# Two-Sided Markets with Negative Externalities*

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## Abstract

In this paper a two-sided-market is analysed in which two platforms compete against each other. One side, the advertisers, exerts a negative externality on the other side, the users. It is shown that in a strategic pricing game there might be too much advertising compared with the social optimum because the negative effect on users is not internalised. An increase in the externality level might lead to higher profits of the platforms because it softens price competition. A model in which platforms have a fixed stock of regular users is compared with one in which they have to compete for users. It can be shown that profits might be higher if platforms have to compete for users because this additional competition effect lessens the price competition for advertisers.

# 1 Introduction

There are many companies which produce services for a group of agents who do not pay for it or pay only a low price. Instead these companies get revenues from advertisers who wish to gain access to potential consumers via the services of these companies. Examples are private radio or television stations which often interrupt their programme to broadcast advertisement. Search engines like Google or Yahoo! or Internet portals often have a multitude of advertisements on their web sites. In the radio or TV cases it is technically not possible to charge listeners/viewers for the broadcasting of programms. In the case of search engines it is not customary to charge users for the services.

This paper studies a model of platform competition in which users dislike advertisement and spent therefore less time to consume services of platforms. Advertisers instead wish to gain attention by users to tempt them to buy their products. It can be shown that in equilibrium the level of advertisement might be too high compared with the socially optimal one because platform pricing does not internalise the externality that users' utility decreases with more advertisement. Concerning platform profits a higher degree of the externality parameter can increase profits because price competition becomes less fierce. The profits of the platforms in two cases, if they must compete for users or if they have a regular user stock are also compared. It turns out that profits might be higher in case of competition for users. The reason is that competition for advertisers is softened because users dislike advertisements and thus prices for advertisers can be increased.

More specifically, we assume that two completely similar platforms compete for user time and advertisers.[1] Advertisers choose one platform exclusively and their profits are increasing in the time users spend on that platform. Users' utility and the time they spend on a platform are decreasing with advertisement. Therefore an

[1]It is not necessary to assume some form of Hotelling differentiation in this model. Profit functions are continuous even in case of homogeneous platforms because of the negative externality.

advertiser causes a negative externality on users' directly and also on all other advertisers on that platform indirectly. If the externality parameter is low all advertisers should advertise from a social point of view because the potential gains from trade of advertisers' products are higher than users' utility loss. With a high externality parameter some of the advertisers should be excluded.

The optimal partition of advertisers among platforms should be even. The intuition is that if one platform has many advertisers the externality on all of them is high. The other platform has only few advertisers and therefore only few advertisers enjoy a low externality. The overall externality can reduced as well as possible with an even partition. In a Nash equlibrium the number of advertisers is too high compared with efficiency for high values of the externality parameter. Platforms do only internalise the indirect externality that one advertiser exerts on other advertisers but not the direct utility loss of users. For low values of the externality parameter the equilbrium and the efficient outcome coincide because all advertisers should advertise.

Platforms' profits also depend on the level of the externality parameter. For low values of this parameter a platform can gain many new advertisers it it lowers it's price. Competition is harsh and profits are low. Thus an increase in the externality parameter can increase profits. If this parameter is already high the reverse is true because the number of advertisers decreases heavily with higher parameter levels.

We also analyse a model in which not only advertisers but also users have to decide exclusively which platform to use. Therefore platforms have to compete for both groups of participants. Despite this additional competition it is possible that profits are higher than without competition for users. The reason is that platforms compete for users by reducing their advertising levels. In doing so platforms set higher prices for advertising. Thus the competition effect on the advertiser market is reduced. Since both platforms increase their prices the number of advertisers at each platform stays the same. The competition for users make competition for advertisers less harsh. This shows that in two-sided markets a higher degree of competition on one side can reduce the competition on the other side.

There are only few papers which analyse two-sided markets with negative externalities. Many papers in the two-sided markets literature are concerned with participants exerting positive externalities on each other like in the market for credit cards. Examples of these papers are Rochet & Tirole (2002) or Wright (2001). In section 6 of their paper Rochet & Tirole (2002) analyse shortly a model in which platforms earns revenues from users and advertisers. Platforms are able to use a two-part tariff for both groups of participants. Rochet & Tirole (2002) show in general that both prices depend on the relations between own- and cross-price elasticities.

Armstrong (2002) analyses competition in two-sided markets in general and give many examples how to model two-sided markets with different features. In section 5.3 of his paper he analyses advertising in newspaper markets. In contrast to the model in my paper readers do not dislike advertising and producers can advertise in both newspapers in the model of Armstrong (2002). He shows that due to this fact if readers buy only a single newspaper in the market equilibrium there is too little advertising because platforms set too high prices. If readers read both newspapers the price of advertisement per reader depends on the number of exclusive readers of a newspaper. The advertiser has to pay the monopoly price for these readers while he gets access to the non exclusive readers for free.[2]

The papers which are closest to the one analysed here are Anderson & Coate (2000) and Gal-Or & Dukes (2002).

Anderson & Coate (2000) analyse a model of TV broadcasting. They are interested in the question if two channels will offer the same or different programs and how much advertisement they will broadcast. They find that dependent on parameter values there can be too few but also too many advertisement and also a too low or too

[2] In section 5.4 Armstrong analyses a model of yellow pages. The common feature with the model considered here is that an additional advertiser exerts a negative externality on all other advertisers. But there are two remarkable differences. First in yellow pages it is the basic service of platforms to match potential consumers with advertisers and not a by-product as in my model. Second a higher number of advertisement might increase user's utility because they have a greater variety of producers they can choose.

For an analysis of a monopoly information gatekeeper see Baye & Morgan (2001).

high variety of programms. In their model viewers suffer from advertising. But this does not result in the consequence that each viewer watches less TV. Instead viewers might switch to their less preferred programme if this has fewer advertisements. As a result an even partition of advertisers is efficient due to keep viewers at their preffered programme. Anderson & Coate (2000) also analyse the case in which viewers can be charged for watching the programms. They find that advertisement levels are usually lower in this case.
In the paper of Gal-Or & Dukes (2002) differentiated TV or radio stations also compete for viewers/listeners and advertisers. They analyse the question under which conditions a merger of two stations can be profitable. In their model consumers are averse to advertising but may profit from advertisements by the fact that they are better informed about prices.[3] If two firms merge this results in a higher level of advertising which can drive producers' prices and profits down. Therefore producers can pay less for advertising. This might render a merger unprofitable.

The remainder of the paper is organised as follows. The next section sets out the basic model. In section 3 the equlibrium of this model is presented and compared with the efficiency result. The basic model is extended in section 4 for the case that platforms compete for advertisers and users. Section 5 gives a short conclusion.

## 2 The Model

The goal is to develop a model in which platforms compete for users (consumers) and advertisers (producers). It is assumed that if platforms are Internet portals, radio stations, or television channels consumers have the hardware to get access to these platforms. Advertising causes a negative externality on users. In the following we describe the basic model. This model is extended in section 4.

[3] A problem in their model is that this gain for viewers/listeners is not included in the utility function. The reason is that this would complicate the model dramatically and would change some results.

*Platforms*

There are two similar platforms $i$, $i = 1, 2$. Consumers cannot be excluded from using the platforms. Therefore platforms cannot make profits directly with consumers' use. Instead platforms only make profits on advertisers. The profit function of platform $i$ is

$$\Pi_i = p_i(p_j)n_i.$$

$p_i$ is the price that platform $i$ is demanding from an advertiser for an advertisement and $n_i$ ist the number of advertisers on platform $i$. Each advertiser can only place one advertisement and has to decide exclusively on which platform she wants to advertise. $p_i$ and $n_i$ depend therefore on the price of platform $j$. It is assumed that platform pricing is linear. We also assume that the costs of platforms are zero.[4]

*Consumers*

There is a mass of consumers $M$. Each consumer decides about the time he wants to spend on each platform. The time on platform $i$ is given by

$$\alpha_i = X - t_A(n_i)^\gamma \qquad i = 1, 2$$

with $\gamma > 0$. It is assumed that a consumer enjoys a utility of one for each unit of time he spends on platform $i$. The utility function of a consumer is therefore $X - t_A(n_i)^\gamma + X - t_A(n_j)^\gamma$. $X$ is the time a consumer would spend on each platform without advertisement on it. Since advertisers cause a negative externality on consumers $\alpha_i$ is decreasing in $n_i$. $t_A$ is a measure of the severeness of the externality. A high $t_A$ means that consumers are very annoyed of advertising.[5] $\gamma$ represents the curvature of $\alpha_i$. If $\gamma = 1$ the $\alpha_i$ function is linear while $\gamma > 1$ ($\gamma < 1$) means a concave (convex) function. E.g. $\gamma > 1$ means that one or two advertisers do not decrease the marginal time spend by a user a lot but time is marginally heavily decreasing if

[4]This assumption is made for simplicity. Relaxing it would change the calculations but not the qualitative results of the model.

[5]Anderson & Coate (2000) in their model call $t_A$ nuisance cost of advertising.

many advertisements are placed on platform $i$. Since $\alpha_i$ represents time of usage it cannot be negative. This means that even if all advertisers which are of mass $N$ are on platform $i$, $\alpha_i$ stays positive e.g. $X - t_A N^\gamma > 0$. Consumers will therefore use both platforms for sure but spend may be different time on them.

The value for each advertisers' product is the same for each consumer. It is $K$ with probability $\beta$ and 0 with probability $1 - \beta$. This modelisation follows Anderson & Coate (2000). Although it is very specific it avoids the problem that advertisement can have a positive value for users. With this modelisation it is not possible in equilibrium that users get positive utility from products they become aware of through advertising.[6]

*Advertisers*

There is a mass of advertisers $N$. No advertiser has ex ante special preferences for one of the platforms. Advertisers decide exclusively on which platform they want to advertise. If platform $i$ is chosen by an advertiser her utility is

$$U_i = M\beta K\alpha_i - p_i.$$

If she decides not to advertise she gets a utility of zero. Each producer sells and advertise her product at a price of $K$ since a lower price does not increase the probability of a sale. For simplicity it is assumed that production costs for advertisements and products are zero. Again this assumption does not change the qualitative results. The value of an advertisement on platform $i$ does positively depend on the time users spend on that platform. The idea is that the more time a user spend on platform $i$ the higher is the possibility that he gets aware of that advertisement and buys the product in the end. The gross value of an advertisement on $i$ is thus $M\beta K\alpha_i$. The advertiser has to pay $p_i$ for an advertisement on $i$.

The structure of the game is the following:

In the first stage the two platforms decide simultaneously about their prices $p_1$ and

[6] See also Gal-Or & Dukes (2002).

$p_2$. In the second stage advertisers decide on which platform they want to advertise if any and consumers spend their time on each platform. Then profits and utilities are realised.

This completes the description of the model. In the following analysis we need one further assumption:

*Assumption 1:* $\beta K(1+\gamma)n_i > 1-\gamma$

The role of this assumption will become clear in the next section.

Before solving the model in the next section I want to describe two markets which fit this modelisation very well:

- Internet portals:

  Consider e.g. search engines like Google and Yahoo!. If no one of them is a technically better search engine there is no preference for users or advertisers for one of both. $X$ can be interpreted as the number of items a user is searching for. Users are disturbed by advertisements on a search engine and thus spend less time on it and switch to the other one.

- Radio stations:

  There are many radio stations which play mainly chart music. No listener or advertiser has ex ante a special preference for one station. But listeners want to listen to music and not to advertisement. Thus they switch to another station if one station is broadcasting advertisement.

# 3 Efficiency and Equlibrium

In this section the optimal number of advertisements on each platform is derived. This result is compared with the equlibrium outcome of the pricing game.

## 3.1 Efficiency

In the analysis of efficiency there are two effects to consider. Firstly a higher number of advertisements increases the possibility of trade of advertisers' products. Secondly a higher number of advertisements decreases users' utility and exerts a negative externality on other advertisers. Thus the welfare is given by

$$WF = M(X - t_A(n_1)^\gamma)\beta K n_1 + M(X - t_A(n_2)^\gamma)\beta K n_2 + M((X - t_A(n_1)^\gamma + (X - t_A(n_2)^\gamma). \tag{1}$$

Differentiating (1) with respect to $n_i$, $i = 1, 2$ yields the first order conditions

$$\frac{\partial WF}{\partial n_i} = M(X - t_A(n_i)^\gamma)\beta K - M t_A \gamma (n_i)^{\gamma-1})\beta K n_i - M t_A \gamma (n_i)^{\gamma-1}) = 0. \tag{2}$$

To be sure that (2) is globally concave the second order condition has to be checked,

$$\frac{\partial^2 WF}{\partial n_i^2} = -M t_A \gamma (n_i)^{\gamma-1}\beta K - M t_A \gamma(\gamma-1)(n_i)^{\gamma-2} - M t_A \gamma (n_i)^{\gamma-1}\beta K - M t_A \gamma (n_i)^{\gamma-1}\beta n_i < 0. \tag{3}$$

This is fulfilled because of assumption 1.

Solving for $n_i$ in (2) we get the following lemma.

**Lemma 1**

The optimal number $n_i, i = 1, 2$ is given by

$$X\beta K = t_A \beta k (1+\gamma)(n_i)^\gamma + t_A \gamma (n_i)^{\gamma-1}. \tag{4}$$

If

$$X\beta K > t_A \beta k (1+\gamma)(N/2)^\gamma + t_A \gamma (N/2)^{\gamma-1} \tag{5}$$

$n_i = \frac{N}{2}$ is optimal.

It is therefore optimal if advertisers partition themselves equally among platforms. The intuition behind this is simple. If we look only at the gains from trade the externality which one advertiser causes on another one is increasing convexly. So if one platform has many advertisers users spend few time on this platform and thus many advertisers gain little attention. To reduce this externality as well as possible it is optimal that each platform has the same number of advertisers. If $X\beta K$ is high which means that the probability and the welfare gains from trade are high all producers should advertise and $n_i = N/2$. If these gains are lower compared to the utility loss of users, $n_1 + n_2 < N$.[7]

## 3.2 Nash-Equilibrium

In this section we solve for the Nash-equlibrium of the pricing game.

Without the externality the model would be similar to a standard Bertrand game since platforms are not differentiated. At first glance it is not obvious how to solve the game. Yet, it turns out that the externality works in a similar way as product differentiation. The game is solvable in a similar way as the product differentiation model of Hotelling.[8]

To see this let us assume first that all $N$ producers advertise. If platform $i$ sets $p_i$ and platform $j$ sets $p_j$ the '$N$th' producer must in equilibrium be indifferent between advertising on $i$ or on $j$[9] otherwise one platform can increase it's price without losing consumers. This '$N$th' advertiser is therefore comparable with the marginal consumer in the product differentiation analysis. If she decides in favour of platform $i$ her externality is $t_A(n_i)^\gamma$ while it is $t_A(N-n_i)^\gamma$ at platform $j$. The marginal advertiser is therefore described by

$$M\beta K(X - t_A(n_i)^\gamma) - p_i = M\beta K(X - t_A(N-n_i)^\gamma) - p_j. \tag{6}$$

[7]Assumption 1 is necesary to guarantee that we have calculated a maximum. If assumption 1 is violated the welfare gains from trade are low compared to the utility loss. In this case platform $i$ should have $n_i = 0$ and $n_j$ should lie between 0 and $N$ dependent on $X, \beta, K$, and $\gamma$.

[8]It should be mentioned that this result is completely different in a model with positive externalities. If in such models buyers (in our model advertisers) can coordinate themselves on that platform which gives them the highest surplus prices would be driven down to zero because of the standard Bertrand argument. For an overview of this literature see Farrell & Klemperer (2001) or Katz & Shapiro (1994).

[9]Since advertisers decide simultaneously one cannot strictly speak about the $N$th advertiser. But in a Nash-equilibrium everyone optimises taking the decisions of all others as given so we can think of the '$N$th' advertiser choosing her platform after the choice of all other advertisers.

Since all advertisers are the same each advertiser is indifferent between platform $i$ and $j$.

Contrary to standard analysis it is not possible to solve (6) for $n_i$ because the externality function can be convex or concave. To get a general solution (6) is solved for $p_i$ which yields

$$p_i = p_j + M\beta K t_A((N - n_i)^\gamma - (n_i)^\gamma). \tag{7}$$

Plugging (7) into the profit function of platform $i$ gives

$$\max_{n_i} \qquad \Pi_i = \{p_j + M\beta K t_A((N - n_i)^\gamma - (n_i)^\gamma)\}n_i. \tag{8}$$

Maximising (8) with respect to $n_i$ yields the first order condition

$$\frac{\partial \Pi_i}{\partial n_i} = M\beta K t_A((N - n_i)^\gamma - (n_i)^\gamma) + p_j - M\beta K t_A \gamma (n_i)^\gamma - M\beta K t_A \gamma n_i (N - n_i)^\gamma = 0. \tag{9}$$

We therefore get a system of four equations (7), (9) and both equations with $i$ and $j$ reversed. It remains to show under which conditions the platforms serve all advertisers and to calculate the equilibrium if $n_1 + n_2 < N$. This is done in the appendix.[10]

**Proposition 1**

If $t_A \leq \frac{X}{1+2\gamma}\left(\frac{2}{N}\right)^\gamma$ in the unique Nash equilibrium $n_i^* = \frac{N}{2}$ and

$$p_i^* = 2^{1-\gamma} M\beta K t_A \gamma N^\gamma. \tag{10}$$

Profits of the platforms are

$$\Pi_i^* = 2^{-\gamma} M\beta K t_A \gamma N^{\gamma+1}. \tag{11}$$

If $\frac{X}{1+2\gamma}\left(\frac{2}{N}\right)^\gamma \leq t_A \leq \frac{X}{1+\gamma}\left(\frac{2}{N}\right)^\gamma$ in the unique Nash equilbrium $n_i^* = \frac{N}{2}$ and

$$p_i^* = M\beta K\left(X - t_A(N/2)^\gamma\right). \tag{12}$$

[10] The method of solution is similar to the product differentiation literature if consumers' gross surplus from buying is such low that firms are local monopolists. See e.g. Gabszewicz & Thisse (1986).

Profits of the platforms are

$$\Pi_i^* = M\beta K \frac{N}{2}\left(X - t_A(N/2)^\gamma\right). \tag{13}$$

If $t_A > \frac{X}{1+\gamma}\left(\frac{2}{N}\right)^\gamma$ in the unique Nash equilbrium

$$n_i = \left(\frac{X}{t_A(1+\gamma)}\right)^{1/\gamma} \tag{14}$$

and

$$p_i^* = \frac{\gamma}{1+\gamma}\beta KMX \tag{15}$$

Profits of the platforms are

$$\Pi_i^* = \beta KM\gamma\left(\frac{1}{t_A}\left(\frac{X}{1+\gamma}\right)^{1+\gamma}\right)^{1/\gamma}. \tag{16}$$

It can be shown that the profit function is continuous but it has two kinks.

It is now possible to compare this result with the efficient outcome.

The efficient result is that $n_i^{eff} = \frac{N}{2}$ if

$$X\beta K > t_A\beta K(1+\gamma)(N/2)^\gamma + t_A\gamma(N/2)^{\gamma-1}. \tag{17}$$

But in the Nash equilibrium $n_i^{eq} = \frac{N}{2}$ if

$$X\beta K > t_A\beta K(1+\gamma)(N/2)^\gamma. \tag{18}$$

So for $t_A\beta k(1+\gamma)(N/2)^\gamma < X < t_A\beta K(1+\gamma)(N/2)^\gamma + t_A\gamma(N/2)^{\gamma-1}$ each platform has $n_i^{eq} = \frac{N}{2}$ advertiser although $n_i^{eff} < \frac{N}{2}$ would be optimal. If $X\beta K \leq t_A\beta K(1+\gamma)(N/2)^\gamma$ the efficient $n_i^{eff}$ is given by $X\beta K = t_A\beta K(1+\gamma)(n_i^{eff})^\gamma + t_A\gamma(n_i^{eff})^{\gamma-1}$ while the $n_i^{eq}$ is given by $X\beta K = t_A\beta K(1+\gamma)(n_i^{eq})^\gamma$.

Thus if $X\beta K \leq t_A\beta K(1+\gamma)(N/2)^\gamma$ the number of advertisers is inefficiently high. The intuition behind this result is that platforms do not take into account that advertisement exerts a negative externality on the users' utility directly. This externality is expressed in the term $t_A\gamma(n_i^{eff})^{\gamma-1}$ in the above expressions. The reason is that platforms profit functions are not affected by that term while the welfare function is. Platforms take fully into account that advertisers cause a negative externality on themselves because users spend less time on the platforms. But the direct effect is not interalised by the firms.

If $t_A$ is low the efficient and the equlibrium outcome coincide. This is the case because the externality is so low that the possible gains from trade are more important than the utility loss of consumers. All producers should advertise.

This suggests that there is scope for policy intervention. If the externality is high it is welfare improving to set an upper bound on the number of advertisements. One example might be broadcasting. In Germany channels face a governmental constraint for the number of minutes of advertisement during one hour. This constraint is usually binding during prime time movies because $t_A$ should be relatively high in this time.[11]

We can derive a comparative static result with respect to the externality.

**Proposition 2**

$\Pi_i^*$ is increasing in $t_A$ as long as $t_A \leq \frac{X}{1+2\gamma}\left(\frac{2}{N}\right)^\gamma$ and decreases in $t_A$ if $t_A > \frac{X}{1+2\gamma}\left(\frac{2}{N}\right)^\gamma$.

This is apparent from the derivations of (11), (13), and (16)

Thus the externality parameter $t_A$ has the same effect as the transportation cost parameter in the Hotelling model. If $t_A$ is low price competition of platforms is very fierce because each platform can win many consumers by slightly undercutting the competitor's price. A higher $t_A$ slightens price competition and increases profits.

If $t_A$ is high profits decrease in $t_A$. The market for advertisers is only partially covered and a lower $t_A$ would increase $n_i$ since prices stay the same because there is no competition. For $\frac{X}{1+2\gamma}\left(\frac{2}{N}\right)^\gamma \leq t_A \leq \frac{X}{1+\gamma}\left(\frac{2}{N}\right)^\gamma$ profits also decrease in $t_A$ because platforms have to lower their prices to keep $N/2$ of the advertisers.

[11] For a detailed discussion of two-sided markets and regulation issues see Evans (2002).

# 4 Exclusive Choice by Advertisers and Consumers

In this section we relax the assumption that all $M$ users are using both platforms. Instead platforms have to compete for users now. Since it is not possible to charge a user fee platforms compete by advertisement levels. So a lower advertisement lewel does now not only increase the time a user spend on a platform but also the number of users. To make profit functions continuous we go back to the Hotelling formulation and assume that platforms are differentiated from a consumers point of view.[12] There is still a mass $M$ of consumers but they are now distributed on a line with length $M$. Platform 1 is located at point 0 and platform 2 is located at point $M$. A user who is located at $s$, $0 \leq s \leq M$, incurs transportation cost $t_U s$ if he uses platform 1 and $t_U(M - s)$ if he uses platform 2. Calculating the marginal user and solving for the user demand of platform $i$ we get

$$m_i = \frac{M}{2} + \frac{M t_A (n_j^\gamma - n_i^\gamma)}{2t_U}. \tag{19}$$

To simplify the analysis it is assumed that the gross utility of users is high enough such that in each equilibrium all user will choose one or the other platform, e.g. $X - t_U/2 - t_A(N)^\gamma > 0$. It is also assumed that the time a user spend on platform $i$ is still $X - t_A(n_i)^\gamma$ and is therefore independent of his location. The location does only determine the platform he uses.

Consider first the case that all producers advertise, $n_1 + n_2 = N$. Calculating the marginal advertiser solving for $p_i$ and plugging this into the profit function of platform $i$ yields

$$\Pi_i = n_i \left(p_j + \beta K M(2m_i - M) + \beta K t_A((M - m_i)(N - n_i)^\gamma - m_i n - i^\gamma)\right), \tag{20}$$

where $m_i$ is given by (19).

Maximising (20) with respect to $n_i$ yields equilibrium prices and profits. The case if not all producers advertise can be calculated in the same way as in the appendix.

[12]If the assumption that platforms are completely similar is kept profit functions are discontinuous and it would be impossible to compare the results with the results from the previous section. Note that platfroms are still completely similar from the advertisers' point of view.

**Proposition 3**

If $X \geq t_A \left(\frac{N}{2}\right)^\gamma \left(\frac{\frac{1}{2}-\gamma+2\gamma\left(\frac{N}{2}\right)^\gamma \frac{t_A}{t_u}}{\frac{1}{2}-2\gamma\left(\frac{N}{2}\right)^\gamma \frac{t_A}{t_u}}\right)$ in the unique Nash equilibrium $n_i^* = \frac{N}{2}$, $m_i^* = \frac{M}{2}$ and

$$p_i^* = M\beta K t_A \gamma \left(\frac{N}{2}\right)^\gamma \left(1 + 2\frac{X}{t_u} - 2\left(\frac{N}{2}\right)^\gamma \frac{t_A}{t_u}\right). \tag{21}$$

Profits of the platforms are

$$\Pi_i^* = M\beta K t_A \gamma \left(\frac{N}{2}\right)^{\gamma+1} \left(1 + 2\frac{X}{t_u} - 2\left(\frac{N}{2}\right)^\gamma \frac{t_A}{t_u}\right). \tag{22}$$

If $t_A \left(\frac{N}{2}\right)^\gamma \left(1+\gamma+\frac{\gamma}{t_u} - \gamma\left(\frac{N}{2}\right)^\gamma \frac{t_A}{t_u}\right) \leq X < t_A \left(\frac{N}{2}\right)^\gamma \left(\frac{\frac{1}{2}-\gamma+2\gamma\left(\frac{N}{2}\right)^\gamma \frac{t_A}{t_u}}{\frac{1}{2}-2\gamma\left(\frac{N}{2}\right)^\gamma \frac{t_A}{t_u}}\right)$ in the unique Nash equilbrium $n_i^* = \frac{N}{2}$, $m_i^* = \frac{M}{2}$ and

$$p_i^* = \beta K \frac{M}{2}\left(X - t_A(N/2)^\gamma\right). \tag{23}$$

Profits of the platforms are

$$\Pi_i^* = \beta K \frac{NM}{4}\left(X - t_A(N/2)^\gamma\right). \tag{24}$$

If $X < t_A \left(\frac{N}{2}\right)^\gamma \left(1+\gamma+\frac{\gamma}{t_U} - \gamma\left(\frac{N}{2}\right)^\gamma \frac{t_A}{t_U}\right)$ in the unique Nash equilbrium $n_i^*$ is given by $X = n_i^\gamma t_A \left(1+\gamma+\frac{\gamma}{t_U}\right) - n_i^{2\gamma}\gamma\frac{t_A^2}{t_U}$, $m_i^* = \frac{M}{2}$ and

$$p_i^* = \beta K \frac{M}{2}\left(X - t_A(n_i^*)^\gamma\right). \tag{25}$$

Profits of the platforms are

$$\Pi_i^* = n_i^* \beta K \frac{M}{2}\left(X - t_A(n_i^*)^\gamma\right). \tag{26}$$

It is now possible to compare profits when platforms compete for users with profits without this competition effect. We have calculated the latter profits in the former section when all M consumers use both platforms. In the section here platforms have to compete for users and in equilibrium each platform gets $\frac{M}{2}$ consumers. So to make a fair comparison it is assumed that without competition each platform has a regular

stock of users of mass $M/2$ independent of $n_i$ or $n_j$.
I want to focus on the case of $X \geq t_A \left(\frac{N}{2}\right)^{\gamma} \left(\frac{\frac{1}{2}-\gamma+2\gamma\left(\frac{N}{2}\right)^{\gamma}\frac{t_A}{t_u}}{\frac{1}{2}-2\gamma\left(\frac{N}{2}\right)^{\gamma}\frac{t_A}{t_u}}\right)$. In this case with competition for users $n_i^* = \frac{N}{2}$ and $m_i^* = \frac{M}{2}$. With X given as above without competition we are either in the first or in the second part of the profit function, this means either $t_A$ is low or it is in some middle range as given in proposition 1. If we are in the first part prices and profits are given by (10) and (11) but we have to take care that each platform has only $\frac{M}{2}$ users which means that profits are $\Pi_i = 2^{-\gamma-1} M\beta K t_A \gamma N^{\gamma+1}$. Comparing this with (22) yields that (22) is higher if $X \geq t_A \left(\frac{N}{2}\right)^{\gamma}$ which is always fulfilled by assumption. In proposition 2 it was shown that platform profits in the second part are lower than the highest profit in the first part and are therefore in any case lower than (22). This yields the following proposition.

**Proposition 4**

If $X \geq t_A \left(\frac{N}{2}\right)^{\gamma} \left(\frac{\frac{1}{2}-\gamma+2\gamma\left(\frac{N}{2}\right)^{\gamma}\frac{t_A}{t_u}}{\frac{1}{2}-2\gamma\left(\frac{N}{2}\right)^{\gamma}\frac{t_A}{t_u}}\right)$ profits of the platforms with competition for consumers is higher than without.

This shows that profits can increase although there is an additional competition effect. What is the intuition behind this result? Platforms compete for user by reducing advertisement levels. This is done by increasing $p_i$. But since both platforms increase their prices no platform loses advertisers and earn higher profits in the end. The competitive effect on user's side softens the competition for advertisers. Thus if negative externalities are present in a two-sided market a higher degree of competition on one side of the market can reduce the competitive effect on the other side.

It is also possible to compare efficiency with and without competition.

**Proposition 5**

If $X \geq t_A \left(\frac{N}{2}\right)^{\gamma} \left(1 + \gamma + \frac{\gamma}{t_U} - \gamma \left(\frac{N}{2}\right)^{\gamma} \frac{t_A}{t_U}\right)$ the number of advertisers in both cases with and without competition is $n_i^* = \frac{N}{2}$.

If $X < t_A \left(\frac{N}{2}\right)^{\gamma} \left(1 + \gamma + \frac{\gamma}{t_U} - \gamma \left(\frac{N}{2}\right)^{\gamma} \frac{t_A}{t_U}\right)$ the number of advertisers in the equilibrium with competition is lower and therefore this equilibrium is more efficient.

This result is quite intuitive. When they have to compete for users platforms want to reduce their amount of advertisement. If X is high they increase prices but the number of advertisers stay the same because advertisement has a high value. If X is low the number of advertisers is recduced. Compared with the case of no competition for users this threshold for X is higher. Thus for all values below this threshold the amount of advertisement is lower with competition than without. Since we know from the previous section that for $X < t_A(1+\gamma)\left(\frac{N}{2}\right)^{\gamma}$ the number of advertisement is inefficiently high the equilibrium with competition is more efficient.

# 5 Conclusion

In this paper a model of platform competition was analysed in which each advertiser exerts a negative externality directly on users and indirectly on all other advertisers on the same platform. It was shown that the number of advertisement in equilibrium might be too high compared with the efficient one. Profits of platforms can increase or decrease in the degree of the externality. If platforms have to compete not only for advertisers but also for users profits can increase since platforms wish to reduce their advertisements and set higher prices.

One way in which the model can be extended is to allow platforms to charge user fees. In this case the model would describe a market for newspapers or magazines and the platforms are publishers. The platforms can make revenues on both sides of the market. Compared with the model analysed in the paper profits should increase. But this is not completely clear. Consider a situation where competition for users is such severe that platforms exclude advertisers. If this is the case it might be possible that the profit gain by users does not offset the loss that platforms incur on advertisers. Platforms are in a prisoner's dilemma situation because if one platform sets a user fee of zero the other one has an incentive to set a positive fee thereby starting the competition effect.[13]

Another interesting suggestion for further research might be to analyse the dynamics of such a two-sided market. Usually if people are used to one Internet portal or read a newspaper for several years they would not switch easily if another one has lower advertisements. So people form some habits. It would be interesting to analyse how such habit formation might change the results. A new platform which enters the market after the others like Google needs a very low level of advertisements to induce consumers to switch. This is what was actually observed by Google. So the question arises if this low level of adverisement does persist or if it will vanish over time.

[13]The effect that duopolists might lose if they can use an additional pricing instrument is also present in the papers of Anderson & Leruth (1993) and Thisse & Vives (1984).

# 6 Appendix

Solving equations (7) and (9) and both equations with $i$ and $j$ reversed yields (10) and (11).

But this is only an equilibrium if it pays for all $N$ advertisers to advertise at this price. This is only the case if

$$(X - t_A(N/2)^\gamma)\beta KM - 2^{1-\gamma} M\beta K t_A \gamma n^\gamma > 0$$

or

$$X - t_A(N/2)^\gamma(1 + 2\gamma) > 0.$$

The next question is what a monopolistic platform would do. It would set

$$p_i = (X - t_A(n_i)^\gamma)\beta KM$$

and the profit function is

$$\max_{n_i} \qquad \Pi_i = (X - t_A(n_i)^\gamma)\beta KMn_i.$$

This yields $n_i = \left(\frac{X}{t_A(1+\gamma)}\right)^{1/\gamma}$ which is equation (14). This $n_i$ is only smaller than $N/2$ if $t_A > \frac{X}{1+\gamma}\left(\frac{2}{N}\right)^\gamma$. Calculating prices and profits yields (15) and (16).

If now $\frac{X}{1+2\gamma}\left(\frac{2}{N}\right)^\gamma \leq t_A \leq \frac{X}{1+\gamma}\left(\frac{2}{N}\right)^\gamma$ a monopolistc platform serves all advertisers. Since there are two platforms the whole market is covered at $p_i^* = M\beta K\left(X - t_A(N/2)^\gamma\right)$ which yields (13).

q.e.d.

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