Switching to a Poor Business Activity: Optimal Capital Structure, Agency Costs and Covenant Rules*

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Abstract

We address the issue of modeling and quantifying the asset substitution problem in a setting where equityholders decisions alter both the volatility and the return of the firm cash flows. Our results contrast with those obtained in models where the agency problem is reduced to a pure risk-shifting problem. We find larger agency costs and lower optimal leverages. We show that covenants that prevent equityholders from adopting an activity with high volatility and low return are value enhancing only when the agency problem is severe enough. Our model highlights the tradeoff between ex-post inefficient behavior of equityholders and inefficient covenant restrictions.

Key words: Capital structure, stockholder-bondholder conflict, covenant rules.

JEL Classification: G30, G32, G33.

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Abstract

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1. Introduction

The asset substitution problem, first documented by Jensen and Meckling (1978), results from the incentives of equityholders to extract value from debtholders by avoiding safe positive net present value projects. This implies a decrease in the value of the firm, as a result of a decrease in the value of the debt and a smaller increase in the value of the equity. This opportunistic behavior of equityholders is incorporated into the price of debt and the ex ante solution to this agency problem is therefore to issue less debt. As a result, the optimal capital structure of the firm highlights the benefit of issuing debt because of tax benefits, and the cost of issuing debt because of both asset substitution problem and bankruptcy costs. It has long been recognized that such a standard stockholder-bondholder conflict might be a key for understanding observed behavior of firms. It is for instance well documented, see Graham (2000), that firms tend to choose large amount of equity in their capital structure and set debt levels well below what would maximize the tax benefits of debt.

Continuous time contingent-claims analysis offers a natural setting for modeling and quantifying the asset substitution effect. The prototype of this approach is the model of Leland (1998), in which equityholders can choose a high or a low volatility level for the firm's assets once the debt is in place. Leland (1998) studies the impact of equityholders' ex post flexibility to choose volatility on the firm's optimal capital structure and finds that agency costs restrict leverage and debt maturity and increase yield spreads. Other results are however more surprising (and somewhat disappointing): agency costs of debt due to the asset substitution effect are about 1.5% which is far less than the tax benefits of debt, bond covenants that restrict equityholders from adopting the high volatility parameter are useless, furthermore the optimal leverage when there is an agency problem is larger than the optimal leverage of a firm that cannot increase risk.

The discussion on asset substitution in a contingent-claims analysis setting has been recently extended in several directions. For instance, Henessy and Tserlukevich (2004) study the role of Warrant in solving agency costs in a setting with dynamic volatility choice. They find that warrants mitigate asset substitution but exacerbate the agency problem of premature default. Childs, Mauer and Ott (2005) provide a numerical model which accommodates both asset substitution and flexibility to increase or decrease the debt level at maturity dates. They find that financing flexibility encourages the use of short term debt and significantly reduces agency costs of investment distortions. Ju and Ou-Yang (2005) show that, in a dynamic model in which the firm issues debt multiple times, the incentives of equityholders to increase volatility of firm's assets are reduced. Other related works on asset substitution are Mello and Parsons (1992), Mauer and Triantis (1994), Parrino and Weisbach (1999), Ericsson (2000), Décamps and Faure-Grimaud (2002), Mauer and Sarkar (2005).

In this paper, we leave aside these meaningful extensions and depart from the existing literature by adopting the view that the asset substitution problem can be also explained by bad investments rather than by simply pure excessive risk taking. According to Bliss (2001) this agency problem may be fundamental: "Poor (apparently irrational) investments are as problematic as excessively risky projects (with positive risk-adjusted returns)". In particular Bliss (2001) reviews several empirical articles that conclude that bank failures are often provoked by bad investments rather than bad luck (and excessive risk taking). This leads us to consider a model in which equityholders can alter both the risk adjusted expected growth rate and the volatility of the cash flows generated by the firm's assets. Specifically, in our model, the firm's activity generates a lognormal cash flows process characterized by a given risk-adjusted expected growth rate and a given volatility. At any time equityholders have the opportunity to switch from a *safe* business activity in place to a *poor* business activity. The adoption of the poor activity lowers the risk adjusted expected growth rate of the cash flows process and increases its volatility. Therefore two problems jointly define asset substitution (i) a pure risk-shifting problem acting on the volatility of the growth rate of the cash flows, and (ii) a first order stochastic dominance problem acting on the risk adjusted expected growth rate of the cash flows. We identify situations where equityholders decide to adopt the poor activity. Such a decision is not socially optimal and generates a loss in the firm value that we analyze. We then investigate how covenant rules written in the debt indenture can reduce the amount of these agency costs.

More precisely, in our model, debt is a coupon bond with infinite maturity and coupon payment offers tax deduction. As in Leland (1998) and many others, we consider endogenous bankruptcy. That is, equityholders have the option to decide when to cease paying the coupon and to declare bankruptcy. The bankruptcy policy is therefore chosen to maximize the value of equity, given the limited liability of equity and the debt structure. Initially, the firm is run with the safe activity. At each instant of time equityholders can switch in an irreversible way to the poor business activity. Switching generates agency costs whose magnitude is defined as the difference between the optimal firm value when the switching policy can be contracted ex ante (before debt is in place) and the optimal firm value when the switching decision policy is taken ex post (that is after debt is in place). In each case the optimal capital structure is characterized by the coupon rate that maximizes the initial firm value. The tradeoff underlying the model is as follows. On the one hand equityholders have incentives to switch to the poor activity because it increases their option value to declare bankruptcy. On the other hand switching entails an opportunity cost since it lowers the (risk adjusted) instantaneous return of the cash flows. We show that a drop of the cash flows can throw equityholders in a gamble for resurrection situation which leads them to choose the poor business activity despite its lower risk adjusted expected return. An alternative interpretation of our model is to see the poor business activity as the result of the decision of the equityholders to cease to monitor the firm's assets, from which it results lower risk adjusted return and larger uncertainty.

Our results contrast with the previous literature where the asset substitution problem is reduced to a pure risk-shifting problem. For example, depending on the severity of the agency problem, agency costs of debt at the optimal leverage can be large (more than 7 %). Accordingly, optimal leverage when an agency problem exists is lower than that of a firm that cannot change its activity. We pursue the analysis recognizing that covenants written in the debt indenture forcing equityholders to go bankrupt modify switching incentives and highly affect the level of agency costs. We show that, the so-called "cash flows based" covenant rule, that triggers bankruptcy as soon as the instantaneous cash flows generated by the firm activity is not sufficient to cover the instantaneous payment to debtholders, eliminates switching incentives but increases agency costs because of premature liquidation. We then introduce a new covenant rule defined as the smallest liquidation trigger such that the switching problem disappears. We show that if the agency problem is severe enough, such a covenant rule can dramatically reduce agency costs (but not eliminate them). On the contrary if the agency problem is not severe enough such a covenant rule increases agency costs and it is better to let equityholders switch to the poor activity and default strategically. Our model highlights the tradeoff between ex-post inefficient equityholders behavior and inefficient covenant restrictions.

The remainder of the paper is organized as follows: Section 2 presents the model, Section 3 analyzes optimal policies followed by equityholders, Section 4 defines and characterizes optimal capital structure and agency costs, Section 5 studies the role of covenants. Section 7 concludes. Proofs are in Appendix.

2. The model

Throughout the paper we denote by $W = (W_t)_{t\geq 0}$ a Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ and by $(\mathcal{F}_t)_{t\geq 0}$ the augmentation with respect to $\mathbb{Q}$ of the filtration generated by W. We denote by $\mathcal{T}$ the set of $\mathcal{F}_t$ adapted stopping times.

2.1. A simple model of the firm.

We start by reviewing a standard model of a firm. The ideas and the results presented in this subsection are those of Leland (1994), Goldstein, Ju and Leland (2001) or more recently Leland and Skarabot (2004). The underlying state variable X is the cash flows generated by the firm's activity (that is the firm's earnings before interest and taxes (EBIT)). We denote by "A" the activity in place and assume that the generated cash flows follow the stochastic differential equation

$$\frac{dX_{t,A}}{X_{t,A}} = \mu_A dt + \sigma_A dW_t, \tag{1}$$

with initial condition $X_{0,A} = x$, where μ_A is the instantaneous risk-adjusted expected growth rate of the cash flows and σ_A the volatility of the growth rate. There is a risk free asset that yields a constant instantaneous rate of return $r > \mu_A$[1]. Markets are complete and the probability $\mathbb{Q}$ denotes the unique risk neutral probability measure. The value of the unlevered firm for a current value x of the cash flows, after paying corporate income taxes, is

$$v_A(x) \;=\; (1-\theta)\mathbb{E}\left[\int_0^\infty e^{-rt} X_{t,A}^x \, dt\right] \;=\; \frac{x}{r-\mu_A}(1-\theta),$$

where θ is a tax rate on corporate income. The total payout rate to all security holders is therefore

$$\delta_A = \frac{(1-\theta)x}{v_A(x)} = r - \mu_A \tag{2}$$

and consequently, the unlevered asset value V under the risk neutral measure $\mathbb{Q}$ follows the process

$$\frac{dV_{t,A}}{V_{t,A}} = (r-\delta_A)dt + \sigma_A dW_t. \tag{3}$$

Note that, because of relation (2), equations (3) and (1) are the same and we could consider as well for state variable the dynamics of the unlevered asset value of the firm. Note also that, inclusive of the payout rate δ_A, the total (risk-adjusted) expected rate of return of the unlevered asset value of the firm is $\delta_A + (r-\delta_A) = r$, as it must be under the risk neutral measure $\mathbb{Q}$.

The firm chooses its initial capital structure consisting of perpetual coupon bond c that remains constant until equityholders endogenously default. In such a simple setting, the firm issues debt so as to take advantage of the tax shields offered for interest expenses. Failure to pay the coupon c triggers immediate liquidation of the firm. At liquidation, a fraction γ of the unlevered firm value is lost as a frictional cost. The liquidation value of the firm is therefore

$$\frac{(1-\theta)(1-\gamma)x}{r-\mu_A}. \tag{4}$$

Taking into account tax benefits and bankruptcy cost, the value of the levered firm is

$$v_A(x) \;=\; \mathbb{E}\left[\int_0^{\tau_L^A} e^{-rt}((1-\theta)X_{t,A}^x + \theta c)\, dt \;+\; e^{-r\tau_L^A}\frac{(1-\theta)(1-\gamma)}{r-\mu_A} X_{\tau_L^A,A}^x\right].$$

where the stopping time τ_L^A defines the bankruptcy policy chosen by equityholders so as to maximize the value of their claim. Formally, the problem of the equityholders is: Find the stopping time $\tau_L^A \in \mathcal{T}$ satisfying

$$E_A(x) \equiv \sup_{\tau\in\mathcal{T}} \mathbb{E}\left[\int_0^{\tau} e^{-rt}(1-\theta)\left(X_{t,A}^x - c\right) dt\right] = \mathbb{E}\left[\int_0^{\tau_L^A} e^{-rt}(1-\theta)\left(X_{t,A}^x - c\right) dt.\right] \tag{5}$$

[1] We assume that the expected present value of the cash flows is positive and finite and therefore that $r > \mu_A$.

Standard computations show that the optimal bankruptcy policy is a trigger policy defined by the stopping time $\tau_L^A = \inf\{t \geq 0 \ s.t \ X_{t,A} = x_L^A\}$ with $x_L^A = -\dfrac{\alpha_A}{1-\alpha_A}\dfrac{c}{r}\dfrac{1}{\nu_A}$ where ν_A denotes the ratio $\dfrac{1}{r-\mu_A}$ and α_A denotes the negative root of the quadratic equation $y(y-1)\frac{\sigma_A^2}{2} + y\mu_A = r$. This implies the following expressions for the equity value $E_A(x)$, the firm value $v_A(x)$ and the debt value $D_A(x)$:

$$\begin{cases} E_A(x) = & (1-\theta)\left\{x\nu_A - \dfrac{c}{r} + \left(\dfrac{c}{r} - x_L^A\nu_A\right)\left(\dfrac{x}{x_L^A}\right)^{\alpha_A}\right\} \ \text{if } x > x_L^A, \\ E_A(x) = & 0 \ \text{ if } x \leq x_L^A \end{cases} \tag{6}$$

and

$$\begin{cases} v_A(x) = & (1-\theta)x\nu_A + \dfrac{\theta c}{r} - \left(\dfrac{\theta c}{r} + x_L^A\gamma(1-\theta)\nu_A\right)\left(\dfrac{x}{x_L^A}\right)^{\alpha_A} \ \text{if } x > x_L^A, \\ \\ v_A(x) = & (1-\gamma)(1-\theta)x\nu_A \ \text{ if } \ x \leq x_L^A \end{cases}$$

The debt value satisfies the relation

$$D_A(x) = v_A(x) - E_A(x) = \mathbb{E}\left[\int_0^{\tau_L^A} e^{-rt}c dt \ + \ e^{-r\tau_L^A}\frac{(1-\theta)(1-\gamma)}{r-\mu_A}X^x_{\tau_L^A,A}\right]$$

or equivalently,

$$\begin{cases} D_A(x) = & \frac{c}{r} - \left(\frac{c}{r} - x_L^A(1-\gamma)(1-\theta)\nu_A\right)\left(\frac{x}{x_L^A}\right)^{\alpha_A} \ \text{if } x > x_L^A, \\ \\ D_A(x) = & (1-\gamma)(1-\theta)x\nu_A \ \text{ if } \ x \leq x_L^A \end{cases}$$

The interpretation of (6) is standard. The equity value is equal to $(\nu_A x - \frac{c}{r})(1-\theta)$, the after tax net present value of equity if equityholders never declare bankruptcy, *plus* the option value associated to the irreversible closure decision at the trigger x_L^A. We denote in the sequel by $x_{PV}^A = \frac{1}{\nu_A}\frac{c}{r}$, the trigger that equalizes to zero the present value of equities under perpetual continuation. Note that, in line with the real option theory, the bankruptcy trigger x_L^A chosen by the equityholders is smaller than the net present value trigger x_{PV}^A.

As usual in such a classical setting, the optimal capital structure is then characterized by the coupon c to be issued that maximizes the initial firm value.

2.2. A simple model of the firm with risk flexibility.

We now extend this standard model of capital structure by considering that, at any time, equityholders have the option to switch to a *poor* business activity (referred as "B" activity) that lowers the drift and increases the volatility of the cash flows. There is no monetary cost to change the activity but the decision to switch is irreversible. Specifically the poor activity "B" generates cash flows ("EBIT") satisfying the stochastic differential equation

$$\frac{dX_{t,B}}{X_{t,B}} = \mu_B dt + \sigma_B dW_t, \tag{7}$$

with $\mu_B < \mu_A$ and $\sigma_B > \sigma_A$. Equivalently, the unlevered asset value V under the risk neutral measure $\mathbb{Q}$ follows the process

$$\frac{dV_{t,B}}{V_{t,B}} = (r - \delta_B)dt + \sigma_B dW_t, \tag{8}$$

where

$$\delta_B = \frac{(1-\theta)x}{v_B(x)} = r - \mu_B, \tag{9}$$

and the value of the unlevered firm is

$$v_B(x) = (1-\theta)\mathbb{E}\left[\int_0^\infty e^{-rt} X_{t,B}^x \, dt\right] = \frac{x}{r-\mu_B}(1-\theta) < \frac{x}{r-\mu_A}(1-\theta).$$

The key inequalities $\mu_A > \mu_B$ and $\sigma_B > \sigma_A$ characterize the tradeoff that drives our model. Because of limited liability equityholders will be tempted to choose the riskier activity (that is the largest possible volatility). However this choice has an opportunity cost since it induces a lower expected return ($\mu_B < \mu_A$). Intuitively, because of this opportunity cost, as long as the cash flows are large enough, changing the activity of the firm (that is switching to the poor activity) is not attractive and equityholders run the firm under the safe activity. However if the cash flows sharply drop, the lower expected return of the high risk activity may not dissuade equityholders from increasing the riskiness of the cash flows. Saying it differently, the lower $\Delta\mu \equiv \mu_A - \mu_B$ with respect to $\Delta\sigma \equiv \sigma_B - \sigma_A$, the larger are the switching incentives of equityholders. Accordingly, after switching, the liquidation value of the firm becomes

$$\frac{(1-\theta)(1-\gamma)x}{r-\mu_B}. \tag{10}$$

To sum up, in our model, equityholders have to decide (i) when to cease the activity in place and switch to the poor activity, (ii) when to liquidate. We refer to these two irreversible decisions as the switching/liquidation policy.

3. Optimal switching/liquidation policy.

In order to study the optimal switching/liquidation policy, we first characterize situations where, whatever the initial value of the cash flows and the coupon c, (i) equityholders optimally decide to run the firm always under the safe activity, and (ii) equityholders immediately adopt the poor activity. We then study the more interesting case where always choosing the safe or the poor activity is not optimal.

In the previous section we derived $E_A(.)$, the equity value assuming equityholders run the firm under the safe activity (and optimally liquidate at time τ_L^A). In the same vein we can obtain $E_B(.)$, the equity value when equityholders run the firm always under the poor activity. We summarize this as follows.

LEMMA 3.1 *Assume equityholders choose the poor activity, (that is the dynamics of the cash flows obeys to the diffusion process (7)) then, the optimal liquidation policy is defined by the random time* τ_L^B *where* $\tau_L^B = \inf\{t \geq 0 \ s.t \ x_t = x_L^B\}$ *with* $x_L^B = -\frac{\alpha_B}{1-\alpha_B}\frac{c}{r}\frac{1}{\nu_B}$. *In this case, the value of equity is defined by the equality*

$$E_B(x) = \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}(1-\theta)(X_{t,B}^x - c)dt\right]$$

or equivalently,

$$\begin{cases} E_B(x) = & (1-\theta)\left\{x\nu_B - \frac{c}{r} + \left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B}\right\} \quad \text{if } x > x_L^B, \\ E_B(x) = & 0 \quad \text{if } x \leq x_L^B \end{cases}$$

where ν_B *denotes the ratio* $\frac{1}{r-\mu_B}$ *and* α_B *denotes the negative root of the quadratic equation* $y(y-1)\frac{\sigma_B^2}{2} + y\mu_B = r$.

The two following lemma identify the cases where equity value $E(x)$ is either $E_B(x)$ (lemma 3.2), or $E_A(x)$ (lemma 3.3).

LEMMA 3.2 *If* $\mu_A = \mu_B$ *and* $\sigma_A < \sigma_B$ *then, equityholders immediately choose the poor activity and liquidate the firm at the trigger* x_L^B.

Here, the switching decision is reduced to a pure risk shifting problem. Equity value is increasing and convex with respect to the cash flows x. In turn, this implies that equity value increases with the volatility of the cash flows. Formally, we have that for all $x \in (0, \infty)$, $E_A(x) < E_B(x)$ (see figure 1). Consequently, equityholders immediately choose the poor activity, (that is the high risk activity), and liquidate at the trigger x_L^B. Note that the liquidation trigger is decreasing with the volatility and we have $x_L^B < x_L^A$. Since equityholders get nothing in the bankruptcy event, a necessary condition for never switching to the high-risk activity being always optimal is clearly $x_L^B > x_L^A$. The following lemma shows that it is also a sufficient condition.

LEMMA 3.3 *If* $x_L^A < x_L^B$ *then, equityholders optimally never choose the poor activity and liquidate at the trigger* x_L^A.

The condition $x_L^A < x_L^B$ ensures that $E_A(x) > E_B(x)$ for all values of x (see figure 2). Equityholders cannot enjoy the high risk activity because the gain from increasing the volatility does not compensate the loss in the expected return.

In these two polar cases the tradeoff between increasing riskiness and decreasing expected return that drives our model is extreme. On the one hand, when increasing risk is costless (that is $\mu_A = \mu_B$) equityholders are better off choosing immediately the riskier activity and then never switch to the low risk activity. On the other hand, when $\Delta\mu$ is large with respect to $\Delta\sigma$, the high risk activity throws down bankruptcy and equityholders optimally always choose the low risk activity.

We now study the more interesting case where neither choosing forever the poor activity or the safe activity is optimal. According to the two previous lemma, a necessary and sufficient condition for that is $x_L^B < x_L^A$ and $\mu_A > \mu_B$. Intuitively, switching to the poor activity is optimal for low values of the cash flows (since for $x_L^B < x < x_L^A$ we have $E_B(x) > 0$ and $E_A(x) = 0$), whereas for sufficiently large values of the cash flows it may be optimal to postpone the switching decision in order to benefit from the larger expected return of the safe activity.

Assuming equityholders start running the firm under the safe activity, their problem is to decide when to switch to the poor activity. Formally, equityholders solve the optimal stopping time problem: Find the stopping times $\tau_S^\star < \tau_L^\star \in \mathcal{T}$ satisfying

$$E(x) \equiv (1-\theta) \sup_{\tau_S \in \mathcal{T}, \tau_L \in \mathcal{T}} \left\{ \mathbb{E}\left[\int_0^{\tau_S} e^{-rt}(X_{t,A}^x - c)dt + \mathbb{E}\left[\int_{\tau_S}^{\tau_L} e^{-rt}(X_{t,B}^{\tau_S, X_{\tau_S,A}^x} - c)dt | \mathcal{F}_{\tau_S} \right] \right] \right\}$$

$$= (1-\theta) \left\{ \mathbb{E}\left[\int_0^{\tau_S^\star} e^{-rt}(X_{t,A}^x - c)dt + \mathbb{E}\left[\int_{\tau_S^\star}^{\tau_L^\star} e^{-rt}(X_{t,B}^{\tau_S^\star, X_{\tau_S^\star,A}^x} - c)dt | \mathcal{F}_{\tau_S^\star} \right] \right] \right\} \qquad (11)$$

where $X_{t,B}^{\tau_S, X_{\tau_S,A}^x}$ denotes the process $X_{t,B}$ that takes value $X_{\tau_S,A}^x$ at time τ_S. We show the following:

Proposition 3.1 *If $x_L^B < x_L^A$ and $\mu_A > \mu_B$ then, equityholders strategically switch to the poor activity at the random time $\tau_S^\star = \inf\{t \geq 0 \ \ s.t \ \ X_t = x_S\}$ and declare bankruptcy at the random time $\tau_L^B = \inf\{t \geq 0 \ \ s.t \ \ X_t = x_L^B\}$. The triggers x_S and x_L^B are defined by the relations*

$$x_S = \left(\frac{(\alpha_B - \alpha_A)\nu_B}{(\nu_A - \nu_B)(1-\alpha_A)(-\alpha_B)} \right)^{\frac{1}{1-\alpha_B}} x_L^B, \quad \text{and} \quad x_L^B = -\frac{\alpha_B}{1-\alpha_B} \frac{c}{r} \frac{1}{\nu_B}.$$

The value of equity is defined by the equalities

$$\begin{cases} E(x) = & (1-\theta)\Big\{x\nu_A - \dfrac{c}{r} - x_S(\nu_A-\nu_B)\left(\dfrac{x}{x_S}\right)^{\alpha_A} \\ & +\left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_A}\left(\frac{x_S}{x_L^B}\right)^{\alpha_B}\Big\} \ \textit{if}\, x > x_S, \\ E(x) = & (1-\theta)\left\{x\nu_B - \dfrac{c}{r} + \left(\dfrac{c}{r} - x_L^B\nu_B\right)\left(\dfrac{x}{x_L^B}\right)^{\alpha_B}\right\} \ \textit{if}\, x_L^B < x \le x_S, \\ E(x) = & 0 \ \textit{if}\, x < x_L^B. \end{cases}$$

Our proposition deserves some comments. First, it shows that the conditions $x_L^B < x_L^A$ and $\mu_A > \mu_B$ are necessary and sufficient for switching from the safe activity to the poor activity being optimal. Second, it shows that the optimal switching policy is characterized by a switching trigger $x_S > x_L^A$ that we derive explicitly[2]. Figure 3 illustrates our proposition. Once the cash flows go below the switching trigger x_S equityholders optimally switch to the poor activity. Because this choice is by assumption irreversible, the equity value is then equal to E_B, the equity value under the poor activity. As long as the cash flows are larger than x_S, the value of the option to switch is strictly positive and $E(x) > E_G(x)$.

In our setting, an approximate measure for the severity of the agency problem is the length of the interval $[x_L^B, x_L^A]$. Indeed the larger $\Delta\sigma$, the larger the length of the interval $[x_L^B, x_L^A]$ and the larger the switching trigger x_S. On the contrary the larger $\Delta\mu$, the lower the distance between x_L^B and x_L^A. Ultimately, when $\Delta\mu$ is too large with respect $\Delta\sigma$, the trigger x_L^B becomes larger than the trigger x_L^A, any incentive to choose the poor activity disappears and, according to lemma 3.3, equityholders always choose the safe activity. It is interesting to compare the switching trigger x_S to the triggers $x_{PV}^A = \frac{c}{r}\frac{1}{\nu_A}$ and $x_{PV}^B = \frac{c}{r}\frac{1}{\nu_B}$ that equalizes to 0 the net present value of equity under perpetual continuation when the firm is run, respectively with the safe activity and with the poor activity. In particular, when $x_{PV}^A < x_S < x_{PV}^B$ the present value of equity evaluated at the switching point x_S is positive under the safe activity but negative under the poor activity. Equityholders nevertheless strategically switch to the poor activity at the trigger x_S because the increase in their option value to declare bankruptcy compensates the loss in the net present value defined by the difference $\nu_A - \nu_B$.

We now give the ex post firm value $v(x)$, that is the value of the firm when equityholders strategically switch at the trigger x_S. We have

$$v(x) \;=\; \mathbb{E}\left[\int_0^{\tau_S} e^{-rt}((1-\theta)X_{t,A}^x + \theta c)\,dt \;+\; e^{-r\tau_S} v_B(X_{\tau_S,A}^x)\right],$$

where

$$v_B(x) = \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}((1-\theta)X_{t,B}^x + \theta c)\,dt \;+\; e^{-r\tau_L^B}(1-\gamma)(1-\theta)\nu_B X_{\tau_L^B,B}^x\right].$$

[2]This last property relies on the irreversibility assumption we made on the decision to switch to the poor business activity. If the switching decision is reversible then, the optimal switching decision is much more difficult to establish and not always defined by a simple threshold strategy as in proposition 3.1

Direct computations yield to

$$\begin{cases} v(x) = & (1-\theta)x\nu_A + \dfrac{\theta c}{r} - (1-\theta)x_S\left(\nu_A - \nu_B\right)\left(\dfrac{x}{x_S}\right)^{\alpha_A} \\ & - \left(\frac{\theta c}{r} + x_L^B\gamma(1-\theta)\nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_A}\left(\frac{x_S}{x_L^B}\right)^{\alpha_B} \quad \text{if } x > x_S, \\ \\ v(x) = & (1-\theta)x\nu_B + \dfrac{\theta c}{r} - \left(\dfrac{\theta c}{r} + x_L^B\gamma(1-\theta)\nu_B\right)\left(\dfrac{x}{x_L^B}\right)^{\alpha_B} \quad \text{if } x_L^B < x \leq x_S, \\ \\ v(x) = & (1-\gamma)(1-\theta)x\nu_B \ \text{ if } \ x \leq x_L^B \end{cases} \tag{12}$$

Let us comment briefly equations (12). For $x \leq x_L^B$ the firm is all-equity financed, is run by the former debtholders and we have $v(x) = (1-\gamma)(1-\theta)\mathbb{E}\left[\int_0^\infty e^{-rt}X_{t,B}^x dt\right] = (1-\gamma)(1-\theta)x\nu_B$. For $x_L^B < x < x_S$, the firm value is equal to the after tax present value of the cash flows when it is run under the poor activity ($(1-\theta)x\nu_B$) plus the present value of tax benefits ($\frac{\theta c}{r}$) minus the discounted expected loss in case of bankruptcy ($\left(\frac{\theta c}{r} + x_L^B\gamma(1-\theta)\nu_B\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B}$). The amount of this loss is equal, at the bankruptcy trigger, to the loss of the tax benefits ($\frac{\theta c}{r}$) plus the loss due to the bankruptcy cost ($x_L^B\gamma(1-\theta)\nu_B$). For $x > x_S$, the additional term $x_S(1-\theta)\left(\nu_A - \nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_A}$ represents the discounted expected loss in net present value that occurs at the switching trigger x_S.

4. Optimal Capital Structure and Agency costs

Equityholders' option to change the activity at the trigger x_S entails loss in value for debtholders and for the whole firm. If equityholders were able to commit to a certain management policy before debt is issued, this problem will disappear. Staying in the tradition of Leland (1998) we define agency costs as the difference between the optimal firm value when the switching policy can be contracted ex ante (before debt is in place) and the optimal firm value when the switching decision policy is taken ex post (that is after debt is in place). In each case the optimal capital structure is characterized by the coupon rate that maximizes the initial firm value. We now turn to the numerical implementation of our model and we analyze in this section, through several examples, properties of the optimal capital structure and the magnitude of the agency costs. Table 1 lists the baseline parameters that support our analysis. Tables 2-3-4-5 report for different values of the couples (μ_A, σ_A) and (μ_B, σ_B) the optimal capital structure for the ex ante case and for the ex post case. The following observations can be made.

1. When the firm's activity policy can be committed ex ante to maximize firm value, equityholders will never switch to the high risk activity. The optimal ex ante firm value coincides in our setting with the optimal firm value when there is no risk flexibility.

The agency costs, that can be very large, are highly sensitive to a change in $\Delta\mu$, the opportunity cost of choosing the high risk activity. Tables 3 and 4 illustrate this point with agency costs dropping from 13.24% to 1.92% for a 2.5% increase of $\Delta\mu$. Accordingly, agency costs increase with $\Delta\sigma$ (that is agency costs increase when equityholders have more incentive to choose the high risk activity). In tables 2 and 3 agency costs increase from 1.02% to 13.24% when $\Delta\sigma$ goes from 5% to 30%.

2. The model predicts that the larger the severity of the agency problem, the lower the optimal leverage ratios. Precisely, optimal leverages in presence of agency costs decrease relative to the ex ante case where there is no risk flexibility. In table 3, leverages drop by more than 35% with respect to the ex ante case where there is no risk flexibility.

3. In our model, agency costs have no significant effect on yield spreads. The reason is that we focus on a pure switching problem between two activities. In particular, we do not consider an additional financing need at the switching trigger nor production costs for generating the cash flows. Remark however that yield spreads are lower in the ex post case than in the ex ante case. This result can be explained noting that optimal leverage in the ex ante case is larger than optimal leverage in the ex post case.

5. Covenants

Following Leland (1998) and many others, we have considered the case of endogenous bankruptcy (equityholders decide the time to go bankrupt). It is however also well documented that covenants written in the debt indenture can trigger bankruptcy. For instance, the so-called "cash flows based" covenant rule triggers bankruptcy as soon as the instantaneous cash flows X_t are not sufficient to cover payments c to debtholders. This is the line followed by Kim et al (1993), Anderson and Sundaresan (1996), Fan and Sundaresan (2000) or Ericsson (2000). The purpose of this section is to study how such covenant rules impact on the magnitude of agency costs, a task that seems to have been neglected by the literature[3]. Under the "cash flows based" covenant rule, the equity value $E_A(x)$ becomes

$$\begin{cases} E_A(x) = & (1-\theta)\left\{x\nu_A - \frac{c}{r} + \left(\frac{c}{r} - c\nu_A\right)\left(\frac{x}{c}\right)^{\alpha_A}\right\} \text{ if } x > c, \\ E_A(x) = & 0 \text{ if } x \leq c \end{cases} \tag{13}$$

It is worth noting that, in Ericsson (2000), equityholders can shift to a high volatility level but cannot alter the instantaneous expected growth rate which is furthermore assumed to be negative (that is, with our notation, $r - \delta_A = \mu_A < 0$ in the previous equation). Under this particular assumption the equity value (13) is convex in the current cash flows x and

[3]Ericsson (2000) is perhaps the only paper that address the issue of the magnitude of the asset substitution problem in a setting where bankruptcy is triggered by a covenant rule

increasing with the volatility. Consequently, equityholders operate immediately at the largest possible volatility. On the contrary, in our analysis we consider, as it is usually the case, positive instantaneous expected growth rates ($0 < \mu_B < \mu_A$). A direct calculus shows that the equity value (13) is concave in the current cash flows x, increasing in the rate of return μ and decreasing in the volatility σ[4]. Thus, equityholders are never tempted by the poor activity and the firm is liquidated at the exogenous trigger $x_L^{CF} = c$. Unfortunately, the fact that equityholders never switch to the poor activity does not imply that agency costs of debt are reduced. Quite on the contrary, numerical results show that rather than triggering premature bankruptcy at the threshold x_L^{CF}, it is socially optimal to let equityholders switch to the poor activity and liquidate at the threshold x_L^B lower than x_L^{CF}. This suggests that less strong covenants that restrict the firm from adopting the poor activity may be useful to reduce agency costs. Based on these remarks we now introduce the "no-switching based" covenant rule defined as the lowest liquidation trigger such that the unique optimal policy for equityholders is never to switch to the poor activity. We show thereafter that, depending on the severity of the agency problem such a covenant can reduce or increase the agency costs of debt.

PROPOSITION 5.2 *The smallest liquidation trigger such that the switching problem disappears is given by*

$$x_L^{NS} = \frac{c}{r}\frac{\alpha_B - \alpha_A}{\nu_A(1-\alpha_A) - \nu_B(1-\alpha_B)}.$$

First, note that $x_L^{NS} < x_L^{CF}$. In words, "cash flows based" covenant rule is not necessary to give equityholders the right incentives never to switch to the poor activity. Triggering bankruptcy at the lower trigger x_L^{NS} is sufficient. Second, remark that $x_L^{NS} \geq x_L^A \Leftrightarrow x_L^A \geq x_L^B$. In words, the liquidation trigger x_L^{NS} is larger than x_L^A, (the optimal liquidation trigger when there is no switching) if and only if equityholders have indeed incentives to switch. This last remark shows that deterring risk shifting incentives is costly for the firm and highlights the tradeoff between ex-post inefficient equityholders behavior and inefficient covenant restrictions. Third, note that the trigger x_L^{NS} is decreasing with the opportunity costs of switching ($\Delta\mu$). That is, when the difference in net present value of the two available activities increases, equityholders have less incentives to switch to the poor activity, and consequently, there is less need to engage in costly covenant restrictions to make them never choose the poor activity.

[4] This point is remarked by Leland (1994) who notes that, when debt is protected by a positive net worth covenant, equityholders will not gain by increasing firm risk and concludes that, in presence of potential agency conflict, protected debt may be the preferred form of financing despite having lower potential tax benefits. Leland(1994) does not however study the magnitude of agency costs at the optimal leverages nor remarks that positive net worth covenant can trigger inefficient premature bankruptcy.

Under the "no-switching based" covenant rule, the ex post value of the firm is given by the following expression:

$$\begin{cases} v(x) = & (1-\theta)x\nu_A + \dfrac{\theta c}{r} - \left(\dfrac{\theta c}{r} + (1-\theta)x_L^{NS}\gamma\nu_A\right)\left(\dfrac{x}{x_L^{NS}}\right)^{\alpha_A} \quad \text{if } x > x_L^{NS}, \\ v(x) = & (1-\gamma)(1-\theta)x\nu_A \ \text{ if } \ x \leq x_L^{NS}. \end{cases}$$

Tables 6-9 compare the optimal capital structure and the magnitude of the agency costs when bankruptcy is endogenous and when bankruptcy is triggered by our "no-switching based" covenant rule. It turns out that the covenant restriction restores some value to the firm only if the agency problem is severe enough. In Table 7 the covenant restriction allows to reduce agency costs by more than 9% (accordingly, optimal leverage increases from 43.84 % to 71.30%). In Table 9 the covenant rule allows to fully eliminate inefficient shifting. However, when the opportunity shifting cost is low ($\Delta\mu$ small) and when the agency problem not important ($\Delta\mu$ large with respect to $\Delta\sigma$) , the covenant restriction may worsen the situation. In table 6 agency costs increase from 1.02% for the endogenous bankruptcy rule to 2.59% for the "no-switching based" covenant rule.

The fact that the "no-switching based" covenant rule worsens the situation when the agency problem is not enough severe suggests to study a less strong covenant restriction that may leave equityholders switch to the poor activity, but still entails liquidation of the firm before equityholders will do (that is before the threshold x_L^B being reached). Precisely, consider a covenant that imposes liquidation at a threshold $x_L \in [x_L^B, x_L^{NS}]$, then equityholders react choosing a corresponding shifting trigger $x_S(x_L)$. The switching trigger $x_S(x_L)$ can be explicitly computed and shown to be decreasing in x_L on the interval $[x_L^B, x_L^{NS}]$ with $x_S(x_L^B) = x_S$ and $x_S(x_L^{NS}) = x_L^{NS}$. This last equality corroborates proposition 5.2 and states that equityholders never switch when liquidation is triggered at x_L^{NS}. We have then numerically compared agency costs when the liquidation policy is defined by the threshold $x_L = x_L^{NS}$ and when liquidation is triggered by $x_L \in [x_L^B, x_L^{NS})$. Our numerical results suggest that the optimal liquidation policy consists of a binary choice $x_L = x_L^B$ or $x_L = x_L^{NS}$. That is, covenant restrictions may be useful only to the extent that they can fully deter the switching problem. However, if the agency problem is not severe enough, covenants worsen the situation and it is optimal to let equityholders acting strategically.

6. Conclusion.

Most of the literature on asset substitution in a contingent-claims analysis setting considers the case in which the growth rate of the cash flows remains constant while the volatility of the cash flows increases by moral hazard. In this paper we adopt the view that the level of agency costs can be also due to bad investments rather than by simply pure excessive risk taking. This leads us to consider a model in which both the drift and the volatility of the cash flows are altered by equityholders decisions. We characterize explicitly equityholders optimal strategies and show using a numerical implementation of the model, that the risk of switching to a poor business activity drastically decreases firm value and optimal leverages. We furthermore investigate the role of positive net worth covenant written on the debt in reducing or exacerbating the magnitude of agency costs. We show that covenants that impede equityholders from switching to the poor business activity are not always value enhancing because they imply premature bankruptcy. We find that when the agency problem is not severe enough it is better letting equityholders switch to the poor business activity and declare bankruptcy strategically. However when the agency problem is severe the "no-switching" covenant rule that we propose dramatically reduces the of agency costs of asset substitution.

7. Appendix

Proof of lemma 3.2 Let denote $\nu = \frac{1}{r-\mu}$, α_σ the negative root of the quadratic equation $\frac{1}{2}\sigma y^2 + (\mu - \frac{1}{2}\sigma^2)y - r = 0$ and $x_L^\sigma = -\frac{\alpha_\sigma}{1-\alpha_\sigma}\frac{c}{r}\frac{1}{\nu}$. A direct computation shows that the mapping $\sigma \longrightarrow x\nu - \frac{c}{r} + \left(\frac{c}{r} - x_L^\sigma \nu\right)\left(\frac{x}{x_L^\sigma}\right)^{\alpha_\sigma}$ is increasing on $(0,\infty)$. Lemma 3.2 is then deduced remarking that, if $\mu_A = \mu_B$, then $x_L^A > x_L^B$ and thus $E_B(x) > E_A(x) = 0 \ \ \forall x_L^B < x < x_L^A$.

Proof of lemma 3.3 A sufficient condition for obtaining our result is $E'_A(x) > E'_B(x)$ for all $x > x_L^B$. We have for all $x > x_L^B$:

$$\begin{aligned}
\frac{1}{1-\theta}\left(xE'_A(x) - xE'_B(x)\right) &= x(\nu_A - \nu_B) + \alpha_A(\frac{c}{r} - x_L^A\nu_A)\left(\frac{x}{x_L^A}\right)^{\alpha_A} \\
&\quad -\alpha_B(\frac{c}{r} - x_L^B\nu_B)\left(\frac{x}{x_L^B}\right)^{\alpha_B} \\
&> x_L^B(\nu_A - \nu_B) + \alpha_A(\frac{c}{r} - x_L^A\nu_A)\left(\frac{x}{x_L^B}\right)^{\alpha_A} \\
&\quad -\alpha_B(\frac{c}{r} - x_L^B\nu_B)\left(\frac{x}{x_L^B}\right)^{\alpha_B} \\
&> \{x_L^B(\nu_A - \nu_B) + \alpha_A(\frac{c}{r} - x_L^A\nu_A) \\
&\quad -\alpha_B(\frac{c}{r} - x_L^B\nu_B)\}\left(\frac{x}{x_L^B}\right)^{\alpha_A} \\
&> \{(\nu_A x_L^A - \frac{c}{r})(1-\alpha_A) \\
&\quad -(\nu_B x_L^B - \frac{c}{r})(1-\alpha_B)\}\left(\frac{x}{x_L^B}\right)^{\alpha_A} = 0
\end{aligned}$$

Proof of proposition 3.1

It follows from the strong Markov property that optimization problem (11) can be rewritten under the form

$$E(x) \equiv \sup_{\tau_S \in \mathcal{T}} \mathbb{E}\left[\int_0^{\tau_S} e^{-rt}(1-\theta)\left(X_{t,A}^x - c\right)dt \ + \ e^{-r\tau_S}E_B(X_{\tau_S,A}^x)\right].$$

The proof[5] of our proposition relies then on the following lemma which shows that the optimal switching strategy is a trigger strategy.

Lemma 7.4 *If*

$$E(x) = \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}(1-\theta)\left(X_{t,B}^x - c\right)dt\right]$$

[5]Our problem is actually a particular case of a more general (and standard) problem in optimal stopping theory which is stated and solved in Theorem 10.4.1, Oksendal (2003). We propose here an elementary proof of our (simple) problem.

then

$$\begin{aligned} E(x-h) &= \sup_{\tau_S \in \mathcal{T}} \mathbb{E}\left[\int_0^{\tau_S} e^{-rt}(1-\theta)\left(X_{t,A}^{x-h}-c\right)dt + e^{-r\tau_S}E_B(X_{\tau_S,A}^{x-h})\right] \\ &= \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}(1-\theta)\left(X_{t,B}^{x-h}-c\right)dt\right] \end{aligned}$$

Proof of the lemma 7.4: Taking advantage from the equalities $X_{t,A}^{x-h} = X_{t,A}^{x} - X_{t,A}^{h}$ and $X_{t,B}^{X_{\tau,A}^{x-h}} = X_{t,B}^{X_{\tau,A}^{x}} - X_{t,B}^{X_{\tau,A}^{h}}$, we deduce from the definitions of $E(x)$ and $E(x-h)$

$$E(x-h) \;\leq\; E(x) \;-\; \inf_{\tau \in \mathcal{T}}\left\{(1-\theta)\mathbb{E}\left[\int_0^{\tau} e^{-rt}X_{t,A}^{h}dt \;+\; e^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{X_{\tau,A}^{h}}dt \,|\, \mathcal{F}_\tau\right]\right]\right\}$$

Moreover,

$$\begin{aligned} \mathbb{E}\left[\int_0^{\tau} e^{-rt}X_{t,A}^{h}dt\right] &= \nu_A\left(h - \mathbb{E}\left[e^{-r\tau}X_{\tau,A}^{h}\right]\right), \\ \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{X_{\tau,A}^{h}}dt \,|\, \mathcal{F}_\tau\right] &= \nu_B\left(X_{\tau,A}^{h} - x_L^B\mathbb{E}\left[e^{-r\tau_L^B} \,|\, \mathcal{F}_\tau\right]\right), \end{aligned}$$

from which we deduce

$$\begin{aligned} &\mathbb{E}\left[\int_0^{\tau} e^{-rt}X_{t,A}^{h}dt \;+\; e^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{X_{\tau,A}^{h}}dt \,|\, \mathcal{F}_\tau\right]\right] \\ &= \nu_A h - (\nu_A-\nu_B)\,\mathbb{E}\left[e^{-r\tau}X_{\tau,A}^{h}\right] - \nu_B x_L^B\mathbb{E}\left[e^{-r\tau}e^{-r\tau_L^B}\right] \end{aligned}$$

Now, from a standard result in optimal stopping theory we have that, $\sup_{\tau \in \mathcal{T}} \mathbb{E}\left[e^{-r\tau}X_{\tau,A}^{h}\right] = h$ which implies that

$$\inf_{\tau \in \mathcal{T}}\left\{\mathbb{E}\left[\int_0^{\tau} e^{-rt}X_{t,A}^{h}dt \;+\; e^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{X_{\tau,A}^{h}}dt \,|\, \mathcal{F}_\tau\right]\right]\right\} \;=\; \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{h}dt\right].$$

We thus obtain

$$E(x-h) \leq \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}(1-\theta)\left(X_{t,B}^{x-h}-c\right)dt\right].$$

As the converse inequality is always satisfied, lemma(7.4) is proved.

Thus, the optimal switching policy is a trigger policy. For a given switching trigger x_S, the equity value is given by standard computations

$$\begin{cases} E(x) = & (1-\theta)\Big\{x\nu_A - \dfrac{c}{r} - x_S(\nu_A-\nu_B)\left(\dfrac{x}{x_S}\right)^{\alpha_A} \\ & + \left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_A}\left(\frac{x_S}{x_L^B}\right)^{\alpha_B}\Big\} \text{ if } x > x_S, \\ E(x) = & (1-\theta)\left\{x\nu_B - \dfrac{c}{r} + \left(\dfrac{c}{r} - x_L^B\nu_B\right)\left(\dfrac{x}{x_L^B}\right)^{\alpha_B}\right\} \text{ if } x_L^B < x \leq x_S, \\ E(x) = & 0 \text{ if } x < x_L^B \end{cases}$$

It is easy to see that this value function reaches its maximum for a value of x_S that does not depend on x, namely

$$x_S = \left(\frac{(\alpha_B - \alpha_A)\nu_B}{(\nu_A - \nu_B)(1-\alpha_A)(-\alpha_B)} \right)^{\frac{1}{1-\alpha_B}} x_L^B > x_L^B.$$

Proof of proposition 5.2

Since by construction $E_A(x_L^{NS}) = E_B(x_L^{NS}) = 0$, a necessary condition for equityholders being not tempted by switching is $E'_A(x_L) > E'_B(x_L)$ where x_L is a liquidation trigger. The minimum liquidation trigger that satisfies this condition is implicitly defined by the equation $x_L E'_A(x_L) = x_L E'_B(x_L)$. This leads to $x_L = x_L^{NS} = \frac{c}{r} \frac{\alpha_B - \alpha_A}{\nu_A(1-\alpha_A) - \nu_B(1-\alpha_B)}$. Conversely, reasoning as in the proof of lemma 3.3, we show that $E_A(x) - E_B(x) \geq 0$ for $x \geq x_L^{NS}$.

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Figures.

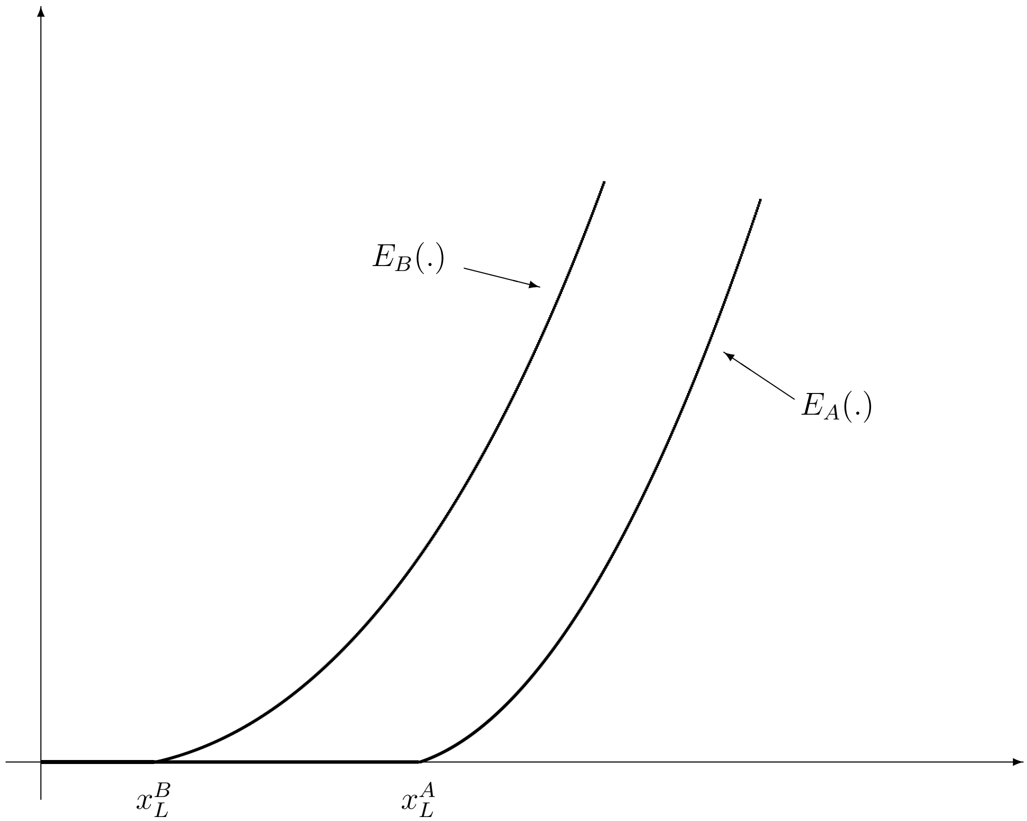

Figure 1 : $\mu_A = \mu_B$ and $\sigma_A < \sigma_B$

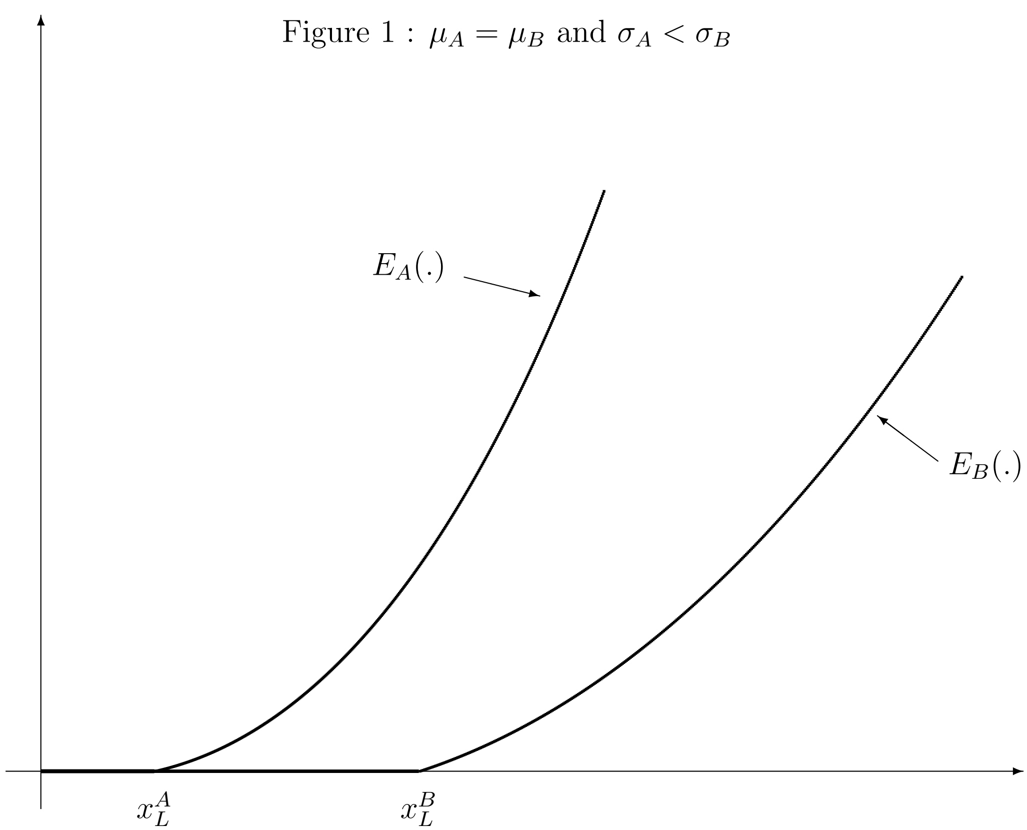

Figure 2 : $x_L^A < x_L^B,\ \mu_A > \mu_B,\ \sigma_A < \sigma_B$

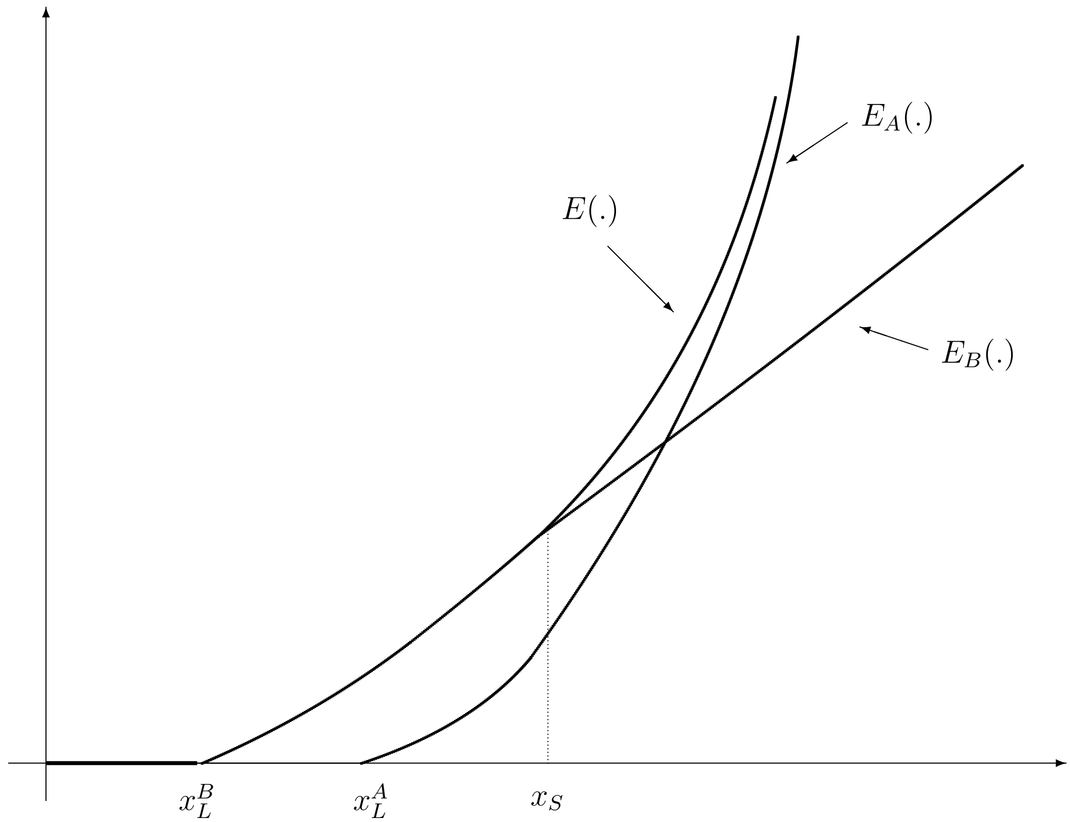

Figure 3 : $x_L^A > x_L^B,\ \mu_A > \mu_B,\ \sigma_A < \sigma_B$

Tables.

Table 1. Parameters for the base case: γ is the bankruptcy cost, θ the tax rate, r the fixed market interest rate and x the normalized initial cash flows value. Values we consider in our analysis are standard in the continuous time corporate finance literature.

Table 1.

γ	θ	r	x
0.4	0.35	0.06	5

Tables 2-5. Optimal capital structure and magnitude of the agency costs for the ex ante case and for the ex post case, for different values of the couples (μ_A, σ_A) and (μ_B, σ_B). In these tables, $v(x)$ is the optimal firm value; $c^0(x)$ is the optimal coupon; L (in percentage of the firm value) is the optimal leverage (D/v) where the debt value D is equal to $v - E$; YS (in basis points) is the yield spread $(c/D - r)$ over the debt; AC (in percentage of the ex ante firm value) is the magnitude of the agency costs.

Table 2.

$\sigma_A = 0.15$	$\Delta\sigma = 5\%$	$\mu_A = 0.015$		$\Delta\mu = 0.5\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex ante	92.05	4.77	74.61	95	–
Ex post	91.11	4.54	72.07	91	1.02

Table 3.

$\sigma_A = 0.1$	$\Delta\sigma = 30\%$	$\mu_A = 0.015$		$\Delta\mu = 0.5\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex ante	96.34	5.03	80.51	49	–
Ex post	83.58	2.37	43.84	47	13.24

Table 4.

$\sigma_A = 0.1$	$\Delta\sigma = 30\%$	$\mu_A = 0.03$		$\Delta\mu = 3\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex ante	148.37	7.88	83.91	33	–
Ex post	145.52	7.32	79.62	32	1.92

Table 5.

$\sigma_A = 0.2$	$\Delta\sigma = 20\%$	$\mu_A = 0.03$		$\Delta\mu = 3\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex ante	135.88	7.08	72.54	118	–
Ex post	132.98	6.34	67.1	111	2.13

Tables 6-9. Optimal capital structure and magnitude of the agency costs when bankruptcy is endogenous and when bankruptcy is triggered by our "no-switching based" covenant rule. In these tables, $v(x)$ is the optimal firm value; $c^0(x)$ is the optimal coupon; L (in percentage of the firm value) is the optimal leverage (D/v) where the debt value D is equal to $v - E$; YS (in basis points) is the yield spread $(c/D - r)$ over the debt; AC (in percentage of the ex ante firm value) is the magnitude of the agency costs.

Table 6.

$\sigma_A = 0.15$ $\Delta\sigma = 5\%$	$\mu_A = 0.015$		$\Delta\mu = 0.5\%$		
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex post case with endogenous bankruptcy	91.11	4.54	72.07	91	1.02
Ex post case with no switching based covenant	89.64	4.19	73.22	38	2.59

Table 7.

$\sigma_A = 0.1$ $\Delta\sigma = 30\%$ $\mu_A = 0.015$ $\Delta\mu = 0.5\%$					
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex post case with endogenous bankruptcy	83.58	2.37	43.84	47	13.24
Ex post case with no switching based covenant	92.50	4.23	71.30	41	3.99

Table 8.

$\sigma_A = 0.1$ $\Delta\sigma = 30\%$ $\mu_A = 0.03$ $\Delta\mu = 3\%$					
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex post case with endogenous bankruptcy	145.52	7.32	79.62	32	1.92
Ex post case with no switching based covenant	147.33	7.61	82.41	27	0.70

Table 9.

$\sigma_A = 0.2$ $\Delta\sigma = 20\%$ $\mu_A = 0.03$ $\Delta\mu = 3\%$					
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex post case with endogenous bankruptcy	132.98	6.34	67.1	111	2.13
Ex post case with no switching based covenant	135.88	7.08	72.54	118	0