Public goods with costly access[¤]

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Abstract

We examine the optimal allocation of excludable public goods with a private access cost that some consumers may not be able to a¤ord. The full-information benchmark is presented …rst. Then, individuals' access costs and income levels are private information. When high income consumers have low access cost, asymmetric information increases the cost of subsidizing the poor for accessing the public good, and inequality increases. When the low access cost consumers have the lower income, subsidizing the poor may involve countervailing incentives, but inequality decreases. Finally, monopoly provision exacerbates underprovision of the poor, particularly of those with low access cost.

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1 Introduction

The economics literature has often stressed the "public", nonrival, nature of information goods and the difficulties they are associated with for the achievement of an optimal allocation of resources (Arrow (1965)). Data files and software goods, for instance, can be centralized and accessed with a computer and a telephone line when needed for producing a service. Alternatively, they can be replicated and installed on personal computers.[1] In either case, the consumption of one individual does not reduce the quantity of the good available for the other individuals. Information goods thus belong to a vast class of public (nonrival) goods, with no obligation of use and the possibility of exclusion. An additional characteristic, which we want to emphasize in this paper, is that access to the good may be costly. In other words, some private good or service must be consumed along with the public good. For information goods this cost may take different forms: connection or telecommunications expenditure, the cost of the personal computer, individual learning costs, etc. Examples of information goods with costly access abound. Cable TV often offers subscription fees supplemented by pay-per-view access charges. In most countries, access to internet requires a local telecommunications charge. Furthermore, it always involves the capital cost of the required equipment. Micropayments for access to websites already exist for some type of services and one can expect their generalization as technology progresses. Beyond these information goods, many other public goods have costly access. Most natural sites, like national parks or beaches, require transportation costs which may make them unaccessible to some consumers.

Goods with these characteristics have been studied by Agnar Sandmo, who considers general technologies available to consumers (Sandmo (1973)) or producers (Sandmo (1972)) for transforming private goods and public

[1] We do not discuss the relevance of these different modes of organization and the optimal choice between them. This is left for future research.

goods into new private goods.[2] In this paper we consider a special case of such technologies,[3] but introduce asymmetric information as an additional and crucial feature. Specifically, we assume that the cost of this technology is private information of the consumers.

The costly access of these public goods raises new redistributive issues. It is not sufficient to make these public goods available when some consumers cannot afford the cost of access. The information technology revolution, combined with the low access costs to internet, have spurred great hopes that LDCs would be able to benefit from all the informational public goods available in the developed world. However, the concern that this information technology revolution might exacerbate rather than mitigate the differences between LDCs and developed countries has been expressed recently (UN report (1999)).[4]

One reason for this disenchantment is the recognition that the private costs of accessing those public goods have often been neglected or, at least, underestimated. In addition to the pricing of these public goods (like the pricing of scientific journals accessible by internet), one must pay attention to the cost of computers, the cost of telecommunications and, last but not least, the usage cost which is highly dependent on the education level.

The importance of the public goods with costly access made available by the information technology suggests that a thorough and specific economic analysis of the allocation, consumption and production of these goods is useful. This paper provides a first step in this direction by studying the optimal allocation of these goods, once the production process is available. This question is considered in a world of asymmetric information, where the resources of some consumers may be insufficient to afford the private cost

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[2] The property that access involves a private, real cost for the consumer distinguishes our setting from the literature on club goods where the access cost is typically a price.

[3] However, because of fixed costs, our technology does not satisfy Sandmo's concavity assumptions.

[4] This concern has led to the strange proposition of taxing e-mail to favor communicating at the world level (UN report (1999)).

associated with the public good consumption (that is when the so-called …nancial constraints are relevant). Along the way, we examine whether the optimal policy involves a (positive or negative) tax on access and/or usage. Furthermore, we study the allocation that emerges if the good is provided by a pro…t maximizing monopoly.[5]

Section 2 presents the model and characterizes the optimal allocation of resources under complete information. When …nancial constraints are irrelevant, we obtain a generalized version of Samuelson (1954)'s conditions and identify two main regimes. In one of the regimes, the no-exclusion regime, all individuals consume the full quantity of available public good. In the other regime, the exclusion regime, some individuals consume less than the available quantity of public good because of the access cost.

Next, we address the case when access may be limited by …nancial constraints. Then, the allocation of resources depends on the correlation between the access costs and the …nancial resources. In the case of positive correlation (the rich are also the ones with low access costs), the connection of the poor may require subsidies which imply a "limited liability" rent. Because of the social cost of this rent the consumption of the poor is reduced. In the case of negative correlation (the poor are the ones with low access costs), the limited liability rent is given to the low cost individual and results in a reduction in the provision of the good.

Section 3 characterizes the distortions implied by asymmetric information on access costs in the absence of …nancial constraints limiting access. It is shown that asymmetric information expands the domain of parameters for which some individuals do not consume all the available public good because of the information rent which must be given up to consumers with low access costs. Furthermore, it is shown that usage is taxed (except possibly

[5] See Drèze (1980) for the regulation of a monopolistic provider of public goods with exclusion. Monopoly provision of an excludable public good has also been studied by Cornes and Sandler (1996, Ch. 8) who do allow for asymmetries of information but have neither private connection costs nor …nancial constraints.

for the low cost type) while access is subsidized.

Section 4 combines asymmetric information and financial constraints and studies the interaction between these two phenomena. In the case of positive correlation the limited liability rent of the poor must now, by incentive compatibility, also be conceded to the rich. Consequently, asymmetric information increases the cost of subsidizing the poor. The distortions in the Samuelson conditions now have two origins.

In the case of negative correlation (the poor are the ones with low access costs), the analysis differs from a classical adverse selection problem in two ways. First, the limited liability rent associated with the financial constraints may induce countervailing incentives.[6] Second, some deviations may not be financially viable; for example an individual with high access cost may not be able to claim he has a low access cost because he cannot afford the bundle allocated to the low cost consumers. This expands the set of implementable allocations.

Finally Section 5 considers two extensions. First, we study a setting where the good is provided by a profit maximizing monopoly. We characterize the monopoly solution and compare it with the optimum in order to analyze the distortions that result from a monopolistic provision of the public good. Second, we show how income effects modify the analysis.

2 The model and the complete information benchmark

2.1 The model

Consider a public good with the following characteristics. There is no obligation of use and exclusion of use is possible. The cost of G units of this public good is given by cG, i.e., the marginal cost is constant. There is a continuum $[0;1]$ of consumers. Each one must incur a fixed cost k and a

[6] On countervailing incentives, see Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995), Jullien (2000), Jeon and Laffont (1999).

variable cost θg to access the public good, that is to consume a level $g > 0$. Individual consumption is less than or equal to the provided level G.

Consumers differ in two respects: marginal cost of access, $\theta \in \{\theta_1; \theta_2\}$ and income, $y \in \{y_1; y_2\}$. For simplicity, we consider a population with two types of individuals only. Type 1, $(\theta_1; y_1)$ representing a proportion ν of consumers and type 2, $(\theta_2; y_2)$, representing a proportion $1 - \nu$. Throughout the paper it is assumed that $\theta_1 < \theta_2$, so that type 1 always refers to the individuals with the lowest cost of access.

Let t_i be the payment made by an agent of type i to enjoy the public good. The agent's utility level is given by

$$U_i = u(g_i) + y_i - \theta_i g_i - k - t_i; \qquad (1)$$

if he accesses the public good (so that $g_i > 0$), and $U_i = y_i$, if he does not access the public good. Since there is no obligation of use, the following participation constraint will always have to be satisfied:

$$u(g_i) - \theta_i g_i - k + y_i - t_i \geq y_i; \quad i = 1; 2; \qquad (P_i)$$

Furthermore, let $1 + \lambda$ be the social cost of public funds.[7]

Finally, when we introduce financial constraints, we assume that the consumption of the numeraire good is restricted to be non-negative so that

$$y_i - \theta_i g_i - k - t_i \geq 0 \quad i = 1; 2; \qquad (F_i)$$

must be satisfied. With quasi-linear preferences these financial constraints are a stylized way to incorporate income effects and, to a limited extent, redistributive considerations.[8]

[7] See Laffont and Tirole (1993) for a discussion of the social cost of public funds.

[8] A major benefit of our approach is that it enables us to provide a full characterization of the results, both with positive and with negative correlation between access costs and incomes; see also Section 4. In the Appendix we show briefly how the results of Sections 2 and 4 can be obtained with a budget constraint (endogenous cost of funds) along with a concave social welfare functions.

2.2 Optimal allocation without financial constraints

We now turn to the determination of the optimal (utilitarian) allocation under full information. Throughout the paper we shall concentrate on situations where it is socially optimal to connect both types of individuals to the public good. This is because we are interested in understanding how informational and financial constraints affect the cost of providing a public good which is so valuable that it should be accessed by all. In this situation, both types consume a positive level of the good and incur the fixed cost of connection. The optimal utilitarian allocation can then be obtained by solving the following problem:

Problem 1

$$\max_{G;g_1;g_2;t_1;t_2} \gamma[u(g_1) - \theta_1 g_1 - k + y_1 - t_1] + (1-\gamma)[u(g_2) - \theta_2 g_2 - k + y_2 - t_2]$$
$$- (1+\lambda)[cG - \gamma t_1 - (1-\gamma)t_2]; \qquad (2)$$

s:t:

$$g_i \leq G \quad i = 1; 2;$$
$$u(g_i) - \theta_i g_i - k + y_i - t_i \geq y_i; \quad i = 1; 2: \qquad (P_i)$$

Recall that the participation constraints, (P_i), must be satisfied because there is no obligation of use. The solution to this problem is presented in the following proposition, where $\Delta\theta = \theta_2 - \theta_1$.

Proposition 1 Under complete information, when the financial constraints are not relevant, the optimal (utilitarian) allocation is characterized by

i) If $\frac{c}{\gamma} < \Delta\theta$; $\quad g_1^* = G^*$ and

$$(1+\lambda)\gamma(u'(G^*) - \theta_1) = (1+\lambda)c; \qquad (3)$$
$$u'(g_2^*) - \theta_2 = 0: \qquad (4)$$

ii) If $\frac{c}{\alpha} > \Delta\theta$; $\quad g_1^* = g_2^* = G^*$ $\quad$ and

$$(1+\lambda)[\alpha(u'(G^*) - \theta_1) + (1-\alpha)(u'(G^*) - \theta_2)] = (1+\lambda)c. \qquad (5)$$

Equations (3) and (5) are the modified Samuelson conditions. In regime ii) we can return to the familiar formula by thinking in terms of net valuations, $u(g_i) - \theta_i g_i$, $i = 1, 2$. The social value of the sum of the marginal rates of substitution (or marginal net valuations) equals the (social) marginal cost.

In regime i) there is a corner solution for consumer 2 whose net valuation is negative at $g_2 = G^*$. The Samuelson condition applies to the consumers of type 1 for whom the resource constraint on the public good is binding. Since there is no obligation of use, the consumption level of 2 maximizes his net valuation and satisfies $u'(g_2^*) = \theta_2$.

The Samuelson conditions are expressed in terms of social costs and benefits. We have not simplified the first-order conditions by $(1+\lambda)$ for this will facilitate the comparisons below. Observe, however, that the solution does not depend on the social cost of public funds. Because social funds are costly, the government extracts as much money as possible from consumers to finance the public good. Under complete information this entails binding participation constraints both individuals. Since the entire surplus is extracted, the benefits to consumers are weighted in the same way as the cost of the public good, namely by $(1+\lambda)$.

Figure 1 illustrates the determination of the solution under regime i) in the (g, t) space. Indifference curves are first increasing (as long as $u'(g_i) > \theta_i$) and then decreasing. The "satiation point" for type 1 is to the right of that for type two.[9] The consumption bundle of each type lies on the indifference curve corresponding to his reservation utility level. The optimal level of $G^* = g_1^*$ is on the indifference curve $U_1 = y_1$ at point B, where the slope (willingness to pay) is $c/\alpha > 0$. Type 2 consumes g_2^* corresponding to point

[9] With quasi-linear preferences, the indifference curves of any given type are parallel; the slope (marginal willingness to pay) depends only on g_i.

A, the maximum of his indifference curve. Observe that the total marginal willingness to pay, at consumed quantities, equals $\nu(c/\nu) + (1 - \nu)0 = c$. Consequently, an extended version of the Samuelson condition continues to hold.

Regime i) occurs as long as A is to the left of B. When the two indifference curves are sufficiently close ($\Delta\theta$ is small) and/or when c/ν is large, the no exclusion regime ii) occurs with both types consuming G^*. This allocation (point (A_n, B_n) in Figure 2) is determined so that the sum of the slopes of the indifference curves (for a given level of G) equals c.

The decentralization of the optimal allocation characterized in Proposition 1 requires two types of instruments. To induce the right public good consumption levels, personalized (Lindahl) prices are needed to account for the different net valuations of consumers. In regime i) these prices are:[10]

$$p_1^* = c/\nu \quad \text{and} \quad p_2^* = 0.$$

In regime ii) they are given by:

$$p_1^* = c + (1 - \nu)\Delta\theta, \quad \text{and} \quad p_2^* = c - \nu\Delta\theta. \tag{6}$$

In both cases, type 1 who has a higher net valuation pays a higher price and the sum of these prices equals the marginal cost of production. Since the cost function is linear, the revenues levied under this pricing scheme cover cost.

Because of the social cost of public funds, the social planner wants to set consumers at their reservation utility level. Consequently, the personalized prices must be complemented by personalized lump sum taxes K_i^*.[11]

[10] Under complete information, the observability of individual consumption is not necessary. To determine the optimal level of public good supply it is sufficient to know the distribution of types. Then, one can decentralize the levels of consumption by rationing type 1 individuals in case i) and both types in case ii).

[11] Graphically the K_i's correspond simply to the intercept of the tangent; see Figure 1.

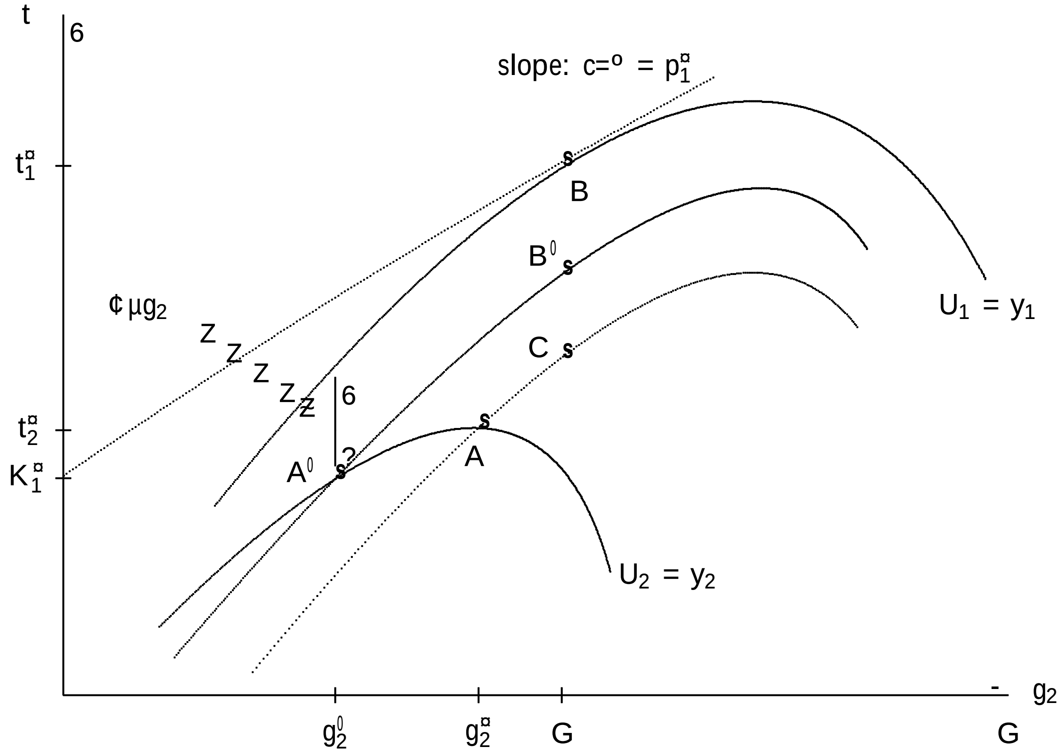

Figure 1: Full information optimum $(A; B)$ and optimum under asymmetric information $(A^0; B^0)$. Both solutions are for regime i) and the …nancial constraints are ignored.

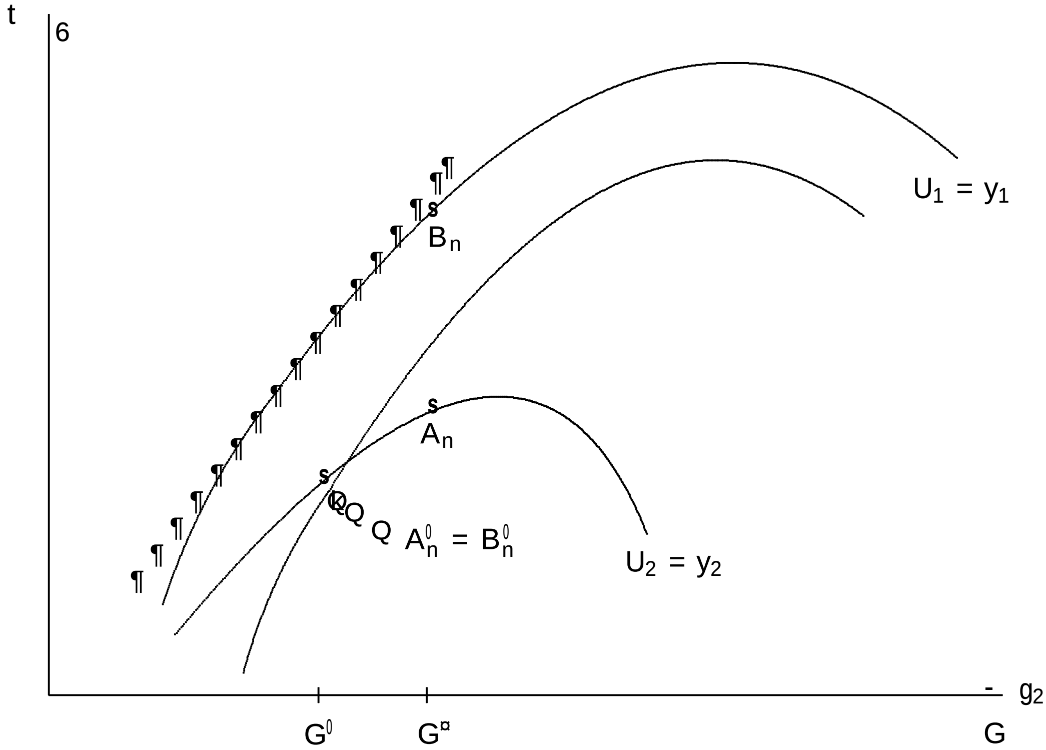

Figure 2: Full information optimum $(A_n; B_n)$ and optimum under asymmetric information $(A'_n; B'_n)$ under regime ii); the financial constraints are ignored. Solutions under regime i).

2.3 Optimal allocation with financial constraints

We now add the financial constraints (F_i) in Problem 1. To understand the interplay between financial and participation constraints, it is useful to combine (P_i) and (F_i) which yields

$$u(g_i) - \theta_i g_i - k - t_i \geq \max\{0; u(g_i) - y_i\}. \tag{7}$$

When $u(g_i) - y_i < 0$ the right-hand-side of (7) is equal to zero and (P_i) is binding, while (F_i) is automatically satisfied. The opposite result, with (F_i) binding, obtains when $u(g_i) - y_i > 0$; this is the situation to which we now turn. We distinguish between two cases.

2.3.1 "Positive correlation"

We use this term to refer to the situation where type 1 is the "good" type for both characteristics, access cost and income; i.e., when $y_1 > y_2$. When only type 2 is financially constrained, t_2 is determined by (F_2) (rather than by (P_2)) while t_1 continues to be determined by P_1. Maximizing welfare, given by (2) subject to (P_1) and (F_2) yields:

Regime i): $G = g_1 > g_2$ with

$$(1+\lambda)\nu(u'(G) - \theta_1) = (1+\lambda)c \tag{8}$$

$$u'(g_2) - \theta_2 = \lambda\theta_2. \tag{9}$$

Since one unit of income for type 2 has social value of $1+\lambda$, the marginal social cost of consumption is $(1+\lambda)\theta_2$. This justifies a downward distortion determined by (9); see (4) for the reference case without distortion.

To have type 2 benefit from the public good, the utilitarian social planner has to concede a rent, which we will refer to as limited liability rent. To see this note that when (F_2) is binding we have

$$U_2 = u(g_2) + y_2 - \theta_2 g_2 - k - t_2 = u(g_2) > y_2;$$

so that the utility of type 2 is above its reservations level.

In the no-exclusion regime, this costly rent affect the level of production. We have:

Regime ii): $G = g_1 = g_2$ with

$$(1+\lambda)[\nu(u'(G)-\theta_1)+(1-\nu)(u'(G)-\theta_2)] = (1+\lambda)c+\lambda(1-\nu)u'(G). \quad (10)$$

The limited liability rent, $u(G)-y_2$, which must be given to type 2 has an expected marginal cost of $\lambda(1-\nu)u'(G)$ which justifies a downward distortion of the supply of public good.

Observe that in both cases the availability of the public good decreases inequality since $U_1 - U_2 = y_1 - y_2 - [u(g_2) - y_2]$.

2.3.2 "Negative correlation"

Now assume that $y_2 > y_1$: type 2 is the "bad" type for the cost but the good type for the income. This describes, for instance, a situation where smart poor consumers have a lower access cost. In the case where only the poor is financially constrained, the problem now consists in maximizing welfare, given by (2), subject to (F_1) and (P_2) which, depending on the relevant regime, yields:

Regime i): $G = g_1 > g_2$ with

$$(1+\lambda)\nu(u'(G)-\theta_1) = (1+\lambda)c+\lambda\nu u'(G); \quad (11)$$

$$u'(g_2)-\theta_2 = 0. \quad (12)$$

Regime ii): $G = g_1 = g_2$ with

$$(1+\lambda)[\nu(u'(G)-\theta_1)+(1-\nu)(u'(G)-\theta_2)] = (1+\lambda)c+\lambda\nu u'(G). \quad (13)$$

Now type 1 must be given a limited liability rent, $u(G)-y_2$, and since he is the one determining the level of public good, there is a downward distortion of production in both regimes. Once again, inequality decreases.

Depending on the "severity" of the financial constraint, two different types of situations can be distinguished. First, when the constraint is not too severe (the income of the constrained individual is not too low), the financially constrained agent can afford to pay for his expenses ($y_i > \theta_i g_i + k$), but the entire surplus is not captured. Second, for a more severely constrained individual, access cost, or even marginal connection cost may have to be subsidized ($y_i < \theta_i g_i + k$). For brevity, we will not explicitly distinguish the two cases in the subsequent discussion; we shall always say that access is subsidized.[12]

3 Asymmetric information

Consider now the case where there is incomplete information: $(\theta_i; y_i)$ is private information of type i. However, in this section we assume that the financial constraints, (F_i), are always satisfied. The optimal allocation is determined by maximizing expected social welfare under participation and incentive constraints:[13]

Problem 2

$$\max_{G;g_1;g_2;t_1;t_2} \quad \nu[u(g_1) - \theta_1 g_1 - k + y_1 - t_1] + (1-\nu)[u(g_2) - \theta_2 g_2 - k + y_2 - t_2]$$
$$- (1+\lambda)[cG - \nu t_1 - (1-\nu)t_2]$$

s.t.

$$u(g_1) - \theta_1 g_1 - k - t_1 \geq 0; \qquad (P_1)$$
$$u(g_2) - \theta_2 g_2 - k - t_2 \geq 0; \qquad (P_2)$$

$$u(g_1) - \theta_1 g_1 - k - t_1 \geq u(g_2) - \theta_1 g_2 - k - t_2; \qquad (IC_1)$$
$$u(g_2) - \theta_2 g_2 - k - t_2 \geq u(g_1) - \theta_2 g_1 - k - t_1; \qquad (IC_2)$$

$$g_i \leq G \quad i = 1; 2:$$

[12] In the negative correlation case with binding financial constraint, a third regime with $G = g_2 > g_1$ cannot be ruled out when the types' access costs are sufficiently close (i.e., when $\Delta\theta < \lambda\theta_1$). To avoid further proliferation of cases, we ignore this regime. Its potential occurrence does not affect our results.

[13] From the Revelation Principle we know that there is no loss of generality in restricting the analysis to pairs of contracts, $(t_1; g_1; t_2; g_2)$, based on the observable variables.

Because of our quasi-linearity assumption, participation and incentive constraints are independent of the income levels. The problem is therefore equivalent to a classical adverse selection problem for a single parameter of adverse selection θ and utility functions given by the net valuations, $u(g_i) - \theta_i g_i$, with the resource constraint $g_i \leq G$, $i = 1, 2$. Consequently, the participation constraint of the bad (high cost) type (θ_2) and the incentive constraint of the good type will be binding.

Using (P_2) and (IC_1), the payments of the two types can then be expressed as follows:

$$\begin{aligned} t_1 &= u(g_1) - \theta_1 g_1 - k - \Delta\theta g_2. \\ t_2 &= u(g_2) - \theta_2 g_2 - k, \end{aligned}$$

An information rent, $\Delta\theta g_2$, must now be conceded to type 1, the good type. The solution to Problem 2 is presented in the following proposition.

Proposition 2 Under incomplete information, when the financial constraints are not relevant, the optimal (utilitarian) allocation is characterized by:

i) If $\frac{c}{\nu} < \Delta\theta \left(1 + \frac{\lambda}{1+\lambda} \frac{\nu}{1-\nu} \right)$, then $g_1 = G$ and

$$(1+\lambda)\nu(u'(G) - \theta_1) = (1+\lambda)c, \tag{14}$$

$$(1-\nu)(1+\lambda)(u'(g_2) - \theta_2) = \lambda\nu\Delta\theta. \tag{15}$$

ii) If $\frac{c}{\nu} > \Delta\theta \left(1 + \frac{\lambda}{1+\lambda} \frac{\nu}{1-\nu} \right)$, then $g_1 = g_2 = G$ and

$$(1+\lambda)[\nu(u'(G) - \theta_1) + (1-\nu)(u'(G) - \theta_2)] = (1+\lambda)c + \lambda\nu\Delta\theta. \tag{16}$$

The right-hand side of (16) now represents the generalized marginal cost which, in addition to $(1+\lambda)c$, includes the expected marginal social cost of the information rent conceded to type 1, $\lambda\nu\Delta\theta$. With this generalized definition of cost, expression (16) can be viewed like a standard Samuelson condition.

However, this interpretation breaks down for (14)–(15), where the public good becomes two dimensional. Equation (14) is the complete information Samuelson condition. Condition (15) says that the sum of social marginal valuations of type 2 consumers equals the expected marginal social cost of the information rent of type 1. However, costs and benefits pertaining to 1 and 2 can no longer be simply added up, as was the case in (16).

To understand the determination of this solution under regime i), let us return to Figure 1. The full information optimum $(A; B)$, is clearly not incentive compatible; the low cost type prefers A to B. A feasible solution is then to offer $(A; C)$, leaving the bundle of type 2 undistorted. However, this implies a (socially) costly information rent for type 1 which can be mitigated by reducing g_2. The optimal solution trades-off rents against distortions in the public good consumption of 2 yielding a solution $A^0; B^0$. Observe that $G = g_1$ is unaffected: at B^0, the marginal valuation of 1 is c/ν, like at the full information optimum. This is the standard no distortion at the top result.

The determination of the solution for the no-exclusion regime ii) is illustrated in Figure 2. We obtain a solution like $(A^0_n; B^0_n)$ with $g_1 = g_2 = G < G^*$ and with the sum of marginal valuations larger than in the full information case.

The distortions in quantities imply that marginal prices may require upward incentive corrections, i.e. marginal taxes on usage. In regime i) they are given by

$$p_1 = \frac{c}{\nu} \qquad p_2 = \frac{\lambda}{1+\lambda}\frac{\nu}{1-\nu}\cdot\theta; \tag{17}$$

with a correction solely for type 2; recall the no distortion at the top property which applies for the low cost type. In case ii), on the other hand, we have

$$p_1 = c + \left(1 - \frac{\nu}{1+\lambda}\right)\cdot\theta \qquad p_2 = c - \frac{\nu}{1+\lambda}\cdot\theta: \tag{18}$$

and both prices are larger than their full information counterparts; see (6). As for the lump sum taxes, it can easily be seen (e.g., from Figures 1 and 2)[14],

[14] Or from a straightforward algebraic argument

that they are in each case smaller than their full information counterparts. This correction can be "interpreted" as a subsidy for access.[15]

With the concept of generalized marginal cost introduced above, one can continue to interpret marginal prices in regime ii) as Lindahl prices. This is because we have the following property:

$$(1+\lambda)[\pi p_1 + (1-\pi)p_2] = (1+\lambda)c + \lambda\pi\Delta\theta.$$

However, this interpretation does not go through in case i).

Asymmetric information thus has the following consequences. First, it expands the region where only type 1 consumes the entire quantity of public good. In that case, the no distortion at the top result implies that the quantity of public good is the same as under complete information. However, to decrease the information rent of type 1, the quantity consumed by type 2 is decreased. Second, in the regime where both types consume all the available quantity, the provided level of public good G is decreased to mitigate the cost of the information rent. To sum up, because of asymmetric information, usage is taxed except for type 1 in case i), while access is subsidized in all cases.

To conclude, let us turn to the issue of inequality. Under complete information the availability of the public good had no impact on inequality; both types remained at their initial utility levels $U_i = y_i$, $i = 1, 2$. Under asymmetric information, however, the low-cost individuals receive a rent and this fosters inequality when they also have the higher income (case referred to as positive correlation). In other words, while the availability of the public good results in a Pareto improvement, it benefits the rich more than the poor. However, when the low-cost type individuals have the lower income, then the availability of the public good decreases inequality.

[15] As usual in nonlinear pricing settings with two types, the optimum cannot be decentralized by a simple menu of two-part tariffs consisting of these marginal prices and lump sum fixed parts.

4 Financial constraints and adverse selection

We are now in a position to address the more difficult case when financial constraints are binding under asymmetric information. The formal problem is affected in two ways. First, we explicitly take into account the financial constraints, (F_i), which require that the consumers be able to have a nonnegative consumption of the private good.

Second, we have to consider the fact that some deviations from truth-telling might not be financially possible. For example, a poor consumer might not be able to claim that he is rich because he cannot afford the bundle designed for the rich. Consequently, there is the possibility that some incentive constraints can be neglected because they are associated with impossible messages.[16]

The optimal allocation thus has to satisfy the following constraints:

Problem 3: constraints

$$u(g_1) - \theta_1 g_1 - k - t_1 \geq 0; \qquad (P_1)$$

$$u(g_2) - \theta_2 g_2 - k - t_2 \geq 0; \qquad (P_2)$$

$$-\theta_1 g_1 - k + y_1 - t_1 \geq 0; \qquad (F_1)$$

$$-\theta_2 g_2 - k + y_2 - t_2 \geq 0; \qquad (F_2)$$

$$u(g_1) - \theta_1 g_1 - k - t_1 \geq u(g_2) - \theta_1 g_2 - k - t_2 \qquad (IC_1)$$
$$\text{if } y_1 - \theta_1 g_2 - k - t_2 \geq 0$$

$$u(g_2) - \theta_2 g_2 - k - t_2 \geq u(g_1) - \theta_2 g_1 - k - t_1 \qquad (IC_2)$$
$$\text{if } y_2 - \theta_2 g_1 - k - t_1 \geq 0.$$

[16] The idea here is that the agent will not send out of equilibrium messages which would result in bankruptcy. Observe that the space of messages of type θ_i, $M(\theta_i)$, is now a function of θ_i. This raises an additional potential problem, namely that the revelation principle might not be valid. However, in the case with only two types, the condition for the validity of the revelation principle (Green and Laffont (1983)) always holds. Consequently, we will not have to worry about this question here.

The objective function and the set of decision variables are the same as in Problems 1 and 2. The solution will depend on the type of correlation which exists between the private costs of access and the levels of income. We consider successively the two cases; for the sake of brevity we shall henceforth concentrate on regime i).

4.1 Case 1: "Positive" correlation

Now, $y_1 > y_2$ and further we assume that y_1 is sufficiently large so that (F_1) can be neglected. We can then expect that the incentive constraint of the good (low cost, high income) type will matter. His deviation to θ_2 is financially viable because (F_2) implies

$$y_1 - \theta_1 g_2 - k - t_2 > y_2 - \theta_2 g_2 - k - t_2 \geq 0.$$

The constraints which will matter are then the incentive constraint of the good type, (IC_1), and either the participation constraint, (P_2), or the financial constraint, (F_2), of the bad type. These last two constraints can be rewritten as:

$$u(g_2) - \theta_2 g_2 - k - t_2 \geq \max\{0; u(g_2) - y_2\}. \qquad (19)$$

Consequently, the following two basic cases can arise:[17]

Case 1.1: $u(g_2) < y_2$

When the solution satisfies $u(g_2) < y_2$ the financial constraints are not binding. For type 2, the relevant constraint is the participation constraint, (P_2) and we return to Problem 2 considered in the previous section. The solution continues to be given by Proposition 2.

[17] Observe that the concavity of the program implies that the solution to Problem 3 must be continuous. Consequently, a regime with three constraints, namely (IC_1), (P_2) and (F_2) binding occurs between the two cases we have listed.

Case 1.2: $u(g_2) > y_2$

Now the solution is affected by the financial constraints. For type 2, the relevant constraint is (F_2) (rather than the participation constraint (P_2)) which implies

$$t_2 = y_2 - \theta_2 g_2 - k.$$

From (IC_1) we have

$$t_1 = u(g_1) - \theta_1 g_1 - k + y_2 - u(g_2) - \Delta\theta g_2.$$

In regime i) the solution is as follows.

$$g_1 = G \qquad (1+\lambda)\alpha(u'(G) - \theta_1) = (1+\lambda)c \tag{20}$$

$$(1+\lambda)(1-\alpha)(u'(g_2) - \theta_2) = \lambda u'(g_2) + \lambda\alpha\Delta\theta. \tag{21}$$

As in Section 2.3, the financial constraint implies a limited liability rent, $u(g_2) - y_2$, for type 2. To maintain incentive compatibility this rent must be added to the rent $\Delta\theta g_2$ which was already conceded to type 1 in the previous case. All of these rents depend only on g_2. Consequently, as long as there is exclusion, only the allocation of g_2 is distorted downward to decrease the rents. The consumption of type 1 and the provided level of public good are the same as in case 1.2. The distortion of g_2 is greater than in case 1.2, because of the limited liability rent, $u(g_2) - y_2$, which must be given up to everybody.

The determination of the solution is illustrated by Figure 3. It differs from Figure 1 in that the financial constraint of type 2, $t_2 \geq y_2 - \theta_2 - k$ is explicitly accounted for. When y_2 is sufficiently large, the asymmetric information solution $A'; B'$ prevails and we are in case 1.1. However, for smaller levels of y_2, A' would violate the constraint. Instead, type 2 could be offered point C with the same level of g_2 but a lower t_2. This yields a costly (limited liability) rent for type 2 which also has to be conceded to type 1. A reduction in g_2 mitigates these rents and we obtain a solution

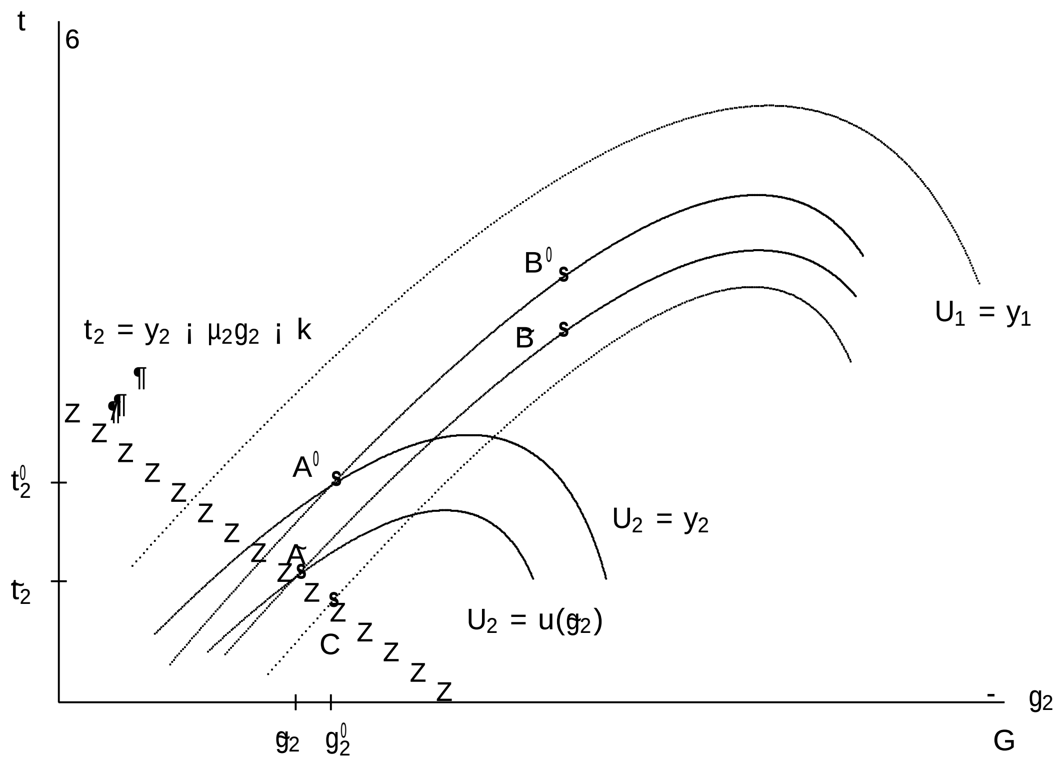

Figure 3: Case of positive correlation, regime i): impact of the …nancial constraint of type 2 (poor, high-cost) individuals. $(A^0; B^0)$ is the (standard) optimum under asymmetric information, which prevails when (F_2) is not binding. When y_2 is su¢ciently small, the …nancial constraint becomes binding and the solution is $(\bar{A}; \bar{B})$.

like $(\bar{A};\bar{B})$, where the solution for type 1 is obtained from the incentive constraint and at the point where the slope of the indifference curve is c=º.

Let us now return to the impact that the availability of the good has on inequality. In the previous section we have shown that in the case of positive correlation inequality increases when the good becomes available under incomplete information. Here the increased inequality due to asymmetric information is unaffected by the financial constraint of the poor.

The main results for the case of positive correlation are summarized in the following proposition.

Proposition 3 Assume that the low-cost type has the higher income. Under asymmetric information, the financial constraint of the poor has the following implications:

(i) When it becomes relevant, it implies further downward distortions which are due to the interaction between financial and incentive constraints. Asymmetric information increases the cost of subsidizing the poor for accessing or consuming the public good.

(ii) While this downward distortion implies a marginal tax on usage, access will be "subsidized" to meet the financial constraint of the poor.

(iii) The increased inequality due to asymmetric information is unaffected by the financial constraint of the poor.

4.2 Case 2: "Negative" correlation

The poor consumers are now the ones with a low cost of access, i.e, they are the good type. The financial constraint of the poor (type 1 now) implies that a limited liability rent $u(g_1) - y_1$ must be given up to type 1.

Further, since type 1 is the good type, an information rent $\Delta\theta g_2$ must also be given up to type 1 because of the incentive constraint (IC_1). This is true as long as this incentive constraint is relevant in the sense that the considered deviation is financially viable for type 1. We shall make this assumption in a first step and reexamine this issue below. Then, the constraints relative to type 1 can be summarized as:

$$u(g_1) - \theta_1 g_1 - k - t_1 \geq \max\{u(g_1) - y_1; \Delta\theta g_2\}. \qquad (22)$$

Like in the case of positive correlation we assume that the rich (now type 2) are sufficiently rich so that we can neglect their financial constraint. Concerning type 2 we will then have to worry about both the participation constraint and the incentive constraint. Three main subcases arise:[18]

Case 2.1: $\Delta\theta g_2 > u(g_1) - y_1$

When the income of type 1 is high enough, the financial constraint, (F_1), does not matter; it is automatically satisfied when the incentive constraint of the good type is satisfied. Consequently, we obtain the same solution as in Section 3. Graphically, this case occurs when the asymmetric information bundle B' is below the financial constraint of type 1; see Figure 4.

Case 2.2: $u(g_1) - y_1 > \Delta\theta g_2$ with (F_1) and (P_2) binding

As y_1 becomes smaller, the incentive constraint of type 1 becomes slack. Instead, the financial constraint of type 1 becomes relevant along with the participation constraint of type 2 which continues to be binding as in case 2.1. For regime i), we obtain the same solution as under complete information when financial constraints are accounted for; see expression (11) and (12) in Section 2.3.

This solution is represented by $(\tilde{A}; \tilde{B})$ on Figure 4. The income level of 1, $\tilde{y}_1$, is now sufficiently low so that B' violates (F_1).[19] Public good supply and the consumption of type 1, $G = g_1$ is now decreased to reduce the limited liability rent of 1. For type 2 on the other hand, we return to

[18] As above, the solution must be continuous. Continuity will be ensured between Cases 2.1 and 2.2 by a regime where (IC_1), (P_2) and (F_1) are binding, and between Cases 2.2 and 2.3 by a regime where (F_1), (P_2) and (IC_2) are binding;see Figure 5

[19] But it is still sufficiently high for the financial constraint not to intersect type 2's reservation utility indifference curve.

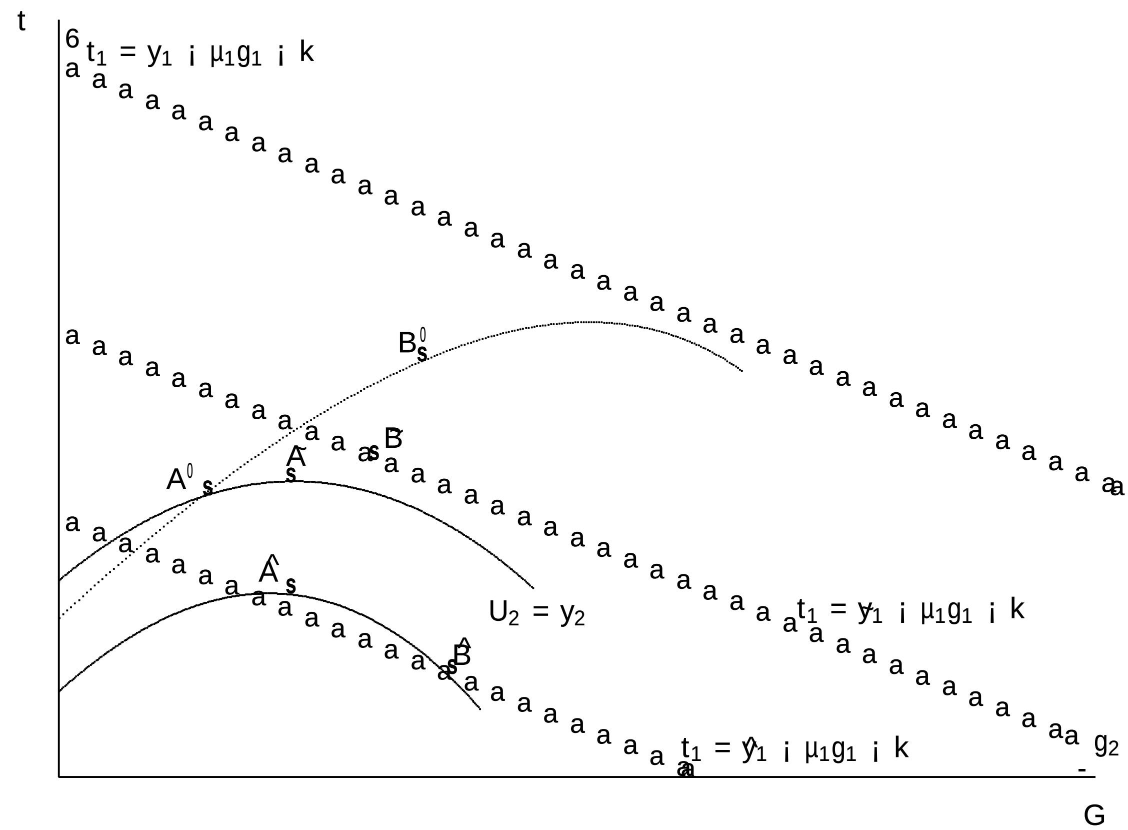

Figure 4: Case of negative correlation, regime i): impact of the financial constraint of type 1 (poor, low-cost) individuals. $(A^0; B^0)$ is the (standard) optimum under asymmetric information, which prevails when (F_1) is not binding. For income level $\tilde{y}_1$ the solution is $(\tilde{A}; \tilde{B})$, with (F_1) and (P_2) binding. For even lower income $\hat{y}_1$, we have countervailing incentives with (IC_2) and (F_1); a possible solution is represented by $(\hat{A}; \hat{B})$.

the full information solution (maximum of the indifference curve). This is because g_2 has no impact on the limited liability rent of type 1 so that there is no longer any reason to distort it.

Note that in both cases 2.1 and 2.2, inequality decreases, either for informational or for financial reasons, because a rent is now conceded to the poor ($\Delta\theta G$ or $u(G) - y_1$).

Case 2.3 $u(g_1) - y_1 > \Delta\theta g_2$ with (F_1) and (IC_2) binding

As y_1 is even smaller, the incentive constraint of type 2 becomes binding; countervailing incentives appear. Because the financial constraint is binding, an increase in g_1 must be accompanied by a decrease in t_1, to subsidize the individual for the higher connection cost. But then the consumption bundle of type 1 becomes increasingly attractive to type 2. For $u(g_1)$ large enough, the rich-high access cost types then want to claim they are poor-low cost types. Consequently, the binding constraints are then the financial constraint of type 1 and the incentive constraint of type 2. A rent must be given up to both types; payments and rents, denoted by R_1 and R_2 can then be expressed as follows:

$$t_1 = y_1 - \theta_1 G - k$$

$$t_2 = u(g_2) - \theta_2 g_2 - k - u(G) + \Delta\theta G + y_1$$

and

$$R_1 = U_1 - y_1 = u(G) - y_1; \tag{23}$$

$$R_2 = U_2 - y_2 = u(G) - y_1 - \Delta\theta G \tag{24}$$

Combining (23) and (24) we obtain $U_2 - U_1 = y_2 - y_1 - \Delta\theta G$. Consequently, inequality continues to decrease when the public good becomes available.

Expressions (23)–(24) show that the rent of individual 1 is increasing in G. This is because of the subsidization of usage mentioned above. More interestingly, the level of G has an ambiguous impact on the rent conceded to individual 2; we have

$$\frac{\partial R_2}{\partial G} = u'(G) - \Delta\theta. \tag{25}$$

The first term on the RHS of (25) is positive: with the binding incentive constraint, the limited liability rent conceded to 1 must also be conceded to 2. The second term, on the other hand, is negative. Recall that type 2 is the high-cost type so that an increase in G makes the consumption bundle of 1 less attractive to him.

We obtain the following characterization for case i), with $g_1 = G$

$$(1+\lambda)\gamma(u'(G) - \theta_1) = (1+\lambda)c + \lambda\gamma u'(G) + \lambda(1-\gamma)(u'(G) - \Delta\theta) \tag{26}$$

$$u'(g_2) - \theta_2 = 0 \tag{27}$$

Since both rents are independent of g_2, only $G = g_1$ is distorted. The sign of the distortion is now ambiguous. When $u'(G) - \Delta\theta > 0$, G is necessarily lower than in the standard asymmetric information case; see Proposition 2. On the other hand, when $u'(G) - \Delta\theta < 0$ a positive distortion cannot be ruled out and arises if the impact on the rents of 2, given by (25), is sufficiently strong, in which case, usage would be subsidized as well as access.[20] These results are also reflected in the marginal price given by

$$p_1 = \frac{c}{\gamma} + \frac{\lambda}{1+\lambda}\left(\lambda u'(G) + \frac{\lambda(1-\gamma)}{\gamma}(u'(G) - \Delta\theta)\right);$$

and $p_2 = 0$.

The solution in Case 2.3 is illustrated by $(\hat{A}; \hat{B})$ on Figure 4. Now, the income level of type 1, $\hat{y}_1$, is so low that a solution like $\tilde{A}; \tilde{B}$ is below the indifference curve $U_2 = y_2$; consequently it violates IC_2. We then obtain the case of countervailing incentives with the added complication, compared to

[20] More precisely, the sign of the distortion depends on the impact on the expected rent, which (up to a constant) is given by

$$\gamma u(G) + (1-\gamma)[u(G) - \Delta\theta G] = u(G) - (1-\gamma)\Delta\theta G.$$

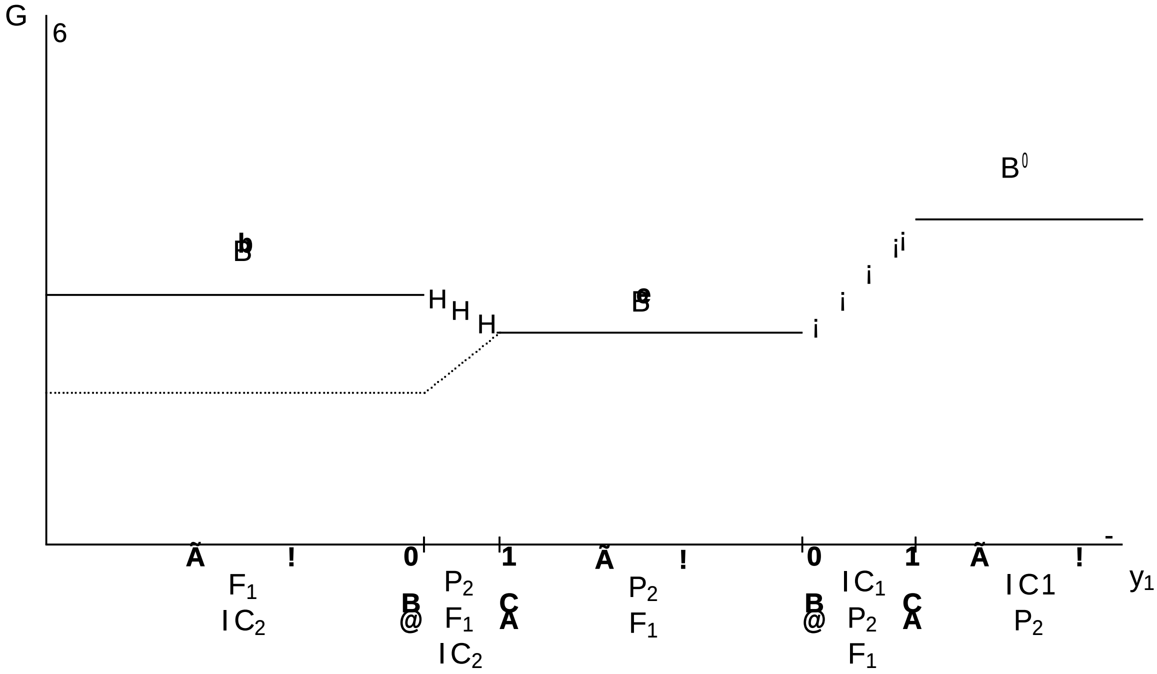

Figure 5: Negative correlation ($y_1 < y_2$), regime i): binding constraints and level of $G = g_1$ as a function of y_1. The solutions B^0, $\tilde{B}$ and $\hat{B}$ correspond to the case represented on Figure 4. The doted line represents the alternative possibility where G would be lower for $\hat{B}$ than for $\tilde{B}$.

the cases encountered in the literature, that type 1 also has a rent (namely the limited liability rent). As explained above, $\hat{B}$ can then be to the right or to the left of B^0 depending on the net impact of G on the rent of type 2.

The three basic cases described above are connected by regimes where three constraints are binding to ensure continuity of the solution. For example, when regime i) prevails in all cases we obtain the alternation of cases and the relationship between $G = g_1$ and y_1 which is depicted on Figure 5. Observe that the three main cases correspond to those represented on Figure 4. Recall that the ranking of $\hat{B}$ and $\tilde{B}$ is not unambiguous; the case where G is lower at $\hat{B}$ than at $\tilde{B}$ can also arise.

So far, we have ignored the financial constraint of the mimicking individual both in (IC_1) and in (IC_2). Let us now examine how, if at all, these constraints do affect the results. For this, we have to consider the financial viability of the deviations envisioned in the incentive constraints.

In Case 2.2, they clearly do not matter because neither of the incentive constraints binds. In Case 2.1, we have to check if the claim of the poor type 1 to be of type 2 is financially viable. Using (P2), one easily shows that this is the case if

$$\Delta\theta g_2 + y_1 \geq u(g_2); \tag{28}$$

a condition which necessarily holds in Case 2.1 which, by definition satisfies $\Delta\theta g_2 + y_1 \geq u(g_1)$.

Finally, in Case 2.3, one can show from (F_1) that the deviation of the rich is viable only if

$$y_2 \geq y_1 + \Delta\theta g_1: \tag{29}$$

In words, the income differential between type 2 and type 1 must be sufficiently large to compensate for the differential in variable costs; recall that type 2 is the high cost type.

If (29) holds nothing is changed. If (29) is violated, (IC_2) is suppressed. Then, since we were in the Case 2.3, the relevant constraints become the financial constraint of type 1 and the participation constraint of type 2. Consequently, we return to Case 2.2 and the countervailing incentives disappear. Observe that the conditions defining the various cases under negative correlation are independent of y_2. A situation where (29) is violated can thus definitively occur.

The main results of this section are summarized in the following proposition.

Proposition 4 Assume that the low-cost type has the lower income. Under asymmetric information, the financial constraint of the poor has the following implication:

(i) As long as countervailing incentives do not occur, subsidizing the poor for using the public good is not more costly under incomplete than under complete information.

(ii) When countervailing incentives occur, it becomes more costly because the limited liability rent of the poor also has to be conceded to the rich; in that case, the distortions are of ambiguous signs.

(iii) In all cases, inequality decreases as the public good becomes available.

5 Extensions

5.1 Monopoly provision

Let us now consider the case where the public good is provided by a profit-maximizing monopoly. This firm has the same information and faces the same constraints as the social welfare maximizer considered above.

Formally the objective function of the monopoly is given by

$$\nu t_1 + (1 - \nu) t_2 - cG;$$

while the constraints are the same as in Problem 3: the participation constraints (P_i), the financial constraints (F_i), the incentive constraints (IC_i) and the feasibility constraints $g_i \leq G$ for $i = 1; 2$.

The analysis of this case is very similar to that of the social optimum. However, there are two main differences. First, the monopoly is even more eager to extract rents from consumers and thus leads to greater distortions. As a matter of fact, the monopoly case can be viewed as the limit of the social optimum as λ grows (so that $\lambda/(1 + \lambda)$ goes to one).

To understand the second difference, recall that limited liability rents were arising when the social welfare maximizer was subsidizing access of poor consumers. In particular, countervailing incentives were occurring when poor consumers with low access costs were favored. The social gain was the (relatively) large rent they obtained from a large consumption of the public good induced by the low access costs. The monopoly, on the other hand, cannot capture this rent. Consequently, it is not interested in favoring the low access cost consumers and the high access cost consumers never want to mimic the low access cost ones. Consequently, the monopoly will exclude the poor much "earlier" than a social planner would do. Summarizing, we have:

Proposition 5 Assume that the public good is provided by a profit maximizing monopoly under asymmetric information.

(i) When the financial constraints are not binding, the distortions that occur under social welfare maximization are exacerbated.

(ii) The public good consumption of any type is never higher than the minimum level for which the financial constraint is binding. Consequently, there is no limited liability rent and the case of countervailing incentives does not occur.

5.2 Income effects

So far we have concentrated on the case of quasi-linear preferences. Consequently, income effects have entered the analysis only in a very stylized way, namely through the financial constraints. We shall now briefly reconsider the SWM's problem under a more general preference structure to illustrate the added difficulties of this general formulation.

Let us now suppose that an agent's utility level is given by

$$V_i = u(g_i; x_i) = u(g_i; y_i - \theta_i g_i - k - t_i). \qquad (30)$$

For simplicity, we assume that $u(g; 0)$ is sufficiently small so that financial constraints are never binding. Furthermore, we shall concentrate on the case where there is no bunching.

The slope of the indifference curves in the $(g; t)$ plane for type $i = 1; 2$ is now given by

$$S_i(g_i; t_i) \equiv \left(\frac{dt_i}{dg_i}\right)_{\bar{V}_i} = \frac{u_g(g_i; x_i) - \theta_i u_x(g_i; x_i)}{u_x(g_i; x_i)}; \qquad (31)$$

where u_g and u_x denote partial derivatives. Assuming normality of both goods it then follows immediately that when $y_1 \geq y_2$ we have

$$S_1(g; t) > S_2(g; t) \quad \forall (g; t). \qquad (32)$$

In words, when the low cost individuals have the higher income, their indifference curves in the (g, t) space are necessarily steeper, at any given point, than those of the high cost individuals. Observe that this inequality remains true when income levels are equal. But it may be reversed in the case where $y_2 > y_1$ It will become clear below that (32) is crucial for determining the sign of the distortions.[21]. In what follows, we restrict our attention to the case where 32 holds.

The maximization problem of a utilitarian SWM can now be stated as follows.

$$\max_{G,g_1,g_2,t_1,t_2} \quad \pi[u(g_1, y_1 - \theta_1 g_1 - k - t_1] + (1-\pi)[u(g_2, y_2 - \theta_2 g_2 - k - t_2] - (1+\lambda)[cG - \pi t_1 - (1-\pi)t_2]$$

s.t.

$$u(g_1, y_1 - \theta_1 g_1 - k - t_1) \geq u(0, y_1); \qquad (P_1)$$
$$u(g_2, y_2 - \theta_2 g_2 - k - t_2) \geq u(0, y_2); \qquad (P_2)$$

$$u(g_1, y_1 - \theta_1 g_1 - k - t_1) \geq u(g_2, y_1 - \theta_1 g_2 - k - t_2); \qquad (IC_1)$$
$$u(g_2, y_2 - \theta_2 g_2 - k - t_2) \geq u(g_1, y_2 - \theta_2 g_1 - k - t_1); \qquad (IC_2)$$

$$g_i \leq G \quad i = 1, 2.$$

The complete information solution can be derived from this problem by neglecting the incentive constraints. The results closely resemble those obtained in the quasi-linear case. In particular, one obtains modified Samuelson conditions which are straightforward extensions of (3) and (4)

Under asymmetric information, depending on the parameters and on the degree of concavity of u a number of cases can arise. When at least one of the participation constraints binds, the solutions closely resemble those presented in the earlier sections. However, we can now also have regimes of a different nature which arise when a single constraint, namely one of the incentive constraints, is binding. It is even possible that none of the constraints is binding in which case the complete information solution is

[21] This "single crossing property" thus plays a role which is similar to the one it plays in a standard two group general income tax problem; see Stiglitz (1982)

revealing and can be achieved under asymmetric information.[22]

Let us now briefly consider the results for the cases where a single incentive constraint binds.

5.2.1 Case 1: (IC_1) is binding

Using the first order conditions of the SWM's problem, and denoting an individual's marginal utility of income by:

$$\alpha_i \equiv \frac{\partial u(g_i;x_i)}{\partial x}$$

the optimal allocation, in the no bunching case with $G = g_1 > g_2$ is characterized by:

$$\nu\left[\frac{u_g(g_1;x_1) - \theta_1\alpha_1}{\alpha_1}\right] = c \tag{33}$$

$$\frac{u_g(g_2;x_2) - \theta_2\alpha_2}{\alpha_2} = \frac{\eta_1\tilde{\alpha}_1}{(1-\nu)\alpha_2}\left[\frac{u_g(g_2;\tilde{x}_1) - \theta_1\tilde{\alpha}_1}{\tilde{\alpha}_1}\right] \tag{34}$$

where η_1 is the Lagrange multiplier associated with (IC_1), while a tilde is used for variables pertaining to the mimicking individuals.[23]

Condition (33) says that the public good consumption of the low cost type (and thus the provided level) is determined by the full information trade-off. This is the well-known "no distortion at the top property".[24]

To interpret (34), observe that it can be written as

$$S_2 = A\tilde{S}_1; \tag{35}$$

where

$$A = \frac{\eta_1\tilde{\alpha}_1}{(1-\nu)\alpha_2};$$

while S is defined by (31). Using the first order conditions one can show that

$$0 < A < 1. \tag{36}$$

[22] However, the case where both incentive constraints are binding cannot arise (at least as long as $y_1 \geq y_2$ because this would imply multiple crossing which is ruled out by (32)).

[23] For instance, $\tilde{x}_1 = y_1 - \theta_1 g_2 - k - t_2$, that is the consumption of the numeraire of a type 1 individual who claims to be of type 2.

[24] There is no distortion at the margin, but because of income effects, the actual level will in general not be equal to the full information solution.

Further observe that S_2 and $\tilde{S}_1$ are evaluated at the same bundle $(g_2; t_2)$. Using (36) and (32), it then follows that (35) implies $S_2 > 0$: the consumption level of type 2 is distorted downward.

5.2.2 Case 2: (IC_2) is binding

Using the first-order conditions and (31), the two relevant conditions characterizing the optimal allocation can be written as follows:

$$S_2 = 0 \tag{37}$$

$$S_1 - \frac{(1+\lambda)c}{\alpha_1 \gamma} = B\tilde{S}_2 \tag{38}$$

where

$$0 < B \equiv \frac{\eta_2 \tilde{\alpha}_2}{\gamma \alpha_1} < 1 \tag{39}$$

Condition (37) is the familiar "no distortion at the top", except that the identity of the "top" individual has changed compared to the previous case. Now it is type 2 towards which no incentive constraint is binding.

Turning to (38), using (32) and (39) one can show that this condition implies

$$S_1 - \frac{c}{\gamma} < 0;$$

so that there is an upward distortion in the public good consumption of the low cost type, and hence also in the supply G.

Summing up, even when none of the participation constraint binds, the results obtained in the quasi-linear case can easily be extended. There are two main differences, though. First, due to income effects the comparison of public good levels between the different cases may be ambiguous, and depend on whether the good is normal or inferior. Second the determination of the conditions under which either of these regimes arises is analytically impossible in general case. Recall that in the quasi-linear case, we were able to obtain a complete and explicit characterization of the different regimes.

6 Conclusion

We have obtained the characterization of the optimal allocation of public goods with access costs when these access costs and incomes are private information and when …nancial constraints may prevent some consumers to access these public goods. We have restricted the analysis to two types. The generalization of our results to several types raises two types of technical di¢culties encountered in incentive theory.

First, we would have to deal with multidimensional adverse selection problems, about access costs and incomes, and we know from the work of Amstrong (1996) and Rochet and Choné (1998) that it is di¢cult to identify the binding constraints. Furthermore, we might have to use stochastic mechanisms to elicit incomes as in Rochet (1984).

Second, it looks as if the necessary and su¢cient conditions for truthful implementability derived in Green and La¤ont (1983) when message spaces depend on private information will not hold in general. This dependence in itself makes the analysis of the relevant incentive constraints even more di¢cult than in the usual multidimensional analysis, and one might have to consider allocation rules not implementable in truthful equilibria.

Beyond these technical problems, the interesting questions lie in the study of imperfect competition in the supply of these goods and in the analysis of innovation. We hope to address these questions in the near future.

Appendix

Let us consider a strictly concave increasing social welfare function $V(.)$ with $V'(0) = +\infty$; so that financial constraints are never an issue. Expected social welfare is now

$$\pi V(u(g_1) - \theta_1 g_1 - k + y_1 - t_1) + (1 - \pi) V(u(g_2) - \theta_2 g_2 - k + y_2 - t_2) \quad \text{(A1)}$$

The social welfare maximizer must now satisfy the budget constraint

$$\pi t_1 + (1 - \pi) t_2 - cG \geq 0 \qquad (\lambda) \qquad \text{(A2)}$$

and, under incomplete information, the incentive constraint (as usual we only need to write the good type's incentive constraint)

$$u(g_1) - \theta_1 g_1 - t_1 \geq u(g_2) - \theta_1 g_2 - t_2 \qquad (\mu) \qquad \text{(A3)}$$

Maximizing (A1) under (A2)–(A3) we obtain the first order conditions:
Case i : $g_1 = G$, $g_2 < G$

$$\pi V'(1)(u'(G) - \theta_1) + \mu(u'(G) - \theta_1) = \lambda c \qquad \text{(A4)}$$

$$(1 - \pi) V'(2)(u'(g_2) - \theta_2) - \mu(u'(g_2) - \theta_1) = 0 \qquad \text{(A5)}$$

$$\pi V'(1) = \lambda \pi - \mu \qquad \text{(A6)}$$

$$(1 - \pi) V'(2) = \lambda(1 - \pi) + \mu \qquad \text{(A7)}$$

from which we derive

$$\pi[u'(G) - \theta_1] = c \qquad \text{(A8)}$$

$$(1 - \pi)(u'(g_2) - \theta_2) = \frac{\mu}{\lambda} \Delta\theta \qquad \text{(A9)}$$

Regime i holds if $c/\pi < \Delta\theta[1 + \mu/\lambda(1 - \pi)]$. Equation (A9) corresponds to (15) with μ/π instead of $\pi\lambda/(1 + \lambda)$. It can also be rewritten

$$u'(g_2) = \theta_2 + \pi \frac{V'(2) - V'(1)}{\pi V'(1) + (1 - \pi) V'(2)} \Delta\theta \qquad \text{(A10)}$$

which remains, however, an implicit expression.

Case ii : $g_1 = g_2 = G$

$$\nu V'(1)(u'(G) - \theta_1) + (1 - \nu)V'(2)(u'(G) - \theta_2) = \lambda c \tag{A11}$$

and also (A6) (A7).

(A11) can be rewritten

$$u'(G) = \nu\theta_1 + (1 - \nu)\theta_2 + c + \frac{\mu}{\lambda}\Delta\theta \tag{A12}$$

which is again similar to (16) with μ/λ instead of $\nu\lambda/(1+\lambda)$. It can also be rewritten

$$u'(G) = \nu\theta_1 + (1 - \nu)\theta_2 + c + \nu(1 - \nu)\frac{V'(2) - V'(1)}{\nu V'(1) + (1 - \nu)V'(2)}\Delta\theta$$

still an implicit expression.

Income effects perturb the generalized Samuelson conditions as financial constraints do in our approach; see Section 5.2 for more general income effects. The benefit of our approach is to yield explicit optimal solutions.

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