Social Value of Innovations, Distortions, and R&D Investments: First Best versus Second Best Equilibria in Growth Models

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Abstract

In this paper, we make a synthesis between the literature on endogenous growth, and the literature on the funding of innovations developed by authors like for instance Arrow (1962), Tirole (1988), Scotchmer (1991, 1999), Dasgupta et *al* (1996), Gallini and Scotchmer (2003). Then, we shed a new light on several questions often studied in these literatures. The first one is related to the social value of innovations. The second one concerns the distortions that prevent the decentralized economy to be optimal. The third one concerns insufficient investments which are observed in research.

Keywords: public good, social value of innovations, distortions, intellectual property rights,.

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1 Introduction

Since the seminal papers of Romer (1990), Grossman and Helpman (1991a), Aghion and Howitt (1992), endogenous growth theorists have focused on a single type of equilibrium. They assume that each innovation, produced by the research and development (R&D) sector, is associated to a particular intermediate good. Each good is then produced by a single producer, who benefits from an intellectual property right, like a patent, to produce and to sell it. In this case, research is privately and indirectly funded by the monopoly profits of intermediate producers. In this type of equilibrium, a market and a price are specified for intermediate goods that embody innovations; however, in our knowledge, neither in the literature on endogenous growth nor in the books of Grossman and Helpman (1991b), Barro and Sala-I-Martin (1995), Aghion and Howitt (1998), any author has defined a market and a price for discoveries. Our question is then the following: why a good which needs scarce resources as inputs in its production technology (i.e. which is costly to produce), has not any price? A consequence of this type of analysis is that we do not know the system of prices that implements the first best optimum; moreover, the equilibrium considered can be interpreted as a second best one since it displays incomplete markets.

Independently of endogenous growth theory, a large literature on the funding of innovations has been developed. Several authors, like Arrow (1962), Tirole (1988), Scotchmer (1991, 1999), Dasgupta et *al* (1996), Gallini and Scotchmer (2003), insist on the difficulty of research funding, in the sense of providing private incentives to innovate, and in achieving an optimal allocation of resources in a decentralized economy, because innovations are public (non-rival) and indivisible goods. These authors propose several schemes to construct equilibria; however, they do not formalize their ideas in a growth model.

The first purpose of this paper is to make a synthesis of these two literatures. To do it, we use the model of Jones and Williams (2000) which is a complete and representative example of the standard growth theory. Inside this model, we formalize the ideas of the authors quoted above, by characterizing the equilibrium with complete markets of the model (the benchmark). Contrary to the standard literature on growth, (in particular, contrary to Jones and Williams (2000)), we distinguish ideas which are public and indivisible goods, from the private and divisible intermediate goods that are produced around them. Then, we account for the case in which intellectual property rights are given to ideas rather than to intermediate goods. On that point we follow Arrow (1962) who writes: "the property rights may be in the information itself, through patents and similar legal devices" (p. 149). Concerning the type of property rights that are used, and the way they must work, we follow Scotchmer (1991) who explains: "a system of property rights that might seem natural would be to protect the first innovator so broadly that licensing is required from all second generations innovators who use the initial technology, whether in research or in production" (p. 32). In other words, we assume that researchers keep the property rights, and license (rent) their discoveries to potential users: if an agent does not pay to acquire a license, he is excluded from its use.

The analysis of this equilibrium allows to compute the system of prices which allows to implement the first order optimum: in this sense, it is a normative solution of the model. However, it can also be considered as a first step to future researches in which ideas would be directly priced. Indeed, since the mid eighties, one can observe an evolution of the intellectual property law in which innovations are directly patented. For instance, Varian (1998) writes: "Up until recently, the US Patent Office and the courts interpreted algorithm as "mathematical formulas" which could not be patented. However, in the mid eighties they reversed this policy and began to issue patents for softwares algorithms. Subsequently the patent office has issued many thousands of software patents". This example concerns the software industry. We can quote other sectors in which this type of patent is used. For instance, Sctochmer (1999) explains that "Incyte, Human Genome Science, Celera sell access to their data-bases at millions of dollars per year per user, providing bioinformatics search tools and other enhancements in addition to gene sequences". Similarly, Henry, Trommetter and Tubiana (2003) say that "Human Genome Science owns a patent on the gene that produces the protein CCR5. The researchers of the National Institutes of Health have used this discovery to produce a medicine for AIDS. They have been obliged to buy a license to use the protein".

The synthesis of the standard literature on growth and of the non-formalized literature on intellectual property rights on ideas allows us to shed a new light on several questions often studied, that is the second purpose of the paper.

The first question concerns the evaluation of the social value of an innovation. We show that the distinction between innovations, which are public goods, and the private intermediate goods which embody them, allows to compute this social value. In particular, we find again the ideas of Scotchmer (1991, 1999), Gallini and Scotchmer (2003) along which research is basically cumulative. This analysis leads to a value which is different from the one computed by Jones and Williams (2000), and also different from the one generally used in Industrial Organization Theory.

The second question concerns the identification of the distortions that prevent the decentralized equilibrium to be optimal. In their analysis, Jones and Williams (2000) explain that, theoretically, it is possible to obtain either excessive or insufficient investments in research compared to the optimum. According to the authors, this result comes from the combination of four distortions that remain in the standard equilibrium analyzed in growth theory, and that have opposite effects on the amount of resources devoted to research: on the one hand, the appropriability effect and the knowledge spill-over effect induce under-investments in research; on the other hand, the creative destruction effect and the duplication (or redundancy) effect, lead to over-investments.[1] We show that the externalities inside the R&D sector are the only remaining distortions with respect to the first best optimum when the equilibrium displays complete markets; the other distortions appear as a result of the assumptions of incomplete markets and no first-price discrimination for intermediate goods.

The third question concerns the empirical observation on the insufficient investments in research: for instance, Jones and Williams (1998) estimate that actual investments are at least four times under what would be socially optimal. Since we show that the duplication effect is the only remaining distortion when the equilibrium displays complete markets, the debate on the factors that induce insufficient investments in research is still opened. A possible alternative explanation for this feature is based on the fact that ideas are public goods: innovators cannot appropriate the entire amount of the surplus they create in the economy, in particular because they face verifiability, excludability, and information problems. For instance, this explanation is consistent with Dasgupta et *al* (1996) who write: "The market mechanism has a tendency to discourage the production of public goods because of an inability on the part of producers to appropriate fully the value of the fruits of their activity" (p. 9).

The remainder of the paper is organized as follows: in Section 2, we describe the model and we study the benchmark equilibrium. In Section 3, we compute the social value of an innovation; we analyze the nature of the distortions exhibited in the standard literature; and, we study the question of under-investments in research. In Section 4, we conclude. The Appendix is gathered in Section 5.

2 Model and Benchmark Equilibrium

2.1 The Model

We consider a similar model to Jones and Williams (2000). The only one difference is that we do not introduce the creative-destruction effect, because we consider a model with horizontal differentiation. Time is continuous and three kinds of goods are produced in the economy: a consumption-capital good ("output"), ideas and intermediate goods. Technologies and preferences are described as follows. Total output, Y_t, produced at time t, is given by

[1] The distortions evoked by Aghion and Howitt (1998), namely the intertemporal spill-over effect, the appropriability effect and the business stealing effect (see chapter 2, p. 79)) recovers from the same type of analysis. The only difference comes from the type of model used to conduct the analysis. In their book, the authors use a model with vertical differentiation whereas we use a one with horizontal differentiation.

$$Y_t = (L_t)^{\alpha} \left(\int_0^{A_t} (x_t(i))^{\rho(1-\alpha)} \, di \right)^{1/\rho}, \tag{1}$$

where $\alpha \in (0,1)$, $0 < \rho < 1/(1-\alpha)$.[2] L_t is the total amount of labor in the economy growing at exogenous and constant rate n, $x_t(i)$ is the variety of capital (intermediate) good i used at time t, and A_t denotes the range of types of intermediate goods that have been produced at time t.

We assume that the production of a capital good requires the preliminary production of an innovation. Then, once a new idea is discovered, an intermediate good can be produced around it. Each good is produced by an intermediate firm that uses physical capital according to a one for one technology: $x_t(i) = k_t(i)$. Since all the amount of physical capital is used to produce the range of types of intermediate goods, we have

$$\int_0^{A_t} x_t(i) \, di = \int_0^{A_t} k_t(i) \, di = K_t, \tag{2}$$

where K_t is the total amount of physical capital in the economy. It is accumulated according to

$$\dot{K}_t = Y_t - C_t - R_t, \tag{3}$$

where C_t is aggregate consumption and R_t is the total amount of resources devoted to research.

Ideas are produced by a large number J, $(j = 1, ..., J)$, of firms whose technologies are given by

$$\dot{A}_{jt} = \xi_t R_{jt} (A_t)^{\phi}, \tag{4}$$

where $\phi < 1$ allows past discoveries to either increase $(\phi > 0)$, or decrease $(\phi < 0)$, current research productivity; $\dot{A}_{jt}$ is the number of innovations produced per unit of time by firm j; R_{jt} represents the amount of resources employed in firm j. The term ξ_t is a productivity factor which is external to each firm. It verifies $\xi_t = \delta (R_t)^{\lambda-1}$, where $R_t = \sum_{j=1}^{J} R_{jt}$ is the total amount of resources used in research; $\delta > 0$ is a constant productivity parameter; $\lambda \in]0,1]$ allows to take into account the possibility of duplication effect or redundancy in research. If we sum over j, then the total number of ideas produced per unit of time is given by

[2] Jones and Williams use a slightly different production function given by $Y_t = (L_t)^{\alpha} \left(\sum_{i=1}^{A_t} (x_t(i))^{\rho(1-\alpha)} \right)^{1/\rho}$. Of course the two approaches are equivalent. However, we prefer to treat A_t as a continuous variable to simplify the calculations.

$$\overset{\bullet}{A}_t = \delta (R_t)^{\lambda} (A_t)^{\phi}. \tag{5}$$

Contrary to what is generally done in the standard literature on endogenous growth, we distinguish an innovation which is a public (non-rival) and indivisible good[3] from the private and divisible intermediate good which embodies it. The non-rival property implies that ideas can be used jointly by many agents. The property of indivisibility implies that once a piece of knowledge has been acquired, there is no value added in acquiring it again.

Formally, an innovation, i, is a point of the segment $[0, A_t]$; we interpret it as a scientific report. For example, we distinguish the medical formula of a vaccine from the vaccine itself. The first good is non-rival. Its marginal cost of production is nil; it can be used again and again without any additional cost. The second good is a private one. Once it has been injected to treat an individual, it is necessary to produce it again.

The preferences of the representative agent are represented by the discounted utility function

$$U = \int_0^{\infty} u(C_t/L_t) e^{-\theta t} dt, \tag{6}$$

where C_t/L_t is per-capita consumption, $\theta > 0$ is the rate of time preferences, $u'(\bullet) > 0$, $u''(\bullet) < 0$, and $\gamma = -u''(\bullet) C_t / \left[u'(\bullet) L_t\right]$ is the inverse of the intertemporal elasticity of substitution.

2.2 Benchmark Equilibrium

The aim of this sub-section is to construct an equilibrium in which prices allow to implement the first best optimum. To do it, we define a price both for innovations (public goods) and for private intermediate goods that embody them. It is clear that generally, in the real world, these prices (in particular the prices of innovations) are not observed. Thus, this equilibrium with complete markets has to be interpreted as a normative solution of the model. In this case, we show that the externalities inside the R&D sector (the duplication effect) are the only distortions that prevent the equilibrium to coincide with the first best optimum. To remove this distortion and to be able to implement an optimal path, we assume that a tax rate, τ_t, is charged on R&D firms.

As we said in the introduction, the analysis conducted in this section is a formalization of ideas already expressed in the literature by Arrow (1962), Dasgupta et *al* (1996), Tirole (1988), Scotchmer (1991, 1999), Gallini and Scotchmer (2003), but which have not retained attention of endogenous growth theorists since the seminal papers of Romer (1990), Grossman and Helpman (1991a) and Aghion and Howitt (1992). In other words,

[3] See Arrow (1962), Dasgupta et *al* (1996) among others who define ideas in the same way.

we characterize an equilibrium which has not been, in our knowledge, studied in the standard literature on growth.

We assume that property rights are given to ideas rather than to intermediate goods (see the citation of Arrow (1962) given in the introduction): innovators license (rent) innovations to potential users. Thus, any agent using a patented innovation rewards directly the researcher or the scientist who has produced it.

Shy (1991) explains that 80 percent of patents are licensed. In the present paper, we study the case of non-exclusive licenses which is consistent with the following example of Arrow (1962): "Suppose, as the result of elaborate tests, some metal is discovered to have a desirable property, say resistance to high heat. Then of course every use of the metal for which this property is relevant would also use this information, and the user would be made to pay for it. But, even more, if another inventor is stimulated to examine chemically related metals for heat resistance, he is using the information already discovered and should pay for it in some measure; and any beneficiary of his discoveries should also pay" (p. 150). (See also the citation of Scotchmer (1991) given in the introduction of the paper).

The public good property of knowledge raises several questions. The first ones are standard in economics literature: they are relative to the possibility to verify which agent uses a discovery; they are linked to the possibility to exclude any agent that does not pay to use an innovation; they concern the information about the willingness to pay of agents to use an innovation. Since we study the normative solution which corresponds to the first best optimum, we assume that there are not any problems of verifiability, excludability and information: in other words, each innovator is able to extract the willingness to pay of any agent which uses his innovation. Note that in this case, we follow for instance Dasgupta et *al* (1996) who write: "It is as well to note here that [...] the producer (or owner) of a piece of information should ideally set different prices for different buyers, because different buyers typically value the information differently. In economics, these variegated prices are called Lindahl prices, in honor of the person who provided the first articulation of this scheme" (p. 10). As said above, it is clear that this normative solution cannot be considered as a positive one: generally we do not observe it in the real world.

In this model, three types of agents use the discoveries produced by the R&D sector: the firm of the output sector, R&D firms and intermediate producers. Each of them buys a license to use an innovation. Formally, we call respectively $v_{Yt}(i)$, $v_{jt}(i)$ and $v_{xt}(i)$, the willingnesses to pay (Lindahl prices) of the final sector, of the R&D firm j, and of any intermediate producer i to use the innovation i at time t. Using the fact that innovations are treated symmetrically, we can write $v_{Yt}(i) \equiv v_{Yt}$, $v_{jt}(i) \equiv v_{jt}$, and $v_{xt}(i) \equiv v_{xt}$. Thus, the instantaneous value of an innovation is given by $v_t = v_{Yt} + v_{At} + v_{xt}$, where $v_{At} = \sum_{j=1}^{J} v_{jt}$. Finally, $V_t = \int_t^{\infty} v_s e^{-\int_t^s r_u du} ds$ is the value of an innovation at time t.

In the present analysis, v_{Yt}, v_{At} and v_{xt} represent, for each agent using an idea, the difference between its profit when it uses a new discovery with its profit when it does not: in this section, we compute the surpluses created by the public, indivisible and infinitely-lived durable good in each sector using it as a productive factor.

The second type of problem raised by the public good property of innovations comes from the non convexity of technologies using ideas as productive factors. On this point, the replication argument states that there are constant returns to scale with respect to private inputs and increasing returns to scale with respect to both private and public inputs. As in a competitive market the payment of private factors fully exhaust revenue, firms are unable to pay for the public good they use.[4] Thus, an equilibrium with perfect competition on private goods markets does not exist if firms that pay to access to innovations are not subsidized. To avoid this situation, we assume that the government subsidizes the willingness to pay of firms. This economic policy, as the tax charged on R&D firms to remove the duplication effect, is funded through a lump-sum tax (or lump-sum transfer), T_t/L_t, charged (or given) to the representative individual. This assumption is the second reason why the normative equilibrium studied here cannot be considered as a positive one. In fact, in the real world, a large part of research is privately funded. To formalize this type of equilibrium, it would be necessary to introduce imperfect competition on the markets of sectors that use innovations (final output, research and intermediate goods): this is out of the scope of this paper.

Remark 1 *: the assumption of imperfect competition used in the seminal papers of Romer (1990), Grossman and Helpman (1991a), Aghion and Howitt (1992), does not solve the problem of existence raised by the property of increasing returns to scale. Indeed, this assumption concerns only the intermediate goods sector, and it allows to get positive profits in order to fund indirectly research. However, in the two other sectors which use the discoveries (final sector and R&D sector), the standard literature assumes perfect competition that is sustainable only because ideas have not any price, i.e. because markets are incomplete.*

We proceed as follows. First, we present the behavior of agents. Second, we derive the solution of the equilibrium. The price of the final homogenous good is normalized to one. The price of the intermediate good of type i, the level of wage, and the interest rate are respectively noted $q_t(i)$, w_t, r_t. There is perfect competition on all private goods markets (Y, L, $x(i)$ for all $i \in [0, A_t]$), including intermediate goods markets. The growth rate of any variable z is noted g_z.

[4] See for instance Kaizuka (1965), Sandmo (1972), Manning et *al* (1985), Feehan (1989), Romer (1990), Jones (2003) among others.

a) The competitive firm of the final sector maximizes its profit given by

$$\pi_{Yt} = (L_t)^{\alpha} \left(\int_0^{A_t} (x_t(i))^{\rho(1-\alpha)} \, di \right)^{1/\rho} - w_t L_t - \int_0^{A_t} q_t(i) \, x_t(i) \, di.$$

The first order conditions yield

$$w_t = \alpha \frac{Y_t}{L_t}, \tag{7}$$

and

$$q_t(i) = (1-\alpha)(L_t)^{\alpha} \left(\int_0^{A_t} (x_t(i))^{\rho(1-\alpha)} \, di \right)^{1/\rho - 1} (x_t(i))^{\rho(1-\alpha)-1}. \tag{8}$$

The willingness to pay to use an innovation at time t, is $v_{Yt} = \partial \pi_{Yt} / \partial A_t$. Using the previous first order conditions and the property of symmetry of intermediate goods, one gets

$$v_{Yt} = \sigma \frac{Y_t}{A_t}, \tag{9}$$

where $\sigma = 1/\rho - (1-\alpha)$.

b) Concerning the intermediate producers, we assume that they behave competitively. Note that this behavior differs from the case described in the standard literature. Since the technology of production of intermediate goods is $x_t(i) = k_t(i)$, it follows that the unit price of an intermediate good is simply $q_t = r_t$. Moreover, the willingness to pay for an innovation is nil. Indeed, if the intermediate producer i does not buy a license to use the idea i, its profit is nil since it cannot produce any good. On the other hand, if it pays to use the idea, its profit is also nil. As a consequence, the willingness to pay to use an innovation is equal to zero, that is to say

$$v_{xt} = 0. \tag{10}$$

Remark 2 *: the result $v_{xt} = 0$ is linked to the constant returns to scale property of the production function for intermediate goods. If the technology is strictly concave with respect to $k_t(i)$, then $v_{xt} > 0$.*

c) In the R&D sector, at each time, the firm j licences the ideas it has produced to each potential user (the final sector and the firms of the R&D sector). Then, it maximizes the sum of the present values of its expected profits given by $\int_0^{\infty} [v_t A_{jt} -$

$(1+\tau_t) R_{jt}]e^{-\int_0^\infty r_u du}dt$, subject to the technology (4). Associating the co-state variable ν_t to the law of motion of ideas in firm j, the Hamiltonian of this problem is

$$\Gamma = [v_t A_{jt} - (1+\tau_t) R_{jt}] e^{-\int_0^t r_u du} + \nu_t \delta (R_t)^{\lambda-1} R_{jt} (A_t)^{\phi}.$$

The first order conditions are: $\partial\Gamma/\partial R_{jt} = \nu_t \delta (R_t)^{\lambda-1} (A_t)^{\phi} - (1+\tau_t) e^{-\int_0^t r_u du} = 0$ (a), and $\partial\Gamma/\partial A_{jt} = -\dot{\nu}_t = v_t . e^{-\int_0^t r_u du}$ (b). The transversality condition is $\lim_{t\to\infty} \nu_t A_{jt} = 0$.

Integrating (b) between t and infinity, we obtain $\int_t^\infty -\dot{\nu}_s ds = \int_t^\infty v_s . e^{-\int_0^s r_u du} ds$, that yields $\nu_t - \nu_\infty = (e^{-\int_0^t r_u du}) \int_t^\infty v_s . e^{-\int_t^s r_u du} ds$. Using the transversality condition and the fact that $A_{jt} > 0$ for all t, we deduce that $\nu_\infty = 0$. Using the fact that $V_t = \int_t^\infty v_s e^{-\int_t^s r_u du} ds$ is the value of an innovation at time t, we can write $\nu_t = V_t . e^{-\int_0^t r_u du}$. Combining this result with (a), one gets $V_t \delta (R_t)^{\lambda-1} (A_t)^{\phi} = (1+\tau_t)$. Multiplying both sides by R_t and using (5), we get

$$\dot{A}_t V_t = (1+\tau_t) R_t. \tag{11}$$

The willingness to pay at time 0 for an innovation used at time t is $\partial\Gamma/\partial A_t = \nu_t \phi \delta (R_t)^{\lambda-1} R_{jt} (A_t)^{\phi-1}$. Multiplying both sides by $e^{-\int_0^t r_u du}$, one gets the willingness to pay at time t for an innovation used at t, that is to say $v_{jt} = \partial\Gamma/\partial A_t = V_t \phi \delta (R_t)^{\lambda-1} R_{jt} (A_t)^{\phi-1}$. Replacing V_t by the expression obtained before, one gets

$$v_{jt} = \frac{\phi (1+\tau_t) R_{jt}}{A_t}. \tag{12}$$

Thus, the willingness to pay at time t for an innovation used at t by all firms of the R&D sector is

$$v_{At} = \frac{\phi (1+\tau_t) R_t}{A_t}. \tag{13}$$

d) For the government, we assume that the budget constraint is balanced at each time. It is given by $T_t = (v_{Yt} + v_{At}) A_t - \tau_t R_t$, where $(v_{Yt} + v_{At}) A_t$ represents the total amount of the subsidy distributed to the sectors that use innovations, and $\tau_t R_t$ is the tax charged on research firms.

e) Finally, the representative household maximizes (6) subject to the budget constraint given by $\dot{a}_t = (r_t - n) a_t + w_t - C_t/L_t - T_t/L_t$, where $a_t = K_t + A_t V_t$ represents the stock of wealth, and r_t is the rate of return of his portfolio. One gets

$$\frac{\dot{u}'}{u'} + \theta + n = \gamma (g_{C_t} - n) + \theta + n = r_t. \tag{14}$$

Now, we can determine the system of prices, the growth rates and the share of output spent in research as a function of the tax rate, τ, in the benchmark equilibrium. Proposition 1 summarizes the results. We focus on balanced growth paths and equilibrium values are denoted by the symbol "$*$".

Proposition 1 *The benchmark equilibrium in which there is perfect competition in all private goods markets, and innovations are rented at their Lindahl prices levels, is characterized by the following system of prices:*

$$\begin{aligned} r^* &= \gamma\left(g_C - n\right) + \theta + n, \\ w_t^* &= \alpha Y_t / L_t, \\ v_{Yt}^* &= \sigma Y_t / A_t, \\ v_{At}^* &= \phi\left(1+\tau\right) R_t / A_t, \\ v_{xt}^* &= 0 \\ V_t^* &= \left(1+\tau\right)\left(R_t\right)^{1-\lambda}\left(A_t\right)^{-\phi}/\delta. \end{aligned}$$

The growth rates of quantities are:

$$\begin{aligned} g_A^* &= \frac{\lambda n}{\left(1-\phi-\lambda\sigma/\alpha\right)}, \\ g_Y^* &= g_C^* = g_R^* = g_K^* = \frac{\sigma}{\alpha} g_A^* + n. \end{aligned}$$

The share of resources allocated to research is:

$$s^* = \frac{\sigma g_A^*}{\left(1+\tau\right)\left(r^* - \left(g_Y^* - g_A^*\right) - \phi g_A^*\right)}.$$

Proof. See Appendix 5.1. ■

For each possible value of the tax rate, τ, there is an associated equilibrium. Only one of these is optimal. Indeed, if we compare the steady-state optimum which is computed in Appendix 5.2 with the characterization of the steady-state equilibrium given in Proposition 1, we obtain the following proposition:

Proposition 2 *If the government chooses* $\tau^o = 1/\lambda - 1 > 0$, *the steady-state benchmark equilibrium is optimal.*

Proposition 2 implies that without any intervention from the government, a research firm is attempted to hire an excessive amount of resources in order to innovate. The reason is that it does not account for the duplication effect in research.

Note that $\partial\tau^o/\partial\lambda < 0$. Thus, the higher is λ (i.e. the lower is the duplication effect measured by this parameter), the lower will be the tax imposed on research firms to implement the optimum. And, if there is not any duplication effect in research, (i.e. if $\lambda = 1$), it is not necessary to tax R&D firms. In that case, $\tau^o = 0$, and the benchmark equilibrium path coincides with the optimal one. Basically, this result can be considered as the proof that the equilibrium with complete markets considered here is the benchmark of the model, in the sense that now we know the system of prices that implements the first best optimum.

3 Social Value of Innovations, Distortions, and Insufficient Investment in R&D

The objective of this section is to clarify several points already studied in endogenous growth theory. First, using the correspondence between benchmark equilibrium and first best optimum, we compute the social value of innovations. Second, comparing the standard equilibrium with incomplete markets with the benchmark one, we show that, except for the duplication effect, all distortions come from the assumptions of incompleteness and no first-price discrimination for intermediate goods. Finally, we give an interpretation to insufficient investment in R&D.

3.1 Social Value of Innovations

We can compute the value of an innovation by putting together the characterization of the optimum (see equation (17) in Appendix 5.2) and the analysis of the benchmark equilibrium (see sub-section 2.2). In the two cases, the instantaneous social value of an innovation is:

$$v_t = \sigma \frac{Y_t}{A_t} + \frac{\phi}{\lambda} \frac{R_t}{A_t}, \tag{15}$$

where $v_{Yt} = \sigma Y_t / A_t$ and $v_{At} = \phi R_t / (\lambda A_t)$ are respectively the Lindahl prices, that is to say the gross surpluses of an innovation, in the final sector and in the R&D sector (we have here $v_{xt} = 0$, see equation (10)). Recall that, since innovations are durable goods, the value of an innovation at each time t is $V_t = \int_t^\infty v_s e^{-\int_t^s r_u du} ds$.

If $\tau^o = 1/\lambda - 1$, the characteristic Keynes-Ramsey condition of optimum, (17),

$$\frac{\lambda \dot{A}_t}{R_t} \left\{ \sigma \frac{Y_t}{A_t} + \frac{\phi}{\lambda} \frac{R_t}{A_t} \right\} + g_Y - g_A = \frac{\dot{u}'}{u'} + \rho + n,$$

corresponds to the condition $v_t / V_t + \dot{V}_t / V_t = r_t$ of the benchmark equilibrium: the term $\lambda \dot{A}_t / R_t$ corresponds to $1/V_t$ (see equation (11)); $\sigma Y_t / A_t$ and $(\phi R_t) / (\lambda A_t)$ corresponds to v_{Yt} and v_{At} (see above); the term $g_Y - g_A$ corresponds to $\dot{V}_t / V_t$; finally, the program of the consumer yields to the usual condition $\dot{u}' / u' + \rho + n = r_t$ (see equation (14)).

This formulation is consistent with ideas already expressed but without any formalization in the literature on growth. For instance, Scotchmer (1999) writes: "The social value of an invention is compounded by the fact that the discovery facilitates or becomes a basis for future discoveries. The incremental value of the future discoveries should be counted as part of the initial innovation's social value created" (p. 1). Gallini and Scotchmer (2003) explain: "The first and most fundamental complexity, articulated by Scotchmer (1991), is that early innovators lay a foundation for later innovations. The later innovations could not be made without the earlier ones. So that the first innovator has enough incentive to invest, she should be given some claim on the profit of the later innovations; otherwise, early innovators could be underrewarded for the social value they create" (p. 65). These authors explain that the social value created by a new idea must also account for the surplus induced in the research activity. More generally it would account for the surplus induced in all sectors using the discoveries as productive factors.

Remark 3 *: the social value (15) is different from the expressions generally obtained in the literature. For instance, Jones and Williams (2000) explain that innovators appropriate only a part of the social value they create. They write: "[...] the gain of the society is given by* $\partial Y_t / \partial A_t = \sigma Y_t / A_t$*, [where* $\sigma = 1/\rho - (1-\alpha)$*]" (p. 72).*[5] *The social value they compute corresponds only to the first term of (15), that is to say* v_{Yt}*. In fact, they neglect the surplus generated in the R&D sector: innovators stand on the "Shoulders of Giants" (Scotchmer (1991)).*

Remark 4 *: the standard Industrial Organization Theory considers the same type of formula: the social value of an innovation is generally the net surplus under the demand curve of the intermediate private good that embodies this innovation.*

In fact, we can verify that the two approaches lead to the same expression. Indeed, integrating (8) between 0 and $x_t(i)$ and using the fact that $x_t(i) = x_t$ for all $i \in [0, A_t]$, one gets the gross surplus $Y_t / (\rho A_t)$. Then, subtracting the price $q_t x_t = (1-\alpha) Y_t / A_t$ gives the net surplus $\sigma Y_t / A_t$, which is exactly the expression of Jones and Williams (2000), that is to say only the first term of the social value given by (15).

Remark 5 *: in sub-section 3.2 below, we recall the standard equilibrium generally studied in endogenous growth theory from the seminal papers of Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and which is also used Jones and Williams (2000). With respect to the benchmark equilibrium, there are two basic assumptions: first,*

[5] We have copied the exact sentence that we can find in the original paper of the authors. However, note that there is a mistake here since $\partial Y_t / \partial A_t = Y_t / (\rho A_t) \neq \sigma Y_t / A_t$. Then, the reader must read $\partial \pi_{Yt} / \partial A_t = Y_t / (\rho A_t)$ instead of the one given in the text.

innovations, i.e. ideas, are not priced; second, innovators cannot practice a first degree price discrimination. Due to incompleteness of markets, the second term of equation (15), $v_{At} = \phi R_t / (\lambda A_t)$, *disappears. Moreover, since there is not any price discrimination, the market value of an innovation reduces to a share of* $v_{Yt} = \sigma Y_t / A_t$. *For instance Jones and Williams (2000) explain that innovators appropriate only a part of the social value they create; they write, "Inventors appropriate the profit flow* $(\kappa - 1)(1 - \alpha)\sigma Y_t/(\kappa A_t)$ *[...]" (p. 65), where* κ *is a constant in their paper.*

3.2 Distortions

The standard literature exhibits several distortions that prevent the decentralized equilibrium to implement the first best optimum. Jones and Williams (2000) count four distortions: first, the duplication effect (the R&D sector does not account for the redundancy of some research projects); second, the intertemporal spill-over effect (innovators do not account that ideas they produce are used to produce new ideas; third, the appropriability effect (innovators appropriate only a part of the social value they create); fourth, the creative-destruction effect, that we have not taken into account in this paper.

The analysis of the benchmark equilibrium shows that, with complete markets, perfect competition in all private goods markets (in particular the market of intermediate goods), and innovations funded at their Lindahl prices level, the only remaining distortion is the redundancy in research: see Proposition 2. The other distortions, described by Jones and Williams (2000), are only relevant in a second best equilibrium context, i.e. in an equilibrium in which innovations are not priced (markets are incomplete), and patents are given to intermediate producers that behave monopolistically. The reasons of this result are the following. Since, innovators are directly rewarded by each agent using their discoveries, they take into account that the ideas they produce are used by other researchers to produce new ideas. Thus, the knowledge spill-over effect is removed. Concerning the appropriability of the surplus created by the production of a new idea, since innovators are able to extract the willingness to pay from agents that use their discoveries, they appropriate the total amount of the social value they create: the appropriability of the surplus is not a remaining distortion.

In the standard equilibrium generally studied in growth theory, for instance by Jones and Williams (2000), ideas are not directly priced. Research is indirectly funded by the monopoly profits on intermediate goods that embody these ideas. These assumptions have two consequences. First, the incompleteness of markets explains the intertemporal spill-over distortion: it well-known that it can be corrected by a subsidy to research (see for instance Barro and Sala-I-Martin (1995), chapter 6). Second, the monopolies on intermediate goods (and not on ideas) explain the appropriability effect: this distortion can be removed by a subsidy on each intermediate demand (see also for instance Barro and Sala-I-Martin (1995)).

3.3 Insufficient research: Why?

In this section, we discuss about the empirical observation suggesting that investments in research are insufficient. In the standard literature on endogenous growth, the question of over or under-investments in research is often asked. According to empirical data, the actual amount of resources devoted to research is insufficient.

On a theoretical side, R&D-based models predict that we may obtain either an excessive or an insufficient allocation of resources in research. This well-known result in the vertical differentiation class of models developed by Grossman and Helpman (1991a) and Aghion and Howitt (1992), has also been obtained by Benassy (1998) in a model "à la Romer (1990)".[6] The question we ask is the following: what is the type of equilibrium considered? In their framework, Jones and Williams (2000) argue that "the main force promoting under-investments is the consumer appropriability problem" (p. 76). As shown in the two previous sub-sections, their analysis is made in a second best equilibrium context, since markets are incomplete. The basic argument is that, since they are unable to practice a first degree price discrimination, the sellers of intermediate goods extract only a part of the surplus generated by these goods (see sub-section 3.1).

Here, we propose an explanation in a perspective of complete markets. Let us consider an economy in which innovations are directly priced: see examples in the introduction. Assume that each innovator licenses his innovation, but that he is not able to extract the whole total surplus given by (15): he faces, for instance, verification, exclusion or informational problems. Our explanation is based on the public nature of innovations. It is not based on the gross markup over marginal cost set to sell an intermediate good as explained by Jones and Williams. Formally, the instantaneous gain from the renting of an innovation is now given by $v_t^F = \eta_Y v_{Yt} + \eta_A v_{At}$, where $\eta_k \in [0,1]$, $k = Y, A$, represent the constant and exogenous shares of the willingnesses to pay that are extracted. The index "F" means fraction.

To facilitate the comparison between the results of this section with those of the optimum, we assume that the government intervenes to remove the redundancy in research. Using the same method than in the preceding section, we can show that the growth rates and the real interest rate keep the same values. However, the share of output devoted to research is now given by

$$s^F = \frac{\eta_Y \sigma g_A}{(1+\tau)\,(r-(g_Y-g_A)-\eta_A \phi g_A)}. \tag{16}$$

[6] Benassy shows that if we modify slightly the technology of the output sector in a model with horizontal differentialtion, it is possible to obtain either an excessive or a too low investment in research compared to the optimum, inducing a too low or a too high long-run growth rate.

From (16), we note that if the duplication effect in research is removed (i.e. $1/(1+\tau) = \lambda$), $s^F \leq s^o$ is always true, whatever the values of η_Y and η_A. As a consequence, if innovators cannot extract the total surplus from agents that use their discoveries, then investments in research are insufficient. The fact that $\partial s^F/\partial \eta_k > 0$, $k = Y, A$, means simply that the amount of resources devoted to research increases with the part of the social value that the R&D sector can appropriate. This result is consistent with Jones and Williams (2000) who explain: "Calibrating the model to micro and macro data, we find that our decentralized economy typically under-invests in R&D relative to what is socially optimal. This is true unless the equilibrium real interest rate is relatively high and the magnitude of duplication effects is simultaneously large" (p. 80).

Note that if the government does not intervene (i.e. if $\tau = 0$), then it is possible to obtain either $s^F < s^o$, or $s^F > s^o$. That is to say, investments in research can be either too high or insufficient compared to the optimum. The reason is the following. There are two driving forces that go in opposite directions and their combination has an ambiguous effect on the final allocation of resources in research. On the one hand, the duplication effect tends to increase the share of output devoted to research. On the other hand, the fact that researchers cannot extract the entire amount of the social value they create leads to a decrease in the share of output devoted to research. When the duplication effect dominates, we obtain an excessive share allocated to research. If not, the reverse happens.

4 Conclusion

In this paper, we made a synthesis between the standard literature on endogenous growth, and the literature (until now, non-formalized) on the funding of innovations developed by authors like, for instance, Arrow (1962), Tirole (1988), Scotchmer (1991, 1999), Dasgupta et *al* (1996), Gallini and Scotchmer (2003). We have distinguished the innovations (public goods) produced by the R&D sector, from the private intermediate goods that embody them. In this case, we have defined a price for both types of goods.

The synthesis of the two above literatures allowed to shed a new light on several questions. First, bringing together the benchmark equilibrium and the first best optimum, we have computed the social value of an innovation. It is the sum of the surpluses of all agents that use it in the economy. This formula allows to recover ideas already expressed, for instance by Scotchmer (1999) and Gallini and Scotchmer (2003). This formula is different from the one computed by Jones and Williams (2000); it is also different from formulas usually used in Industrial Organization Theory. Second, we examined the questions related to the nature of the distortions that prevent the decentralized economy to achieve the optimum: except for the duplication effect in research, we have shown that the distortions discussed by endogenous growth theorists are relevant only in a second best context, i.e. when ideas are not priced. Third, we have given an alternative interpretation to the insufficient investments that are empirically observed in research. Using an equilibrium with complete markets, we have explained that excludability, verifiability and informational problems can prevent innovators to appropriate the total amount of the surplus they create. That is, our interpretation is based on the public good nature of innovations themselves rather than on the existence of potential distortions which are relevant in some equilibria but not in others.

5 Appendix

5.1 Characterization of the benchmark equilibrium

We focus on steady-states. First, we compute the value of the growth rates of quantities. To do it, we use (3), (5), the conditions (7), (8), (11), (14) and the property of symmetry of intermediate goods in the final sector that implies $x_t(i) = x_t$ for all $i \in [0, A_t]$, $Y_t = (A_t)^{1/\rho} (L_t)^{\alpha} (x_t)^{(1-\alpha)}$ (see (1)), and $K_t = A_t x_t$ (see (2)). Then, one can show that $g_Y = g_C = g_R = g_K = \sigma g_A/\alpha + n$, where $\sigma = 1/\rho - (1-\alpha)$, and $g_A = \lambda n/(1 - \phi - \lambda\sigma/\alpha)$ (see also Jones and Williams (2000)).

Now, we determine the value of the share of resource, $s = R_t/Y_t$, devoted to research at steady-state. As mentioned in the text, in this case, the tax-rate must be constant: we write $\tau_t = \tau$ for all t. Using (7) and (11), we obtain $\dot{V}_t/V_t = g_Y - g_A$. Using (7), (8) and the willingnesses to pay of agents to use an innovation ((9) and (13)), we obtain $v_t/V_t = g_A\sigma/[(1+\tau)s] + g_A\phi$. Combining the two previous results with the fact that $r_t = \dot{V}_t/V_t + v_t/V_t = \dot{u}'/u' + \theta + n$, (see equation (14)), one gets the value s^* given in Proposition 1.

The prices are given by equations (7) to (14).

5.2 Characterization of the steady-state optimum

The social planner's problem is to maximize (6) subject to (1) to (3). Using the property of symmetry of intermediate goods in the final sector, the Hamiltonian of this problem is

$$\begin{aligned}\Gamma &= u(C_t/L_t)e^{-\theta t} + \mu_t\left[(A_t)^{1/\rho}(L_t)^{\alpha}(x_t)^{(1-\alpha)} - C_t - R_t\right] \\ &\quad + \nu_t\delta(R_t)^{\lambda}(A_t)^{\phi}/(1+\psi) + \xi_t(K_t - A_t x_t).\end{aligned}$$

The first order conditions are: $u'(C_t/L_t)e^{-\theta t} = L_t\mu_t$ (a), $\mu_t(1-\alpha)Y_t/x_t = \xi_t A_t$ (b), $\mu_t = \nu_t\lambda\dot{A}_t/R_t$ (c), $\xi_t = -\dot{\mu}_t$ (d), $\mu_t Y_t/(\rho A_t) - \xi_t x_t + \nu_t\phi\dot{A}_t/A_t = -\dot{\nu}_t$ (e).

Assume that the economy is at steady-state. In this case, we know that the growth rates are constant and have the same value than in the case of the benchmark equilibrium (the model is semi-endogenous).

Differentiating equations (a) and (c) with respect to time yield $\dot{u}'/u' + \theta + n = -\dot{\mu}_t/\mu_t$ (f) and $g_{\mu t} = g_{\nu t} - g_{Rt} + g_{At}$ (g). Combining equations (b) and (e), one gets $\mu_t Y_t[\rho - (1-\alpha)]/A_t + \nu_t \phi \dot{A}_t/A_t = -\dot{\nu}_t$; dividing each side by ν_t, and putting μ_t/ν_t in factor, one obtains $\mu_t\{Y_t[\rho - (1-\alpha)]/A_t + (\nu_t/\mu_t)\phi \dot{A}_t/A_t\}/\nu_t = -\dot{\nu}_t/\nu_t$. Finally, using (c), (f) and (g), one gets

$$\frac{\lambda \dot{A}_t}{R_t}\left\{\sigma \frac{Y_t}{A_t} + \frac{\phi}{\lambda}\frac{R_t}{A_t}\right\} + g_Y - g_A = \frac{\dot{u}'}{u'} + \theta + n. \tag{17}$$

where $\sigma = 1/\rho - (1-\alpha)$. Simplifying the expression of (17), we obtain $s^o = \sigma\lambda g_A/[\gamma(g_C - n) + \theta + n - (g_Y - g_A) - \phi g_A]$ (see also Jones and Williams (2000), p. 72).

6 References

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