). This signals that, at the implementation stage, regulatory bodies of liberalized industries should not be restricted to access and use information about the sole regulated firms. They should rather be allowed to extract and make use of information about the overall industry.

Still concerning implementation, we suggest that the regulatory target be progressively approached by applying the scheme iteratively over time, hinging on past data about market activities. We show that, when this strategy is indeed followed, the extended cap we propose exhibits the desirable property of being robust to small errors in the weights attached to the regulated firm's price and quality.

Our analysis further predicts that, in the particular case in which each consumer demands a single product unit, the incumbent is optimally regulated as if it were still a monopolist, provided its policy choices do not affect the follower's quality. Indeed, in the absence of quality effects, the optimal scheme collapses onto a standard price-and-quality cap that refers to the sole regulated firm. More precisely, this is a "standard" price-cap, where the price is replaced by the average *generalized* price of the individuals patronizing the incumbent. This may come out as a useful result for those sectors, such as (passenger) transportation, where unit demand is a reasonable assumption but relevant markets are difficult to define.

The remainder of the article is organized as follows. Section 2 presents the framework. Section 3 illustrates the impact of the incumbent's actions on the competitor's decisions and characterizes the optimal policy in a mixed Stackelberg oligopoly. Sec-

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yield. Noticeably, in our environment, the first-best outcome is beyond reach even without budget requirements and under "perfect" partial regulation.

[7]Within the domain of literature about mixed oligopolies, Bös [12] reaches the conclusion that a public firm facing the requirement to operate at zero profits should stick to a *modified Ramsey-pricing rule*. In turn, exploring a homogeneous-product Stackelberg game with the public firm in the leader's position, Beato and Mas Colell [6] show that the solution to this game involves average-cost pricing for the public firm. The latter anticipates the competitor's policy choices, setting quantity so as to break even at equilibrium.

tion 4 shows how this policy target can be decentralized by means of an appropriate price-and-quality cap. The latter is further explored in the unit demand case, which returns especially useful and intuitive insights. Section 5 concludes. Most of the mathematical details are relegated to the Appendices.

2 The model

We consider an industry where two firms provide vertically differentiated products. The two providers play a Stackelberg game. One firm, denoted L (for leader), is subject to regulation. The other firm, denoted F (for follower), is not. This framework allows us to closely represent situations in which the leader is the incumbent of a previously fully regulated industry, whereas the follower is a newcomer that entered the market after liberalization.

Firms' strategic variables are price and quality (respectively, p_i and q_i, $i = L, F$). To capture the circumstance that the latter are adjusted over the same time horizon, we suppose that either operator chooses its own price and quality simultaneously. We further take price and quality to be observable and verifiable, which is actually the case in several sectors, as aforementioned.

Under the previous assumptions, in the market game *à la* Stackelberg, firm L sets p_L and q_L anticipating its competitor's policy choice. In turn, firm F selects p_F and q_F taking the leader's price and quality as given.

The goods that firms provide are perfect substitutes, except for the difference in quality. We suppose that the leader's product is superior to the follower's $(q_L > q_F)$. It is however noteworthy that, *mutatis mutandis*, the analysis would be analogous in the converse case. Consumers are heterogeneous in their valuation for quality, which is represented by a parameter θ. More precisely, sticking to a quasi-linear framework, we assume that the net surplus a consumer of characteristic θ derives from the consumption of x units of quality q bought at unit price p writes

$$v_\theta(x, p, q) = u(x) - (p - \theta q)x. \tag{1}$$

The parameter θ is distributed over the interval $\left[\underline{\theta}, \overline{\theta}\right]$, with $\overline{\theta} > \underline{\theta} \geq 0$ and according to a continuous density function $f(\theta)$. The associated cumulative distribution function is denoted $F(\theta)$. Given her quality valuation, a θ-consumer patronizing firm $i \in \{L, F\}$ faces the so-called generalized price $\widetilde{p}_i(\theta) \equiv (p_i - \theta q_i)$,

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[7]In a more general formulation, one could allow the marginal valuation of quality to depend on the quality level, namely $\theta(q)$. However, imposing the restriction that $\theta(q) = \theta$, $\forall q$, does not affect the very nature of results, as long as variations in the quality level do not yield significant variations in the marginal valuation of quality.

that is the unit price p_i net of the benefits θq_i associated with product quality. A θ-consumer prefers to purchase the good from firm i, rather than from firm $j \neq i$, whenever by doing so he/she bears a smaller generalized price $(\widetilde{p}_i(\theta) < \widetilde{p}_j(\theta))$. Observe that, by construction, no consumer finds it beneficial to patronize both firms, as usual in environments with vertical differentiation.

Assuming that the regulated firm sells its (higher-quality) product at a (weakly) higher price, as compared to the competitor $(p_L \geq p_F)$, the marginal consumer, who is indifferent between the two operators, is characterized by the parameter value

$$\theta_m \equiv \frac{p_L - p_F}{q_L - q_F}. \tag{2}$$

Thus, individuals whose θ exceeds θ_m patronize firm L, whereas individuals whose θ is smaller than θ_m patronize firm F. We abstract from the possibility that some of the potential consumers abstain from making any purchase, an unlikely case for some "basic" services like telecommunications or daily transport services.

2.1 Consumer valuation of quality, demand and surplus

The demand of a θ-consumer is pinned down by maximizing (1) with respect to x. This yields

$$\frac{\partial u}{\partial x} = \widetilde{p}_i(\theta), \tag{3}$$

where $\widetilde{p}_i(\theta) = \arg\min\{\widetilde{p}_L(\theta), \widetilde{p}_F(\theta)\}$. Individual consumption $x_i(p_i, q_i; \theta)$ appears to be a function of the sole generalized price, $\widetilde{p}_i(\theta)$, of the consumed commodity. Since $\widetilde{p}_i(\theta)$ decreases with θ and provided $\widetilde{p}_i(\theta_m) = \widetilde{p}_j(\theta_m)$, the ranking of consumers in terms of quality valuation is reflected by their ranking in terms of individual consumption x_θ. Formally, $x_{\theta_1} \leq x_{\theta_2}$ whenever $\theta_1 \leq \theta_2$.

Relying upon (3), it is possible to establish the relationship between the impacts on consumption of marginal changes in price and quality. To see this, observe first that (3) holds for any p_i and q_i. Differentiating both sides with respect to p_i and to q_i and combining the two equations, we obtain

$$\frac{\partial x_i / \partial q_i}{-\partial x_i / \partial p_i} = \theta, \quad i = L, F. \tag{4}$$

This evidences that, for the demand of a θ-consumer to remain unchanged as price p_i is increased by one unit, quality q_i should be raised by an amount equal to the individual marginal valuation for quality, namely θ. It also follows that a consumer with a strictly higher quality valuation patronizing the same firm would consider an *increase* in (p_i, q_i) that leaves a θ-consumer indifferent as strictly beneficial. Conversely, a consumer with a strictly lower valuation would find it detrimental.

Opposite appreciations would arise if a *decrease* in (p_i, q_i) that leaves a θ-consumer indifferent were considered.

Firms' aggregate demands are immediately obtained by summing over the relevant ranges of θ's. Formally,

$$X_F(\mathbf{p}, \mathbf{q}) = \int_{\underline{\theta}}^{\theta_m(\mathbf{p}, \mathbf{q})} x_F(p_F, q_F; \theta) f(\theta) d\theta, \tag{5a}$$

$$X_L(\mathbf{p}, \mathbf{q}) = \int_{\theta_m(\mathbf{p}, \mathbf{q})}^{\overline{\theta}} x_L(p_L, q_L; \theta) f(\theta) d\theta, \tag{5b}$$

where $\mathbf{p}$ and $\mathbf{q}$ denote the vector of prices and qualities respectively. They display the rather standard properties that we briefly recall hereafter. For any $i, j \in \{L, F\}$:

1. $(\partial X_i / \partial p_i) < 0$: demand for firm i's product decreases with its own price p_i ;

2. $(\partial X_i / \partial q_i) > 0$: demand for firm i's product increases with its own quality q_i ;

3. $(\partial X_i / \partial p_j) > 0$: demand for firm i's product increases with the rival price p_j ;

4. $(\partial X_i / \partial q_j) < 0$: demand for firm i's product decreases with the rival quality q_j.

It is also straightforward to obtain aggregate consumer surplus as a function of prices and qualities. At this aim, we plug individual demands, as pinned down by (3), into the surplus function (1) and sum over the relevant ranges of θ's. This ultimately returns

$$V(\mathbf{p}, \mathbf{q}) = \int_{\underline{\theta}}^{\theta_m} v_\theta(x_\theta, p_F, q_F) f(\theta) d\theta + \int_{\theta_m}^{\overline{\theta}} v_\theta(x_\theta, p_L, q_L) f(\theta) d\theta. \tag{6}$$

2.2 Technologies and profits

We denote $C_i(X_i, q_i)$ the cost function of firm $i \in \{L, F\}$. This function is assumed to be continuous and increasing in both production level and quality. In formal terms $(\partial C_i / \partial X_i) > 0$ and $(\partial C_i / \partial q_i) > 0$, $i = L, F$. We further assume that $C_i(\cdot, +\infty) = +\infty$. This says that high quality products are so costly to improve that perfect products ($q_i = +\infty$) are never actually offered on the market. We finally assume that the firms never find it profitable to decrease the quality of their products down to zero. Taken together, these hypotheses ensure an interior solution to the determination of quality. Formally, there exists a finite $\overline{q}$ such that $0 < q_i < \overline{q}$, $\forall i$. Firm i's profit function is written

$$\pi_i(\mathbf{p}, \mathbf{q}) = p_i X_i - C_i(X_i, q_i), \quad i = L, F. \tag{7}$$

3 Characterization of the optimal policy

As a first step of our analysis, we characterize the *optimal policy*, which is to be taken as the regulatory target. This is the policy that would materialize in a mixed duopoly where a welfare-maximizing (public) firm were to play the market game as a Stackelberg leader *vis-à-vis* a profit-maximizing (private) competitor, under a non-negative profit constraint. As usual in Stackelberg games, the analysis is performed backward.

3.1 The price-and-quality policy of firm F

As a follower, firm F takes the price and quality of firm L as given and optimizes its own accordingly. Let $\varepsilon_F \equiv (p_F/X_F)\,(-\partial X_F/\partial p_F)$ be (the absolute value of) the demand elasticity with respect to its own price. The price-and-quality policy that maximizes firm F's profits is pinned down in the lemma hereafter.

Lemma 1 *The price-and-quality policy that maximizes $\pi_F\left(\mathbf{p},\mathbf{q}\right)$ is characterized by the following pair of conditions*

$$\frac{p_F - (\partial C_F/\partial X_F)}{p_F} = \frac{1}{\varepsilon_F}, \tag{8a}$$

$$p_F \frac{\partial X_F}{\partial q_F} = \frac{\partial C_F}{\partial q_F} + \frac{\partial C_F}{\partial X_F}\frac{\partial X_F}{\partial q_F}. \tag{8b}$$

Equation (8a) is the standard Lerner formula. It evidences that the follower acts as a monopolist *vis-à-vis* the "residual demand", X_F. Moreover, according to equation (8b), quality q_F is chosen so that marginal benefits from quality improvements (the left-hand side) equate marginal costs (the right-hand side). The latter are expressed by the sum of the direct costs of quality (the first term) and the indirect costs (as reflected in the second term), which follow from the demand increments induced by quality raise.

Rearranging (8b) and combining it with (8a) yields

$$\frac{\partial X_F/\partial q_F}{-\partial X_F/\partial p_F} = \frac{1}{X_F}\frac{\partial C_F}{\partial q_F}. \tag{9}$$

Interestingly, by analogy with equation (4), the ratio in the left-hand side of (9) can be interpreted as the aggregate marginal valuation of quality by firm F's clients. In turn, the right-hand side of (9) represents the average cost of a marginal increase in quality for this same firm. Although firm F is a profit maximizer, no distortion is introduced by the choices it makes in terms of quality. Given consumer valuation,

further quality improvement would not appear to be worth its costs[8].

3.2   The socially optimal policy

We now move to characterize the bundle of price and quality that firm L, the regulated operator, should implement so as to pursue social interests without incurring budgetary losses. In formal terms, the optimal (p_L, q_L)-pair is pinned down by maximizing the social welfare function

$$W(\mathbf{p}, \mathbf{q}) = V(\mathbf{p}, \mathbf{q}) + \pi_L(\mathbf{p}, \mathbf{q}) + \pi_F(\mathbf{p}, \mathbf{q}) \tag{10}$$

subject to the non-negative profit constraint

$$\pi_L(\mathbf{p}, \mathbf{q}) \geq 0, \tag{11}$$

taking into account that p_F and q_F obey the rules in (8a) and (8b)[9]. Let λ the Lagrange multiplier associated with (11). Further denote

$$\widetilde{\theta}_L \equiv \int_{\theta_m}^{\overline{\theta}} \frac{x_L(p_L, q_L; \theta)}{X_L} \theta f(\theta) \, d\theta,$$

$$\widetilde{\theta}_F \equiv \int_{\underline{\theta}}^{\theta_m} \frac{x_F(p_F, q_F; \theta)}{X_F} \theta f(\theta) \, d\theta,$$

the weighted average of quality valuations by the clients of firm L and F respectively. The price-and-quality pair that is optimal for the collectivity is characterized by the proposition hereafter.

Proposition 1 *Under Lemma 1, the leader's price-and-quality policy that maximizes (10) subject to (11) is characterized by the following pair of conditions:*

$$\frac{d\pi_L}{dp_L} = \frac{1}{1+\lambda}\left[X_L - \left(\frac{\partial X_F/\partial p_L}{-\partial X_F/\partial p_F}\right)X_F + X_F\frac{dp_F}{dp_L} - \widetilde{\theta}_F X_F\frac{dq_F}{dp_L}\right] \tag{12a}$$

$$\frac{d\pi_L}{dq_L} = \frac{-1}{1+\lambda}\left[\widetilde{\theta}_L X_L + \left(\frac{\partial X_F/\partial q_L}{-\partial X_F/\partial p_F}\right)X_F - X_F\frac{dp_F}{dq_L} + \widetilde{\theta}_F X_F\frac{dq_F}{dq_L}\right]. \tag{12b}$$

Equation (12a) and (12b) embody the variations induced both in follower's profits

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[8]This does *not* mean that consumer and firm's objectives are perfectly aligned, even when attention is restricted to the quality dimension. In fact, were the price lower, the demand would be larger. As a result, the average cost of quality would be smaller, calling for a strict improvement in terms of quality.

[9]As the follower is not subject to regulatory control, the regulator does not need to be concerned with its viability. Yet, to warrant that the incumbent does face an unregulated competitor, we implicitly take the follower's profits to be non-negative at the optimal policy.

π_F and in consumer net surplus V by a raise in price p_L and quality q_L respectively.

Specifically, the marginal impact of a raise in p_L on π_F is given by the term $\left(\frac{\partial X_F / \partial p_L}{-\partial X_F / \partial p_F} \right) X_F$ in (12a). It is the product of the marginal rates at which the leader's and the follower's price are to be substituted for the size of the market served by the follower not to vary, and the very same market size, X_F. Similarly, the marginal impact of a raise in q_L on π_F is represented by $\left(\frac{\partial X_F / \partial q_L}{-\partial X_F / \partial p_F} \right) X_F$ in (12b). It is the product of the marginal rate at which the leader's quality and the follower's price are to be substituted for the size of the market served by the follower not to vary, and, again, the very same market size, X_F. This evidences the relevance of cross-price and cross-quality effects $(\partial X_F / \partial p_L$ and $\partial X_F / \partial q_L)$ for the determination of the leader's optimal policy.

Besides, strategic interactions across firms are to be accounted for, as captured by the terms (dp_F / dp_L), (dq_F / dp_L), (dp_F / dq_L) and (dq_F / dq_L). Their presence is due to the circumstance that, with firm L the market leader, variations in its price and quality affect consumer utility not only in a direct way, but also through the impact on the rival's price and quality. Observe that the terms under scrutiny are systematically weighed by the follower's demand. They are further weighed by the average valuation of quality of the follower's clients, whenever interactions with quality q_F are concerned.

The apparent complexity of (12a) and (12b) may induce one to consider the definition of the optimal policy as a purely theoretical exercise, with no practical value. If the optimal policy does not find an explicit expression, exact implementation is indeed likely to be beyond reach. This makes more striking the results we present hereafter.

4 Decentralization through an extended price cap

In this section, we propose an extended version of the standard quality-adjusted price-cap scheme. More precisely, we unveil a construct that allows to decentralize the optimal policy of Proposition 1. After developing a general analysis, we specifically focus on environments in which consumers display unit demand.

4.1 The general case

Assume that the incumbent is left free to choose both price and quality, provided a *price-and-quality cap* is satisfied. Formally, let firm L pin down the pair (p_L, q_L) that maximizes its profits $\pi_L (\mathbf{p}, \mathbf{q})$ subject to the constraint

$$\alpha p_L - \beta q_L \leq P + \gamma p_F - \delta q_F. \tag{13}$$

As long as $\alpha > 0$, the regulatory constraint is tightened by an increase in price p_L. With $\beta > 0$, it is relaxed by an increase in quality q_L. These effects are mitigated or enhanced through their impact on the decisions to be made by the follower, which are explicitly considered in the extended price cap (the right-hand side of (13)). Define μ as the Lagrange multiplier associated with (13). The following proposition summarizes how the regulator should set the weights and the ceiling P for (12a) and (12b) to be enforced (see Appendix A for mathematical details).

Proposition 2 *In the ideal price-and-quality cap, weights are given by*

$$\alpha = X_L^{PR} - \nu X_F^{PR}, \tag{14a}$$

$$\beta = \widetilde{\theta}_L^{PR} X_L^{PR} - \theta_m^{PR} \nu X_F^{PR}, \tag{14b}$$

$$\gamma = -X_F^{PR}, \tag{14c}$$

$$\delta = -\widetilde{\theta}_F^{PR} X_F^{PR}, \tag{14d}$$

where

$$\nu \equiv \left(\frac{\partial X_F^{PR}/\partial p_L}{-\partial X_F^{PR}/\partial p_F} \right) = \left[1 + \frac{q_L^{PR} - q_F^{PR}}{x_m^{PR} f\left(\theta_m^{PR}\right)} \int_{\underline{\theta}}^{\theta_m^{PR}} \left(\frac{-\partial x_F^{PR}}{\partial p_F} \right) f\left(\theta\right) d\theta \right]^{-1}. \tag{15}$$

Moreover, if P is set so that $\pi_L \left(\mathbf{p}^{PR}, \mathbf{q}^{PR} \right) = 0$, then $\mu = 1/\left(1 + \lambda^{PR} \right)$.

Proposition 2 tells that, for the extended cap in (13) to implement the bundle (p_L, q_L) characterized in Proposition 2, it suffices (i) to set coefficients α, β, γ and δ as in (14a) to (14d) and (ii) to decrease P enough to wash out firm L's profits. The presence of the superscript PR indicates that the exact values are those obtained at the optimal policy, which is decentralized under the *Partial Regulatory* regime.

According to (14a), the appropriate price weight in the cap is given by the difference between two terms. The first term is the regulated firm's quantity evaluated at $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR} \right)$, namely X_L^{PR}. The second term consists in firm F's quantity evaluated at $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR} \right)$, namely X_F^{PR}, as multiplied by the coefficient ν that reflects product differentiation. Observe that ν is smaller than one[10]. Thus, firm L's output is given a larger relevance than firm F's output in the composition of the price weight. That is to say, the price weight α is obtained by subtracting from the regulated firm's quantity X_L^{PR} (the "standard" weight in cap formulae) a fraction of the quantity of

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[10]In Appendix A, we show that the coefficient ν is the ratio between the marginal variation in X_F^{PR} induced by an increase in p_L and the (absolute value of the) overall (marginal and inframarginal) variation in the same quantity X_F^{PR} as induced by an increase in p_F. The ratio is smaller than 1 since the (cross) effect of price p_L on the follower's demand is lower than the (own) effect of price p_F. Its specific magnitude depends on the difference between cross and own-price effects.

the (unregulated) competitor, νX_F^{PR}.

Similarly, the quality weight β, as defined by (14b), is given by the difference between two terms. The first term, $\widetilde{\theta}_L^{PR} X_L^{PR}$, is an aggregate measure of the quality appreciation by firm L's consumers. The second term is linked to the appreciation of quality by firm F's consumers. However, since the sole marginal clients of firm F are concerned by changes in q_L, the quality appreciation refers to θ_m^{PR} and *not* to $\widetilde{\theta}_F$. Note that, if qualities are observable (as assumed), the parameter θ_m can easily be computed. Interestingly enough, this marginal quality valuation is to be multiplied by νX_F^{PR}, the exact same part of α that refers to firm F. The coefficient is to be calculated by using the whole demand for firm F's product, X_F^{PR} (which is possibly observable), and not the consumption by firm F's marginal clients (which is not).

The downsizing of X_F^{PR} and $\theta_m^{PR} X_F^{PR}$ respectively in (14a) and (14b), which depends upon the coefficient ν, relates to three elements. Firstly, *ceteris paribus*, the smaller the quality spread between products $(q_L - q_F)$, the larger ν (and so the smaller α and β) . This suggests that less regulatory pressure needs to be exerted on firm L when products are not very differentiated. Indeed, in that case, the leader is disciplined by fierce competitive pressure. Secondly, the higher the marginal demand $x_m^{PR} f\left(\theta_m^{PR}\right)$, the weaker the regulation. A similar argument applies here: having a large amount of individuals indifferent between operators signals that, given prices and qualities, products are almost "perfect substitutes", so that competition is again a substitute for regulation. Lastly, the smaller the term $\int_{\underline{\theta}}^{\theta_m} \left(\frac{-\partial x_F^{PR}}{\partial p_F}\right) f\left(\theta\right) d\theta$, the higher ν, meaning that a relatively soft regulation is required when the entrant lacks market-power.

A clear message emerges from (14a) and (14b). Firstly, a proper regulation of firm L cannot spare reference to its competitor, firm F. Secondly, the larger the market share of the unregulated firm, the lower the required regulatory pressure on the regulated firm. In fact, if X^{PR} denotes the market size at $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR}\right)$, the optimal weight (14a) attached to price p in the price-cap formula can be rewritten $\alpha = \left[X^{PR} - (1 + \nu) X_F^{PR}\right]$. This also means that competition has a bigger impact on markets than it appears when considering the sole market share of the unregulated firm $\left(X_F^{PR}\right)$. This further evidences an important feature of our extended cap: it accounts for (and adapts to) the transition of liberalized sectors from regulated monopolies to increasingly more competitive, and thus less heavily regulated, markets. In particular, it captures the magnitude of the competitive effect through the coefficient ν that expresses the degree of product differentiation, as aforementioned. Lastly, the higher θ_m^{PR}, the lower the rewards for the regulated firm's product quality.

At the implementation stage, in the same vein as in Vogelsang and Finsinger [34], De Fraja and Iozzi [20], Billette de Villemeur [9] and Billette de Villemeur

and Vinella [11][11], one can conceive that the target $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR}\right)$ can be approached through an iterative process. More precisely, as for a "standard" price-cap, information on past market performance can be used to update the weights in the constraint at each step; and firm's profits may be progressively reduced by adjusting P till the point where $\pi_L = 0$. It is worth to mention that such a regulatory scheme exhibits a robustness property, which we state in the following proposition (see Appendix B for mathematical details).

Proposition 3 *Define $W\left(\alpha, \beta, \gamma, \delta\right)$ as the welfare level that is achieved when α, β, γ and δ are fixed as in (14a) to (14d) and P is progressively decreased until $\pi_L\left(\mathbf{p}, \mathbf{q}\right) = 0$. Variations in α and β have zero first-order effects:*

$$\frac{dW\left(\alpha, \beta, \gamma, \delta\right)}{d\alpha} = 0,$$
$$\frac{dW\left(\alpha, \beta, \gamma, \delta\right)}{d\beta} = 0.$$

According to Proposition 3, variations in the price weight and in the quality weight around their optimal values have no first-order effect on welfare. This means that, in terms of the welfare it yields, the scheme is robust to the possibility that the regulator be unable to set α and β exactly as dictated by (14a) and (14b).

4.2   The unit demand case

We now turn to explore environments in which each customer allocates a single unit of consumption to his/her preferred operator[12]. This specification does fit numerous real-world situations where regulation is actually performed. Consider, for instance, (passenger) transportation markets. With regard to the latter, demand is naturally modelled as a discrete choice across available alternatives. The unit demand assumption corresponds to adopting trips as consumption units and considering that alternatives are attached to the various ways to reach the envisioned (single) destination. Note also that a quality attribute like travel time is both observable and verifiable, hence it can be used to regulate (say) the operator that manages one of two competing modes.

In the unit demand framework, neither changes in price nor in quality impact

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[11]The works by Vogelsang and Finsinger [34], De Fraja and Iozzi [20] and Billette de Villemeur [9] belong to the wide family of papers that elaborate converging schemes of price regulation under monopoly. Billette de Villemeur and Vinella [11] propose a converging scheme of partial quantity regulation under Cournot oligopoly.

[12]Recall that, for each individual, the preferred operator is the one that ensures the lower generalized price, given his/her personal valuation for quality.

infra-marginal consumer decisions. Thus changes affect marginal consumers only. Nevertheless, as we show in Appendix C, this does *not* mean that regulation can be tailored to the sole characteristics of marginal consumers. Yet, in this setting, any lost consumer for one firm is a gain for its competitor. This involves that $\nu = 1$. Hence, the following proposition can be stated.

Proposition 4 *In environments where consumers display unit demand, the optimal weights to be attached to the leader's price and quality are given by*

$$\alpha^U = X_L^{PR} - X_F^{PR}, \tag{16a}$$
$$\beta^U = \widetilde{\theta}_L^{PR} X_L^{PR} - \theta_m^{PR} X_F^{PR}. \tag{16b}$$

According to (16a), the appropriate price weight is now simply the difference between the optimal quantity of the regulated and the unregulated firm. Moreover, (16b) says that the quality weight reduces to the difference between the aggregate quality appreciation of the regulated firm's consumers and that of the competitor's, which still embodies the sole quality appreciation of the marginal consumers.

Using (16a) and (16b) and recalling that $\left(p_L - \theta_m^{PR} q_L\right) = \left(p_F - \theta_m^{PR} q_F\right)$ by definition of the marginal type, the regulatory constraint can be rewritten as

$$\widetilde{p}_L X_L^{PR} \leq P - (\theta_m^{PR} - \widetilde{\theta}_F^{PR}) q_F X_F^{PR}. \tag{17}$$

This formulation evidences that the strategic interactions between firms do not yield any change in the fundamental structure of the incentives to be given to the firm. That is, in the end, what matters is $\widetilde{p}_L$, the average generalized price associated with the services provided by the regulated firm. This average generalized price $\widetilde{p}_L = (p_L - \widetilde{\theta}_L^{PR} q_L)$ involves an estimate of the marginal value of quality, namely $\widetilde{\theta}_L^{PR}$, which is simply the average quality valuation of the leader's consumers.

Integrating by parts to obtain

$$\int_{\underline{\theta}}^{\theta_m^{PR}} \theta f\left(\theta\right) d\theta = \theta_m^{PR} F\left(\theta_m^{PR}\right) - \int_{\underline{\theta}}^{\theta_m^{PR}} F\left(\theta\right) d\theta$$

and rearranging, (17) further becomes

$$\widetilde{p}_L X_L^{PR} \leq P - q_F \int_{\underline{\theta}}^{\theta_m^{PR}} F\left(\theta\right) d\theta. \tag{18}$$

As (18) shows, the regulatory constraint requires that, *ceteris paribus*, the average generalized price of the regulated firm, as weighted with the latter's optimal quantity,

be smaller the higher the quality provided by the unregulated competitor. That is, while price p_F plays no role in the scheme, the regulatory pressure to be exerted on firm L strengthens as q_F raises. Remarkably, this also involves that the second term in the right-hand side of (18) can be safely removed as long as the leader's policy decisions have no impact on the follower's quality choice. For instance, this would be the case in environments where unregulated operators offer some minimum quality level (say, because they obey some given standard) that does not adjust to variations in the incumbent's policy. In such contexts, the conclusion hereafter can be drawn.

Proposition 5 *In environments where consumers display unit demand and the policy chosen by the regulated firm does not affect the quality provided by the unregulated competitor, the ideal price-and-quality cap reduces to*

$$p_L - \widetilde{\theta}_L^{PR} q_L \leq \overline{p}. \tag{19}$$

According to Proposition 5, absent quality effects on the follower side, optimal regulation is tantamount to imposing a "standard" quality-adjusted price-cap. Albeit the regulatory policy does account for its impact on the whole industry, the target allocation can be decentralized by looking at the incumbent only: firm L can be regulated as if it were a monopolist. This may come out to be of importance with regard to industries where relevant markets are difficult to define.

As (19) evidences, with unit demand and no quality effect on the follower side, the regulator is only concerned with the quality weight, which is found to be $\beta^U = \widetilde{\theta}_L^{PR}$. This means that the incentive scheme hinges upon a single parameter, to be *exogenously* set. If quality is to be taken into account by the regulator, this parameter is the simplest information one can think of. This is indeed an *average* marginal valuation of quality by the consumers of the regulated firm, upon which the regulator may legitimately and more easily collect data. For instance, stated preferences can be (and are indeed largely) used to form time value estimates in passenger transport sectors. De Fraja and Iozzi [20] point out that a difficulty arises at the implementation stage of the ideal quality-adjusted price-cap they characterize with regard to monopolies. The difficulty streams from the circumstance that, in the implementation scheme they propose, quality valuation is *endogenously* determined by computing at each step consumer marginal surplus $(\partial V/\partial q)$[13]. To circumvent this problem, they need to introduce an additional constraint that further limits

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[13]Specifically, this creates a problem in terms of convergence of the regulatory algorithm to the second-best monopoly prices and qualities.

the regulated firm's choices. By contrast, and along the actual practice, we suggest that the social valuation of quality ought to be a policy attribute chosen and made public by the regulator. There are two reasons for that. First, small biases in the estimation of the quality valuation $\widetilde{\theta}_L^{PR}$ would not be an issue, since they would have no (first-order) effect on welfare (see Proposition 3). Second, with a fixed coefficient for quality valuation, our price-and-quality scheme would result less prone to manipulations and could be implemented in a more transparent way. It would thus be more likely to obtain public support.

5   Concluding remarks

There are essentially three insights to be drawn from our analysis.

Firstly, in a partially regulated industry, the regulatory agency should be able to hinge upon information on the whole industry. Information about the sole regulated firm does not appear to allow for efficient regulation, in general. In the extended price-and-quality cap we have looked at, appropriate weights depend on the (optimal) quantities provided by both the regulated incumbent and the follower, despite the latter is not directly concerned by regulation. This is necessary for the regulatory scheme to be adjusted to the competitive pressure the regulated firm bears.

Secondly, the sole situations in which regulators can legitimately focus on one firm are those where consumers typically purchase a single unit of the patronized good and the quality of the unregulated firm's products does not react to variations in the incumbent's policy. In those frameworks only, the optimal scheme relates exclusively to the regulated firm, which can thus be approached as if it were a monopolist.

Thirdly, price-cap regulation is robust to small errors in the weights attached to the regulated firm's decision variables. It thus appears reasonable to hinge upon such regulatory mechanisms to account for the quality dimension of market interactions, despite they may rely upon (possibly imperfect) statistical estimates.

Comparing our findings with those in Brennan [13], it emerges that, in general, the optimal cap would not be the same if the regulated incumbent were to compete with an unregulated fringe. As previously recalled, Brennan establishes that, as long as fringe profits are included in social welfare, pure price-cap regulation of an incumbent competing with a passive fringe is to refer to the former's output only. From our results it should be clear that, in general, the same cannot be said with regard to price-and-quality cap regulation of an incumbent that faces strategic competitor(s) whose profits are taken to contribute to social welfare. Our study thus extends that of Brennan to partially regulated industries in which unregulated

competitors are endowed with market power.

All along the analysis, we have taken firms to behave non cooperatively. In practice, they might have an incentive to collude so as to undo the partial regulatory policy. That asymmetrically regulated firms can profitably coordinate against the regulator is shown by Aubert and Pouyet [5] in Bayesian environments with adverse selection. It would thus be useful to study the ideal price-and-quality cap with regard to collusive settings. This is left for further research.

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A Optimal policy and ideal cap

The first-order condition that follows from the constrained maximization of (10) with respect to price p_L is given by

$$\frac{d\pi_L}{dp_L} = \left(\frac{-1}{1+\lambda}\right)\left(\frac{dV}{dp_L} + \frac{d\pi_F}{dp_L}\right). \tag{20}$$

The left-hand side of (20) is the *total* derivative of firm $L's$ profits with respect to p_L, which also accounts for the indirect effects on π_L of a variation in p_L. Indeed, the latter induces adjustments in the rival's price and quality, p_F and q_F. Formally, we have

$$\frac{d\pi_L}{dp_L} = \frac{\partial\pi_L}{\partial p_L} + \frac{\partial\pi_L}{\partial p_F}\frac{dp_F}{dp_L} + \frac{\partial\pi_L}{\partial q_F}\frac{dq_F}{dp_L}.$$

Similarly, the impact on consumer surplus of a change in price p_L can be decomposed as

$$\frac{dV}{dp_L} = \frac{\partial V}{\partial p_L} + \frac{\partial V}{\partial p_F}\frac{dp_F}{dp_L} + \frac{\partial V}{\partial q_F}\frac{dq_F}{dp_L}. \tag{21}$$

With the leader serving the market segment $\left[\theta_m, \overline{\theta}\right]$ and the follower the segment $\left[\underline{\theta}, \theta_m\right)$, Roy's identity yields

$$\frac{\partial V}{\partial p_L} = -\int_{\theta_m}^{\overline{\theta}} x_L\left(p_L, q_L; \theta\right) f\left(\theta\right) d\theta = -X_L \tag{22a}$$

$$\frac{\partial V}{\partial p_F} = -\int_{\underline{\theta}}^{\theta_m} x_F\left(p_F, q_F; \theta\right) f\left(\theta\right) d\theta = -X_F \tag{22b}$$

$$\frac{\partial V}{\partial q_L} = \int_{\theta_m}^{\overline{\theta}} x_L\left(p_L, q_L; \theta\right) \theta f\left(\theta\right) d\theta = \widetilde{\theta}_L X_L \tag{22c}$$

$$\frac{\partial V}{\partial q_F} = \int_{\underline{\theta}}^{\theta_m} x_F\left(p_F, q_F; \theta\right) \theta f\left(\theta\right) d\theta = \widetilde{\theta}_F X_F. \tag{22d}$$

By contrast, since $(\partial\pi_F/\partial p_F) = 0$ and $(\partial\pi_F/\partial q_F) = 0$, the derivative of firm $F's$ profits with respect to p_L simplifies as

$$\begin{aligned}
\frac{d\pi_F}{dp_L} &= \frac{\partial\pi_F}{\partial p_L} \\
&= \left(p_F - \frac{\partial C_F}{\partial X_F}\right)\frac{\partial X_F}{\partial p_L} \\
&= X_F\left(\frac{\partial X_F/\partial p_L}{-\partial X_F/\partial p_F}\right). \tag{23}
\end{aligned}$$

Plugging (22a), (22b) and (22d) into (21) and then (23) and (21) into (20), we ultimately obtain (12a).

Turning now to the second relevant dimension, the first-order condition for a

constrained maximum of (10) with respect to quality q_L is given by

$$\frac{d\pi_L}{dq_L} = \left(\frac{-1}{1+\lambda}\right)\left(\frac{dV}{dq_L} + \frac{\partial\pi_F}{\partial q_L}\right). \tag{24}$$

A similar analysis yields the following decomposition of the variation in consumer surplus

$$\frac{dV}{dq_L} = \widetilde{\theta}_L X_L - X_F \frac{dp_F}{dq_L} + \widetilde{\theta}_F X_F \frac{dq_F}{dq_L}. \tag{25}$$

Again, relying upon the first-order conditions of firm F's profit-maximization, we can write

$$\frac{\partial\pi_F}{\partial q_L} = X_F\left(\frac{\partial X_F/\partial q_L}{-\partial X_F/\partial p_F}\right). \tag{26}$$

Replacing (25) and (26) into (24), we ultimately obtain (12b).

From (12a) and (12b), the optimal weights α and β are found to be as follows

$$\begin{aligned}
\alpha &= X_L^{PR} - X_F^{PR}\left(\frac{\partial X_F^{PR}/\partial p_L}{-\partial X_F/\partial p_F}\right), \\
\beta &= \widetilde{\theta}_L^{PR} X_L^{PR} + X_F^{PR}\left(\frac{\partial X_F^{PR}/\partial q_L}{-\partial X_F/\partial p_F}\right).
\end{aligned}$$

The derivatives $\left(\partial X_F^{PR}/\partial p_L\right)$ and $\left(\partial X_F^{PR}/\partial q_L\right)$ reflect only marginal variations, as firm F's inframarginal customers are not concerned by changes in firm L's price and quality. Therefore, we have

$$\begin{aligned}
\frac{\partial X_F^{PR}}{\partial p_L} &= x_m^{PR} f\left(\theta_m^{PR}\right)\frac{\partial\theta_m^{PR}}{\partial p_L} \\
&= \frac{x_m^{PR} f\left(\theta_m^{PR}\right)}{q_L - q_F}
\end{aligned} \tag{27}$$

together with

$$\begin{aligned}
\frac{\partial X_F^{PR}}{\partial q_L} &= x_m^{PR} f\left(\theta_m^{PR}\right)\frac{\partial\theta_m^{PR}}{\partial q_L} \\
&= -\theta_m^{PR}\frac{x_m^{PR} f\left(\theta_m^{PR}\right)}{q_L - q_F} \\
&= -\theta_m^{PR}\frac{\partial X_F^{PR}}{\partial p_L},
\end{aligned} \tag{28a}$$

where $x_m^{PR} f\left(\theta_m^{PR}\right)$ measures consumption by marginal clients, *i.e.* consumers char-

acterized by quality valuation $\theta_m^{PR} \equiv \left(p_L^{PR} - p_F^{PR} \right) / \left(q_L^{PR} - q_F^{PR} \right)$. We also have

$$
\begin{aligned}
\frac{\partial X_F^{PR}}{\partial p_F} &= \int_{\underline{\theta}}^{\theta_m^{PR}} \frac{\partial x^{PR}}{\partial p_F} f(\theta)\, d\theta + x_m^{PR} f\left(\theta_m^{PR}\right) \frac{\partial \theta_m^{PR}}{\partial p_F} \\
&= \int_{\underline{\theta}}^{\theta_m^{PR}} \frac{\partial x^{PR}}{\partial p_F} f(\theta)\, d\theta - \frac{x_m^{PR} f\left(\theta_m^{PR}\right)}{q_L - q_F}.
\end{aligned} \tag{29}
$$

Thus

$$
\begin{aligned}
\left(\frac{\partial X_F^{PR}/\partial p_L}{-\partial X_F/\partial p_F} \right) &= \frac{\frac{x_m^{PR} f\left(\theta_m^{PR}\right)}{q_L - q_F}}{\int_{\underline{\theta}}^{\theta_m^{PR}} \left(\frac{-\partial x^{PR}}{\partial p_F} \right) f(\theta)\, d\theta + \frac{x_m^{PR} f\left(\theta_m^{PR}\right)}{q_L - q_F}} = \nu, \\
\left(\frac{\partial X_F^{PR}/\partial q_L}{-\partial X_F/\partial p_F} \right) &= -\theta_m^{PR} \left(\frac{\partial X_F^{PR}/\partial p_L}{-\partial X_F/\partial p_F} \right) = -\theta_m^{PR} \nu.
\end{aligned}
$$

The weights α and β can thus be rewritten as

$$
\begin{aligned}
\alpha &= X_L^{PR} - \nu X_F^{PR}, \\
\beta &= \widetilde{\theta}_L^{PR} X_L^{PR} - \theta_m^{PR} \nu X_F^{PR}.
\end{aligned}
$$

The other weights in Proposition 2 follow straightforwardly.

B   Robustness of the scheme

Since $\pi_L = 0$, we can compute

$$
\frac{dW}{d\alpha} = \frac{d}{d\alpha}\left(V + \pi_F \right).
$$

We have

$$
\begin{aligned}
\frac{dV}{d\alpha} &= \left(\frac{\partial V}{\partial p_L} + \frac{\partial V}{\partial p_F}\frac{dp_F}{dp_L} + \frac{\partial V}{\partial q_F}\frac{dq_F}{dp_L} \right) \frac{dp_L}{d\alpha} + \left(\frac{\partial V}{\partial q_L} + \frac{\partial V}{\partial p_F}\frac{dp_F}{dq_L} + \frac{\partial V}{\partial q_F}\frac{dq_F}{dq_L} \right) \frac{dq_L}{d\alpha} \\
&= \left(-X_L - X_F \frac{dp_F}{dp_L} + \widetilde{\theta}_F X_F \frac{dq_F}{dp_L} \right) \frac{dp_L}{d\alpha} + \left(\widetilde{\theta}_L X_L - X_F \frac{dp_F}{dq_L} + \widetilde{\theta}_F X_F \frac{dq_F}{dq_L} \right) \frac{dq_L}{d\alpha}
\end{aligned}
$$

together with

$$
\begin{aligned}
\frac{d\pi_F}{d\alpha} &= \frac{\partial \pi_F}{\partial p_L}\frac{dp_L}{d\alpha} + \frac{\partial \pi_F}{\partial q_L}\frac{dq_L}{d\alpha} \\
&= X_F \left(\frac{\partial X_F/\partial p_L}{-\partial X_F/\partial p_F} \right) \frac{dp_L}{d\alpha} + X_F \left(\frac{\partial X_F/\partial q_L}{-\partial X_F/\partial p_F} \right) \frac{dq_L}{d\alpha}.
\end{aligned}
$$

Hence, we can write

$$
\frac{dW}{d\alpha} = \left[-X_L - X_F \frac{dp_F}{dp_L} + \widetilde{\theta}_F X_F \frac{dq_F}{dp_L} + X_F \left(\frac{\partial X_F / \partial p_L}{-\partial X_F / \partial p_F} \right) \right] \frac{dp_L}{d\alpha} \tag{30}
$$
$$
+ \left[\widetilde{\theta}_L X_L - X_F \frac{dp_F}{dq_L} + \widetilde{\theta}_F X_F \frac{dq_F}{dq_L} + X_F \left(\frac{\partial X_F / \partial q_L}{-\partial X_F / \partial p_F} \right) \right] \frac{dq_L}{d\alpha}.
$$

Proceeding similarly with β, we can also write

$$
\frac{dW}{d\beta} = \frac{d}{d\beta} (V + \pi_F)
$$
$$
= \left[-X_L - X_F \frac{dp_F}{dp_L} + \widetilde{\theta}_F X_F \frac{dq_F}{dp_L} + X_F \left(\frac{\partial X_F / \partial p_L}{-\partial X_F / \partial p_F} \right) \right] \frac{dp_L}{d\beta} \tag{31}
$$
$$
+ \left[\widetilde{\theta}_L X_L - X_F \frac{dp_F}{dq_L} + \widetilde{\theta}_F X_F \frac{dq_F}{dq_L} + X_F \left(\frac{\partial X_F / \partial q_L}{-\partial X_F / \partial p_F} \right) \right] \frac{dq_L}{d\beta}.
$$

B.1 Imperfections in α

With $\pi_L = 0$, we can write

$$
\frac{d\pi_L}{dp_L} \frac{dp_L}{d\alpha} + \frac{d\pi_L}{dq_L} \frac{dq_L}{d\alpha} = 0,
$$

which yields

$$
\frac{dp_L}{d\alpha} = - \frac{d\pi_L / dq_L}{d\pi_L / dp_L} \frac{dq_L}{d\alpha}.
$$

The firm's first-order conditions with respect to p_L and q_L are written

$$
\frac{d\pi_L}{dp_L} = \mu \left(\alpha - \gamma \frac{dp_F}{dp_L} + \delta \frac{dq_F}{dp_L} \right) \tag{32}
$$
$$
\frac{d\pi_L}{dq_L} = -\mu \left(\beta + \gamma \frac{dp_F}{dq_L} - \delta \frac{dq_F}{dq_L} \right). \tag{33}
$$

Hence, we can write

$$
\frac{dp_L}{d\alpha} = \frac{\beta + \gamma \frac{dp_F}{dq_L} - \delta \frac{dq_F}{dq_L}}{\alpha - \gamma \frac{dp_F}{dp_L} + \delta \frac{dq_F}{dp_L}} \frac{dq_L}{d\alpha}.
$$

For the optimal values of the parameters α, β, γ and δ as defined by (14a)-(14d), we have

$$
\frac{dp_L}{d\alpha} = \frac{\widetilde{\theta}_L^{PR} X_L^{PR} - X_F^{PR} \frac{dp_F}{dq_L} + \widetilde{\theta}_F^{PR} X_F^{PR} \frac{dq_F}{dq_L} - \theta_m^{PR} \nu X_F^{PR}}{X_L^{PR} + X_F^{PR} \frac{dp_F}{dp_L} - \widetilde{\theta}_F^{PR} X_F^{PR} \frac{dq_F}{dp_L} - \nu X_F^{PR}} \frac{dq_L}{d\alpha}.
$$

Plug this into (30), together with the two identities established above

$$
\left(\frac{\partial X_F^{PR} / \partial p_L^{PR}}{-\partial X_F^{PR} / \partial p_F^{PR}} \right) = \nu \quad and \quad \left(\frac{\partial X_F^{PR} / \partial q_L^{PR}}{-\partial X_F^{PR} / \partial p_F^{PR}} \right) = -\theta_m^{PR} \nu,
$$

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to obtain:

$$\frac{dW^{PR}}{d\alpha} = 0.$$

B.2 Imperfections in β

With $\pi_L = 0$, we can write

$$\frac{d\pi_L}{dp_L}\frac{dp_L}{d\beta} + \frac{d\pi_L}{dq_L}\frac{dq_L}{d\beta} = 0,$$

which yields

$$\frac{dp_L}{d\beta} = -\frac{d\pi_L/dq_L}{d\pi_L/dp_L}\frac{dq_L}{d\beta}.$$

Replacing from (32) and (33) returns

$$\frac{dp_L}{d\beta} = \frac{\beta + \gamma\frac{dp_F}{dq_L} - \delta\frac{dq_F}{dq_L}}{\alpha - \gamma\frac{dp_F}{dp_L} + \delta\frac{dq_F}{dp_L}}\frac{dq_L}{d\beta}$$

Substituting into (31), for the optimal values of the parameters α, β, γ and δ as defined by (14a)-(14d), we obtain

$$\frac{dW^{PR}}{d\beta} = 0.$$

C Regulation and quality valuation of marginal consumer

Assume that firm L and firm F produce the high and low-quality good respectively. This means that firm L serves consumers with a marginal valuation of quality θ in $\left[\theta_m, \overline{\theta}\right]$, while firm F serves consumers with marginal variation in $[\underline{\theta}, \theta_m)$. Remind that the threshold valuation θ_m is given by the ratio $\left[(p_L - p_F)/(q_L - q_F)\right]$.

By definition, in the unit demand case, price and quality changes have an impact on the marginal consumer only. As a result,

$$\frac{\partial X_F}{\partial p_F} = f(\theta_m)\frac{\partial \theta_m}{\partial p_F} = -\frac{f(\theta_m)}{q_L - q_F} \tag{34a}$$

$$\frac{\partial X_F}{\partial q_F} = f(\theta_m)\frac{\partial \theta_m}{\partial q_F} = \theta_m\frac{f(\theta_m)}{q_L - q_F}. \tag{34b}$$

One thus obtains

$$\frac{\partial X_F}{\partial q_F} = -\theta_m\frac{\partial X_F}{\partial p_F},$$

that is to say,

$$\frac{\partial \theta_m}{\partial q_F} = -\theta_m\frac{\partial \theta_m}{\partial p_F}. \tag{35}$$

We hereafter check whether and, if so, under which conditions, a similar relationship holds true when changes in p_L and q_L are considered.

Observe that, in the framework under scrutiny, changes made by the leader have an impact on the choices of the follower. This explains why the impact of changes in p_L and q_L on the marginal consumer are not straightforward. A standard decomposition leads to

$$
\begin{aligned}
\frac{d\theta_m}{dp_L} &= \frac{\partial\theta_m}{\partial p_L} + \frac{\partial\theta_m}{\partial p_F}\frac{\partial p_F}{\partial p_L} + \frac{\partial\theta_m}{\partial q_F}\frac{\partial q_F}{\partial p_L} \\
&= \frac{1}{q_L - q_F}\left(1 - \frac{\partial p_F}{\partial p_L} + \theta_m\frac{\partial q_F}{\partial p_L}\right) \quad (36\text{a}) \\
\frac{d\theta_m}{dq_L} &= \frac{\partial\theta_m}{\partial q_L} + \frac{\partial\theta_m}{\partial p_F}\frac{\partial p_F}{\partial q_L} + \frac{\partial\theta_m}{\partial q_F}\frac{\partial q_F}{\partial q_L} \\
&= \frac{1}{q_L - q_F}\left(-\theta_m - \frac{\partial p_F}{\partial q_L} + \theta_m\frac{\partial q_F}{\partial q_L}\right). \quad (36\text{b})
\end{aligned}
$$

If, as assumed, firm F profit maximization has an interior solution, p_F and q_F are determined by the system of first-order conditions

$$
\begin{aligned}
\left(p_F - \frac{\partial C_F}{\partial X_F}\right)\frac{f(\theta_m)}{q_L - q_F} &= F(\theta_m) \\
\left(p_F - \frac{\partial C_F}{\partial X_F}\right)\frac{f(\theta_m)}{q_L - q_F} &= \frac{1}{\theta_m}\frac{\partial C_F}{\partial q_F}.
\end{aligned}
$$

This yields

$$
\theta_m F(\theta_m) = \frac{\partial C_F}{\partial q_F}. \quad (37)
$$

Differentiating both sides of (37) with respect to p_L and q_L returns the following pair of equalities:

$$
\begin{aligned}
\frac{d\theta_m}{dp_L}F(\theta_m) + \theta_m f(\theta_m)\frac{d\theta_m}{dp_L} &= \frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dp_L} + \frac{\partial^2 C_F}{\partial q_F \partial X_F}\left(\frac{\partial X_F}{\partial p_F}\frac{dp_F}{dp_L} + \frac{\partial X_F}{\partial q_F}\frac{dq_F}{dp_L}\right) \\
\frac{d\theta_m}{dq_L}F(\theta_m) + \theta_m f(\theta_m)\frac{d\theta_m}{dq_L} &= \frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dq_L} + \frac{\partial^2 C_F}{\partial q_F \partial X_F}\left(\frac{\partial X_F}{\partial p_F}\frac{dp_F}{dq_L} + \frac{\partial X_F}{\partial q_F}\frac{dq_F}{dq_L}\right).
\end{aligned}
$$

Making use of both (34a) and (34b), they can be respectively rewritten

$$
\frac{d\theta_m}{dp_L} = \frac{\frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dp_L} - \frac{\partial^2 C_F}{\partial q_F \partial X_F}\frac{f(\theta_m)}{q_L - q_F}\left(\frac{dp_F}{dp_L} - \theta_m\frac{dq_F}{dp_L}\right)}{F(\theta_m) + \theta_m f(\theta_m)} \quad (38\text{a})
$$

$$
\frac{d\theta_m}{dq_L} = \frac{\frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dq_L} - \frac{\partial^2 C_F}{\partial q_F \partial X_F}\frac{f(\theta_m)}{q_L - q_F}\left(\frac{dp_F}{dq_L} - \theta_m\frac{dq_F}{dq_L}\right)}{F(\theta_m) + \theta_m f(\theta_m)}. \quad (38\text{b})
$$

Remind that both left-hand sides were defined in (36a) and (36b). This allows us to rewrite both equations as a function of the sole follower reactions to leader decision

changes, *i.e.* (dp_F/dp_L), (dq_F/dp_L), (dp_F/dq_L) and (dq_F/dq_L). More precisely, we obtain

$$\frac{dp_F}{dp_L} - \theta_m \frac{dq_F}{dp_L} = \frac{F(\theta_m) + \theta_m f(\theta_m) - (q_L - q_F)\frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dp_L}}{F(\theta_m) + \theta_m f(\theta_m) - f(\theta_m)\frac{\partial^2 C_F}{\partial q_F \partial X_F}} \qquad (39a)$$

$$\frac{dp_F}{dq_L} - \theta_m \frac{dq_F}{dq_L} = \frac{-\theta_m[F(\theta_m) + \theta_m f(\theta_m)] - (q_L - q_F)\frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dq_L}}{F(\theta_m) + \theta_m f(\theta_m) - f(\theta_m)\frac{\partial^2 C_F}{\partial q_F \partial X_F}}. \qquad (39b)$$

If $C_F(\cdot)$ is linear in q_F, so that $(\partial^2 C_F/\partial q_F^2) \equiv 0$, then it further follows that

$$\frac{dp_F}{dq_L} - \theta_m \frac{dq_F}{dq_L} = -\theta_m \left(\frac{dp_F}{dp_L} - \theta_m \frac{dq_F}{dp_L}\right).$$

As a result, (38a) and (38b) yield

$$\frac{d\theta_m}{dq_L} = -\theta_m \frac{d\theta_m}{dp_L}, \qquad (40)$$

an equality that exactly mirrors the relationship (35) obtained for the follower.

More generally, from (37) we know that

$$\begin{aligned}
\frac{\partial^2 C_F}{\partial q_F^2} &= \frac{\partial}{\partial q_F}[\theta_m F(\theta_m)] \\
&= \frac{\theta_m}{q_L - q_F}[F(\theta_m) + \theta_m f(\theta_m)].
\end{aligned}$$

From the unit-demand assumption, we have $X_F = F(\theta_m)$ so that (37) also yields

$$\frac{\partial^2 C_F}{\partial q_F \partial X_F} = \frac{\partial}{\partial X_F}[\theta_m F(\theta_m)] = \theta_m.$$

Plugging these two last results into (38a) and (38b), one obtains

$$\frac{d\theta_m}{dp_L} = \frac{\theta_m}{q_L - q_F}\left(\frac{dq_F}{dp_L} - \frac{f(\theta_m)}{F(\theta_m) + \theta_m f(\theta_m)}\left(\frac{dp_F}{dp_L} - \theta_m \frac{dq_F}{dp_L}\right)\right) \qquad (41a)$$

$$\frac{d\theta_m}{dq_L} = \frac{\theta_m}{q_L - q_F}\left(\frac{dq_F}{dq_L} - \frac{f(\theta_m)}{F(\theta_m) + \theta_m f(\theta_m)}\left(\frac{dp_F}{dq_L} - \theta_m \frac{dq_F}{dq_L}\right)\right), \qquad (41b)$$

while substituting those same results into (39a) and (39b) yields

$$\frac{dp_F}{dp_L} - \theta_m \frac{dq_F}{dp_L} = \left(1 + \frac{\theta_m f(\theta_m)}{F(\theta_m)}\right)\left(1 - \theta_m \frac{dq_F}{dp_L}\right) \qquad (42a)$$

$$\frac{dp_F}{dq_L} - \theta_m \frac{dq_F}{dq_L} = -\theta_m\left(1 + \frac{\theta_m f(\theta_m)}{F(\theta_m)}\right)\left(1 + \frac{dq_F}{dq_L}\right). \qquad (42b)$$

Combining (41a) with (42a) and (41b) with (42b) returns

$$
\frac{d\theta_m}{dp_L} = \frac{\theta_m}{q_L - q_F} \left(\left(1 + \frac{\theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)} \right) \frac{dq_F}{dp_L} - \frac{f\left(\theta_m\right)}{F\left(\theta_m\right)} \right)
$$

$$
\frac{d\theta_m}{dq_L} = \frac{\theta_m}{q_L - q_F} \left(\left(1 + \frac{\theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)} \right) \frac{dq_F}{dq_L} + \frac{\theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)} \right).
$$

Therefore

$$
\left(\frac{d\theta_m}{dq_L} + \theta_m \frac{d\theta_m}{dp_L} \right) = \frac{\theta_m}{q_L - q_F} \left(1 + \frac{\theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)} \right) \left(\frac{dq_F}{dq_L} + \theta_m \frac{dq_F}{dp_L} \right), \qquad (43)
$$

that is $(d\theta_m/dq_L) = [-\theta_m (d\theta_m/dp_L)]$ if and only if

$$
\theta_m = \frac{-\left(dq_F/dq_L\right)}{\left(dq_F/dp_L\right)}.
$$

In words, even in the unit demand case, an increase of one unit in q_L is *not* equivalent to a decrease of θ_m units in p_L, in general. Unless costs can be assumed to be linear in quality, the impact of price changes and that of quality changes are *not* related to each other through marginal consumer quality valuation θ_m.