Advertising, Competition and Entry in Media Industries[1]

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Abstract

This paper presents a model of media competition with free entry when media platforms are financed both from advertising receipts and customers' subscriptions. We establish a relationship between the equilibrium levels of prices, advertising and entry, the welfare maximizing levels, and the advertising technology. Under constant or increasing returns to scale in the audience size, we find an excessive level of entry and an insufficient level of advertising. We then extend the analysis along several dimensions. The relation between advertising receipts and subscription receipts depends on whether media platforms act as quantity setters or price setters in the market for advertising, leading to higher subscription prices in the latter case. We then compare pay media with free media, showing in particular that when consumers dislike advertising, imposing a non-negativity constraint on the subscription price raises profits and entry, reduces consumers' surplus and reduces advertising. We also discuss the role of endogenous content quality when prices are bound by a non-negativity constraint, showing that results are intermediate between the pay and the free media model.
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1 Introduction

In many media industries, firms receive revenues from two sources: advertising and sales. For example, consumers of newspapers, magazines, cable TV, coded broadcast TV, etc. have to pay to read articles or to view programs that include ads. In these activities, services and ads increase the total revenue of the media owner but they can be hardly increased simultaneously because of scarce space on the supply side and of advertisement disutility on the demand side. In other media industries where it is too costly to exclude consumers, for example websites or free broadcasted TV, advertising is the only revenue. This paper analyses how this two-sided financing influences quantity and price competition as well as conditions of entry and exit.[1]

How many independent firms can be active in the media industry is a very actual problem, as it can be seen from the press industry for instance.[2] Recent innovations, such as digitalization or Internet, have considerably reduced the limitation on transmission channels, providing new opportunities for entry. New media, new business models, and media convergence have drastically changed the media market conditions. In a context where the number of media firms that can be distributed increases, socially excessive entry may become a real concern. Our paper aims at providing some light on the ongoing process by discussing the role of advertising from a two-sided market perspective.

Digital technologies have also deeply changed both the way information is transmitted and the way a specific audience can be targeted. The model we use in this paper takes account of this technological advance by assuming that returns from advertising for announcers and the disutility of advertising for households are not necessarily linear functions of the number of announcements. With a linear advertising technology, the revenues from advertising are competed away through lower subscription prices and equilibrium profits are the same as in a model with no advertising. This profit equivalence breaks when the advertising technology is not linear: profits are higher and there is more entry if the advertising revenue per subscriber decreases with the audience. We then analyze the implications for entry and efficiency, and extend the analysis to various relevant contexts, incorporating price setting in the advertising market, free media and endogenous content.

[1] See Jullien (2005) for an introduction to two-sided markets. For more detailed analysis, see Caillaud-Jullien (2001, 2003), Rochet-Tirole (2003), Armstrong (2006).

[2] See section 7 in Anderson and Gabszewicz (2005).

The literature on financing of media from two sides includes papers by Gabszewicz, Laussel and Sonnac (2001a,b) and Dukes and Gal-Or (2003) who analyze the impact of advertising on the level of differentiation.[3] They find that advertising reduces differentiation between media platforms. More recently, Peitz and Valletti (2004) compare both the advertising intensity and the level of differentiation when media platforms offer free services and when the subscription price is positive. Our model does not allow for endogenous differentiation apart from the impact of entry, but we also conclude that free services are associated with larger advertising levels when customers dislike advertising. The analysis of Anderson and Coate (2005) focuses on welfare issues. They show that equilibrium advertising levels can be above or below socially optimal levels and that media platforms can provide too many or too few programs. Armstrong (2006) presents the comparison between an advertising quantity game and an advertising price game in the Hotelling model. His results are akin to ours for a fixed number of firms.

Most of these models do not endogenize the number of active media firms (or only from monopoly to duopoly). Exceptions are Choi (2006) who analyzes the case of free media with constant returns to scale in the advertising technology, and Seabright and Weeds (2005) for the case without advertising. Our analysis extends these works in several directions, including pay media, price competition and returns to audience scale for advertising, leading to new and key insights on profitability, entry and welfare, and also pointing to the limits of assuming linearity of the advertising technology.

To analyze the effects of advertising on pricing and entry strategies, we consider the Salop-Vickrey model of horizontal differentiation. In a first stage, the media platforms must decide whether to enter the market or not; if they enter, they incur a fixed entry cost. In a second stage, media platforms decide how much space will be sold to advertisers and compete in subscription prices to attract consumers. In the benchmark case where advertising exhibits constant returns to scale in the audience size and the unit value of an ad is just a fixed value per customer, the level of entry is the same as without advertising and consumers are better off due to lower subscription prices. Welfare analysis indicates that entry is excessive and that the level of advertising is insufficient. The level of entry is smaller but we obtain the same qualitative welfare results when a larger audience implies a larger unit

[3]See also Häckner and Nyberg (2000), Nilssen and Sorgard (2000), Ferrando, Gabszewicz, Laussel and Sonnac (2004).

value per customer of an ad. In the opposite case where increasing the audience decreases the unit value per customer of an ad, the effect on firms' profit and consumers' surplus is directly linked to this ad unit value. Compared to a world without advertising, media platforms always benefit from advertising and the level of entry is larger, but welfare results are ambiguous.

On the advertising market, media platforms can either impose an advertising level (quantity setting) or an advertising price (price setting). In the quantity setting game, media platforms fix advertising volumes and the advertising price is adjusted by the market at the willingness to pay of the marginal advertiser. The demand for advertising is thus independent of the demand for media services. In the price setting game, the demand for advertising depends on both the advertising price and the audience of the media. An increase in any price has then a complex impact due to the two-sided nature of the interaction between the customers and the supply of ads. At equilibrium, the amount of advertising is the same in the two games but the equilibrium subscription price is different. In the price setting game, the subscription price is higher when consumers dislike ads (and lower when they like it). The profit of the media platforms and the level of entry are thus larger when media platforms set a unit price for advertising than when they set the advertising quantity, or equivalently when they use a pricing rule that ties the price of an ad to its true value.

The paper focuses on the interaction between two sources of revenue but we also consider the case where media firms cannot charge their audience directly and only get resources through advertising. The comparison between a pay media model and a free media model is driven by the sign of the subscription price in the pay media model. When consumers dislike advertising, profits are higher, advertising is less important and consumers' surplus lower in the free media model if the subscription price is negative. In particular imposing a non-negativity constraint on subscription prices can only stimulate entry.

We then consider additional content provision as an alternative to a negative equilibrium subscription price, by assuming that subscription prices are non-negative but consumers' perceived quality can be raised at a positive cost. The introduction of quality in a media context has been developed recently and independently by Seabright and Weeds (2005), who derive in particular the free entry equilibrium with endogenous quality in the case where there is no advertising, and by Armstrong and Weeds (2005) in a duopoly model with advertising showing that quality is higher in a pay media context than in a free media one. The focus of these papers is different from ours as quality is analyzed as an additional instrument for media platforms, while we use it only when the subscription price vanishes. Due to a higher cost of raising the audience with content improvement, we obtain less advertising in a free media model with quality investment than in a model with a negative subscription price.

The paper is organized as follows. In section 2, we present the hypotheses and notations of the model. In section 3, we analyze the long run equilibrium of an industry with advertising resources when media firms choose the quantity of advertising space. Section 4 is devoted to alternative modes of competition between platforms, in particular the case where media platforms set the unit price of advertising, the case where media are supplied for free and the case where content is endogenous. Section 5 concludes.

2 Model setting

We consider an oligopoly with n media firms (or platforms) supplying customers uniformly distributed along a circle of length 1. Platform i sells its services (news, movies, etc.) at flat price p_i and receives additional revenues from the broadcasting of advertisements a_i. When the customer located at point x subscribes to platform i located at x_i, his net utility is

$$u(x; x_i, a_i) - p_i = \bar{u} - t|x - x_i| - \lambda(a_i) - p_i \tag{1}$$

where t is the unit cost of remoteness, $\bar{u}$ is the willingness to pay for the media services and $\lambda(.)$ is the disutility from ads. We allow disutility (the "nuisance effect" in Anderson and Coate (2005)) to have a U shape with respect to the number of ads a_i. A small amount of ads may thus raise the willingness to pay[4] but beyond some point the impact becomes negative. More precisely, the function $\lambda(a)$ is assumed to be convex, with $\lambda(0) = 0$ and $\lambda'(a)$ tends to $+\infty$ when a goes to infinity.

From standard arguments of the Vickrey-Salop's model[5], when the n platforms are evenly distributed on the circle and $(n-1)$ charge price p_n and

[4]Indeed, some customers enjoy some types of ads. It is the case for magazines specialized in fashion or cars as well as for some specific events: "the Super Bowl is a showcase for television commercials, and more than a quarter of viewers tune in just to watch the ads" (The Economist, April 2nd 2005, 'A survey of consumer power', p.3).

[5]See for example Tirole (1988) chapter 7.

advertise quantity a_n, one can easily derive from (1) the demand for services to firm i:

$$q_i = \frac{1}{n} + \frac{\lambda(a_n) - \lambda(a_i) + p_n - p_i}{t}. \tag{2}$$

The operating profit of platform i is then

$$\pi_i = q_i(p_i - c) + r(q_i, a_i), \tag{3}$$

where c is the unit operating cost and $r(.,.)$ is the revenue from ads.

Advertisers are producers of goods and services who post ads on the media support with the objective to attract buyers. They can post one ad per platform. The relationship between media customers and advertisers is synthesized by the inverse demand $s(q_i, a_i)$ which is the willingness to pay for one advertising message aimed at reaching the q_i customers of the media. It is increasing with the audience q_i and decreasing with the advertising volume a_i. The direct demand function for advertising spots is denoted by $a_i = A(s_i, q_i)$.

Let us define

$$v(q_i, a_i) \equiv \frac{s(q_i, a_i)}{q_i}$$

the willingness to pay for an ad per customer at platform i. Define also

$$\varphi(q_i, a_i) \equiv v(q_i, a_i)a_i$$

the advertising revenue per customer of i. The total revenue from advertisement is then

$$r(q_i, a_i) = q_i\varphi(q_i, a_i) = s(q_i, a_i)a_i$$

that is assumed to be concave in a_i. We also assume that for $q \leq 1$, v_q has a constant sign and $|v_q|$ and $|v_{qq}|$ are uniformly bounded and small enough (see the appendix for details). In particular $r(.,.)$ is increasing with q_i. Moreover, we assume that $\varphi_a(q_i, 0) = v(q_i, 0) > \lambda'(0)$ and $\varphi_{aa} < 0$, which will ensure that the advertising expenditure is strictly positive.

We do not provide a detailed analysis of how the producers decide to advertise in order to inform/persuade the customers to buy their goods and services but here is a hint on how the advertising demand function can be generated. There is a continuum of advertisers indexed by γ who want to advertise their products, where γ is uniformly distributed on $[0, 1]$. Each of them may post one ad. The ad of advertiser γ reaches the q customers of the media, which generates a net value $qv(q, \gamma)$ to advertiser γ. The function $v(q, \gamma)$ is an index of the average efficiency of advertising, decreasing with γ. It depends on the characteristics of the media and on the geographical and sociological dispersion of the households. At unit price s, advertiser γ will post an ad iff $qv(q, \gamma) \geq s$. Therefore the marginal advertiser is $\hat{\gamma}$ such that $qv(q, \hat{\gamma}) = s$, and the total demand for posting ads to one platform is defined as $a = \int_0^{\hat{\gamma}} d\gamma$. Thus the indirect demand function for advertising is defined by $s(q, a) = qv(q, a)$.

Depending on the characteristics of the media, $v(q, a)$ can have different forms. In the benchmark case, $v(q, a)$ is a constant in q. Since the willingness to pay for the product is the same among consumers, manufacturer γ is willing to pay up to $qv(q, a) = qv(0, a)$ to post an ad received by q potential clients.

If there are some increasing returns from the audience size, we have $v_q(q, a) > 0$. This is the case for instance when potential customers belong to a sub-group within the set of households, and risk averse advertisers benefit from better predictability with a large audience.[6] By contrast, when $v_q(q, a) < 0$, there are decreasing returns in advertising, say because potential buyers are concentrated and well identified and an increase in the size of the audience of the media brings less targeted customers.

In the next section, we analyze a game with the following timing: platforms simultaneously set the price p_i for the service they propose to customers and the level a_i of advertising. As an example, one may think of a platform that has prespecified space for advertising, or a magazine with a fixed number of pages of advertising. As noted by Peitz and Valletti (2005), this may also be viewed as the result of advertising prices being contingent on the audience of medias.

[6] Consider a risk averse advertiser and two consumers. Only one consumer is a potential client who generates a payoff $\pi = 1$ if she see the ads, but she is not identified. The other consumer generates nothing. When $q = 1$, the ad generates a sale with probability $1/2$ and by risk aversion the willingness to pay is $v(1, 1) < \frac{1}{2}$. When $q = 2$, an ad generates 1 for sure and the willingness to pay per consumer is $v(2, 1) = \frac{1}{2} > v(1, 1)$. More generally if there are X potential consumers within a total population N, the return from one ad on a sample of size qN, $q < 1$, randomly selected follows an hypergeometric distrisbution with mean qX and a variance $q(1-q)\frac{X(N-X)}{N-1}$ that increases at a lower rate than q and decreases for $q > 1/2$.

We assume for the rest of the paper that the market is covered, so that a symmetric equilibrium obtains when the mass of customer on each site is $q_n^* = \frac{1}{n}$, while the equilibrium price is p_n^* and the level of advertising is a_n^*. For the sake of conciseness, we present only the first order conditions of the media platforms' optimal strategies. Second order and global existence conditions are in Appendix A. Existence requires that the media be differentiated enough, which translates into the following assumption:

Assumption For all q and $a^*(q) = \arg\max_a(\varphi(q,a) - \lambda(a))$, the following condition holds:

$$t \geq 2\varphi_q + q\varphi_{qq} - q\frac{(\varphi_{aq})^2}{\varphi_{aa} - \lambda''}. \tag{4}$$

3 Competing media

We begin with the calculation of the equilibrium prices fixed by platforms for the service they provide and the equilibrium quantities of ads they accept to publish with an emphasis on the role of returns to scale in the advertising technology. We then address the determination of long run equilibrium and we compare the results with the socially optimal levels.

3.1 Equilibrium price and advertising levels

The first order conditions for the maximization of profit (3) are evaluated at $p_i = p_n$ and $a_i = a_n$ to define the equilibrium level of price and the equilibrium level of advertising. This writes as

$$\begin{aligned} \left(\frac{\partial q}{\partial p}\right)(p_n - c) + q_n + \left(\frac{\partial q}{\partial p}\right) r_q(q_n, a_n) &= 0, \\ \left(\frac{\partial q}{\partial a}\right)(p_n - c) + r_a(q_n, a_n) + \left(\frac{\partial q}{\partial a}\right) r_q(q_n, a_n) &= 0. \end{aligned}$$

The equilibrium levels then verify

$$p_n^* = c + \frac{t}{n} - r_q(\frac{1}{n}, a_n^*), \tag{5}$$

and

$$\varphi_a(\frac{1}{n}, a_n^*) = \lambda'(a_n^*). \tag{6}$$

Increasing the amount of advertising a for a given clientele q generates a marginal revenue per customer φ_a and requires to reduce the subscription price by λ' so as to maintain the clientele. The marginal revenue per customer must thus be equal to the marginal disutility of advertising per customer.

Notice that the platform may set an advertising level that would not be perceived as a nuisance by consumers, even at the margin, i.e. such that $\lambda'(a) < 0$. Whenever the revenue per customer decreases with total advertising at the level that maximizes the utility of customers, the platform chooses to restrict advertising below this level.

Since r is increasing with q, the equilibrium price is below $c + \frac{t}{n}$. The subscription equilibrium price is lowered by an audience effect of advertising. As stated in Proposition 1, the price of services does not necessarily cover the marginal cost.

Proposition 1 *When the number of sites becomes large, the subscription price is below marginal cost.*

Proof. It is sufficient to observe that $r_q(\frac{1}{n}, a_n^*)$ converges to $\varphi(0, a_\infty^*) > 0$ when the number of platforms becomes large. ■

Under constant return in the advertising technology, the equilibrium hedonic price $p_n^* + \lambda(a_n^*)$ is the same as in a Salop-Vickrey model with cost $c - \varphi(a_n^*) + \lambda(a_n^*)$.[7] The possibility of generating revenue on the advertising market leads the media platform to maximize the joint surplus of the pair platform-client, which entails a reduction in price. Overall this operation is neutral for the platforms and beneficial to consumers. The impact of advertising financing is then independent of the number of platforms, as each platform chooses the advertising level a^* and reduces its price by $\varphi(a^*)$. When returns are not constant, the advertising level depends on the number of platforms. A standard comparative static analysis gives the following result:

Proposition 2 *The level of advertising decreases, increases, or does not change with the number of platforms as $\varphi_{aq} > 0, \varphi_{aq} < 0$ or $\varphi_{aq} = 0$ respectively.*

[7] See Armstrong and Vickers (2001).

Proof. We have $a_n^* = a^*(1/n)$. Totally differentiating condition (6) and using second order conditions show that the sign of $\frac{da^*(q)}{dq}$ is the sign of φ_{aq}. The results then follows from the fact that increasing n reduces the audience per site. ■

Indeed what matters for the level of advertising is the marginal revenue per customer which may decrease or increase with the size of the audience.

3.2 Advertising technology and entry

If we insert condition (5) into (1), the difference between the equilibrium level $u(n,x) - p_n$ of net utility for the consumer located at x when there are n active platforms financed by ads and the net utility of the same viewer absent advertising is given by

$$u(n,x) - p_n - (\bar{u} - c - t|x - x_i| - \frac{t}{n}) = r_q(\frac{1}{n}, a_n^*) - \lambda(a_n^*).$$

Similarly, the difference between the equilibrium level of (variable) profit of the individual media platform financed by advertising and the profit of the same media platform without ads is given by (using $r - qr_q = -q\varphi_q$)

$$\pi(n) - \frac{t}{n^2} = -\frac{\varphi_q(\frac{1}{n}, a_n^*)}{n^2}. \quad (7)$$

The following proposition describes how the advertising technology determines the impact of ads on platforms' profits and consumers' surplus.

Proposition 3 *Compared with the no-advertising case, for a fixed number of platforms :*

i) under constant returns to scale in the audience ($\varphi_q = 0$), platforms' profits are not affected by advertising and the consumers' surplus is higher;

ii) under increasing returns to scale ($\varphi_q > 0$), platforms' profits are lower and the consumers' surplus is higher than without advertising.

iii) under decreasing returns to scale ($\varphi_q < 0$), platforms' profits are higher; the consumers' surplus is higher when n is large enough, or when $|\varphi_q|$ is small enough, or when $\lambda(a_n^*) < 0$.

Proof. The results are immediate for profits. The change in consumer surplus has the sign of $r_q - \lambda$, which is clearly positive if $\lambda < 0$. Now suppose that $\lambda > 0$. Then, using $r = q\varphi$, and $\varphi = av(q,a)$ and since $\frac{\lambda'}{n} = r_a$, we have

$$r_q - \lambda = r_q - \frac{\lambda}{\lambda'}\frac{r_a}{q} = \varphi\left(1 - \frac{\lambda}{a\lambda'}\right) + q\varphi_q - \frac{\lambda}{\lambda'}av_a.$$

Given that $\lambda(a_n^*)$ is positive and $\lambda(.)$ is convex, $1 > \frac{\lambda}{a\lambda'} > 0$ and $v_a < 0$. Therefore the effect of advertising is positive when returns to scale in the audience are constant or increasing ($\varphi_q \geq 0$). When returns to scale are decreasing, consumers benefit from advertising only when $q\varphi_q$ is close to 0. When n is large, note that $q\varphi_q$ converges to zero so that the effect is positive. ■

The effect of advertising on platforms' profit crucially depends on the advertising technology. Advertising increases the revenue of the platform but it also has a negative impact on the subscription price. The total effect is given by the sign of φ_q. When advertising exhibits constant returns to scale in the audience, the net effect is null.

For customers, the effect of advertising depends on the sign of $(r_q - \lambda)$. If the "price effect" r_q is higher than the "nuisance effect" λ, customers are better off when media can provide advertising. Notice that $r_q - \lambda$ is the derivative with respect to q of $\max_a (r(q,a) - \lambda(a)q)$, which is the revenue net of the nuisance and thus the total surplus generated by advertising for the media and its customers. Thus the customers benefit from advertising whenever the maximal surplus that the platform and its clients can derive from advertising is increasing with q.

Let us now consider the level of entry. In the long run, the equilibrium number of platforms is given approximately by $\pi(n) = k$, where k is the fixed cost of entry. Condition (4), which guarantees the existence of the equilibrium, implies also that the profit is decreasing with respect to n. Thus there exists a unique equilibrium number of entrants.[8] Let n^* be the level of entry in the quantity game. From (7), the number of active platforms in the long run equilibrium is given by

$$\frac{t}{n^{*2}} - k = \varphi_q(\frac{1}{n^*}, a_{n^*}^*)\frac{1}{n^{*2}}. \tag{8}$$

from which we can assert the following:

[8] Monotonicity of the equilibrium profit with respect to n is given by $\frac{\partial\pi(n)}{\partial n} < 0$, which imposes that $2t > 2\varphi_q(\frac{1}{n}, a_n) + \frac{1}{n}\varphi_{qq}(\frac{1}{n}, a_n) - \frac{1}{n}\frac{\varphi_{qa}(\frac{1}{n},a_n)^2}{\varphi_{aa}(\frac{1}{n},a_n) - \lambda''(a_n)}$.

Proposition 4 *The number of active firms is the same as in a world without advertising when there are constant returns to scale in the audience. The level of entry is higher (resp. lower) with advertising than without under decreasing (resp. increasing) returns to scale in the audience.*

Proof. Immediate from Proposition 3. ■

Thus, assessing the nature of the advertising technology is key to understand the impact of advertising on the market structure. In some cases, this may result in less entry than with a ban of advertising.

3.3 Welfare

To determine welfare, we assume that there is no informational externality between consumers related to advertising such as word of mouth. We also assume that the goods offered by the producers who pay for advertising are not competing. Following Anderson and Coate (2005), welfare can be computed as the sum of the consumers' surplus, the advertisers' surplus and the platforms' profit. Social welfare can then be written as follows:

$$W(n,a) = [\bar{u} - 2n \int_0^{\frac{1}{2n}} txdx - \lambda(a)] + \int_0^a v(\frac{1}{n},\gamma)d\gamma - nk - c. \tag{9}$$

The expression in the square bracket represents consumers' benefits from consuming the media services given a number n of media platforms and a quantity of advertising a. The term $\int_0^a v(\frac{1}{n},\gamma)d\gamma$ represents manufacturers' benefits from advertising. The last two terms are media platforms' costs.

The first order condition with respect to n defines the optimal number of media platforms $n^o(a)$ for a given a:

$$\frac{t}{4n^o(a)^2} - k = \frac{1}{n^o(a)^2}\int_0^a v_q(\frac{1}{n^o(a)},\gamma)d\gamma \tag{10}$$

The optimal advertising quantity a_n^o, given a number n of platforms and a mass $1/n$ of customers per site is

$$v(\frac{1}{n},a_n^o) = \lambda'(a_n^o) \tag{11}$$

The optimal level of advertising is such that the willingness to pay for the marginal advertiser (the marginal social benefit of advertising) equals the marginal disutility of advertising (the marginal social cost).

A first relation between the socially optimal levels and the equilibrium levels is stated in the following lemma. Denote $n^*(a)$ the solution of equation (8) for a given value of a.

Lemma 1 *Comparing the advertising and entry levels at market equilibrium and at welfare maximum, we have:*

i) $a_n^o > a_n^*$ *for all* n.

ii) $n^o(a) < n^*(a)$ *for all* a *when* $v_{qa} \leq 0$.

Proof. *See Appendix B.* ■

The first part results from the exercise of market power by media platforms on the market for advertising. Instead of equating the increase in advertising disutility $\lambda'(.)$ with the willingness to pay for an ad per customer $v(.)$ like in (11), media platforms equate $\lambda'(.)$ and the marginal revenue of advertising $\varphi_a = v + av_a > v$ (see (6)). Thus for a given number of media platforms, there is too little advertising. The second part is a standard result of the Salop-Vickrey model. Because of a business stealing effect, for a given level of advertising, there are too many entrants.

The overall comparison between the free entry equilibrium welfare and the optimum is delicate, as it involves several effects. The next proposition characterizes the welfare analysis for the case of constant and increasing returns to scale in the audience.

Proposition 5 *Assume that consumers dislike advertising. When* a_n^* *is non-decreasing* ($\varphi_{qa} \geq 0$) *and* $v_{qa} \leq 0$, *there are more entry and less advertising than at the welfare optimum.*

Proof. See Appendix B. ■

This result of underprovision of advertising, as pointed out by Armstrong (2006), comes from the fact that each media platform acts has a local monopoly in the advertising market.

4 Alternative modes of competition

In reaction to technological innovations, media business models have been quite innovating in the recent years. Because new forms of competition may lead to different outcomes, we need to understand how our analysis is affected when the competition between platforms is takes alternative forms . In what follows, we modify the model along three dimensions that seem to be the most relevant: the price as a strategic variable on the market for advertising, free media platforms, and the quality dimension. For the sake of conciseness we restrict attention in this section to the case where consumers dislike advertising, as stated below:[9]

Assumption: The disutility $\lambda(a)$ is increasing with a.

4.1 Price *vs.* quantity on the advertising market

There are many instances where media platforms can let the amount a_i of advertising adjust to prices without affecting the quality and quantity of the service they supply. For instance, newspapers or magazines can adjust the number of pages, websites can adjust their ergonomics. Here we consider what happens if the platforms choose a subscription price p_i and an advertising price s_i.The demand for advertising addressed to firm i is then $a_i = A(s_i, q_i)$ and is jointly determined with q_i.

Intuitively, the subscription demand is less sensitive to the subscription price than in the previous case. To understand this effect, suppose that media platform i increases its subscription price by 1 unit. Absent advertising, this would reduce the demand for its service by $\frac{1}{t}$. The immediate effect is to raise the demand of customers to adjacent platforms. As a consequence the level of advertising at adjacent platforms also increases. But this reduces the attractiveness of adjacent platforms so that the final reduction is smaller than $\frac{1}{t}$. There is thus a *feedback effect* through the adjustment of advertising levels.

One difficulty is that the demand to platform i now depends on the strategies of all the other platforms. When it changes its subscription price, the

[9] Alternatively we could assume that platforms always set advertising levels at a point where the marginal impact on consumers is negative. This seems a reasonable assumption for the vast majority of media.

change in the level of advertising at adjacent platforms $i+1$ and $i-1$ affects the demand of media services to the next platforms $i+2$ and $i-2$, and thereby the level of advertising they face. This in turn affects the next platforms and so on. Following this reasoning we see that demand for both services and advertising to all platforms are affected. We refer to this as a *propagation effect* along the circle. The next proposition compares the subscription price and the quantity of advertising under price setting with p_n^* and a_n^* determined in the former section under quantity setting.

Proposition 6 *When platforms set the price of advertising, for a given* n*, the equilibrium level of advertising is* a_n^**, and there exists* $\theta_n^* > 1$ *such that the equilibrium subscription price is* $p_n^* + \frac{t}{n}(\theta_n^* - 1)$.

Proof. See Appendix B. ■

Like in the quantity game, the subscription equilibrium price is lowered by the audience effect of advertising.[10] Since consumers dislike ads, because the volume of advertising at all platforms is endogenous, it is harder to capture market shares, which mitigates competition. This effect, that appears through $\theta_n^* > 1$, tends to raise the equilibrium price and platforms are better off in the "price setting game" as compared with the "quantity setting game". Of course for a given n, consumers are worse off, and advertisers obtain the same surplus.

We show in Crampes et al. (2005) that θ_n^* is of the order of $\sqrt{n}$. As a consequence, when n is large, both the media platforms and the consumers (and of course the advertisers) are better off compared to the no-advertising case, which was not the case in the quantity game.

The main consequence of the reduction in the level of competitiveness is that entry is boosted:

Proposition 7 *There is more entry when media platforms set the price of advertising than when they set the volume of advertising.*

Proof. The proof is straightforward given that there is a unique number of entrant in the quantity setting game, and profits are positive for $n \leq n^*$ since $\theta_n^* > 1$. ■

[10] In the case were $\lambda' < 0$, the price is smaller than when platforms set advertising quantity (as $\theta_n^* < 1$), while for $\lambda' = 0$ it is the same. This generalizes similar results obtained by Armstrong (2005), for a two-firm model, and thus without the propagation effect.

An immediate consequence is that if advertising increases with the audience, there is also less advertising in the long-run equilibrium when platforms set the advertising price. In particular:

Corollary 1 *When $\varphi_{qa} \geq 0$ and $v_{qa} \leq 0$, the quantity setting model is socially preferable to the price setting model.*

Proof. See Appendix B. ■

Notice that given that $\varphi_{qa} = v_q + av_{qa}$, the conditions require that $v_q \geq 0$, which means constant or increasing returns in the advertising technology.[11] Under decreasing returns to scale, the choice between the two models is more difficult. In particular, if entry is excessive, we may obtain too much entry in the price setting game in comparison to the quantity setting game but we may obtain an advertising level closer to the socially optimal one in the former.

4.2 Free media *vs.* pay media

Media may be distributed for free for two reasons. First, it may be technically difficult to monitor access to a media, as for instance to free-to-air TV. Second, the equilibrium subscription price may be negative and it may be difficult to support negative prices, for instance when the concerned population is small and the rest of the population can acquire the good at no cost and decide not to consume it (with thus no benefit to advertisers). Then the service will be offered for free and financed through advertising revenue solely. Typical examples of this are the emergence of free newspapers in recent years, and the free search engines and website on Internet.

When consumers access freely to media services, results can be derived by adjusting the first order condition of the quantity game to the constraint $p_n = 0$. The equilibrium level of advertising is $\hat{a}_n = \arg\max_a r(q, a) - cq$ where $q = \frac{1}{n} + \frac{\lambda(a_n) - \lambda(a)}{t}$. The first order condition gives [12]

$$r_a(\frac{1}{n}, \hat{a}_n) - \frac{\lambda'(\hat{a}_n)}{t}\left(r_q(\frac{1}{n}, \hat{a}_n) - c\right) = 0. \tag{12}$$

[11] If v is separable $v(q, a) = g(q)G(a)$, where $G' < 0$; then the conditions are verified exactly when $v_q \geq 0$.

[12] A detailed analysis can be found in Choi (2006).

The trade-off is between the increase in the volume of advertising and the reduction in the price of advertising induced by the reduction in audience. The comparison between $\hat{a}_n$ and the level a_n^* of advertising when the platform has a non zero subscription price is based on the observation that equation (6) can be rewritten as

$$r_a(\frac{1}{n}, a_n^*) - \frac{\lambda'(a_n^*)}{t}(r_q(\frac{1}{n}, a_n^*) - c) = \frac{\lambda'(a_n^*)}{t} p_n^*. \tag{13}$$

Lemma 2 *Assume that (12) has a unique solution. Then* $\hat{a}_n < a_n^*$ *(resp.* $> a_n^*$*) when* $p_n^* < 0$ *(resp.* > 0*).*

Proof. The LHS of (12) estimated at a_n^* has the same sign as p_n^*. The Lemma then results from the fact that (12) is decreasing at $\hat{a}_n$ by the second order condition. ■

Thus, given that customers dislike advertising, a pay-media generates less advertising when equilibrium prices are positive.

Of particular interest is the case where media platforms can charge subscription prices but these prices have to be non-negative. Then the equilibrium price is the maximum of zero and p_n^*, and the level of advertising is the minimum of a_n^* and $\hat{a}_n$. Imposing a non-negativity constraint on subscription price can thus only reduce the level of advertising.

The next question is whether platforms benefit from imposing positive subscription prices to consumers and whether consumers are better off or worse off in the free-media model. We show in Appendix B that the benefit of imposing prices to consumers depends on returns to scale in the advertising technology.

Proposition 8 *Assume that* $\varphi_{aq} \geq 0$*. If* $p_n^* < 0$*, free-media profits are larger than pay-media profits and free-media consumers' surplus is lower than pay-media surplus.*

Proof. See Appendix B. ■

The case where the equilibrium subscription price is constrained to be non-negative leads to interesting conclusions. Now the media service is either free or sold at a positive price. Then profits are either equal to or larger than profits in the case were prices can be negative. Moreover the advertising level is unchanged or smaller.

Corollary 2 *Assume that $\varphi_{aq} \geq 0$. There is (weakly) more entry and less advertising if prices are constrained to be non-negative than if prices are not constrained.*

Proof. There is more entry since for given n, profits are equal to or larger than profits in the unconstrained price regime. From lemma 2, advertising is $\min \{a_n^*, \hat{a}_n\}$ and it is decreasing in n. ■

Thus the excessive entry / insufficient advertising result of section 3.3 is reinforced.

Corollary 3 *Assume that $\varphi_{aq} \geq 0$ and $v_{qa} \leq 0$. Then total welfare is lower when a non-negativity constraint is imposed on prices.*

Proof. Immediate from Corollary 2 and Proposition 5. ■

In the case where media services are constrained to be free, the result of excessive entry and insufficient advertising is not evident. Indeed Choi (2006) shows in a model with constant returns in the advertising technology that entry and advertising can be either excessive or insufficient.

Finally let us point that the analysis of the equilibrium with free access is similar when platforms set the price of advertising, accounting for the factor θ. The new feature is that the transport cost t has to be replaced by $\theta_n^* t$ for the pay-media case, and $\hat{\theta}_n t$ for the free-media case, where the coefficients θ_n^* and $\hat{\theta}_n$ are functions of the advertising levels a_n^* and $\hat{a}_n$. The conclusions are then similar to those obtained when platforms set the volume of advertising.

4.3 Quality *vs.* negative price

When firms are constrained to set the service price to zero, they may opt for alternative competition strategies that would not be profitable with flexible prices. One such alternative is to increase the content either in volume or in quality.[13]

To examine the issue, let us assume that prices are constrained to be non-negative, and that investing z_i per customer allows a platform to raise the utility of its customers by an amount ρz_i, where ρ is a parameter smaller than

[13] The provision of quality in a broadcasting context is developed in Armstrong and Weeds (2005) and Seabright and Weeds (2005). They consider quality as a complementary instrument to subscription prices.

1. In this case the subscription demand (2) is $q_i = \frac{1}{n} + \frac{\lambda(a_n)-\lambda(a_i)+p_n-\rho z_n-p_i+\rho z_i}{t}$, and the profit (3) becomes $(p_i - z_i - c)\, q_i + r(q_i, a_i)$.

If $\rho = 1$, we can relabel an equivalent price $\widetilde{p}_i = p_i - z_i$ that can be positive or negative. The situation would then be the same as before, interpreting negative price as quality provision.

Now assume that $\rho < 1$. As reducing the price is more profitable than increasing z_i, it is immediate that a platform will not include an extra content if its price is positive. Therefore the equilibrium is the same as before if $p_n^* \geq 0$, with $z_n = 0$.

When $p_n^* < 0$, the equilibrium is obtained at a zero subscription price. The profit of a platform i choosing a zero subscription price can be defined as

$$\begin{aligned} \pi_i &= q_i((-z_i) - c) + r(q_i, a_i) \\ \text{where } q_i &= \frac{1}{n} + \frac{\lambda(a_n)/\rho - \lambda(a_i)/\rho + (-z_n) - (-z_i)}{t/\rho}. \end{aligned}$$

Thus we can treat $(-z_i)$ as a subscription price in a game where the disutility from ads is $\lambda(a)/\rho$ and the transport cost is t/ρ, and platforms choose a non-negative subscription price $(-z_i)$ and the advertising level a_i. Thus when $z_n > 0$, the equilibrium level of adverting is $a_n^\rho < a_n^*$ that solves:

$$\varphi_a(\frac{1}{n}, a_n^\rho) = \lambda'(a_n^\rho)/\rho,$$

and by (5) the level of z_i is given by

$$z_n^* = -\left[c + \frac{t}{\rho n} - r_q(\frac{1}{n}, a_n^\rho)\right].$$

We thus obtain:

Proposition 9 *Suppose that $p_n^* < 0$ and subscription prices cannot be negative. Then platforms set $p_n = 0$ and increase content by $\max\{z_n^*, 0\}$. Assuming that (12) has a unique solution, the advertising level is $\max\{a_n^\rho, \hat{a}_n\}$.*

Proof. The first part of the proposition is immediate given the remarks above. Observe that from (13) $z_n^* = -[c + \frac{t r_a(\frac{1}{n}, a_n^\rho)}{\lambda'(a_n^\rho)} - r_q(\frac{1}{n}, a_n^\rho)]$. We observe that $z_n^* = 0$ is obtained at $a_n^\rho = \hat{a}_n$, and since (12) is decreasing at $\hat{a}_n$, we have that $z_n^* > 0$ implies $a_n^\rho > \hat{a}_n$. ■

An equilibrium with a zero price and no extra content emerges when

$$c + \rho\frac{t}{n} - r_q\left(\frac{1}{n}, a_n^\rho\right) \geq 0 \geq c + \frac{t}{n} - r_q\left(\frac{1}{n}, a_n^*\right),$$

thus when ρ is large enough.

The effect of allowing for extra content is thus to restore the role of negative prices but at the cost of higher disutility for consumers for given (negative) unit revenue and also less differentiation. Comparing with the previous cases, this softens the negative impact on advertising of the constraint on subscription prices. The reason is that the relative cost of preserving the audience by reducing advertising is smaller than before, as the alternative is to increase z which is more costly than reducing the price p. The qualitative features of the equilibrium are however the same as before, adjusting for coefficient ρ.

Notice also that, assuming that $\varphi_{aq} \geq 0$, profit is lower at $z_n^* > 0$ (a negative price) than at $z_n = 0$: platforms would be better off without the possibility to raise content. As a consequence, we can assert the following proposition:

Proposition 10 *Assume that $\varphi_{aq} \geq 0$, that subscription prices cannot be negative and that platforms can improve content. Then there is more entry and less advertising than with negative prices, and less entry and more advertising than with no extra content.*

Proof. The same proof as for proposition 8 shows that profit lies between these two cases, so that entry is intermediate. The result on advertising follows from corollary 2 and the monotonicity of a_n^*. ■

Thus the possibility to add content leads to an intermediate conclusion but does not change the nature of the analysis.

5 Conclusion

In contexts where media platforms are financed both from advertisers and viewers, the relation between advertising receipts and sales receipts, and the level of entry and welfare have been shown to depend in a critical way on the advertising technology. Under reasonable conditions, consumers benefit from advertising due to lower subscription prices. The equilibrium level of advertising is suboptimal. As compared with a no-advertising benchmark, media profits are the same with a linear advertising technology, but they are higher under decreasing returns in the audience size, and they are lower under increasing returns. The level of entry is excessive and adjusts in a similar way, with more entry under decreasing returns in the advertising technology.

We have also seen that whether media platforms set the advertising space or the advertising price matters, with higher profits and prices and less efficiency in the latter case.

Is it socially better to let media platforms set a subscription price? Our analysis suggests that despite the fact that advertising intensifies competition on the subscription market, unconstrained equilibrium subscription prices are too high from a consumers' welfare perspective. Thus a constraint that raises the equilibrium prices is detrimental to consumers, while a move from positive to zero subscription prices is more likely to benefit consumers.

We also emphasized the fact that extra content provision may act as a substitute to negative subscription prices. The analysis of the model with endogenous content suggests that the relevant model for the analysis of free media may intermediate been the pay-media model and the free-media model.

One lesson is that the specific nature of the advertising technology is crucial to understanding the interactions between platforms, the profitability of the market and the pattern of entry. The nature of nonlinearity in advertising markets should ultimately be traced back to the type of support, the type of audience, and the advertised products. It would thus be useful to understand how this varies for different medias.

Accounting for nonlinearity in the advertising technology appears to be the most important for issues related to profits. One issue related to profits that we did not addressed is investment. The importance of future advertising revenue is clear for instance for the development of the new generation of mobile telephony (3G). Again the nature of the advertising technology will be a key determinant of the ability of telecommunication operators to recoup their investments.

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A Global existence conditions

In the Salop-Vickrey model ($a \equiv 0$), the profit function is concave and first-order conditions are sufficient for equilibrium. Notice that in the case $\varphi_q \equiv 0$, the level of advertising is $a^*(0)$ for any price p set by the media, so that the model is equivalent to a Salop-Vickrey model with marginal cost $c - \varphi(a^*(0))$; see (5) in the text. Thus first-order conditions are sufficient to prove existence.

This may not be the case in our general model due to nonlinearity in advertising resources which changes the gains from increasing the size of the audience. The sufficient conditions for the existence of the equilibrium are given in the following lemma.

Lemma 3 *Under condition* (4), (p_n^*, a_n^*) *is an equilibrium if and only if a media platform does not benefit from setting* (p_i, a_i) *such that* $q_i = \frac{3}{n}$, *which reduces to*

$$t \geq 3n \int_{\frac{1}{n}}^{\frac{3}{n}} \varphi_q(q, a^*(q))dq - 2\varphi_q(\frac{1}{n}, a_n^*).$$

Proof. Consider the symmetrical equilibrium with n active media platforms.

Define

$$\hat{\pi}_1(q,t) = (t(\frac{1}{n} - q) - c + p_n^* + \lambda\,(a_n^*))q + \max_a(r(q,a) - \lambda\,(a)\,q).$$

Let us now consider the deviation of firm i. Consider first a deviation leading to $q < \frac{2}{n}$. Then firm i faces a demand

$$q = \frac{\lambda(a_n^*) - \lambda(a) + p_n^* - p}{t} + \frac{1}{n}.$$

The maximal profit it can obtain while selling $q < \frac{2}{n}$ is then given by $\hat{\pi}_1(q,t)$. If the function $\hat{\pi}_1(q,t)$ is concave, deviations with $q < \frac{2}{n}$ are not profitable. Notice that

$$\hat{\pi}_1(q,t) = \hat{\pi}_1(q, \frac{t}{2}) + \frac{t}{2}\left(\frac{1}{n} - q\right)q$$

is concave if $\hat{\pi}_1(q, \frac{t}{2})$ is concave.

At $q = \frac{2}{n}$, there is a discontinuity because if a customer at the distance $\frac{1}{n}$ buys from the media, so do all customers at a distance below $\frac{3}{2n}$. Therefore q jumps at $\frac{3}{n}$. Assume now that a consumer located at a distance $x \in [\frac{j-1}{n}, \frac{j}{n}]$ from firm i is indifferent between purchasing from firm i and purchasing from the firm located at the distance $\frac{j}{n}$ from firm i. We have that

$$\lambda\left(a_n^*\right) + p_n^* + t(\frac{j}{n} - x) = \lambda\left(a\right) + p + tx.$$

Then firm i faces a demand

$$q = \frac{\lambda(a_n^*) - \lambda(a) + p_n^* - p}{t} + \frac{j}{n}.$$

Thus the media platform can sell $q \in [\frac{2j-1}{n}, \frac{2j}{n}]$ when announcing a price $p = t(\frac{j}{n} - q) + \lambda(a_n^*) - \lambda(a) + p_n^*$. The profit of the deviating firm is given by

$$\hat{\pi}_j(q) = (t(\frac{j}{n} - q) - c + p_n^* + \lambda\left(a_n^*\right))q + \max_a (r(q,a) - \lambda\left(a\right)q).$$

This can be rewritten as

$$\begin{aligned} \hat{\pi}_j(q) &= \hat{\pi}_1(q, \frac{t}{2}) + \frac{t}{2}(\frac{2j-1}{n} - q)q \\ &\leq \hat{\pi}_1(q, \frac{t}{2}), \end{aligned}$$

where the equality holds at $q = \frac{2j-1}{n}$.

Observe next that $\hat{\pi}_1(\frac{1}{n}, t) = \hat{\pi}_1(\frac{1}{n}, \frac{t}{2})$, so that comparing the equilibrium profit with the deviation profit when $q > \frac{3}{n}$ amounts to comparing $\hat{\pi}_1(\frac{1}{n}, \frac{t}{2})$ and $\hat{\pi}_1(q, \frac{t}{2})$.

Notice that $\hat{\pi}_1(q, \frac{t}{2})$ is increasing with q at $q = \frac{1}{n}$. When $\hat{\pi}_1(q, \frac{t}{2})$ is concave $\hat{\pi}_1(q, \frac{t}{2}) \leq \hat{\pi}_1(\frac{1}{n}, \frac{t}{2})$ for all $q \geq \frac{3}{n}$ if this is true for $\frac{3}{n}$.

The sufficient conditions to guarantee that firm i prefers not to deviate are thus

i) The function $\hat{\pi}_1(q, \frac{t}{2})$ is concave.

ii) $\hat{\pi}_1(\frac{1}{n}, \frac{t}{2}) \geq \hat{\pi}_1(\frac{3}{n}, \frac{t}{2})$.

Let us now establish the condition for the concavity of $\hat{\pi}_1(q, \frac{t}{2})$. We evaluate $\frac{\partial^2 \hat{\pi}_1(q,\frac{t}{2})}{\partial q^2}$ at $a = a^*(q)$ where $a^*(q)$ is defined by $\varphi_a(q, a^*(q)) = \lambda'(a^*(q))$. Using $\frac{da^*(q)}{dq} = -\frac{\varphi_{aq}}{\varphi_{aa}-\lambda''}$, we obtain

$$\frac{\partial^2 \hat{\pi}_1(q, \frac{t}{2})}{\partial q^2} = -t + 2\varphi_q + q\varphi_{qq} + q\varphi_{qa}\left(-\frac{\varphi_{aq}}{\varphi_{aa} - \lambda''}\right),$$

which gives condition (4). Moreover the second order condition writes

$$t + 2\varphi_q(\frac{1}{n}, a_n^*) \geq 3n\left(\max_a(\varphi(\frac{3}{n}, a) - \lambda(a)) - \max_a(\varphi(\frac{1}{n}, a) - \lambda(a))\right)$$

which gives the condition in the lemma 3. ■

Observe that condition (4) is verified under constant returns to scale in the audience. Else, it requires that the first and second derivatives of φ with respect to the audience be close enough to zero. For n large, the sufficient condition reduces to

$$t \geq 4\varphi_q(0, a_\infty^*),\ \textit{where } a_\infty^* = \lim_{n\to+\infty} a_n^* > 0 \text{ solves } \varphi_a(0, a_\infty^*) = \lambda'(a_\infty^*).$$

B Proofs of Lemmas and Propositions

Proof of lemma 1.

For a given n, the advertising equilibrium level satisfies equation

$$v\left(\frac{1}{n}, a_n^*\right) + a_n^* v_a(\frac{1}{n}, a_n^*) = \lambda'(a_n^*),$$

and the socially optimal level is given by

$$v(\frac{1}{n}, a_n^o) = \lambda'(a_n^o).$$

Since v is decreasing with a, we obtain that $a_n^* < a_n^o$.

Recall that the level of entry in the quantity model $n^*(a)$ is given by

$$\frac{t}{n^2} - k = av_q(\frac{1}{n}, a)\frac{1}{n^2}.$$

The condition defining $n^o(a)$ is

$$\frac{t}{4n^2} - k = \frac{1}{n^2}\int_0^a v_q(\frac{1}{n},\gamma)d\gamma \tag{14}$$

Observe that when $v_{qa} \le 0$, $av_q(\frac{1}{n},a) \le \int_0^a v_q(\frac{1}{n},\gamma)$ so that $\frac{t}{n^2} - av_q(\frac{1}{n},a)\frac{1}{n^2} > \frac{t}{4n^2} - \frac{1}{n^2}\int_0^a v_q(\frac{1}{n},\gamma)d\gamma$. And thus $n^*(a) > n^o(a)$. ■

Proof of proposition 5.

Assume $v_{qa} \le 0$ and $\varphi_{qa} \ge 0$. We use lemma 1 to obtain the result. The sign of $\frac{dn^*}{da}$ is given by the sign of $-\frac{\frac{\partial \pi(n)}{\partial n}}{\frac{\partial \pi(n)}{\partial a}}$ which is negative, smaller than $\frac{\partial a_n^*}{\partial n} < 0$. This implies that for $n \ge n^*$, $n^*(a_n^*) \le n$. Then $a_n^o > a_n^*$ implies that if $n \ge n^*$, then $n > n^*(a_n^*) > n^*(a_n^o) > n^o(a_n^o)$. Thus we must have $n^o < n^*$ and as a consequence $a^o = a_{n^o}^o > a_{n^o}^* > a_n^*$. ■

Proof of proposition 6.

Starting from a symmetric situation $p_n = p$ and $s_n = s$, $q_n = \frac{1}{n}$ and $a_n = a$, suppose that media platform $i = 1$ changes its prices p_1 and s_1. By symmetry, the consequences of these changes will be the same for media platform j and $n + 2 - j$. Using the demand functions for services

$$q_i = \frac{\lambda(a_{i+1}) + \lambda(a_{i-1}) - 2\lambda(a_i) + p_{i+1} + p_{i-1} - 2p_i}{2t} + \frac{x_{i+1} - x_{i-1}}{2}$$

and the demand function for advertising $a_i = A(s_i, q_i)$ we obtain

$$\begin{aligned} dq_1 &= \frac{\lambda'(a)da_2 - dp_1 - \lambda'(a)da_1}{t} \\ da_1 &= A_s ds_1 + A_q dq_1 \end{aligned} \tag{15}$$

$$\begin{aligned} dq_2 &= \frac{dp_1 + \lambda'(a)da_1 + \lambda'(a)da_3 - 2\lambda'(a)da_2}{2t} \\ da_2 &= A_q dq_2 \end{aligned} \tag{16}$$

$$\begin{aligned} dq_j &= \frac{\lambda'(a)da_{j-1} + \lambda'(a)da_{j+1} - 2\lambda'(a)da_j}{2t} \\ da_j &= A_q dq_j \text{ for } j \ge 3. \end{aligned} \tag{17}$$

The case $\lambda'(a) = 0$ is immediate, so assume that $\lambda'(a) > 0$. Assume first that there are $n = 2m$ media, then platform $m+1$ is facing m and $m+2 = n+2-m$. In that case, we can write $da_{m+2} = da_{n-m} = da_m$ and by (17)

$$\begin{aligned} dq_{m+1} &= \frac{2\lambda'(a)da_m - 2\lambda'(a)da_{m+1}}{2t} \\ da_{m+1} &= A_q dq_{m+1} \end{aligned}$$

Assume now that there are $n = 2m+1$ platform, then platform $m+1$ is facing m and $m+2 = n+2-(m+1)$. In that case, we have $da_{m+2} = da_{n-m} = da_{m+1}$ so that by (17)

$$\begin{aligned} dq_{m+1} &= \frac{\lambda'(a)da_m - \lambda'(a)da_{m+1}}{2t} \\ da_{m+1} &= A_q dq_{m+1} \end{aligned}$$

We can rewrite condition (16) as

$$-\frac{dp_1}{\lambda'(a)} = da_1 - 2\left(\frac{t}{\lambda'(a)A_q} + 1\right) da_2 + da_3 \tag{18}$$

and condition (17) as

$$0 = da_{j-1} - 2\left(\frac{t}{\lambda'(a)A_q} + 1\right) da_j + da_{j+1},\ j = 3, .., m. \tag{19}$$

Consider platform $m + 1$:

if $n = 2m$, we know that $da_{m+2} = da_m$; therefore

$$\left(\frac{t}{\lambda'(a)A_q} + 1\right) da_{m+1} = da_m \tag{20}$$

if $n = 2m + 1$, we know that $da_{m+2} = da_{m+1}$; therefore

$$2\left(\frac{t}{\lambda'(a)A_q} + 1\right) da_{m+1} = da_m + da_{m+1} \tag{21}$$

Denote by α (resp. β) the smaller (resp. larger) root of

$$0 = 1 - 2\left(\frac{t}{\lambda'(a)A_q} + 1\right) x + x^2.$$

Observe that when $\lambda'(a) > 0$, the equation admits two (positive) roots, and when $\lambda'(a) < 0$, the equation admits two (negative) roots only when $\lambda'(a) > \frac{t}{2A_q}$. Let

$$da_j = x_j\alpha^{j-1} + y_j\beta^{j-1} \tag{22}$$

From (18)

$$-\frac{dp_1}{\lambda'(a)} - da_1 = (-2\left(\frac{t}{\lambda'(a)A_q} + 1\right)x_2\alpha + x_3\alpha^2) + (-2\left(\frac{t}{\lambda'(a)A_q} + 1\right)y_2\beta + y_3\beta^2).$$

From (19), for $j = 3, ..., m$

$$0 = (x_{j-1} - 2\left(\frac{t}{\lambda'(a)A_q} + 1\right)x_j\alpha + x_{j+1}\alpha^2)\alpha^{j-2} + (y_{j-1} - 2\left(\frac{t}{\lambda'(a)A_q} + 1\right)y_j\beta + y_{j+1}\beta^2)\beta^{j-2}$$

For $m+1$, from (20), (21) and (22)

$$\begin{aligned} 0 = &\left(x_m - \left(\frac{t}{\lambda'(a)A_q} + 1\right)x_{m+1}\alpha\right)\alpha^{m-1} \\ &+ \left(y_m - \left(\frac{t}{\lambda'(a)A_q} + 1\right)y_{m+1}\beta\right)\beta^{m-1} \; if \;\; n = 2m \end{aligned}$$

$$\begin{aligned} 0 = &\left(x_m - \left(\frac{2t}{\lambda'(a)A_q} + 1\right)x_{m+1}\alpha\right)\alpha^{m-1} \\ &+ \left(y_m - \left(\frac{2t}{\lambda'(a)A_q} + 1\right)y_{m+1}\beta\right)\beta^{m-1} \; if \;\; n = 2m+1 \end{aligned}$$

Set $x_j = x$ and $y_j = y$. We obtain

$$-\frac{dp_1}{\lambda'(a)} - da_1 = -x - y \tag{23}$$

$$\begin{aligned} 0 &= x\left(1 - \left(\frac{t}{\lambda'(a)A_q} + 1\right)\alpha\right)\alpha^{m-1} + y\left(1 - \left(\frac{t}{\lambda'(a)A_q} + 1\right)\beta\right)\beta^{m-1} \; if \;\; n = 2m \\ 0 &= x\left(1 - \left(\frac{2t}{\lambda'(a)A_q} + 1\right)\alpha\right)\alpha^{m-1} + y\left(1 - \left(\frac{2t}{\lambda'(a)A_q} + 1\right)\beta\right)\beta^{m-1} \; if \;\; n = 2m+1 \end{aligned}$$

or

$$x\left(\frac{1-\alpha^2}{2}\right)\alpha^{m-1} = -y\left(\frac{1-\beta^2}{2}\right)\beta^{m-1}\ if\ \ n=2m \tag{24}$$

$$x\left(\alpha-\alpha^2\right)\alpha^{m-1} = -y\left(\beta-\beta^2\right)\beta^{m-1}\ if\ \ n=2m+1 \tag{25}$$

From (23) and (24) we can compute x and y corresponding to the case $n = 2m$. Then injecting those values into (22) for $j = 2$, we obtain:

$$da_2 = -\left(\frac{dp_1}{\lambda'(a)}+da_1\right)\left(\frac{\alpha\left(\beta^{m-1}-\beta^{m+1}\right)-\beta\left(\alpha^{m-1}-\alpha^{m+1}\right)}{\left(\alpha^{m-1}-\alpha^{m+1}\right)-\left(\beta^{m-1}-\beta^{m+1}\right)}\right) \tag{26}$$
$$if\ \ n=2m$$

$$da_2 = -\left(\frac{dp_1}{\lambda'(a)}+da_1\right)\left(\frac{\alpha\left(\beta^{m}-\beta^{m+1}\right)-\beta\left(\alpha^{m}-\alpha^{m+1}\right)}{\left(\alpha^{m}-\alpha^{m+1}\right)-\left(\beta^{m}-\beta^{m+1}\right)}\right) \tag{27}$$
$$if\ \ n=2m+1$$

Notice that $\alpha\beta = 1$. So we can also write (26) and (27) as

$$\begin{aligned}\lambda'(a)da_2 &= \left(\lambda'(a)da_1+dp_1\right)\frac{\alpha^{\frac{n}{2}-1}+\beta^{\frac{n}{2}-1}}{\alpha^{\frac{n}{2}}+\beta^{\frac{n}{2}}} \\ &= \left(\lambda'(a)da_1+dp_1\right)\frac{\alpha^{n-1}+\alpha}{\alpha^n+1}\quad \text{for any } n, \text{ even or odd.}\end{aligned}$$

Using (15), we write the variation of quantity as

$$\begin{aligned}dq_1 &= -\frac{\lambda'(a)da_1+dp_1}{\theta t}, \\ da_1 &= A_s ds_1 + A_q dq_1\end{aligned}$$

where $\theta = \frac{\alpha^n+1}{(1-\alpha)(1-\alpha^{n-1})} > 1$.

The first order conditions for equilibrium are then the same as before but replacing t by $\theta_n t$ where θ_n^* is computed for $a = a_n^*$. ■

Proof of corollary 1.

In both cases, $a = a_n^*$.

$$\begin{aligned}\frac{dW(n,a_n^*)}{dn} &= \frac{t}{4n^2}-\frac{1}{n^2}\int_0^{a_n^*} v_q(\frac{1}{n},\gamma)d\gamma - k + \left(v(\frac{1}{n},a_n^*)-\lambda'\left(a_n^*\right)\right)\frac{da_n^*}{dn} \\ &\leq \frac{t}{4n^2}-\frac{1}{n^2}v_q(\frac{1}{n},a_n^*) - k - a_n^* v_a(\frac{1}{n},a_n^*)\frac{da_n^*}{dn}\ if\ v_{qa}\leq 0.\end{aligned}$$

But $\varphi_{qa} \leq 0$ implies that $\frac{da_n^*}{dn} \leq 0$ and thus $a_n^* v_a(\frac{1}{n}, a_n^*)\frac{da_n^*}{dn} > 0$, and the fact that $\pi(n)$ is decreasing and equation 8 imply that $\frac{t}{4n^2} - \frac{1}{n^2}v_q - k < 0$ fro $n > n^*$. Thus $\frac{dW(n,a_n^*)}{dn} < 0$ for $n > n^*$. Since the price setting model leads to $n \geq n^*$, the social welfare is lower than in the quantity model (strictly if $n > n^*$). ■

Proof of proposition 8.

Both for free-media and pay-media we have:

$$p - c = \frac{t}{\lambda'(a)}r_a(a, \frac{1}{n}) - r_q(a, \frac{1}{n}) = \frac{t}{\lambda'}\frac{1}{n}\varphi_a - \varphi - \frac{1}{n}\varphi_q$$

and profits

$$\pi = r + \frac{1}{n}(p - c) = \frac{1}{n^2}\left(\frac{t}{\lambda'}\varphi_a - \varphi_q\right).$$

We then have

$$\frac{\partial}{\partial a}\left(\frac{t}{\lambda'}\varphi_a - \varphi_q\right) = \left(\frac{t}{\lambda'}\varphi_{aa} - \frac{t\lambda''}{\lambda'^2}\varphi_a - \varphi_{aq}\right) < 0,$$

when $\varphi_{aq} ge 0$ and since $\lambda'(a) > 0$. Profit is larger under free-media if advertising is lower.

Consumer surplus is a decreasing function of $\lambda(a) + p$ (with $p = 0$ in the free-media model) and the sign of $\lambda'\frac{da}{dp} + 1$ indicates whether consumer surplus increases or decreases when the price increases.

We have from (13) that

$$\begin{aligned}\frac{da}{dp} &= \frac{-1}{\varphi_a + q(\varphi_{aq} - \frac{t}{\lambda'}\varphi_{aa} + \frac{t\lambda''}{\lambda'}\varphi_a)} \\ &= \frac{-1}{\varphi_a + q[\frac{\partial}{\partial a}(\varphi_q - \frac{t}{\lambda'}\varphi_a)]}\end{aligned}$$

Thus

$$\lambda'(a)\frac{da}{dp} + 1 = 1 - \frac{\lambda'}{\varphi_a + q[\frac{\partial}{\partial a}(\varphi_q - \frac{t}{\lambda'}\varphi_a)]} \tag{28}$$

Now observe that $\varphi_a > \lambda'$ when $p > p_n^*$ and that $\frac{\partial}{\partial a}(\varphi_q - \frac{t}{\lambda'}\varphi_a) > 0$ when $\varphi_{aq} \geq 0$ so that $\frac{\lambda'}{\varphi_a + q[\frac{\partial}{\partial a}(\varphi_q - \frac{t}{\lambda'}\varphi_a)]} < 1$ if $p > p_n^*$. We thus have that $\lambda(a) + p$ increases between p_n^* and 0 when $p_n^* < 0$. ■