Are Monetary Models with Exogenous Money Growth Rule Able to Match the Taylor Rule?

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Abstract

This paper questions the ability of three monetary models to quantitatively reproduce a Taylor rule when considered as a useful way to describe monetary facts. Using a standard econometric procedure, we estimate the relation between the nominal interest rate, the expected inflation and the output gap under these monetary models with exogenous money growth rule. We show that the estimated relation under the flexible price model – implying Fisherian inflation premia – is close to the empirical estimates from actual data. Conversely both the sticky prices and the limited participation models – consistent with the monetary transmission mechanism and the liquidity effect – are not able to match estimates of the Taylor rule from actual data.

Keywords: flexible prices, sticky prices, limited participation, Taylor rule

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Introduction

In a famous paper, Taylor [1993] showed that US monetary policy after 1986 is well characterized by a simple rule wherein the interest rate – the nominal Federal funds rate – responds positively to the inflation rate and to the output gap. This rule presents the attractive empirical feature to be robust over periods, monetary regimes and countries (see Clarida, Gali and Gertler [1998], [2000] and Taylor [1999]). For example, the estimated nominal interest rate and the estimated expected inflation parameters generally have the expected sign and are significant. However, previous empirical results suggest that the estimated parameter of the expected output gap is marginally significant for the Volcker–Greenspan era (see Clarida, Gali and Gertler [2000]). Various interpretations may be given to the estimated Taylor rule (see Taylor [1999]). First, the estimated rule can be considered as a guideline – or explicit formula – for the central bank to follow when making monetary policy decisions. In this case, the Taylor rule describes how a central bank sets the nominal interest rate in response to economic variables (the inflation rate and the output gap). Second, it can be used as a way to examine several episodes of monetary history. In this case, the estimated rule does not necessarily represent the "true" central bank behavior, but rather a useful description of the monetary policy in different historical time periods.

Over the recent years, there has been a great resurgence of interest on how to conduct monetary policy. Fundamentally, the central bank controls only the volume of bank reserves. To do this, the central bank buys or sells Treasury bills in the open market, thereby either taking reserves away from banks or giving banks reserves. The central bank can use this one instrument to control some measure of the money supply (M_1, M_2 or any other measure of the money stock the central bank can invent). Indeed, in the 1970s the U.S. Federal Reserve bank did just that by creating many measures of money. Alternatively, the central bank can control short-term interest rates, the federal funds rate in the U.S. case which is the rate banks pay to borrow reserves. Then, the central bank can target bank reserves, some definition of the money stock, or the short-term interest rates. Whatever they do, they can just use one instrument. A consensus has emerged about the fact that monetary policy should be focused on the control of inflation using the short term interest rates as instrument. However, the bank central's objective for open market operations has varied over the years, over monetary regimes and countries. Therefore, the question remains open. In this paper, considering the theoretical consensus described below, we use the second interpretation of the Taylor rule – *i.e.* a useful way to describe monetary policy and monetary facts –.

Whatever the monetary policy is, the problem is to identify and to estimate consistently the assumed central bank behavior. Any monetary rule is estimated using aggregate data which are the realization of the economic equilibrium. Therefore, the econometrician must use a set of relevant instrumental variables in order to identify and estimate the structural equation that summarizes the central bank behavior. For example, empirical studies on the Taylor rule generally use four–lagged inflation, output and nominal interest rates as instrumental variables (see Clarida et al. [1998] and [2000]). Using the same econometric procedure, we estimate the relation between the nominal interest rate, the expected inflation and the output gap under three monetary models with an exogenous money growth rule.[1] This paper examines under this methodology the Volcker–Greenspan monetary era using the framework of an interest rate rule for monetary policy as described by Taylor [1999]. The aim of this paper is to judge of the ability of monetary models with exogenous money growth rule – consistent with different monetary stylized facts we are going to describe – to reproduce the estimated Taylor rule.

Before the exposition of the monetary models, let us first review a set of empirical monetary facts that basically motivate our specification choices.

[1]Our approach is similar to Auray and Fève [2002] and Salyer and Van Gaasbeck [2002], but it extends these previous studies in several ways. In Auray and Fève [2002] , we only consider monetary models without capital accumulation. Moreover, the Taylor rule is simpler as it only includes expected inflation. In their paper, Salyer and Van Gaasbeck [2002] only consider a limited participation model and a Taylor rule without nominal interest rate inertia.

A first part of the empirical literature describes the long–run monetary facts examining data over long periods (see Lucas [1980], Barro [1990], Poole [1994] and McCandless and Weber [1995]). They show that the growth rates of the money supply and the general price level are highly correlated for different money definitions (M_1 or M_2) over a period running from 1960 to 1990. Therefore, these studies support the quantity theory of money. Under this theory, rapid money growth is associated with high inflation as well as with high interest rates. Indeed, the interest rates behavior can similarly be understood in terms of Fisherian inflation premia. More recently, Monnet and Weber [2001], using data since 1960 to 1998 for about 40 countries, show that money and interest rates are positively related, supporting, therefore, the Fisher equation view.

Another part of the empirical literature attempts to uncover the effects of monetary policy over the business cycle using the Vector Autoregression methodology. This empirical literature usually finds a non–neutrality property of money in the short–run. More specifically, three main stylized facts seem to emerge from empirical studies: following a contractionary monetary policy, *(i)* there is a persistent decline in real GDP; *(ii)* prices are almost non responsive in the very short–run but decrease and *(iii)* the nominal interest rate rises. *(i)* together with *(ii)* constitute the so–called monetary transmission mechanism and *(i)* together with *(iii)* define the liquidity effect. These results seem to be robust to different identification schemes (see e.g. Sims [1992], Christiano, Eichenbaum and Evans [1996] and [1999] , Sims and Zha [1995] and Leeper, Sims and Zha [1996]). Consequently, any structural model that could plausibly be used for monetary policy analysis should be able to account for these two mechanisms.

In order to account for these monetary facts, our experiment is conducted using a flexible prices model – implying a Fisherian inflation premia –, a sticky prices model – consistent with the monetary transmission mechanism – and a limited participation model – reproducing a liquidity effect –. Moreover, any theoretical analysis needs to reconcile these facts with empirical studies on monetary policy rules. The central question that this paper studies is the following : *Are monetary models with exogenous money growth rule –implying either Fisherian inflation premia, the monetary transmission mechanism or the liquidity effect – able to match the estimated Taylor rule?*

Flexible prices monetary models, using either a cash–in–advance constraint (see e.g. Lucas and Stokey [1983] or Cooley and Hansen [1989] for an application) or the money in the utility function, imply that following a positive money injection output drops and the nominal interest rate rises. Indeed, in these models, the individuals attempt to escape the inflation tax the money injection creates by decreasing their consumption and increasing their leisure. Therefore the output drops. Further, since households postpone consumption and save more, the nominal interest rate raises. For a large part, this result may be explained by the existence of the inflation tax. Therefore, this model is consistent with the Fisherian inflation premia but not with the monetary transmission mechanism nor the liquidity effect. Hence, most of the models that have attempted to provide with a better representation of the effects of monetary policy on aggregate dynamics have tried to weaken the inflation tax.

A first approach has been to impose price stickiness, which prevents firms from instantaneously adjusting prices in response to monetary policy shocks (see e.g. Hairault and Portier [1993b], Rotemberg and Woodford [1992], Chari, Kehoe and McGrattan [2000], Christiano, Eichenbaum and Evans [1997] among others). This assumption breaks the inflation tax so that following a positive money injection the output rises and prices do not — obviously — fully respond on impact. Therefore, this model is consistent with the monetary transmission mechanism, but is still counterfactual with regard to the liquidity effect in the short run.

A second approach has been to assume limited participation. Lucas [1990], Christiano [1991] and Fuerst [1992] introduce limited participation in monetary model implying that households cannot adjust to any changes in financial market circumstances. Then any money injection disproportionately translates in higher supply of loanable funds, which puts downward pressure on the nominal interest rate. Access to credit being easier, firms can increase their scale of operations and the output rises. Following a positive money injection output rises and nominal interest declines. Therefore, this model is able to reproduce the liquidity effect found in the data.

We show that under the flexible prices model, the estimated relation between the nominal interest rate, the expected inflation and the output gap is close to the empirical estimates from actual data spanning the period 1979.3–2001.1. Conversely, the sticky prices and the limited participation model are not able to reproduce the relation between the nominal interest rate, the inflation rate and the output gap found in the data.

The paper is organized as follows. A first section presents the three monetary models. The second section presents the evaluation method and discusses the empirical results. A last section offers some concluding remarks.

1 The Three Monetary Models

This section describes the main ingredients that characterize the three monetary economies.

1.1 The Cash–in–Advance model

Households

The economy is comprised of a unit mass continuum of identical infinitely lived agents. Each household has preferences over consumption and leisure represented by the following intertemporal utility function:

$$\mathcal{U} = E_0 \left\{ \sum_{t=0}^{\infty} (\beta^{\star})^t \left(\log(C_t) - h_t\right) \right\} \tag{1}$$

where E_0 denotes the conditional expectation operator at time $t = 0$. $\beta^{\star} \in (0,1)$ is the discount factor (adjusted for growth), C_t and h_t respectively denote the household's consumption services and hours worked in period t.

The intertemporal budget constraint of the household is thus given by:

$$M_{t+1} + B_{t+1} + P_t I_t + P_t C_t \leq M_t + N_t + R_{t-1} B_t + W_t h_t + P_t Q_t K_t \tag{2}$$

The household enters period t with some nominal balances, M_t, that corresponds to its money demand at the end of period $t-1$. The household supplies her labor on the labor market at the real wage rate W_t/P_t and rents out capital at the real rental rate, Q_t. During the period the households gets a lump-sum transfer of cash from the monetary authorities equal to N_t and interest payments from bond holdings $R_{t-1}B_t$. These revenues are used to consume C_t, to invest, I_t, get money and bonds for the next period, M_{t+1} and B_{t+1}. Investment is used to form capital according to the following law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

where $\delta \in (0,1)$ is the depreciation rate. Money is held because the household faces a cash–in–advance constraint of the form:

$$P_t C_t \leq M_t + N_t + R_{t-1} B_t - B_{t+1} \tag{4}$$

The problem of the representative household is then to choose her consumption–savings, labor and real balances plans to maximize (1) subject to (2)–(4), given the initial conditions. The household's optimal behavior is then given by the set of first order conditions:

$$\frac{P_t}{W_t} = \beta^\star E_t \left[\frac{P_{t+1}}{W_{t+1}} (Q_{t+1} + 1 - \delta) \right]$$

$$1 = \beta^\star E_t W_t \left[\frac{1}{P_{t+1} C_{t+1}} \right]$$

$$\frac{1}{P_t C_t} = \beta^\star E_t R_t \left[\frac{1}{P_{t+1} C_{t+1}} \right]$$

Firms

The representative firm produces an homogenous good that can be either invested or consumed using the constant returns to scale technology, represented by the following Cobb–Douglas production function:

$$Y_t \leq A_t K_t^{\alpha} (\Gamma_t h_t)^{1-\alpha} \tag{5}$$

where K_t is the aggregate capital stock and h_t corresponds to hours worked. Γ_t denotes Harrod neutral technological progress, and is assumed to growth at rate $\gamma - 1 > 0$. The level of the technology A_t is described by the following autoregressive process of order one:

$$\log(A_{t+1}) = \rho_a \log(A_t) + (1 - \rho_a) \log(\overline{A}) + \sigma_a \varepsilon_{a,t+1}$$

where $\sigma_a > 0$, $|\rho_a| < 1$ and $\varepsilon_{a,t}$ is a Gaussian white noise with unit variance.

Each period, the firm determines its demand for factor inputs from profit maximization:

$$\max_{K_t, h_t} A_t K_t^{\alpha} (X_t h_t)^{1-\alpha} - \frac{W_t}{P_t} h_t - Q_t K_t$$

The first order conditions are given by :

$$\begin{aligned} Q_t &= \alpha \frac{Y_t}{K_t} \\ \frac{W_t}{P_t} &= (1-\alpha) \frac{Y_t}{h_t} \end{aligned}$$

Government

The government finances a part of government expenditures on the good using lump sum taxes. The government policy includes sequences of nominal transfers net of taxes N_t and nominal government debt B_t that satisfies the following government budget constraint:

$$P_t G_t + N_t = M_{t+1} - M_t + B_{t+1} - R_{t-1} B_t$$

where the initial stock of government debt, B_0, is given. The stationary component of government expenditures is assumed to follow an exogenous stochastic process of the form:

$$log(g_{t+1}) = \rho_g log(g_t) + (1-\rho_g) log(\overline{g}) + \sigma_g \epsilon_{g,t+1}$$

where $\sigma_g > 0$, $|\rho_g| < 1$ and $\epsilon_{g,t}$ is a Gaussian white noise with unit variance.

Money Supply

Money is assumed to grow at a rate $(\mu_t - 1)$:

$$M_{t+1} = \mu_t M_t \tag{6}$$

We assume that μ_t follows an exogenous stochastic process of the following form:

$$log(\mu_{t+1}) = \rho_\mu log(\mu_t) + (1-\rho_\mu) log(\overline{\mu}) + \sigma_\mu \epsilon_{\mu,t+1}$$

where $\sigma_\mu > 0$, $|\rho_\mu| < 1$ and $\epsilon_{\mu,t}$ is a gaussian white noise with unit variance. We finally assume that the money created in period t is entirely distributed to households:

$$(\mu_t - 1) M_t = N_t$$

Equilibrium

An equilibrium of this economy is a sequence of prices $\mathcal{P}_t = \{P_t, W_t, q_t, R_t\}_{t=0}^{\infty}$ and a sequence of quantities $\mathcal{Q}_t = \{C_t, h_t, Y_t, I_t, K_{t+1}, M_{t+1}, B_{t+1}, G_t, N_t\}_{t=0}^{\infty}$ such that for a given sequence of prices $\mathcal{P}_t$, the sequence $\{C_t, h_t, K_{t+1}, M_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ maximizes households' utility, the sequence $\{h_t, K_t\}_{t=0}^{\infty}$ maximizes the profit of the representative firm and for a given sequence of quantity $\mathcal{Q}_t$, the sequence $\{P_t, r_t, W_t\}_{t=0}^{\infty}$ clears all markets, and money is supplied according to (6).

1.2 The Sticky Prices model

This model differs from the cash–in–advance model described in the previous section as it incorporates monopolistic competition among intermediate–goods producers (see Blanchard and Kiyotaki [1987], Ball and Romer [1991], Beaudry and Devereux [1995] and

King and Watson [1996] among others). If prices are free to adjust, one–time, permanent changes in the level of the money supply will induce proportional changes in all prices, leaving the real equilibrium unaffected. Price stickiness remains critical in generating significant real effects of money (see Chari et al. [2000]). In this section, we add price stickiness by assuming that producers engage in one–period, staggered price setting. We omit any discussion of household's behavior as it is symmetric with before.

Final good firm

In each period, a final good, Y_t, is produced by a perfectly competitive firm using inputs supplied by a continuum of intermediate–goods–producing firms indexed by $i \in (0,1)$ using the CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^\theta\right]^{\frac{1}{\theta}} \text{ with } 0 < \theta \leq 1$$

where $Y_t(i)$ is the input of intermediate good i. Let $P_t(i)$ and P_t denote, respectively, the price of the intermediate good i and of the final good, the demand for good i expresses as:

$$Y_t^d(i) = \left[\frac{P_t}{P_t(i)}\right]^{\frac{1}{1-\theta}} Y_t$$

from which we get the following expression for the aggregate price index:

$$P_t = \left[\int_0^1 P_t(i)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$$

Firms

There is a continuum of firm, $i \in (0,1)$, each of which produces a particular good by means of capital and labor according to a constant returns–to–scale technology, represented by the production function:

$$Y_t(i) = A_t K_t(i)^\alpha (\Gamma_t h_t(i))^{1-\alpha} \text{ with } \alpha \in (0,1) \tag{7}$$

where $K_t(i)$ and $h_t(i)$ respectively denote the physical and labor input used by firm i in the production process. A_t is an exogenous stationary stochastic shock whose properties are the same than in the cash–in–advance model. Assuming that each firm i operates under perfect competition in the input markets, the firm determines its production plan so as to minimize its total cost:

$$\min_{\{K_t(i),\, h_t(i)\}} = W_t h_t(i) + q_t K_t(i)$$

subject to equation (7). q_t is the nominal rental rate of capital. This yields to the following expression for total costs:

$$\mathcal{MC}_t Y_t(i)$$

where the real marginal cost $\mathcal{MC}_t$ is given by

$$\mathcal{MC}_t = \frac{W_t^{1-\alpha} q_t^{\alpha}}{\chi A_t}$$

with $\chi = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. The demand for each input is determined by:

$$q_t K_t(i) = \alpha \mathcal{MC}_t Y_t(i)$$
$$W_t h_t(i) = (1-\alpha) \mathcal{MC}_t Y_t(i)$$

Producers are monopolistically competitive on the good market, and therefore set prices for the good they produce. Since the price setting is independent of any firm characteristic, all firms choose the same price. The intermediate price index is given by:

$$P_t = \frac{1}{\theta} E_{t-1} \left[\frac{1}{A_t} q_t^{\alpha} W_t^{(1-\alpha)} \alpha^{-\alpha} (1-\alpha)^{(\alpha-1)} \right] \tag{8}$$

According to the price stickiness assumption, the price equation (8) is the only modification compared to the cash–in–advance model described in the previous section.

Equilibrium

An equilibrium of this economy is a sequence of prices $\{\mathcal{P}_{t=0}^{\infty}\} = \{P_t, W_t, q_t, R_t\}_{t=0}^{\infty}$ and a sequence of quantities $\{\mathcal{Q}_{t=0}^{\infty}\} = \{\mathcal{Q}^{\mathcal{H}}{}_{t=0}^{\infty},\ \mathcal{Q}^{\mathcal{F}}{}_{t=0}^{\infty}\}$ with $\{\mathcal{Q}^{\mathcal{H}}{}_{t=0}^{\infty}\} = \{C_t, I_t, B_{t+1}, K_t, h_t, G_t, M_{t+1}, N_t\}$ and $\{\mathcal{Q}^{\mathcal{F}}{}_{t=0}^{\infty}\} = \{Y_t, Y_t(i), K_t(i), h_t(i);\ i \in (0,1)\}$ such that for a given sequence of prices

$\{\mathcal{P}_t\}$, and a sequence of shocks, $\{\mathcal{Q}^{\mathcal{H}}{}_{t=0}^{\infty}\}$ is a solution to the representative household's problem, $\{\mathcal{Q}^{\mathcal{F}}{}_{t=0}^{\infty}\}$ is a solution to the representative firms' problem and for a given sequence of quantity $\mathcal{Q}_t$ and a sequence of shocks, the sequence $\{\mathcal{P}_{t=0}^{\infty}\}$ clears all markets, and money is supplied according to (6). Finally, prices satisfy equation (8).

1.3 The Limited Participation model

Lucas [1990], Christiano [1991] and Fuerst [1992], among others, have proposed to introduce limited participation in the model implying that households cannot adjust their behavior to any changes in financial market circumstances. Then, any money injection disproportionately translates in higher supply of loanable funds, which puts downward pressure on the nominal interest rate. Access to credit being cheaper, firms can increase their scale of operations and the output rises. This limited participation model is similar to Christiano [1991] and Fuerst [1992]. We just describe the main ingredients of this model which differ from the cash-in–advance model.

Households

At the beginning of the period t, money supply M_t is held by households in the form of cash (M_t^c) and deposits (M_t^d), such that total money held by households is given by:

$$M_t = M_t^c + M_t^d \tag{9}$$

M_t^c can be interpreted as money held in checking account that yield zero interest rate and M_t^c as money held in saving accounts that yields a positive nominal interest rate $(R_t - 1) > 0$. A checking account is needed by each household as cash to purchase goods such that each household faces the following cash–in–advance constraint:

$$P_t C_t \leq M_t^c \tag{10}$$

The nominal income of households is composed of wage income $W_t h_t$, interest income associated to money deposits $(R_t - 1)M_t^d$, and dividends from firms F_t, and financial intermediaries, B_t. These revenues are used to consume and get money for the next period. The budget constraint then takes the following form:

$$M_{t+1}^c + M_{t+1}^d = R_t M_t^d + W_t h_t + F_t + B_t - P_t C_t + M_t^c \tag{11}$$

The problem of an household is then to choose her consumption–savings, labor and money holdings plans, $\{c_t, h_t, M_{t+1}^c, M_{t+1}^d\}_{t=0}^{\infty}$, to maximize (1) subject to (9)–(11), and given the stochastic process for $\{P_t, W_t, R_t, B_t, F_t\}_{t=0}^{\infty}$ and the initial conditions $M_0^c, M_0^d > 0$.

Firms

The capital stock is owned by firms and labor is rented at the nominal wage W_t. Firms borrow money from a financial intermediary at interest rate R_t in order to finance the wage bill $W_t h_t$. After payments of interests to financial intermediaries, wage bill and capital expenditures, the dividends distributed at the household are:

$$F_t = P_t Y_t - R_t W_t h_t - P_t I_t$$

Firms choose the contingency plan $\{Y_t, h_t, K_{t+1}, P_t, F_t\}_{t=0}^{\infty}$ to maximize the expected value of the dividend flow:

$$E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{t+1} F_t \right\}$$

subject to the constraints (3), (5), (6) and given the stochastic process for $\{P_t, W_t, R_t, \Lambda_t, A_t\}_{t=0}^{\infty}$ and the initial condition for capital stock $K_0 \geq 0$. As firms act in the best interest of the shareholders, the stochastic discount factor Λ_{t+1} corresponds to the representative household's relative valuation of cash across time:

$$\Lambda_{t+1} = \frac{(\beta^{\star})^{t+1}}{P_{t+1} C_{t+1}}$$

Financial intermediaries

At the beginning of the period t, financial intermediaries supply a quantity of money which comes from the loanable funds provided by the households M_t^d and a lump sum cash injection N_t from the monetary authorities. Loan market clearing thus requires:

$$W_t h_t = M_t^d + N_t$$

At the end of the period, financial intermediaries have to repay the interests for the money loans by the households. Consequently, profits flow for the financial intermediaries are given by $B_t = R_t N_t$.

Government

The government finances government expenditures on the good using lump sum taxes. The government policy satisfies the following government budget constraint:

$$P_t G_t + N_t = M_{t+1} - M_t$$

Equilibrium

An equilibrium of this economy is a sequence of prices $\mathcal{P}_t = \{P_t, W_t, q_t, R_t\}_{t=0}^{\infty}$ and a sequence of quantities $\mathcal{Q}_t = \{C_t, h_t, Y_t, I_t, K_{t+1}, M_{t+1}^c, M_{t+1}^d, B_t, G_t, N_t, F_t\}_{t=0}^{\infty}$ such that for a given sequence of prices $\mathcal{P}_t$, the sequence $\{C_t, h_t, M_{t+1}^c, M_{t+1}^d\}_{t=0}^{\infty}$ maximizes households' utility, the sequence $\{h_t, K_{t+1}\}_{t=0}^{\infty}$ maximizes the profit of the representative firm and for a given sequence of quantity $\mathcal{Q}_t$, the sequence $\{P_t, r_t, W_t\}_{t=0}^{\infty}$ clears all markets.

2 The Estimated Taylor Rule

This section presents the method of quantitative evaluation we retain here. We evaluate the ability of each model to reproduce a Taylor type rule estimated from the actual data. The benchmark calibration is then presented and we discuss the empirical results. The flexible prices, the sticky prices and the limited participation models are denoted CIA, SP and LP hereafter.

2.1 Quantitative Evaluation

We use the three models as a DGP which allows to reproduce some features of actual data, which is taken as the realization of an unknown – to the econometrician – stochastic process. The features we are interested in include a set of conditional moments on the nominal interest rate, the inflation rate and the output gap. More precisely, we specify a Taylor type rule that aims at describing the joint behavior of nominal interest rate, inflation rate and output gap, given a set of restrictions based on the information set. The specification that we use takes the following simple form:

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + \eta E_t \widehat{\pi}_{t+1} + \varphi E_t \widehat{y}_{t+1} \tag{12}$$

where the hat denotes the percentage of deviation from the long run value.[2] This Taylor type rule incorporates the lagged interest rate, the expected inflation rate and the expected output gap. Equation (12) is rather standard as we follow the specification proposed by Clarida et al. [2000]. Moreover, as in Clarida et al. [2000], the output gap is obtained in deviation from a quadratic trend. Previous empirical results suggest that the estimated parameter of expected output gap φ is marginally significant for the Volcker–Greenspan era (see Clarida et al. [2000]). Conversely, the estimates of η and ρ are significant and positive in most cases (see Taylor [1999] and Clarida et al. [2000]). The specification combines a partial adjustment of actual interest rate to the target and a target specification which includes the expected inflation and the expected output gap. Equation (12) can be specified as a partial adjustment to the target:

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho) \widehat{R}_t^{\star} \tag{13}$$

where

$$\widehat{R}_t^{\star} = \left(\eta^{\star} E_t \widehat{\pi}_{t+1} + \varphi^{\star} E_t \widehat{y}_{t+1}\right)$$

and $\eta^{\star} = \eta/(1-\rho)$ and $\varphi^{\star} = \varphi/(1-\rho)$. We essentially retain the first specification (12), as we faced numerous numerical failures when we estimate the Taylor type rule (13) under the three structural models. Equation (12) can be expressed in terms of observable:

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + \eta \widehat{\pi}_{t+1} + \varphi \widehat{y}_{t+1} + \varepsilon_{t+1}$$

where $\varepsilon_{t+1} = -\eta \left(\widehat{\pi}_{t+1} - E_t \widehat{\pi}_{t+1}\right) - \varphi \left(\widehat{y}_{t+1} - E_t \widehat{y}_{t+1}\right)$. Let Z_t denotes a vector of instrument known in period t. This instruments verifies a set of orthogonality conditions:

$$E\left(\varepsilon_{t+1} \otimes Z_t\right) = 0$$

[2] As each variable that enters in the Taylor type rule (12) are demeaned, we do not introduce a constant term.

or equivalently

$$E\left(\left(\widehat{R}_t - \rho\widehat{R}_{t-1} - \eta\widehat{\pi}_{t+1} - \varphi\widehat{y}_{t+1}\right)\otimes Z_t\right) = 0 \tag{14}$$

where $\otimes$ denotes the kronecker product. Equation (14) is the basis of the GMM estimation of the parameters $\psi = \{\rho, \eta, \varphi\}$ given a weighting matrix that accounts for possible serial correlation. Let us introduce the empirical counterpart of (14)

$$g_T = \frac{1}{T}\sum\left(\left(\widehat{R}_t - \rho\widehat{R}_{t-1} - \eta\widehat{\pi}_{t+1} - \varphi\widehat{y}_{t+1}\right)\otimes Z_t\right)$$

where T denotes the size of the sample. A GMM estimator $\widehat{\psi}_T$ of ψ is a value of ψ that minimizes the following quadratic loss function:

$$\widehat{\psi}_T = \operatorname{Arg}\min_{\psi} g_T' W_T g_T$$

where W_T represents the weighting matrix. Necessary conditions for identification impose that the number of instrument will be greater than equal to the number of parameters to be estimated. We closely follow previous empirical studies as our instrument set includes

$$Z_t = \left\{\widehat{R}_{t-1}, ..., \widehat{R}_{t-4}, \widehat{\pi}_{t-1}, ..., \widehat{\pi}_{t-4}, \widehat{y}_{t-1}, ..., \widehat{y}_{t-4}\right\}$$

This leads to nine overidentifying restrictions that we can test in order to check for the validity of this specification given the set of instrumental variables. Note that we need an estimate of the weighting matrix W_T before estimating ψ and we need an estimate of ψ before estimating this matrix W_T. An approach suggested by Hansen [1982] is to iterate back and forth between parameter estimation and weighting matrix estimation until a fixed point for ψ is reached. A last point concerns the weighting matrix estimation procedures. We use the Newey and West [1987] estimate with a lag length equals to four. Consider now $\widehat{\psi}_T$ an estimate of ψ for a sample of size T using equations (14). The estimated value summarize a set of conditional moments for the joint process $\{\widehat{R}_t, \widehat{R}_{t-1}, \widehat{\pi}_{t+1}, \widehat{y}_{t+1}\}$. If the J statistic does not reject the model, we can consider that this simple parametric and parsimonious model account for most of the features of the nominal interest rate, inflation rate and output gap. We use this description of the data in order to assess the fit of various monetary models with respect to the specification (12). For each model, we proceed as follows:

Step 1 From *(i)* the log–linear approximation of equilibrium conditions about the deterministic steady state, *(ii)* the vector of the structural parameters θ (see table 1), *(iii)* initials conditions for the predetermined and exogenous variables and *(iv)* the shocks, N simulated paths for a sample of size T, denoted $\tilde{R}_t^i(\theta)$, $\tilde{\pi}_t^i(\theta)$ and $\tilde{y}_t^i(\theta)$ are performed ($i = 1, ..., N$ and $t = 1, ..., T$).

Step 2 We add the deterministic growth component to the simulated cyclical output $\tilde{y}_t^i(\theta)$ and we obtained the output gap – denoted $\tilde{\hat{y}}_{t+1,i}(\theta)$ ($i = 1, ..., N$ and $t = 1, ..., T$) – as in actual data, *i.e.* in deviation from a quadratic trend.

Step 3 An estimate $\tilde{\psi}_T^i(\theta)$ ($i = 1, ..., N$) for ψ minimizes the quadratic form

$$\tilde{\psi}_T^i(\theta) = \text{Arg}\min_{\psi} \tilde{g}'_{T,i}(\theta)\tilde{W}_{T,i}\tilde{g}_{T,i}(\theta)$$

where $\tilde{g}_{T,i}(\theta)$ is the simulated sample analog of (14):

$$\tilde{g}_T(\theta) = \frac{1}{T}\sum\left(\left(\tilde{\hat{R}}_{t,i}(\theta) - \rho\tilde{\hat{R}}_{t-1,i}(\theta) - \eta\tilde{\hat{\pi}}_{t+1,i}(\theta) - \varphi\tilde{\hat{y}}_{t+1,i}(\theta)\right) \otimes \tilde{Z}_{t,i}(\theta)\right)$$

where $\tilde{Z}_{t,i}$ the set of simulated instrumental variables under each structural model:

$$\tilde{Z}_{t,i} = \left\{\tilde{\hat{R}}_{t-1,i}(\theta), ...\tilde{\hat{R}}_{t-4,i}(\theta), \tilde{\hat{\pi}}_{t-1,i}(\theta), ..., \tilde{\hat{\pi}}_{t-4,i}(\theta), \tilde{\hat{y}}_{t-1,i}(\theta), ..., \tilde{\hat{y}}_{t-4,i}(\theta)\right\}$$

The weighting matrix $\tilde{W}_{T,i}$ is estimated using the Newey and West estimate using the same lag length than in the actual data.

Step 4 From these N simulations,we construct the average:

$$\tilde{\psi}_T^N(\theta) = \frac{1}{N}\sum_{i=1}^{N}\tilde{\psi}_T^i(\theta)$$

The simulations (Step 1 to 4) are only used to compute $\psi_T(\theta)$ because direct calculations are complicated. As N becomes large, we may expect that $\tilde{\psi}_T^N(\theta) \approx \psi_T(\theta)$. $\psi_T(\theta)$ represents the implied estimate of the Taylor type rule for a sample of size T. If there exist a small sample biais, this estimated value departs from the value of the parameter implied by each model. However, as T becomes large, we have $\psi_T(\theta) \approx \psi(\theta)$. In what follows, we consider these two estimates of the Taylor rule, denoted $\psi_T(\theta)$ and $\psi(\theta)$.

The estimated value of ψ obtained under each model is then compared to the one obtained from the data. Moreover, using the covariance matrix $\widehat{\Sigma}_T$ of $\widehat{\psi}_T$,

$$\widehat{\Sigma}_T = \left(\left.\frac{\partial g_T'}{\partial \psi}\right|_{\psi=\widehat{\psi}_T} \widehat{W}_T \left.\frac{\partial g_T}{\partial \psi'}\right|_{\psi=\widehat{\psi}_T} \right)^{-1}$$

where $\widehat{W}_T$ is the estimated weighting matrix at convergence, one may construct a Wald statistic

$$\mathcal{W} = \left(\widehat{\psi}_T - \tilde{\psi}_T^N(\theta)\right) \widehat{\Sigma}_T^{-1} \left(\widehat{\psi}_T - \tilde{\psi}_T^N(\theta)\right)'$$

In our case, the three parameters of interest are ρ, η and φ. The statistic is thus asymptotically distributed as a chi-square with three degrees of freedom. Note also that we can construct a Wald statistic associated to each element of ψ separately. This latter allows to locate some failures of the structural models with regard to specific empirical features of the estimated Taylor type rule. In this case, the Wald statistic is distributed as a chi-square with one degree of freedom.

2.2 Calibration

Before any simulation experiments, we need to fix the structural parameters (see Table 1). This concerns the deep parameters which represent preferences and technology and the parameters associated to the forcing variables. It is worth noting that the three models share exactly the same parameters, the difference in their outcomes coming from different assumptions about market arrangements. The discount factor is set to 0.989 implying a long run real interest rate equals to 4.4% per year. The capital share is fixed to 33%. With linear utility in leisure, we do not have to fix the time spent to economic activity for the CIA and SP models. Conversely, long run constraints in the LP model imposes to fix this time spent. We consider that individual allows 70% of their time endowment excluding sleeping hours to economic activities. The share of government spending is fixed to 20% of GDP. The parameters of the forcing variables are fixed in two ways. First concerning the technological and the government spending shocks, we follows the previous calibrations. Second, concerning the money growth we estimate an autoregressive

Table 1: Calibration

Parameters	Value	Description
$\beta^{\star}$	0.989	Subjective discount factor
δ	0.019	Rate of depreciation of the capital stock
α	0.333	Capital share
γ	1.015	Steady state growth rate
h	0.700	Time spend to productive activity
g	0.200	Share of public spending
Money supply		
M_1		
ρ	0.618	Persistence of money growth
σ_m	0.0122	s.e. of money shock
γ_m	1.0126	Rate of growth of money supply
M_2		
ρ	0.562	Persistence of money growth
σ_m	0.0122	s.e. of money shock
γ_m	1.0145	Rate of growth of money supply
Technological shock		
ρ_a	0.95	Persistence of technological shock
σ_a	0.0079	s.e. of technological shock
Government spending shock		
ρ_a	0.97	Persistence of government spending shock
σ_a	0.02	s.e. of government spending shock

process of order one for a sample period 1979:3–2001.1 with quarterly frequency data. We consider alternatively M_1 and M_2 as measures of money stock.[3] It is worth noting that the autoregressive parameters for M_1 is larger than the one frequently used in quantitative exercises.

2.3 Empirical Results

We first present the estimated Taylor from actual data and then discuss whether the three monetary models are consistent with the interest rule estimated from actual the data.

The data are quarterly time series spanning the period 1979:3–2001.1. We use as the interest rate the Federal Funds rate expressed in annual rate. The measure of inflation is the annualized rate of change in the GDP deflator between two subsequent quarters.[4] The output gap is obtained as the deviation of the logarithm of GDP from a quadratic trend.

Table 2 reports the GMM estimates of the Taylor type rule. In the first column, we report the estimated value from actual data and the standard errors are in parentheses.[5] All the estimated parameters have the expected sign: the nominal interest rate displays some inertia, the parameter associated to expected inflation is positive and exceeds unity and the parameter associated to output gap is positive and significant. The parameters of the target are very close to previous estimates: $\widehat{\eta}/(1-\widehat{\rho}) = 1.52$ and $\widehat{\varphi}/(1-\widehat{\rho}) = 0.25$.

We now discuss the implementation of the evaluation method. The estimate $\psi_T(\theta)$ is

[3] The data were obtained from the Federal Reserve Data Bases available at `http://www.stls.frb.org/fred/data/monetary/m1sl` and `m2sl`. Quarterly data are obtained by quarter average.

[4] The data were obtained from the Federal Reserve Data Bases. Federal fund rate: `http://www.stls.frb.org/fred/data/irates/fedfunds`, monthly frequency, average of daily figures. The quarterly rate is defined as the quarter average. GDP deflator: `http://www.stls.frb.org/fred/data/gdp/gdp` for GDP in current dollars divided by `http://www.stls.frb.org/fred/data/gdp/gdpc1` for the GDP of chained 1996 dollars, S.A..

[5] Note also that we report the estimated parameters of the Taylor rule (12). It follows that the estimated values of η and φ do not represent the parameters associated to the target.

obtained using $N = 100$ simulations[6] were used for a sample size equals to 87. Simulated values are redrawn from the same seed values for each evaluation of the three structural models. In order to reduce the effect of initial conditions, simulated samples include 100 initial points which are subsequently discarded in the estimation. The estimate $\psi(\theta)$ is obtained using $T = 10000$.

Tables 2 and 3 report the GMM estimates of the Taylor type rule under the three models. We summarize the main empirical findings as follow.

First, the estimated values for the lagged nominal interest rate and the inflation rate – using either M_1 or M_2 – have the expected positive sign under the flexible prices model. Moreover these values are close to those obtained under actual data. However, the estimated value for the expected output gap is close to zero.

Second, the values for the lagged nominal interest rate and the expected inflation rate under the sticky prices model have the right sign. Nevertheless, the value of the expected inflation rate is significantly greater than the one obtained on actual data.

Third, the estimated value of the expected inflation rate under the limited participation model does not have the expected sign. Moreover, the nominal interest rate displays too weak inertia.

Fourth, the P–value of the Wald statistics clearly shows that any of the model we consider is not able to match simultaneously the conditional moments on the nominal interest rate, the expected inflation rate and the expected output gap at standard level. However, the highest P–value of the Wald statistic associated to all elements of ψ is obtained under the cash–in–advance model. Moreover, the P–value of the Wald statistic associated to each element of ψ shows that the cash–in–advance is able to match the conditional moments on the expected inflation using M_1 as measure of the money stock. Finally, this statistics clearly shows that the sticky prices and the limited participation model do not match

[6]We also performed the estimation with a larger number of simulations, but the results are left unaffected by such a change.

Table 2: Empirical Results ($\widehat{\psi}_T$ and $\psi_T(\theta)$)

Parameters	Historical	CIA model	SP model	LP Model
		M_1		
ρ	0.6984	0.5664	0.5331	0.2448
	(0.043)	[0.21%]	[0%]	[0%]
η	0.4715	0.4450	0.8758	-3.3800
	(0.090)	[76.9%]	[0%]	[0%]
φ	0.0771	0.0014	0.0007	-0.0197
	(0.027)	[0.48%]	[0.44%]	[0%]
$\mathcal{W}(\rho, \eta, \varphi)$		[0%]	[0%]	[0%]
		M_2		
ρ	0.6984	0.5187	0.4970	0.2210
	(0.043)	[0%]	[0%]	[0%]
η	0.4715	0.4398	0.8051	-2.7487
	(0.090)	[72.5%]	[0%]	[0%]
φ	0.0771	0.0014	0.0005	-0.0167
	(0.027)	[0.48%]	[0%]	[0%]
$\mathcal{W}(\rho, \eta, \varphi)$		[0%]	[0%]	[0%]

Note : Standard–error in parentheses. P–value of the Wald statistic in brackets.

Table 3: Empirical Results ($\widehat{\psi}_T$ and $\psi(\theta)$)

Parameters	Historical	CIA model	SP model	LP Model
		M_1		
ρ	0.6984	0.5626	0.5029	0.4392
	(0.043)	[0.16%]	[0%]	[0%]
η	0.4715	0.3540	1.1671	-1.0757
	(0.090)	[19.3%]	[0%]	[0%]
φ	0.0771	-0.0021	0.0013	-0.0731
	(0.027)	[0.31%]	[0.47%]	[0%]
$\mathcal{W}(\rho, \eta, \varphi)$		[0%]	[0%]	[0%]
		M_2		
ρ	0.6984	0.5178	0.4693	0.3761
	(0.043)	[0%]	[0%]	[0%]
η	0.4715	0.2737	1.1174	-0.9076
	(0.090)	[2.84%]	[0%]	[0%]
φ	0.0771	-0.0008	0.0012	-0.0637
	(0.027)	[0.37%]	[0.47%]	[0%]
$\mathcal{W}(\rho, \eta, \varphi)$		[0%]	[0%]	[0%]

Note : Standard–error in parentheses. P–value of the Wald statistic in brackets.

the conditional moments both on *(i)* each element of ψ taken separately and on *(ii)* all elements of ψ.

Fifth, the comparison of the GMM estimates in Tables 2 and 3 shows that there exists a small sample biais concerning the parameter of expected inflation. Conversely, this biais is small for parameter of lagged nominal interest rate.

3 Concluding Remarks

This paper questions the ability of monetary models, implying different monetary effects – Fisherian inflation premia, monetary transmission mechanism or liquidity effect – to quantitatively reproduce a Taylor rule when considered as a way to describe monetary facts. Using the standard econometric procedure, we estimate the relation between the nominal interest rate, the expected inflation and the output gap under three monetary models with exogenous money growth rule. In the flexible prices model, we show that the estimated relation between the nominal interest rate, the expected inflation and the output gap does not depart so much from empirical estimates from actual data spanning the Volcker–Greespan period. Conversely, the sticky prices and the limited participation models are not able to reproduce the relation between the nominal interest rate, the inflation rate and the output gap found in the data. If the Taylor rule is taken as a serious description of monetary facts, our results surprisingly conclude that a monetary model with flexible prices – inconsistent both with the monetary transmission mechanism and the liquidity effect – provides the better match of actual data.

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