Regulation and Taxation of a Monopoly generating Externalities

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Abstract

We consider a model with a monopolist that generates externalities. Externalities depend on either the volume of services provided X, or the number of clients N, or both. We study the socially optimal prices and propose a regulatory mechanism to decentralize the optimum allocation. We then compare the merits of this approach and of optimal taxation and discuss how regulatory policy should be amended if both taxation and regulation coexist.

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1 Long summary

While often regulated monopolists operate in industries where externalities abound, the implications of their presence have seldom been investigated by the regulation literature. Moreover, the industries in question are also subject to taxation, which means that some distortions are introduced by state intervention. Both the presence of externalities and taxes have important implications for providers and regulators alike. In particular, what is the optimal structure of prices when we account for externalities (e.g. network effects), and/or for taxation? And, furthermore, is regulation to be preferred to taxation, or does the opposite hold true?

In this paper we consider the possibility that externalities depend on either the volume of services provided X (an instance is lower quality of service as an effect of greater of use of a network), or the number of clients N (as is usually assumed for modelling "network externalities"), or both. We assume that producers have access to two-part tariffs, so they can price both access to the service (i.e. connection to the network) and intensity of use. Consistently, we will assume that the state may tax both access and consumption.

We first study the consumer behaviour and decompose the effect of price changes into a direct effect and an indirect effect, that follows from the presence of the externality. This decomposition allows to identify the impact of externalities on market demand properties. Then we turn to a rather standard normative analysis and characterize in turn the first-best situation, the profit-maximising prices and the second-best allocation.

First-best prices are shown to equate the marginal costs of production net of the marginal impact of consumption on welfare. The study of profit-maximisation and of second-best allocation lead us to introduce the concept of virtual connection cost and virtual marginal costs. These "virtual costs" are the costs faced by the firm when the composed effects of externalities are accounted for. We provide explicit formulae for these costs and analyze how they depend on the various externalities at work. At this point we are able to show that profit-maximising prices essentially do not differ from those provided by the Lerner formula. Similarly, the second-best allocation is characterised by optimal prices that have a Ramsey flavour. However an additional correction has to be made to account for the (direct) impact of externalities on social welfare.

Interestingly enough, the second-best allocation can be implemented trough a decentralised regulatory mechanism. In fact. we show that it is sufficient to impose an "extended global price-cap". This scheme does only require accounting data and an estimate of the marginal impact of both commodities on social welfare. We conclude by comparing the merits of this approach and of optimal taxation and discuss how regulatory policy should be amended if both taxation and regulation coexist.

2 Introduction

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This paper is organised as follows. Next section introduces the model and studies the market demand. We then turn to the study of the First-best (Section 3), the profit-maximising price structure (Section 4) and the socially optimum prices, when the producer is required to break-even (Section 5). We then introduce a regulatory mechanism that allows to decentralize the later (second-best) allocation. Last section concludes.

3 The model

A monopolist delivers to N consumers a service produced in quantity X at a cost $C(X, N)$. The service is sold at a unit price b. In addition each client is charged a fee equal to a for having access to the service.

Let $S_\theta(x_\theta, X, N)$ be the (gross) surplus of a client of type θ, where x_θ denotes his consumption. Net individual surplus is obtained by subtracting individual expenditure for the service. Since the demand of inframarginal consumers does not depend on a, we can write

$$x_\theta (b, X, N) = \arg\max_{x \geq 0} \{S_\theta (x, X, N) - (a + bx)\}, \tag{1}$$

whenever the corresponding net surplus

$$V_\theta (a, b) = S_\theta [x_\theta (b, X, N), X, N] - [a + bx_\theta (b, X, N)]. \tag{2a}$$

is higher than $S_\theta (0, X, N)$, i.e. when the client finds it beneficial to patronize the firm.

Assume that the population of types is distributed over $[0, +\infty]$ according to the density function $g(\theta)$ and the cumulative distribution function $G(\theta)$. We also assume that the (gross) surplus S_θ is increasing with θ,that is $\partial S_\theta / \partial \theta \geq 0$. As a result of the envelope theorem, the net surplus V_θ is also increasing with θ. Let θ_m be the type of the marginal consumer, who is indifferent between consuming the service or not, i.e.:

$$\max_x \{S_{\theta_m} (x, X, N) - (a + bx)\} = S_{\theta_m} (0, X, N). \tag{3}$$

Cosnumers with $\theta \leq \theta_m$ do not consume, while those with $\theta \geq \theta_m$ find it profitable to get access to the services. Aggregate demand is

$$X (a, b) = \int_{\theta_m}^{+\infty} x_\theta (a, b) g(\theta) d\theta, \tag{4}$$

while the number of consumers is

$$N (a, b) = \int_{\theta_m}^{+\infty} g(\theta) d\theta. \tag{5}$$

Given the assumed externalities, a preliminary step is a careful study of the impact of price changes on demand.

3.1 Consumer Behaviour

3.1.1 Impact of changes in the access fee a :

A change in a induces a shift in the marginal type θ_m, hence a change in the number of consumer N. More precisely, we know from equation (5) that

$$\frac{dN}{da} = -g\left(\theta_m\right)\frac{d\theta_m}{da}, \tag{6}$$

where, from the monotonicity of V_θ, the derivative $\left(d\theta_m/da\right)$ is certainly positive.

As already pointed out, the access fee a has no direct impact on the individual demand of inframarginal consumers. However it does impact $x_\theta\left(b, X, N\right)$ indirectly as a consequence of the externalities. To assess this changes we consider the first-order condition that follows from the consumer program (1) that writes:

$$b = \frac{\partial S_\theta\left(x, X, N\right)}{\partial x}. \tag{7}$$

Differentiating with respect to a gives

$$\frac{dx_\theta}{da} = \left(\frac{-\partial^2 S_\theta}{\partial x^2}\right)^{-1}\left[\frac{\partial^2 S_\theta}{\partial x \partial X}\frac{dX}{da} + \frac{\partial^2 S_\theta}{\partial x \partial N}\frac{dN}{da}\right]. \tag{8}$$

We now turn to the study of the aggregate demand. By definition,

$$\frac{dX}{da} = \int_{\theta_m}^{+\infty}\frac{dx_\theta}{da}g\left(\theta\right)d\theta - g\left(\theta_m\right)x_{\theta_m}\frac{d\theta_m}{da}.$$

By using equation (8) and (6), the latter equation can be rewritten[1] as

$$\frac{dX}{da} = \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\frac{dN}{da}, \tag{9}$$

where

$$\begin{aligned} E_{xX} &= \int_{\theta_m}^{+\infty}\left(\frac{-\partial^2 S_\theta}{\partial x^2}\right)^{-1}\frac{\partial^2 S_\theta}{\partial x \partial X}g\left(\theta\right)d\theta, \\ E_{xN} &= \int_{\theta_m}^{+\infty}\left(\frac{-\partial^2 S_\theta}{\partial x^2}\right)^{-1}\frac{\partial^2 S_\theta}{\partial x \partial N}g\left(\theta\right)d\theta. \end{aligned}$$

[1] Note that, in order to do so, we need to assume that externalities are "sufficiently small" so that $E_{xX} < 1$. This appears to be quite reasonable, since it amounts to suppose that cross-effects are smaller (in absolute value) than direct effects.

3.1.2 Impact of changes in the price b :

The effect of the price b on the consumption of X can be decomposed into a marginal effect and an infra-marginal effect:

$$\frac{dX}{db} = -x_{\theta_m} g\left(\theta_m\right) \frac{d\theta_m}{db} + \int_{\theta_m}^{+\infty} \frac{dx_\theta}{db} g\left(\theta\right) d\theta, \tag{10}$$

where, from equation (5) ,we know that the marginal effect can be rewritten as:

$$-x_{\theta_m} g\left(\theta_m\right) \frac{d\theta_m}{db} = x_{\theta_m} \frac{dN}{db}. \tag{11}$$

The effect of the price b on the consumption of an infra-marginal individual consumer can be decomposed in turn as a direct effect and as an indirect effect, the latter resulting from the presence of externalities. Indeed, differentiating with respect to b equation (7) that defines individual consumption gives

$$1 = \left(\frac{\partial^2 S_\theta}{\partial x^2}\right) \frac{dx_\theta}{db} + \left(\frac{\partial^2 S_\theta}{\partial x \partial X}\right) \frac{dX}{db} + \left(\frac{\partial^2 S_\theta}{\partial x \partial N}\right) \frac{dN}{db}.$$

It follows that

$$\frac{dx_\theta}{db} = \left(\frac{-\partial^2 S_\theta}{\partial x^2}\right)^{-1} \left[\left(\frac{\partial^2 S_\theta}{\partial x \partial X}\right) \frac{dX}{db} + \left(\frac{\partial^2 S_\theta}{\partial x \partial N}\right) \frac{dN}{db} - 1\right]$$

so that

$$\int_{\theta_m}^{+\infty} \frac{dx_\theta}{db} g\left(\theta\right) d\theta = \frac{\partial \widehat{X}}{\partial b} + E_{xX} \frac{dX}{db} + E_{xN} \frac{dN}{db} \tag{12}$$

where $\left(\partial \widehat{X} / \partial b\right)$ is nothing but the direct effect of b on the demand of infra-marginal consumers, given by

$$\frac{\partial \widehat{X}}{\partial b} = \int_{\theta_m}^{+\infty} \left(\frac{\partial^2 S_\theta}{\partial x^2}\right)^{-1} g\left(\theta\right) d\theta = \int_{\theta_m}^{+\infty} \frac{\partial x_\theta}{\partial b} g\left(\theta\right) d\theta,$$

while the two other terms on the right-hand side of (12) reflect the indirect effect of b on the demand of infra-marginal consumers.

Plugging (11) and (12) into (10) yields to:

$$\frac{dX}{db} = \frac{1}{1 - E_{xX}} \frac{\partial \widehat{X}}{\partial b} + \left(\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right) \frac{dN}{db}. \tag{13}$$

4 First-best optimum

Let us start by characterizing the first-best allocation, that is the allocation that maximizes total surplus (sum of consumer surplus and profits). At this point the provider is not required to break even. We thus implicitly assume that fixed costs can be financed at no efficiency cost through a subsidy financed from the general budget. Such a solution is usually not feasible in practice. Nevertheless it provides us with an interesting benchmark.

We consider two formulations of the problem. The first one is direct and intuitive: we optimize with respect to individual decisions (access and consumption) and directly derive the optimal allocation. It appears that the latter can be decentralised by the means a two-part tariff. This allows us to propose a second approach that uses prices as decision variables. It is more complicated in a first-best setting but it will simplify the second-best problem significantly.

4.1 Direct approach

Social welfare writes as the difference between aggregate social surplus and total costs:

$$W_1 = \int_0^{+\infty} S_\theta\left(x_\theta, X, N\right) g\left(\theta\right) d\theta - C\left(X, N\right). \tag{14}$$

By definition, X and N verify

$$\begin{aligned} X &= \int_0^{+\infty} x_\theta g\left(\theta\right) d\theta, \\ N &= \int_0^{+\infty} 1_{x_\theta > 0} g\left(\theta\right) d\theta. \end{aligned}$$

Consider first the determination of individual consumption. Differentiating (14) with respect to x_θ yields the following first-order condition:

$$\frac{\partial S_\theta}{\partial x_\theta} + \int_0^{+\infty} \frac{\partial S_\theta}{\partial X} g\left(\theta\right) d\theta = \frac{\partial C}{\partial X}. \tag{15}$$

Given the pricing scheme, consumer maximizing behavior implies $(\partial S_\theta / \partial x_\theta) = b$ if $x_\theta > 0$. Substituting this expression into (15) yields

$$b = \frac{\partial C}{\partial X} - E_X, \tag{16a}$$

where E_X denote the marginal impact of X on aggregate (gross) surplus:

$$E_X = \int_0^{+\infty} \frac{\partial S_\theta}{\partial X} g(\theta)\, d\theta. \tag{17}$$

We now shift to the access decision. Differentiating (14) with respect to θ_m, making use of the characterization of the marginal type (3) in our pricing scheme, of the two relations $(dN/d\theta_m) = -g(\theta_m)$ and $(dX/d\theta_m) = -g(\theta_m)\, x_{\theta_m}$, and of the first order condition (16a) just obtained, we isolate this second first-order condition (See Appendix 9.1):

$$a = \frac{\partial C}{\partial N} - E_N, \tag{18}$$

where E_N denote the marginal impact of N on aggregate (gross) surplus:

$$E_N = \int_0^{+\infty} \frac{\partial S_\theta}{\partial N} g(\theta)\, d\theta. \tag{19}$$

Expressions (18) and (16a) do not come as a surprise. They show that the first-best allocation can be decentralized through prices and have a number of interesting implications. First, despite the externalities, the service should be sold at the same price whatever the quantity consumed. Second, both prices a and b do not depend on consumer characteristics. Even if we allowed for perfect discrimination, it would not be desirable (on efficiency grounds) to charge different prices to different types; this is because social marginal costs do not depend on type. Third, both prices generally differ from marginal costs. In particular, if externalities are negative, the prices just obtained are higher than the corresponding marginal costs; as a result, the efficient allocation requires the monopolist to make strictly positive margins. In this case, marginal cost pricing would indeed imply over-consumption. Conversely, if the externality terms $\not{E}_N$ and/or E_X are positive, the first-best allocation requires access and/or consumption to be subzidized.

Finally, one can easily verify that this pricing policy does not necessarily allow the provider to break even. For instance, in the presence of constant marginal costs, the provider may be unable to cover the fixed cost. Consequently, the first-best solution may not feasible if the provider faces a break-even constraint. One then has to adopt a second-best solution where prices are set above marginal cost in order to recover fixed cost. This is studied in Section **??**. However, to facilitate the transition to the second-best setting, it is interesting to consider an alternative specification of the first-best problem.

Observe that the first-best access pricing rule (18) that defines a depends on the very fact that equation (16a) holds (i.e. that the consumption price b is set correctly). By contrast, the first-best pricing rule (16a) that defines b does not require equation (18) to hold. It follows that, if the access price is exogenously determined, the (consumption) pricing rule (16a) can still be used. In particular, it continues to hold true if prices are restricted to be linear i.e. $a = 0$.

4.2 Indirect approach

Alternatively we express total surplus as a function of prices (rather than quantities) which then also become our decision variables. The objective function is then given by:

$$\begin{aligned} W_2 &= \int_0^{+\infty} V_\theta(a,b)\, g(\theta)\, d\theta \\ &\quad + aN(a,b) + bX(a,b) - C\left[X(a,b), N(a,b)\right]. \end{aligned}$$

The impact of prices on the indirect (net) utility function write

$$\begin{aligned} \frac{dV_\theta}{da} &= \frac{\partial S_\theta}{\partial X}\frac{dX}{da} + \frac{\partial S_\theta}{\partial N}\frac{dN}{da} - 1_{\theta \geq \theta_m}, \\ \frac{dV_\theta}{db} &= \frac{\partial S_\theta}{\partial X}\frac{dX}{db} + \frac{\partial S_\theta}{\partial N}\frac{dN}{db} - 1_{\theta \geq \theta_m} x_\theta (a,b). \end{aligned}$$

Thus, differentiating W_2 with respect to a and b and rearranging yields the FOCs :

$$\begin{aligned} \frac{dW_2}{da} &= \left(a - \frac{\partial C}{\partial N} + E_N\right)\frac{dN}{da} + \left(E_X + b - \frac{\partial C}{\partial X}\right)\frac{dX}{da} \\ \frac{dW_2}{db} &= \left(a - \frac{\partial C}{\partial N} + E_N\right)\frac{dN}{db} + \left(E_X + b - \frac{\partial C}{\partial X}\right)\frac{dX}{db} \end{aligned}$$

that yield the very same marginal cost pricing conditions (18) and (16a). Not surprisingly, both approaches thus yield the same results. While the direct approach is convenient in a first-best setting it is difficult to handle when a budget constraint is introduced.

5 Profit-maximising price structure

The firm may be subject to a taxation scheme (τ_X, τ_N) so that the general expression for its profits is given by:

$$\Pi = (a - \tau_N) N + (b - \tau_X) X - C(X, N). \tag{20}$$

Profit maximization gives thus rise to the following system of F.O.Cs:

$$\frac{d\Pi}{da} = N + \left(a - \frac{\partial C}{\partial N} - \tau_N\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X} - \tau_X\right)\frac{dX}{da} = 0, \tag{21}$$

$$\frac{d\Pi}{db} = X + \left(a - \frac{\partial C}{\partial N} - \tau_N\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X} - \tau_X\right)\frac{dX}{db} = 0. \tag{22}$$

5.0.1 Profit-maximising price a

By using (9) , equation (21) can be rewritten to characterize the optimal access price a as

$$a - \frac{\partial C}{\partial N} - \tau_N + \left(b - \frac{\partial C}{\partial X} - \tau_X\right)\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}} = \frac{a}{\epsilon_N}, \tag{23}$$

where ϵ_N is the "price-elasticity" of number of users N with respect to the access price a (i.e. the price-elasticity of the demand for connection)

$$\epsilon_N = \left(-\frac{a}{N} \frac{dN}{da} \right). \tag{24}$$

Observe that (23) is nothing but a "standard" Lerner formula.

To see that, it may be useful to consider the case without taxes and externalities. In this case the equation (23) that characterizes the profit maximizing price a simplifies to

$$\frac{a - \frac{\partial C}{\partial N} + \left(b - \frac{\partial C}{\partial X}\right) x_{\theta_m}}{a} = \frac{1}{\epsilon_N}. \tag{25}$$

While connection is priced at a, the marginal cost of an additional connection to the provider $(\partial C / \partial N)$ is compensated by benefits derived from the consumption x_{θ_m} of the additional (marginal) consumer. It is "as if" the firm were contemplating a "virtual cost" of connection

$$\frac{\partial C}{\partial N} - \left(b - \frac{\partial C}{\partial X} \right) x_{\theta_m}, \tag{26}$$

which is lower than the marginal cost of connection $(\partial C / \partial N)$. Indeed, when there are no externalities, it can be shown easily that $b > (\partial C / \partial X)$. Thus when a two part pricing scheme is used, the access price a should be lower than what a bold (but erroneous) application of the Lerner principle would predict.

In the presence of taxes, the firm bases its decisions on prices net of taxes; and in presence of externalities the connection of an additional consumer also impacts on the behaviour of other consumers, so this "virtual cost" contemplated by the firm is somewhat more complex than (26). However both (23) and (25) have the very same interpretation. Indeed, equation (9) makes it clear that the ratio $(x_{\theta_m} + E_{xN}) / (1 - E_{xX})$ that appears in (23) is nothing but the marginal change in demand that result from extending access to an additional consumer,i.e:

$$\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}} \equiv \left(\frac{dX}{da} / \frac{dN}{da} \right).$$

In other words, (23) may be rewritten as

$$\frac{a - \widetilde{C}_N}{a} = \frac{1}{\epsilon_N}$$

where the "virtual cost" of connection $\widetilde{C}_N$ is defined by

$$\widetilde{C}_N = \frac{\partial C}{\partial N} + \tau_N - \left(b - \frac{\partial C}{\partial X} - \tau_X\right) \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}. \tag{27}$$

5.0.2 Profit-maximising price b

In the same manner as a change in a impacts on the (aggregate) quantity of services X sold by the firm, a variation in b impacts on the desirability of connection, hence on the number of consumers N who actually get access to services. A simple rewriting of (22) yields to a modified or "virtual" marginal cost of services that writes

$$\widetilde{C}_X = \frac{\partial C}{\partial X} + \tau_X - \left[a - \left(\frac{\partial C}{\partial N} + \tau_N\right)\right] \left(\frac{dN}{db} / \frac{dX}{db}\right) \tag{28}$$

that enters into a "standard" Lerner formula

$$\frac{b - \widetilde{C}_X}{b} = \frac{1}{\epsilon_X} \tag{29}$$

where ϵ_X is the standard price elasticity:

$$\epsilon_X = \left(-\frac{b}{X} \frac{dX}{db}\right). \tag{30}$$

The interpretation of 29 strictly parallels the interpretation given above with reference to connections. However, while the impact of an additional connection on total consumption is relativily easy to determine, the impact of an increase in total consumption on the number of consumers is less easy to evaluate. Indeed, while $\left(\frac{dN}{da} / \frac{dX}{da}\right)$ admits an explicit formulation, there is no closed form for $\left(\frac{dN}{db} / \frac{dX}{db}\right)$. So the formulae (28) and (29) are rather provided to illustrate the mechanisms at hand (and enhance their similarity) than for actual use. Thus we now turn to a more convenient formulation of the optimal pricing rule, which holds true when the price a is set to its profit-maximizing level (23), i.e. when both prices a and b can be used as instruments by the monopolist.

Building on the analysis of consumer behaviour conducted above, in particular on equation (13), we rewrite now the FOC (22) as

$$\begin{aligned} 0 &= X + \left(b - \frac{\partial C}{\partial X} - \tau_X\right)\left(\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right) \\ &\quad + \left(a - \frac{\partial C}{\partial N} - \tau_N + \left(b - \frac{\partial C}{\partial X} - \tau_X\right)\left(\frac{x_{\theta_m} + E_{xN}}{1-E_{xX}}\right)\right)\frac{dN}{db} \end{aligned} \tag{31}$$

If the price a is set to its profit-maximizing level (23), the later equation boils down to:

$$0 = X + \left(b - \tau_X - \frac{\partial C}{\partial X}\right)\left(\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right) - N\left(\frac{dN}{db}/\frac{dN}{da}\right).$$

Plugging into this equation the expression (51) derived in appendix, one gets:

$$\frac{b - \widehat{C}_X}{b} = \frac{1 - x_{\theta_m}/\overline{x}}{\widehat{\epsilon_X}} \tag{32}$$

where $\overline{x} = X/N$ is average consumption,

$$\widehat{C}_X = \frac{\partial C}{\partial X} + \tau_X - N\left(\frac{\partial S^+_{\theta_m}}{\partial X} - \frac{\partial S^-_{\theta_m}}{\partial X}\right) \tag{33}$$

is an alternative definition of virtual cost and

$$\widehat{\epsilon_X} = -\frac{b}{X}\frac{\partial \widehat{X}/\partial b}{1 - E_{xX}} \tag{34}$$

is the price elasticity of infra-marginal consumers when the changes induced by a variation in the number of consumers N are ignored (or cannot be accounted for).

Note that, despite its seeming complexity, the elasticity $\widehat{\epsilon_X}$ appears to be much simpler to estimate than the standard price elasticity ϵ_X since it can be based on data streaming from the sole actual consumers.

The Lerner formula (32) merits a few comments. First the "apparent elasticity" that is considered, namely $\widehat{\epsilon_X} / (1 - x_{\theta_m}/\overline{x})$, is always larger[2] than $\widehat{\epsilon_X}$, whatever the nature of the externalities. Second, everything happens as if the firm where contemplating a virtual marginal cost $\widehat{C}_X$. If there are no taxes and furthermore either (i) there are no externalities or (ii) economic agents bear the cost / enjoy the benefit of the externalities that derive from X whether they are connected or not, this virtual marginal cost $\widehat{C}_X$ is exactly identical to plain marginal cost $(\partial C / \partial X)$. If instead the externality that derive from X only accrues to those who have access, then the virtual marginal cost may be lower or higher than $(\partial C / \partial X)$ depending on the sign of $\left(\partial S_\theta^+ / \partial X\right)$. Interestingly, this result contrasts with those obtained for the virtual cost to be considered when pricing access. In the absence of externalities, $\widetilde{C}_N$ is indeed always strictly smaller than $(\partial C / \partial N)$. However, the elasticity ϵ_N was not corrected in the formula that defines the profit-maximising price a. As a result, in the absence of externalities, both the access price a and the marginal price b should be lower than what would predict the bold (and erroneous) application of the standard Lerner formula.

6 Socially Optimum Prices

We now turn to the second-best solution which consists in maximizing W_2 subject to the operator break even constraint $\Pi \geq 0$. Let $\mathcal{L}$ be the Lagrangean expression associated with this problem while λ is the multiplier of the break-

[2] However, the more heterogeneous the population, the smaller the correction. In a very heterogeneous population, the consumption of the marginal consumer as compared to the total (average) consumption is indeed negligible.

even constraint. We obtain the following first-order conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial a} &= \int_0^{+\infty} \left[\frac{\partial S_\theta}{\partial X}\frac{dX}{da} + \frac{\partial S_\theta}{\partial N}\frac{dN}{da} - 1_{\theta \geq \theta_m}\right] g(\theta)\, d\theta \\
&\quad + (1+\lambda)\left[N + (a - \tau_N - c)\frac{dN}{da} + \left(b - \tau_X - \frac{\partial C}{\partial X}\right)\frac{dX}{da}\right] \quad (35) \\
\frac{\partial \mathcal{L}}{\partial b} &= \int_0^{+\infty} \left[\frac{\partial S_\theta}{\partial X}\frac{dX}{db} + \frac{\partial S_\theta}{\partial N}\frac{dN}{db} - x_\theta(a,b)\, 1_{\theta \geq \theta_m}\right] g(\theta)\, d\theta \\
&\quad + (1+\lambda)\left[X + (a - \tau_N - c)\frac{dN}{db} + \left(b - \tau_X - \frac{\partial C}{\partial X}\right)\frac{dX}{db}\right] \quad (36)
\end{aligned}$$

To rearrange and interpret these conditions, we make use of the notations introduced in (17), (19), (24), (28), (30), as well as in (34), and build on previous results, in particular equations (9) and (13). This allows us to obtain for the socially optimal price a the following characterization (See Appendix 9.3):

$$a-(c+\tau_N)+\left(b-\tau_X-\frac{\partial C}{\partial X}\right)\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}} = \frac{\lambda}{1+\lambda}\frac{a}{\epsilon_N}-\frac{1}{1+\lambda}\left[E_N+E_X\left(\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right)\right] \quad (37)$$

or

$$a-\widetilde{C}_N = \frac{\lambda}{1+\lambda}\frac{a}{\epsilon_N}-\frac{1}{1+\lambda}\left[E_N+E_X\left(\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right)\right] \quad (38)$$

Similarily, the optimal price b should obey the equation

$$b-\widetilde{C}_X = \frac{\lambda}{1+\lambda}\frac{b}{\epsilon_X}-\frac{1}{1+\lambda}\left[E_X+E_N\left(\frac{dN}{db}/\frac{dX}{db}\right)\right]. \quad (39)$$

As for the equation (29) that defines the profit-maximising price b, this formulation, although usefull to highlight the mechanisms at hand, have the drawback to rest on the ratio $\left(\frac{dN}{db}/\frac{dX}{db}\right)$ which is a priori difficult to estimate . However, if a is indeed an instrument, so equation (37) holds true, the latter expression can be rewritten as (See Appendix 9.3)

$$b-\left(\frac{\partial C}{\partial X}+\tau_X\right)+\frac{1}{1+\lambda}E_X+\frac{\lambda}{1+\lambda}N\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right) = \frac{\lambda}{1+\lambda}\left(1-\frac{x_{\theta_m}}{X/N}\right)\frac{b}{\widehat{\epsilon_X}} \quad (40)$$

or

$$b-\widehat{C}_X = \frac{\lambda}{1+\lambda}\left(1-\frac{x_{\theta_m}}{X/N}\right)\frac{b}{\widehat{\epsilon_X}}-\frac{1}{1+\lambda}\left[E_X-N\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)\right]. \quad (41)$$

In what follows we will use a star to denote the second-best solution derived in this section: (a^*, b^*).

7 Decentralization and global price-cap

So far we have concentrated on the pricing policy that would be chosen by a welfare maximizing (and well-informed) regulator. Let us now examine how this solution can be decentralized through a regulatory policy when the regulator faces a profit-maximizing provider. In other words, we study how the socially optimal prices (a, b) as defined by (37) and (39) can be achieved as a solution to the provider's profit maximization problem. It is plain that in the absence of regulation, the (monopoly) operator will generally not choose the socially optimal policy.[3] Some regulatory intervention is thus necessary to achieve the optimal outcome. The question is then, how "tight" this regulation has to be. Specifically, is it necessary to regulate every single price, or is some more "global" regulation sufficient?

To address these questions, we study the problem of a firm which is subject to a global price cap, i.e., a constraint imposing an upper limit on the weighted average of its prices. Throughout the section we consider price cap formulae under which the weights (of the different prices) are exogenous for the provider.

Let the provider maximize its profits as defined in (20) subject to the global price-cap constraint given by

$$\alpha a + \beta b \leq \bar{p} + \varphi N + \psi X, \qquad (42)$$

where α and β are the weights of goods access and service, respectively.

Let $\mathcal{L}$ be the Lagrangean expression of the provider's problem while μ is the

[3] Except of course when the maximum achievable profit is equal to zero. In that case, the budget constraint can only be met if profit is maximized. Profit maximization and welfare maximization *subject to a break even constraint* then yield the same result.

multiplier of the constraint (42). The first-order conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial a} = N + \left(a - \frac{\partial C}{\partial N} - \tau_N + \mu\varphi\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X} - \tau_X + \mu\psi\right)\frac{dX}{da} - \mu \tag{43}$$

$$\frac{\partial \mathcal{L}}{\partial b} = X + \left(a - \frac{\partial C}{\partial N} - \tau_N + \mu\varphi\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X} - \tau_X + \mu\psi\right)\frac{dX}{db} - \mu \tag{44}$$

Equation (43) rewrites simply as (See appendix 9.4) :

$$a - \widetilde{C}_N = \left(1 - \mu\frac{\alpha}{N}\right)\frac{a}{\epsilon_N} - \mu\left(\varphi + \psi\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right). \tag{45}$$

Again, if a is endogenous (can be chosen by the firm) so that it obeys (45), equation (44) rewrites (See appendix 9.4)

$$\begin{aligned} & b - \left(\frac{\partial C}{\partial X} + \tau_X\right) + \mu\psi + N\left(1 - \mu\frac{\alpha}{N}\right)\left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right) \\ = & \left(1 - \mu\frac{\beta}{X}\right)\left[1 - \left(\frac{1 - \mu\alpha/N}{1 - \mu\beta/X}\right)\frac{x_{\theta_m}}{X/N}\right]\frac{b}{\widehat{\epsilon_X}} \end{aligned} \tag{46}$$

The decentralization of the second-best solution requires that the solution (a, b) defined by (45) and (46) solves (35)–(36) for the appropriate value of μ. Comparing (38) and (41), the expressions determining the second-best solution to (45) and (46), we show that this is the case when

$$\mu = \frac{1}{1 + \lambda^*} \tag{47}$$

and

$$\begin{aligned} \alpha &= N(a^*, b^*), \quad \beta = X(a^*, b^*), \\ \varphi &= E_N, \quad \psi = E_X. \end{aligned} \tag{48}$$

In words, the appropriate weights are simply equal to the aggregate demand levels at the second-best solution. Once these weights are determined, one can set $\bar{p}$ such that profits goes to zero. One readily verifies that (47) is then automatically also satisfied.

8 References

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9 Appendix

9.1 First-Best Allocation

The welfare function writes

$$W_1 = \int_0^{\theta_m} S_\theta\left(0, X, N\right) g\left(\theta\right) d\theta + \int_{\theta_m}^{+\infty} S_\theta\left(x_\theta, X, N\right) g\left(\theta\right) d\theta - C\left(X, N\right).$$

Differentiate with respect to θ_m :

$$\begin{aligned}
\frac{dW_1}{d\theta_m} &= \left[S_{\theta_m}\left(0, X, N\right) - S_{\theta_m}\left(x_{\theta_m}, X, N\right)\right] g\left(\theta_m\right) \\
&\quad + \int_0^{+\infty} \left[\frac{\partial S_\theta}{\partial X}\frac{dX}{d\theta_m} + \frac{\partial S_\theta}{\partial N}\frac{dN}{d\theta_m}\right] g\left(\theta\right) d\theta - \frac{\partial C}{\partial X}\frac{dX}{d\theta_m} - \frac{\partial C}{\partial N}\frac{dN}{d\theta_m} \\
&= \left[S_{\theta_m}\left(0, X, N\right) - S_{\theta_m}\left(x_{\theta_m}, X, N\right)\right] g\left(\theta_m\right) \\
&\quad + \left(E_X - \frac{\partial C}{\partial X}\right)\frac{dX}{d\theta_m} + \left(E_N - \frac{\partial C}{\partial N}\right)\frac{dN}{d\theta_m} \\
&= g\left(\theta_m\right)\left\{\left[S_{\theta_m}\left(0, X, N\right) - S_{\theta_m}\left(x_{\theta_m}, X, N\right)\right] - \left[\left(E_X - \frac{\partial C}{\partial X}\right) x_{\theta_m} + \left(E_N - \frac{\partial C}{\partial N}\right)\right]\right\}
\end{aligned}$$

From the first FOC $(16a)$, we know that $b = \partial C/\partial X - E_X$, hence

$$\frac{dW_1}{d\theta_m} = -g\left(\theta_m\right)\left[S_{\theta_m}\left(x_{\theta_m}, X, N\right) - S_{\theta_m}\left(0, X, N\right) - bx_{\theta_m} + \left(E_N - \frac{\partial C}{\partial N}\right)\right]$$

and since the marginal consumer is such that $S_{\theta_m}\left(x_{\theta_m}, X, N\right) - \left(a + bx_{\theta_m}\right) = S_{\theta_m}\left(0, X, N\right)$, one gets the FOC :

$$a = \frac{\partial C}{\partial N} - E_N$$

9.2 Relationship between changes in N that follow from changes in prices

By differentiating with respect to a equation (3) that defines the marginal type θ_m, it follows that

$$\begin{aligned}
& \frac{\partial S_\theta^+}{\partial x}\frac{dx_{\theta_m}}{da} + \frac{\partial S_\theta^+}{\partial X}\frac{dX}{da} + \frac{\partial S_\theta^+}{\partial N}\frac{dN}{da} + \frac{\partial S_\theta^+}{\partial \theta}\frac{d\theta_m}{da} - \left(1 + b\frac{dx_{\theta_m}}{da}\right) \\
= \; & \frac{\partial S_\theta^-}{\partial X}\frac{dX}{da} + \frac{\partial S_\theta^-}{\partial N}\frac{dN}{da} + \frac{\partial S_\theta^-}{\partial \theta}\frac{d\theta_m}{da}
\end{aligned}$$

where S_θ^+ stands for $S_{\theta_m}(x_{\theta_m}, X, N)$, i.e. the surplus function of the marginal consumer who actually gets access to the service, while S_θ^- stands for $S_{\theta_m}(0, X, N)$, i.e. the surplus of the marginal consumer who actually opts for not accessing the service.[4] By the envelope theorem, this boils down to:

$$\left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)\frac{dX}{da} + \left(\frac{\partial S_\theta^+}{\partial N} - \frac{\partial S_\theta^-}{\partial N}\right)\frac{dN}{da} + \left(\frac{\partial S_\theta^+}{\partial \theta} - \frac{\partial S_\theta^-}{\partial \theta}\right)\frac{d\theta_m}{da} = 1. \quad (49)$$

Similarily, differentiating with respect to b equation (3) gives

$$\begin{aligned} & \frac{\partial S_\theta^+}{\partial x}\frac{dx_{\theta_m}}{db} + \frac{\partial S_\theta^+}{\partial X}\frac{dX}{db} + \frac{\partial S_\theta^+}{\partial N}\frac{dN}{db} + \frac{\partial S_\theta^+}{\partial \theta}\frac{d\theta_m}{db} - \left(x_{\theta_m} + b\frac{dx_{\theta_m}}{db}\right) \\ = \; & \frac{\partial S_\theta^-}{\partial X}\frac{dX}{db} + \frac{\partial S_\theta^-}{\partial N}\frac{dN}{db} + \frac{\partial S_\theta^-}{\partial \theta}\frac{d\theta_m}{db} \end{aligned}$$

hence

$$\left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)\frac{dX}{db} + \left(\frac{\partial S_\theta^+}{\partial N} - \frac{\partial S_\theta^-}{\partial N}\right)\frac{dN}{db} + \left(\frac{\partial S_\theta^+}{\partial \theta} - \frac{\partial S_\theta^-}{\partial \theta}\right)\frac{d\theta_m}{db} = x_{\theta_m}. \quad (50)$$

From (6) and (9), equation (49) rewrites:

$$\left[\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right) + \left(\frac{\partial S_\theta^+}{\partial N} - \frac{\partial S_\theta^-}{\partial N}\right) - \frac{1}{g(\theta_m)}\left(\frac{\partial S_\theta^+}{\partial \theta} - \frac{\partial S_\theta^-}{\partial \theta}\right)\right]\frac{dN}{da} = 1.$$

From (11) and (13) ,equation (49) rewrites:

$$\begin{aligned} x_{\theta_m} = \; & \left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)\frac{1}{1 - E_{xX}}\frac{\partial \widehat{X}}{\partial b} \\ & + \left[\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right) + \left(\frac{\partial S_\theta^+}{\partial N} - \frac{\partial S_\theta^-}{\partial N}\right) - \frac{1}{g(\theta_m)}\left(\frac{\partial S_\theta^+}{\partial \theta} - \frac{\partial S_\theta^-}{\partial \theta}\right)\right]\frac{dN}{db} \end{aligned}$$

hence

$$\frac{dN}{db} / \frac{dN}{da} = x_{\theta_m} - \left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)\left[\frac{1}{1 - E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right] \quad (51)$$

[4] Remind that the marginal consumer is precisely indifferent between accessing or not so that the values of both functions are equals. Of course, the differentials generally differ.

9.3 Second_best

9.3.1 Computation of equations (37) – (38)

Making use of equation (9) and of notations (17) and (19), the FOC condition (35) yields directly

$$-\lambda N = \left(E_N + E_X \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right) \frac{dN}{da} + (1+\lambda)\left[(a - \tau_N - c) + \left(b - \tau_X - \frac{\partial C}{\partial X}\right) \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right] \frac{dN}{da}$$

With the elasticity ϵ_N defined in (24) ,this rewrites (37) . Equation (38) is obtained by plugging the definition of the virtual connection cost introduced in (27) .

9.3.2 Computation of equations (40) – (41)

Making use of equation (13) and of notations (17) and (19), the FOC condition (36) writes directly

$$\begin{aligned} -\lambda X &= (1+\lambda)\left[\left(b - \tau_X - \frac{\partial C}{\partial X}\right) + \frac{1}{1+\lambda} E_X\right] \frac{1}{1 - E_{xX}} \frac{\partial \widehat{X}}{\partial b} \\ &+ \left(E_N + E_X \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right) \frac{dN}{db} \\ &+ (1+\lambda)\left[(a - \tau_N - c) + \left(b - \tau_X - \frac{\partial C}{\partial X}\right)\left(\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right)\right] \frac{dN}{db} \end{aligned}$$

From (35) we know that:

$$-\lambda N = \left(E_N + E_X \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right) \frac{dN}{da} + (1+\lambda)\left[(a - \tau_N - c) + \left(b - \tau_X - \frac{\partial C}{\partial X}\right) \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right] \frac{dN}{da}.$$

hence condition (36) rewrites

$$-\lambda X = (1+\lambda)\left[\left(b - \tau_X - \frac{\partial C}{\partial X}\right) + \frac{1}{1+\lambda} E_X\right] \frac{1}{1 - E_{xX}} \frac{\partial \widetilde{X}}{\partial b} - \lambda N \left(\frac{dN}{db} / \frac{dN}{da}\right).$$

By using (51) ,one gets

$$\begin{aligned} -\lambda X &= (1+\lambda)\left[\left(b - \tau_X - \frac{\partial C}{\partial X}\right) + \frac{1}{1+\lambda} E_X\right] \frac{1}{1 - E_{xX}} \frac{\partial \widetilde{X}}{\partial b} \\ &\quad - \lambda N \left(x_{\theta_m} - \left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)\left[\frac{1}{1 - E_{xX}} \frac{\partial \widetilde{X}}{\partial b}\right]\right). \end{aligned}$$

With the price elasticity of infra-marginal consumers $\widehat{\epsilon_X}$ defined in (34) this rewrites (40). Equation (41) is obtained by plugging the definition of the virtual marginal cost introduced in (33).

9.4 Regulation and Global Price Cap

9.4.1 Computation of equation (45)

By using (9), equation (43) rewrites directly as:

$$-N+\mu\left[\alpha-\left(\varphi+\psi\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right)\frac{dN}{da}\right]=\left[a-\frac{\partial C}{\partial N}-\tau_N+\left(b-\frac{\partial C}{\partial X}-\tau_X\right)\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right]\frac{dN}{da}. \tag{52}$$

It follows that

$$a-\frac{\partial C}{\partial N}-\tau_N+\left(b-\frac{\partial C}{\partial X}-\tau_X\right)\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}=\left(1-\mu\frac{\alpha}{N}\right)\frac{a}{\epsilon_N}-\mu\left(\varphi+\psi\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right).$$

With the definition (27) of the virtual connection cost $\widetilde{C}_N$, one gets directly (45).

9.4.2 Computation of equation (46)

Plugging (13) and (51) into (44) gives

$$\begin{aligned}
0 &= X-\mu\beta+\left(b-\frac{\partial C}{\partial X}-\tau_X+\mu\psi\right)\left(\frac{1}{1-E_{xX}}\frac{\partial\widehat{X}}{\partial b}\right)\\
&\quad+\left[a-\frac{\partial C}{\partial N}-\tau_N+\mu\varphi+\left(b-\frac{\partial C}{\partial X}-\tau_X+\mu\psi\right)\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right]\frac{dN}{db}\\
&= X-\mu\beta+\left(b-\frac{\partial C}{\partial X}-\tau_X+\mu\psi\right)\left(\frac{1}{1-E_{xX}}\frac{\partial\widehat{X}}{\partial b}\right)\\
&\quad+\left[a-\frac{\partial C}{\partial N}-\tau_N+\mu\varphi+\left(b-\frac{\partial C}{\partial X}-\tau_X+\mu\psi\right)\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right]\\
&\quad\times\left(x_{\theta_m}-\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)\left[\frac{1}{1-E_{xX}}\frac{\partial\widetilde{X}}{\partial b}\right]\right)\frac{dN}{da}.
\end{aligned}$$

Now from (52), the later expression rewrites

$$\begin{aligned}
0 &= X\left[\left(1-\mu\frac{\beta}{X}\right)-\left(1-\mu\frac{\alpha}{N}\right)\frac{x_{\theta_m}}{X/N}\right]\\
&\quad+\left(b-\frac{\partial C}{\partial X}-\tau_X+\mu\psi+N\left(1-\mu\frac{\alpha}{N}\right)\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)\right)\left(\frac{1}{1-E_{xX}}\frac{\partial\widehat{X}}{\partial b}\right)
\end{aligned}$$

that gives directly (46).