From Coal to Clean Energy: Hotelling with a Limit on the Stock of Externalities

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Abstract

The Kyoto Protocol aims to stabilize the concentration of carbon in the atmosphere, which is mainly caused by the burning of nonrenewable resources such as coal in power generation. We ask how a ceiling on the stock of emissions may affect the textbook Hotelling model. We show that when the ceiling is binding, both the low-cost nonrenewable resource (coal) and the high-cost renewable resource (solar energy) may be used jointly. Emissions may be reduced at any given time through abatement or by replacing coal with solar energy, but not both. If energy demand declines in the long run, we obtain a zigzag pattern of resource use: coal is used first, followed by the joint use of coal and clean solar energy when the ceiling is tight, reverting to coal again when emissions are no longer binding, and finally to solar energy when coal is exhausted.

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1 Introduction

The Kyoto Protocol proposes a limit on carbon emissions which are primarily caused by the burning of nonrenewable fossil fuels such as coal, oil and natural gas (IPCC (2001)). Although the Protocol prescribes carbon limits only for the developed countries, its stated long-term goal is stabilizing the concentration of carbon in the atmosphere.[1] In this paper, we ask how a concentration target or equivalently, a ceiling on the global stock of emissions affects the standard Hotelling model with a polluting nonrenewable resource and a clean renewable resource (Hotelling (1931)). Is the renewable resource used only when all of the nonrenewable resource is exhausted or earlier? Is there joint extraction of both the resources? How does increasing (or decreasing) demand affect the order of extraction? These questions need to be addressed in order to understand the impacts of any international agreement on the extraction of fossil fuel resources and the long-run transition to clean, renewable energy.

There have been important theoretical contributions on the more general problem of global warming, but none have investigated how a limit on the stock of emissions may alter the sequence of extraction of the fossil fuel and the backstop resource over time. The literature has mainly relied on models that specify damage functions caused by the use of nonrenewable resources.[2] Forster (1980) first studied the analysis of pollution in a model of nonrenewable resources. Pollution has a negative effect on the utility function, but the clean substitute

[1] The Protocol has of late become mired in international politics, but there is universal agreement that the only way to combat global warming is through achieving a target concentration of carbon in the atmosphere. The details of the Protocol - what proportion of emission reductions could be achieved through trading in carbon permits or through domestic means and whether developing countries will participate and in what form - is very much open to debate. For a comprehensive review of its current status, see McKibbin and Wilcoxen (2002). The Protocol imposes caps on annual emission flows from each participating country. Implicitly, the concentration target is translated into an annual cap for each nation, which may be adjusted over time depending on new information on the benefits and costs of warming, the relationship between carbon concentration and temperature rise, and other parameters (UNFCCC, 1997). In this paper we model this phenomenon by assuming an upper bound on the stock of aggregate emissions.

[2] A ceiling on the stock of emissions may be considered a special case of an increasing and convex damage function in which damages are negligible until the stock reaches a threshold level (the ceiling) and sufficiently high beyond. In general, there is a high degree of uncertainty with respect to the precise shape of the damage function because the actual costs of climate change are difficult to estimate (McKibbin and Wilcoxen, 2002). Another justification for using a ceiling is that most operational climate change models have used Kyoto-type ceilings to examine the economic effects of global warming (e.g., see Zhang (1998)).

plays no role except in the terminal phase. Other studies such as Sinclair (1994) and Ulph and Ulph (1994) have examined the time profile of the carbon tax in an infinite horizon framework but without a backstop resource.[3] Hoel and Kverndokk (1996) and Tahvonen (1997) analyze the path of optimal carbon taxes in a model with a nonrenewable resource and a clean backstop. Using stock-dependent extraction costs, they show that there may be a period of simultaneous extraction of the nonrenewable and renewable resource. Toman and Withagen (2000) use a general equilibrium framework to examine the role of economic incentives in managing the stock of pollution arising from use of a polluting input but they do not have resource scarcity in their model. Fisher et al. (2002) also do not consider resource scarcity in modeling the relationship between the pollution stock and the development of a clean technology. Most empirical work on global warming either assumes a general equilibrium framework that does not explicitly recognize the scarcity of fossil fuels or models the problem by imposing exogenous carbon taxes (e.g., see Manne and Richels (1991) and Chakravorty, Roumasset and Tse (1997)).

This paper combines two features that are critical to the understanding of the long-run impacts of any international agreement to limit fossil fuel emissions. The first is the scarcity of the nonrenewable resource, which drives up its price over time. The second is the ceiling placed on aggregate emissions from consumption of the resource. As we shall see below, the scarcity of the fossil fuel drives the dynamics of pollution accumulation and the ultimate transition to the cleaner backstop. However, the constraint on the stock of emissions causes the renewable resource to be used even though the cheaper nonrenewable has not yet been exhausted. Once this constraint is no longer binding, the solution reverts to the standard Hotelling case.

For illustrative purposes, let coal be the polluting nonrenewable resource used in electricity generation and solar energy be the clean renewable resource.[4] The burning of coal produces carbon emissions. The stock of carbon in the atmosphere must satisfy an upper limit that is imposed exogenously, for instance, by an international agreement. Carbon emissions can be

[3] Farzin (1996) and Gjerde et al. (1999) address the optimal timing of a carbon tax under threshold effects beyond which the damages become irreversible. Farzin and Tahvonen (1996) model the carbon cycle and study its implications for the carbon tax.

[4] This illustration is reasonable because coal-burning for electricity is indeed a major contributor to greenhouse gas emissions. For example, coal accounts for 89% of all carbon emissions from the U.S. electricity sector (EIA, 2002).

abated at cost through sequestration, e.g., by forests.[5]

When energy demand is stationary, we show that coal is used until the stock of carbon reaches the ceiling. At the ceiling, there may be two possible solutions. Coal may be extracted at a constant rate until it becomes scarce and its extraction rate begins to fall. Then the ceiling ceases to be binding and coal use follows a strict Hotelling path until exhaustion. However, if the cost of solar power is relatively low, we get an alternative solution in which both coal and solar energy are extracted simultaneously at the ceiling until the former gets exhausted. These results are somewhat contrary to the standard Hotelling notion of a switch from a cheap nonrenewable to a costly renewable resource.

We consider the case when demand for energy is increasing over time, such as from an increase in per capita consumption or growth in population. The sequence of resource use depends upon the relative costs of coal and solar power, the abundance of coal, whether coal is highly (or mildly) polluting, and whether the imposed carbon ceiling is high or low. Only coal may be used when the ceiling is binding, or both coal and solar power may be used jointly. Or the ceiling may begin with the exclusive use of coal and end with joint extraction of both coal and solar power. When energy demand *decreases* over time, which is a likely scenario in the long run,[6] we obtain a counter-intuitive result: coal is used at first, then both resources are used at the ceiling, followed again by exclusive use of coal until it is exhausted and finally, a complete transition to solar energy. The more expensive but clean renewable resource is used at the ceiling so that the stock of emissions is kept within limits, and the cheaper nonrenewable resource is extracted later when the ceiling is no longer binding.[7]

When emissions can be abated through sequestration, we show that it is never optimal to use solar energy and sequester carbon simultaneously. If the cost of sequestration is higher

[5] Emissions can also be reduced by other means such as improving energy efficiency or farming practices that retain carbon in the soil. Sequestration through forestry and other land-use changes ("carbon sinks") are allowed under the Kyoto Protocol. Baselines and procedures for measuring changes in land-use practices are currently being established (UNFCCC (1997)).

[6] Energy demand may decrease over time if there is a decline in global population and the inevitable levelling off of per capita energy consumption in the developing countries. Recent population projections have significantly downgraded earlier estimates of global population growth and the level at which world population will begin a steady decline (United Nations (2002), Lutz, et al (2001)).

[7] This result violates Herfindahl's (1967) theorem of "least cost first," which suggests that in partial equilibrium, nonrenewable resources must be used in order of increasing extraction cost (see Amigues et al. (1998) for another recent example).

than the cost of the renewable resource, there will never be any sequestration. In general, however cheap abatement may be, it is never done before the pollution ceiling is attained or when the ceiling ceases to be binding. Abatement takes place only during the period when the ceiling is binding. When abatement costs are relatively low, it starts exactly at the instant the ceiling is hit.

These results suggest that if an agreement such as the Kyoto Protocol were to be implemented, the need to stabilize emissions may imply the joint use of fossil fuels and renewable energy. Renewables such as solar energy may be employed in electricity generation even though they are costly relative to coal. However, once global populations and energy demand peak and then begin a decline, the emissions ceiling may not be binding any longer. We may then abandon the expensive renewable energy and revert back to using fossil fuels exclusively, until the final transition to clean energy sources. The relative abundance of coal over the other fossil fuels (oil and natural gas) may suggest that the second period with exclusive use of coal may be an extended one.

Section 2 describes the Hotelling model with a ceiling on the stock of carbon. Section 3 develops intuition by focusing on special cases including stationary and increasing (decreasing) demand. Section 4 considers carbon sequestration. Section 5 concludes the paper.

2 The Hotelling Model with a Ceiling on the Stock of Emissions

The economy uses two resources: coal and solar power for electricity generation. They are perfect substitutes: i.e., if x_t and y_t are their respective extraction (consumption) rates, then aggregate energy consumption at time t is given by $q_t = x_t + y_t$.[8] The gross surplus or utility from electricity consumption at time t is $u(q,t)$. We assume that this utility function is strictly increasing and strictly concave in q, i.e., $u_1(q,t) = \partial u/\partial q > 0$, and $u_{11}(q,t) = \partial^2 u/\partial q^2 < 0$.

We consider the case in which demand for electricity increases exogenously over time (denoted by IN) and compare it with the opposite case of decreasing demand (denoted by DN).[9] Demand increases (decreases) if utility and marginal utility are both increasing (decreasing) and if u is bounded from above (below) by $\overline{u}(\underline{u})$. That is, $u_2 = \partial u/\partial t > 0 (< 0)$,

[8] The time subscript on q_t may be omitted whenever convenient.

[9] The latter may happen with a decline in world population so that aggregate energy consumption falls even

$u_{12} = \partial^2 u/\partial q \partial t > 0(< 0)$ and $\lim\limits_{t \uparrow +\infty} u(q,t) = \overline{u}(q)(\underline{u}(q))$. Furthermore, for $IN(DN)$, $u_{122} = \partial^3 u(q,t)/\partial q \partial t^2 < 0(> 0)$. This regularity condition is useful for characterizing the optimal path and implies that the marginal utility at a given q is an increasing and concave function of time for IN, and a decreasing and convex function for DN.

Let the given initial stock of coal be denoted by X_0. Then the residual stock at time t is given by X_t, so that $\dot{X}_t = -x_t$. Its average extraction cost c_e is assumed to be constant.[10] Coal is assumed to be scarce even for DN. That is, let x_{ct} be its consumption level for which the marginal utility equals the marginal cost, so that $u_1(x_{ct}, t) = c_e$. Then we assume that the stock necessary to sustain this path (infinite in the IN case) is higher than X_0.[11]

When used, each unit of coal emits a constant ζ units of carbon. Let Z_t be the stock of carbon at any time t. Then the flow of carbon at time t, z_t is proportional to the extraction rate of coal, $z_t = \zeta x_t$. As is standard in the literature, we assume that the natural regeneration capacity of the atmosphere is proportional to the stock of carbon Z_t (see e.g., Kolstad and Krautkraemer (1993)). Let the unit cost of abatement, i.e., carbon sequestration be given by c_a. With abatement, the stock of pollution changes by $\dot{Z}_t = \zeta x_t - a_t - \alpha Z_t$, with $\zeta x_t - a_t \geq 0$, where a_t is the amount of pollution abated at time t, and $\alpha > 0$ is a constant. Let $\overline{x}(a_t) = (a_t + \alpha \overline{Z})/\zeta$. Then $\overline{x}(a_t)$ is the extraction rate of coal at the ceiling, when a_t units are being abated through sequestration. Increased abatement will imply that more coal can be used and still satisfy the ceiling. Define the corresponding marginal utility as $\overline{p}_{et}(a_t) = u_1(\overline{x}(a_t), t)$. Since $\partial \overline{p}_{et}(a_t)/\partial t = u_{12}(\overline{x}(a_t), t)$ and $\partial^2 \overline{p}_{et}(a_t)/\partial t^2 = u_{122}(\overline{x}(a_t), t)$, then for any given a_t, $\overline{p}_{et}(a_t)$ is increasing and concave for IN and decreasing and convex for DN. When there is no abatement, then $a_t = 0$, and it is useful to define $\overline{x}(0)$ and $\overline{p}_{et}(0)$ as $\overline{x}$ and $\overline{p}_{et}$ respectively.

Let c_r be the unit cost of solar energy in electricity and $\overline{\theta}$ be the time at which $\overline{p}_{et} = c_r$ both for $IN(\overline{p}_{e0} < c_r < \overline{p}_{e\infty})$ and for $DN(\overline{p}_{e\infty} < c_r < \overline{p}_{e0})$. This is the time at which the marginal utility at the ceiling (with no abatement) equals the unit cost of solar power. Note that if $\overline{p}_{et} < c_r$, solar energy is relatively costly at the ceiling. However, $\overline{p}_{et}$ varies with time. This relationship between $\overline{p}_{et}$ and c_r is a key determinant of resource use under the ceiling, as we see below. Assume a constant available flow of solar energy, $\overline{y}$. It is considered nonstorable so that the portion unused, $\overline{y} - y_t$ is lost. Its average delivery cost c_r is assumed constant,

with the expected increases in per capita energy demand from the developing countries.

[10] The extraction cost is interpreted as a delivery cost, i.e., inclusive of processing and transportation.

[11] In other words, $\lim\limits_{t \uparrow +\infty} \int_0^t x_{c\tau} d\tau > X_0$.

with $c_e < c_r$.[12]

We assume that solar energy is abundant. Let y_{ct} be the consumption rate for which the marginal utility equals the marginal cost of solar power, $u_1(y_{ct}, t) = c_r$. Then $y_{ct} \leq \overline{y}$, the flow of solar energy is sufficient to meet demand at its marginal cost. Finally, note that x_{ct} and y_{ct} are time increasing for IN and time decreasing for DN.[13]

2.1 The Optimization Problem

The social planner's problem (defined by P) is to maximize the net social welfare by choosing the quantities of coal and solar energy as well as carbon abated at any given instant:

$$(P) \quad \max_{\{(x_t, y_t, a_t),\ t \geqslant 0\}} \int_0^{+\infty} \{u(x_t + y_t, t) - c_e x_t - c_a a_t - c_r y_t\} e^{-\rho t} dt \tag{1a}$$

$$\tag{1b}$$

$$s.t. \quad \overset{\bullet}{X}_t = -x_t \ , \quad X_0 \text{ given}, \ \ X_t \geqslant 0, \tag{1c}$$

$$\zeta x_t - a_t \geqslant 0, \tag{1d}$$

$$x_t \geqslant 0, \tag{1e}$$

$$a_t \geqslant 0, \tag{1f}$$

$$\overset{\bullet}{Z}_t = \zeta x_t - a_t - \alpha Z_t \ , \ \ Z_0 < \overline{Z} \text{ given}, \ \ \overline{Z} - Z_t \geqslant 0, \text{ and} \tag{1g}$$

$$y_t \geqslant 0. \tag{1h}$$

The corresponding current value Lagrangian for (P) is given by

$$\begin{aligned} L_t \;=\; & u(x_t + y_t, t) - c_e x_t - c_a a_t - c_r y_t - \lambda_t x_t + \mu_t [\zeta x_t - a_t - \alpha Z_t] \\ & + \nu_t [\bar{Z} - Z_t] + \gamma_{et} x_t + \overline{\gamma}_{at} [\zeta x_t - a_t] + \underline{\gamma}_{at} a_t + \gamma_{rt} y_t \end{aligned}$$

[12] The marginal utility for small consumption rates is taken to be greater than c_r, so that solar energy is used after coal is exhausted, and as we show later, sometimes before, i.e., there exists $\epsilon > 0$ such that $\lim_{q \downarrow 0} u_1(q, t) \geq c_r + \epsilon$, $\forall t$.

[13] Because $dx_{ct}/dt = -u_{12}(x_{ct}, t)/u_{11}(x_{ct}, t)$ and $dy_{ct}/dt = -u_{12}(y_{ct}, t)/u_{11}(y_{ct}, t)$.

so that the necessary conditions are

$$u_1 - c_e - \lambda_t + \zeta\mu_t + \gamma_{et} + \zeta\overline{\gamma}_{at} = 0, \tag{2}$$

$$u_1 - c_r + \gamma_{rt} = 0, \tag{3}$$

$$-c_a - \mu_t - \overline{\gamma}_{at} + \underline{\gamma}_{at} = 0, \tag{4}$$

together with the complementarity slackness conditions

$$\nu_t \geq 0, \bar{Z} - Z_t \geq 0, \text{ and } \nu_t \left[\bar{Z} - Z_t\right] = 0, \tag{5}$$

$$\gamma_{et} \geq 0, x_t \geq 0, \text{ and } \gamma_{et} x_t = 0, \tag{6}$$

$$\gamma_{rt} \geq 0, y_t \geq 0, \text{ and } \gamma_{rt} y_t = 0, \tag{7}$$

$$\overline{\gamma}_{at} \geq 0, \zeta x_t - a_t \geq 0, \text{ and } \overline{\gamma}_{at}\left[\zeta x_t - a_t\right] = 0, \text{ and} \tag{8}$$

$$\underline{\gamma}_{at} \geq 0, a_t \geq 0, \text{ and } \underline{\gamma}_{at} a_t = 0. \tag{9}$$

The dynamics of the two shadow prices, λ_t and μ_t are given by

$$\dot{\lambda}_t = \rho\lambda_t \ (i.e., \lambda_t = \lambda_0 e^{\rho t}), \tag{10}$$

and

$$\dot{\mu}_t = (\rho + \alpha)\mu_t + \nu_t, \tag{11}$$

where λ_t is the scarcity rent of coal and μ_t, which is non-positive, is the shadow price of the stock of carbon. Finally, the transversality conditions at infinity are

$$\lim_{t \uparrow +\infty} e^{-\rho t}\lambda_t X_t = \lambda_0 \lim_{t \uparrow +\infty} X_t = 0, \tag{12}$$

and

$$\lim_{t \uparrow +\infty} e^{-\rho t}\nu_t Z_t = 0. \tag{13}$$

2.2 Interpretation of the Necessary Conditions

In order to characterize the solution, it is useful to disaggregate the above problem by considering three different price paths for coal: the "pure" Hotelling price path, the price path when the carbon stock constraint is binding, and the price path with both a binding stock constraint and non-zero abatement.

2.2.1 The Hotelling Price Path

For any initial scarcity rent $\lambda_0 \in (0, c_r - c_e)$, let $\tilde{x}_t(\lambda_0)$ solve $u_1(x_t, t) = c_e + \lambda_0 e^{\rho t} \equiv \tilde{p}_t(\lambda_0)$.[14] That is, $\tilde{x}_t(\lambda_0)$ is the optimal extraction rate of coal with the carbon constraint nonbinding at time t and never binding at any future time period $\tau > t$.[15] In other words, $\tilde{p}_t(\lambda_0)$ is the equilibrium price path of coal in the Hotelling model with no pollution. For intuitive ease, let us call it the "Hotelling price path." Then the time derivative of $\tilde{x}_t(\lambda_0)$ is given by $\partial\tilde{x}_t(\lambda_0)/\partial t = \frac{-u_{12}+\rho\lambda_0 e^{\rho t}}{u_{11}}$. For IN, $u_{12} > 0$, hence this derivative cannot be signed. However for DN, $u_{12} < 0$, hence $\tilde{x}_t(\lambda_0)$ decreases with time. When energy demand is increasing, the extraction rate may increase or decrease with time. When demand is decreasing, the extraction rate must, as in the fixed demand model, decrease with time. In general, $\partial\tilde{x}_t(\lambda_0)/\partial\lambda_0 = \frac{e^{\rho t}}{u_{11}} < 0$, so that extraction decreases with the initial scarcity rent. In the limit, $\lim_{\lambda_0 \downarrow 0} \tilde{x}_t(\lambda_0) = x_{ct}$, i.e., there is no scarcity effect and extraction occurs at marginal cost.

Define $\tilde{\theta}(\lambda_0)$ as the time at which the Hotelling price path equals the cost of solar power, $\tilde{p}_t(\lambda_0) = c_r$. That is, $\tilde{\theta}(\lambda_0) = \rho^{-1}\left[\log(c_r - c_e) - \log\lambda_0\right]$. Without the carbon constraint, the optimal value of λ_0 is the solution to the supply equals demand condition $\int_0^{\tilde{\theta}(\lambda_0)} \tilde{x}_t(\lambda_0)dt = X_0$.

2.2.2 Hotelling Prices with a Ceiling on the Stock of Carbon

Given some Z_0, let $\tilde{Z}_t(\lambda_0)$ be the stock of carbon induced by $\tilde{x}_t(\lambda_0)$, so that $d\tilde{Z}_t(\lambda_0)/dt = \zeta\tilde{x}_t(\lambda_0) - \alpha\tilde{Z}_t(\lambda_0)$ and $\tilde{Z}_0(\lambda_0) = Z_0$. Then $\tilde{Z}_t(\lambda_0)$ is the stock in the time interval $\left[0, \tilde{\theta}(\lambda_0)\right]$ if there is no binding ceiling and no abatement. Beyond $\tilde{\theta}(\lambda_0)$ only solar energy is used and there is no pollution, so that $\tilde{Z}_t$ declines steadily to zero. For the problem to be meaningful, let us assume that the ceiling is binding at some time. Then the Hotelling extraction path $\tilde{x}_t(\lambda_0)$ is no longer optimal at the beginning of the planning period. For any $\lambda_0 \in [0, \bar{p}_{e0} - c_e]$,define

[14] If such a solution exists, zero otherwise. This caveat holds for future definitions and is not repeated.

[15] provided that $\tilde{p}_t(\lambda_0) < c_r$, and λ_0 be the correct initial scarcity rent.

$\tilde{\tau}(\lambda_0)$ as the time at which $\tilde{p}_t(\lambda_0) = \bar{p}_{et}$, i.e., the time at which the Hotelling price equals marginal utility when the ceiling is binding and there is no abatement.[16]

For any $\lambda_0 \in [0, c_r - c_e]$ and $\mu_0 \in (-[c_r - (c_e + \lambda_0)]/\zeta, 0)$, let $\hat{x}_t(\lambda_0, \mu_0)$ solve $u_1(x_t, t) = c_e + \lambda_0 e^{\rho t} - \zeta\mu_0 e^{(\rho+\alpha)t} \equiv \hat{p}_t(\lambda_0, \mu_0)$. The extraction path $\hat{x}_t(\lambda_0, \mu_0)$ is the optimal consumption rate of coal if until time t the carbon ceiling has been nonbinding,[17] but will be binding in the future. As in the case for $\tilde{x}_t(\lambda_0)$, how the extraction rate $\hat{x}_t(\lambda_0, \mu_0)$ changes with time cannot be determined for IN but is decreasing with time for DN.[18] Let us call $\hat{p}_t(\lambda_0, \mu_0)$ the "Hotelling price with externality." Then define $\hat{\theta}(\lambda_0, \mu_0)$ as the time at which this price path equals the cost of solar energy, i.e., $\hat{p}_t(\lambda_0, \mu_0) = c_r$. Let $\widehat{Z}_t(\lambda_0, \mu_0)$ be the stock of carbon generated by $\widehat{x}_t(\lambda_0, \mu_0)$.[19] For any $\lambda_0 \in [0, \overline{p}_{et} - c_e]$ and $\mu_0 \in (-[\overline{p}_{et} - (c_e + \lambda_0)] \ / \ \zeta, \ 0)$ we denote by $\widehat{\tau}(\lambda_0, \mu_0)$ the time at which $\widehat{p}_t\,(\lambda_0, \mu_0) = \overline{p}_{et}$.[20]

2.2.3 Hotelling Prices with Abatement and a Ceiling on the Stock of Carbon

By (4), $-\mu_t = c_a - \overline{\gamma}_{at} + \underline{\gamma}_{at} = 0$. Suppose $Z_t = \bar{Z}$ so that the stock of carbon is at the ceiling. Then the extraction rate of coal is given by the Hotelling price $c_e + \lambda_0 e^{\rho t}$ when $x_t < \bar{x}$ (no abatement is needed) and by $c_e + \lambda_t - \zeta\mu_t$ (from (2)) if $x_t > \bar{x}$ (abatement is needed). When abatement a_t is strictly positive, it must equal $\zeta(x_t - \bar{x})$. Then the constraint $\zeta x_t - a_t \geq 0$ is never tight so that $\overline{\gamma}_{at} = 0$. Since $a_t > 0$, $\underline{\gamma}_{at} = 0$. Substituting from (4) in (2) yields $u_1 = c_e + \zeta c_a + \lambda_0 e^{\rho t} \equiv \breve{p}_t(\lambda_0)$, where $\breve{p}_t(\lambda_0)$ represents the price of coal when the carbon ceiling is tight and excess emission is being abated. Let $\breve{x}_t(\lambda_0)$ be the corresponding extraction path for coal.[21] For coal to be used, the equilibrium price $\breve{p}_t(\lambda_0)$ must be lower than the cost of solar energy, c_r. As before, extraction $\breve{x}_t(\lambda_0)$ decreases with time for DN, but is indeterminate for IN. Let $\breve{\theta}(\lambda_0)$ be the time at which $\breve{p}_t(\lambda_0) = c_r$. Then $\breve{\theta}(\lambda_0) = \rho^{-1}[\log(c_r - c_e - \zeta c_a) - \log \lambda_0]$. Finally, let $\breve{\tau}(\lambda_0)$ be the time at which $\breve{p}_t(\lambda_0) = \overline{p}_{et}$.

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[16]Note that $\tilde{p}_t(\lambda_0)$ is increasing and convex. When demand is stationary, $\bar{p}_{et}$ is constant. It is increasing and concave for IN and decreasing and convex for DN. Thus $\tilde{\tau}(\lambda_0)$ is well defined and unique for each case, and $\tilde{p}_t(\lambda_0) < (>) \bar{p}_{et}$ for $t < (>) \tilde{\tau}(\lambda_0)$.

[17]so that $\nu_\tau = 0$, $\tau \in [0, t]$ and $\mu_t = \mu_0 e^{(\rho+\alpha)t}$ from (11).

[18]In fact, $\partial\hat{x}_t(\lambda_0, \mu_0)/\partial t = \frac{-u_{12} + \rho\lambda_0 e^{\rho t} - (\rho+\alpha)\mu_0 e^{(\rho+\alpha)t}}{u_{11}}$,which cannot be signed if $u_{12} > 0$.

[19]so that $\frac{\partial \widehat{Z}_t(\lambda_0, \mu_0)}{\partial t} = \zeta\widehat{x}_t(\lambda_0, \mu_0) - \alpha\widehat{Z}_t(\lambda_0, \mu_0)$ and $\widehat{Z}_0(\lambda_0, \mu_0) = Z_0$.

[20]Under the regularity assumption on $u(\cdot)$, $\widehat{\tau}(\lambda_0, \mu_0)$ is well defined and unique and $\widehat{p}_t\,(\lambda_0, \mu_0) < (>) \overline{p}_{et}$ according to whether $t < (>) \widehat{\tau}(\lambda_0, \mu_0)$.

[21]It solves $u_1(x_t, t) = \breve{p}_t(\lambda_0)$.

3 Resource Substitution when Sequestration is Costly

In this section we develop intuition by focusing on several special cases when there is no abatement of emissions at equilibrium. This is plausible if sequestration is prohibitively costly. In section 4, we consider the case when abatement is economically feasible. Here we start with the simplest case - when demand is fixed and then discuss increasing (decreasing) demand over time.

3.1 Demand is Stationary

With a fixed demand and no abatement, we get the standard Hotelling model with a pollution ceiling. Without abatement the carbon stock grows with emissions net of natural decay, $\dot{Z}_t = \zeta x_t - \alpha Z_t$. At the ceiling, $\overline{x} = \alpha\overline{Z} \ / \ \zeta$. Consider the case when $y_c < \overline{x}$, i.e., the maximum extraction rate of solar energy is lower than the extraction rate of coal at the ceiling.[22] This only happens if $\bar{p}_e < c_r$, i.e., the price of coal at the ceiling is lower than the cost of solar energy (see Fig. 1). When the carbon stock is at the ceiling, coal can supply all of the electricity demanded. Quadrant 1 (in Fig. 1) shows the relevant price paths and quadrant 4 shows resource use over time. The sequence of resource use can be completely described by four different phases. In phase I, the true marginal cost for consumption of coal is given by the curve $\widehat{p}_t\left(\lambda_0, \mu_0\right) = c_e + \lambda_0 e^{\rho t} - \zeta\mu_0 e^{(\rho+\alpha)t}$, which is the Hotelling price with externality. Note from quadrant 4, that the carbon emitted in this phase is higher than $\overline{x}$, the rate at which the flow is exactly neutralized by natural decay. Thus the stock of carbon increases over time from some initial level $Z_0 < \overline{Z}$. However, at time $\widehat{\tau}(\lambda_0, \mu_0)$, the stock reaches the ceiling $\overline{Z}$. This is the beginning of phase II, in which the consumption of coal equals $\overline{x}$, and the stock of carbon is binding at the ceiling $\overline{Z}$. The price of coal equals $\overline{p}_e$ and is constant in this phase, which ends at time $\widetilde{\tau}(\lambda_0)$. This marks the beginning of phase III, in which coal use declines from $\overline{x}$ and the ceiling is no longer binding. This is a transition phase from coal to solar energy. The price path is strictly Hotelling, given by $c_e + \lambda_0 e^{\rho t} \equiv \widetilde{p}_t(\lambda_0)$. There are no externality costs since the carbon ceiling is never binding beyond $\widetilde{\tau}(\lambda_0)$. Finally at time $\widetilde{\theta}\left(\lambda_0\right)$, the price of coal equals the cost of solar energy. At this point, coal gets exhausted and

[22] Since $\overline{p}_{et}$, x_{ct} and y_{ct} are now constant, we drop the subscript t.

the economy is supplied by solar power, $q_t = y_c$.[23]

In both phases I and II, the optimal solution could be achieved through a tax on carbon emissions equal to $-\mu_t$ per unit of carbon, or equivalently, $-\zeta\mu_t$ per unit of coal. The tax grows at a constant rate $\rho + \alpha$ until the target $\bar{Z}$ is reached. Then it declines steadily to zero during the period when the ceiling is binding. The quota or number of permits for coal at time t equal x_t, with a permit price of $-\zeta\mu_t$. Equivalently, the number of carbon permits is ζx_t, with price $-\mu_t$.[24]

Fig 1 here

Now consider the alternative case in which $y_c > \overline{x}$, i.e., the extraction rate of coal at the ceiling is lower than the maximum extraction rate of solar energy. In this case, $\bar{p}_e > c_r$, so that solar power becomes economical at a price that is lower than the equilibrium price when the ceiling is binding (see Fig. 2). As in the previous case, in phase I, coal is extracted and the stock of carbon is increasing. However, since $\overline{x} < y_c$, and the Hotelling price with externality increases over time, it must equal the cost of solar power c_r at or before the ceiling is achieved. It does not make economic sense to use solar energy *before* the ceiling is achieved, since it is more costly than coal. Thus the ceiling becomes tight exactly at the instant $\widehat{\theta}\,(\lambda_0, \mu_0)$ when the Hotelling price with externality $\widehat{p}_t\,(\lambda_0, \mu_0)$ equals c_r, the cost of solar energy. In phase II, there is joint consumption of both resources with the deficit $y_c - \overline{x}$ supplied by solar energy. The equilibrium price of electricity is constant and the pollution stock is binding over an interval until coal is exhausted at time $\widetilde{\theta}\,(\lambda_0)$. The key difference between this case and the previous one is that here solar energy is used at the ceiling and there is no Hotelling transition from the stage when the ceiling is binding to the backstop. Carbon emissions decline from their maximum level to zero at $\widetilde{\theta}\,(\lambda_0)$.[25]

[23] In the Appendix we check that the necessary conditions are satisfied for increasing (and decreasing) demand and pollution abatement, which subsumes all of the cases discussed here.

[24] Analogous interpretations could be made for each of the cases analyzed below.

[25] Several other interesting cases may arise if we consider the flow of solar power (say $\overline{y}$) to be limited, i.e., $\overline{y} < y_c$. These are developed elsewhere for the stationary demand case (see Chakravorty, Magne and Moreaux (2003)). The focus of the present paper is to examine resource substitution under non-stationary demand and abatement.

Fig 2 here

3.2 Demand is Increasing Over Time

Consider the case when electricity demand increases over time. This case is reasonable to explore since global energy demand is expected to increase at least for the next several decades both from increases in population as well as in energy consumption per capita, especially in the developing countries. As in the stationary demand scenario above, it will be convenient to develop the solution for three distinct cases: when the cost of solar power is high, low and medium relative to $\overline{p}_{et}$, the limit price when the carbon ceiling is binding, i.e., $\overline{p}_{e\infty} < c_r$; $c_r < \overline{p}_{e0}$ and $\overline{p}_{e0} < c_r < \overline{p}_{e\infty}$ where $\overline{p}_{e\infty} = \lim\limits_{t\uparrow+\infty} \overline{p}_{et}$.

3.2.1 High Cost Solar Power ($\overline{p}_{e\infty} < c_r$)

This case is illustrated in Fig. 3. Since demand is increasing over time, $\overline{p}_{et}$ is increasing and concave, as discussed earlier. The pattern of resource substitution is somewhat analogous to the stationary demand case shown in Fig. 1. When the ceiling is binding, the marginal utility must equal $\overline{p}_{et}$. Since the initial stock is lower than the cap, $Z_0 < \overline{Z}$, there must exist an initial period during which the carbon constraint is slack and Z_t is increasing. The price path is given by the Hotelling price with externality, $\widehat{p}_t(\lambda_0, \mu_0)$ and only coal supplies energy, i.e., $q_t = x_t = \widehat{x}_t(\lambda_0, \mu_0)$. At $\widehat{\tau}(\lambda_0, \mu_0)$ the carbon ceiling is attained and a second phase begins, during which coal use is constrained at $\overline{x}$, $q_t = x_t = \overline{x}$, up to the date $\widetilde{\tau}\,(\lambda_0)$ when $\overline{p}_{et} = \widetilde{p}_t\,(\lambda_0)$.[26] In this phase the price of coal is increasing but its consumption is constant. Next we get the transition stage during which coal is consumed along the pure Hotelling path and the ceiling is no longer binding until time $\widetilde{\theta}\,(\lambda_0)$ when $\overline{p}_{et} = c_r$. At $\widetilde{\theta}\,(\lambda_0)$ coal is exhausted and the economy switches to solar energy.[27]

Fig 3 here

[26] As noted earlier, $\widehat{x}_t(\lambda_0, \mu_0)$ is not necessarily monotonic but if $\widehat{p}_t\,(\lambda_0, \mu_0) < \overline{p}_{et}$, it implies that $\widehat{x}_t\,(\lambda_0, \mu_0) > \overline{x}$ and $Z_0 < \overline{Z}$. Then $\widehat{Z}_t\,(\lambda_0, \mu_0)$ is monotonically increasing up to the time at which $\widehat{Z}_t\,(\lambda_0, \mu_0) = \overline{Z}$.

[27] λ_0 and μ_0 are determined as the solution to the system of the two following equations:

3.2.2 Low Cost Solar Power ($c_r < \overline{p}_{e0}$)

This case is illustrated in Fig. 4 and is similar in spirit to Fig. 2. Since $\overline{p}_{et} > c_r$, during the time interval in which the carbon ceiling is tight, coal is unable to supply all of the demand and the more expensive solar energy must be used. Thus, in the first phase only coal is used. This phase ends at time $\widehat{\theta}(\lambda_0, \mu_0)$ when the Hotelling price with externality equals the cost of solar energy, $\widehat{p}_t(\lambda_0, \mu_0) = c_r$. At $\widehat{\theta}(\lambda_0, \mu_0)$, coal use falls from $\widehat{x}_{\widehat{\theta}(\lambda_0,\mu_0)}(\lambda_0, \mu_0)$ down to $\overline{x}$ while solar use jumps from 0 to $y_{c\widehat{\theta}(\lambda_0,\mu_0)} - \overline{x}$, so that consumption is continuous. During this second phase $\left[\widehat{\theta}(\lambda_0, \mu_0), \widetilde{\theta}\,(\lambda_0)\right]$ both resources are simultaneously used. At $\widetilde{\theta}\,(\lambda_0)$ coal is exhausted. The extraction rate of coal falls from $\overline{x}$ to zero while the use of solar jumps from $y_{c\widetilde{\theta}(\lambda_0)} - \overline{x}$ to $y_{c\widetilde{\theta}(\lambda_0)}$ and the consumption path is continuous. Beginning from $\widetilde{\theta}\,(\lambda_0)$, only solar energy is used.

Fig 4 here

3.2.3 Medium Cost Solar Power ($\overline{p}_{e0} < c_r < \overline{p}_{e\infty}$)

There may be three different solutions for this case. The first two are as in the low and high cost cases described above. The third one is specific to the present case. Let us discuss

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i. the supply of coal must equal cumulative consumption:

$$\int_0^{\widehat{\tau}(\lambda_0,\mu_0)} \widehat{x}_t(\lambda_0, \mu_0)dt + \overline{x}\left[\widetilde{\tau}\,(\lambda_0) - \widehat{\tau}(\lambda_0, \mu_0)\right] + \int_{\widetilde{\tau}(\lambda_0)}^{\widetilde{\theta}(\lambda_0)} \widetilde{x}_t\,(\lambda_0)\,dt = X_0; \tag{14}$$

ii. the carbon ceiling is attained at date $\widehat{\tau}(\lambda_0, \mu_0)$:

$$\widehat{Z}_{\widehat{\tau}(\lambda_0,\mu_0)}(\lambda_0, \mu_0) = \overline{Z}. \tag{15}$$

If these two conditions are satisfied, then the consumption path defined above is continuous. In the Appendix we use the general model to show that there exist values of the other dual variables, υ_t, γ_{et} and γ_{rt} such that all the necessary conditions are satisfied.

these cases intuitively. Fig. 5 shows the solution that is similar to the high cost case (Fig. 3). Note that the time $\overline{\theta}$ at which $\overline{p}_{et} = c_r$ could be sufficiently "distant" ($\overline{\theta} > \widetilde{\theta}\,(\lambda_0)$) so that only coal is being extracted at the ceiling and during the transition to the backstop. Several factors may lead to this outcome, such as a sufficiently high stock of coal, a large cost differential between coal and solar power, or relatively "clean" quality coal (small ζ).

Fig 5 here

On the other hand, an outcome similar to the low cost case (Fig. 4) may arise ($\overline{\theta} < \widehat{\theta}(\lambda_0, \mu_0)$), as shown in Fig. 6. Then during the period when the ceiling is binding $\left[\widehat{\theta}(\lambda_0, \mu_0), \widetilde{\theta}\,(\lambda_0)\right]$, both resources are used. Coal is exhausted at time $\widetilde{\theta}\,(\lambda_0)$. This is likely to happen if the stock of coal is small (leading to a higher scarcity rent), the cost of solar is relatively low or the pollution content of coal is high.

Fig 6 here

Now consider the case illustrated in Fig. 5. If X_0 is sufficiently high, the time period during which coal consumption is constrained by $\overline{x}$ is so long that it must end after $\overline{\theta}$. Once $\overline{p}_{et} > c_r$ then the low cost coal is used at its maximal rate $\overline{x}$ and the balance is supplied by solar power (see Fig. 7). During a first phase $[0, \widehat{\tau}\,(\lambda_0, \mu_0)]$, only coal is used. At $\widehat{\tau}\,(\lambda_0, \mu_0)$ the carbon ceiling is attained. During a second phase, from $\widehat{\tau}\,(\lambda_0, \mu_0)$ up to $\overline{\theta}$, coal is used while solar power is still too expensive. The latter becomes economical at $\overline{\theta}$ and during the third phase $\left[\overline{\theta}, \widetilde{\theta}\,(\lambda_0)\right]$ both resources are used, the lower cost coal at its maximal rate $x_t = \overline{x}$ and solar power making up the deficit $y_t = y_{ct} - \overline{x}$. At $\widetilde{\theta}\,(\lambda_0)$ coal is exhausted and solar power supplies energy exclusively.

Fig 7 here

In general, when the renewable resource is moderately costly, we get three distinct cases: (i) only the nonrenewable resource is used at the ceiling, followed by a Hotelling transition to the renewable resource; (ii) both nonrenewable and renewable resources are used at the ceiling until the former is exhausted and emissions decline, and (iii) a combination of the above two: the ceiling stage itself has two phases: one with the exclusive use of the nonrenewable resource, followed by use of both resources until the nonrenewable resource gets exhausted.

3.3 Demand is Decreasing Over Time

Although somewhat counter-intuitive, the case of decreasing demand may be important in the long-run, if for instance, global fertility rates and population decline faster than expected, as has been predicted recently by several projections (Lutz, Sanderson and Scherbov (2001)).[28] With decreasing demand, $\overline{p}_{et}$ decreases with time, hence $\overline{p}_{e0} > \overline{p}_{e\infty}$. Again, it is convenient to classify the solution according to the cost of solar energy relative to the marginal utility at the carbon ceiling.

3.3.1 High Cost Solar Power ($\overline{p}_{e0} < c_r$)

With demand decreasing over time, $\overline{p}_{et}$ is decreasing and convex (see Fig. 8). The consumption path is similar to the case $\overline{p}_{e\infty} < c_r$ under IN, except that here the marginal utility at the ceiling $\overline{p}_{et}$ is decreasing in the period $[\widehat{\tau}(\lambda_0, \mu_0), \widetilde{\tau}(\lambda_0)]$ during which the consumption of coal is constrained by $\overline{x}$. The price path increases along the Hotelling path with externality, then decreases during the period the ceiling is binding, and increases again along the pure Hotelling price until exhaustion. Beyond time $\widetilde{\theta}(\lambda_0)$,solar energy is used. Equilibrium energy prices first increase, then decrease at the ceiling and increase again. Energy consumption is always decreasing except at the ceiling when it is constant.

Fig 8 here

[28] World population is expected to rise from its present level of 6 billion to about 9 billion in 2070, then decline to 8.4 billion in 2100. Fertility rates are falling below replacement levels not only in the developed countries but also in some 74 intermediate-fertility developing countries. With the general aging of these societies, energy demand will likely follow this declining trend, albeit with a time lag that accounts for an increase in energy consumption per capita for residents of developing countries.

3.3.2 Low Cost Solar Power ($c_r < \overline{p}_{e\infty}$)

The marginal utility at the ceiling $\overline{p}_{et}$ is now everywhere greater than the cost of solar energy c_r (see Fig. 9). This case is similar to that of $\overline{p}_{et} > c_r$ under IN. The only difference is that here energy consumption is strictly decreasing. At the ceiling, solar energy is used jointly with coal. Because of decreasing demand, consumption of solar energy declines at the ceiling (from $\widehat{\theta}\left(\lambda_0, \mu_0\right)$ until $\widetilde{\theta}\left(\lambda_0\right)$) and jumps up when coal is exhausted.

Fig 9 here

3.3.3 Medium Cost Solar Power ($\overline{p}_{e\infty} < c_r < \overline{p}_{e0}$)

As in the case $\overline{p}_{e0} < c_r < \overline{p}_{e\infty}$ under IN, we again have three solutions, two of which are similar to the previous two cases and are not discussed separately. We only describe the third path, illustrated in Fig. 10. This case is unique because both the low cost coal and the high cost solar energy are exploited simultaneously for a period followed by exclusive use of coal and then solar energy. The backstop resource is used during two disjoint time periods.

Consider the case illustrated in Fig. 8. If X_0 is sufficiently large, then the period during which $x_t = \overline{x}$ is long enough so that $\widehat{\theta}\left(\lambda_0, \mu_0\right) < \widehat{\tau}\left(\lambda_0, \mu_0\right) < \overline{\theta}$ as illustrated in Fig. 10, and the resource use profile has five phases. During the first phase $\left[0, \widehat{\theta}(\lambda_0, \mu_0)\right]$ only coal is used until the carbon ceiling is attained. At $t = \widehat{\theta}(\lambda_0, \mu_0)$, $c_r < \overline{p}_{et}$ and $\widehat{x}_t(\lambda_0, \mu_0) = y_{ct} > \overline{x}$ so that the extraction of coal must fall from $\widehat{x}_t(\lambda_0, \mu_0)$ to $\overline{x}$ while the consumption of solar power jumps from 0 to $y_{ct} - \overline{x}$. During the second phase $\left[\widehat{\theta}(\lambda_0, \mu_0), \overline{\theta}\right]$ both resources are used. In the third phase $\left[\overline{\theta}, \widetilde{\tau}\left(\lambda_0\right)\right]$ solar energy is no longer exploited since demand has declined and it is relatively more expensive. Coal is used at $\overline{x}$ and the ceiling is tight. During the fourth phase $\left[\widetilde{\tau}\left(\lambda_0\right), \widetilde{\theta}(\lambda_0)\right]$ the carbon ceiling is no longer binding and coal consumption declines. It is finally exhausted at $t = \widetilde{\theta}(\lambda_0)$ and only solar energy is used subsequently. The expensive solar power is used along with the cheaper coal, followed by a period during which only the cheap coal is used (first at the maximal rate, then lower) followed by a terminal phase with solar energy.

Fig 10 here

4 Reducing Emissions through Sequestration

All the cases examined above can be viewed as special cases of the general model with abatement when the unit cost of sequestration is high enough for it not to be economical. For instance, as shown in Fig.3, a sufficiently high abatement cost c_a would imply that the Hotelling price with emissions and abatement, $\breve{p}_t(\lambda_0)$ is higher than the Hotelling price with emissions, $\widehat{p}_t(\lambda_0, \mu_0)$ until the former intersects the cost of solar energy c_r at time $\breve{\theta}(\lambda_0)$. Beyond this point, it is cheaper to "reduce" emissions by replacing a unit of coal with a unit of solar energy.[29]

We can develop intuition about the timing of emission reductions by considering two typical cases: a medium and a low cost of sequestration, c_a.[30]

4.1 Sequestration under Increasing and Decreasing Demand

4.1.1 High Cost Solar Power

When energy demand is increasing over time, and sequestration is not prohibitively expensive, there may be two cases to consider. The first one is shown in Fig. 11. Coal is used at the beginning until the carbon stock hits the ceiling. When the ceiling is attained, coal continues to be the sole supplier at $\overline{x}$ until sequestration becomes economical at time $\breve{\tau}_1(\lambda_0)$. Beyond this point, the carbon stock is still at the ceiling, but the excess emission generated ($x_t > \overline{x}$) is abated. After time $\breve{\tau}_2(\lambda_0)$, sequestration becomes too expensive and is discontinued, while coal is extracted again at $\overline{x}$ at the ceiling. Finally, as in the cases without abatement, the ceiling becomes nonbinding with a rise in the price of coal, until it is exhausted and substituted by solar energy. When the cost of sequestration is low, shown in Fig. 12, it

[29] By definition, $\breve{p}_t(\lambda_0)$ is parallel to the Hotelling price path $\widetilde{p}_t(\lambda_0)$ because $\breve{p}_t(\lambda_0) = \widetilde{p}_t(\lambda_0) + \zeta c_a$.

[30] The high cost case has been subsumed in the cases described in the previous section.

is economical exactly at the instant the ceiling becomes binding. The rest of the resource use and pollution stock profile mimics the previous case just described.

Fig 11 here

Fig 12 here

4.1.2 Low Cost Solar Power

A typical case when sequestration is relatively cheap and energy demand is decreasing is shown in Fig. 13. Abatement starts at the instant the stock hits the ceiling and is replaced by solar energy at time $\breve{\theta}(\lambda_0)$. At this instant the extraction of coal falls discontinuously from $\breve{x}_t(\lambda_0)$ to $\overline{x}$.

Fig 13 here

We can make some general observations on the timing of carbon sequestration. Sequestration emerges as an alternative to the use of renewable energy. There is never a period with active sequestration *and* use of solar energy. Since the real cost of abatement rises at an exponential rate, it may be cheaper than the renewable resource in the beginning but will eventually become more expensive. Moreover, carbon will only be sequestered during a period when the stock is at its ceiling. It will never be done *before* the ceiling becomes binding because then there is no immediate economic benefit from sequestering carbon, and it can be postponed to the future, which is profitable given the positive discount rate.[31]

[31] The complete solution for high, medium and low abatement costs under increasing and decreasing demand are not provided here, but can be obtained from the authors. The results are a combination of the cases with increasing and decreasing demand without abatement together with the low and medium cost abatement cases described in this paper.

5 Concluding Remarks

This paper is a first attempt at extending Hotelling theory to the case when the stock of externalities from a fossil fuel is limited. We consider both increasing and decreasing demand, as well as the possibility of abatement. One general result is that in all cases, the stock of pollution builds up over time followed by an interval in which the constraint is binding. Beyond this interval, the emission stock declines to zero as energy supply shifts from the exclusive use of the nonrenewable to that of the renewable resource. However, the details of this transition differ from case to case. If the renewable backstop is cheap or the nonrenewable resource is highly polluting, or the ceiling is low, the renewable may be used along with the nonrenewable resource exactly at the instant the ceiling is attained. This path is followed until the nonrenewable resource is exhausted. In another case when the nonrenewable resource is abundant or mildly polluting, or the backstop is expensive, the supply of energy at the ceiling is provided only by the nonrenewable resource, followed by a transition phase when extraction declines and the ceiling is nonbinding, until the nonrenewable resource is exhausted and is replaced by the renewable resource. One particular case of decreasing demand is unique because both resources may be used at the ceiling, followed by the exclusive use of the nonrenewable, and then the renewable resource in the terminal period. It suggests two disjoint periods of time when the renewable resource may be used, which is somewhat unusual in Hotelling theory.

Modeling a nonrenewable resource with a pollution ceiling is a first step towards developing theory that can examine substitution across multiple energy resources (oil, coal, natural gas and renewables) under agreements like the Kyoto Protocol. Empirical trends such as the recent transition from coal burning to the cleaner natural gas in power generation can be better understood in a model with multiple nonrenewable resources each with different emission characteristics.[32]

However, useful policy insights can be obtained even from the simple model developed here. One implication is that the standard Hotelling solution of a transition from a polluting fossil fuel to a clean renewable resource may be overly simplistic when there is a ceiling on the stock of emissions. There is a strong case for use of the renewable resource *during* the period

[32] The theoretical literature on this topic is sparse, but a few empirical models have been developed using the multiple resource framework (Nordhaus (1979), Chakravorty et al (1997)).

when the ceiling is tight, even though the cost of the backstop is higher than that of the fossil fuel and the latter has not been exhausted. Thus, solar or other renewable technologies may need to be used to supplement the use of fossil fuel resources, even if they are not economical in terms of the unit cost of energy.[33] More importantly, since coal is relatively more abundant than oil and gas, and it is realistic to expect global energy demand to peak and then decline over the long-term, the joint extraction of fossil fuels and solar energy may be feasible, if an international agreement were in place. We may then use expensive solar energy for a time when the ceiling is binding then revert back to a "Hotelling" world with coal as the primary source of energy. Empirical work needs to be done using the Hotelling framework to see which of the cases considered here are likely given plausible parameter values.

Reducing emissions through abatement, such as through carbon sequestration by forests, emerges as a clear alternative to renewable energy. If the cost of abatement is sufficiently high, it will never be used. The renewable resource will be employed jointly with the nonrenewable resource when the stock achieves a maximum. When abatement is economically feasible, it is used only during the period when the emissions ceiling is binding.[34] Over time, the true cost of abatement increases exponentially, and eventually the renewable resource becomes cheaper. Thus, abatement technologies compete with the renewable resource and only one or the other will be employed at any given time. The joint use of the two is not possible.

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[33] Conventional wisdom, which suggests that renewables cannot be used because they are currently costlier than fossil fuels, may be misplaced.

[34] One policy insight from our analysis is that the emission caps (as defined by the Kyoto Protocol) may vary over time and the time intervals over which sequestration is necessary may be determined endogenously.

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6 Appendix

6.1 Adjoint variables

We check that all the first order conditions (2)-(13) are satisfied by the values of $\gamma_{et}, \gamma_{rt}, \mu_t$ and ν_t, for the various cases discussed in the paper. In each case, coal is exhausted in finite time and $\lim_{t\uparrow+\infty} Z_t = 0$, so that the transversality conditions are satisfied.

- IN with $\overline{p}_{e\infty} < c_r$:

$$\gamma_{et} = \begin{cases} 0 & , \ t \in \left[0, \widetilde{\theta}(\lambda_0)\right) \\ \widetilde{p}_t(\lambda_0) - c_r & , \ t \in \left[\widetilde{\theta}(\lambda_0), +\infty\right) \end{cases}$$

$$\gamma_{rt} = \begin{cases} c_r - \widehat{p}_t(\lambda_0, \mu_0) & , \ t \in [0, \widehat{\tau}(\lambda_0, \mu_0)) \\ c_r - \overline{p}_{et} & , \ t \in [\widehat{\tau}(\lambda_0, \mu_0), \widetilde{\tau}(\lambda_0)) \\ c_r - \widetilde{p}_t(\lambda_0) & , \ t \in \left[\widetilde{\tau}(\lambda_0), \widetilde{\theta}(\lambda_0)\right) \\ 0 & , \ t \in \left[\widetilde{\theta}(\lambda_0), +\infty\right) \end{cases}$$

$$\mu_t = \begin{cases} \mu_0 e^{(\rho+\alpha)t} & , \ t \in [0, \widehat{\tau}(\lambda_0, \mu_0)) \\ -\zeta^{-1}\left[\overline{p}_{et} - \widetilde{p}_t(\lambda_0)\right] & , \ t \in [\widehat{\tau}(\lambda_0, \mu_0), \widetilde{\tau}(\lambda_0)) \\ 0 & , \ t \in [\widetilde{\tau}(\lambda_0), +\infty) \end{cases}$$

$$\nu_t = \begin{cases} 0 & , \ t \notin \left[\widetilde{\tau}(\lambda_0), \widetilde{\theta}(\lambda_0)\right) \\ -\zeta^{-1}\left[(\rho+\alpha)\left(\overline{p}_{et} - \widetilde{p}_t(\lambda_0)\right) - \left(\dot{\overline{p}}_t - \dot{\widetilde{p}}_t(\lambda_0)\right)\right] & , \ t \in \left[\widetilde{\tau}(\lambda_0), \widetilde{\theta}(\lambda_0)\right) . \end{cases}$$

Since $\overline{p}_{et} - \widetilde{p}_t(\lambda_0)$ is decreasing within the time interval $[\widehat{\tau}(\lambda_0, \mu_0), \widetilde{\tau}(\lambda_0))$, then $\nu_t > 0$.

- IN with $c_r < \overline{p}_{e0}$:

$$\gamma_{et} = \begin{cases} 0 & , \ t \in \left[0, \widetilde{\theta}(\lambda_0)\right) \\ \widetilde{p}_t(\lambda_0) - c_r & , \ t \in \left[\widetilde{\theta}(\lambda_0), +\infty\right) \end{cases}$$

$$\gamma_{rt} = \begin{cases} c_r - \widehat{p}_t(\lambda_0, \mu_0) & , \ t \in \left[0, \widehat{\theta}(\lambda_0, \mu_0)\right) \\ 0 & , \ t \in \left[\widehat{\theta}(\lambda_0, \mu_0), +\infty\right) \end{cases}$$

$$
\mu_t = \begin{cases} \mu_0 e^{(\rho+\alpha)t} & , \ t \in \left[0, \widehat{\theta}(\lambda_0, \mu_0)\right) \\ -\zeta^{-1}\left[c_r - \widetilde{p}_t(\lambda_0)\right] & , \ t \in \left[\widehat{\theta}(\lambda_0, \mu_0), \widetilde{\theta}(\lambda_0)\right) \\ 0 & , \ t \in \left[\widetilde{\theta}(\lambda_0), +\infty\right) \end{cases}
$$

$$
\nu_t = \begin{cases} 0 & , \ t \notin \left[\widehat{\theta}(\lambda_0, \mu_0), \widetilde{\theta}(\lambda_0)\right) \\ -\zeta^{-1}\left[(\rho+\alpha)\left(c_r - \widetilde{p}_t(\lambda_0)\right) + \dot{\widetilde{p}}_t(\lambda_0)\right] & , \ t \in \left[\widehat{\theta}(\lambda_0, \mu_0), \widetilde{\theta}(\lambda_0)\right). \end{cases}
$$

- IN with $\overline{p}_{e0} < c_r < \overline{p}_{e\infty}$, illustrated in Fig. 5 :

$$
\gamma_{et} = \begin{cases} 0 & , \ t \in \left[0, \widetilde{\theta}(\lambda_0)\right) \\ \widetilde{p}_t(\lambda_0) - c_r & , \ t \in \left[\widetilde{\theta}(\lambda_0), +\infty\right) \end{cases}
$$

$$
\gamma_{rt} = \begin{cases} c_r - \widehat{p}_t(\lambda_0, \mu_0) & , \ t \in \left[0, \widehat{\tau}(\lambda_0, \mu_0)\right) \\ c_r - \overline{p}_{et} & , \ t \in \left[\widehat{\tau}(\lambda_0, \mu_0), \overline{\theta}\right) \\ 0 & , \ t \in \left[\overline{\theta}, +\infty\right) \end{cases}
$$

$$
\mu_t = \begin{cases} \mu_0 e^{(\rho+\alpha)t} & , \ t \in \left[0, \widehat{\tau}(\lambda_0, \mu_0)\right) \\ -\zeta^{-1}\left[\overline{p}_{et} - \widetilde{p}_t(\lambda_0)\right] & , \ t \in \left[\widehat{\tau}(\lambda_0, \mu_0), \overline{\theta}\right) \\ -\zeta^{-1}\left[c_r - \widetilde{p}_t(\lambda_0)\right] & , \ t \in \left[, \overline{\theta}, \widetilde{\theta}(\lambda_0)\right) \\ 0 & , \ t \in \left[\widetilde{\theta}(\lambda_0), +\infty\right) \end{cases}
$$

$$
\nu_t = \begin{cases} 0 & , \ t \notin \left[0, \widehat{\tau}(\lambda_0, \mu_0)\right) \cup \left[\widetilde{\theta}(\lambda_0), +\infty\right) \\ -\zeta^{-1}\left[(\rho+\alpha)\left(\overline{p}_{et} - \widetilde{p}_t(\lambda_0)\right) - \left(\dot{\overline{p}}_t - \dot{\widetilde{p}}_t(\lambda_0)\right)\right] & , \ t \in \left[\widehat{\tau}(\lambda_0, \mu_0), \overline{\theta}\right) \\ -\zeta^{-1}\left[(\rho+\alpha)\left(c_r - \widetilde{p}_t(\lambda_0)\right) + \dot{\widetilde{p}}_t(\lambda_0)\right] & , \ t \in \left[\overline{\theta}, \widetilde{\theta}(\lambda_0)\right). \end{cases}
$$

- DN with $\overline{p}_{e\infty} < c_r < \overline{p}_{e0}$, illustrated in Fig. 10 :

$$
\gamma_{et} = \begin{cases} 0 & , \ t \in \left[0, \widetilde{\theta}(\lambda_0)\right) \\ \widetilde{p}_t(\lambda_0) - c_r & , \ t \in \left[\widetilde{\theta}(\lambda_0), +\infty\right) \end{cases}
$$

$$
\gamma_{rt} = \begin{cases} c_r - \widehat{p}_t(\lambda_0, \mu_0) & , \ t \in \left[0, \widehat{\theta}(\lambda_0, \mu_0)\right) \\ 0 & , \ t \in \left[\widehat{\theta}(\lambda_0, \mu_0), \overline{\theta}\right) \cup \left[\widetilde{\theta}(\lambda_0), +\infty\right) \\ c_r - \overline{p}_{et} & , \ t \in \left[, \overline{\theta}, \widetilde{\tau}(\lambda_0)\right) \\ c_r - \widetilde{p}_t(\lambda_0) & , \ t \in \left[\widetilde{\tau}(\lambda_0), \widetilde{\theta}(\lambda_0)\right) \end{cases}
$$

$$\mu_t = \begin{cases} \mu_0 e^{(\rho+\alpha)t} & , \ t \in \left[0, \widehat{\theta}\left(\lambda_0, \mu_0\right)\right) \\ -\zeta^{-1}\left[c_r - \widetilde{p}_t\left(\lambda_0\right)\right] & , \ t \in \left[\widehat{\theta}\left(\lambda_0, \mu_0\right), \overline{\theta}\right) \\ -\zeta^{-1}\left[\overline{p}_{et} - \widetilde{p}_t\left(\lambda_0\right)\right] & , \ t \in \left[, \overline{\theta}, \widetilde{\tau}\left(\lambda_0\right)\right) \\ 0 & , \ t \in \left[\widetilde{\tau}\left(\lambda_0\right), +\infty\right) \end{cases}$$

$$\nu_t = \begin{cases} 0 & , \ t \notin \left[0, \widehat{\theta}\left(\lambda_0, \mu_0\right)\right) \cup \left[\widehat{\tau}\left(\lambda_0\right), +\infty\right) \\ -\zeta^{-1}\left[\left(\rho+\alpha\right)\left(c_r - \widetilde{p}_t\left(\lambda_0\right)\right) + \dot{\widetilde{p}}_t\left(\lambda_0\right)\right] & , \ t \in \left[\widehat{\theta}\left(\lambda_0, \mu_0\right), \overline{\theta}\right) \\ -\zeta^{-1}\left[\left(\rho+\alpha\right)\left(\overline{p}_{et} - \widetilde{p}_t\left(\lambda_0\right)\right) - \left(\dot{\overline{p}}_{et} - \dot{\widetilde{p}}_t\left(\lambda_0\right)\right)\right] & , \ t \in \left[\overline{\theta}, \widehat{\tau}\left(\lambda_0\right)\right). \end{cases}$$

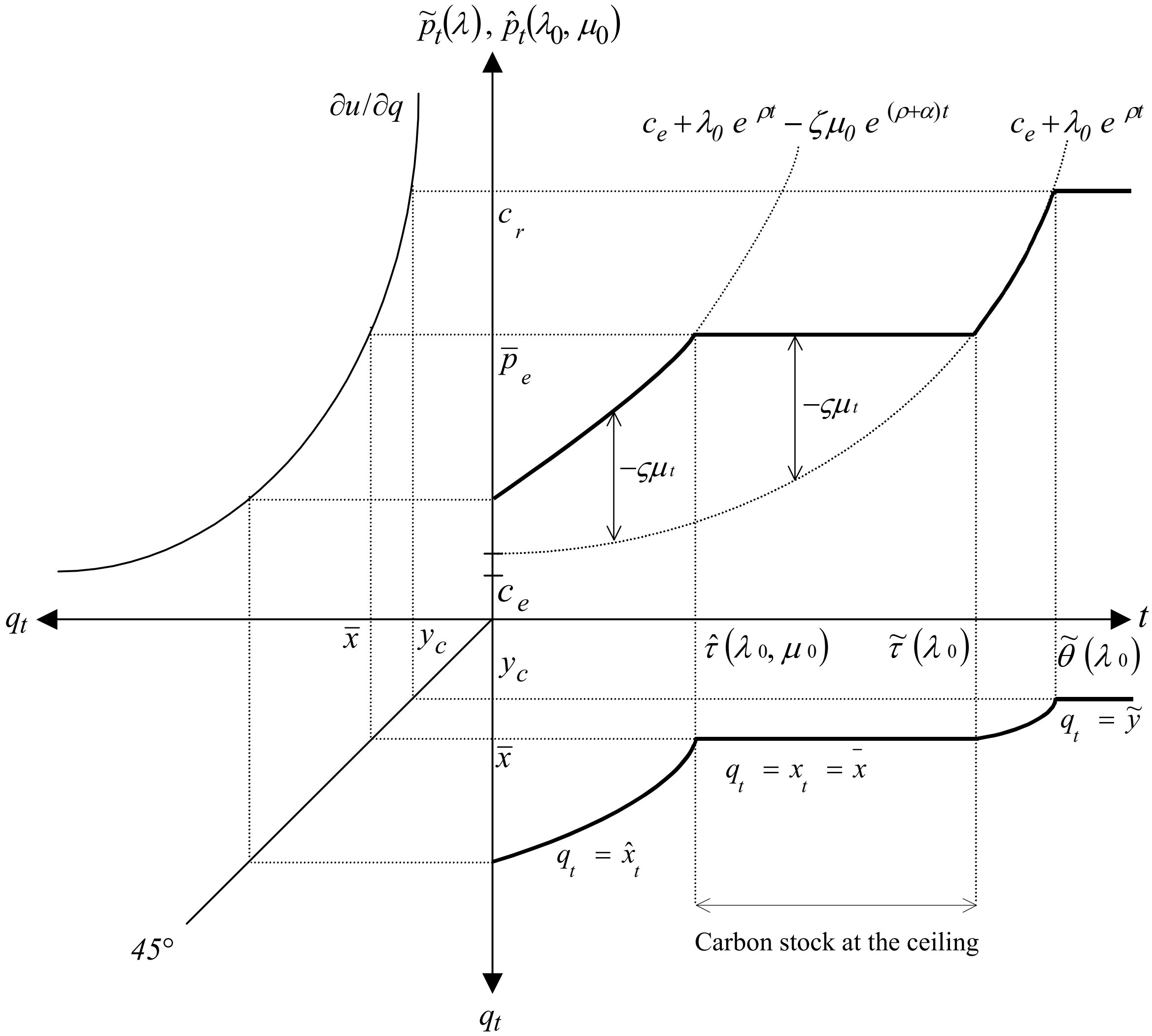

Figure 1: Stationary Demand ($\bar{p}_e < c_r$) - only coal is used at the ceiling.

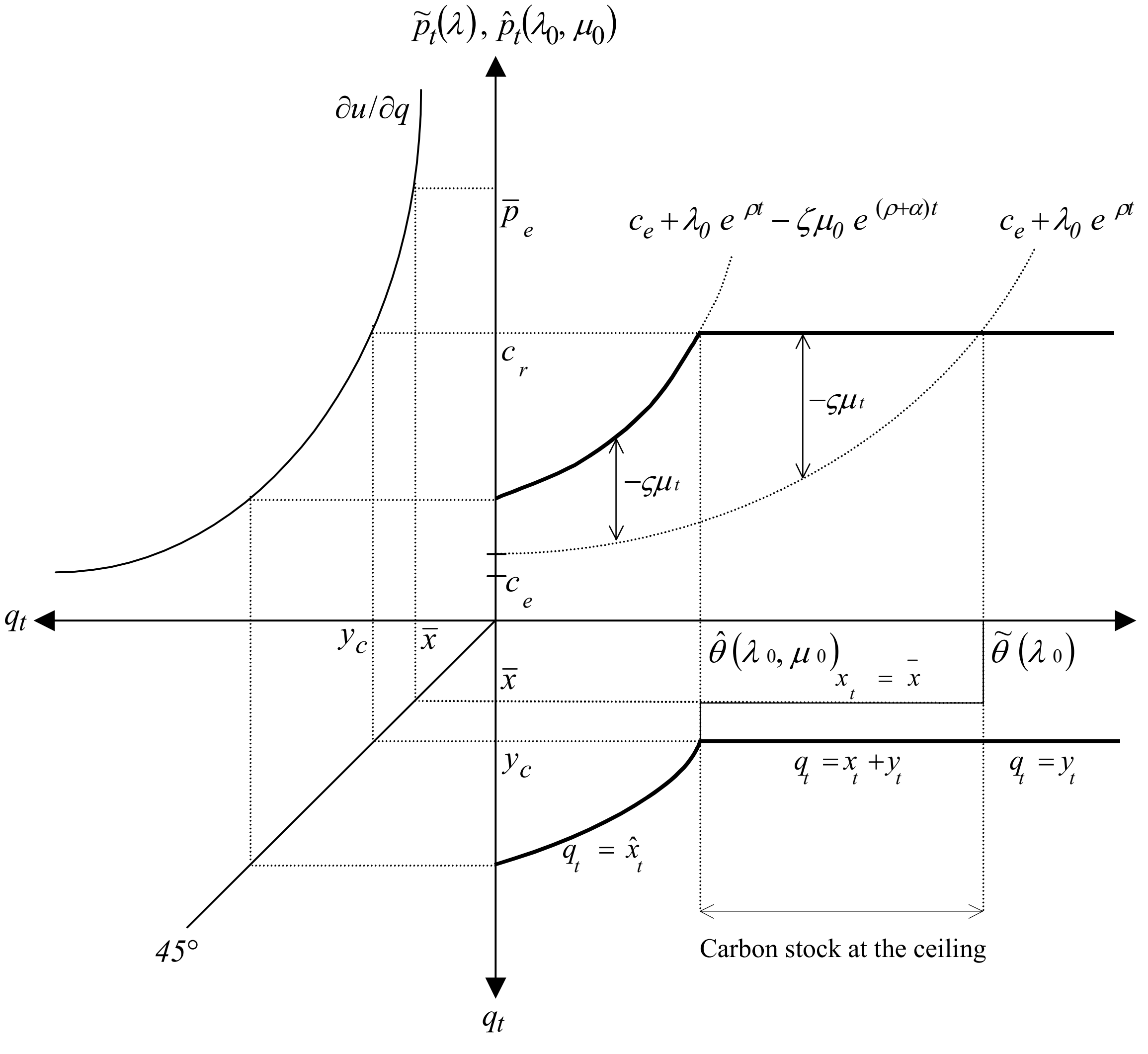

Figure 2: Stationary Demand ($\bar{p}_e > c_r$) - both coal and solar energy are used at the ceiling.

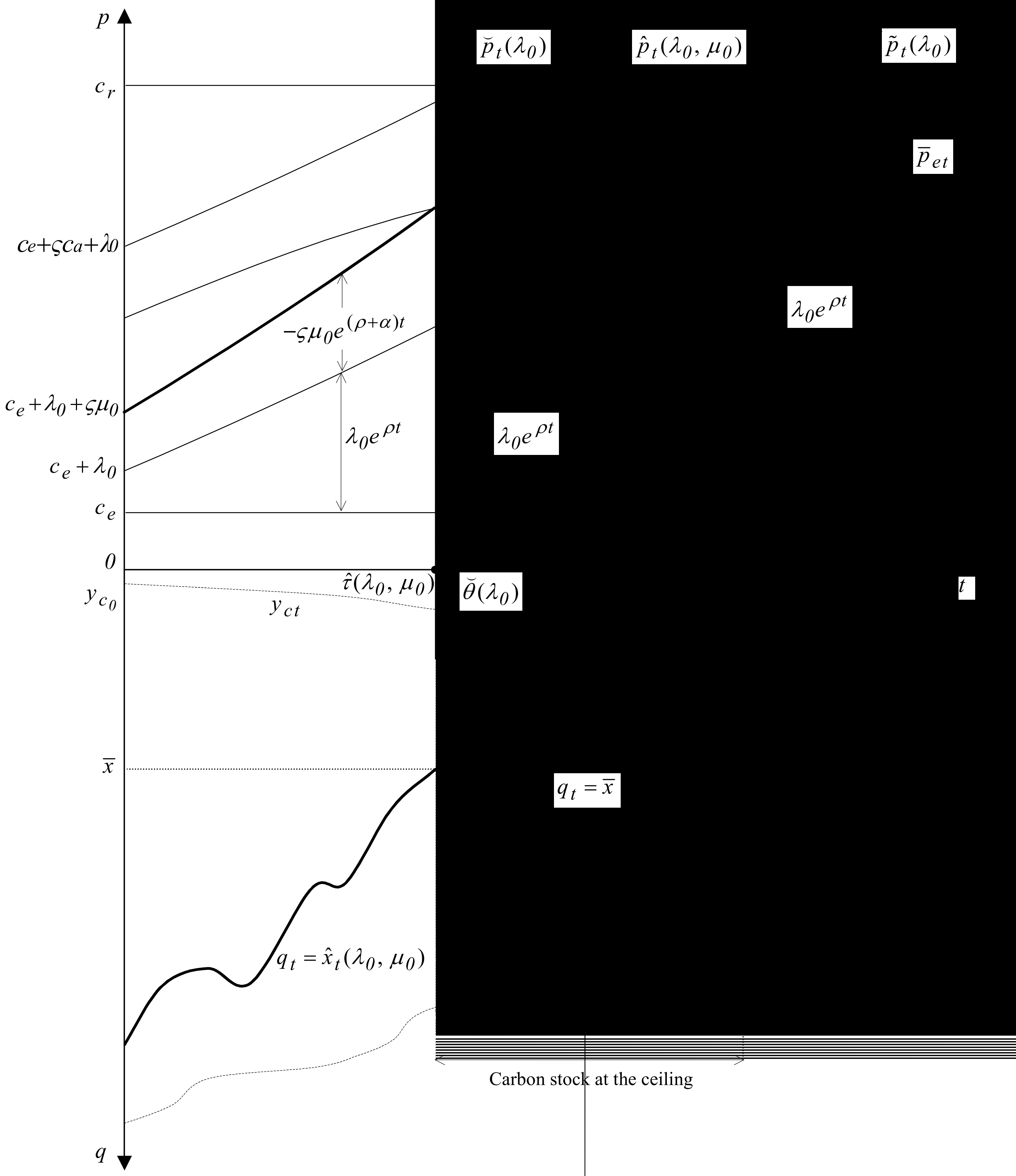

Figure 3: Increasing Demand ($\overline{p}_{e\infty} < c_r$) - only coal is used at the ceiling.

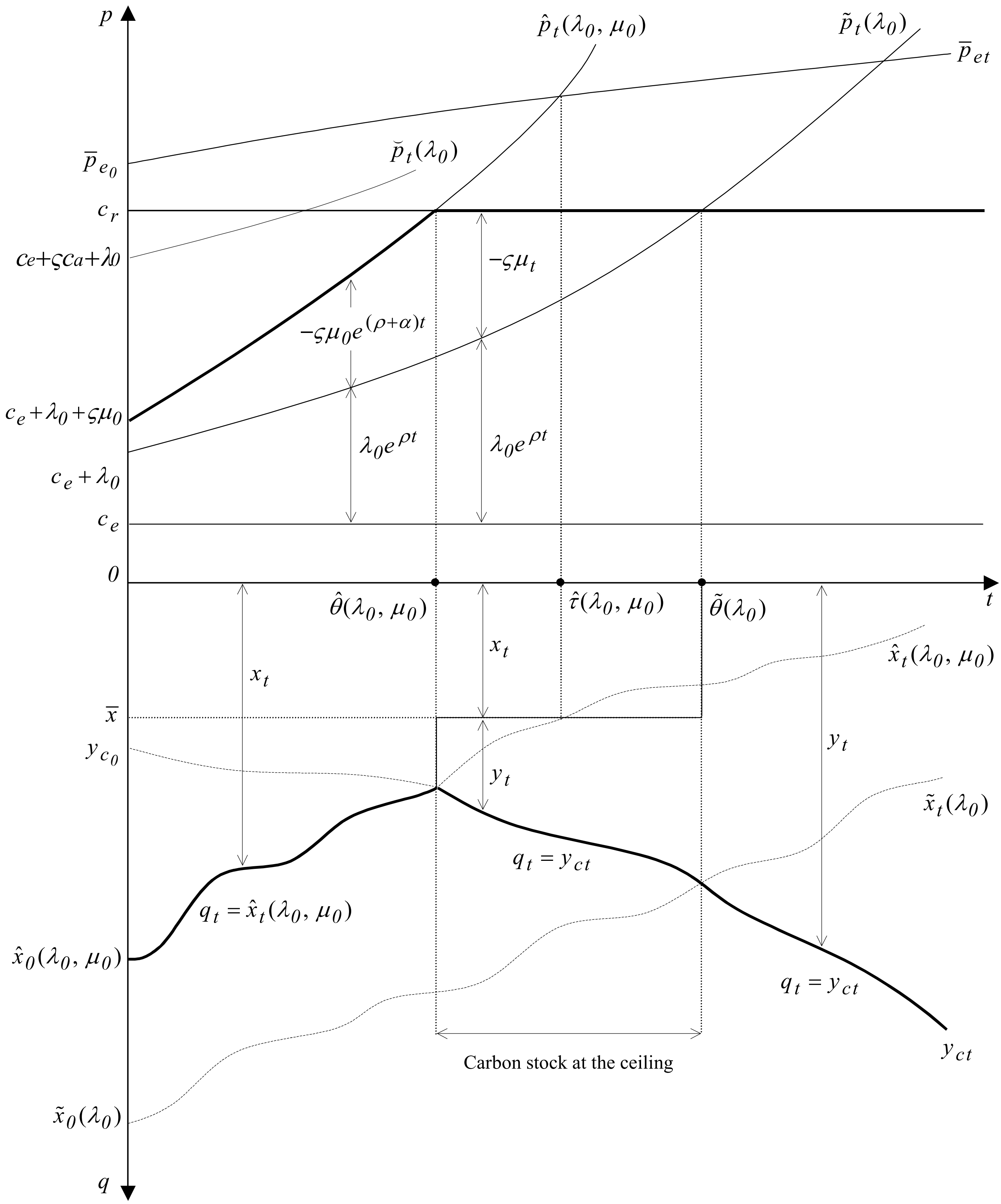

Figure 4: Increasing Demand ($c_r < \overline{p}_{e0}$) - both coal and solar energy are used at the ceiling.

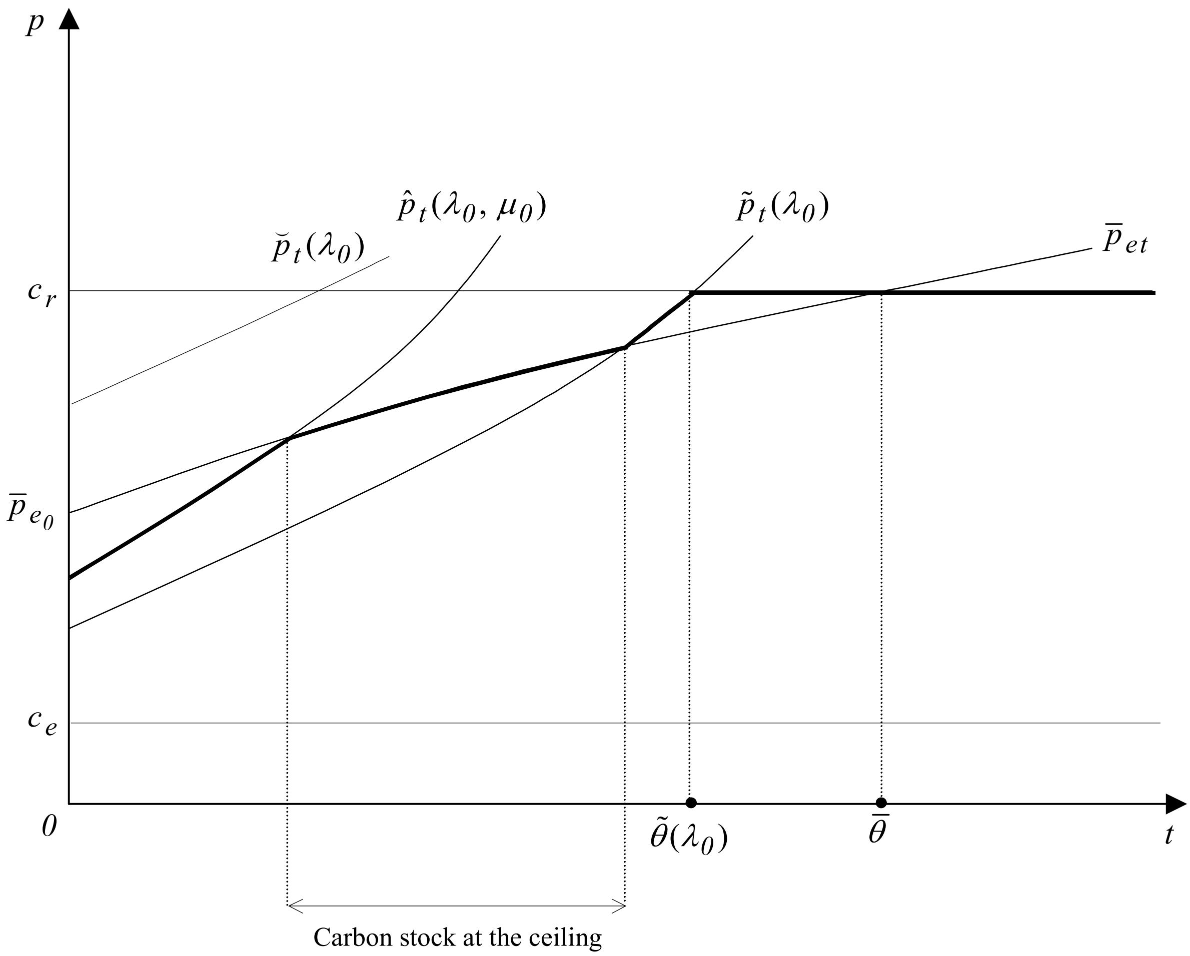

Figure 5: Increasing Demand ($\overline{p}_{e0} < c_r < \overline{p}_{e\infty}$) - only coal is used at the ceiling.

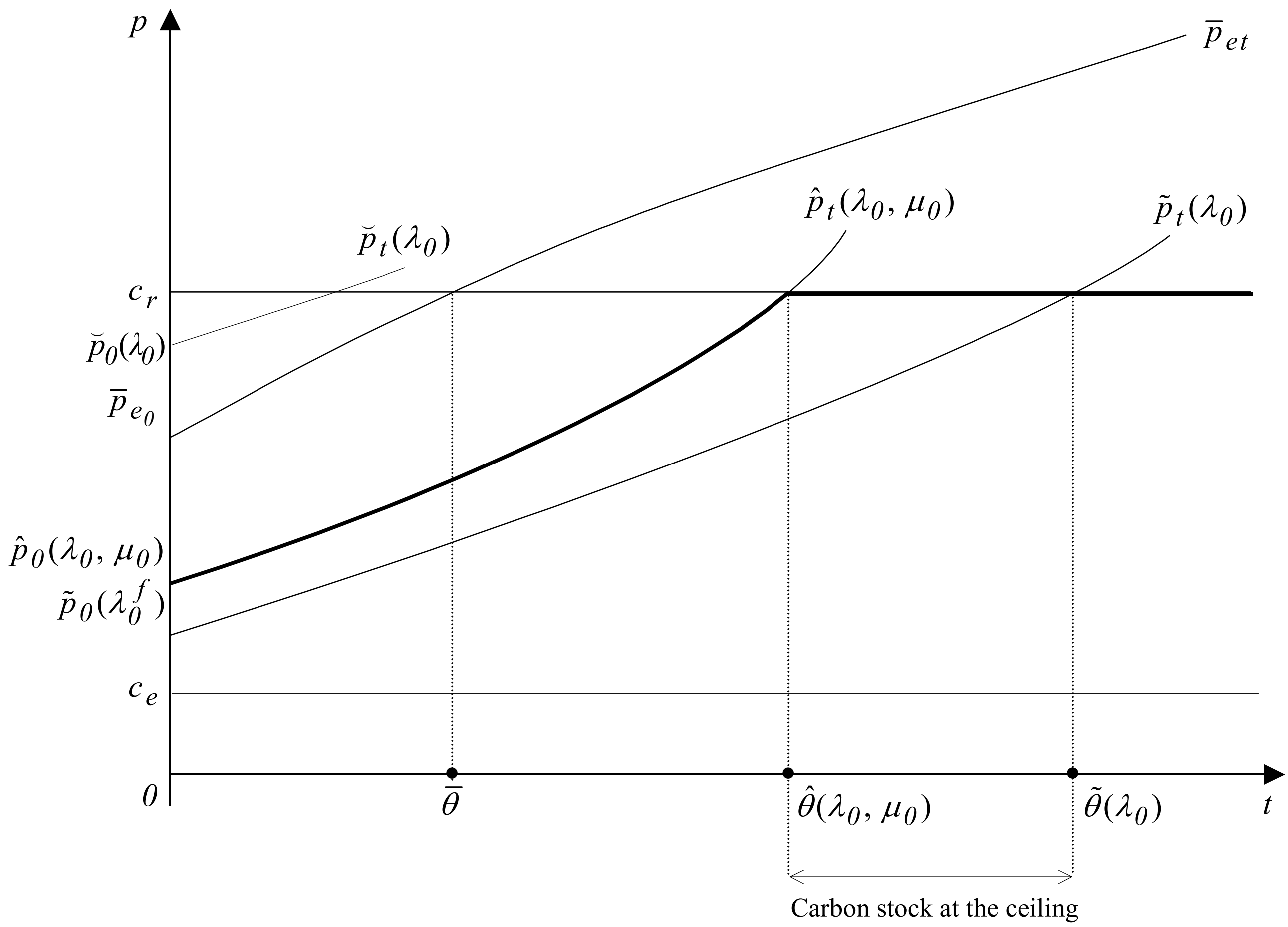

Figure 6: Increasing Demand ($\overline{p}_{e0} < c_r < \overline{p}_{e\infty}$) - both resources are used at the ceiling.

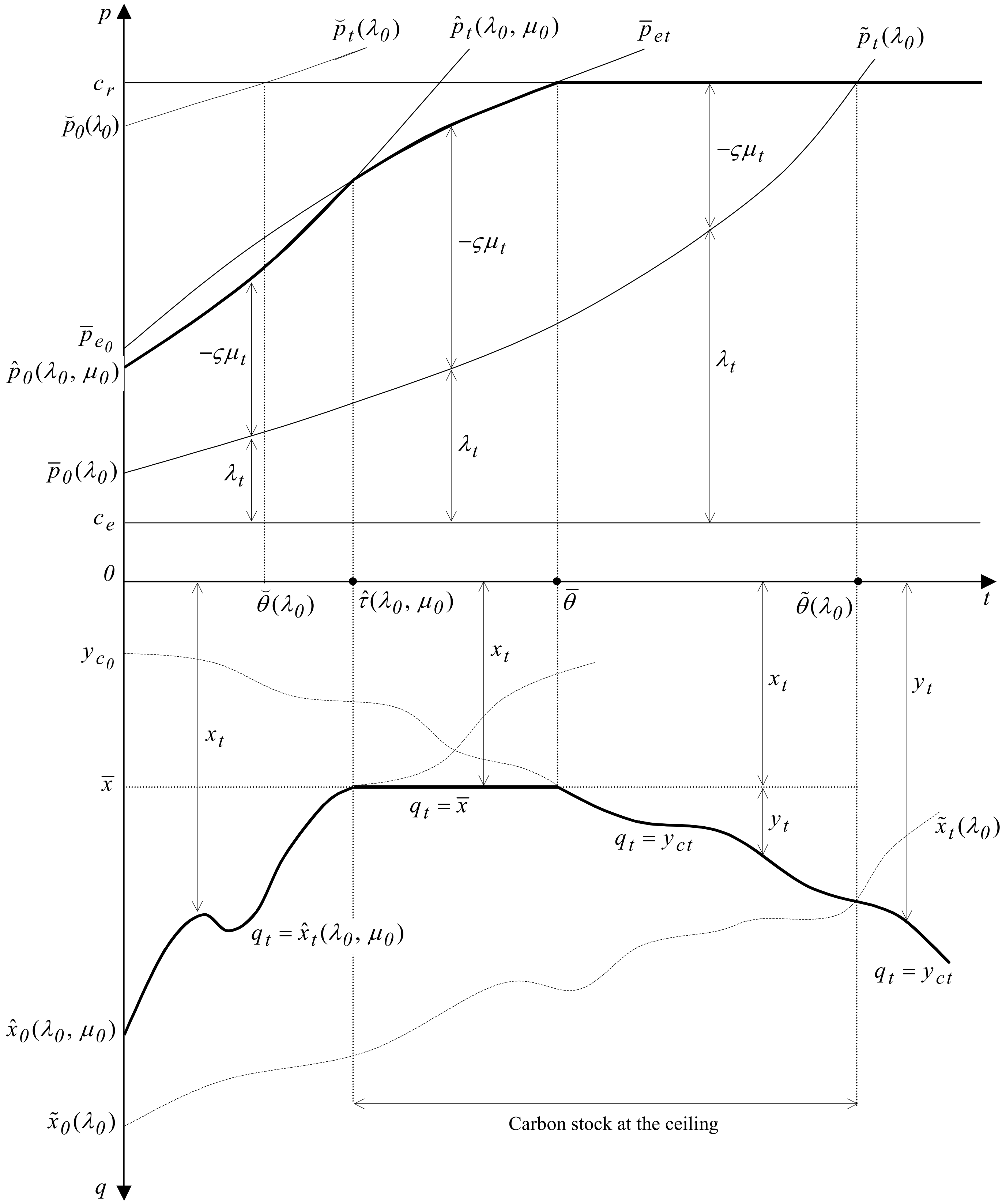

Figure 7: Increasing Demand ($\overline{p}_{e0} < c_r < \overline{p}_{e\infty}$) - the ceiling has two phases - coal is used first, followed by both resources.

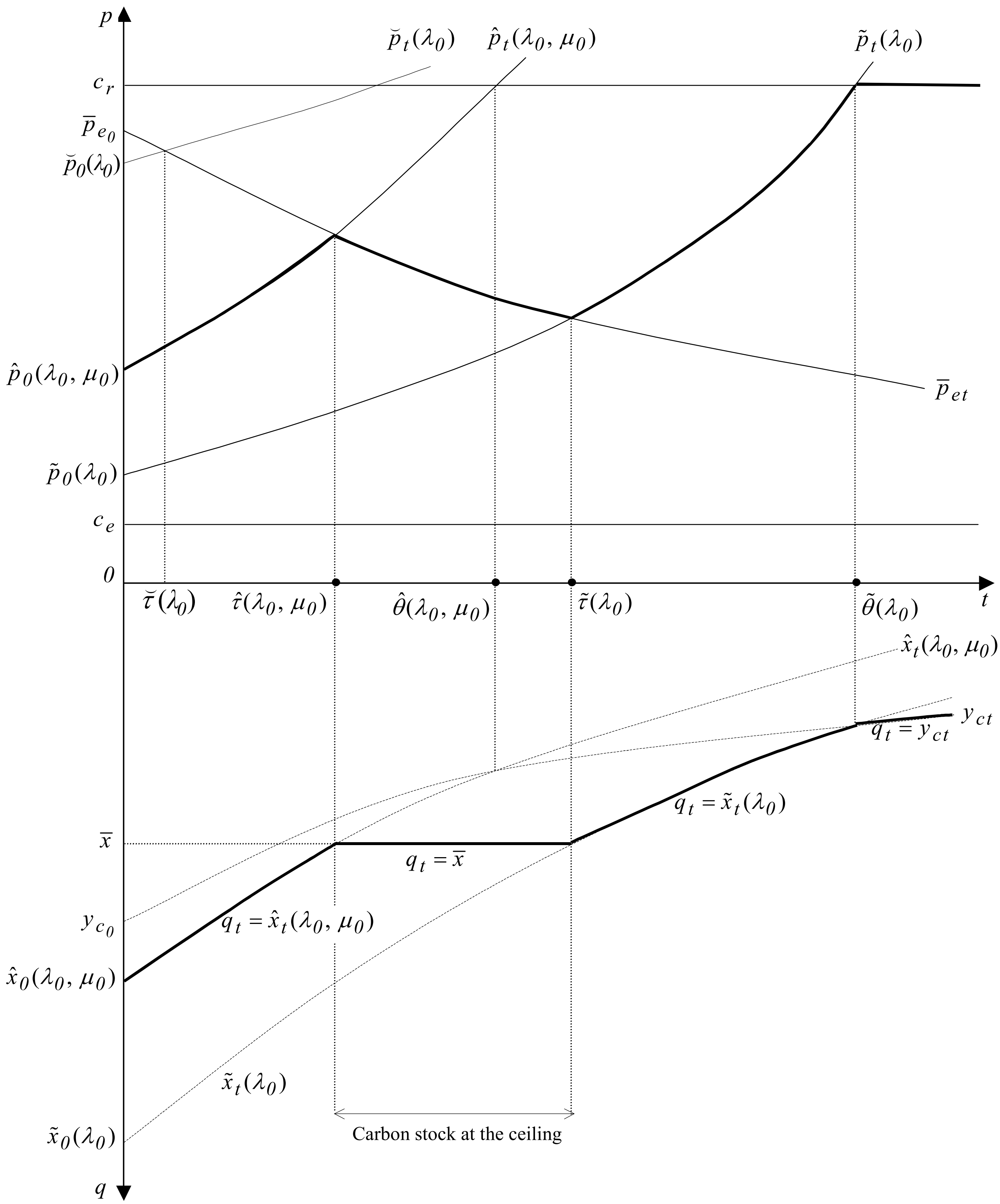

Figure 8: Decreasing Demand ($\overline{p}_{e0} < c_r$) - only coal is used at the ceiling.

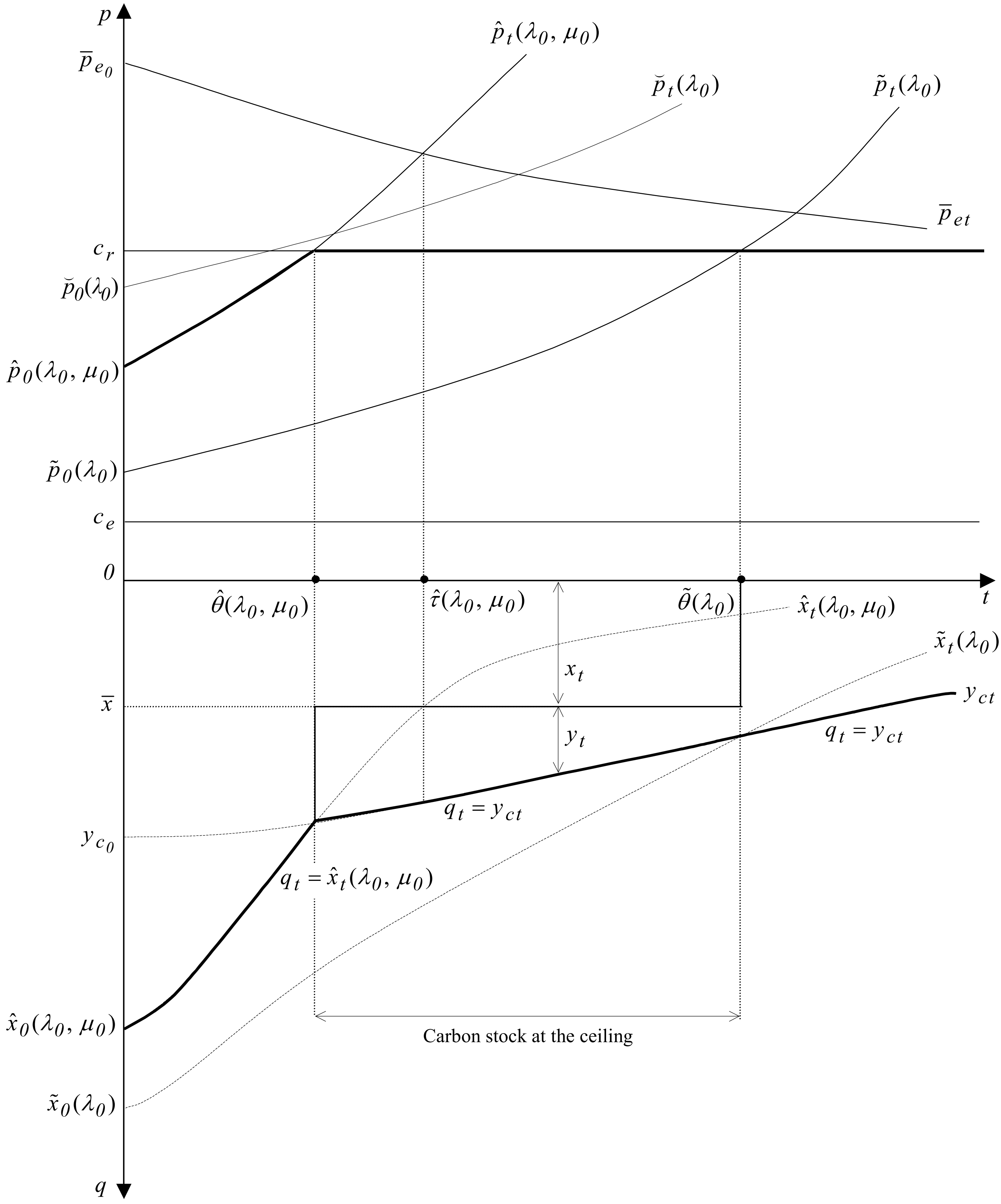

Figure 9: Decreasing Demand ($\overline{p}_{e\infty} > c_r$) - both resources are used at the ceiling.

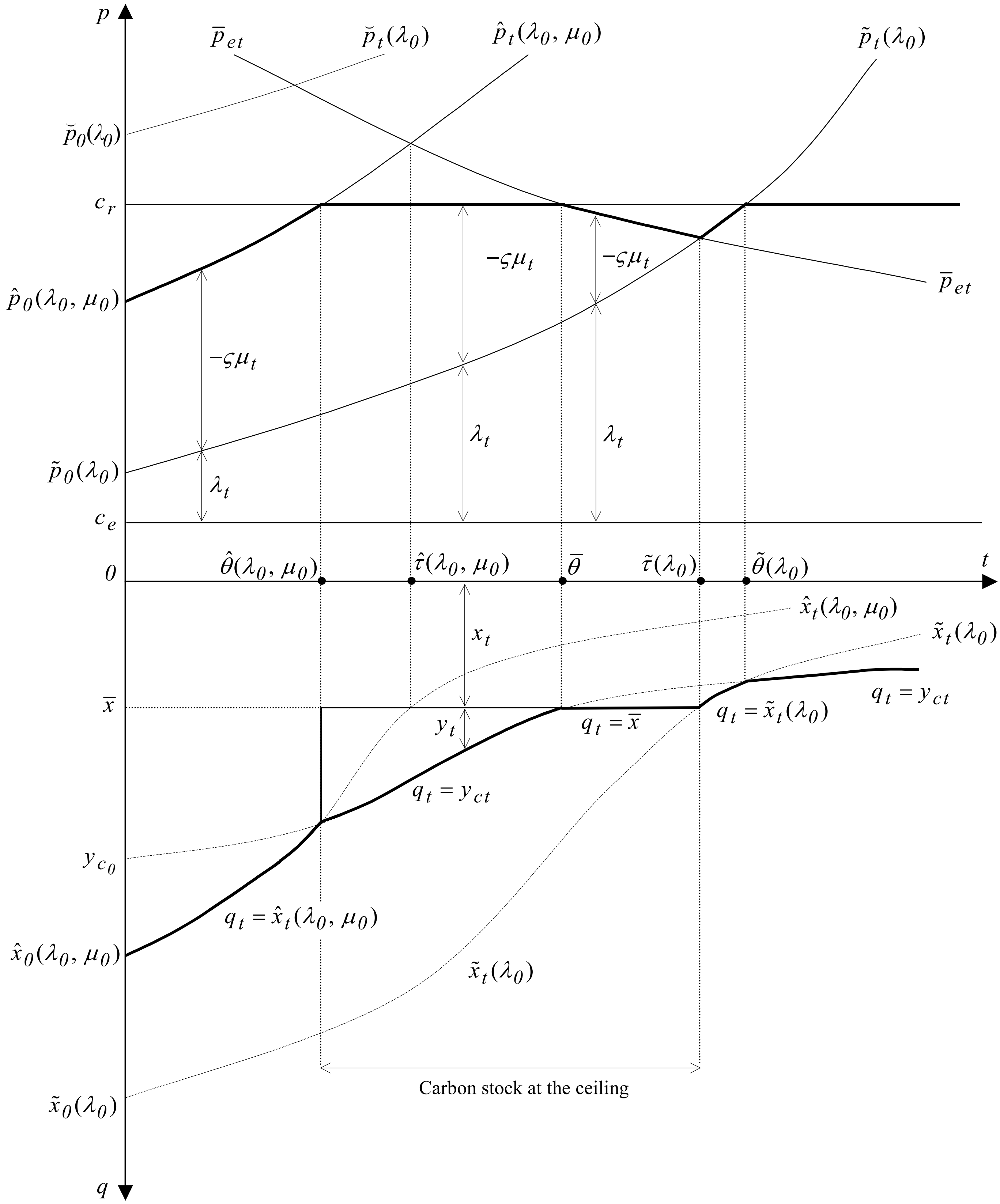

Figure 10: Decreasing Demand ($p_{e0} > c_r > p_{e\infty}$) - solar energy is used during two disjoint time periods.

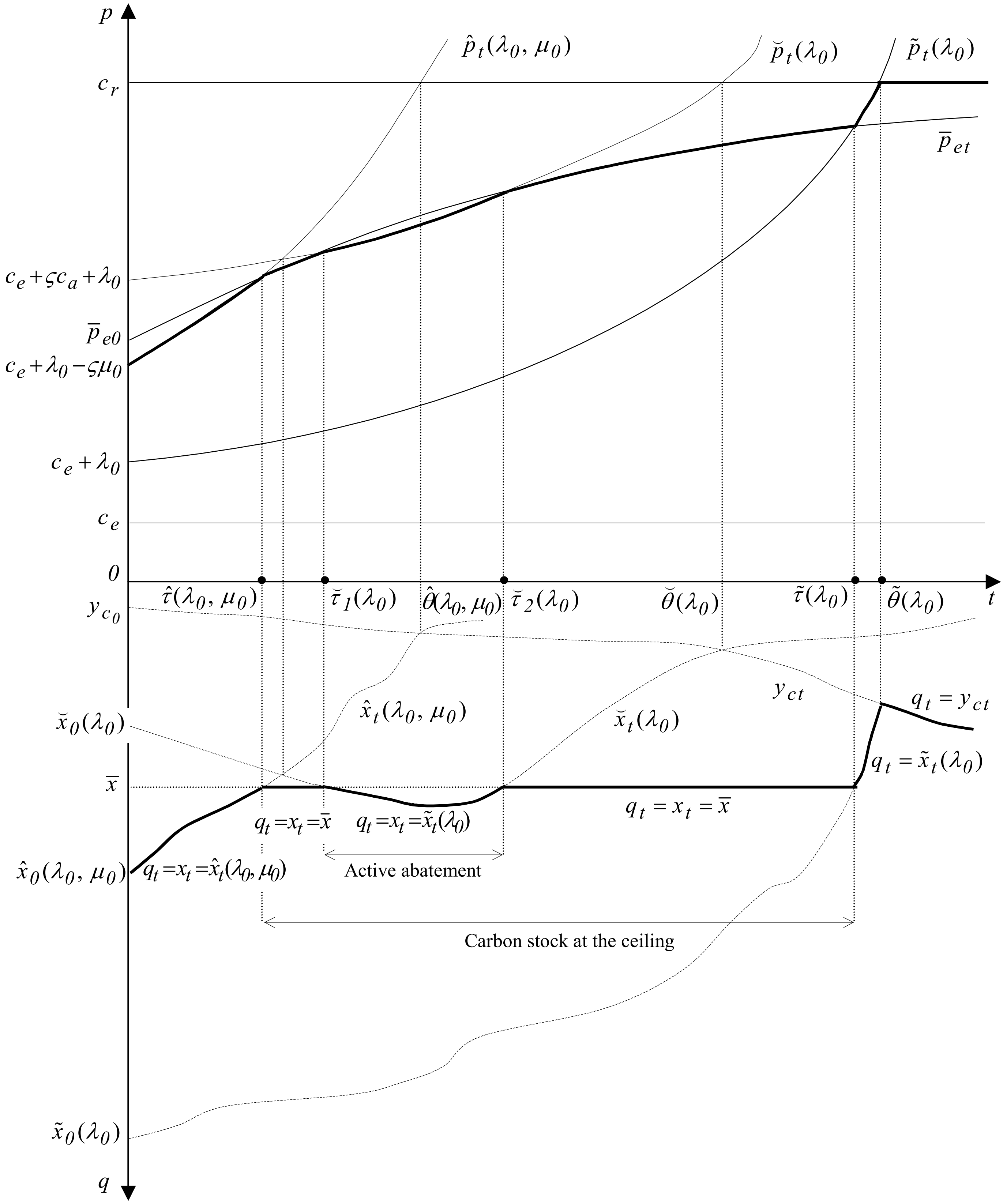

Figure 11: Carbon is sequestered strictly within the period when the ceiling is binding.

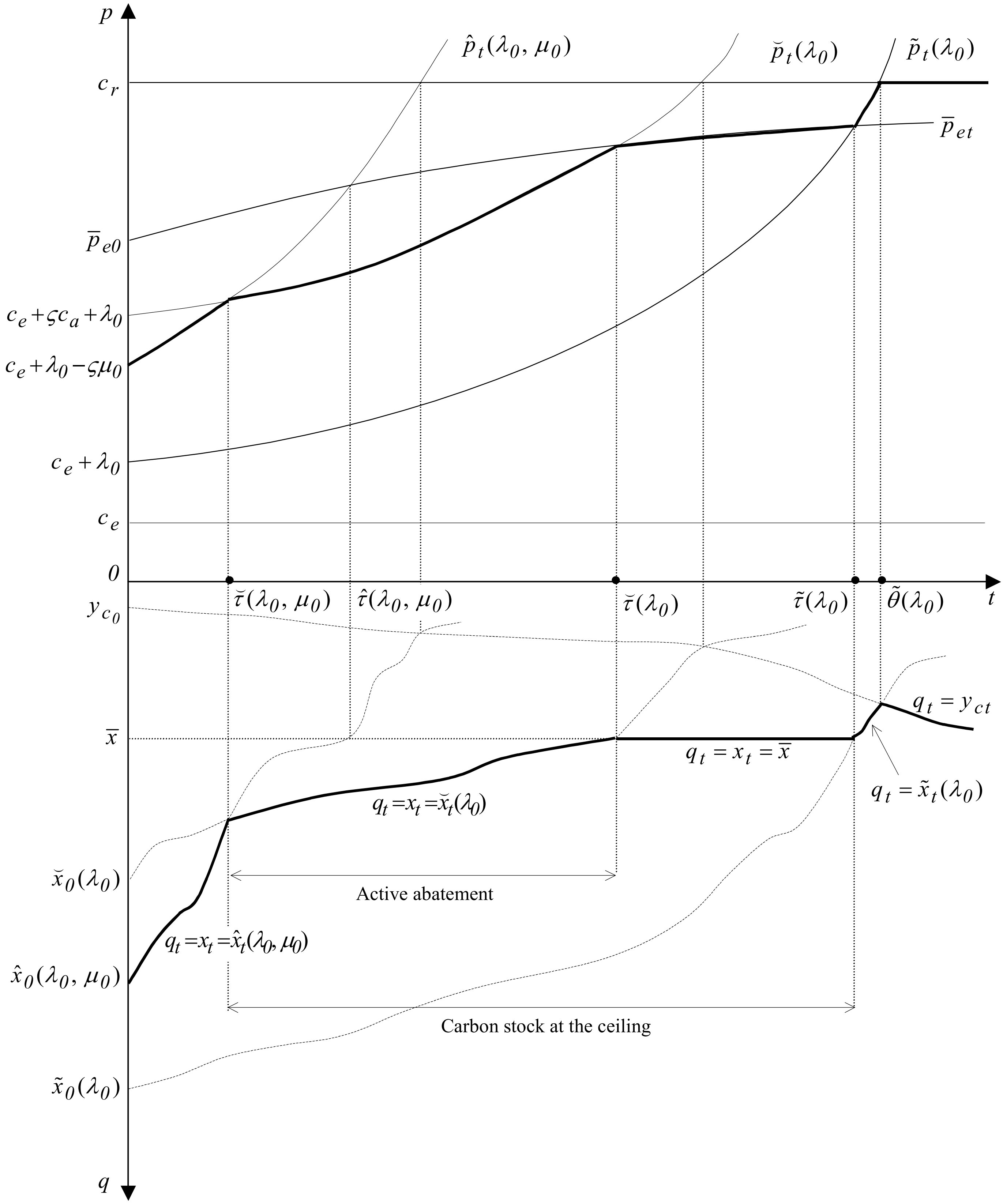

Figure 12: Sequestration starts exactly at the instant the ceiling becomes binding.

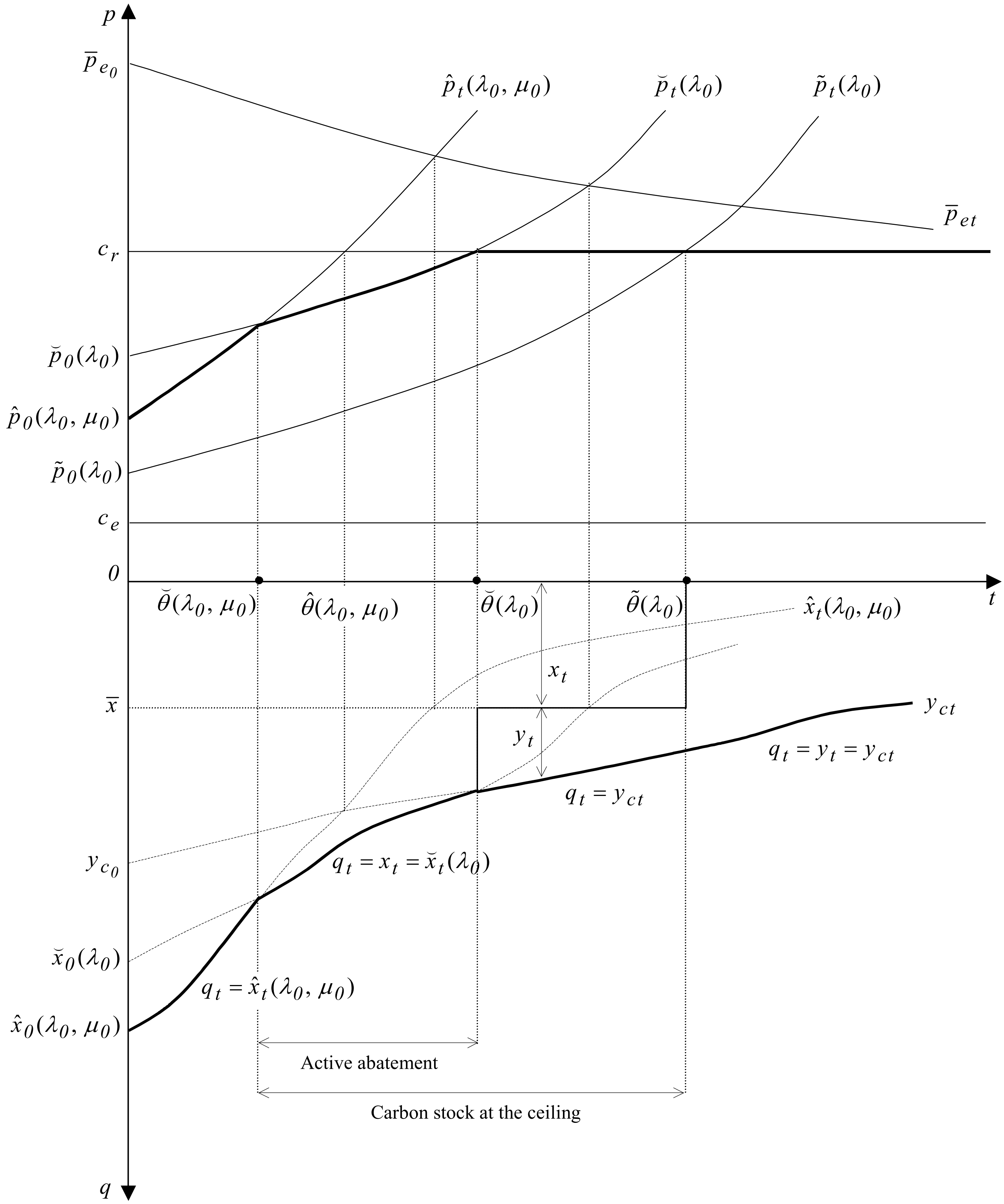

Figure 13: Sequestration starts when the ceiling becomes binding but is replaced by solar energy.