

Some Perils of Policy Rule Regression

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Abstract

This paper shows that an estimated policy rule under a model with exogenous policy leads to indeterminacy when placed in the original model. Moreover, we stress the potential policy misidentification involved by this methodology. We illustrate this finding with a monetary model with exogenous money growth rule. An estimated Taylor type rule under this structural model leads to indeterminacy and sunspot equilibria. We conclude that the use of policy rule regression for economic policy evaluation can lead to spurious recommendations.

Keywords: exogenous and endogenous policy rules, policy rule regression, limited- and full-information estimation methods, indeterminacy

JEL Class.: C22, E40, E52

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Introduction

“GMM approaches based on Euler equations were designed to deliver only part of an economic model. Their virtue and liability is that they are based on partial specifications of an econometric model [...] This is both a strength and a weakness [...] [They] allow an econometrician to learn about something without having the need to learn about everything [...] [H]owever, [they] limit the questions that can be answered by an empirical investigation. For instance, the analysis of hypothetical interventions or policy changes typically requires a fully specified dynamic, stochastic general equilibrium model.”

Lars Peter Hansen (2002)

The aim of this paper is to point out some perils¹ of policy rule regression, especially when the regression is conducted from a single equation. As L.P. Hansen states, in order to fully recognize the effects of any economic policy, a complete dynamic exercise within a general equilibrium model must be performed. Disregarding a complete modelling may lead to spurious conclusions about the effects of economic policy.

In recent years, there has been an increasing interest about the identification of policy rules that the economic authorities are assumed to follow. Notably, applied monetary economics estimates single equations using instrumental variables in order to describe the reaction function of the central bank, which is assumed to respond to changes in expected inflation and the output gap away their targets. For example, Clarida, Galí and Gertler (2000) estimate a monetary policy rule using GMM and document how the change in their estimation accounts for economic (monetary) stability.

The work of Clarida, Galí and Gertler has been criticized by recent contributions as it does not pay sufficient attention to the fact that their single equation would be probably part of a system (see Linde (2002), Lubik and Schorfheide (2004)). Moreover, a single equation approach suffers from a lack of identification (especially when weak instruments are used), contrary to full-information environments (see Mavroeidis (2004a), (2004b), Nason and Smith (2003)). Another problematic issue is presented in Beyer and Farmer (2003, 2004), where it is shown that a single equation approach tell us nothing about the real nature of aggregate fluctuations, *i.e.*, a given policy may be either associated to

¹The reader could easily find some analogy with the paper of Benhabib, Schmitt-Grohe and Uribe (2001b).

fundamental shocks or to sunspot shocks.

In this paper, we present two perils of policy rule regression. The first one is that nothing ensures that the estimated parameters could represent a unique type of structural behavior of the agents. These estimates may correspond only to aggregate co-movements that characterize the equilibrium of the economy. In this case, the estimated policy rule is just a reduced form of a model where the empirical parameters are not policy invariant, thus illustrating the famous *Lucas critique*. However, this first peril is not so problematic if the estimated policy rule is not taken too seriously, *i.e.*, if the estimation is considered only as a useful description of the data. The second – and more important – peril comes when the estimated rule is taken seriously and becomes a policy recommendation. In this case, the estimated rule when placed in the original model creates indeterminacy and sunspot fluctuations.

In order to illustrate these two types of perils, we consider a simple linear rational expectations model with an exogenous policy rule. The stationary solution will be given by a linear relation between the endogenous variable and the exogenous one, that can be stated or represented in terms of any of them. Suppose now that an econometrician wants to estimate a policy rule using actual data (generated by the model with exogenous policy) on these two variables. She could believe that the estimated policy rule is *endogenous*, not paying attention to the true (exogenous) character of the policy function. If this estimation is interpreted as the structural behavior of policy-makers, this will constitute the first peril. Moreover, when the econometrician uses the estimated policy function in order to study the implied aggregate dynamics of the model economy, our results show immediately that the equilibrium is indeterminate and that aggregate fluctuations can be driven by sunspots. This is the second (and more serious) peril of policy rule regression.

We then extend this simple model in order to account for measurement errors or multiple exogenous variables. It turns out that if the measurement errors (or the additional exogenous variables) explain a large part of the variations of the endogenous variable, we have no longer indeterminacy, since the policy variable does not play any role at equilibrium.

Finally, we consider an illustration of these two perils within the Taylor rule regression that is performed in applied monetary economics. Specifically, we consider an exogenous money growth rule together with technological and government spending shocks in a standard cash-in-advance (CIA) economy. A calibration exercise shows that the estimated

parameters of the Taylor type rule are similar to those founded in the empirical literature for the U.S. economy. A quantitative exercise shows that the policy rule regression principle is not sufficiently informative about the structural behavior of the Federal Reserve. Moreover, the estimated Taylor type rule leads to real indeterminacy.

The paper is organized as follows. Section 1 presents a simple rational expectations model with an exogenous policy rule and the principle of policy rule regression with a forward-looking policy. Conditions for indeterminacy under the estimated policy function are presented. Section 2 covers the extensions to account for measurement errors and multiple exogenous variables. Section 3 contains the Taylor rule's application. Finally, the last section presents some concluding remarks.

1 The model and Policy Rule Regression

We first briefly present a simple rational expectations model. We then discuss the principle of policy rule regression and the implied aggregate dynamics with the estimated policy rule.

1.1 The model

We consider a representation of an economy that expresses a single endogenous variable y_t in period t as a linear function of the conditional (on the available information at time t) expectation of this same variable in period $t + 1$ and a single exogenous variable x_t . The model is deliberately stylized in order to deliver clear results about the policy rule regression. This economy takes the form:

$$y_t = aE_t y_{t+1} + b x_t \tag{1}$$

where $b \neq 0$ and $|a| < 1$. When x_t is purely exogenous, this last assumption implies that the rational expectations model is determinate. A stationary solution would be thus obtained iterating (1) forward.² E_t is the conditional expectation operator with respect to the current and past values of $\{x_t, x_{t-1}, \dots\}$. Note that we use here the minimal information set sequence, but it has no consequences as the inclusion of the current and past values of $\{y_t, y_{t-1}, \dots\}$ in the information set is redundant and extraneous processes like sunspots do not matter in the determinate case. The parameters a and b are given scalars that are supposed to be known. In what follows, we are not concerned with the identification

²Following Gourieroux, Laffont and Monfort (1982), among others, this simple model has been widely used in order to study the solutions of linear rational expectations models (see e.g. Blanchard and Fisher (1989), Broze and Szafarz (1991) and Farmer (1999)).

and estimation of these two parameters, but only with the quantitative implications of the solution for policy rule regression. We consider that x_t follows an exogenous rule of the form:

$$x_t = \rho x_{t-1} + \sigma \varepsilon_t \quad \text{with } \varepsilon_t \sim iid(0, 1) \quad (2)$$

where $|\rho| < 1$ and $\sigma > 0$. The assumption that x_t is AR(1) is widely used in macroeconomic dynamics. It allows to simply represent the time series behavior of the forcing variable. Moreover, one must remark that x_t can be correlated with y_t . In this case, we have just to modify (1), taking into account the feedback effect of x_t on y_t . The AR(1) specification then represents the pure exogenous component in x_t .

Without loss of generality, we omit a constant term both in (1) and (2) since the endogenous and exogenous variables can be considered as deviations from their average value. Equation (1) defines either a linear or a log-linear representation of the equilibrium conditions in a dynamic model. Moreover, equation (1) can represent either a linear or a log-linear approximation to the equilibrium conditions of a non-linear model. Despite its simplicity, this simple linear representation embodies several model economies: the Cagan model, the monetary model of exchange rate determination, asset pricing models, dynamic factor demand models, business cycle models, monetary models with a cash-in-advance constraint³, and so on.

Using forward substitutions, the endogenous variable is expressed by

$$y_t = bE_t \sum_{i=0}^T a^i x_{t+i} + a^{T+1} E_t y_{t+T+1}$$

Excluding explosive paths, *i.e.*,

$$\lim_{T \rightarrow \infty} a^{T+1} E_t y_{t+T+1} = 0$$

the forward solution takes the form:

$$y_t = bE_t \sum_{i=0}^{\infty} a^i x_{t+i}$$

Using the exogenous policy rule (2), the solution is

$$\begin{aligned} y_t &= b \left(\sum_{i=0}^{\infty} a^i \rho^i \right) x_t \\ &= \frac{b}{1 - a\rho} x_t \quad \forall t \end{aligned} \quad (3)$$

This reduced form expresses the endogenous variable y_t as a linear function of the exogenous variable x_t . This solution shows that when the parameter ρ of the exogenous policy

³See section 3 for an illustration

rule changes, the reduced form parameter $b/(1 - a\rho)$ will be evidently affected. The lack of policy invariance of the reduced form parameter is a basic illustration of the *Lucas critique*. Note also that the reverse representation of (3) also holds for each period:

$$x_t = \frac{1 - a\rho}{b}y_t \quad \forall t \quad (4)$$

This latter equation that expresses the exogenous variable as a linear function of the endogenous variable is on the basis of our demonstration of the perils of policy rule regression that we will discuss further.

1.2 Policy Rule Regression and aggregate dynamics

We now present the principle of policy rule regression. The estimation method that we use follows closely the usual practice of many empirical studies which aim to identify and estimate policy rule parameters.

Suppose that an econometrician wishes to estimate a relationship between the forcing and the endogenous variables. She assumes that the policy rule takes the form:

$$x_t = \eta E_t y_{t+1} \quad (5)$$

This type of policy rule, extensively used in applied monetary economics (see e.g. Clarida, Galí and Gertler (2000)), sets down that the policy variable x_t reacts to expected changes of the endogenous variable.⁴ Note that this equation can be expressed in terms of observed variables:

$$x_t = \eta y_{t+1} + \varepsilon_{t+1}^y \quad (6)$$

where $\varepsilon_{t+1}^y = \eta(E_t y_{t+1} - y_{t+1})$, and $E_t \varepsilon_{t+1}^y = 0$. An empirical issue concerns the econometric method that the practitioner can use in order to identify and estimate the policy parameter η . A full information estimation method requires to specify the complete model, to find out its solution and compute its reduced form. The advantage of such approach is that the policy equation is determined consistently as the parameters a , b and η are estimated using the cross-equation restrictions created by the structural model. Nevertheless, this approach faces some limits, as it implies to *correctly* specify the whole model. The estimation of the parameters is, therefore, model dependant.⁵ To a certain

⁴We have also investigated the estimation of static and backward-looking policy rules. The results about the estimated policy rule and the implied aggregate dynamics are rather similar. See the appendix A–B for more details.

⁵Moreover, the feedback effect of the policy rule (5) implies that the dynamic properties of the equilibrium will depend on the (unknown to the econometrician) policy parameter η , given the values of a and b in (1). Depending on the value of η , the equilibrium can be either determinate ($|a + b\eta| < 1$) or indeterminate ($|a + b\eta| > 1$), leaving a room for sunspot disturbances in this latter situation. The stochastic dimension will thus differ: When the equilibrium is determinate, equations (1) and (5) will lead to a regular solution, whereas when the equilibrium is indeterminate, the reduced form will be stochastically driven by a sunspot shock.

extent, this is why most of empirical studies prefer to use a limited information estimation method. The main advantage of such method is that the econometrician does not have to specify the complete Data Generating Process (DGP, hereafter). So, she does not have to formulate arbitrary assumptions about functional forms, market arrangements, constraints and the distribution of shocks, in order to infer something about the policy parameters (see Hansen (2002) for a short discussion).

In order to estimate the policy function (5), the econometrician uses a set of instrumental variables that are assumed to be sufficiently informative about the policy behavior. In order to avoid the endogeneity problem, empirical studies use a set of predetermined – or weekly exogenous – instrumental variables. We follow here exactly the same empirical strategy. For simplicity and tractability, we assume that the econometrician uses a single instrument. Since we are interested in the co-movements between x_t and y_t , the necessary condition for identification of the parameter η is fulfilled. Let z_t denote a single instrument known in period t . This instrument verifies the following orthogonality condition

$$E(\varepsilon_{t+1}^y z_t) = 0$$

or equivalently

$$E((x_t - \eta y_{t+1}) z_t) = 0 \tag{7}$$

Equation (7) is the basis of the GMM estimation (or IV estimation in this simple case) of the parameter η . As the number of orthogonality conditions is equal to the number of parameter of interest, it follows that the GMM estimator is free from any weighting matrix and can be obtained directly from (7). As previously mentioned, the instrument is commonly a pre-determined variable, so let us assume that $z_t = y_{t-1}$. The orthogonality condition (7) becomes :

$$E((x_t - \eta y_{t+1}) y_{t-1}) = 0 \tag{8}$$

From (8), the GMM estimator $\hat{\eta}$ of η is given by :

$$\hat{\eta} = \frac{Cov(x_t, y_{t-1})}{Cov(y_{t+1}, y_{t-1})}$$

In what follows, we will use this formula in order to compute the parameter estimate of the policy function.⁶ Be aware on the fact that the econometrician observes data about x_t and y_t that are generated by (2) and (3), *i.e.* a model with an exogenous policy rule. Additionally, it is worth noting that the results are the same if we choose x_{t-1} as an

⁶Note that equation is obtained using covariances between x and y rather than expectations. The two formulations are strictly equivalent as the model does not introduce constant terms both in (1) and (2).

instrumental variable, since the reduced form (3) or (4) represents a one-to-one mapping. Given the solution (3) and the exogenous process (2), the GMM estimator $\hat{\eta}$ of the policy function parameter η is given by:

$$\begin{aligned}\hat{\eta} &= \frac{Cov(x_t, y_{t-1})}{Cov(y_{t+1}, y_{t-1})} \\ &= \frac{\left(\frac{b}{1-a\rho}\right) \rho V(x_t)}{\left(\frac{b^2}{(1-a\rho)^2}\right) \rho^2 V(x_t)} \\ &= \frac{1-a\rho}{b\rho}\end{aligned}\tag{9}$$

The estimated policy parameter $\hat{\eta}$ depends on the deep parameters (a, b) and the parameter of the forcing variable ρ . As previously mentioned, the lack of policy invariance of the estimated reduced form parameter provides an illustration of the *Lucas critique*. The limited information estimation method ignores the true DGP and the cross-equation restrictions created by the structural model. This estimated policy rule obviously does not represent any endogenous policy behavior, but rather it describes (perhaps usefully) an equilibrium relationship between the exogenous and the endogenous variables. This result represents the first peril of policy rule regression.

Figure 1 illustrates the GMM estimate (9) of the policy parameter. For the ease of the exposition, this figure shows the case of a bounded distribution of the innovation ε_t . The solid line represents the *expected* linear function between the variables x_t and y_{t+1} at the equilibrium of the model economy, *i.e.* $E_t y_{t+1} = \frac{b}{1-a\rho} \rho x_t$, implied by (2) and (3), or equivalently by (6) and (9). The shifts in the policy function are associated to the news in expectations, *i.e.* to the unexpected changes of the exogenous variable given by

$$y_{t+1} = \frac{b}{1-a\rho} (\rho x_t + \varepsilon_{t+1})$$

Thus, in the equilibrium path, x_t and y_{t+1} are related by the previous defined mapping, disturbed by the realizations of ε_{t+1} . This is a graphic illustration of the first peril, where an external observer could actually think that the policy variable x_t reacts to changes of $E_t y_{t+1}$, even though the true policy is exogenous.

For the second peril, we now explore the consequences of this misidentification of the policy rule regression – when considered as a recommendation – on the dynamic properties of the model economy. Given the estimated policy rule parameter (9) and the form of the policy rule (5), the dynamic properties of the economy is obtained after substitution of

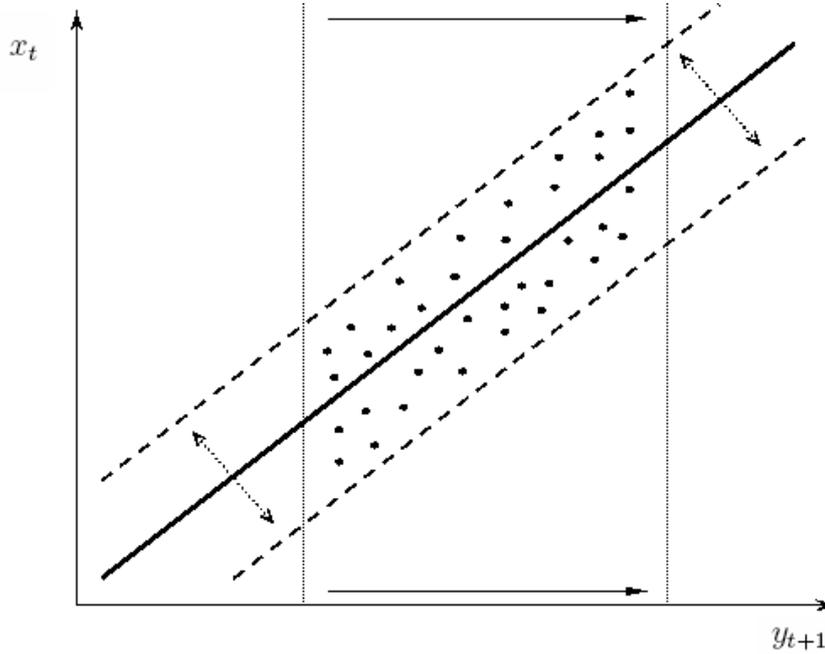


Figure 1: Policy rule regression

the estimated policy rule in (1):

$$y_t = aE_t y_{t+1} + \frac{1 - a\rho}{\rho} E_t y_{t+1}$$

This equation rewrites,

$$\rho y_t = E_t y_{t+1}$$

Since $|\rho| < 1$, *i.e.* the exogenous variable follows a stationary process, the equilibrium under the estimated (believed wrongly endogenous by the econometrician) policy rule is indeterminate. A forward-looking policy rule under the model with exogenous policy leads to indeterminacy and sunspot equilibria. Note that this property of indeterminacy holds even when $a = 0$, because the policy rule introduces an expectation about the endogenous variable of the next period. This additional result represents the second peril of the policy rule regression.

In order to better understand this finding, let us consider for simplicity the model economy when $a = 0$ and $b = 1$. In this case, the solution is simply $y_t = x_t$ and the estimated policy rule parameter corresponds to the linear projection of x_t on x_{t+1} . Given the stationary representation of the exogenous policy (2), the estimated policy rule parameter is $1/\rho$. As the reduced form is $y_t = x_t$, equation (1) rewrites $y_t = (1/\rho)E_t y_{t+1}$. For any stationary exogenous policy rule (2), the equilibrium is thus indeterminate.

2 Extensions of the Original Model

Most of dynamic models with rational expectations involves so numerous restrictions that it is costly to specify them correctly. Many researchers argue that such models are too stylized to explain many features of the data. One approach to solve this limitation is to augment the solution of the model with a serially error term. This error term accounts in part for measurement errors, omitted variables or incorrect specifications of the structural model. An alternative approach is to introduce additional structural disturbances in the model. The aim of this section is to question the robustness of our previous results when additional disturbances are considered.⁷ We first consider measurement errors and then we present the multiple variables case.

2.1 Measurement Errors

The solution is now augmented with an error term:

$$y_t = y_t^* + u_t \quad (10)$$

where y_t^* is given by (3) and the exogenous variable x_t follows the same stationary stochastic process (2). The error term u_t is assumed to be a stationary and serially correlated process:

$$u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_t^u \quad \text{with } \varepsilon_t^u \sim iid(0, 1) \quad \text{and} \quad |\rho_u| < 1 \quad (11)$$

This approach is used by many studies that attempt to estimate dynamic stochastic models (see Altug (1989), Mc Grattan (1994), Hall (1996), Mc Grattan, Rogerson and Wright (1997), Ireland (2004)). Following Sargent (1989), some studies interpret the error term as measurement errors, but it can be viewed as well as capturing the distance between the solution found by the researcher and the rational expectations solution embodied by the actual DGP.

Consider now the estimation of the policy function (5) when the data are generated according to (2), (3), (10) and (11), and provided that ε_t and ε_t^u are not correlated for all t . The GMM estimator $\hat{\eta}$ of η is given by:

$$\hat{\eta} = \frac{1 - a\rho}{b\rho} \nu \quad (12)$$

⁷We have also investigated the robustness of our results against other specifications of our simple model. For example, when equation (1) is replaced by $y_t = aE_t y_{t+1} + bE_t x_{t+1} \equiv aE_t y_{t+1} + b\rho x_t$, the quantitative implications for policy rule regression and aggregate dynamics are exactly the same. Moreover, if we consider $y_t = aE_{t-1} y_{t+1} + b x_t$ instead of (1), the results are left unaffected (see appendix C for more details).

with

$$\nu = \frac{(b^2/(1-a\rho)^2)\rho^2V(x_t)}{(b^2/(1-a\rho)^2)\rho^2V(x_t) + \rho_u^2V(u_t)},$$

where $V(x_t) = \sigma^2/(1-\rho^2)$ and $V(u_t) = \sigma_u^2/(1-\rho_u^2)$. It is worth noting that ν can be expressed as well in terms of the conditional variance of y_t on the exogenous variable and the error term, since $V(y_t/x_t) = (b^2/(1-a\rho)^2)V(x_t)$ and $V(y_t/u_t) = V(u_t)$. If the contribution of x_t to the variance of y_t is large (*e.g.*, the measurement errors are small, or the model is roughly well specified), then we have $\nu \simeq 1$ and the GMM estimator is very close to the single exogenous variable case. Note that when the error term u_t is *iid* ($\rho_u = 0$) and/or the variance of this term is zero ($\sigma_u = 0$), we have $\nu = 1$ and we retrieve the GMM estimator of the single exogenous variable case. Conversely, if the measurement errors explain a large part of the variance of y_t , specifically, if $\rho_u^2/\rho^2V(y_t/u_t) \gg V(y_t/x_t)$, we have $\nu \simeq 0$ and thus $\hat{\eta} \simeq 0$.

This latter situation is illustrated by figure 2 where the shifts in the policy rule are the result of two shocks, where those of the measurement errors are more important. The large band represents the expected – conditional on u_t – equilibrium relationship between x_t and y_{t+1} given by

$$E_t y_{t+1} = \frac{b}{1-a\rho} \rho x_t + \rho_u u_t$$

This band incorporates the fact that u_t is a persistent ($\rho_u \neq 0$) stationary process. The innovations of the policy function are bounded by the support of the shock term $\frac{b}{1-a\rho} \varepsilon_{t+1} + \varepsilon_{t+1}^u$. Finally, the horizontal line is the policy rule regression with an estimated value $\hat{\eta}$ of η close to zero. In this case, the econometrician could believe that the policy variable x do not react on average to an expected change of the endogenous variable y . In fact, even when there exist an equilibrium relation between these two variables, it is heavily weakened by the importance of the measurement errors in the model.

We now study the implied dynamics of the model economy. After the substitution of (5) and (12) into (1) and (10), the dynamics of the economy is given by:

$$y_t = aE_t y_{t+1} + \nu \left(\frac{1-a\rho}{\rho} \right) E_t y_{t+1} + u_t$$

The last term is unimportant for the dynamic properties as it involves an exogenous stationary variable. It follows that if

$$\left| \frac{\rho}{a\rho + (1-a\rho)\nu} \right| < 1$$

the equilibrium is indeterminate. When $\nu = 1$, *i.e.* when the exogenous variable x_t totally accounts for the variance of the endogenous variable y_t , we retrieve the previous result

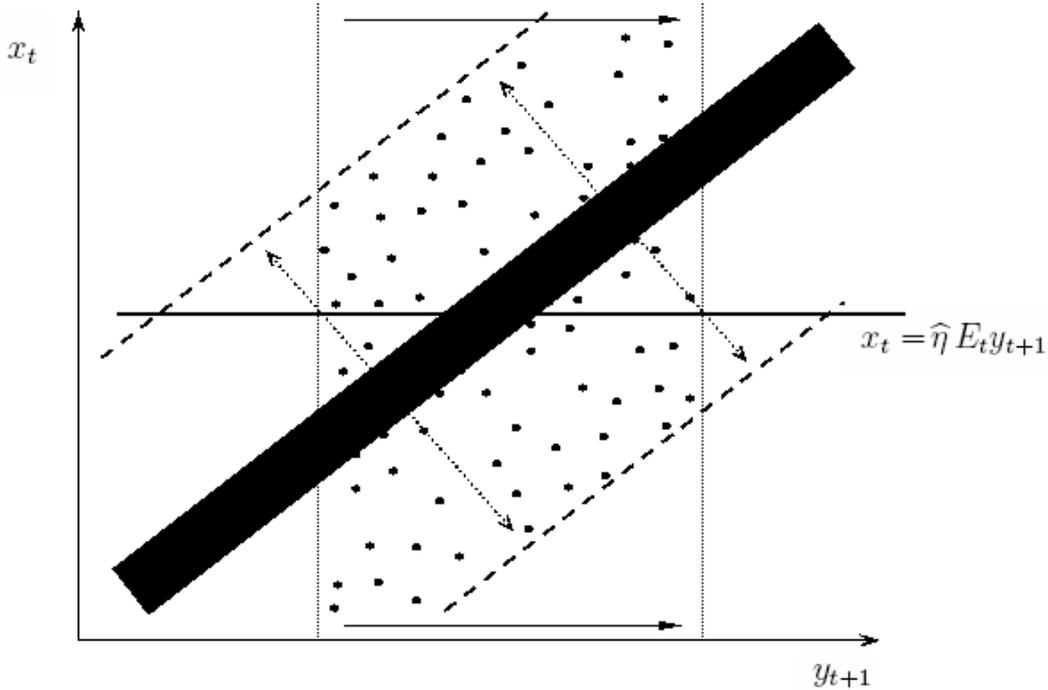


Figure 2: Policy rule regression with measurement errors

that policy rule regression implies indeterminacy. Conversely, when $\nu = 0$ (or equivalently $\hat{\eta} = 0$), *i.e.* when the variance of the endogenous variable y_t is only explained by the error term u_t , indeterminacy occurs less likely as $y_t = aE_t y_{t+1} + u_t$ and $|a| < 1$.

2.2 Multiple exogenous variables

Most of macroeconomic models include several shocks – productivity, government spending, tastes, money supply – in order to improve the specification of the endogenous variable.⁸ Moreover, a typical exercise in the business cycle literature is to identify the various sources of aggregate fluctuations and thus to evaluate their relative contribution. We thus extend the previous model to the case of multiple exogenous variables:

$$y_t = aE_t y_{t+1} + \sum_{j=1}^m b_j x_{j,t} \quad (13)$$

where as before $|a| < 1$ and each exogenous variable $x_{j,t}$ follows an $AR(1)$ process

$$x_{j,t} = \rho_j x_{j,t-1} + \sigma_j \varepsilon_{j,t} \quad \text{with } \varepsilon_{j,t} \sim iid(0, 1) \quad (14)$$

where $|\rho_j| < 1$ and $\sigma_j > 0$. We assume that the innovations verify

$$E(\varepsilon_{j,t} \varepsilon_{j',t}) = 0 \quad \forall j \neq j'$$

⁸This section is very similar to the previous one, as the measurement error is replaced by additional shocks with the same dynamic properties.

So, the exogenous variables are uncorrelated. Note that the information set includes now the present and the past values of all these sequences. From the independence of the exogenous variables, if y_t^j for $j = 1, \dots, m$ is a stationary solution of

$$y_t^j = aE(y_{t+1}^j/x_t^j, x_{t-1}^j, \dots) + b_j x_t^j$$

then $y_t = \sum_{j=1}^m y_t^j$ is a stationary solution of (13). Iterating (13) forward and using the exogenous policy rules (14), the solution is given by

$$y_t = \sum_{j=1}^m \frac{b_j}{1 - a\rho_j} x_{j,t} \quad (15)$$

Now, suppose that the econometrician does the same job as before. She estimates an endogenous policy function for one variable among the m ones. Let $x_{1,t}$ be the selected variable, so the policy function takes the form

$$x_{1,t} = \eta E_t y_{t+1} \quad (16)$$

The GMM estimator $\hat{\eta}$ of η is, given that the instrumental variable is y_{t-1} :

$$\hat{\eta} = \frac{b_1 \rho_1 / (1 - a\rho_1) V(x_{1,t})}{\sum_{j=1}^m b_j^2 \rho_j^2 / (1 - a\rho_j)^2 V(x_{j,t})}$$

where $V(x_{j,t}) = \sigma_j^2 / (1 - \rho_j^2)$ for $j = 1, \dots, m$. This estimator rewrites

$$\hat{\eta} = \frac{1 - a\rho_1}{b_1 \rho_1} \nu \quad (17)$$

where

$$\nu = \frac{V(y_t/x_{1,t})}{V(y_t/x_{1,t}) + \sum_{j=2}^m (\rho_j/\rho_1)^2 V(y_t/x_{j,t})}$$

Since $V(y_t/x_{j,t})$ denotes the variance of y_t conditional on $x_{j,t}$, ν can be interpreted as the weighted contribution of $x_{1,t}$ to the variance of y_t . This is clear when all the weights are the same, *i.e.* when $\rho_1 = \dots = \rho_m$ and ν simply reduces to

$$\nu = \frac{V(y_t/x_{1,t})}{V(y_t)}$$

If the contribution of $x_{1,t}$ to the variance of y_t is large, we have $\nu \simeq 1$ and the GMM estimator is the same as the one obtained in the single exogenous variable case. Moreover, when $\rho_j = 0$ or $\sigma_j = 0$ for $j = 2, \dots, m$ we have $\nu = 1$ and we retrieve the GMM estimator of the single exogenous variable case. Conversely, if the contribution of $x_{1,t}$ to the variance of y_t is very small, we have $\nu \simeq 0$ and $\hat{\eta} \simeq 0$ (see figure 2 for an illustration).

We now study the implied dynamics of the model economy. After the substitution of (16) and (17) into (13), the dynamics of the economy is given by:

$$y_t = aE_t y_{t+1} + \frac{1 - a\rho_1}{\rho_1} \nu E_t y_{t+1} + \sum_{j=2}^m b_j x_{j,t}$$

The last term is unimportant for the dynamic properties as it involves stationary exogenous variables. It follows that if

$$\left| \frac{\rho_1}{a\rho_1 + (1 - a\rho_1)\nu} \right| < 1$$

the equilibrium is indeterminate. The interpretation is the same as before. It is worth noting that if $V(y_t/x_{1,t}) \rightarrow 0$, indeterminacy occurs less often, since $x_{1,t}$ does not play an important role in the determination of y_t .

Another aspect of policy rule regression in the case of multiple exogenous variables concerns the choice of the instrument. First, if the econometrician uses one of the other exogenous variables $x_{j,t}$ for $j = 2, \dots, m$ as instrument, then the GMM estimator becomes zero. This results from the independence of the exogenous variables. In such a case, the lack of feedback effects leaves unaffected the dynamic properties of the original model. Second, if she uses the exogenous variable $x_{1,t}$ itself as instrument, the IV estimator is the same as in the model with a single exogenous variable. In this case, indeterminacy occurs as $|\rho_1| < 1$. It follows that the property of indeterminacy of the the estimated policy rule is instrument dependant.

3 An Application: The Estimated Taylor Rule

Since Taylor (1993), abundant empirical evidence and some theoretical explorations have defended the use of the so called interest rate policy rule, in order to control prices and stabilize the economy. Further, it is *implicitly* assumed in applied monetary economics that an active interest rate rule, *i.e.*, a raise of the nominal interest rate by more than a one-by-one basis to an increase of expected inflation, must rely naturally on an accommodating money supply. This rule has been characterized as *endogenous*, where it is thought that the central bank systematically fights excessive inflation by manipulating its instruments. Therefore, an increasing literature, notably Taylor (1999) and Clarida, Galí and Gertler (2000), identifies these rules as good and stabilizing policies, or fine examples of *leaning against the wind* by central banks.

However, some sceptics have argued that the estimation of such rules might not provide sufficient information about the true behavior of policy-makers. Remarkably, Hetzel

(2000) criticizes the Taylor principle in that the estimated rules describe rather the final interest rate equilibrium relationship, and not the central bank's performance. This is, an inflation coefficient above one might not necessarily entails an aggressive policy nor a good policy, but could also be the result of a pure reaction of the agents given their expectations.

A minor interest has been left to discussion about the problems that may arise when there is a misidentification of monetary policy given these interest rate rules. Our claim in this section is to show how, within a simple framework, this misidentification can actually occur when the Taylor regression is considered. We assume explicitly an *exogenous* monetary policy rule and we let an econometrician to infer the true behavior of monetary authorities.

3.1 A Monetary Model

Consider an economy⁹ composed by a unit mass of infinite-lived expected utility maximizer agents, indexed by $i \in [0, 1]$. There is a competitive final-good sector, which is labor intensive. For the ease of the analysis, let us assume that there is no capital accumulation. Additionally, there exist a government who collects lump-sum taxes, consumes the final good, and provides money, bonds and lump-sum transfers to the agents in the economy.

Households

A representative household carries from period $t - 1$ the nominal balances M_t , and the total revenues from bond holdings $R_{t-1}B_t$, where R is the *gross* nominal interest rate. In period t , the household has a disposable nominal income $W_t h_t - T_t$ per h_t worked hours, where W_t is the nominal wage and the T_t denotes lump-sum taxes. Additionally, the household receives from the government the lump-sum transfer N_t . All the revenues are used to buy the consumption good, C_t , money balances and bonds for the next period. Consequently, its budget constraint is

$$B_{t+1} + M_{t+1} + P_t C_t + T_t \leq W_t h_t + R_{t-1} B_t + M_t + N_t \quad (18)$$

The household holds cash in order to buy the consumption bundle and the bond holdings in period t . Therefore, the CIA constraint has the form

$$P_t C_t + T_t + B_{t+1} \leq M_t + R_{t-1} B_t + N_t \quad (19)$$

⁹We consider for simplicity of the exposition a flexible price model. We obtain the same *quantitative* results with a sticky price model, *i.e.* when prices are set before the observation of the shocks of the period. Appendix C simply illustrates this result where the conditional expectation operator E_t in (1) is replaced by E_{t-1} .

Let us assume that the instantaneous utility function of the household over consumption and labor takes the form $U(C_t, h_t) = \log(C_t) - h_t$. Therefore, the household maximization problem is simply

$$\max_{C_\tau, h_\tau, M_{\tau+1}, B_{\tau+1}} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} (\log(C_\tau) - h_\tau)$$

subject to (18) and (19). The subjective discount factor is $\beta \in (0, 1)$, and E_t denotes the expectation operator conditional on the information set available at period t . The optimal behavior in consumption, labor and bond holdings yields

$$\frac{1}{W_t} = \frac{1}{\beta} E_t \frac{1}{P_{t+1} C_{t+1}}$$

and

$$R_t = E_t \frac{W_t}{P_t C_t}$$

Final-good sector

We consider a representative firm whose output, Y_t , is produced by the technology

$$Y_t = A_t h_t$$

where A_t represent a technology shock. The real wage is thus given by $W_t/P_t = A_t$.

Government

The budget constraint of the government is given by

$$P_t G_t - T_t = M_{t+1} - M_t - N_t + B_{t+1} - R_{t-1} B_t$$

where G_t is the real government spending, and M_0 and B_0 are given. The money supply is exogenously given and follows the simple money growth rule:

$$M_{t+1} = \gamma_t M_t$$

Shocks

We consider three exogenous disturbances: technology, money growth, and government spending shocks. All of them are assumed to follow AR(1) processes, *i.e.*

$$\log(A_t) = \rho_a \log(A_{t-1}) + (1 - \rho_a) \log(\bar{A}) + \sigma_{\varepsilon, a} \varepsilon_{a,t} \quad (20)$$

$$\log(\gamma_t) = \rho_\gamma \log(\gamma_{t-1}) + (1 - \rho_\gamma) \log(\bar{\gamma}) + \sigma_{\varepsilon, \gamma} \varepsilon_{\gamma,t} \quad (21)$$

$$\log(G_t) = \rho_G \log(G_{t-1}) + (1 - \rho_G) \log(\bar{G}) + \sigma_{\varepsilon, G} \varepsilon_{G,t} \quad (22)$$

where $\varepsilon_{j,t} \sim iid(0, 1)$, with $E(\varepsilon_{j,t} \varepsilon_{j',t}) = 0 \quad \forall \quad j \neq j'$. Further, $\sigma_{\varepsilon, j} > 0$ and $\rho_j \in (0, 1)$ for all $j \in \{a, \gamma, G\}$.

Equilibrium

An equilibrium is a set of prices $\{P_t, W_t, R_t\}$ and an allocation $\{C_t, Y_t, h_t, M_{t+1}, B_{t+1}\}$, such that given prices, and all shocks, this allocation maximizes profits, households' utility, and clears all markets every period. A log-linear approximation around the deterministic steady state characterizes the solution of the model in terms of the forcing variables denoted by the vector $\theta = [\rho_a \ \rho_\gamma \ \rho_G \ \sigma_{\varepsilon,a} \ \sigma_{\varepsilon,\gamma} \ \sigma_{\varepsilon,G}]'$, and the private share, namely $\omega = \bar{C}/\bar{Y}$, *i.e.* the ratio between consumption and output at the steady state. The equilibrium of the economy is summarized by the following equations (hat letters are the deviations from the steady state):

$$\hat{y}_t = \omega \hat{c}_t + (1 - \omega) \hat{g}_t \quad (23)$$

$$\hat{a}_t = E_t [\hat{\pi}_{t+1} + \hat{c}_{t+1}] \quad (24)$$

$$\hat{R}_t = \hat{a}_t - \hat{c}_t \quad (25)$$

$$\hat{\pi}_{t+1} = \hat{\gamma}_{t+1} - \hat{y}_{t+1} + \hat{y}_t \quad (26)$$

with (20), (21) and (22). Equations (23)–(26) collapse into the single expression

$$\hat{y}_t = \hat{a}_t - E_t \left[\frac{1 - \omega}{\omega} \left(\hat{y}_{t+1} - \hat{g}_{t+1} \right) + \hat{\gamma}_{t+1} \right] \quad (27)$$

If $\omega > \frac{1}{2}$, the regular or determinate equilibrium gives the following solutions for output, inflation, and the interest rate:¹⁰

$$\hat{y}_t = \alpha_0 \hat{a}_t - \alpha_1 \hat{\gamma}_t + \alpha_2 \hat{g}_t \quad (28)$$

$$\hat{\pi}_t = \hat{\gamma}_t + \alpha_1 (\hat{\gamma}_t - \hat{\gamma}_{t-1}) - \alpha_0 (\hat{a}_t - \hat{a}_{t-1}) - \alpha_2 (\hat{g}_t - \hat{g}_{t-1}) \quad (29)$$

$$\hat{R}_t = -\beta_0 \hat{a}_t + \beta_1 \hat{\gamma}_t + \beta_2 \hat{g}_t \quad (30)$$

where

$$\begin{aligned} \alpha_0 &= \frac{\omega}{\omega(1 - \rho_a) + \rho_a} \\ \alpha_1 &= \frac{\omega \rho_\gamma}{\omega(1 - \rho_\gamma) + \rho_\gamma} \\ \alpha_2 &= \frac{(1 - \omega) \rho_G}{\omega(1 - \rho_G) + \rho_G} \\ \beta_0 &= \frac{\alpha_0 - \omega}{\omega} \\ \beta_1 &= \frac{\alpha_1}{\omega} \\ \beta_2 &= \frac{(1 - \omega) - \alpha_2}{\omega} \end{aligned}$$

with $\alpha_k, \beta_k \in (0, 1)$ for $k = 0, 1, 2$, as long as $\rho_j \in (0, 1)$ for all $j \in \{a, \gamma, G\}$ and $\omega > \frac{1}{2}$.

¹⁰This imposes that the share of public spending in output must be strictly less than 50%.

3.2 The Taylor Regression

We use the model (28)–(30) with exogenous money growth rule, technology and government shocks – (20)–(22) – as the true DGP. We are interested in the conditional moments of the nominal interest rate and inflation, characterized by the equilibrium of the economy. Assume that an econometrician, in charge to evaluate the role of monetary policy, and who observes only the realizations of the stochastic process generated by this DGP, wants to estimate the following Taylor type rule

$$\hat{R}_t = \eta E_t \hat{\pi}_{t+1} \quad (31)$$

We consider this rule, that relates the nominal interest rate only with inflation, as the implied dynamic properties will depend on a single parameter. Moreover, previous empirical results suggest that the estimates of η are unambiguously significant and positive for the Volcker–Greenspan era (see Clarida et al. 2000). This rule can be expressed in terms of observables as:

$$\hat{R}_t = \eta \hat{\pi}_{t+1} + \varepsilon_t$$

with $\varepsilon_t = \eta(E_t \hat{\pi}_{t+1} - \hat{\pi}_{t+1})$. In the Taylor’s principle literature, the kind of rules as (31) are usually estimated using GMM. Let z_t denotes a single instrument known in period t , that must verify the following orthogonality condition

$$E(\varepsilon_t z_t) = 0$$

or equivalently

$$E\left[\left(\hat{R}_t - \eta \hat{\pi}_{t+1}\right) z_t\right] = 0$$

The latter defines a simple IV estimator. As in previous empirical works, let us consider the instrumental variable $\hat{\pi}_{t-1}$,¹¹. The GMM estimator is thus given by:

$$\hat{\eta} = \frac{Cov\left(\hat{R}_t, \hat{\pi}_{t-1}\right)}{Cov\left(\hat{\pi}_{t+1}, \hat{\pi}_{t-1}\right)}$$

The estimator $\hat{\eta}$ depends on the parameters of the forcing variables θ and ω within the monetary model.

Proposition 1 *The GMM estimator $\hat{\eta}(\theta, \omega)$ of η is given by:*

$$\hat{\eta}(\theta, \omega) = \frac{n_a(\theta, \omega) + n_\gamma(\theta, \omega) + n_G(\theta, \omega)}{d_a(\theta, \omega) + d_\gamma(\theta, \omega) + d_G(\theta, \omega)}$$

¹¹Clarida, Galí and Gertler (2000) include lagged inflation rates up to four lags. To keep tractable results, we do not introduce over-identifying conditions.

where

$$\begin{aligned}
n_a(\theta, \omega) &= \beta_0 \alpha_0 (1 - \rho_a) \rho_a \sigma_{\varepsilon, a}^2 (1 - \rho_G^2) (1 - \rho_\gamma^2) \\
n_\gamma(\theta, \omega) &= \beta_1 (1 + \alpha_1 (1 - \rho_\gamma)) \rho_\gamma \sigma_{\varepsilon, \gamma}^2 (1 - \rho_G^2) (1 - \rho_a^2) \\
n_G(\theta, \omega) &= -\beta_2 \alpha_2 (1 - \rho_G) \rho_G \sigma_{\varepsilon, G}^2 (1 - \rho_a^2) (1 - \rho_\gamma^2) \\
\\
d_a(\theta, \omega) &= -\alpha_0^2 (1 - \rho_a)^2 \rho_a \sigma_{\varepsilon, a}^2 (1 - \rho_G^2) (1 - \rho_\gamma^2) \\
d_\gamma(\theta, \omega) &= \left\{ \rho_\gamma - \alpha_1 (1 - \rho_\gamma)^2 (1 + \alpha_1) \right\} \rho_\gamma \sigma_{\varepsilon, \gamma}^2 (1 - \rho_G^2) (1 - \rho_a^2) \\
d_G(\theta, \omega) &= -\alpha_2^2 (1 - \rho_G)^2 \rho_G \sigma_{\varepsilon, G}^2 (1 - \rho_a^2) (1 - \rho_\gamma^2)
\end{aligned}$$

Proof: The moments are deduced from the solution equations (29) and (30). Since all the exogenous shocks are independent from each other, $E(x_t x'_{t+s}) = Cov(x_t x'_{t+s}) = 0 \quad \forall x \neq x'$. Therefore, we have only to worry about the autocovariances of these shocks. Some algebra let us express these moments as

$$\begin{aligned}
Cov(\hat{R}_t, \hat{\pi}_{t-1}) &= \beta_0 \alpha_0 (1 - \rho_a) \rho_a V(\hat{a}_t) + \beta_1 (1 + \alpha_1 (1 - \rho_\gamma)) \rho_\gamma V(\hat{\gamma}_t) \\
&\quad - \beta_2 \alpha_2 (1 - \rho_G) \rho_G V(\hat{g}_t)
\end{aligned}$$

and

$$\begin{aligned}
Cov(\hat{\pi}_{t+1}, \hat{\pi}_{t-1}) &= -\alpha_0^2 (1 - \rho_a)^2 \rho_a V(\hat{a}_t) + \{ \rho_\gamma - \alpha_1 (1 - \rho_\gamma)^2 (1 + \alpha_1) \} \rho_\gamma V(\hat{\gamma}_t) \\
&\quad - \alpha_2^2 (1 - \rho_G)^2 \rho_G V(\hat{g}_t)
\end{aligned}$$

where $V(\hat{a}_t) = \sigma_{\varepsilon, a}^2 / (1 - \rho_a^2)$, $V(\hat{\gamma}_t) = \sigma_{\varepsilon, \gamma}^2 / (1 - \rho_\gamma^2)$ and $V(\hat{g}_t) = \sigma_{\varepsilon, G}^2 / (1 - \rho_G^2)$. Further simplifications lead us to the estimator $\hat{\eta}(\theta, \omega)$ stated in Proposition 1. \square

Proposition 1 provides the GMM estimator of the Taylor type rule when the three shocks are considered. In such a case, the expression of $\hat{\eta}(\theta, \omega)$ does not deliver a simple interpretation of the estimated parameter of the policy function. Nevertheless, there exist simplifications with clearer interpretations.

Corollary 1 *In the absence of technological and government shocks, and with a zero long-run value on government spending, i.e. when $\theta \equiv \theta' = [0 \quad \rho_\gamma \quad 0 \quad 0 \quad \sigma_{\varepsilon, \gamma} \quad 0]'$ and $\omega = 1$, the GMM estimator $\hat{\eta}$ of η rewrites*

$$\hat{\eta}(\theta', 1) = \frac{1}{\rho_\gamma}$$

In this single shock case, the coefficient of the expected inflation will be greater than one as long as the growth rate of money be a stationary stochastic process. Since only the terms related to $V(\hat{\gamma}_t)$ do not vanish, an equivalent interpretation is that the weight of this term

placed on the denominator is smaller. Thus, we should expect that if the weights of $V(\hat{a}_t)$ and $V(\hat{g}_t)$ are particularly small on $Cov(\hat{R}_t, \hat{\pi}_{t-1})$ and $Cov(\hat{\pi}_{t+1}, \hat{\pi}_{t-1})$, then $\hat{\eta}(\theta, \omega)$ could be still greater than one.

Corollary 1 provides a clear illustration of the first peril in the policy rule regression principle, *i.e.* an econometrician could interpret $\hat{\eta} > 1$ as an aggressive interest rate rule (with an accommodating growth rate of money), when in fact it is an exogenous money growth rule that prevails. Therefore, we conclude that the single-equation analysis is not robust to avoid policy misidentifications. This will be shown clearly in section 3.3. For the moment, let us illustrate the consequences of the second peril, *i.e.* when the policy regression estimation is taken seriously and becomes a policy recommendation.

Indeterminacy

Assume now that, convinced that an aggressive interest rate rule is good for the stabilization of the economy, the central bank decides to abandon the constant-mean growth rate of money and instead introduces an accommodating rate, namely $\tilde{\gamma}_t$. The money growth process is now endogenous, and is intended to sustain the forward-looking policy rule (31).

The new system that determines the rational expectations equilibrium, incorporates this information and replaces the AR(1) process of money growth by the interest rate rule (31). Once again, the equilibrium conditions collapse into a single equation:

$$\hat{y}_t = \eta E_t \hat{y}_{t+1} + \omega(1 - \eta)\hat{a}_t + (1 - \omega)(1 - \eta\rho_G)\hat{g}_t \quad (32)$$

Following previous studies, notably Calrstrom and Fuerst (2000), equation (32) shows that an aggressive forward-looking interest rate rule, *i.e.* with some $|\eta| > 1$, generates real indeterminacy or an irregular equilibrium.¹²

3.3 A Calibration Exercise

The aim of this section is to show that the simple monetary model described in section 3.1 can actually match the empirical moments of the estimated Taylor rule for the U.S. economy. Table 1 presents the parametrization that we consider in this quantitative exercise. These are values that could be believed as standard in the business cycle literature (see the note below table 1 for a discussion).

Table 2 presents the calibrated value $\hat{\eta}$ and the estimation of Clarida et al. (2000) as the benchmark for the empirical estimate of η . This tables also considers three calibrated

¹²see also Benhabib, Schmitt-Grohe and Uribe (2001a)

Table 1: Parameters Values for calibration

Parameters	Values	Source
$(\rho_a, \sigma_{\varepsilon,a})$	(0.98, 0.0072)	King and Rebelo (1999)
$(\rho_\gamma, \sigma_{\varepsilon,\gamma})$	(0.49, 0.0089)	Cooley and Hansen (1995)
$(\rho_G, \sigma_{\varepsilon,G})$	(0.96, 0.0210)	Christiano and Eichenbaum (1992)
ω	0.823*	<i>ibid.</i>

Note : The technology coefficients are the standard ones used in the RBC literature for the U.S., here updated by King and Rebelo (1999) over the sample period 1947:1–1996:4, quarterly observations. Cooley and Hansen (1995) reports the parameters of an AR(1) process for the M1 growth over the period 1954:1–1991:2. Christiano and Eichenbaum (1992) measures the government spending as real government purchases of goods and services minus real investment; its estimation period covers 1955:3–1983:4.

*In Christiano and Eichenbaum (1992) the average ratio between government spending and output equals 0.177, therefore we set the private share on 0.823. Nevertheless, our main results hold if we take instead the average ratio between private consumption and output, which equals 0.56.

values $\hat{\eta}$ that depend on three different specifications about the origin of the shocks. First, we assume that the three shocks are present, and the private share is fixed according to Table 1. Second, we remove the government disturbance and we set $\omega = 1$. With this, we intend to give a first look at the effect of the government spending shock on the calibrated parameter. Finally, we assume that the unique source of disturbance is the money growth process, and we set again $\omega = 1$. At the bottom of Table 2 is the baseline estimation of expected inflation of Clarida et al. (2000) for the Volscker–Greenspan era.

The similarity between the calibrated and the estimated parameter is evident. Thus, the risk of possible policy misidentification is sustained. At this point, a word of caution is necessary. Table 2 does not tell us that the actual DGP of the U.S. economy is in fact the simple flexible–price monetary model presented above, but that even this kind of model can match the empirical $\hat{\eta}$ of the policy rule estimation for the U.S. Moreover, it is possible that other types of monetary models, with more realistic assumptions in their fundamentals, could match as well this parameter estimate. It must be clear by now that the single–equation analysis tell us very little about the true policy behavior of the central bank, and that the limited information estimation method by itself is unable to screen the actual monetary rules of the authorities.

A second question that we have to answer is: Why the calibrated value $\hat{\eta}$ did not change significantly when the technological and government spending shocks were included? According to section 2.2, this is precisely what we should expect if the contributions of the technological and government shocks explain just a small part of the total variance of inflation and the nominal interest rate. Table 3 presents the variance decomposition

Table 2: The calibrated and estimated $\hat{\eta}$

Shocks	
Governement, technological and money shocks	2.14*
No government, only technological and money shocks	2.04
No government, only money shocks	2.11
Baseline estimate of Clarida et al. (2000)	2.15

Note: The estimation of Clarida et al. (2000) corresponds to the tenures of Paul Volcker and Alan Greenspan over the sample period 1979:3–1996:4.

*If we consider the alternative measure of the private share mentioned in the note of table (1), equal to 0.56, the calibrated coefficient changes to 2.3541.

Table 3: Variance decomposition (in %)

Horizon	Inflation			Nominal interest rate		
	Technology	Money	Government	Technology	Money	Government
1	16.6	77.4	6.0	0.0	99.9	0.1
2	16.2	77.9	5.9	0.0	99.8	0.2
3	16.1	78.1	5.9	0.0	99.8	0.2
4	16.0	78.1	5.9	0.0	99.7	0.3
5	16.0	78.1	5.9	0.0	99.7	0.3
⋮						
100	16.1	78.0	5.9	0.0	99.0	1.0

of these variables accounting for the contributions of the three shocks. In both cases, the money growth process largely explains the forecast error variances of these variables (in the short and in the long run). After 100 periods, the money shock justifies 78 and 99 per cent of the variance of inflation and the interest rate, respectively.

Finally, let us explore the aggregate dynamic properties when the estimated Taylor type rule is placed in the original model. From equation (32) and Table 2, we see immediately that the estimated value $\hat{\eta}$ of η leads to real indeterminacy as $\hat{\eta} \simeq 2 (> 1)$. This illustrates again the second peril of policy rule regression.

4 Concluding Remarks

This paper provides a general overview of the policy rule regression program. Our main finding is that, within the framework of a rational expectations model, policy rule regression that uses a limited information estimation method tells us very little about the

actual structural behavior of policy-making. In fact, the parameter estimates of such program account only for the description of an equilibrium relationship between the endogenous and exogenous variables, and cannot be associated with a unique kind of policy rules. In order to fully identify the effects of economic policy, a complete exercise within a structural model is required.

We identified two perils in the implementation of actual policy regression. The first one is policy misidentification. Although this threat is not too important when it is limited to a pure description of the policy behavior, its consequences become serious when it is transformed in a policy advice. This is the second peril, and deals with spurious recommendations potentially able to create real indeterminacy and sunspot equilibria. In fact, in the monetary model considered, an estimated aggressive forward-looking interest rate rule will always lead to real indeterminacy when placed in the original model.

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APPENDIX

A Static Policy Rule

Consider the following static policy rule

$$x_t = \eta_s y_t$$

We apply the same empirical methodology dscribed in section 1.2. Notably, we use y_{t-1} as the instrumental variable. The GMM estimator $\hat{\eta}_s$ of η_s is given by:

$$\begin{aligned}\hat{\eta}_s &= \frac{Cov(x_t, y_{t-1})}{Cov(y_t, y_{t-1})} \\ &= \frac{1 - a\rho}{b}\end{aligned}$$

After replacement into (1), one gets:

$$y_t = aE_t y_{t+1} + (1 - a\rho)y_t$$

or equivalently if $a \neq 0$

$$\rho y_t = E_t y_{t+1}$$

As $|\rho| < 1$, the equilibrium under the estimated policy rule is indeterminate.

B Backward Policy Rule

Consider now a backward policy rule

$$x_t = \eta_b y_{t-1}$$

y_{t-1} is again the instrumental variable as in sections 1.2 and A. The GMM estimator $\hat{\eta}_b$ of η_b is given by:

$$\begin{aligned}\hat{\eta}_b &= \frac{Cov(x_t, y_{t-1})}{V(y_{t-1})} \\ &= \rho \left(\frac{1 - a\rho}{b} \right)\end{aligned}$$

After replacement into (1), one obtains:

$$y_t = aE_t y_{t+1} + \rho(1 - a\rho)y_{t-1}$$

The dynamic properties of the economy is characterized by a second order linear difference equation. The equilibrium is indeterminate if the two roots of the characteristic polynomial lie inside the unit circle. The two (real) roots of the characteristic polynomial are:

$$\lambda_1 = \rho \quad \text{and} \quad \lambda_2 = \frac{1 - a\rho}{a}$$

It follows that indeterminacy occurs when $|\lambda_2| < 1$, *i.e.* when $|a| > 1/(1 + \rho)$. Note that when indeterminacy occurs, the stable root λ_2 is positive and the model displays persistence, provided $\rho \in (0, 1)$.

C Another specification of equation (1)

Consider now that the economy takes the form:

$$y_t = aE_{t-1}y_{t+1} + bx_t \tag{33}$$

where $b \neq 0$ and $|a| < 1$. The policy rule is always given by (2). The stationary solution is

$$y_t = \frac{b\rho}{1 - a\rho}x_{t-1} + b\varepsilon_t$$

or equivalently

$$y_t = \frac{b}{1 - a\rho}x_t - \frac{ab\rho}{1 - a\rho}\varepsilon_t$$

The GMM estimator $\hat{\eta}$ of the forward policy function parameter η (see equation (5)) is given by:

$$\begin{aligned}\hat{\eta} &= \frac{Cov(x_t, y_{t-1})}{Cov(y_{t+1}, y_{t-1})} \\ &= \frac{\left(\frac{b\rho}{1 - a\rho}\right) V(x_t) (1 - a\rho(1 - \rho^2))}{\left(\frac{b^2\rho^2}{(1 - a\rho)^2}\right) V(x_t) (1 - a\rho(1 - \rho^2))} \\ &= \frac{1 - a\rho}{b\rho}\end{aligned}$$

The dynamic properties of the economy is obtained after substitution of the estimated policy rule in (33):

$$y_t = aE_{t-1}y_{t+1} + \frac{1 - a\rho}{\rho}E_t y_{t+1}$$

Taking the expectation operator E_{t-1} , this equation rewrites,

$$\rho E_{t-1}y_t = E_{t-1}y_{t+1}$$

Since $|\rho| < 1$, *i.e.* the exogenous variable follows a stationary process, the equilibrium under the estimated policy rule is indeterminate.