Decentralized equilibrium analysis in a growth model with directed technical progress and climate change mitigation*

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February 14, 2008

Abstract

The paper considers an endogenous growth model with two dedicated R&D sectors and climate change mitigation. First, we characterize the optimum analytically. Second, we propose a methodology that allows to analyze the equilibrium in a decentralized economy. Since knowledge is not embodied into intermediate goods, its price is defined in an alternative way (as a part of its social value, which is equal to the sum of its marginal profitabilities in all sectors using it). Moreover, the two types of market failures arising in our setting, i.e. the pollution from fossil resource use and the incomplete appropriability of surplus in research activities, are corrected by two economic policy instruments: a carbon tax and a research subsidy for each R&D sector. Third, we determine the optimal policies. Finally, we illustrate the theoretical model using some calibrated functional specifications. In particular, we investigate the effects of various combinations of public policies (including the optimal ones) by determining the deviation of each corresponding equilibrium from the "laisser-faire" benchmark.

JEL classification: H23, O32, Q43, Q54, Q55.

Keywords: Climate change, exhaustible resource, backstop, energy, induced technical change, knowledge pricing.

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*We are very grateful to David Popp for having provided us with the code and the detailed documentation of ENTICE-BR.

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1 Introduction

Emerging energy technologies, such as clean coal or renewable energy, are crucial for a cost-effective climate change mitigation policy. The relevant appraisal of a climate policy should thus include the appropriate incentives for R&D investments in carbon-free energies that will drive the substantial technical improvements necessary to their large scale deployment[1] (see Energy Journal, 2006, Special issue on endogenous technical change and the economics of atmospheric stabilization). The strand of literature on economic growth and climate change contains mostly optimization models (see for instance Bosetti et al., 2006; Edenhofer et al., 2005, 2006; Gerlagh 2006; Gerlagh and Van Der Zwaan 2006; Popp, 2004, 2006a, 2006b). The more often in those models, the analysis focuses on the optimal trajectories together with the system of prices and economic policies that implements the optimum. A complementary approach to these questions consists in characterizing the equilibrium in the associated decentralized economy, as suggested for instance by Edenhofer et al. (2006) who writes: "Therefore, designing a general intertemporal equilibrium version of MIND for a comparison with the social planner solution would be the natural next step".

The study of the decentralized economy offers one major advantage: it allows for the entire characterization of the continuum of all existing equilibria and not only the optimal one. Indeed, a particular equilibrium is associated with each feasible vector of policy instruments. The approach followed in this paper gives some insights on how the economy reacts to policy changes: when the economy faces one or several market failures, e.g. pollution or insufficient research effort, this characterization of market equilibria reveals crucial for measuring the impacts of economic tools such as environmental taxes, pollution permits or research subsidies. Because of budgetary, socioeconomic or political constraints, the enforcement of first best optimum is usually difficult to achieve for the policy-maker that would rather implement second-best solutions. For instance, Hart (2007) examines what should be the second-best carbon tax if research subsidies were below their first-best levels.

The objective of this paper is to complete the literature mentioned above by setting up a general equilibrium analysis, that includes explicitly both the optimal outcome and the decentralized equilibrium. However, the main difficulty of this approach lies in the way the research activity is modeled, in particular the type of innovation goods which are developed

[1] In 2004, wind and solar energy represented roughly 0.4% of world energy supply (IEA, 2006).

as well as their pricing. In the standard endogenous growth theory (Aghion and Howitt, 1998; Romer, 1990; Grossman and Helpman, 1991), when an innovation is produced, it is associated with a particular intermediate good. Research is funded by the monopoly profits of intermediate producers who benefit from an exclusive right, like a patent, for the production and the sale of these goods. However, this methodology has two inconvenients. Firstly, the more often, embodying knowledge into intermediate goods becomes inextricable in more general computable endogenous growth models with pollution and/or natural resources such as the ones previously mentioned. In addition, those technical difficulties are emphasized when dealing with several research sectors, i.e. when there are several types of specific knowledge, each of them being dedicated to a particular input (resource, labor, capital, backstop...) as it is proposed in Acemoglu (2002). Secondly, new pieces of knowledge, or new ideas, are not necessary associated with tangible intermediate goods. In particular, in new technology sectors as biotechnology or software industries, they are directly embodied into non-tangible goods that Quah (2001) and Scotchmer (2005) call knowledge goods, or information goods. As clearly explained in Scotchmer (2005)[2], these goods have the same property as knowledge: there are non rival. That is why databases, softwares, business plans, ..., are today directly protected by intellectual property rights.

To circumvent those obstacles, we assume the absence of tangible intermediate goods in research sectors, as it is done for instance by Gerlagh and Lise (2005), Edenhofer et al. (2006) and Popp (2004, 2006a). Therefore, in an equilibrium framework, it reveals necessary to directly price pieces of knowledge. To do it, we formalize ideas previously developed by Arrow (1962), Tirole (1988), Scotchmer (1991), Gallini and Scotchmer (2003) for instance. More recently, Grimaud and Rougé (2005)[3] have adapted such a formalization in growth models with polluting resources and environmental concerns. Based on this literature, we propose a method that consists in three points.

First, we define the optimal price of one unit of specific knowledge (associated with the energy or backstop R&D sectors) as the sum of the marginal profitabilities of this unit in each sector using this specific knowledge: this is the social value of an innovation.

Second, by referring to several empirical studies (see for instance Jones, 1995; Jones and Williams, 1998; Popp, 2004, 2006a), we assume that, in the decentralized economy, the

[2] "By information goods, we usually mean computer software and entertainment products stored in digital form, such as music. Information goods have a feature that sets them apart from ordinary private goods. They are public goods in the technical sense meant by economists: use by one person does not preclude the use by any other person and does not cost additional resources, except the small cost of distributing them. That is, the use of such good is non rival."

[3] See also Grimaud and Tournemaine (2007).

equilibrium price of knowledge is in fact equal to a given proportion of this optimal value, usually on the order of a quarter to a third. This is justified in the standard literature by the presence of several distortions that prevent the decentralized equilibrium to implement the first-best optimum. Jones and Williams (2000) count four of them. i) *the duplication effect*: the R&D sector does not account for the redundancy of some research projects; ii) *the intertemporal spillover effect*: inventors do not account for that ideas they produce are used to produce new ideas; iii) *the appropriability effect*: inventors appropriate only a part of the social value they create; iv) *the creative-destruction effect*. The overall effect of those distortions causes the market value of an innovation to be lower than the social one.

Third, we assume that the R&D sectors can be subsidized in order to reduce the gap between these social and market values[4].

We develop an endogenous growth model in which energy services can be produced from a polluting non-renewable resource as well as a clean backstop. As in Popp (2006a), we introduce two R&D sectors, the first one improving the efficiency of energy production and the second one, the efficiency of the backstop. With this respect, we have to consider two types of market failures: the pollution from fossil resource consumption and the research spillovers in each R&D sector. That is why, in the decentralized equilibrium, we introduce two kinds of economic policy instruments in accordance: a tax on the fossil fuel use[5] and a research subsidy for the energy and backstop R&D sectors. There is an equilibrium associated to each vector of instruments, which allows to study the impact of one or several policy changes on the equilibrium trajectories. Clearly, when public instruments are optimally set, the equilibrium of the decentralized economy coincides with the first best optimum.

We illustrate our methodology by using the ENTICE-BR model (Popp, 2006b) and we find three main streams of results. First, the fact that the optimal policy instruments, which have been computed analytically, implement numerically the optimal trajectories confirms the consistency of the equilibrium concept used here, in particular regarding the

[4]According to the OECD Science, Technology and R&D Statistics, publicly-funded energy R&D in 2004 among OECD countries amounted to 9.72 billion US$, which represented 4% of overall public R&D budgets. In the United States, energy investments from the private sector have shrunk during the last decade; governmental funding currently represents 76% of total US energy R&D expenditures (Nemet and Kammen, 2007).

[5]Fischer et al. (2003) and Nordhaus (2006) analyze the relative advantage of such a carbon tax as compared to a tradable emission permit system. In an earlier paper, Pizer (2002) argues that, when uncertainties about climate change mitigation costs are accounted for, price controls are much more efficient as long as damages are not too abrupt.

computation of the social values of innovations for each R&D sector. Second, the numerical comparison of the optimum and the equilibrium situation of "laisser-faire" allows us to measure the impact of the optimal instruments on the vector of prices and quantities of all economic sectors. In addition, since the analysis is undertaken at a decentralized level, we are able to dissociate those effects according to the various sectors: energy, backstop, fossil fuel, R&D. Third, our methodology renders possible the impact study of any economic policy on all variables, prices and quantities. In particular, we isolate the effects of the environmental policy from the ones of the research policies and vice versa. For instance, we show that an increase in the carbon tax has insignificant effect on the R&D activities. It reduces the flow of fossil fuel extraction and stimulates the backstop penetration. It also implies a rent transfer from the resource-holders to the government. Moreover, a research policy in a given R&D sector increases knowledge accumulation in this sector, but has no effect on the level of knowledge in the other R&D sector. Whatever the R&D sector, granting research always reduces the fossil fuel extraction and increases the backstop use.

The article is organized as follows. Section 2 presents the theoretical model. In section 3, we determine the optimal solutions owing to five characterizing conditions. Section 4 studies the decentralized economy. We first analyze the behavior of each agent in the economy. Next, we characterize the equilibrium solutions owing to five conditions and we compute the equilibrium prices for any policy levels. In section 5, we implement the first best optimum by comparing the two corresponding sets of characterizing conditions, which allows us to determine the optimal policies. In section 6, we derive a selection of numeric results focusing on i) the simultaneous effects of all the optimal policies (i.e. comparison between the optimum and the "laisser-faire" equilibrium), and ii) the differentiated effects of one policy, the other ones being given[6]. We conclude in section 7.

2 The model

We consider an economy in which, at each time t, a quantity Q_t^G of a homogeneous good is produced according to the following technology:

$$Q_t^G = Q(K_t, E_t, L_t, A_t), \tag{1}$$

where K_t is the amount of physical capital used within the production process, E_t is the flow of energy services, L_t, $L_t \equiv L_0 e^{\int_0^t g_{L,s} ds}$, denotes labor and A_t, $A_t \equiv A_0 e^{\int_0^t g_{A,s} ds}$, is an

[6]The functional specifications used for numerical computations are provided in the appendix.

efficiency index that measures the total productivity of factors. Growth rates $g_{L,t}$ and $g_{A,t}$ are exogenously given. Since, as we will see later, climate change affects global income and not utility, Q_t^G is in fact the final output that we would get without any environmental damage, i.e. the gross output. Function $Q(.)$ is assumed to be increasing and concave in each of his arguments and exhibits constant returns to scale.

As in Popp (2006a), production of energy services requires some specific knowledge $H_{E,t}$, fossil fuels F_t and a backstop energy source B_t:

$$E_t = E(H_{E,t}, F_t, B_t). \tag{2}$$

Production function $E(.)$ is increasing and concave in each argument and the backstop and the fossil fuel are supposed to be imperfect substitutes. $H_{E,t}$ represents a stock of specific technological knowledge dedicated to energy production process[7].

The fossil fuel end product is obtained from some carbon-based non-renewable resource and some specific investment[8]:

$$F_t = F(Q_{F,t}, Z_t), \tag{3}$$

where $Q_{F,t}$ is the amount of final product devoted to the production of fossil fuel and Z_t, $Z_t \equiv \int_0^t F_s ds$, is the cumulative extraction of the exhaustible resource from the initial date up to t. We assume that function $F(.)$ is increasing and concave in Q_F, decreasing and convex in Z, and that the fossil fuel extraction is constrained by a ceiling $\bar{Z}$: $Z_t \leq \bar{Z}$, $\forall t \geq 0$.[9]

The backstop resource is produced from specific investment and knowledge:

$$B_t = B(Q_{B,t}, H_{B,t}), \tag{4}$$

where $B(.)$ is an increasing and concave function in $Q_{B,t}$, the amount of final product

[7] In a model "à la Romer" with tangible intermediate goods, (2) would write $E_t = E\left[\int_0^{H_{E,t}} f(x_{j,t}^E)dj, F_t, B_t\right]$, where $x_{j,t}^E$ is the j^{th} intermediate good and $f(.)$ is an increasing and strictly concave function.

[8] An appreciable difference with the DICE stream of models lies in the definition of such a production function which, in fact, replaces the cost (or price) function of the fossil fuel. In Nordhaus and Boyer (2000) or in Popp (2004, 2006a), such a full cost function is equal to the full extraction cost augmented by the scarcity rent that depends on Z_t. By making this transformation, this utility/technology canonical model allows for an endogenous determination of the resource market price when solving the equilibrium (see section 4 below). However, we will analytically specify function $F(.)$ in such a way that there exists a correspondence with the cost function mentioned above and such that the calibration of the DICE model still applies.

[9] Here, the capacity constraint of the exhaustible resource is not characterized by the limited availability of initial stocks, but by the decreasing relationship between the flow of produced fossil fuel and the amount of resource that has already been extracted. Then, resource scarcity is not physically but economically captured.

that is devoted to the backstop production sector, and in $H_{B,t}$, the stock of knowledge pertaining to the backstop[10].

In this model, there are two stocks of knowledge, H_E and H_B, each associated with a specific R&D sector (i.e. the energy and the backstop ones). Here, in the energy (resp. the backstop) R&D sector, we consider that each innovation is a non-rival, indivisible and infinitely durable good which is simultaneously used by the energy (resp. backstop) production sector and by the R&D sector in question. Formally, it is a point on the segment $[0, H_{E,t}]$ (resp. $[0, H_{B,t}]$). We now specify the dynamics of these two stocks. At each time t, the stock of knowledge in sector i, $i = \{B, E\}$, evolves as follows:

$$\dot{H}_{i,t} = H^i(R_{i,t}, H_{i,t}), \tag{5}$$

where $R_{i,t}$ is the R&D investment into sector i, i.e. the amount of final output that is devoted to R&D sector i, and $H^i(.)$ an innovation function assumed to be increasing and concave in each argument. Then, the stock of knowledge $H_{i,t}$ increases due to increases in R&D effort and in already accumulated knowledge.

Pollution is generated by fossil fuel use. Let α be the unitary carbon content of fossil fuel such that, without any abatement policy, the carbon flow released into the atmosphere would be equal to αF_t. Let G_0 be the stock of carbon in the atmosphere at the beginning of the planning period, G_t the stock at time t and ζ, $\zeta > 0$, the natural rate of decay, so that[11]:

$$\dot{G}_t = \alpha F_t - \zeta G_t. \tag{6}$$

As in the DICE model (see also Farzin and Tahvonen, 1996), the atmospheric carbon concentration does not directly enter the damage function. In fact, the increase in carbon concentration drives the global mean temperature away from a given state – here the 1990 level – and the difference between this state and the present global mean temperature should be taken as an index of climate change. Let T_t denote this difference, whose dynamics is governed by the following state equation:

$$\dot{T}_t = \Phi(G_t) - mT_t, \tag{7}$$

[10] Again, in a model with tangible intermediate goods, (4) would write $B_t = B\left[Q_{B,t}, \int_0^{H_{B,t}} g(x^B_{i,t})di\right]$.

[11] In the analytical treatment of the model, we assume for the sake of clarity that the emission and natural decay rates are constant, despite what the DICE model recommends. However, in the numerical simulations, we adopt the carbon cycle characterization from DICE, that represents the carbon enhances between the oceans and the atmosphere. Based on Nordhaus and Boyer (2000), Goulder and Mathai (2000) estimate parameters α and ζ that take into account the inertia of the climatic system. They state that only 64% of current emissions actually contribute to the augmentation of atmospheric CO_2 and that the portion of current CO_2 concentration in excess is removed naturally at a rate of 0.8% per year.

where $\Phi(.)$ is an increasing and concave function that links the atmospheric carbon concentration to the dynamics of temperature (i.e. the radiative forcing as characterized in Nordhaus and Boyer, 2000) and m, $m > 0$, is a constant parameter[12].

Damage affects society through the global income. We denote by $D(T_t)$ the instantaneous penalty rate induced by temperature increases, with $D'(T_t) < 0$. The net output, Q_t^N, when taking into account climate change effects is:

$$Q_t^N = D(T_t) \times Q_t^G, \tag{8}$$

The final output is devoted to either aggregated consumption C_t, fossil fuel production $Q_{F,t}$, backstop production $Q_{B,t}$, investment in physical capital I_t or in the two R&D sectors $R_{E,t}$ and $R_{B,t}$:

$$Q_t^N = C_t + Q_{F,t} + Q_{B,t} + I_t + R_{E,t} + R_{B,t}. \tag{9}$$

The dynamic equation of the physical capital stock is:

$$\dot{K}_t = I_t - \delta K_t, \tag{10}$$

where δ, $\delta > 0$, is the capital depreciation rate.

Finally, the social welfare function W_t is defined as:

$$W_t = \int_0^t U(C_s) e^{-\int_0^s \rho_\tau d\tau} ds = \int_0^t L_s u(c_s) e^{-\int_0^s \rho_\tau d\tau} ds, \tag{11}$$

where ρ_t, $\rho_t \equiv \rho_0 e^{-g_\rho t}$, is the instantaneous social rate of time preferences, g_ρ is the constant declining rate of ρ_t, $U(C_t)$ is the instantaneous utility function from aggregated consumption, $c_t \equiv C_t/L_t$ is the per capita consumption and $u(c_t)$ is the per capita instantaneous utility function. As usual, functions $U(.)$ and $u(.)$ are increasing, concave and satisfy Inada conditions. The model is summarized in Figure 1.

[Figure 1 here]

To conduct numerical simulations, we assign functional specifications to the utility and technological functions so as to obtain a calibrated model. Those functional forms are listed in Appendix A1.

[12] As for the dynamics of the atmospheric carbon stock, the state equation (7) replaces in fact a more complex and general set of dynamic equations which considers two measures of temperature – the atmospheric temperature and the lower oceanic one – and the interactions between both. Kriegler and Bruckner (2004) have recourse to such simplified dynamics by using a log function for Φ and by estimating the associated parameter m. However, for numerical simulations, we keep the DICE formulation that fully describes temperature variations.

3 Welfare analysis

The social planner problem consists in choosing $\{C_t, Q_{F,t}, Q_{B,t}, R_{E,t}, R_{B,t}\}_{t=0}^{\infty}$ that maximizes W_∞, as defined by (11), subject to constraints (1)-(10). After eliminating the co-state variables, the first order conditions reduce to the five characteristic conditions of Proposition 1 below, which hold at each time t.

Proposition 1 *At each time t, an optimum is characterized by the following five conditions:*

$$\begin{aligned}\left[D(T_t)Q_E E_F - \frac{1}{F_{Q_F}}\right] U'(C_t)e^{-\int_0^t \rho_s ds} + \int_t^\infty \frac{F_Z}{F_{Q_F}} U'(C_s)e^{-\int_0^s \rho_x dx} ds & \\ +\alpha \int_t^\infty \left[\int_s^\infty D'(T_x)Q_x U'(C_x)e^{-\left[m(x-s)+\int_0^x \rho_y dy\right]}dx\right] \Phi'(G_s)e^{-\zeta(s-t)}ds &= 0 \end{aligned} \tag{12}$$

$$D(T_t)Q_E E_B B_{Q_B} = 1 \tag{13}$$

$$D(T_t)Q_K - \delta = \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} \tag{14}$$

$$H^B_{H_B} + \frac{H^B_{R_B} B_{H_B}}{B_{Q_B}} - \frac{\dot{H}^B_{R_B}}{H^B_{R_B}} = \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} \tag{15}$$

$$H^E_{H_E} + \frac{H^E_{R_E} E_{H_E}}{E_B B_{Q_B}} - \frac{\dot{H}^E_{R_E}}{H^E_{R_E}} = \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} \tag{16}$$

where J_X stands for the partial derivative of function $J(.)$ with respect to X.

Proof. See Appendix A2.

Equation (12) reads as a particular version of the Hotelling rule in this model, which takes into account the carbon accumulation in the atmosphere, the dynamics of temperatures and their effects on output. We will see later (cf. equation (38) in Proposition 2) that this equation allows for the computation of the optimal tax on the fossil fuel. Equation (13) tells that the marginal productivity of specific input $Q_{B,t}$ equals its marginal cost. The three last equations are Keynes-Ramsey conditions. Equation (14) characterizes the optimal trade-off between physical capital K_t and consumption C_t, as in more standard growth models. Equation (15) (resp. (16)) characterizes the same kind of optimal trade-off between specific investment into backstop R&D sector, $R_{B,t}$ (resp. energy R&D sector, $R_{E,t}$) and consumption.

4 Decentralized equilibrium

In the decentralized economy, we assume that all sectors, except R&D sectors, are perfectly competitive. The price of output Q_t^N is normalized to one and $p_{F,t}$, $p_{B,t}$, $p_{E,t}$, w_t and r_t are the prices at date t of fossil fuel, backstop, energy, labor and the interest rate on financial market, respectively. We also assume that the representative household holds capital and rents it to the final good producer at a rental price R_t. Standard arbitrage conditions imply $R_t = r_t + \delta$. Moreover, in order to correct the two types of distortions involved by the model (pollution and research spillovers in each R&D sector), we introduce two types of policy tools: an environmental tax, τ_t, on the resource use and two subsidies, $\sigma_{B,t}$ and $\sigma_{E,t}$, for the backstop and energy research sectors, respectively.

4.1 Behavior of agents

4.1.1 The final good sector

At each time t, the firm chooses $\{K_t, E_t, L_t\}_{t=0}^{\infty}$ that maximizes its profit function $\Pi_t^Q = D(T_t)Q_t^G - p_{E,t}E_t - w_tL_t - (r_t+\delta)K_t$, subject to (1). The first order conditions are:

$$\frac{\partial \Pi_t^Q}{\partial K_t} = 0 \Rightarrow r_t = D(T_t)Q_K - \delta \tag{17}$$

$$\frac{\partial \Pi_t^Q}{\partial E_t} = 0 \Rightarrow p_{E,t} = D(T_t)Q_E \tag{18}$$

$$\frac{\partial \Pi_t^Q}{\partial L_t} = 0 \Rightarrow w_t = D(T_t)Q_L. \tag{19}$$

4.1.2 The energy sector

At each time t, the energy producer maximizes $\Pi_t^E = \left[p_{E,t}E_t - p_{F,t}^m F_t - p_{B,t}B_t\right]$ subject to (2), where $p_{F,t}^m$ is the fossil fuel market price, i.e. the price which is paid by the firm and which includes the environmental tax τ_t. This tariff is assumed to be additive: $p_{F,t}^m = p_{F,t}^s + \tau_t$. However, our results can easily be extended to the case of an ad-valorem tax τ_t^a: $p_{F,t}^m = p_{F,t}^s(1+\tau_t^a)$. The first order conditions write:

$$\frac{\partial \Pi_t^E}{\partial F_t} = 0 \quad \Rightarrow p_{E,t} = \frac{p_{F,t}^m}{E_F} = \frac{p_{F,t}^s + \tau_t}{E_F} \tag{20}$$

$$\frac{\partial \Pi_t^E}{\partial B_t} = 0 \quad \Rightarrow p_{E,t} = \frac{p_{B,t}}{E_B}. \tag{21}$$

Those conditions determine respectively the inverse demand functions for fossil fuel and backstop.

4.1.3 The fossil resource sector

The program of the fossil fuel producer writes:

$$\max_{\{Q_F,t\geq 0\}} \int_0^\infty \left(p^s_{F,t}F_t - Q_{F,t}\right) e^{-\int_0^t r_s ds} dt \quad \text{s.t. (3) and } Z_t = \int_0^t F_s ds,$$

where $p^s_{F,t}$ denotes the selling price of the fossil resource, i.e. the price which is received by the resource-holder and which thus does not include the carbon tax. Static and dynamic first order conditions are:

$$(p^s_{F,t}F_{Q_F} - 1)e^{-\int_0^t r_s ds} + \eta_t F_{Q_F} = 0 \tag{22}$$

$$p^s_{F,t}F_Z e^{-\int_0^t r_s ds} + \eta_t F_Z = -\dot{\eta}_t, \tag{23}$$

together with the transversality condition $\lim_{t\to\infty} \eta_t Z_t = 0$. Replacing p_F into (23) by its expression coming from (22), it comes:

$$\dot{\eta}_t = -\frac{F_Z}{F_{Q_F}} e^{-\int_0^t r_s ds}. \tag{24}$$

By integrating (24) and using (22) again, we find:

$$\eta_t = \int_t^\infty \frac{F_Z}{F_{Q_F}} e^{-\int_0^s r_u du} ds \tag{25}$$

and then:

$$p^s_{F,t} = \frac{1}{F_{Q_F}} - \int_t^\infty \frac{F_Z}{F_{Q_F}} e^{-\int_t^s r du} ds. \tag{26}$$

4.1.4 The backstop sector

At each time t, the backstop producer maximizes its profit $\Pi^B_t = [p_{B,t}B_t - Q_{B,t}]$, subject to technological constraint (4). The first order condition determines, for each time t, the inverse demand function for specific investment $Q_{B,t}$:

$$p_{B,t} = \frac{1}{B_{Q_B}}. \tag{27}$$

4.1.5 The R&D sectors

We have seen (cf. section 2) that each innovation is a non-rival, indivisible and infinitely durable piece of knowledge. Since it is not embodied into tangible intermediate goods, it cannot be financed by the sale of these goods. However, in order to fully describe the equilibrium, we need to find a way to assess the price received by the inventor for each piece of knowledge. We use a method that consists in three points. i) In each research sector, we determine the *social* value of an innovation. Since an innovation is a public good, this social value is the sum of marginal profitabilities of this innovation in all sectors which use it. If the inventor was able to extract the willingness to pay of each user, he would receive this social value and the first best optimum would be implemented[13]. ii) In reality, as mentioned in introduction, there are some distortions that constrain the inventor to extract only a part of this social value[14]. This implies that the *market* value (without subsidy) is lower than the social one. iii) The research sectors are eventually subsidized in order to reduce the gap between the social and the market values of innovations.

Let us apply this three-steps procedure to the energy R&D sector, for instance. Each innovation produced by this sector is used by the sector itself as well as by the energy sector. Thus, at each date t, the instantaneous social value of this innovation is $\bar{v}_{H_E,t} = v^E_{H_E,t} + v^{H_E}_{H_E,t}$, where $v^E_{H_E,t}$ and $v^{H_E}_{H_E,t}$ are the marginal profitabilities of this innovation in the energy production sector and in the energy R&D sector, respectively. The social value of this innovation at t is $\bar{V}_{H_E,t} = \int_t^\infty \bar{v}_{H_E,s} e^{-\int_t^s r_x dx} ds$. We assume that, without any public intervention, only a share γ_E of the social value is paid to the innovator, with $0 < \gamma_E < 1$. However, the government can decide to grant this R&D sector by applying a non-negative subsidy rate $\sigma_{E,t}$. Note that if $\sigma_{E,t} = 1 - \gamma_E$, the market value matches the social one. The instantaneous market value (including subsidy) is:

$$v_{H_E,t} = (\gamma_E + \sigma_{E,t}) \bar{v}_{H_E,t}, \tag{28}$$

and the market value at date t is:

$$V_{H_E,t} = \int_t^\infty v_{H_E,s} e^{-\int_t^s r_x dx} ds. \tag{29}$$

By analogy, the instantaneous social value of an innovation in the backstop R&D sector is $\bar{v}_{H_B,t} = v^B_{H_B,t} + v^{H_B}_{H_B,t}$, where $v^B_{H_B,t}$ and $v^{H_B}_{H_B,t}$ are the marginal profitabilities of an innovation in the backstop sector and in the backstop R&D sector, respectively. Then, $\bar{V}_{H_B,t} = \int_t^\infty \bar{v}_{H_B,s} e^{-\int_t^s r_x dx} ds$ is the social value of an innovation at date t, and $V_{H_B,t} = \int_t^\infty v_{H_B,s} e^{-\int_t^s r_x dx} ds$ is the market value (including subsidy), in which $v_{H_B,t} = (\gamma_B + \sigma_{B,t}) \bar{v}_{H_B,t}$, with $0 < \gamma_B < 1$ and $\sigma_{B,t} \geq 0$. Here also, if $\sigma_{B,t} = 1 - \gamma_B$, the market value and the social one coincide. Note that differentiating (29) (and the corresponding equation for $V_{H_B,t}$) with respect to time leads to the usual arbitrage relation:

$$r_t = \frac{\dot{V}_{H_i,t}}{V_{H_i,t}} + \frac{v_{H_i,t}}{V_{H_i,t}}, \quad \forall i = \{B, E\}, \tag{30}$$

[13] This result will be proved by Proposition 3 below. In fact, what we call social value is the sum of the Lindahl prices associated with the innovations.

[14] For instance, Jones and Williams, 1998, estimate that actual investment in research are at least four times below what would be socially optimal; on this point, see also Popp, 2006a.

which reads as the equality between the rate of return on the financial market and the rate of return on the R&D sector i.

We can now analyze the behaviors of the R&D sectors. At each time t, each sector i, $i = \{B, E\}$, supplies the flow of innovations $\dot{H}_{i,t}$ at price $V_{H_i,t}$ and demands some specific investment $R_{i,t}$ at price 1, so that the profit function to be maximized is $\Pi_t^{H_i} = V_{H_i,t} H^i(R_{i,t}, H_{i,t}) - R_{i,t}$. The first order condition implies:

$$\frac{\partial \Pi_t^{H_i}}{\partial R_{i,t}} = 0 \quad \Rightarrow V_{H_i,t} = \frac{1}{H^i_{R_i}}. \tag{31}$$

The marginal profitability for specific knowledge of R&D sector i is:

$$v^{H_i}_{H_i,t} = \frac{\partial \Pi_t^{H_i}}{\partial H_{i,t}} = V_{H_i,t} H^i_{H_i} = \frac{H^i_{H_i}}{H^i_{R_i}}, \quad \forall i = \{B, E\}. \tag{32}$$

Finally, in order to determine the social and the market values of an innovation in both research sectors, we need to know the marginal profitabilities of innovations in the energy and backstop production sectors. From (21) and (27), those values are given by:

$$v^E_{H_E,t} = \frac{\partial \Pi_t^E}{\partial H_{E,t}} = \frac{E_{H_E}}{E_B B_{Q_B}}, \tag{33}$$

$$v^B_{H_B,t} = \frac{\partial \Pi_t^B}{\partial H_{B,t}} = \frac{B_{H_B}}{B_{Q_B}}. \tag{34}$$

Therefore, the instantaneous market values (including subsidies) of innovations are:

$$v_{H_E,t} = (\gamma_E + \sigma_{E,t}) \left(\frac{\partial \Pi_t^E}{\partial H_{E,t}} + \frac{\partial \Pi_t^{H_E}}{\partial H_{E,t}} \right) = (\gamma_E + \sigma_{E,t}) \left[\frac{E_{H_E}}{E_B B_{Q_B}} + \frac{H^E_{H_E}}{H^E_{R_E}} \right] \tag{35}$$

and

$$v_{H_B,t} = (\gamma_B + \sigma_{B,t}) \left(\frac{\partial \Pi_t^B}{\partial H_{B,t}} + \frac{\partial \Pi_t^{H_B}}{\partial H_{B,t}} \right) = (\gamma_B + \sigma_{B,t}) \left[\frac{B_{H_B}}{B_{Q_B}} + \frac{H^B_{H_B}}{H^B_{R_B}} \right]. \tag{36}$$

4.1.6 The household and the government

The representative household maximizes W_∞ subject to the following dynamic budget constraint: $\dot{K}_t = rK_t + w_t L_t + \Pi_t - C_t - T_t^a$, where Π_t is the total profits gained in the economy (including the resource rent) at time t and T_t^a is a lump-sum tax (subsidy free) that allows to balance the budget constraint of the government. This maximization leads to the following condition:

$$\rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} = r_t \Rightarrow U'(C_t) = U'(C_0) e^{\int_0^t (\rho_s - r_s) ds}. \tag{37}$$

Finally, assuming that the government's budget constraint holds at each time t, then it writes:

$$T_t^a + \tau_t F_t = \left(\frac{\sigma_{B,t}}{\gamma_B + \sigma_{B,t}}\right) V_{H_B,t}\dot{H}_{B,t} + \left(\frac{\sigma_{E,t}}{\gamma_E + \sigma_{E,t}}\right) V_{H_E,t}\dot{H}_{E,t}.$$

4.2 Characterization of the decentralized equilibrium

From the previous analysis of individual behaviors, we can now characterize an equilibrium in the decentralized economy, which is done by the following Proposition:

Proposition 2 *For a given triplet of policies $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$, the equilibrium conditions can be summed up as follows:*

$$\begin{aligned}
\left[D(T_t)Q_E E_F - \tau_t - \frac{1}{F_{Q_F}}\right] U'(C_t)e^{-\int_0^t \rho_s ds} &+ \int_t^{\infty} \frac{F_Z}{F_{Q_F}} U'(C_s)e^{-\int_0^s \rho_x dx} ds = 0 && (38)\\
D(T_t)Q_E E_B B_{Q_B} &= 1 && (39)\\
D(T_t)Q_K - \delta &= \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} && (40)\\
-\frac{\dot{H}_{R_B}^B}{H_{R_B}^B} + (\gamma_B + \sigma_{B,t})\left[\frac{B_{H_B}H_{R_B}^B}{B_{Q_B}} + H_{H_B}^B\right] &= \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} && (41)\\
-\frac{\dot{H}_{R_E}^E}{H_{R_E}^E} + (\gamma_E + \sigma_{E,t})\left[\frac{E_{H_E}H_{R_E}^E}{E_B B_{Q_B}} + H_{H_E}^E\right] &= \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} && (42)
\end{aligned}$$

and the equilibrium corresponding prices are:

$$\begin{aligned}
r_t^* &= D(T_t)Q_K - \delta && (43)\\
w_t^* &= D(T_t)Q_L && (44)\\
p_{F,t}^{s*} &= \frac{1}{F_{Q_F}} - \int_t^{\infty} \frac{F_Z}{F_{Q_F}} e^{-\int_t^s r_x dx} ds && (45)\\
p_{B,t}^* &= \frac{1}{B_{Q_B}} && (46)\\
p_{E,t}^* &= \frac{p_{F,t}^{s*} + \tau_t}{E_F} = \frac{p_{B,t}^*}{E_B} = D(T_t)Q_E && (47)\\
v_{H_B,t}^* &= (\gamma_B + \sigma_{B,t})\left[\frac{B_{H_B}}{B_{Q_B}} + \frac{H_{H_B}^B}{H_{R_B}^B}\right] \; ; \quad V_{H_B,t}^* = \frac{1}{H_{R_B}^B} && (48)\\
v_{H_E,t}^* &= (\gamma_E + \sigma_{E,t})\left[\frac{E_{H_E}}{E_B B_{Q_B}} + \frac{H_{H_E}^E}{H_{R_E}^E}\right] \; ; \quad V_{H_E,t}^* = \frac{1}{H_{R_E}^E}. && (49)
\end{aligned}$$

Proof. See Appendix A3.

Equations (38)-(42) are related to the quantities $Q_{F,t}$, $Q_{B,t}$, I_t, $R_{B,t}$ and $R_{E,t}$, respectively. They have to be compared one by one to equations (12)-(16) of Proposition 1 which characterize the optimum. In particular, by analyzing condition (38) and the optimal corresponding one (12), we will be able to compute the tax that implements the first best optimum (see next section). Equation (43) gives the interest rate and equations (44)-(49), the equilibrium prices of L_t, F_t, E_t, $H_{B,t}$ and $H_{E,t}$, respectively. A particular equilibrium is associated to a given triplet of policies $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$ and the set of equations given by Proposition 2 allows to compute quantities and prices for this equilibrium. If the triplet of policy tools is optimal, this set of equations gives the same quantities as the ones obtained from Proposition 1; it also gives the first best prices.

5 Implementation of the optimum

5.1 Determination of the first-best optimal policies

Recall that for a given set of public policies, a particular equilibrium is characterized by conditions (38)-(42) of Proposition 2. This equilibrium will be said to be optimal if it satisfies the optimum characterizing conditions (12)-(16) of Proposition 1. By analogy between these two sets of conditions, we can show that there exists a single triplet $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$ that implements the optimum.

Since conditions (13) and (14) have the same expressions as (39) and (40) respectively, we only have to compare the three remaining conditions of each proposition. First, by identifying (12) to (38) and using (37), the level of the additive environmental tax that implements the optimum – referred to as the first-best optimal tax from now on – is defined by:

$$\tau_t^o = -\frac{\alpha}{U'(C_t)} \left\{ \int_t^{\infty} \Phi'(G_t) e^{-\zeta(s-t)} \left[\int_s^{\infty} D'(T_x) Q_x U'(C_x) e^{-m(x-t)-\int_t^x \rho_y dy} dx \right] ds \right\}. \tag{50}$$

The interpretation of (50) is quite standard. This expression reads as the ratio between the marginal social cost of climate change – the marginal damage in terms of utility coming from the consumption of an additional unit of final good – and the marginal utility obtained by consuming this unit, i.e. the marginal rate of substitution between pollution and consumption. Equivalently, that corresponds to the social cost of one unit of carbon in terms of final good.

Next, the correspondence between the equilibrium characterizing condition (41) (resp.

(42)) and the optimum characterizing condition (15) (resp. (16)) is achieved if and only if $\sigma_{i,t}$ is equal to $1-\gamma_i$, $i=\{B,E\}$, i.e. if both sectors are fully subsidized. These results are summarized in Proposition 3 below.

Proposition 3 *The equilibrium defined in Proposition 2 is optimal if and only if the triplet of policies* $\{\sigma_{B,t},\sigma_{E,t},\tau_t\}_{t=0}^{\infty}$ *is such that* $\sigma_{B,t}=1-\gamma_B$, $\sigma_{E,t}=1-\gamma_E$ *and* $\tau_t=\tau_t^o$, *for all* $t\geq 0$.

Following Jones (1995), we set γ_B and γ_E equal to 0.3, so that optimal levels of research grants in both sectors correspond to $\sigma_B=\sigma_E=0.7$. Using the specified model as introduced in Appendix A1, the first-best optimal carbon tax can be illustrated by Figure 2.

[Figure 2 here]

This tax τ_t^o starts from some low 5 US$ per ton of carbon and follows an inverted U-shape trajectory, reaching around 90 US$ by 2100, 200$ in 250 years, before plummeting. As we will see later, this carbon policy increases the delivered price of the resource, i.e. the market price including the carbon tax (p_t^m). We will see also that this more expensive fossil energy provides strong incentives for adopting alternative energy supply.

5.2 Second-best analysis: an example

Proposition 3 gives the unique triplet of policies $\{\sigma_{B,t},\sigma_{E,t},\tau_t\}_{t=0}^{\infty}$ that restores the first-best optimum; nevertheless, the model allows to characterize any second-best optimum. As an illustration, assume that research cannot be subsidized at the first-best level ($\sigma_i=1-\gamma_i$, for $i=\{B,E\}$). Then, Figure 2 exhibits the second-best profile of the carbon tax in two cases[15]: the case of zero-research grants ($\sigma_i=0$) and the intermediate case where $\sigma_i=(1-\gamma_i)/2=0.35$. We observe that the lower the research subsidies, the higher the second-best environmental tax. We do not develop this example any further and leave the analysis of alternative second-best approaches for future work. Rather, we devote the next section to the analysis of the key variables sensitivity to various public policies at the equilibrium.

[15]The second-best tax comes from the maximization of the welfare subject to two types of constraints: i) the decentralized equilibrium described in section 4 prevails and ii) research subsidies are not set at their first-best levels.

6 Impacts of economic policies

In this section, we use the analytical model developed so far to conduct numerical simulations. We appraise the impacts of environmental and research policies on all variables – prices and quantities – in the decentralized equilibrium and we emphasize their transmission channels. We proceed as follows. In sub-section 6.1, we compare the so-called "laisser-faire" case (here after "LF"), that consists in determining the outcome in the decentralized economy without neither climate nor research policy, and the optimal outcome of the model. In other words, starting from the equilibrium with any public policy, we analyze the effect of the simultaneous introduction of optimal environmental tax and research grants. In order to quantify how sensitive are the economic variables to the environmental policy, we next analyze in sub-section 6.2 the impacts of a carbon tax variation from 0 to τ^o on the equilibrium trajectories, given zero-research grants. In sub-section 6.3, given a zero-carbon tax, we analyze how the trajectories evolve when i) σ_B and σ_E are simultaneously fixed at their first-best optimal levels; ii) either σ_B or σ_E is optimal while the other one is set equal to 0, i.e. when the research policy focuses on a single sector, the remaining one being not subsidized at all.

We adopt the following notations that will help us pointing at various facts when describing graphs. $\Delta^{\tau,\sigma}|_X$ stands for the change in variable X due to a simultaneous increase of τ from 0 to τ^o and of $\sigma_B = \sigma_E$ from 0 to 0.7. Those changes are illustrated in figures 3-7 by a shift from the "LF" trajectories to the "Optimum" trajectories. $\Delta^{\tau}|_X$ is the change of X due to an increase in τ from 0 to τ^o, given $\sigma_B = \sigma_E = 0$ (i.e. shifts from "LF" to "Opti tax" on the figures). Finally, given $\tau = \tau^o$, $\Delta^{\sigma_B,\sigma_E}|_X$ denotes the change in variable X due to a simultaneous increase of σ_B and σ_E from 0 to 0.7, and $\Delta^{\sigma_i}|_X$ the change in variable X due to an increase of σ_i from 0 to 0.7, with $\sigma_j = 0$, for $i, j = \{B, E\}$ and $i \neq j$ (i.e. shifts from "LF" to "Opti subs.", "Subs. B" and "Subs. E", respectively). Table 1 summarizes the findings from our sensitivity analysis conducted consequently, i.e. provides the signs of the Δ.

6.1 Optimum vs laisser-faire

In both R&D sectors, as seen from Figure 3, the implementation of optimal policies clearly translates into higher research investments and then, into much faster knowledge accumulation: $\Delta^{\tau,\sigma}|_{R_i} > 0$ and $\Delta^{\tau,\sigma}|_{H_i} > 0$, for $i = \{B, E\}$. The innovation selling prices $V_{H_E,t}$

	Optimum vs Laisser-faire	Environmental policy	R&D policies		
X	$\Delta^{\tau,\sigma}\vert_X$	$\Delta^{\tau}\vert_X$	$\Delta^{\sigma_B,\sigma_E}\vert_X$	$\Delta^{\sigma_B}\vert_X$	$\Delta^{\sigma_E}\vert_X$
H_B	$+$	$+$(weak)	$+$	$+$	$\sim$
H_E	$+$	$\sim$	$+$	$\sim$	$+$
R_B	$+$	$+$	$+$	$+$	$-$
R_E	$+$	$+$(weak)	$+$	$-$	$+$
F	$-$	$-$	$-$	$-$	$-$
B	$+$	$+$	$+$	$+$	$-$
E	$+$	$-$	$+$	$+$	$+$
T	$-$	$-$	$-$	$-$	$-$
Q^G	$+$	$-$	$+$	$+$	$+$
Q^N	$+$	first $-$, next $+$	$+$	$+$	$+$
V_{H_B}	$+$	$+$	$+$	$+$	$-$
V_{H_E}	$-$	$\sim$	$-$	$\sim$	$-$
p_F^m	$+$	$+$	$-$	$-$	$-$
p_F^s	$-$	$-$	$-$	$-$	$-$
p_B	$-$	$\sim$	$-$	$-$	$\sim$
p_E	$-$	$+$	$-$	$-$	$-$

Table 1: Summary of economic policy effects

and $V_{H_B,t}$ follow diverging time-paths (see Figure 4): $V_{H_E,t}$ decreases over time, while $V_{H_B,t}$ follows a reverse upward trend, at least for the first two centuries. The optimal instruments shift the price of an innovation dedicated to energy efficiency below its laisser-faire counterpart: $\Delta^{\tau,\sigma}|_{V_{H_E}} < 0$. Simultaneously, they shift the selling price of innovations dedicated to the backstop production above the laisser-faire level: $\Delta^{\tau,\sigma}|_{V_{H_B}} > 0$. As will be seen in sub-section 6.3, those results are caused by the R&D policies.

[Figures 3 and 4]

In the fossil fuel sector, the introduction of optimal policies implies a reduction of the instantaneous flow of extraction: $\Delta^{\tau,\sigma}|_F < 0$ (see Figure 5(a)). The resource market price increases, whereas its selling price diminishes: $\Delta^{\tau,\sigma}|_{p_F^m} > 0$ and $\Delta^{\tau,\sigma}|_{p_F^s} < 0$ (see Figure 5(b)). This overall effect on fuel prices is mainly due to the environmental policy and will be commented in sub-section 6.2.

[Figure 5]

In the backstop and energy sectors, the price of carbon-free energy $p_{B,t}$, as well as of final energy $p_{E,t}$, is reduced: $\Delta^{\tau,\sigma}|_{p_B} < 0$ and $\Delta^{\tau,\sigma}|_{p_E} < 0$; their respective consumption

B_t and E_t are intensified, overriding the fossil use reduction: $\Delta^{\tau,\sigma}|_B > 0$ and $\Delta^{\tau,\sigma}|_E > 0$ (see Figure 6).

[Figure 6]

Finally, the optimal time-paths of temperature variation start diverging from the laisser-faire case by the middle of the century: $\Delta^{\tau,\sigma}|_T < 0$ (see Figure 7(a)). In 2100, the optimal temperature variation is 9.4% lower than the no-climate-policy case, reaching almost 5 degrees in the very long run. The same observation holds for the net world product Q^N (see Figure 7(b)): it is positively affected by optimal policies by the end of this century, the most prominent benefit from the no-intervention case occurring only later on. This benefit overshoots 2% only around 2130.

[Figure 7]

6.2 Sensitivity to environmental tax

As shown in Figure 3, the effect of the environmental tax on the specific investments in both R&D sectors are positive: $\Delta^{\tau}|_{R_i} > 0$ for $i = \{B, E\}$. However, this effect is particularly weak in the energy R&D sector. This implies, in the backstop R&D sector, a positive effect on the accumulation of knowledge ($\Delta^{\tau}|_{H_B} > 0$), but, because of the relative inertia of the discovery function H^i, not significant enough to really confirm the robustness of the Porter hypothesis (in the sense that environmental regulation does not stimulate green research). We can just say that our model does not invalidate it. In the energy R&D sector, variations of the carbon tax let quite unchanged the rhythm of knowledge accumulation: $\Delta^{\tau}|_{H_E} \approx 0$. Moreover, the choice of the environmental tax affects positively the price of innovations in the green research sector ($\Delta^{\tau}|_{V_{H_B}} > 0$), but seems to have no impact on the prices in the energy research sector ($\Delta^{\tau}|_{V_{H_E}} \approx 0$), as shown by Figure 4.

As far as the resource market is concerned, reinforcing the carbon tax level throughout the entire time horizon is shifting the fossil fuel market price upward, and then the resource use downward, as depicted in Figure 5: $\Delta^{\tau}|_{p_F^m} > 0$ and $\Delta^{\tau}|_F < 0$. However, it is worth observing that the selling price of the fossil resource is decreasing: $\Delta^{\tau}|_{p_F^s} > 0$. This reduction implies a rent transfer from the resource-holder to the government. The idea that environmental taxes generally imply some redistributive effects in addition to the expected efficiency gains has already been evoked by economists, such as Nordhaus and Boyer (2000)[16]. Our framework provides an unambiguous characterization of those redistributive effects and allows for the assessment of their extent.

Concerning the backstop sector, the backstop price reveals unaffected by the environmental policies, i.e. $\Delta^{\tau}|_{p_B} \approx 0$, since its production cost – that partly depends on the specific level of knowledge in the backstop R&D sector H_B – remains fairly constant. However, the demand for the backstop is stimulated by such policies (see Figure 6(a)): $\Delta^{\tau}|_B > 0$. Indeed, in the energy sector, since firms face a higher fossil fuel price, they substitute the backstop for the polluting resource. Nonetheless, this more intensive backstop use is not sufficient to maintain the laisser-faire level of energy consumption (see Figure 6(b)): $\Delta^{\tau}|_E < 0$. This last result comes from the increase of the energy market price when a carbon tax is levied on the fossil fuel use: $\Delta^{\tau}|_{p_E} > 0$.

Finally, since this penalizes energy production – and thus final output production, ceteri paribus – the gross product Q^G is reduced when an optimal carbon tax is implemented. Moreover, the reduced carbon intensity of the global economy stemming from the carbon tax policy, slows down the increase in the temperature variation as shown in Figure 7(a), and in turn, reduces the environmental damage. The overall effect on the net world product Q^N, as compared with the laisser-faire case, corresponds to a negative impact of the tax until 2190, and a positive impact next. Such an environmental policy would thus generate more net product in the short run only if it was initially combined with an appropriate green research policy.

6.3 Sensitivity to research subsidies

When implementing a research policy, the regulator can act either on the sole energy R&D sector, on the sole backstop R&D sector, or on both sectors simultaneously. Then, a complete analysis of the effect of such a policy requires the dissociation of the joint effects from the marginal ones on each sector. For this matter, we proceed to two kinds of comparisons. For a given zero-level of carbon tax (i.e. $\tau_t = 0$ for all t), we analyze how the trajectories evolve when i) σ_E and σ_B are simultaneously increased from 0 to 0.7; ii)

[16] Nordhaus and Boyer (2000) justify the existence of a potential rent transfer by the fact that fossil fuel availability is generally limited (i.e. resources are scarce) and their supply curves are relatively price-inelastic. As they mention p.54, "In the limited case of perfectly price-inelastic supply of carbon-energy with zero extraction costs, carbon taxes may have no economic effect at all and would simply redistribute rents from the resource owners to the government". The situation described in our model is not so extreme since the resource will never be exhausted given the specified extraction technology. Nevertheless, beyond the efficiency effects, we still observe a rent effect.

either σ_E or σ_B is modified while the other one is set to 0, i.e. when the research policy focuses on a single sector, the remaining one being not subsidized at all.

Let us begin with the R&D sectors. We have previously noted (cf. sub-section 6.2) that the environmental policy has only small effects on the level of knowledge. Thus, the overall effect of optimal policies on R&D sectors, as mentioned in sub-section 6.1, should be essentially due to the research policies. A simultaneous increase in σ_B and σ_E makes the specific investments in both R&D sectors higher (see Figure 3) and thus, the knowledge accumulation faster: $\Delta^{\sigma_B,\sigma_E}|_{R_i} > 0$ and $\Delta^{\sigma_B,\sigma_E}|_{H_i} > 0$, for $i = \{B, E\}$. However, if we decompose the aggregate effect according to each sector, we observe that there is no cross-sector effects: $\Delta^{\sigma_B,\sigma_E}|_{R_i} \approx \Delta^{\sigma_i}|_{R_i}$ and $\Delta^{\sigma_j}|_{R_i} \approx 0$, for $i, j = \{B, E\}$ and $i \neq j$. The same observations apply to H_i.

The innovation selling prices being equal to the marginal costs of innovations, apprehending how they are affected by the optimal policy proves difficult. Contrary to the stock of knowledge in each R&D sector, the innovation selling prices do not react the same way to any research policy, as depicted in Figure 4: $\Delta^{\sigma_B,\sigma_E}|_{V_{H_E}} < 0$ and $\Delta^{\sigma_B,\sigma_E}|_{V_{H_B}} > 0$. From (31) and appendix A1, we have:

$$V_{H_i,t} = \frac{1}{H^i_{R_i}} = \frac{R_{i,t}}{b_i H^i(R_{i,t}, H_{i,t})}, \text{ for } i = \{B, E\}\,.$$

For any $i = \{B, E\}$, both the numerator and the denominator of this ratio increase. Nevertheless, when $i = E$, $R_{E,t}$ grows less than $b_E H^E(.)$, which leads to a decrease of the marginal cost of innovation in the energy R&D sector, along with its selling price $V_{H_E,t}$. Alternatively, when $i = B$, $R_{B,t}$ grows more than $b_B H^B(.)$, which leads to the opposite result. Those complex interactions stem from general equilibrium mechanisms.

Some further analysis of the R&D policy make the effect of each type of R&D subsidy on the innovation selling prices clearer. First, $\Delta^{\sigma_E}|_{V_{H_E}} \approx \Delta^{\sigma_B,\sigma_E}|_{V_{H_E}}$ and $\Delta^{\sigma_B}|_{V_{H_E}} \approx 0$: there is no cross-sector effects on the energy R&D sector, i.e. σ_B has no effect on $V_{H_E,t}$. Second, $\Delta^{\sigma_B}|_{V_{H_B}} > \Delta^{\sigma_B,\sigma_E}|_{V_{H_B}} > 0$ and $\Delta^{\sigma_E}|_{V_{H_B}} < 0$. In this case, strong cross-sector effects are occurring. When the only backstop R&D sector is subsidized, the increase in the innovation selling price in backstop R&D is higher than the increase which is observed when both R&D sectors are subsidized. Moreover, subsidizing the only energy R&D causes the backstop innovation price to move in the opposite direction, i.e. to decrease.

In turn, we examine the simultaneous effects of research policies on the fossil fuel, the backstop and the energy sectors, and we try to give some intuitions on the results. Since a simultaneous increase in σ_B and σ_E stimulates the knowledge accumulation in both sectors, this directly reduces the production costs of the backstop and the energy services, as well as their respective market prices p_B and p_E: $\Delta^{\sigma_B,\sigma_E}|_{p_B} < 0$ and $\Delta^{\sigma_B,\sigma_E}|_{p_E} < 0$. This implies an increase in the backstop and energy productions: $\Delta^{\sigma_B,\sigma_E}|_B > 0$ and $\Delta^{\sigma_B,\sigma_E}|_E > 0$ (Figure 6). Since the backstop is relatively less costly than the fossil fuel (p_B/p_F decreases), then the energy producers substitute the former for the latter: $\Delta^{\sigma_B,\sigma_E}|_F < 0$ (Figure 5(a)). The demand for the fossil fuel being reduced, its price decreases: $\Delta^{\sigma_B,\sigma_E}|_{p_F} < 0$ (Figure 5(b)). Remark that an increase in σ_E reduces the backstop production, but leaves its market price unchanged: $\Delta^{\sigma_E}|_B < 0$ (which implies $\Delta^{\sigma_B}|_B > \Delta^{\sigma_B,\sigma_E}|_B > 0$) and $\Delta^{\sigma_E}|_{p_B} \approx 0$.

Finally, the temperature changes as well as the net final output (taking into account environmental damages) are positively affected by a rise in energy subsidies in any sector (see Figure 7).

7 Conclusion

This paper establishes the template of a climate change integrated assessment model, capable of defining the decentralized outcome, i.e. the equilibrium, of a given climate policy architecture. One of the main features of the model lies in the analytical derivation of the innovation prices. In our context, those innovations are dedicated to knowledge accumulation in two sectors: the backstop energy sector and the energy efficiency sector. Since knowledge is not embodied into intermediate goods, its price is defined in an alternative way (as a part of its social value that is equal to the sum of its marginal profitabilities in all sectors using it).

Another key feature of the model lies in its ability and suitability to assess various economic policies. As the economy encompasses three market distortions, i.e. the pollution from fossil resource consumption and the two research spillovers, two types of economic policy instruments are implemented: a tax on the fossil fuel use and a research subsidy for each R&D sector. As one obtains a distinct equilibrium for each vector of instruments, we are able to test for any policy architectures, including suboptimal carbon taxes and research subsidies. This should be of particular interest for studying second best policy in the context of climate change mitigation.

We use a calibrated version of the model to simulate the socially optimum outcome and compare it to its laisser-faire counterpart in the decentralized economy. We assess the impacts on all economic and environmental variables and characterize the efficiency of the policy measures, and particularly the efficiency of the R&D funding that have to be devoted to energy technologies. The laisser-faire situation results in some additional gross world product losses of 1.6% in the long run, as compared to the socially desirable outcome. We exhibit the significant influence of R&D activities aiming at reducing the polluting fossil energy use. This setting advocates for higher subsidies dedicated to renewable energies, and, to a lower extent, for subsidies aiming at improving energy efficiency. This mainly comes from the underlying assumption on the potential improvements of energy efficiency that are limited to 20%, suggesting that improvement in energy efficiency would rather be a short term option for tackling the climate change issue, while bringing the backstop energy to the market is more beneficial in the longer term.

The natural extension of the model will consist in introducing a richer set of climate mitigation options such as the possibility of capturing and storing the carbon in geological formations. One might also introduce biofuel energy, the feedstock then encompassing the features of a renewable resource. The specificities of nuclear energy may also be incorporated in our model. The flexibility of the tool at hand allows for the modeling of specific knowledge stocks for each of the energy supply technologies.

Finally, the calibration of this model may require some further adjustment. In this respect, alternative functional forms may be experienced (See Nordhaus's comment on Stern review and the accompanying data update – Nordhaus, 2007). Moreover, as suggested by the IPCC report (IPCC, 2000), a number of plausible scenarios may arise in the future. The DICE model calibration may be revised so as to match more closely the GDP projections of other long term studies. In particular, it would be worthwhile analyzing the effects of a more sustained long term growth. An enhanced world economic growth would turn into more intensive fossil energy use, at least in the early decades where the renewable energy does not exhibit sufficient cost reduction. Besides the increased externality resulting from more rapid climate change, the modified economically recoverable resource base may, in turn, confront us to lower fossil resource availabilities in the long run. The effect on the fossil fuel prices and the incentive for increased investment in clean energy R&D deserves some further investigation.

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Appendix

A1. Analytical specification and calibration of the model

To characterize analytically our model, we use a mix of functional forms considered in the DICE and ENTICE-BR models:

$$
\begin{aligned}
Q(K,E,L,A) &= AK^{\gamma}E^{\beta}L^{1-\gamma-\beta}, \quad \text{with} \quad \beta,\gamma \in (0,1) \\
g_i &= \left(\frac{g_{i0}}{d_i}\right)\left(1-e^{-d_i t}\right), \quad \text{with} \quad d_i > 0, \, \forall i = \{A, L\} \\
E(H_E,F,B) &= \left[(\alpha_H H_E)^{\rho_H} + (F^{\rho_B} + B^{\rho_B})^{\frac{\rho_H}{\rho_B}}\right]^{\frac{1}{\rho_H}}, \quad \text{with} \quad \alpha_H, \rho_H, \rho_B \in (0,1) \\
H^i(R_i,H_i) &= a_i R_i^{b_i} H_i^{\phi_i}, \quad \text{with} \quad a_i > 0, \text{ and } b_i, \phi_i \in [0,1], \, \forall i = \{E, B\} \\
D(T) &= \frac{1}{1+a_1 T + a_2 T^2}, \quad \text{with} \quad a_1 < 0 \text{ and } a_2 > 0 \\
U(C) &= k_1 L \log\left(\frac{C}{L}\right) + k_2, \quad \text{with} \quad k_1, k_2 > 0 \\
\Phi(G) &= \epsilon_1 \frac{\log(G/\epsilon_2)}{\log 2} + O(t),
\end{aligned}
$$

where $O(t) = \epsilon_3 t - \epsilon_4$ for $t < \bar{t}$, $O(t) = \epsilon_5$ otherwise, $\epsilon_i > 0$, $i = \overline{1,5}$. We also consider the following production functions:

$$
\begin{aligned}
F(Q_F, Z) &= \frac{Q_F}{c_F + \alpha_F \times (Z/\bar{Z})^{\eta_F}}, \quad \text{with} \quad c_F, \alpha_F, \eta_F > 0 \\
B(Q_B, H_B) &= Q_B \times \frac{H_B^{\eta_B}}{\alpha_B}, \quad \text{with} \quad \alpha_B, \eta_B > 0.
\end{aligned}
$$

For numerical computations, we use the same values of exogenous parameters as in the ENTICE-BR model [17]. Since we have transformed the cost functions of fossil fuel and backstop into production functions, we also specify the parameters of these production functions in such a way that the calibration of the ENTICE-BR model still applies to our model. Finally, we consider a finite time horizon starting at date $t_0 = 1990$ and ending at $T = t_0 + 350$.

[17] For the sake of simplicity, the exogenous land use emissions have been omitted. Those emissions are likely small (see Nordhaus, 2007) and would alter neither our qualitative nor our quantitative results.

A2. Proof of Proposition 1

Let H be the discounted value of the Hamiltonian of the optimal program (we drop time subscripts for notational convenience):

$$
\begin{aligned}
H &= U(C)e^{-\int_0^t \rho ds} + \lambda D(T)Q\left\{K, E\left[H_E, F(Q_F, Z), B(Q_B, H_B)\right]\right\} \\
&\quad -\lambda\left(C + Q_F + Q_B + \sum_i R_i + \delta K\right) + \sum_i \nu_i H^i(R_i, H_i) \\
&\quad +\mu_G(\alpha F - \zeta G) + \mu_T[\Phi(G) - mT] + \eta F.
\end{aligned}
$$

The associated first order conditions are:

$$
\begin{aligned}
\frac{\partial H}{\partial C} &= U'(C)e^{-\int_0^t \rho ds} - \lambda = 0 && (51)\\
\frac{\partial H}{\partial Q_F} &= \lambda[D(T)Q_E E_F F_{Q_F} - 1] + \alpha\mu_G F_{Q_F} + \eta F_{Q_F} = 0 && (52)\\
\frac{\partial H}{\partial Q_B} &= \lambda[D(T)Q_E E_B B_{Q_B} - 1] = 0 && (53)\\
\frac{\partial H}{\partial R_i} &= -\lambda + \nu_i H^i_{R_i} = 0, \quad i = \{B, E\} && (54)\\
\frac{\partial H}{\partial K} &= \lambda[D(T)Q_K - \delta] = -\dot{\lambda} && (55)\\
\frac{\partial H}{\partial H_i} &= \lambda D(T)Q_E E_{H_i} + \nu_i H^i_{H_i} = -\dot{\nu}_i, \quad i = \{B, E\} && (56)\\
\frac{\partial H}{\partial G} &= -\zeta\mu_G + \mu_T\Phi'(G) = -\dot{\mu}_G && (57)\\
\frac{\partial H}{\partial T} &= \lambda D'(T)Q - m\mu_T = -\dot{\mu}_T && (58)\\
\frac{\partial H}{\partial Z} &= \lambda D(T)Q_E E_F F_Z + \alpha\mu_G F_Z + \eta F_Z = -\dot{\eta} && (59)
\end{aligned}
$$

and the transversality conditions are:

$$
\begin{aligned}
\lim_{t\to\infty} \lambda K &= 0 && (60)\\
\lim_{t\to\infty} \nu_i H_i &= 0, \quad i = \{B, E\} && (61)\\
\lim_{t\to\infty} \mu_G G &= 0 && (62)\\
\lim_{t\to\infty} \mu_T T &= 0 && (63)\\
\lim_{t\to\infty} \eta Z &= 0 && (64)
\end{aligned}
$$

First, we show how to obtain condition (12), the less evident one. From (52), we have:

$$
\alpha\mu_G + \eta = -\lambda\frac{[D(T)Q_E E_F F_{Q_F} - 1]}{F_{Q_F}},
$$

where $\lambda = U'(C)e^{-\int_0^t \rho ds}$ from (51). Substituting this expression into (59) and after simplifications, we get the following differential equation:

$$\dot{\eta} = -\frac{F_Z}{F_{Q_F}} U'(C) e^{-\int_0^t \rho ds}.$$

Integrating this expression and using transversality condition (64), we obtain:

$$\eta = \int_t^{\infty} \frac{F_Z}{F_{Q_F}} U'(C) e^{-\int_0^s \rho du} ds. \tag{65}$$

From (51) and (58), we have:

$$\dot{\mu}_T = m\mu_T - D'(T)QU'(C)e^{-\int_0^t \rho ds}.$$

Solution of such a differential equation is given by:

$$\mu_T = e^{mt}\left[\mu_{T,0} - \int_0^t D'(T)QU'(C)e^{-\left(ms+\int_0^s \rho dx\right)} ds\right].$$

Using (63), this expression becomes:

$$\mu_T = e^{mt}\int_t^{\infty} D'(T)QU'(C)e^{-\left(ms+\int_0^s \rho dx\right)} ds = \int_t^{\infty} D'(T)QU'(C)e^{-\left[m(s-t)+\int_0^s \rho dx\right]} ds. \tag{66}$$

Now, let us consider condition (57). Using transversality condition (62), this differential solution is solved for:

$$\mu_G = e^{\zeta t}\int_t^{\infty} \mu_T \Phi'(G) e^{-\zeta s} ds = \int_t^{\infty} \mu_T \Phi'(G) e^{-\zeta(s-t)} ds, \tag{67}$$

where μ_T is defined by (66).

Finally, condition (12) is equivalent to condition (52) when replacing λ, μ_G and η by their expressions coming from (51), (67) and (65) respectively, and dividing each side of the equation by F_{Q_F}.

Second, the characterizing condition (13) is directly provided by (53). To continue, remark that (51) implies:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{U}'(C)}{U'(C)} - \rho. \tag{68}$$

Then, condition (14) is simply obtained from (55) and (68).

Finally, differentiating (54) with respect to time and using (13), (56) and (68), we get the characterizing conditions (15) and (16), which concludes the proof.

A3. Proof of proposition 2

The first characterizing condition (38) is obtained by replacing η into (22) by its expression coming from (25) and by noting that $p_F^s = p_E E_F - \tau$ from (20), where $p_E = D(T)Q_E$ from (18). Second, combining (27), (21) and (18) leads to condition (39). Next, using (17) and (37), we directly get condition (40). Finally, the differentiation of (31) with respect to time leads to:

$$\frac{\dot{V}_{H_i}}{V_{H_i}} = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}}, \quad i = \{B, E\}.$$

Substituting this expression into (30) and using (28), (31) and (32), it comes:

$$r = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}} + (\gamma_i + \sigma_i) H^i_{R_i} \left(v^i_{H_i} + \frac{H^i_{H_i}}{H^i_{R_i}} \right), \quad \forall i = \{B, E\}.$$

We thus obtain the two last characterizing equilibrium conditions (41) and (42) by replacing into this last equation $v^B_{H_B}$ and $v^E_{H_E}$ by their expressions coming from (34) and (33) respectively.

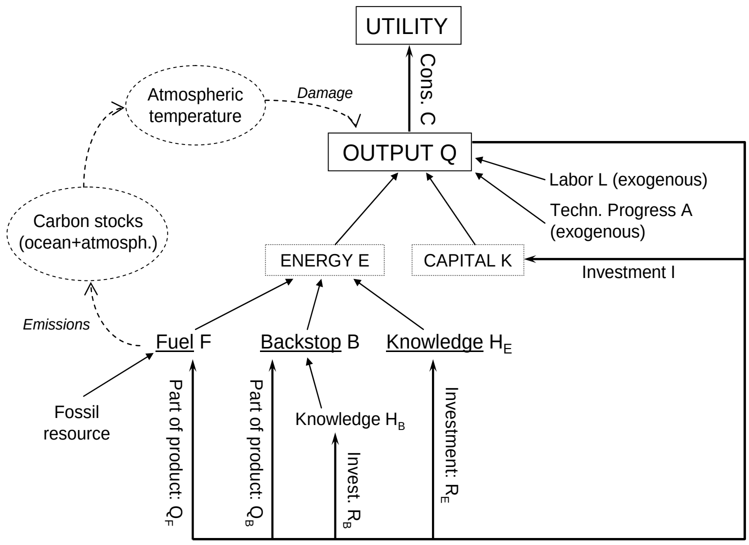

Figure 1: Description of the model

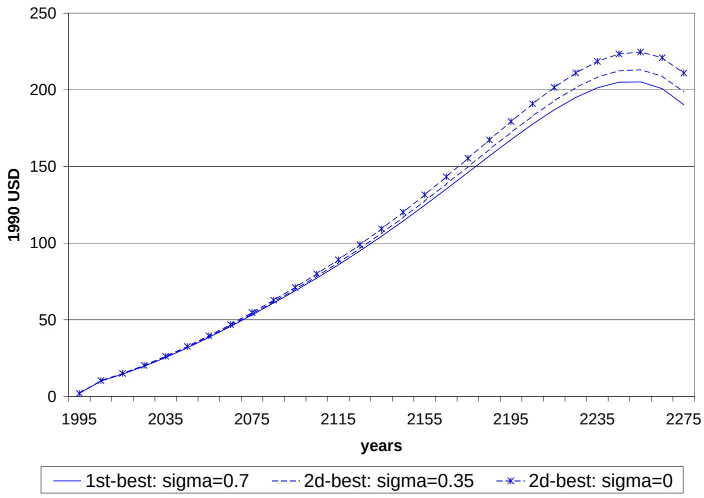

Figure 2: Optimal carbon taxes

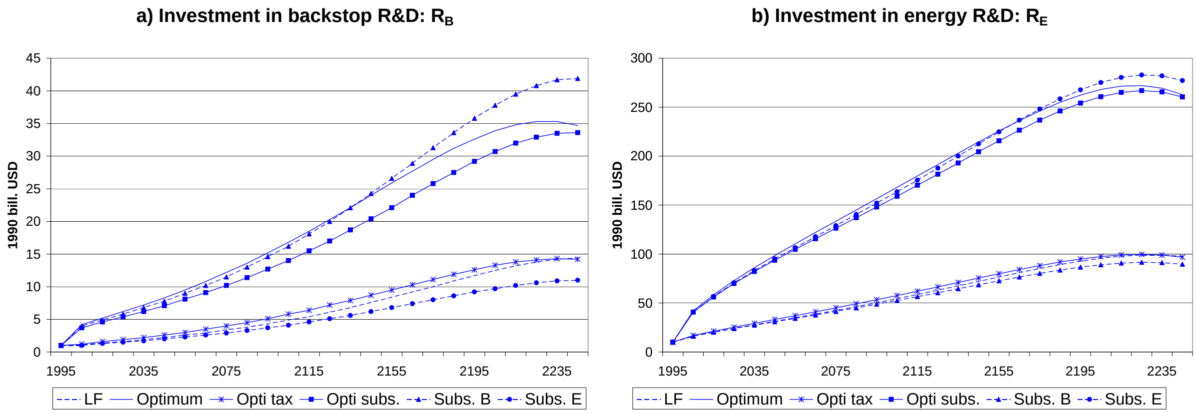

Figure 3: Effect of public policies on research investments

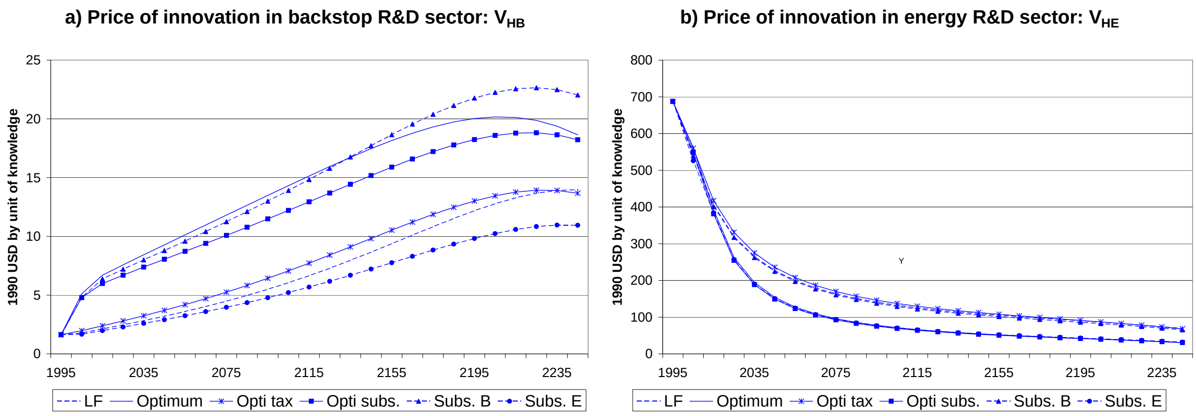

Figure 4: Effect of public policies on innovation selling prices

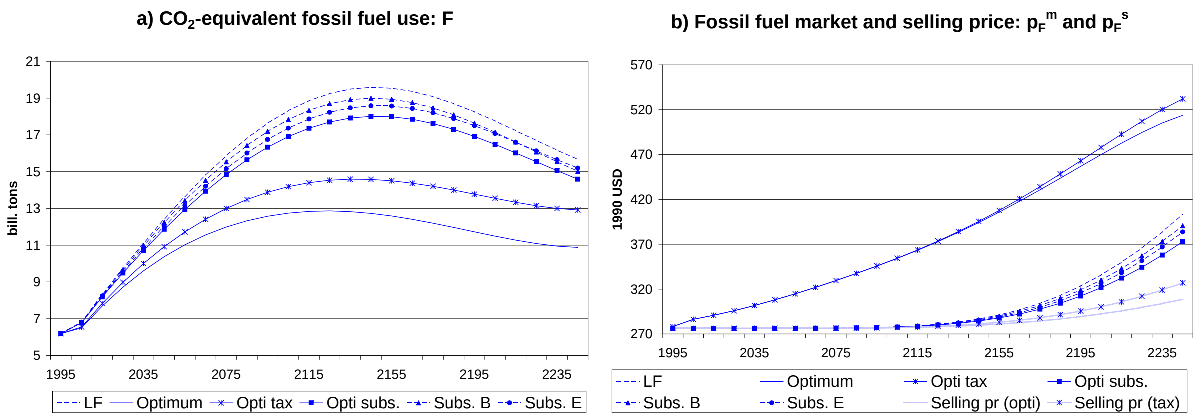

Figure 5: Effect of public policies on the fossil fuel sector

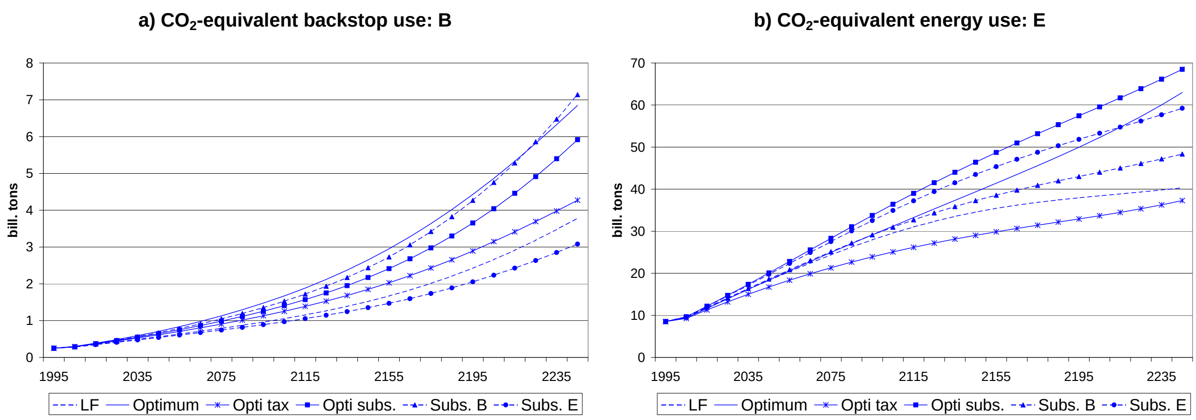

Figure 6: Effect of public policies on the backstop and energy use

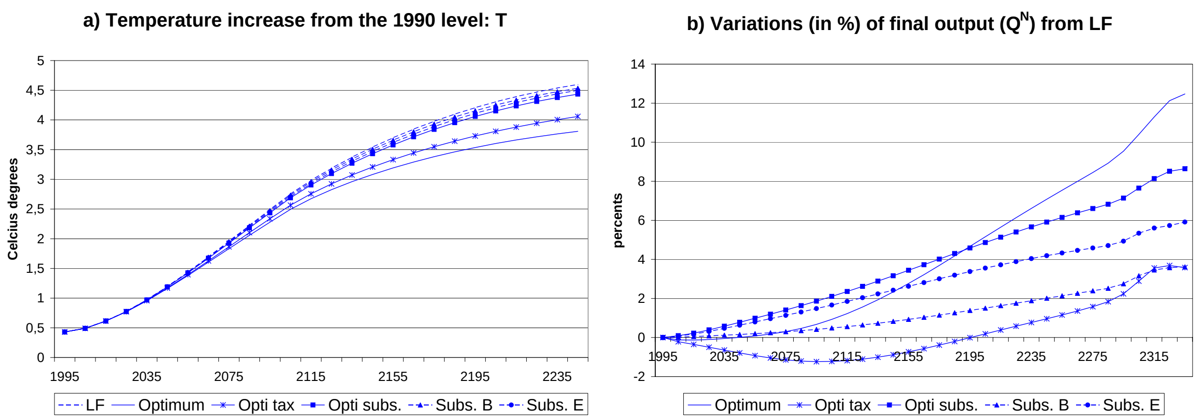

Figure 7: Effect of public policies on atmospheric temperature and the final input