The Incentives to Cooperate in Local Public Goods Supply: A Repeated Game with Imperfect Monitoring*

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Abstract: This paper applies the tacit coordination framework to the public economic context. We develop a two-country model where each invests in a local public good which produces positive externalities. Each country's effort investment is private information and cannot be directly observed. It is only inferred from the observed level of public output which is a function of investment effort devoted towards its production plus a random shock. In a repeated game setting, we characterize the condition for the existence of a *cut-off trigger strategy equilibrium* such that the two countries have no incentives to deviate from the full cooperation. We then analyze how the optimal value of the cut-off point changes with the spillover and discount parameters. Finally, we show that increasing (from 0) the correlation between the two country-specific shocks gives rise to a manipulation of information thereby restricting the prospects for cooperation.

Keywords: Local Public Goods, Externality, Uncertainty, Repeated Game.

JEL Classification: H7, C73

1 Introduction

In the presence of spillovers, decentralized provision of local public goods leads to an inefficient outcome. If, however, local jurisdictions have repeated interactions, they may tacitly cooperate so as to internalize cross-border externalities. Indeed, according to the "folk theorems" cooperation can be sustained as a Nash equilibrium of a repeated game by strategies of reciprocity as long as one's partner does not discount the future too heavily. Our objective in this paper is then to explore the consequences of imperfect information with respect to the cost levels of the local public goods provided by neighboring jurisdictions on the sustainability of efficient outcomes in an infinitely repeated game.

The literature on local public good provision typically deals with a perfect information situation. In several circumstances, however, there is an information asymmetry with respect to local cost conditions. One can think of the costs of research and development activities or the costs of investment in environmental quality. For example, Cornes and Silva (2000, 2002) refer to an analysis of Mäler (1991) who noted about the Acid rain problem in Europe that "*the control costs and environmental damage in one particular country is known to that country only*". It is also well recognized in the literature on fiscal federalism that private information with respect to both preferences and cost conditions poses important problems for the design of transfer schemes at the central level (e.g., Costello (1993)). Clearly, imperfect information about technology and local cost conditions may impose significant constraints on the ability of local jurisdictions to achieve an efficient outcome. In a dynamic framework, it restricts the effectiveness of the threat of non-cooperation in the future in order cooperate today since deviations from cooperation are only imperfectly observed due to private effort investments in public goods.

We develop a two-country model where each provides a local public good. These public goods produce spillover benefits which are enjoyed by residents of the other country. The output of the public good in each country is a function of the cost of effort devoted towards its production plus a country-specific shock. While public output in each country is perfectly observed, the cost of effort in each country is private information and thereby cannot be directly observed in the other country. Finally, we consider that the situation is repeated over time so that the two countries may tacitly cooperate and internalize externalities through decentralized strategies of reciprocity. Specifically, we focus in this paper on *cut-off trigger strategy equilibria.* In other words, each country produces the optimal level of effort as long as each country's realized public output is above a certain level. If realized public output in one country (or both) falls below this level or cut-off point, the two countries revert to the static non-cooperative outcome forever. We characterize a necessary and sufficient condition for the existence of such a cut-off point (also called the trigger public output) such that cooperation can be sustained. We then show that this cut-off point decreases both in the discount parameter and in the spillover parameter. In other words, as the spillover effect or the patience of countries increase, it becomes more likely that two countries maintain cooperation. We analyze, in turn, the impact of the correlation between the two country-specific shocks on the incentives for each country to produce the optimal level of effort. Interestingly, we show that increasing the correlation coefficient from 0 reduces the prospects for cooperation in the sense that it decreases, for a given level of the cut-off point, the marginal benefit of exerting effort compared to the case of uncorrelated shocks. Indeed, shock-interdependence gives rise to a manipulation of information which leads each country to provide a lower level of public investment than that it would provide if correlation were absent.

The present paper is related to the general problem of tacit cooperation in a dynamic game setting with imperfect monitoring initiated by Porter (1983) and Green and Porter (1984). Although, this problem has been extensively analyzed in dynamic oligopoly models, it has not been explored in a model of provision of local public goods. For example, McMillan (1979) and more recently Pecorino (1999) analyze a repeated game setting for the private provision of public good but without uncertainty. Recently, public economists have examined the implications of imperfect information with respects to the costs of providing local public services, in particular, for the allocation of resources between member states of a federation (see, e.g. Lockwood (1999) and references therein). Here, we analyze the implications of information asymmetry with respect to cost levels of local public goods on the ability of independent jurisdictions to sustain cooperation in a repeated game setting.

Our analysis is also related to the literature on inter-governmental yardstick competition initiated by Salmon (1987) and Besley and Case (1995).[1] Indeed, when there is shock-interdependence the evaluation of government policy in one country depends on government performance in neighboring countries. In a static setting, yardstick competition mechanisms that rely on an informational externality in general help to enhance efficiency. In a dynamic setting, we show that such a mechanism may give rise to a manipulation of information thereby restricting the prospects for future cooperation.

The paper is organized as follows. We begin in Section 2 by presenting the model. In Section

[1] This theory has been further developed by, among others, Sand-Zantman (2004), Belleflamme and Hindriks (2005), Revelli (2006), and Besley and Smart (2007).

3, we characterize the cut-off trigger strategy equilibrium of the repeated game with imperfect monitoring. In Section 4, we examine how shock-interdependence affects this equilibrium outcome. Section 4 offers a brief conclusion.

2 The Stage Game

We consider a world consisting of two geographical countries. Each country has a population size normalized to 1 and there is no mobility across countries. In each country all individuals have identical endowments y and consume a private good and two public services, each one associated with a particular country. A given level of investment e_j in the jth country will give the following level of public services in that country.

$$q_j = e_j + \varepsilon_j \qquad j = 1, 2 \tag{1}$$

where e_j is the level of investment or effort (number of civil servants, level of infrastructure) chosen by the government of the jth country. e_j is not observable by the citizens of country k. ε_j is a random shock that follows a normal law with mean 0 and variance σ^2 for $j = 1, 2$. Individual private consumption in country j is $x_j = y - c(e_j)$ where $c(e_j)$ is the cost of producing e_j units of effort investments in public goods. We assume a convex cost function as it follows

$$c(e_j) = e_j^2/2, \qquad j = 1, 2. \tag{2}$$

Individuals in the two countries have the same preferences for private and public consumption. These preferences are represented by a linear utility function

$$U_j = x_j + \frac{[q_j + \beta q_k]}{1+\beta}, \qquad j \neq k \tag{3}$$

where β represents the intensity of cross-country spillovers related to public service provision. When $\beta = 0$, citizens care only about the public good in their own country, while when $\beta = 1$ they care equally about public spending in both countries. We also assume that exogenous income y is sufficiently high to always allow positive consumption of the private good. This implies together with linearity of preferences that there are no wealth effects.

Let W_j be the expected level of public goods surplus in country j. We have $W_j = E\left[\frac{[q_j+\beta q_k]}{1+\beta}\right] - e_j^2/2 = \frac{[e_j+\beta e_k]}{1+\beta} - e_j^2/2$ since ε_j has a 0 mean for $j = 1, 2$. Suppose first, that each country maximizes its own expected surplus W_j with respect to e_j given the other country's choice of effort investment. In the Nash equilibrium, both countries invest $1/(1+\beta)$. This gives the following equilibrium level of expected public services surplus for both countries $W^N = (1+2\beta) / \left[2(1+\beta)^2\right]$ which is decreasing in the spillover parameter.

If, however, both countries manage to cooperate, they maximize the sum of the expected surplus i.e. $W_1 + W_2$ with respect to both e_1 and e_2. The Bowen-Lindhal-Samuelson condition gives the common optimal level of effort investment i.e. $e^* = 1$. The level of expected public goods surplus, in that case, is then $W^C = 1/2$ which is independent of the size of the spillover effect. This comes from the symmetry of the model and from our specification of the utility function that is normalized with the spillover parameter.

If each country's effort were publicly observable, it would be straightforward to show that the efficient outcome can be supported as a trigger strategy equilibrium when the future is important. Specifically, if one country (let say country 1) defects from the cooperative outcome, it would choose the same level of effort investment as in the Nash outcome i.e. $1/(1+\beta)$. Hence, the equilibrium level of surplus of the country that defects would be $W_1^D = [1+2\beta(1+\beta)] / \left[2(1+\beta)^2\right]$ which is increasing in the spillover parameter. Let $0 < \delta < 1$ be the discount factor of both countries. Then the optimal outcome is attainable in every period with infinite Nash reversion if and only if $\delta \geq 1/2$.

3 Cooperation Under Imperfect Monitoring

Consider now the repeated game with imperfect monitoring. The two countries meet each period to play the stage game described above, where each country has the objective of maximizing its expected discounted stream of public good surplus. When entering a period, a country observes only the history of its own level of effort and realized public output in the two countries. Following Green and Porter (1984) and Fudenberg, Levine, and Maskin (1994), we restrict attention to those equilibria in which countries' strategies only depend on realized public outputs and not on their own private history of policy schedule. Such strategies are called public strategies and such equilibria are called perfect public equilibria (PPE).

Formally, in the stage game, each country $j = 1, 2$ chooses a level of effort e_j from a finite set E_i. Each profile of level of efforts $e \in E = E_1 \times E_2$ induces a probability distribution over the publicly observed outcomes. Let $q_t = (q_{1t}, q_{2t})$ be the vector of realized public output in period t and h_t the history of realized public output up to date t i.e. $h_t = (q_1, q_2, ...q_{t-1})$. Let H_t be the set of potential public histories at period t. A strategy for country j in period t is denoted $\sigma_{jt} : H_t \to E_i$. Let σ_t a strategy profile in period t and let σ represent a sequence of such strategy profile, $t = 1, 2, ...\infty$. Each strategy profile generates a probability distribution over histories and thus also generates a distribution over sequences of stage-game payoff vectors. The two countries discount future with a common discount factor δ, and country j's objective in the repeated game is to maximize the expected value of the discounted sum of his stage game payoffs i.e. $v_i = \sum_{t=0}^{\infty} \delta^t W_j \left(\sigma_t \left(h_t\right)\right)$.

As is typical in repeated games, there can be many perfect public equilibria in our game many of which can involve complicated strategies. We then make two restrictions. First, as in Green and Porter (1984), we consider equilibria with two levels of effort in public investment and with symmetric strategies. In addition, we constraint the two countries to choosing either the non-cooperative level of effort or the Pareto optimal level of effort. We then presuppose, as in Green and Porter (1984), special forms for the cooperative and punishment phases. More precisely, as a part of their strategies, the two countries must decide when to produce the cooperative or the non-cooperative level of effort as a function of public histories.

We also consider, for the moment, that the two country-specific shocks are independently distributed over time and across countries. Hence, high public output realization in a particular country would tend to suggest that this country has produced the optimal level of effort while low realization would tend to suggest that this country has defected. Abreu, Pearce and Stacchetti (1986, 1990) show that if the conditional distribution of the public signal given effort satisfies the Monotone Likelihood Ratio Property (MLRP) then a tail test is the optimal statistical criterion for the players to adopt.[2] This implies the existence of a critical level of the observable public output denoted $\widehat{q}$ such that if public output falls below this value, then punishment is triggered. We finally assume that the punishment length is infinite.

Each country then use the following cut-off strategy : (i) to produce the optimal level of effort e^* in the first period and to continue to do so as long as the observed level of public output in each country is as high as $\widehat{q}$; (ii) if public output in one or both countries falls below $\widehat{q}$ at some period t, then to produce the non-cooperative level of effort $\widetilde{e}$ in all subsequent periods. The probability of maintaining cooperation next period is then given by

$$\mu(e_1, e_2) = Prob[\varepsilon_1 \geq \widehat{q} - e_1].Prob[\varepsilon_2 \geq \widehat{q} - e_2]. \tag{4}$$

Using the properties of the normal distribution and denoting Φ the cumulative of a standard normal law (with mean equal to 0 and variance equal to 1), this probability can be written as it follows

$$\mu(e_1, e_2) = \left(1 - \Phi\left[\frac{\widehat{q} - e_1}{\sigma}\right]\right).\left(1 - \Phi\left[\frac{\widehat{q} - e_2}{\sigma}\right]\right). \tag{5}$$

[2] Formally, let $F(q\,|e)$ the cumulative distribution of public output given effort, with density $f(q\,|e)$. It satisfies the MLRP if for two values of effort e_1 and e_2 with $e_1 > e_2$, we have that $f(q\,|e_1)\,/f(q\,|e_2)$ is increasing in q.

The expected present discounted value of public good surplus in each country in period t is given by

$$V_t^C = W(e_1, e_2) + \delta\left[\mu(e_1, e_2)V_{t+1}^C + (1 - \mu(e_1, e_2))\, V_{t+1}^N\right]. \tag{6}$$

This value equals present period expected payoff plus expected future payoffs in present discounted value. Next period, either cooperation is continued (with probability $\mu(e_1, e_2)$) or the implicit contractual agreement is broken (with probability $1-\mu(e_1, e_2)$) in which case both countries revert to the static non-cooperative equilibrium forever (leading to an intertemporal utility of V_{t+1}^N).

When the two countries produce the optimal level of effort, i.e. $e_1 = e_2 = e^*$, the present discounted value of each country's payoff under cooperation in a stationary regime is

$$V^C = \frac{W^C + \delta(1 - \mu(e^*))\, V^N}{1 - \delta\mu(e^*)}. \tag{7}$$

where $\mu(e^*) \equiv \mu(e^*, e^*)$.

Similarly, the present discounted value of each country's payoff under non-cooperation in a stationary regime is

$$V^N = \frac{W^N}{1-\delta}. \tag{8}$$

Therefore, using (7) and (8), we have that

$$V^C - V^N = \frac{W^C - W^N}{1 - \delta\mu(e^*)}. \tag{9}$$

We can now analyze the optimal behavior of each country. Let us suppose that country 2 produces the optimal level of effort i.e. $e_2 = e^*$. Then, the necessary first-order condition for $e_1 = e^*$ to be country 1's best-response is

$$\frac{\partial V_t^C}{\partial e_1}\Big|_{e_1=e^*} = \frac{\partial W(e_1, e^*)}{\partial e_1}\Big|_{e_1=e^*} + \delta\left[V_{t+1}^C - V_{t+1}^N\right]\frac{\partial \mu(e_1, e^*)}{\partial e_1}\Big|_{e_1=e^*} = 0. \tag{10}$$

The time invariant nature of our framework implies that if the cooperative level of effort is an optimal strategy for country 1 today, it will also be an optimal strategy for that country in the future. Hence, this condition may be written equivalently as

$$\frac{\partial V^C}{\partial e_1}\Big|_{e_1=e^*} = \frac{-\beta}{1+\beta} + \delta\left[V^C - V^N\right]\frac{\partial \mu(e_1, e^*)}{\partial e_1}\Big|_{e_1=e^*} = 0. \tag{11}$$

The first term of the above expression represents the expected marginal benefit from under-producing effort investments in public goods. When country 1 decreases its effort below the optimal level of effort, it free-rides onto the other country and the expected marginal benefit of deviation is increasing in the spillover parameter. The second term corresponds to the expected marginal loss in future payoffs from possibly triggering a Nash reversion. This expected marginal cost of deviation is the product of two terms. The first term corresponds to the expected difference between the intertemporal utility of cooperation and the intertemporal utility of non-cooperation. The second term corresponds to the marginal probability that the game remains in the cooperative phase. As shown below, when country 1 decreases its effort below the optimal level of effort, it contributes to decrease the marginal probability of remaining in the cooperative phase. Therefore, the optimal level of effort $e^* = 1$ is a best-response to the predicted action of country 2 when the expected marginal benefit exactly balance the marginal cost from under-producing public investments. Recalling that $W^N = [1+2\beta] / \left[2(1+\beta)^2\right]$ and $W^C = 1/2$ and using (9), the necessary first-order condition given by (11) can then be rewritten as it follows[3]

$$\frac{\partial V^C}{\partial e_1}|_{e_1=e^*} = 0 \Leftrightarrow -1 + \frac{\delta\beta}{2(1+\beta)} \frac{1}{1-\delta\mu(e^*)} \frac{\partial \mu(e_1, e^*)}{\partial e_1}|_{e_1=e^*} = 0. \tag{12}$$

This equilibrium condition imposes some restrictions on the level of the trigger output $\widehat{q}$ and on the structural parameters δ, β and σ. Let first characterize the marginal probability that the game remains in the cooperative phase. Using (5), it is easy to see that

$$\frac{\partial \mu(e_1, e^*)}{\partial e_1}|_{e_1=e^*} = \frac{e^{-\frac{(\widehat{q}-e^*)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}.(1 - \Phi\left[\frac{\widehat{q}-e^*}{\sigma}\right]) > 0. \tag{13}$$

Let note $G = \widehat{q} - e^*$ be the difference between the trigger output and the optimal level of effort. Then using (5) and (13), (12) can then be rewritten as $R(G) = 0$ with

$$R(G) = -2(1+\beta)\left(1 - \delta\left[1 - \Phi\left(\frac{G}{\sigma}\right)\right]^2\right)\sigma\sqrt{2\pi} + \delta\beta e^{-\frac{G^2}{2\sigma^2}}\left(1 - \Phi\left(\frac{G}{\sigma}\right)\right) \tag{14}$$

Again, the expected marginal return to a country from decreasing its effort balances exactly the marginal increase in risk of incurring a loss in returns by triggering a reversion to the non-cooperative outcome.

In the Appendix, we show that the $R(G)$ function given by (14) is single-peaked with a unique maximum denoted $\widetilde{G}$ and that it has the shape as shown in Figure 1. (14) has then either no solutions or two solutions, depending on whether $R\left(\widetilde{G}\right)$ is negative or positive.

INSERT FIGURE 1

[3] From the proof of Proposition 1, we have that $\widehat{q} - e^* < 0$ which implies that the second-order condition is satisfied when the first-order condition is satisfied. Hence, the objective function is quasi-concave and (12) do represent the best response of country 1. Indeed, the sign of $\frac{\partial^2 V^C}{\partial e_1^2}$ is the same as the sign of $\frac{\partial^2 \mu(e_1,e^*)}{\partial e_1^2}$. Using (5), we have that $\frac{\partial^2 \mu(e_1,e^*)}{\partial e_1^2}|_{e_1=e^*} = \frac{(\widehat{q}-e^*)}{\sigma^3\sqrt{2\pi}} e^{-\frac{(\widehat{q}-e^*)^2}{2\sigma^2}} (1 - \Phi\left[\frac{\widehat{q}-e^*}{\sigma}\right]) < 0$.

Let δ^* be the discount factor that satisfies $R(G^*) = 0$. The following Proposition, which is proved in the Appendix, characterizes the existence of a cut-off trigger strategy equilibrium.

Proposition 1 : (i) *There exists a cut-off point $\widehat{q}$ such that the two countries have no incentives to deviate from the implicit contractual agreement if and only if $R\left(\widetilde{G}\right) \geq 0$ i.e. if and only if $\delta \geq \delta^*$.* (ii) *If it exists, the cut-off point $\widehat{q}$ is decreasing both in the spillover parameter β and in the discount parameter δ; the impact of an increase in the variance of each shock σ on $\widehat{q}$ is, however, indeterminate.*

When $R\left(\widetilde{G}\right) \geq 0$, $R(G) = 0$ has two solutions and the smaller solution G^* yields the optimal cut-off point $\widehat{q}$ which in turn determines the critical value of the discount parameter above which cooperation between the two countries can be sustained as perfect public equilibrium.[4] Unfortunately, on can not obtain an explicit solution for δ^*. Hence, in order to get a sharper result, one may use a stronger sufficient condition i.e. $R(0) \geq 0$ which necessarily implies that $R\left(\widetilde{G}\right) \geq 0$. As shown in the Appendix, The condition $R(0) \geq 0$ reads as $\overline{\delta} = \frac{4(1+\beta)\sigma\sqrt{2\pi}}{\beta+(1+\beta)\sigma\sqrt{2\pi}}$. With this specification, it can be then easily verified that $\overline{\delta}$ is lower than 1 if and only if the variance of each shock is sufficiently small. Indeed, when the variance of each shock increases, it becomes more to difficult to infer the behavior of each country which in turn makes tacit cooperation very difficult or impossible to sustain.

Proposition 1 says that, even though there is imperfect monitoring, the efficient outcome can be sustained by the players' threats to revert to the Nash equilibrium in case of a deviation from the efficient path as long as the two countries do not discount the future too heavily. However, compared to the case of perfect monitoring, tacit cooperation works less well since it can break down with a positive probability in every period and this almost surely happens in the long run even on the equilibrium path. Put another way, in equilibrium punishment is not triggered by the inference that one country deviated in the previous period. Rather, each country correctly presumes that its partner produced the optimal level of effort and that public service provision was low because of a negative shock. Reversion to the non-cooperative outcome is however necessary in this case because if the punishment did not occur when public output was low, the two countries would not have any incentives to cooperate.

Proposition 1 also establishes the comparative statics for the cut-off point which are intuitive.

[4] In a degenerate case, when $R\left(\widetilde{G}\right) = 0$, it has one solution. When it has two solutions, the two solutions have the same impact on efforts but the probability that the game remains in the cooperative phase is higher with the lower solution G^* (hence with the lower value of $\widehat{q}$) than with the larger solution.

When cooperation is feasible, the admissible value of the difference between the observed level of public service in each country and the optimal level of investment is decreasing both in the spillover parameter and in the discount parameter. In other words, cooperation will more likely be sustained if the countries care more about the future (greater δ) or if public good spillovers are more important (greater β). Indeed, the value of the cut-off point that enforces cooperation between the two countries must balance two objectives. On the one hand, it must be sufficiently high to give the countries an incentive to cooperate. On the other hand, it must be sufficiently low to decrease the probability of triggering a punishment inappropriately. If δ or β increases, the two countries have more incentives to cooperate and consequently the optimal trigger output must be lower to account for the possibility of bad shocks.

The impact of the variance of each shock on the cut-off point $\widehat{q}$ is, however, indeterminate. Indeed, increasing the variance has two conflicting effects on the cut-off value. First, it raises the risk of triggering a punishment inappropriately. This effect calls for a lower value of the cut-off point. Second, it diminishes the marginal impact of each country's effort on the resulting public output. This effect calls for a higher cut-off value in order to preserve the incentives to cooperate. The net effect is indeterminate and we cannot assess the impact of uncertainty on the degree of stringency of cut-off rules. In particular, when the public signal in each country becomes less informative (greater σ), it does not necessarily make the maintenance of cooperation less likely.

4 Imperfect Monitoring with Correlation

In this Section, we suppose that there exists a cut-off trigger strategy equilibrium and we analyze the impact of the correlation between the two country-specific shocks on this equilibrium outcome. Indeed, one may pretend that neighboring jurisdictions face a similar socioeconomic environment and are likely to experience similar shocks. In the context of our framework, such an informational externality would thus make it possible to infer more accurately each country's effort in public investment. It is then tempting to conclude, that shock-interdependence would enhance efficiency. Interestingly, we show that a small correlation gives rise to a manipulation of information that can undermine the standard positive effect of correlation on the agency problem.

Before proceeding, it might be useful to give an intuition of this result. As in the model without correlation, a tail test is the optimal statistical criterion for the countries to adopt. However, shock-interdependence brings some information which allows (with the use of observable variables) to make inference on actions with much higher precision than without shock-interdependence. Indeed, when shocks are independently distributed across countries, the best estimate of each country's effort is the observed level of public output. With correlated noise, however, the observed level of public output in one country can be compared to that in the other country to estimate more accurately each country's effort. Reversion to the non-cooperative outcome is then triggered when the estimated level of effort in one country (or both) - given the observed level of public output in the two countries - is lower than some threshold.

Now, let us suppose that each country believes that its partner produces the optimal level of effort. Suppose further that one country (let say country 2) in fact behaves in this way but that country 1 considers the possibility of deviating from the first-best level of effort. If country 1 indeed decides to shirk, it leads to decrease the expected level of public output observed in country 1. This in turn diminishes the probability that the cut-off rule associated to country 1 is satisfied. But a low realization of the public signal in country 1 gives rise to the belief that this country incurred a negative shock. With positive correlation between country-specific shocks, this leads to the belief that country 2 also incurred a negative shock. This in turn results in overestimation of country 2's effort which increases the probability that the cut-off rule associated to country 2 is satisfied. Hence, when country 1 deviates it becomes less likely that the cut-off rule associated to country 1 is satisfied but it also becomes more likely that cut-off rule associated to country 2 is satisfied. The net effect might be positive so that country 1 may have an incentive to decrease its effort.

We now present the formal analysis of the impact of the correlation between region-specific shocks on the incentives to cooperate. As before, public output q_j in the jth country is given by $q_j = e_j + \varepsilon_j$. We now consider that $(\varepsilon_1, \varepsilon_2)$ follows a normal law with mean equal to 0 and a variance-covariance matrix equal to

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \tag{15}$$

where ρ is the correlation coefficient.

Each country now uses the following cut-off strategy : (i) to produce the optimal level of effort e^* in the first period and to continue to do so as long as the estimated level of effort in each country, given the observed levels of public output in both countries, is as high as $\widehat{q}$; (ii) if the estimated level of effort in one or both countries fall below $\widehat{q}$ at some period t, then to produce the non-cooperative level of effort $\widetilde{e}$ in all subsequent periods.

Let $\hat{e}_j$ be the estimated level of effort of country j given the equilibrium behavior of the other country e^* and the observed levels of public output in both countries q_1 and q_2. We have

$\widehat{e}_j = E[q_j - \varepsilon_j | q_1, q_2, e^*]$ and the cut-off rule associated to each country is then

$$\hat{e}_1 \geq \hat{q} \Leftrightarrow q_j - E[\varepsilon_j | q_1, q_2, e^*] \geq \hat{q}. \tag{16}$$

Let $\varepsilon_j^e = q_j - e^*$ be the inference made on the shock in country j under the belief that this country produces the optimal level of effort e^*. Since $q_j = e_j + \varepsilon_j$, we have that $\varepsilon_j^e = e_j - e^* + \varepsilon_j$. If country j indeed produces the optimal level of effort i.e. $e_j = e^*$, then the shock in that country is perfectly inferred from the observation of q_j. In this case, we then have $\varepsilon_j^e = \varepsilon_j$. Now, let us suppose that country 1 considers deviating from e^* by producing some effort $e_1 < e^*$ but that country 2 indeed produces e^*. The cut-off rule associated to country 1 is

$$\begin{aligned} \hat{e}_1 \geq \hat{q} \quad &\Leftrightarrow \quad E[q_1 - \varepsilon_1 | q_1, q_2, e^*] \geq \hat{q} \\ &\Leftrightarrow \quad q_1 - E[\varepsilon_1 | \varepsilon_2^e = q_2 - e^*] \geq \hat{q}. \end{aligned} \tag{17}$$

where $E[\varepsilon_1 | \varepsilon_2^e = q_2 - e^*] = \rho \varepsilon_2^e = \rho \varepsilon_2$ since $e_2 = e^*$. We then have

$$\hat{e}_1 \geq \hat{q} \Leftrightarrow \varepsilon_1 - \rho \varepsilon_2 \geq \hat{q} - e_1. \tag{18}$$

The cut-off rule associated to country 2 is

$$\begin{aligned} \hat{e}_2 \geq \hat{q} \quad &\Leftrightarrow \quad E[q_2 - \varepsilon_2 | q_1, q_2, e^*] \geq \hat{q} \\ &\Leftrightarrow \quad q_2 - E[\varepsilon_2 | \varepsilon_1^e = q_1 - e^*] \geq \hat{q}. \end{aligned} \tag{19}$$

Assuming that country 2 produces the optimal level of effort and that country 2 believes that country 1 also behaves optimally, country 1 can manipulate the inference made on its own shock since $\varepsilon_1^e = e_1 - e^* + \varepsilon_1$. We then have $E[\varepsilon_2 | \varepsilon_1^e = q_1 - e^*] = \rho \varepsilon_1^e = \rho\,[e_1 - e^* + \varepsilon_1]$. The cut-off rule associated to country 2 is then

$$\hat{e}_2 \geq \hat{q} \Leftrightarrow \varepsilon_2 - \rho \varepsilon_1 \geq \hat{q} - e^* + \rho(e_1 - e^*). \tag{20}$$

To characterize the probability that the game remains in the cooperative phase in the presence of shock-interdependence, let note $x = \varepsilon_1 - \rho \varepsilon_2$ and $y = \varepsilon_2 - \rho \varepsilon_1$. Both x and y follow a normal law with mean equal to 0, variance equal to $(1 - \rho^2)\sigma^2$ and covariance equal to $-\rho(1 - \rho^2)\sigma^2$. Let $f(x, y)$ be the joint density, $f(x)$ and $f(y)$ the marginal densities and $f(y|x)$ the conditional density.

Using (18) and (20), cooperation is continued if and only if

$$\left\{ \begin{array}{lll} x & \geq & \hat{q} - e_1 \\ y & \geq & \hat{q} - e^* + \rho(e_1 - e^*) \end{array} \right. . \tag{21}$$

Since x and y are correlated, the probability that the game remains in the cooperative phase is then

$$\mu(e_1, e^*) = \int_{\hat{q}-e_1}^{+\infty} \left[\int_{\hat{q}-e^*+\rho(e_1-e^*)}^{+\infty} f(y|x)dy \right] f(x)dx. \tag{22}$$

As in the analysis without correlation, country 1 trades off the expected static benefit of deviation and the expected marginal cost from increasing the probability of triggering infinite reversion to the Nash outcome. Therefore, the necessary first-order condition for $e_1 = e^*$ to be country $1'$s best response to the predicted action of country 2 is still given by (12). Our purpose here is to investigate how an increase in the correlation coefficient ρ affects this equilibrium condition. Note that the correlation coefficient has an impact both on the equilibrium probability $\mu(e^*, e^*)$ and on the marginal probability $\frac{\partial \mu}{\partial e_1}(e_1, e^*)$ of remaining in the cooperative phase. Unfortunately, we cannot obtain a general result as to the effect of the correlation coefficient on these probabilities. However, one can obtain the following interesting local result.

Lemma 1 : *Increasing the correlation between the two signals of public output from* 0 *decreases both the marginal probability and the equilibrium probability of remaining in the cooperative phase.*

The proof of this Lemma is given in the Appendix. Again, increasing the correlation between the two signals of public output from zero increases the prospect for information manipulation. When country 1 deviates, it becomes less likely that the cut-off rule associated to country 1 is satisfied but it also becomes more likely that the cut-off rule associated to country 2 is satisfied. This is because a low realization of the public signal in country 1 is (mis)interpreted as the occurrence of a negative shock in that country. This leads to the belief that country 2 also incurred a negative shock because the country-specific shocks are positively correlated. There is thus less chance that the estimated level of effort in country 2 falls below the cut-off point. It turns out that when the correlation coefficient is small, this last effect dominates the first effect. Hence, country 1 has an incentive to make a lower level of effort in order to increase the marginal probability of remaining in the cooperative phase.

In addition, increasing the correlation coefficient from 0 also decreases the equilibrium probability of maintaining cooperation. Though an increase in the correlation decreases global uncertainty it also makes the two continuation rules more contradictory. Indeed, $x = \varepsilon_1 - \rho\varepsilon_2$ and $y = \varepsilon_2 - \rho\varepsilon_1$ move in opposite directions as ρ raises. To get an intuition of that result, assume that country 1 benefits from a positive shock while country 2 incurs a negative shock. In this case, the cut-off rule associated to country 1 is more likely to be satisfied while the reverse holds for country 2. There are two reasons for this. First, this country incurred a negative shock. Second, given the positive correlation between country-specific shocks and the occurrence of a positive shock in country 1, the level of effort in country 2 is underestimated. It turns out that, for small values of the correlation coefficient, the overall effect is negative which decreases the equilibrium probability of remaining in the cooperative phase. It is worth pointing out that this type of explanation is valid only for small values of the correlation coefficient. As shock-interdependence becomes more important, it is less likely to have a positive shock in one country and a negative shock in the other and the two continuation rules become less contradictory.

To sum up, increasing the correlation coefficient has the effect of shifting down the $R(G)$ function as depicted in Figure 2. (Recall that $R(G)$ given by (14) characterizes the necessary first-order condition for $e_1 = e^*$ to be country 1's best-response to $e_2 = e^*$).

INSERT FIGURE 2

As shown in the Appendix, the following Proposition directly follows from Lemma 1.

Proposition 2 : *Increasing the correlation between the two signals of public output from* 0 *reduces the prospects for maintaining cooperation as a perfect public equilibrium.*

As shown in figure 2, the value of the cut-off point $\widehat{q}$ that prevails in the model without shock-interdependence is too low to achieve cooperation as a perfect public equilibrium when the two region-specific shocks are positively correlated. As explained above, the cross-correlation in the noisy public realization of the public good changes the inference process in a way that it decreases the marginal benefit of exerting effort. Therefore, introducing correlation between region-specific shocks requires a larger value of the cut-off point with respect to the case of uncorrelated shocks. This in turn makes cooperation more difficult to sustain in the sense that, in every period, there is more chance that the observed level of public output in one country (or both) falls below the cut-off point. It then reduces the length of cooperation as well as each country's present discounted payoff.

Since the existence of a cut-off trigger strategy equilibrium (which is supposed this Section) is characterized by the difference between the value of the cut-off point and the level of effort investment that the two countries try to enforce on the equilibrium path, one can equivalently state the following. For a given level of the cut-off point, the cooperative level of effort investment that is possible to sustain is lower with shock-interdependence than without which also reduces each country's present discounted payoff.

5 Conclusions

We have analyzed in this paper, the possibility to sustain cooperation between two countries that make a public investment with cross-border externality and within a context of imperfect information. Even though the level of public investment provided by each country is imperfectly observed, it is shown that efficiency in local public goods provision can be sustained as a (stationary) perfect public equilibrium through a simple cut-off trigger strategy. In the absence of correlation between country-specific shocks, our comparative static results are quite intuitive. The two countries are more likely to be able to sustain cooperation if they do not discount the future too heavily or if public good spillovers are large. Introducing a marginal correlation between country-specific shocks, however, restricts the possibility of implementing intertemporal cooperation because it gives rise to a manipulation of information.

The simplicity of the framework analyzed in this paper is attractive but might be criticized on several fronts. First, as is common in this type of model, the two countries are able to sustain cooperation in public goods provision on the equilibrium path until a bad realization of the public signal. Each period, there is thus a positive probability of triggering permanent reversion to the non-cooperative outcome inappropriately. One could instead construct public strategy equilibria with punishment phases that last a fixed number of periods as in Green and Porter (1984). But this would not change our comparative statics results although punishment periods which are finite would strengthen the condition of existence of a cut-off trigger strategy equilibrium. Second, the two countries have strong incentives to renegotiate and continue with their relationship when the public signals in one country (or both) falls below the cut-off point (especially considering that both countries did not deviate on the equilibrium path). Put another way, the equilibrium is not renegotiation-proof. In turn, if the possibility of renegotiation is anticipated by the two countries, this will destroy their incentives to cooperate in the first place. A thorough investigation of this issue for our analysis of tacit cooperation in local public goods supply would be interesting for future research.

6 Appendix

6.1 Proof of Proposition 1

We first prove that the $R(G)$ has a unique maximum. The derivative of $R(G)$ given by (14) with respect to G is given by

$$R'(G) = -2(1+\beta)\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{G}{\sigma}\right)\right]2\delta\frac{e^{-\frac{G^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} - \delta\beta e^{-\frac{G^2}{2\sigma^2}}\frac{e^{-\frac{G^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} - \delta\beta\left(1-\Phi\left(\frac{G}{\sigma}\right)\right)\frac{G}{\sigma^2}e^{-\frac{G^2}{2\sigma^2}}.$$

Simplifying this expression, one find

$$R'(G) = -\delta e^{-\frac{G^2}{2\sigma^2}}\left[\left[1-\Phi\left(\frac{G}{\sigma}\right)\right]\left[4(1+\beta)+\frac{\beta G}{\sigma^2}\right]+\frac{\beta e^{-\frac{G^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\right].$$

Let $\widetilde{G}$ such that $R'(\widetilde{G}) = 0$. For this equality to be satisfied, we must have $4(1+\beta)+\frac{\beta\widetilde{G}}{\sigma^2} < 0$ which implies that $\widetilde{G} < 0$.

Calculating the second derivative of $R(G)$ with respect to G, one find

$$\begin{aligned} R''(G) &= \delta\frac{G}{\sigma^2}e^{-\frac{G^2}{2\sigma^2}}\left[\left[1-\Phi\left(\frac{G}{\sigma}\right)\right]\left[4(1+\beta)+\frac{\beta G}{\sigma^2}\right]+\frac{\beta e^{-\frac{G^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\right] \\ &\quad -\delta e^{-\frac{G^2}{2\sigma^2}}\left[-\frac{e^{-\frac{G^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\left[4(1+\beta)+\frac{\beta G}{\sigma^2}\right]+\left[1-\Phi\left(\frac{G}{\sigma}\right)\right]\frac{\beta}{\sigma^2}-\frac{G}{\sigma^2}\frac{\beta}{\sigma\sqrt{2\pi}}e^{-\frac{G^2}{2\sigma^2}}\right] \end{aligned}$$

We then have

$$R''(G) = -\frac{G}{\sigma^2}R'(G) - \delta e^{-\frac{G^2}{2\sigma^2}}\left[-\frac{e^{-\frac{G^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\left[4(1+\beta)+\frac{\beta G}{\sigma^2}\right]+\left[1-\Phi\left(\frac{G}{\sigma}\right)\right]\frac{\beta}{\sigma^2}-\frac{\beta G}{\sigma^2}\frac{e^{-\frac{G^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\right].$$

This implies

$$R''\left(\widetilde{G}\right) = -\delta e^{-\frac{\widetilde{G}^2}{2\sigma^2}}\left[-\frac{e^{-\frac{\widetilde{G}^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\left[4(1+\beta)+\frac{2\beta\widetilde{G}}{\sigma^2}\right]+\left[1-\Phi\left(\frac{\widetilde{G}}{\sigma}\right)\right]\frac{\beta}{\sigma^2}\right]$$

If $R''\left(\widetilde{G}\right) \leq 0$, then the $R(G)$ function has a maximum in $\widetilde{G}$ provided that the first-order condition is satisfied i.e. $R'\left(\widetilde{G}\right) = 0$. Again, $R'(\widetilde{G}) = 0$ implies that $4(1+\beta)+\frac{\beta\widetilde{G}}{\sigma^2} < 0$ which implies that $4(1+\beta)+\frac{2\beta\widetilde{G}}{\sigma^2} < 0$. Hence, for every point that satisfies the first-order condition, the second derivative is negative i.e. $R''\left(\widetilde{G}\right) \leq 0$. Therefore, the $R(.)$ function has a unique maximum in $\widetilde{G}$. (If the $R(.)$ function had more than one maximum, the $R(.)$ function would have a minimum and hence a point that would satisfy the first-order condition and where the second derivative would have been positive).

Consequently, a necessary and sufficient condition for the existence of a cut-off point $\widehat{q}$ such that the two countries have no incentives to deviate from the cooperative agreement is $R\left(\widetilde{G}\right) \geq 0$ i.e.

$$R\left(\widetilde{G}\right) = -2(1+\beta)\left(1-\delta\left[1-\Phi\left(\frac{\widetilde{G}}{\sigma}\right)\right]^2\right)\sigma\sqrt{2\pi}+\delta\beta e^{-\frac{\widetilde{G}^2}{2\sigma^2}}\left(1-\Phi\left(\frac{\widetilde{G}}{\sigma}\right)\right) \geq 0.$$

In this case and as shown in figure 1, $R(G) = 0$ has two solutions and the smaller solution $G^* < \widetilde{G} < 0$ with $G^* = \widehat{q} - e^*$ is the optimal cut-off point. (In a degenerate case, $R(G) = 0$ has a unique solution $G^* = \widetilde{G} < 0$).

Since $R(G)$ given by (14) is increasing in δ, this cut-off point in turn determines a lower bound for the discount parameter δ^* such that the two countries have no incentives to deviate from the cooperative agreement. Unfortunately, on can not obtain an explicit solution for δ^*. Hence, in order to get a sharper result, one may use a stronger sufficient condition i.e. $R(0) \geq 0$ which necessarily implies that $R\left(\widetilde{G}\right) \geq 0$. The condition $R(0) \geq 0$ reads as

$$-2(1+\beta)\left(1-\frac{\delta}{4}\right)\sigma\sqrt{2\pi} + \frac{\delta\beta}{2} \geq 0.$$

This condition can then be rewritten as it follows

$$\delta \geq \overline{\delta} \equiv \frac{4(1+\beta)\sigma\sqrt{2\pi}}{\beta + (1+\beta)\sigma\sqrt{2\pi}}.$$

We now turn to the proof of part (ii). Using the implicit function theorem, we have: $\frac{\partial G^*}{\partial \beta} = -\frac{\partial R/\partial\beta}{\partial R/\partial G^*}$. Under the condition of existence, the denominator of the above expression is positive i.e. $R'(G^*) > 0$ since the R function has a unique maximum $\widetilde{G} > G^*$. The derivative of $R(G^*)$ with respect to β is given by

$$\frac{\partial R(G^*)}{\partial \beta} = -2\left(1-\delta\left[1-\Phi\left(\frac{G^*}{\sigma}\right)\right]^2\right)\sigma\sqrt{2\pi} + \delta e^{-\frac{G^{*2}}{2\sigma^2}}\left(1-\Phi\left(\frac{G^*}{\sigma}\right)\right).$$

We can express the above expression as a function of $R(G^*)$ i.e.

$$\frac{\partial R(G^*)}{\partial \beta} = R(G^*) + 2\beta\left(1-\delta\left[1-\Phi\left(\frac{G^*}{\sigma}\right)\right]^2\right)\sigma\sqrt{2\pi} + \delta(1-\beta)e^{-\frac{G^{*2}}{2\sigma^2}}\left(1-\Phi\left(\frac{G^*}{\sigma}\right)\right).$$

Since $R(G^*) = 0$ by definition, we have that $\frac{\partial R(G^*)}{\partial \beta} > 0$. Therefore, we have $\frac{\partial G^*}{\partial \beta} < 0$.

Similarly, the change in the equilibrium value of G^* following a change in the discount parameter δ is given by $\frac{\partial G^*}{\partial \delta} = -\frac{\partial R/\partial\delta}{\partial R/\partial G^*}$. Again $R'(G^*) > 0$ and it is easily verified that $R(G^*)$ is increasing in δ. Specifically, we have

$$\frac{\partial R(G^*)}{\partial \delta} = \left[1-\Phi\left(\frac{G^*}{\sigma}\right)\right]\left[\beta e^{-\frac{G^{*2}}{2\sigma^2}} + 2(1+\beta)\left[1-\Phi\left(\frac{G^*}{\sigma}\right)\right]\right] > 0.$$

This implies that $\frac{\partial G^*}{\partial \delta} < 0$.

Finally, the change in the equilibrium value of G^* following a change in the variance of each country-specific shock σ is given by $\frac{\partial G^*}{\partial \sigma} = -\frac{\partial R/\partial\sigma}{\partial R/\partial G^*}$. Again $R'(G^*) > 0$ and we also have

$$\begin{aligned}\frac{\partial R(G^*)}{\partial \sigma} &= -2(1+\beta)\left(1-\delta\left[1-\Phi\left(\frac{G^*}{\sigma}\right)\right]^2\right)\sqrt{2\pi} + 4(1+\beta)\delta\left(1-\Phi\left(\frac{G^*}{\sigma}\right)\right)\frac{G^*}{\sigma}e^{-\frac{G^{*2}}{2\sigma^2}} \\ &\quad + \delta\beta\frac{G^{*2}}{\sigma^3}e^{-\frac{G^{*2}}{2\sigma^2}}\left(1-\Phi\left(\frac{G^*}{\sigma}\right)\right) + \delta\beta\frac{G^*}{\sigma^2\sqrt{2\pi}}e^{-\frac{G^{*2}}{\sigma^2}}\end{aligned}$$

Simplifying and expressing the above expression as a function of $R(G^*)$, we have

$$\begin{aligned}\frac{\partial R(G^*)}{\partial \sigma} &= \frac{R(G^*)}{\sigma} - \frac{\delta\beta}{\sigma}e^{-\frac{G^{*2}}{2\sigma^2}}\left(1-\Phi\left(\frac{G^*}{\sigma}\right)\right) + 4(1+\beta)\delta\left(1-\Phi\left(\frac{G^*}{\sigma}\right)\right)\frac{G^*}{\sigma}e^{-\frac{G^{*2}}{2\sigma^2}} \\ &\quad + \delta\beta\frac{G^{*2}}{\sigma^3}e^{-\frac{G^{*2}}{2\sigma^2}}\left(1-\Phi\left(\frac{G^*}{\sigma}\right)\right) + \delta\beta\frac{G^*}{\sigma^2\sqrt{2\pi}}e^{-\frac{G^{*2}}{\sigma^2}}\end{aligned}$$

$R(G^*) = 0$ by definition of G^*. Rearranging the above expression, we then have

$$\frac{\partial R(G^*)}{\partial \sigma} = \frac{\delta}{\sigma} e^{-\frac{G^{*2}}{2\sigma^2}} \left(1 - \Phi\left(\frac{G^*}{\sigma}\right)\right) \left[G^* \left(4(1+\beta) + \frac{\beta G^*}{\sigma^2}\right) - \beta\right] + \frac{\delta \beta G^*}{\sigma^2 \sqrt{2\pi}} e^{-\frac{G^{*2}}{\sigma^2}}$$

The second term of the above expression is negative since $G^* < 0$. The sign of the first term is the same as the sign of $\left[G^* \left(4(1+\beta) + \beta \frac{G^*}{\sigma^2}\right) - \beta\right]$. From the proof of the existence of cut-off point, we have that the $R(.)$ function has a unique maximum $\widetilde{G}$ that satisfies $R'\left(\widetilde{G}\right) = 0$ which implies that $4(1+\beta) + \beta \frac{G^*}{\sigma^2} < 0$. The sign of the term in brackets is then indeterminate.

6.2 Proof of Lemma 1

Let us start by studying the impact of the correlation coefficient on the equilibrium probability given by

$$\mu(e^*) = \int_{\hat{q}-e^*}^{+\infty} \left[\int_{\hat{q}-e^*}^{+\infty} f(y|x)dy\right] f(x)dx$$

where

$$f(x) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma\sqrt{1-\rho^2}} \exp^{-\frac{x^2}{2\sigma^2(1-\rho^2)}} \text{ and } f(y|x) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma(1-\rho^2)} \exp^{-\frac{(y+\rho x)^2}{2\sigma^2(1-\rho^2)^2}}.$$

Differentiating the equilibrium probability of remaining in the cooperative phase with respect to the correlation coefficient, yields

$$\frac{\partial \mu(e^*)}{\partial \rho} = \int_{\hat{q}-e^*}^{+\infty} \frac{df(x)}{d\rho} \left[\int_{\hat{q}-e^*}^{+\infty} f(y|x)dy\right] dx + \int_{\hat{q}-e^*}^{+\infty} f(x) \left[\int_{\hat{q}-e^*}^{+\infty} \frac{df(y|x)}{d\rho} dy\right] dx.$$

We focus on an increase in ρ from 0. It is easily verified that

$$\frac{df(x)}{d\rho}|_{\rho=0} = 0 \text{ and } \frac{df(y|x)}{d\rho}|_{\rho=0} = -\frac{1}{\sqrt{2\pi}\sigma} \frac{\exp^{-\frac{y^2}{2\sigma^2}}}{\sigma^2} xy.$$

Therefore,

$$\frac{\partial \mu(e^*)}{\partial \rho}|_{\rho=0} = -\left[\frac{1}{\sigma} \int_{\hat{q}-e^*}^{+\infty} \frac{\exp^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} x dx\right]^2 < 0.$$

Let us turn to the impact of the correlation coefficient on the marginal probability $\frac{\partial \mu(e_1, e^*)}{\partial e_1}$ of remaining in the cooperative phase. Using (22), we have that

$$\frac{\partial \mu(e_1, e^*)}{\partial e_1} = \left[\int_{\hat{q}-e^*+\rho(e_1-e^*)}^{+\infty} f(y|\hat{q}-e_1)dy\right] f(\hat{q}-e_1) - \rho \int_{\hat{q}-e_1}^{+\infty} f(\hat{q}-e^*+\rho(e_1-e^*)|x) f(x)dx.$$

Let evaluate this expression at $e_1 = e^*$. We have

$$\frac{\partial \mu(e_1, e^*)}{\partial e_1}|_{e_1=e^*} = \left[\int_{\hat{q}-e^*}^{+\infty} f(y|\hat{q}-e^*)dy\right] f(\hat{q}-e^*) - \rho \int_{\hat{q}-e_1}^{+\infty} f(\hat{q}-e^*|x) f(x)dx.$$

Differentiating the above expression with respect to ρ yields

$$\begin{aligned}
\frac{\partial\left(\frac{\partial \mu(e_1, e^*)}{\partial e_1}|_{e_1=e^*}\right)}{\partial \rho} &= \frac{\partial f(\hat{q}-e^*)}{\partial \rho} \int_{\hat{q}-e^*}^{+\infty} f(y|\hat{q}-e^*)dy + f(\hat{q}-e^*) \int_{\hat{q}-e^*}^{+\infty} \frac{\partial f(y|\hat{q}-e^*)}{\partial \rho} dy \\
&\quad - \int_{\hat{q}-e^*}^{+\infty} f(\hat{q}-e^*|x) f(x)dx - \rho \int_{\hat{q}-e^*}^{+\infty} \frac{\partial f(\hat{q}-e^*|x)}{\partial \rho} f(x)dx \\
&\quad - \rho \int_{\hat{q}-e^*}^{+\infty} f(\hat{q}-e^*|x) \frac{\partial f(x)}{\partial \rho} dx
\end{aligned}$$

Now let evaluate this expression at $\rho = 0$, we have

$$\frac{\partial(\frac{\partial\mu(e_1,e^*)}{\partial e_1}|_{e_1=e^*})}{\partial\rho}|_{\rho=0} = \frac{\partial f(\hat{q}-e^*)}{\partial\rho}|_{\rho=0}\int_{\hat{q}-e^*}^{+\infty} f(y|\hat{q}-e^*)dy$$
$$+f(\hat{q}-e^*)\int_{\hat{q}-e^*}^{+\infty}\frac{\partial f(y|\hat{q}-e^*)}{\partial\rho}|_{\rho=0}dy - \int_{\hat{q}-e^*}^{+\infty} f(\hat{q}-e^*|x)f(x)dx.$$

Again, it is easily verified that $\frac{\partial f(\hat{q}-e^*)}{\partial\rho}|_{\rho=0} = 0$ and $\frac{\partial f(y|\hat{q}-e^*)}{\partial\rho}|_{\rho=0} = -\frac{(\hat{q}-e^*)}{\sigma^2}\frac{\exp^{-\frac{y^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}y$. Hence, the above expression can be rewritten as it follows

$$\frac{\partial(\frac{\partial\mu(e_1,e^*)}{\partial e_1}|_{e_1=e^*})}{\partial\rho}|_{\rho=0} = -\frac{(\hat{q}-e^*)}{\sigma^2}f(\hat{q}-e^*)\int_{\hat{q}-e^*}^{+\infty}\frac{\exp^{-\frac{y^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}ydy - f(\hat{q}-e^*)\int_{\hat{q}-e^*}^{+\infty}\frac{\exp^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}dx.$$

One can equivalently rewrite the above expression as

$$\frac{\partial(\frac{\partial\mu(e_1,e^*)}{\partial e_1}|_{e_1=e^*})}{\partial\rho}|_{\rho=0} = -f(\hat{q}-e^*)\left[\frac{(\hat{q}-e^*)}{\sigma}\int_{\frac{\hat{q}-e^*}{\sigma}}^{+\infty}\frac{\exp^{-\frac{y^2}{2}}}{\sqrt{2\pi}}ydy + \int_{\frac{\hat{q}-e^*}{\sigma}}^{+\infty}\frac{\exp^{-\frac{x^2}{2}}}{\sqrt{2\pi}}dx\right].$$

Let us study the term in brackets. Let $Z \equiv \frac{\hat{q}-e^*}{\sigma}$ and let define the $H(.)$ function as

$$H(Z) = Z\int_Z^{+\infty}\frac{\exp^{-\frac{t^2}{2}}}{\sqrt{2\pi}}tdt + \int_Z^{+\infty}\frac{\exp^{-\frac{t^2}{2}}}{\sqrt{2\pi}}dt.$$

When $\rho = 0$, it has been shown in the previous section that $\hat{q} < e^*$. Therefore, Z is negative. We then study the $H(.)$ function for all negative values of Z.
Since $H''(Z) = \frac{Z\exp^{-\frac{Z^2}{2}}}{\sqrt{2\pi}}(Z^2-2)$, H' is first decreasing and then increasing over $]-\infty, 0]$. In addition, it is easily verified that $H'(Z) = \int_Z^{+\infty}\frac{\exp^{-\frac{t^2}{2}}}{\sqrt{2\pi}}tdt - (Z^2+1)\frac{\exp^{-\frac{Z^2}{2}}}{\sqrt{2\pi}}$ is equal to 0 at the boundaries. Therefore, H' is non positive for all $Z \leq 0$ which implies that $H(Z)$ is a decreasing function. Since, $H(0) = 1/2 > 0$, it follows that $H(Z) \geq 0$ for all $Z \leq 0$. We then have that $\frac{\partial(\frac{\partial\mu(e_1,e^*)}{\partial e_1}|_{e_1=e^*})}{\partial\rho}|_{\rho=0} < 0$.

6.3 Proof of Proposition 2

The proof of Proposition 2 directly follows from Lemma 1. The fundamental equation (12), which characterizes the first-order condition for $e_1 = e^*$ to be country 1's best-response to $e_2 = e^*$, is an increasing function of both $\mu(e^*)$ and $\frac{\partial\mu(e_1,e^*)}{\partial e_1}|_{e_1=e^*}$. Hence, increasing the correlation coefficient from 0 implies that $\frac{\partial(\frac{\partial V_t^C}{\partial e_1}|_{e_1=e^*})}{\partial\rho}|_{\rho=0} < 0$. This has the effect of shifting down the $R(G)$ function as depicted in Figure 2.

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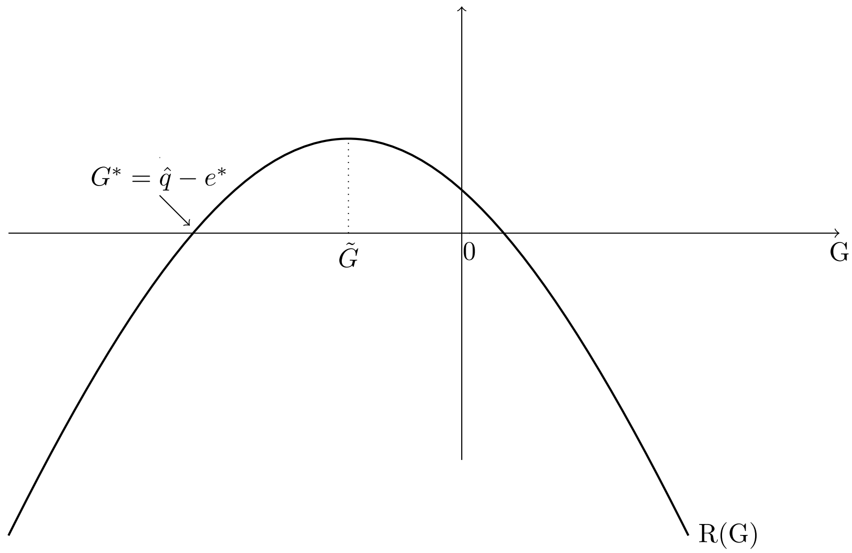

FIGURE 1

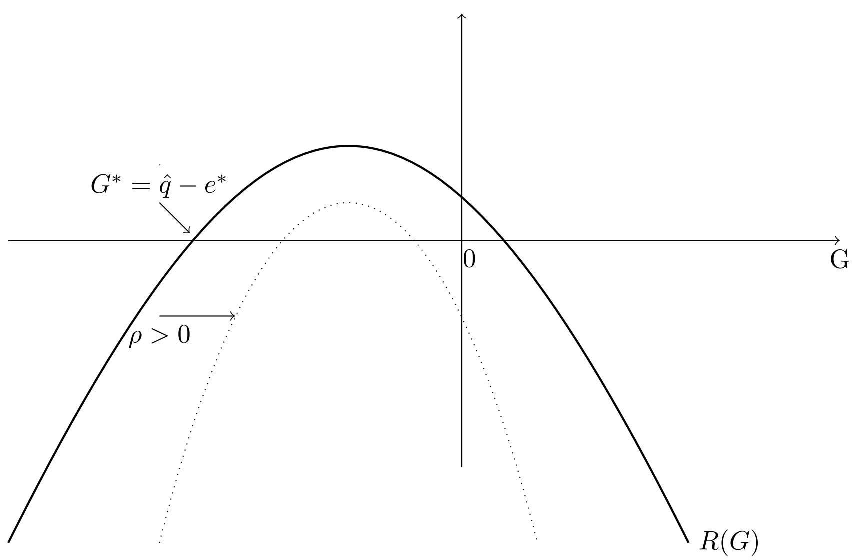

FIGURE 2