Supplementary Material for "Bilateral Control with Vertical Contracts"

Patrick Rey

University of Toulouse (IDEI and GREMAQ)

and

Thibaud Vergé

University of Southampton

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Abstract

This Web Appendix presents complete version of some of the proofs of the paper plus proofs to some of the claims made in the paper.

Section A of this Appendix analyses the problem of non-existence of equilibrium with passive beliefs when contracts are *interim observable.*

Section B presents the complete proof of proposition 4 (Equilibrium with wary beliefs in the price competition setting with interim unobservable contracts).

Finally, section C proposes the complete analysis of the interim observability case and presents the proof of proposition 5 of the paper.

A Interim Observability with Passive Beliefs

We show in this section that in the linear demand case there exists no passive beliefs equilibrium when contracts are interim observable and the substitutability parameter is large $\left(\beta > \widehat{\beta} \simeq 0.806\right)$.

A.1 Quantity competition

At the last stage of the game, each retailer sets the quantity it buys and resells, having observed the contract received (and accepted) by its competitor. Given the linear demand, the retail equilibrium is unique and "symmetric"; quantities and retail profits are given by:

$$q_i = q^C(w_i, w_j) = \frac{2-\beta-2w_i+\beta w_j}{4-\beta^2}, \tag{A1}$$

$$\pi_i^R = \pi^C(w_i, w_j) = \left(\frac{2-\beta-2w_i+\beta w_j}{4-\beta^2}\right)^2. \tag{A2}$$

When receiving an offer (f_i, w_i), R_i anticipates that R_j received the equilibrium offer and therefore faces a marginal cost equal to w_j^*. R_i thus accepts the offer if $f_i \leq \pi^C\left(w_i, w_j^*\right)$. M chooses w_1^* and w_2^* so as to maximize its profit given the acceptable franchise fees:

$$\left(w_1^*, w_2^*\right) = \underset{(w_1,w_2)}{\arg\max} \pi_P(w_1, w_2),$$

where

$$\pi_P(w_1, w_2) = (w_1 - c)\, q^C(w_1, w_2) + (w_2 - c)\, q^C(w_2, w_1) + \pi^C\left(w_1, w_2^*\right) + \pi^C\left(w_2, w_1^*\right).$$

The first-order conditions lead to a unique candidate equilibrium:

$$w_1^* = w_2^* = \frac{-\beta^2 + \left(4-\beta^2\right)c}{2\left(2-\beta^2\right)},$$

while the second-order derivatives are given by:

$$\partial_{11}^2 \pi_P = \frac{-4\left(2-\beta^2\right)}{\left(4-\beta^2\right)^2} \text{ and } \partial_{12}^2 \pi_P = \frac{2\beta}{4-\beta^2}.$$

Second-order conditions ($\partial_{11}^2\pi_P \leq 0$ and $\left|\partial_{11}^2\pi_P\right| \geq \partial_{12}^2\pi_P$) are therefore satisfied only if $\beta \leq \widehat{\beta}$, where $\widehat{\beta} \simeq 0.806$ is the unique solution between 0 and 1 of the equation $4 - 4\beta - 2\beta^2 + \beta^3 = 0$. For $\beta > \widehat{\beta}$, the manufacturer's profit function is not concave and there thus exists no equilibrium with passive beliefs.

A.2 Price competition

The proof is similar for the case of price competition. Equilibrium retail prices and profits are given by:

$$p_i = p^B(w_i, w_j) = \frac{(1-\beta)(2+\beta) + 2w_i + \beta w_j}{4-\beta^2}, \tag{A3}$$

$$\pi_i^R = \pi^B(w_i, w_j) = \frac{\left((1-\beta)(2+\beta) - \left(2-\beta^2\right) w_i + \beta w_j\right)^2}{\left(4-\beta^2\right)^2 \left(1-\beta^2\right)}. \tag{A4}$$

The manufacturer's profit π_P thus becomes, as a function of wholesale prices:

$$\begin{aligned}
\pi(w_1, w_2) &= (w_1 - c) D\left(p^B(w_1, w_2), p^B(w_2, w_1)\right) + \pi^B(w_1, w_2^*) \\
&\quad + (w_2 - c) D\left(p^B(w_2, w_1), p^B(w_1, w_2)\right) + \pi^B(w_2, w_1^*).
\end{aligned}$$

The candidate equilibrium is:

$$w_1^* = w_2^* = \frac{\beta^2 + \left(4-\beta^2\right) c}{4},$$

and second-order derivatives are:

$$\partial_{11}^2 \pi_P = \frac{-4\left(2-\beta^2\right)}{\left(4-\beta^2\right)^2 \left(1-\beta^2\right)} \text{ and } \partial_{12}^2 \pi_P = \frac{2\beta}{\left(4-\beta^2\right)\left(1-\beta^2\right)}.$$

Second-order conditions are therefore again satisfied if and only if $\beta \leq \widehat{\beta}$.

B Price Competition and Interim Unobservability

We study here price competition with *interim* unobservability, and focus on beliefs $W_j(w_i)$ that depend only on the wholesale price (not on the franchise fee). R_i's best reply to the R_j's anticipated retail price $P_j(W_j(w_i))$, for $i \neq j = 1, 2$, is then given by:

$$P_i(w_i) = \arg\max_{p_i}(p_i - w_i)D(p_i, P_j(W_j(w_i))).$$

The manufacturer chooses the equilibrium wholesale prices, w_1^* and w_2^*, so as to maximise its profit

$$\begin{aligned}\pi_P(w_1, w_2) = \quad & (w_1 - c) D(P_1(w_1), P_2(w_2)) + (P_1(w_1) - w_1) D(P_1(w_1), P_1(W_1(w_1))) \\ & + (w_2 - c) D(P_2(w_2), P_1(w_1)) + (P_2(w_2) - w_2) D(P_2(w_2), P_2(W_2(w_2)));\end{aligned}$$

where the wary beliefs satisfy $\partial_1 \pi_P(W_1(w), w) = 0$ and $\partial_2 \pi_P(w, W_2(w)) = 0$.

B.1 Any symmetric equilibrium retail price is strictly lower than the monopoly price

Focusing on symmetric equilibria, the manufacturer's program can be rewritten as follows:

$$(w^*, w^*) = \arg\max_{(w_1, w_2)} \pi_P(w_1, w_2),$$

where

$$\begin{aligned}\pi_P(w_1, w_2) = \quad & \pi^M(P(w_1), P(w_2)) + (P(w_1) - w_1)(D(P(w_1), P(W(w_1))) - D(P(w_1), P(w_2))) \\ & + (P(w_2) - w_2)(D(P(w_2), P(W(w_2))) - D(P(w_2), P(w_1)))\end{aligned}$$

and

- $\pi^M(p_1, p_2) = (p_1 - c) D(p_1, p_2) + (p_2 - c) D(p_1, p_2);$
- the retailers' pricing strategy is

$$P(w) = \arg\max_p (p - w) D(p, P(W(w))); \qquad (P)$$

- and the wary beliefs $W(w)$ are such that

$$\partial_1 \pi_P(W(w), w) = 0. \qquad (W)$$

B.1.1 The equilibrium retail price is lower than the monopoly price $\left(p^* \leq p^M\right)$

Let us first show that any symmetric equilibrium retail price must be lower than the monopoly price, characterized by $\partial_1 \pi^M \left(p^M, p^M\right) = 0$.

- **First-order condition of the manufacturer's maximization program:**

The first-order condition of the manufacturer's program is $\partial_1 \pi_P (w^*, w^*) = 0$, that is, with $p^* = P(w^*)$,

$$\left[\partial_1 \pi^M (p^*, p^*) + (p^* - w^*) \partial_2 D (p^*, p^*) (W'(w^*) - 1)\right] P'(w^*) = 0.$$

- **Step 1:** $P'(w^*) \neq 0$

The first-order condition of the retailers' program writes as:

$$(P(w) - w) \partial_1 D (P(w), P(W(w))) + D (P(w), P(W(w))) = 0. \tag{B1}$$

Differentiating (B1) at $w = w^*$ yields:[1]

$$\begin{aligned} (P'(w^*) - 1) \partial_1 D + (p^* - w^*) (\partial^2_{11} D + \partial^2_{12} D.W'(w^*)) P'(w^*) & \\ (\partial_1 D + \partial_2 D.W'(w^*)) P'(w^*) &= 0, \end{aligned}$$

or

$$\left[2\partial_1 D + \partial_2 D.W'(w^*) + (p^* - w^*) \left(\partial^2_{11} D + \partial^2_{12} D.W'(w^*)\right)\right] P'(w^*) = \partial_1 D. \tag{B2}$$

$\partial_1 D < 0$ then implies $P'(w^*) \neq 0$.

- **Step 2:** $p^* \leq p^M$

Since $P'(w^*) \neq 0$, the first-order condition of the manufacturer's program simplifies to

$$\partial_1 \pi^M (p^*, p^*) = -(p^* - w^*) \partial_2 D (p^*, p^*) (W'(w^*) - 1). \tag{B3}$$

Differentiating (W) with respect to w yields:

$$\partial^2_{11} \pi_P (W(w), w) W'(w) + \partial^2_{12} \pi_P (W(w), w) = 0, \tag{B4}$$

which, evaluated at $w = w^*$, leads to:

$$\partial^2_{11} \pi_P (w^*, w^*) .W'(w^*) + \partial^2_{12} \pi_P (w^*, w^*) = 0 \Leftrightarrow W'(w^*) = -\frac{\partial^2_{12} \pi_P (w^*, w^*)}{\partial^2_{11} \pi_P (w^*, w^*)}.$$

[1] The derivatives of the demand function D are all evaluated at (p^*, p^*).

The second-order conditions of the manufacturer's program thus requires $|W'(w^*) \leq 1|$.

Evaluating $(B1)$ at $w = w^*$ yields

$$-(p^* - w^*)\,\partial_1 D(p^*, p^*) = D(p^*, p^*),$$

and thus $p^* > w^*$. Finally, since $\partial_2 D(p^*, p^*) > 0$:

- either $W'(w^*) < 1$, in which case $(B3)$ implies $\partial_1 \pi^M(p^*, p^*) < 0$; the concavity of the function π^M then ensures that $p^* < p^M$;
- or $W'(w^*) = 1$, in which case $p^* = p^M$.

B.1.2 The monopoly price is not an equilibrium price $(\Leftrightarrow W'(w^*) \neq 1)$.

In order to sustain the monopoly price as an equilibrium price, the equilibrium wholesale price $\left(w^* = w^M\right)$ must satisfy:

$$D\left(p^M, p^M\right) + \left(p^M - w^M\right)\partial_1 D\left(p^M, p^M\right) = 0 \text{ and } W'\left(w^M\right) = 1.$$

The second condition implies that $\partial_{11}^2 \pi_P\left(w^M, w^M\right) + \partial_{12}^2 \pi_P(w^*, w^*) = 0$, and we thus need to look at third-order effects. We now show that the gain from a symmetric deviation $\left(w^M + \varepsilon, w^M + \varepsilon\right)$ is strictly positive for $\varepsilon > 0$ (small enough), thereby ruling out w^M as a possible equilibrium wholesale price. The gain from such a deviation is:

$$\delta(\varepsilon) = \pi_P(w^M + \varepsilon, w^M + \varepsilon) - \pi_P(w^M, w^M).$$

If w^M is a symmetric equilibrium wholesale price, since $\partial_{11}^2 \pi_P\left(w^M, w^M\right) + \partial_{12}^2 \pi_P\left(w^M, w^M\right) = 0$ and $\delta'(0) = 0$, we also have $\delta''(0) = 0$. Using the symmetry of the profit function π_P, the third-order derivative is given by:

$$\delta'''(0) = 2\partial_{111}^3 \pi_P(w^M, w^M) + 6\partial_{112}^3 \pi_P(w^M, w^M). \tag{B5}$$

Differentiating $(B4)$ with respect to w at $w = w^M$ yields, using $W'\left(w^M\right) = 1$ and the symmetry of the profit function π_P:

$$\partial_{111}^3 \pi_P\left(w^M, w^M\right) + 3\partial_{112}^3 \pi_P\left(w^M, w^M\right) + \partial_{11}^2 \pi_P\left(w^M, w^M\right) W''\left(w^M\right) = 0. \tag{B6}$$

Using $(B6)$, we can rewrite $(B5)$ as:

$$\delta'''(0) = -2\partial_{11}^2 \pi_P\left(w^M, w^M\right) W''\left(w^M\right).$$

We thus need to show that $W''\left(w^M\right) > 0$.

The beliefs $W(w)$ are such that $\partial_1 \pi_P (W_1(w), w) = 0$, that is:

$$\begin{aligned} & P'(W(w)) \left[\partial_1 \pi^M (P(W(w)), P(w)) + (P(w) - w) \partial_1 D (P(w), P(W(w))) \right. \\ & + (P(W(w)) - W(w)) \left(\partial_1 D (P(W(w)), P(W(W(w))) - \partial_1 D (P(W(w)), P(w))\right] \\ & + (P'(W(w)) - 1) \left[D (P(W(w)), P(w)) - D (P(W(w)), P(W(W(w)))\right] \\ & + P'(W(W(w))) W'(W(w)) (P(W(w)) - w) \partial_2 D (P(W(w)), P(W(W(w))) \quad = 0. \end{aligned}$$

Differentiating this equation with respect to w at $w = w^M$ (using $\partial_1 \pi^M \left(p^M, p^M\right) = 0$ and $W'\left(w^M\right) = 1$) leads to:

$$W''\left(w^M\right)\left(p^M - w^M\right) \partial_2 D\left(p^M, p^M\right) = -P'\left(w^M\right)\left(\partial_{11}^2 \pi^M\left(w^M, w^M\right) + \partial_{12}^2 \pi^M\left(w^M, w^M\right)\right).$$

Thus $W''\left(w^M\right)$ has the same sign as $P'\left(w^M\right)$. Evaluating $(B2)$ at $w = w^M$ leads to:[2]

$$P'\left(w^M\right)\left(2\partial_1 D + \partial_2 D + \left(p^M - w^M\right)\left(\partial_{11}^2 D + \partial_{12}^2 D\right)\right) = \partial_1 D.$$

The strict concavity of the profit function $\left(p_1 - w^M\right) D(p_1, p_2) + \left(p_2 - w^M\right) D(p_2, p_1)$ ensures that

$$\partial_1 D + \partial_2 D + \left(p^M - w^M\right)\left(\partial_{11}^2 D + \partial_{12}^2 D\right) < 0,$$

which in turns establishes $P'\left(w^M\right) > 0$ and concludes the proof.

B.2 Existence (and uniqueness) with polynomial beliefs

We now restrict attention to the linear demand case:

$$D(p_i, p_j) = \frac{1 - \beta - p_i + \beta p_j}{1 - \beta^2}.$$

For each retailer's maximization program, the first-order condition is then necessary and sufficient and writes as:

$$2P_i(w_i) - \beta P_j(W_j(w_i)) = 1 - \beta + w_i. \tag{P_i}$$

Using (P_i) we thus have:

$$D(P_i(w_i), P_j(W_j(w_i))) = \frac{P_i(w_i) - w_i}{1 - \beta^2}.$$

[2] The derivatives of the demand function being evaluated at $\left(p^M, p^M\right)$.

R_i's beliefs are such that $\partial_2 \pi_P (w_i, W_j(w_i)) = 0$, that is:

$$\begin{aligned}((1-\beta)c + \beta w_i - W_j(w_i)) P_j'(W_j(w_i)) + 1 - \beta - P_j(W_j(w_i)) + \beta P_i(w_i) \\ +2\left(P_j'(W_j(w_i)) - 1\right)(P_j(W_j(w_i)) - W_j(w_i)) &= 0.\end{aligned} \tag{W_j}$$

Let us now consider the polynomial solutions to the system consisting of equations $((W_i),(P_i))_{i=1,2}$. We denote by n_i and m_i the degrees of the polynomials $W_i(w_j)$ and $P_i(w_i)$, and by $\omega_{i,k}$ and $p_{i,k}$ the coefficients of their terms of degree k:

$$W_i(w) = \sum_{k=0}^{n_i} \omega_{i,k} w^k \text{ and } P_i(w) = \sum_{k=0}^{m_i} \pi_{i,k} w^k.$$

- **Step 1: any polynomial solution is affine** $(0 \leq m_1, m_2, n_1, n_2 \leq 1)$

Consider (P_i):

$$\underbrace{2P_i(w_i)}_{\text{deg}=m_i} - \underbrace{\beta P_j(W_j(w_i))}_{\text{deg}=m_j n_j} \quad = \quad \underbrace{1-\beta+w_i}_{\text{deg}=1}.$$

Three cases can arise:

1. $m_i < m_j n_j$. This implies $m_i = 0$ and $m_j = n_j = 1$. Then (W_i) reduces to

$$1 - \beta - \pi_{i,0} + \beta P_j(w_j) - 2(\pi_{i,0} - W_i(w_j)) = 0,$$

   and thus $n_i = 1$.

2. $m_i > m_j n_j$. This implies $m_i = 1$ and $m_j n_j = 0$. Thus, either $m_j = 0$ or $m_j > 0$ and $n_j = 0$.

   (a) The case $m_j = 0$ is similar to case 1 (reverting the roles of i and j).

   (b) If $m_j > 0$ then $n_j = 0$ and (P_j) reduces to:

$$\begin{aligned} 2P_j(w) &= \beta P_i(W_i(w)) + 1 - \beta + w \\ &= \beta(\pi_{i,0} + \pi_{i,1} W_i(w)) + 1 - \beta + w. \end{aligned}$$

   and therefore $m_j = \max(n_i, 1)$. If $n_i \leq 1$, then no degree exceeds 1. The only remaining case is $m_j = n_j \geq 2$. Since $m_i = 1$ and $n_j = 0$, equation (P_i) leads to

$$P_i(w) = \frac{1}{2}(1 - \beta + \beta P_j(\omega_{j,0}) + w) \Rightarrow P_i'(w) = \frac{1}{2}.$$

   Differentiating (W_i) and (P_j) twice then yields respectively

$$\beta P_j''(w) = \frac{1}{2} W_i''(w) \text{ and } \beta W_i''(w) = 4P_j''(w),$$

   implying $2\beta^2 P_j''(w) = W_i''(w) = 4P_j''(w)$ ($\neq 0$ since $n_i = m_j \geq 2$), a contradiction.

3. $m_i = m_j n_j \geq 1$. In this case, either $m_j = m_i n_i \geq 1$ or all degrees are equal or lower than 1 (simply inverting the roles played by i and j in cases 1 and 2).

But $m_i = m_j n_j \geq 1$ and $m_j = m_i n_i \geq 1$ imply $n_i = n_j = 1$ and $m_j = m_i = m \geq 1$. The only interesting case is when $m \geq 2$. Then (W_j) yields:

$$\underbrace{\left((1-\beta)c + \beta w_i - W_j(w_i)\right) P_j'(W_j(w_i)) + 1 - \beta - P_j(W_j(w_i)) + \beta P_i(w_i)}_{\deg \leq m}$$
$$\underbrace{+2\left(P_j'(W_j(w_i)) - 1\right)\left(P_j(W_j(w_i)) - W_j(w_i)\right)}_{\deg = 2m-1 \geq 3} = 0,$$

which contradicts $m > 1$.

This concludes the proof and shows that polynomial solutions must be affine.

- **Step 2: any equilibrium with affine wary beliefs satisfies $\pi_{1,1} = \pi_{2,1}$ and $\omega_{1,1} = \omega_{2,1}$.**

With affine beliefs, (P_i) reduces to

$$2\left(\pi_{i,0} + \pi_{i,1} w\right) - \beta\left(\pi_{j,0} + \pi_{j,1}\left(\omega_{j,0} + \omega_{j,1} w\right)\right) = 1 - \beta + w,$$

and since it holds for any w, it implies

$$2\pi_{i,0} - \beta\pi_{j,0} = 1 - \beta + \beta\pi_{j,1}\omega_{j,0}, \tag{B7}$$
$$2\pi_{i,1} - \beta\pi_{j,1}\omega_{j,1} = 1. \tag{B8}$$

$(B8_i)$ and $(B8_j)$ yield

$$\pi_{i,1} = \frac{2 + \beta\omega_{j,1}}{4 - \beta^2\omega_{1,1}\omega_{2,1}},$$

and thus:

$$\left(4 - \beta^2\omega_{1,1}\omega_{2,1}\right)\left(\pi_{i,1} - \pi_{j,1}\right) = \beta\left(\omega_{j,1} - \omega_{i,1}\right). \tag{B9}$$

Similarly, (W_j) implies:

$$2\left(\pi_{j,1}^2 - 3\pi_{j,1} + 1\right)\omega_{j,0} = -1 + \beta - (1-\beta)c\pi_{j,1} + \left(3 - 2\pi_{j,1}\right)\pi_{j,0} - \beta\pi_{i,0}, \tag{B10}$$
$$2\left(\pi_{j,1}^2 - 3\pi_{j,1} + 1\right)\omega_{j,1} = -\beta\left(\pi_{i,1} + \pi_{j,1}\right). \tag{B11}$$

Using $(B8)$ to replace $\pi_{j,1}\omega_{j,1}$ in $(B11)$ yields:

$$6+\beta^2\left(\pi_{i,1}+\pi_{j,1}\right)+4\pi_{i,1}\pi_{j,1}+2\beta\omega_{j,1}=12\pi_{i,1}-2\pi_{j,1}. \tag{B12}$$

Substracting $(B12_j)$ to $(B12_i)$, we have:

$$5(\pi_{i,1}-\pi_{j,1})=\beta\left(\omega_{j,1}-\omega_{i,1}\right), \tag{B13}$$

which, combined with $(B9)$, imposes:

$$\left(1+\beta^2\omega_{1,1}\omega_{2,1}\right)\left(\pi_{i,1}-\pi_{j,1}\right)=0. \tag{B14}$$

But the second-order conditions of the manufacturer's program impose $0\leq\omega_{1,1}\omega_{2,1}\leq 1$.[3] Therefore, $(B14)$ imposes $\pi_{1,1}=\pi_{2,1}=\pi_1$ and thus $\omega_{1,1}=\omega_{2,1}=\omega_1$.

Given the symmetry, $(B8)$ and $(B11)$ simplify to

$$\begin{aligned} \beta\omega_1\pi_1 &= 2\pi_1-1, & \text{(B15)}\\ \left(\pi_1^2-3\pi_1+1\right)\omega_1 &= -\beta\pi_1. & \text{(B16)}\end{aligned}$$

- **Step 3: there exists a unique pair (π_1^*,ω_1^*) satisfying $(B15)$ and $(B16)$ as well as second-order conditions.**

Let us use $(B15)$ to eliminate ω_1 in $(B16)$:

$$\left(\pi_1^2-3\pi_1+1\right)(2\pi_1-1)=-\beta^2\pi_1^2 \tag{B17}$$

$$\Leftrightarrow 2\pi_1^3-\left(7-\beta^2\right)\pi_1^2+5\pi_1-1=0. \tag{B18}$$

The left-hand side is a polynomial φ of degree 3 such that:

$$\varphi(0)=-1<0<\varphi\left(\frac{1}{2}\right)=\frac{\beta^2}{4}\text{ and }\varphi(1)=-(1-\beta^2)<0<\varphi\left(+\infty\right).$$

Therefore, φ has three roots: one in $\left]0,\frac{1}{2}\right[$, one in $\left]\frac{1}{2},1\right[$ and one in $]1,+\infty[$.

Using the retailers's responses, the manufacturer's profit can be expressed as

$$\begin{aligned}\pi_P\left(w_1,w_2\right)= &\left[(w_1-c)D(P_1(w_1),P_2(w_2))+\frac{(P_1(w_1)-w_1)^2}{1-\beta^2}\right.\\ &\left.+(w_2-c)D(P_2(w_2),P_1(w_1))+\frac{(P_2(w_2)-w_2)^2}{1-\beta^2}\right].\end{aligned} \tag{B19}$$

[3] Beliefs satisfy $\partial_1\pi^P\left(W_1(w),w\right)=0$ and $\partial_2\pi^P\left(w,W_2(w)\right)=0$. Therefore, $\omega_{i,1}=-\partial^2_{12}\pi^P/\partial^2_{ii}\pi^P$ and the second-order conditions of the manufacturer's program impose $\omega_{1,1}\omega_{2,1}=\left(\partial^2_{12}\pi^P\right)^2/\partial^2_{11}\pi^P\partial^2_{22}\pi^P>0$.

Therefore:

$$\begin{aligned}\partial_1\pi_P(w_1,w_2) &= \frac{\pi_1}{1-\beta^2}\left(-(w_1-c)+\beta(w_2-c)\right)+D(P_1(w_1),P_2(w_2))\\ &+\frac{2}{1-\beta^2}(\pi_1-1)(P_1(w_1)-w_1),\end{aligned}$$

and

$$\begin{aligned}\partial^2_{11}\pi_P &= \frac{2}{1-\beta^2}\left(\pi_1^2-3\pi_1+1\right),\\ \partial^2_{12}\pi_P &= \frac{2}{1-\beta^2}\beta\pi_1.\end{aligned}$$

A first necessary condition is $\partial^2_{11}\pi_P \leq 0$, that is $\pi_1^2-3\pi_1+1\leq 0$. Together with $(B17)$, it implies

$$2\pi_1-1>0 \Leftrightarrow \pi_1>\frac{1}{2}. \tag{B20}$$

A second necessary condition is $\left(\partial^2_{11}\pi_P\right)^2 \geq \left(\partial^2_{12}\pi_P\right)^2$, which is equivalent to

$$\left(\pi_1^2-3\pi_1+1\right)^2-\beta^2\pi_1^2\geq 0$$

$$\begin{aligned}&\Leftrightarrow \underbrace{-\left(\pi_1^2-3\pi_1+1\right)(2\pi_1-1)-\beta^2\pi_1^2}_{=0 \text{ from } (B17)}-\pi_1(1-\pi_1)\left(\pi_1^2-3\pi_1+1\right)\geq 0\\ &\Leftrightarrow \pi_1(1-\pi_1)\geq 0 \Leftrightarrow 0\leq\pi_1\leq 1. \qquad\qquad (B21)\end{aligned}$$

Together, $(B20)$ and $(B21)$ impose that the solution of $(B18)$ is the unique root of φ in $\left]\frac{1}{2},1\right[$. $(B15)$ then uniquely defines ω_1^*:

$$\omega_1^*=\frac{2\pi_1^*-1}{\beta\pi_1^*}>0.$$

- **Step 4: the solution of the overall program, if it exists, is symmetric.**

Substracting $(B7_j)$ from $(B7_i)$ and $(B10_j)$ from $(B10_i)$ yields respectively:

$$\begin{aligned}(2+\beta)(\pi_{1,0}-\pi_{2,0}) &= \beta\pi_1(\omega_{1,0}-\omega_{2,0}),\\ 2\left(\pi_1^2-3\pi_1+1\right)(\omega_{1,0}-\omega_{2,0}) &= (3+\beta-2\pi_1)(\pi_{1,0}-\pi_{2,0}),\end{aligned}$$

thus implying

$$2(2+\beta)\left(\pi_1^2-3\pi_1+1\right)(\pi_{1,0}-\pi_{2,0})=\beta\pi_1(3-\beta-2\pi_1)(\pi_{1,0}-\pi_{2,0}).$$

But then $\pi_1^2-3\pi_1+1<0$ and $\frac{1}{2}<\pi_1<1$ imply $\pi_{1,0}=\pi_{2,0}$ and thus $\omega_{1,0}=\omega_{2,0}$.

- **Step 5: there exists a unique solution.**

Given the symmetry, $(B7)$and $(B10)$ reduce to:

$$\begin{aligned} (2-\beta)\pi_0 - \beta\pi_1\omega_0 &= 1-\beta, && \text{(B22)} \\ (3-\beta-2\pi_1)\pi_0 - 2\left(\pi_1^2 - 3\pi_1 + 1\right)\omega_0 &= 1-\beta+(1-\beta)c\pi_1. && \text{(B23)} \end{aligned}$$

The determinant is

$$-2(2-\beta)\left(\pi_1^2 - 3\pi_1 + 1\right) + \beta\pi_1(3-\beta-2\pi_1) > 0.$$

It is positive since $(\pi_1^2 - 3\pi_1 + 1) < 0$ and $\frac{1}{2} < \pi_1 < 1$. Therefore, $(B22)$ and $(B23)$ uniquely define π_0^* and ω_0^* as functions of π_1. The equilibrium retail price is then

$$p^* = \frac{1-\beta+w^*}{2-\beta},$$

where

$$w^* = W(w^*) = \omega_0^*(\pi_1^*) + \omega_1^*(\pi_1^*)\, w^* = \frac{\omega_0^*(\pi_1^*)}{1-\omega_1^*(\pi_1^*)}.$$

C Interim Observability with Wary Beliefs[4]

We assume in this section that contracts are *interim* observable: contract offers are initially secret (acceptance decisions are therefore based on beliefs) but retailers observe the accepted contracts before competing (in prices or in quantities) on the final market. The equilibrium of the retail competition subgame is therefore the solution of a standard Cournot $\left(q_i = q^C(w_i, w_j)\right)$ or Bertrand-fashion $\left(p_i = p^B(w_i, w_j)\right)$ competition game for which the firms face costs equal to w_i and w_j. In what follows, we denote by $q^R(w_i, w_j)$, $p^R(w_i, w_j)$ and $\pi^R(w_i, w_j)$, the retail quantity, price and profit emerging from the retail competition subgame.

- In the case of quantity competition:

$$q^R(w_i, w_j) = q^C(w_i, w_j) \text{ and } p^R(w_i, w_j) = P\left(q^R(w_i, w_j), q^R(w_j, w_i)\right).$$

- In the case of price competition:

$$p^R(w_i, w_j) = p^B(w_i, w_j) \text{ and } q^R(w_i, w_j) = D\left(p^B(w_i, w_j), p^B(w_j, w_i)\right).$$

- In both cases the retail profit is given by:

$$\pi^R(w_i, w_j) = \left(p^R(w_i, w_j) - w_i\right) q^R(w_i, w_j).$$

Let us denote by $\pi_I(w_1, w_2)$, the industry profit in any of these two cases, that is:

$$\pi_I(w_1, w_2) = \left(p^R(w_1, w_2) - c\right) q^R(w_1, w_2) + \left(p^R(w_2, w_1) - w_2\right) q^R(w_1, w_2).$$

In what follows, we make the following assumptions:

(H_1) The demand function is such that, both in the Bertrand-like and the Cournot-like frameworks, $q^R(w_i, w_j)$ and $\pi^R(w_i, w_j)$ are strictly decreasing in w_i and strictly increasing in w_j. Moreover, the direct effect dominates, that is, $q^R(w, w)$ and $\pi^R(w, w)$ are strictly decreasing in w.

(H_2) The industry profit function $\pi_I(w_1, w_2)$ is strictly concave in (w_1, w_2) and reaches its maximum at a unique symmetric point $\left(w^M, w^M\right)$.

[4]This section presents the complete analysis of the interim observability case briefly presented in the paper (including the proof of proposition 5).

When being offered a contract $t_i = (f_i, w_i)$, R_i again expects M to offer and R_j to accept a tariff $T_j(t_i)$ given by:

$$\begin{aligned} T_j(t_i) &= \arg\max_{(w_j,f_j)} (w_i - c) q^R(w_i, w_j) + f_i + (w_j - c) q^R(w_j, w_i) + f_j \\ \text{s.t.} &: \quad f_j \leq \pi^R(w_j, W_i(t_j)) . \end{aligned}$$

Clearly, the solution of this program does not depend on f_i and, since the objective function is strictly increasing in f_j, the constraint must be binding. The rival's anticipated contract is thus given by:

$$W_j(w_i) = \arg\max_{w_j} (w_i - c) q^R(w_i, w_j) + (w_j - c) q^R(w_j, w_i) + \pi^R(w_j, W_i(w_j)) \tag{C1}$$

and

$$F_j(w_i) = \pi^R(W_j(w_i), W_i(W_j(w_i))) .$$

In contrast with the Cournot-like case with *interim* unobservability, the objective function in $(C1)$ is no longer separable in w_i and w_j. This implies that the beliefs will now depend on w_i. Wary beliefs thus differ from passive beliefs.

M chooses wholesale prices w_1^* and w_2^* that maximize its profit $\pi_P(w_1, w_2)$ given the acceptable franchise fees, with:

$$\begin{aligned} \pi_P(w_1, w_2) = \quad & (w_1 - c) q^R(w_1, w_2) + \pi^R(w_1, W_2(w_1)) \\ & + (w_2 - c) q^R(w_2, w_1) + \pi^R(w_2, W_1(w_2)) . \end{aligned}$$

Note that beliefs satisfy:

$$\partial_1 \pi_P [W_1(w), w] = 0 \text{ and } \partial_2 \pi_P [w, W_2(w)] = 0 \tag{C2}$$

and that equilibrium wholesale prices (w_1^*, w_2^*) satisfy $w_1^* = W_1(w_2^*)$ and $w_2^* = W_2(w_1^*)$.

The following proposition provides some characterization of wary beliefs equilibria.

Proposition 5 *When contracts are interim observable, wary beliefs no longer coincide with passive beliefs. If retailers have wary beliefs, then:*

(i) in any symmetric equilibrium, the equilibrium retail price is strictly lower than the monopoly price and the manufacturer therefore does not obtain the monopoly profit; and

(ii) if demand is linear, there exists a unique equilibrium with polynomial beliefs and this equilibrium is symmetric.

C.1 The equilibrium price is strictly lower than the monopoly price

In the symmetric case, the manufacturer's profit function rewrites as:

$$\begin{aligned}\pi_P(w_1,w_2)=\pi_I(w_1,w_2) &+ \pi^R(w_1,W(w_1))-\pi^R(w_1,w_2)\\ &+ \pi^R(w_2,W(w_2))-\pi^R(w_2,w_1);\end{aligned}$$

with the wary beliefs $W(w)$ being given by:

$$\partial_1\pi_P(W(w),w)=0. \tag{C3}$$

- **Step 1: any symmetric equilibrium price is lower than the monopoly price**

Focusing on symmetric equilibria, the first-order condition of the manufacturer's maximization program is:

$$\begin{aligned}\partial_1\pi_I(w^*,w^*)+\partial_1\pi^R(w^*,w^*)+\partial_2\pi^R(w^*,w^*)\,W'(w^*)&\\ -\partial_1\pi^R(w^*,w^*)-\partial_2\pi^R(w^*,w^*)&=0,\end{aligned}$$

which simplifies into:

$$\partial_1\pi_I(w^*,w^*)=-\partial_2\pi^R(w^*,w^*)\left(W'(w^*)-1\right). \tag{C4}$$

Differentiating $(C3)$ yields:

$$\partial^2_{11}\pi_P[W(w),w]\,W'(w)+\partial^2_{12}\pi_P[W(w),w]=0. \tag{C5}$$

Evaluating $(C5)$ at $w=w^*$ leads to

$$W'(w^*)=-\frac{\partial^2_{12}\pi_P(w^*,w^*)}{\partial^2_{11}\pi_P(w^*,w^*)}.$$

The second-order conditions of the manufacturer's maximisation program thus imply $|W'(w^*)|\leq 1$. Since $\partial_2\pi^R(w^*,w^*)>0$, we have $\partial_1\pi_I(w^*,w^*)\leq 0$. Assumption (H_2) then implies that $w^*\leq w^M$ and, under assumption (H_1), this leads to:

$$q^R(w^*,w^*)\geq q^M\left(w^M,w^M\right)\Leftrightarrow p^R(w^*,w^*)\leq p^M.$$

- **Step 2: any symmetric equilibrium price is strictly lower than the monopoly price**

The monopoly outcome can be sustained at an equilibrium with wary beliefs if and only if $w^* = w^M$, that is, for $W'\left(w^M\right) = 1$. To show that this cannot be an equilibrium, consider the symmetric deviations $\left(w^M + \varepsilon, w^M + \varepsilon\right)$, and check that the gain from such deviations is strictly positive for $\varepsilon > 0$ small enough This gain is:

$$\delta\left(\varepsilon\right) = \pi_P(w^M + \varepsilon, w^M + \varepsilon) - \pi_P(w^M, w^M).$$

If w^M is a symmetric equilibrium wholesale price, since:

$$\delta'\left(0\right) = 0 \text{ and } \partial^2_{11}\pi_P\left(w^M, w^M\right) + \partial^2_{12}\pi_P\left(w^M, w^M\right) = 0,$$

we also have $\delta''\left(0\right) = 0$. We thus need to compute the third-order derivative, which using the symmetry of the profit function π_P is given by:

$$\delta'''\left(0\right) = 2\partial^3_{111}\pi_P(w^M, w^M) + 6\partial^3_{112}\pi_P(w^M, w^M). \tag{C6}$$

Differentiating $(C5)$ with respect to w at $w = w^M$, using $W'\left(w^M\right) = 1$ and the symmetry of the profit function π_P leads to:

$$\partial^3_{111}\pi_P(w^M, w^M) + 3\partial^3_{112}\pi_P(w^M, w^M) + 2\partial^2_{11}\pi_P\left(w^M, w^M\right)W''\left(w^M\right) = 0 \tag{C7}$$

Using $(C7)$, $(C6)$ rewrites as:

$$\delta'''\left(0\right) = -2\partial^2_{11}\pi_P\left(w^M, w^M\right)W''\left(w^M\right).$$

We thus need to show that $W''\left(w^M\right) > 0$.

The wary beliefs $W\left(w\right)$ are given by $\partial_1\pi_P\left[W(w), w\right] = 0$, or:

$$\begin{aligned}\partial_1\pi_I\left(W\left(w\right), w\right) + \partial_1\pi^R\left(W\left(w\right), W\left(W\left(w\right)\right)\right) + \partial_2\pi^R\left(W\left(w\right), W\left(W\left(w\right)\right)\right)W'\left(W\left(w\right)\right) \\ -\partial_1\pi^R\left(w, W\left(w\right)\right) - \partial_1\pi^R\left(w, W\left(w\right)\right) &= 0.\end{aligned} \tag{C8}$$

Differentiating $(C8)$ with respect to w at $w^* = w^M$, and using $W'\left(w^M\right) = 1$ leads to:[5]

$$\begin{aligned}\partial^2_{11}\pi_I + \partial^2_{12}\pi_I + \partial^2_{11}\pi^R + \partial^2_{12}\pi^R + \partial^2_{21}\pi^R + \partial^2_{22}\pi^R \\ +\partial_2\pi^R.W''\left(w^M\right) - \partial^2_{11}\pi^R - \partial^2_{12}\pi^R - \partial^2_{11}\pi^R - \partial^2_{12}\pi^R &= 0,\end{aligned}$$

[5]All the derivatives of the profit functions π_I and π^R are evaluated at $\left(w^M, w^M\right)$.

which, using the symmetry of the profit function π^R, simplifies into:

$$\partial_2 \pi^R\left(w^M, w^M\right) W''\left(w^M\right) = -\left(\partial^2_{11}\pi_I\left(w^M, w^M\right) + \partial^2_{12}\pi_I\left(w^M, w^M\right)\right).$$

Assumptions (H_1) and (H_2) then ensure that $W''\left(w^M\right) > 0$, which concludes the proof of section (i) of proposition 5.

C.2 Existence under Quantity Competition

We now prove the existence and the uniqueness of an equilibrium with polynomial wary beliefs in the Cournot-like framework. When being offered a wholesale price w_i, R_i expects M to offer and R_j to accept a wholesale price $W_j\left(w_i\right)$ and a franchise fee $F_j\left(w_i\right)$. R_i thus accept the contract $\left(f_i, w_i\right)$ if $f_i \leq \pi^R\left(w_i, W_j\left(w_i\right)\right)$. Wary beliefs must satisfy:

$$F_j(w_i) = \pi^R\left(w_j, W_i\left(w_j\right)\right)\Big|_{w_j = W_j(w_i)} \tag{C9}$$

and

$$W_j(w_i) = \arg\max_{w_j}\left[(w_i - c)q^R\left(w_i, w_j\right) + (w_j - c)q^R\left(w_j, w_i\right) + \pi^R\left(w_j, W_i\left(w_j\right)\right)\right],$$

where:

$$\begin{aligned} q^R\left(w_i, w_j\right) &= q^C\left(w_i, w_j\right) = \frac{2 - \beta - 2w_i + \beta w_j}{4 - \beta^2}, \\ \text{and } \pi^R\left(w_i, w_j\right) &= \pi^C\left(w_i, w_j\right) = \left(\frac{2 - \beta - 2w_i + \beta w_j}{4 - \beta^2}\right)^2. \end{aligned}$$

The first-order condition characterizing the beliefs is thus:

$$\begin{aligned} &-(2-\beta)\left(\beta^2 - \left(4 - \beta^2\right)c\right) + 2\beta\left(4 - \beta^2\right)w_i - 4\beta W_i\left(W_j\left(w_i\right)\right) \\ &-4\left(2 - \beta^2\right)W_j\left(w_i\right) + 2\beta\left(2 - \beta - 2W_j\left(w_i\right) + \beta W_i\left(W_j\left(w_i\right)\right)\right)W_i'\left(W_j\left(w_i\right)\right) = 0. \end{aligned} \tag{W_j}$$

We focus here on polynomial wary beliefs, of the form:

$$W_1(w) = \sum_{k=0}^{n_1}\omega_{1,k}w^k \text{ and } W_2(w) = \sum_{k=0}^{n_2}\omega_{2,k}w^k,$$

and characterize the unique perfect Bayesian equilibrium with such beliefs. We first show below that beliefs are affine and symmetric. We then check that there exists a unique equilibrium.

- **Step 1: any polynomial wary belief is affine** $(n_1 = n_2 = 1)$.

Condition (W_j) rewrites as:

$$\begin{aligned}\underbrace{-\left(2-\beta\right)\left(\beta^2-\left(4-\beta^2\right)c\right)+2\beta\left(4-\beta^2\right)w_i}_{\deg=1}-\underbrace{4\left(2-\beta^2\right)W_j\left(w_i\right)}_{\deg=n_j}\\ -\underbrace{4\beta W_i\left(W_j\left(w_i\right)\right)}_{\deg=n_in_j}+\underbrace{2\beta\left(2-\beta\right)W_i'\left(W_j\left(w_i\right)\right)}_{\deg=(n_i-1)n_j}\\ -\underbrace{4\beta W_j\left(w_i\right)W_i'\left(W_j\left(w_i\right)\right)}_{\deg=n_in_j}+\underbrace{2\beta^2W_i\left(W_j\left(w_i\right)\right)W_i'\left(W_j\left(w_i\right)\right)}_{\deg=(2n_i-1)n_j}&=0.\end{aligned}$$

This implies $n_j \geq 1$. And $n_i > 1$ would imply $2n_i - 1 \geq \max(n_i, 3)$, in which case the last term would dominate, a contradiction. (W_1) and (W_2) thus impose $n_1 = n_2 = 1$, that is, the beliefs are of the form $W_i(w) = \omega_{i,0} + \omega_{i,1}w$.

- **Step 2: any equilibrium with affine wary beliefs is such that** $\omega_{1,1} = \omega_{2,1}$

Focusing on the linear terms, (W_1) and (W_2) impose:

$$-\beta\left(4-\beta^2\right)+\left(2\left(2-\beta^2\right)+4\beta\omega_{2,1}-\beta^2\omega_{2,1}^2\right)\omega_{1,1} = 0, \tag{C10}$$

$$-\beta\left(4-\beta^2\right)+\left(2\left(2-\beta^2\right)+4\beta\omega_{1,1}-\beta^2\omega_{1,1}^2\right)\omega_{2,1} = 0. \tag{C11}$$

Substracting these two conditions yields:

$$\left(\omega_{1,1}-\omega_{2,1}\right)\left(2\left(2-\beta^2\right)+\beta^2\omega_{1,1}\omega_{2,1}\right)=0. \tag{C12}$$

Differentiating $(C2)$ with respect to w implies:

$$\partial_{11}^2\pi_P\omega_{1,1}+\partial_{12}^2\pi_P = 0, \tag{C13}$$

$$\partial_{12}^2\pi_P+\partial_{22}^2\pi_P\omega_{2,1} = 0. \tag{C14}$$

The second-order conditions of the manufacturer's program impose $\partial_{11}^2\pi_P, \partial_{22}^2\pi_P \leq 0$. Therefore,

$$\omega_{1,1}\omega_{2,1}=\frac{\left(\partial_{12}^2\pi_P\right)^2}{\partial_{11}^2\pi_P\partial_{22}^2\pi_P}\geq 0$$

and $(C12)$ thus imposes $\omega_{1,1} = \omega_{2,1} = \omega_1$. Condition $(C10)$ thus simplifies to:

$$\beta\left(4-\beta^2\right)+\left(-2\left(2-\beta^2\right)-4\beta\omega_1+\beta^2\omega_1^2\right)\omega_1=0. \tag{C15}$$

- **Step 3: there exists a unique ω_1^* satisfying $(C15)$ and the second-order conditions**

The second-order cross derivative of the manufacturer's profit π_P is positive:

$$\partial_{12}^2 \pi_P = \frac{2\beta}{4-\beta^2} > 0.$$

Given that $\omega_{1,1} = \omega_{2,1} = \omega_1$, the second-order conditions of the manufacturer's program are $\partial_{11}^2 \pi_P \leq 0$ and $\partial_{11}^2 \pi_P + \partial_{12}^2 \pi_P \leq 0$. Together with $\partial_{12}^2 \pi_P > 0$ and $(C13)$, they imply $0 \leq \omega_1 \leq 1$. Since the left-hand side of $(C15)$ is a third-degree polynomial function $\phi(\omega_1)$ satisfying:

$$\phi(-\infty) < 0, \phi(0) > 0 > \phi(1) = -(1+\beta)(2-\beta)^2 \text{ and } \phi(+\infty) > 0,$$

$(C15)$ has a unique solution ω_1^* in $[0,1]$.

- **Step 4: there exists a unique equilibrium, which is symmetric**

Focusing on the constant terms and using $\omega_{1,1} = \omega_{2,1} = \omega_1$, conditions (W_1) and (W_2) impose:

$$\begin{aligned}
(2-\beta\omega_1^*)\,\omega_1^*\omega_{1,0} - \left(4-\beta^2\right)\omega_{2,0} &= -\frac{(2-\beta)\left(\beta^2 - \left(4-\beta^2\right)c - 2\beta\omega_1^*\right)\omega_1^*}{2\beta}, \\
(2-\beta\omega_1^*)\,\omega_1^*\omega_{2,0} - \left(4-\beta^2\right)\omega_{1,0} &= -\frac{(2-\beta)\left(\beta^2 - \left(4-\beta^2\right)c - 2\beta\omega_1^*\right)\omega_1^*}{2\beta}.
\end{aligned}$$

Substracting these two conditions leads to:

$$\left(4-\beta^2 - 2\omega_1^* + \beta\omega_1^{*2}\right)(\omega_{1,0} - \omega_{2,0}) = 0.$$

$\omega_1^* \in [0,1]$ thus implies $\omega_{1,0} = \omega_{2,0} = \omega_0^*$ and the above two conditions reduce to:

$$\omega_0^* = \frac{(2-\beta)\left(\beta^2 - \left(4-\beta^2\right)c - 2\beta\omega_1^*\right)\omega_1^*}{2\beta\left(4-\beta^2 - 2\omega_1^* + \beta\omega_1^{*2}\right)}.$$

This shows that there exists a unique solution to the overall program, and that this solution is symmetric. The equilibrium wholesale price is defined by

$$w^* = W(w^*) = \omega_0^* + \omega_1^* w^* \Leftrightarrow w^* = \frac{\omega_0^*}{1-\omega_1^*}.$$

C.3 Existence under Price Competition

The analysis of the Bertrand-like framework is very similar to that of the Cournot-like framework, replacing the quantity q^R and the retailer's profit π^R by:

$$\begin{aligned} p^R(w_i, w_j) &= p^B(w_i, w_j) = \frac{(1-\beta)(2+\beta) + 2w_i + \beta w_j}{4-\beta^2}, \\ \text{and } \pi^R(w_i, w_j) &= \pi^B(w_i, w_j) = \frac{\left((1-\beta)(2+\beta) - \left(2-\beta^2\right) w_i + \beta w_j\right)^2}{\left(4-\beta^2\right)^2\left(1-\beta^2\right)}. \end{aligned}$$

The first-order conditions (W_i) characterizing the beliefs become:

$$\begin{aligned} &(2+\beta)(1-\beta)\left(\beta^2 + \left(4-\beta^2\right)c\right) \\ 2\beta &\left[\left(4-\beta^2\right) w_i - \left(2-\beta^2\right) W_i\left(W_j\left(w_i\right)\right)\right] - 4\left(2-\beta^2\right) W_j\left(w_i\right) \\ +2\beta &\left[(2+\beta)(1-\beta) - \left(2-\beta^2\right) W_j\left(w_i\right) + \beta W_i\left(W_j\left(w_i\right)\right)\right] W_i'\left(W_j\left(w_i\right)\right) = 0. \end{aligned} \tag{W_i}$$

Retracing the same steps as above, it can be checked again that the only wary beliefs equilibrium is symmetric and involves affine beliefs, of the form $W(w) = \omega_0^* + \omega_1^* w$, where ω_1^* is the unique solution in $[0,1]$ to:

$$\beta\left(4-\beta^2\right) + \left(2\left(2+\beta^2\right) + 2\beta\left(2-\beta^2\right)\omega_1 + \beta^2\omega_1^2\right)\omega_1 = 0, \tag{C16}$$

and ω_0^* is given by:

$$\omega_0^* = \frac{(2+\beta)(1-\beta)\left(\beta^2 + \left(4-\beta^2\right)c + 2\beta\omega_1^*\right)\omega_1^*}{2\beta\left(4-\beta^2 - \left(2-\beta^2-\beta\omega_1^*\right)\omega_1^*\right)}. \tag{C17}$$

The equilibrium wholesale price is again defined by

$$w^* = W(w^*) = \omega_0^* + \omega_1^* w^*.$$