Ambiguity, Learning, and Asset Returns

Nengjiu Ju[1] Jianjun Miao[2]

[1]Department of Finance
HKUST

[2]Department of Economics
Boston University

[2]Department of Finance
HKUST

September 2008

Rational Expectations Hypothesis

- There exists an objective probability law governing the state process
- Economic agents know this law which coincides with their subjective beliefs
- Imposing rational expectations removes from the need for separately specifying subjective probabilities, thereby simplifying model specification.
- Learning to be rational

What can go wrong?

- Knight (1921) and Keynes (1936)
- Ellsberg Paradox and ambiguity
- An urn contains 30 red balls and 60 white and black balls

	R	B	W
f	10	0	0
g	0	10	0
f'	10	0	10
g'	0	10	10

- EU implies $f \succ g \Rightarrow f' \succ g'$
- But experiment evidence reveals that $f \succ g$ but $g' \succ f'$

What can go wrong?

- Model misspecification
- Statistical ambiguity
- Averse to model ambiguity (uncertainty)
- Robustness
- Hansen and Sargent

What can go wrong?

- Asset pricing puzzles and empirical failure
  - Equity premium, riskfree rate and equity volatility puzzles
  - Countercyclical variation of equity premia and equity volatility
  - Procyclical variation of price-dividend ratio
  - Long-horizon predictability and serial correlation of excess returns

What we do?

- Set up a Lucas-style model by departing from REH
- Apply the smooth ambiguity model developed by Klibanoff et al (2005, 2008)
- Analyze quantitative asset pricing implications of learning under ambiguity
- Provide a new explanation of asset pricing puzzles

Main model ingredients

- Assume hidden Markov switching consumption process
  - The agent learns about hidden states under ambiguity
  - Posterior state beliefs are a state variable driving asset return dynamics
  - Non-Bayesian approach: irreducibility of compound distributions
- Assume the agent is ambiguous about hidden states
  - Recursive smooth ambiguity utility (Klibanoff et al. (2005, 2008))
  - It is tractable and permits separation between ambiguity and ambiguity aversion
  - Ambiguity aversion helps propagate and amplify shocks to the dynamics of asset returns

Main results

- Propose and analyze two tractable parametric specifications: log-exponential and power-power
- Under reasonable calibration, both can match first moments of equity premium and riskfree rate
- Only power-power specification can generate dynamic asset pricing phenomena observed in data
- The standard Bayesian learning model may worsen asset pricing puzzles
- Ambiguity aversion is not simply reinforcement of risk aversion

Related literature

- **Decision theory:** Gilboa and Schmeidler (1989), Epstein and Schneider (2003), Klibanoff et al. (2005, 2008), Segal (1987)
- **Applications of multiple-priors model:** Chen and Epstein (2002), Epstein and Miao (2003), Epstein and Schneider (2007), Epstein and Wang (1994), Leippold et al. (2007)
- **Robustness approach:** Anderson et al. (2002), Hansen (2007), Hansen and Sargent (2007, 2008)
- **Distorted beliefs:** Abel (2002), Brandt et al. (2004), Cecchetti et al. (2000)
- **Other explanations:** Campbell and Cochrane (1999) and Bansal and Yaron (2004)

Setup

- A representative-agent Lucas-style pure exchange model
- The agent trades one aggregate stock and one bond
- Hidden Markov switching dividend process

$$\log\left(\frac{D_{t+1}}{D_t}\right) = \kappa_{z_{t+1}} + \sigma\varepsilon_{t+1}, \; D_0 \text{ given},$$

  where ε_t is iid standard normal and the state $z_t \in \{1, 2, ..., N\}$ follows a N state Markov chain with transition matrix (λ_{ij})
- Assume $\kappa_1 > \kappa_2 > ... > \kappa_N$
- In equilibrium aggregate consumption is equal to dividends

State beliefs

- Let $\mu_t(j) = \Pr\left(z_{t+1} = j | s^t\right)$
- Prior beliefs μ_0 given
- Bayesian updating

$$\mu_{t+1} = B\left(\log\left(D_{t+1}/D_t\right), \mu_t\right)$$

  for some vector of functions B
- The standard Bayesian approach implies that the posterior and likelihood can be reduced to a predictive distribution
- The key idea of learning under ambiguity is irreducibility of compound distributions

Static smooth ambiguity preferences

Utility function

$$\phi^{-1}\left(\int_{\Pi}\phi\left(\mathbb{E}_{\pi}u\left(C\right)\right)d\mu\right),\quad \forall C:S\to\mathbb{R}_{+},$$

- u: risk attitude
- ϕ: ambiguity attitude
- Π: set of prob measures or models
- μ: subjective prior over possible models

Ambiguity attitude

- Use expected utility as the ambiguity neutrality benchmark
- DM is ambiguity averse iff ϕ is concave
- constant absolute ambiguity aversion (CAAA) utility:

$$\phi(x) = -e^{-\frac{x}{\theta}}, \ \theta > 0,$$

  where $1/\theta$ is the parameter of CAAA.
- constant relative ambiguity aversion (CRAA) utility:

$$\phi(x) = \frac{x^{1-\alpha}}{1-\alpha}, \ x > 0, \ \alpha > 0, \neq 1$$

  where α is the parameter of CRAA.

Special cases: CAAA

- When $\theta \to 0$, converge to Gilboa and Schmeidler (1989)

$$\inf_{\pi \in \Pi} \mathbb{E}_\pi u(C).$$

- Robustness (Hansen and Sargent) and risk-sensitivity (Tallarini (2000))

$$\begin{aligned} & \phi^{-1}\left(\mathbb{E}_\mu \phi\left(\mathbb{E}_\pi u(C)\right)\right) \\ = & \min_{m \geq 0, \mathbb{E}_\mu[m]=1} \mathbb{E}_\mu\left[m \mathbb{E}_\pi u(C)\right] + \theta \mathbb{E}_\mu\left[m \log m\right] \\ = & -\theta \log \mathbb{E}_\mu \exp\left(-\frac{\mathbb{E}_\pi u(C)}{\theta}\right) \end{aligned}$$

Resolving Ellsberg Paradox

- An urn contains 30 red balls and 60 white and black balls

	R	B	W
f	10	0	0
g	0	10	0
f'	10	0	10
g'	0	10	10

- Beliefs: (1/3,0,2/3) with prob 1/2 and (1/3,2/3,0) with prob 1/2
- Bayesian approach with linear ϕ does not help

Resolving Ellsberg Paradox

- Strictly concave ϕ
- $f \succ g$ because

$$
\begin{aligned}
u(10)/3 &= \phi^{-1}\left(\frac{1}{2}\phi(u(10)/3) + \frac{1}{2}\phi(u(10)/3)\right) \\
&> \phi^{-1}\left(\frac{1}{2}\phi(0) + \frac{1}{2}\phi(u(10) \times 2/3)\right)
\end{aligned}
$$

- $g' \succ f'$ because

$$
\begin{aligned}
u(10) \times 2/3 &= \phi^{-1}\left(\frac{1}{2}\phi(\frac{2}{3}u(10)) + \frac{1}{2}\phi(\frac{2}{3}u(10))\right) \\
&> \phi^{-1}\left(\frac{1}{2}\phi(\frac{1}{3}u(10) + \frac{2}{3}u(10)) + \frac{1}{2}\phi(\frac{1}{3}u(10))\right)
\end{aligned}
$$

Recursive smooth ambiguity model

- Time $t = 0, 1, 2, ...$
- Period state space S, full space S^{∞}.
- History $s^t = \{s_0, s_1, s_2, ..., s_t\}$ with s_0 given.
- Unknown parameter z in Z
- Utility

$$V_t\left(C; s^t\right) = u\left(C_t\right) + \beta\phi^{-1}\left(\int_Z \phi\left(\int_S V_{t+1}\left(C; s^t, s_{t+1}\right) d\pi_z\left(s_{t+1}|s^t\right)\right) d\mu\left(z|s^t\right)\right)$$

Pricing kernel

$$M_{t+1,z} = \frac{\phi'\left(\mathbb{E}_{\pi_{t,z}}\left[V_{t+1}(C)\right]\right)}{\phi'\left(\phi^{-1}\left(\mathbb{E}_{\mu_t}\left[\phi\left(\mathbb{E}_{\pi_{t,z}}\left[V_{t+1}(C)\right]\right)\right]\right)\right)} \frac{\beta u'(C_{t+1})}{u'(C_t)}$$

Log-exponential specification

- $u(c) = \log(c)$ and ϕ is CAAA
- Bellman equation

$$J(W_t, \mu_t) = \max_{C_t, \psi_t} \log(C_t) -$$
$$\beta\theta \log\left(\sum_j \mu_t(j) \exp\left(-\frac{1}{\theta}\mathbb{E}_{t,j}\left[J(W_{t+1}, \mu_{t+1})\right]\right)\right)$$

Log-exponential specification

- The equilibrium stock price and return are given by

$$P_t = \frac{\beta}{1-\beta} D_t, \; R_{e,t+1} = R_{m,t+1} = \frac{1}{\beta} \frac{D_{t+1}}{D_t}.$$

- The equilibrium bond return is given by

$$\frac{1}{R_{f,t+1}} = \sum_j \mu_t(j) \mathbb{E}_{t,j}\left[M_{t+1,j}\right],$$

where the pricing kernel is given by

$$M_{t+1,j} = \beta \frac{C_t}{C_{t+1}} \frac{\exp\left(-\frac{1}{\theta}\mathbb{E}_{t,j}\left[J\left(W_{t+1}, \mu_{t+1}\right)\right]\right)}{\sum_j \mu_t(j) \exp\left(-\frac{1}{\theta}\mathbb{E}_{t,j}\left[J\left(W_{t+1}, \mu_{t+1}\right)\right]\right)}.$$

Intuition

- Distorted beliefs $\hat{\mu}_t(j) = \mu_t(j)\, m^*_{t,j}$,

$$\frac{1}{R_{f,t+1}} = \sum_j \hat{\mu}_t(j)\, \mathbb{E}_{t,j}\left[\beta \frac{C_t}{C_{t+1}}\right],$$

- The ambiguous averse agent puts relatively more weight on smaller continuation values than larger values under distorted beliefs
- An increase in the degree of ambiguity aversion implies a first-order stochastic dominated shift of state beliefs
- This pessimism induces the agent to save more for future consumption

Power-power specification

- $u(C) = C^{1-\gamma}/(1-\gamma)$, and ϕ is CRAA
- When $\gamma > 1$, utility function is not well defined
- We adopt

$$V_t(C; s^t) = \Big[C_t^{1-\gamma} +$$

$$\beta \left\{ \int \left(\int V_{t+1}^{1-\gamma}(C; s^t, s_{t+1})\, d\pi_z(s_{t+1}|s^t) \right)^{1-\alpha} d\mu(z|s^t) \right\}^{\frac{1}{1-\alpha}} \Big]^{\frac{1}{1-\gamma}}$$

- Ordinally equivalent (Epstein and Zin (1989, 1991))

Power-power specification: Pricing

- The equilibrium stock price and return are given by

$$P_t = \varphi(\mu_t) D_t, \; R_{e,t+1} = R_{m,t+1} = \frac{D_{t+1}}{D_t} \frac{1+\varphi(\mu_{t+1})}{\varphi(\mu_t)},$$

where the function φ satisfies

$$1 = \sum_j \mu_t(j) \left(\mathbb{E}_{t,j} \left[\frac{1+\varphi(\mu_{t+1})}{\varphi(\mu_t)} \beta \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right] \right)^{1-\alpha}.$$

- The pricing kernel is given by

$$M_{t+1,j} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\mathbb{E}_{t,j} \left[R_{m,t+1} \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \right)^{-\alpha}.$$

Intuition

- μ_t is a state variable driving stock returns
- Pessimism induces the agent to save more and invest less in the stock
- Ambiguity aversion has different effects for the $\gamma > 1$ case and for the $\gamma < 1$ case
- Wealth and substitution effects

Calibration

- Use data from Cecchetti et al. (2000)
- For log-exponential specification, choose $(\beta, 1/\theta)$ to match mean values of equity premium and riskfree rate
- For power-power specification, choose (β, γ, α) to match mean values of equity premium and riskfree rate and their correlation
- Require $\beta \in (0, 1)$ and $\gamma \in (0, 10)$

Table 1. Maximum likelihood estimates of the consumption process

λ_{11}	λ_{22}	κ_1	κ_2	σ
0.978	0.516	2.251	−6.785	3.127

Notes: The numbers in the last three columns are expressed in percentage. This table is taken from Table 2 in Cecchetti et al. (2000).

Table 2. Stylized facts of equity and short-term bond returns using annual observations from 1871-1993

A. First and second moments as a percentage

Mean equity premium	μ_{eq}	5.75
Mean risk-free rate	r_f	2.66
Standard deviation		
Equity premium	$\sigma(\mu_{eq})$	19.02
Risk-free rate	$\sigma(r_f)$	5.13
Correlation	$\rho_{eq,f}$	-0.24

B. Predictability and persistence of excess returns

Horizon	Regression slope	R^2	Variance ratio
1	0.148	0.043	1.000
2	0.295	0.081	1.038
3	0.370	0.096	0.921
5	0.662	0.191	0.879
8	0.945	0.278	0.766

Notes: The regression slope and R^2 are for regressions of the $k-$year ($k = 1, 2, 3, 5, 8$) ahead equity premium on the current log dividend-price ratio. The variance ratio is the variance of the $k-$year equity premium dividend by k times the variance of the one-year equity premium. This table is taken from Table 1 in Cecchetti et al. (2000).

Table 3. Unconditional Moments for the Log–Exponential Case

1	2	3	4	5	6	7	8	9	10	11
								$r_f = r_f^* + \Delta r_f^L + \Delta r_f$		
$1/\theta$	r_f	$\sigma(r_f)$	r_e	μ_{eq}	$\sigma(\mu_{eq})$	$\frac{\sigma(M)}{E[M]}$	μ_{eq}^*	r_f^*	Δr_f^L	Δr_f
Panel A: Baseline parameter values: $\beta = 0.940$, $1/\theta = 1.292$										
	2.660	0.914	8.410	5.750	3.853	3.792	0.133	8.276	-0.002	-5.614
Panel B: $\beta = 0.940$										
0.00	8.274	0.817	8.410	0.136	3.816	0.037	0.133	8.276	-0.002	0.000
0.25	7.990	1.107	8.410	0.420	3.831	0.185	0.133	8.276	-0.002	-0.283
0.50	7.369	1.399	8.410	1.041	3.874	0.560	0.133	8.276	-0.002	-0.904
0.75	6.027	1.477	8.410	2.383	3.905	1.436	0.133	8.276	-0.002	-2.246
1.00	4.318	1.260	8.410	4.092	3.883	2.614	0.133	8.276	-0.002	-3.955
1.25	2.867	0.962	8.410	5.543	3.856	3.644	0.133	8.276	-0.002	-5.407
1.50	1.791	0.699	8.410	6.619	3.846	4.420	0.133	8.276	-0.002	-6.483
1.75	1.028	0.496	8.410	7.382	3.849	4.976	0.133	8.276	-0.002	-7.246
2.00	0.495	0.347	8.410	7.915	3.857	5.367	0.133	8.276	-0.002	-7.778
Panel C: $\beta = 0.98$										
0.00	3.892	0.784	4.023	0.131	3.662	0.037	0.128	3.895	-0.002	0.000
0.25	1.398	1.396	4.023	2.625	3.747	1.675	0.128	3.895	-0.002	-2.494
0.50	-2.582	0.606	4.023	6.605	3.690	4.612	0.128	3.895	-0.002	-6.474
0.75	-4.011	0.206	4.023	8.034	3.712	5.705	0.128	3.895	-0.002	-7.903
1.00	-4.477	0.068	4.023	8.500	3.728	6.064	0.128	3.895	-0.002	-8.369
1.25	-4.628	0.022	4.023	8.651	3.734	6.181	0.128	3.895	-0.002	-8.520
1.50	-4.676	0.007	4.023	8.699	3.737	6.219	0.128	3.895	-0.002	-8.569
1.75	-4.692	0.002	4.023	8.715	3.737	6.231	0.128	3.895	-0.002	-8.584
2.00	-4.697	0.001	4.023	8.720	3.738	6.235	0.128	3.895	-0.002	-8.589
Panel D: $\beta = 0.92$										
0.00	10.668	0.835	10.807	0.139	3.901	0.037	0.136	10.670	-0.003	0.000
0.25	10.476	1.047	10.807	0.331	3.909	0.131	0.136	10.670	-0.003	-0.192
0.50	10.154	1.275	10.807	0.653	3.933	0.314	0.136	10.670	-0.003	-0.514
0.75	9.565	1.465	10.807	1.242	3.968	0.670	0.136	10.670	-0.003	-1.103
1.00	8.567	1.516	10.807	2.240	3.990	1.308	0.136	10.670	-0.003	-2.101
1.25	7.311	1.396	10.807	3.496	3.981	2.146	0.136	10.670	-0.003	-3.357
1.50	6.096	1.186	10.807	4.711	3.958	2.979	0.136	10.670	-0.003	-4.572
1.75	5.064	0.964	10.807	5.743	3.940	3.698	0.136	10.670	-0.003	-5.604
2.00	4.235	0.763	10.807	6.572	3.931	4.283	0.136	10.670	-0.003	-6.433

Notes: Except for the numbers in Columns 1 and 7, all numbers are in percentage. The variables in the first row and columns 2-6 are defined as in Table 2. $\sigma(M)/E[M]$ is the ratio of the standard deviation to the mean of the pricing kernel. r_f^* and μ_{eq}^* are the mean riskfree rate and the mean equity premium for benchmark model I. r_f^L is the mean riskfree rate for benchmark model II. $\Delta r_f^L = r_f^L - r_f^*$ denotes the change of the mean riskfree rate due to learning only. $\Delta r_f = r_f - r_f^L$ denotes change of the riskfree rate due to ambiguity.

Table 4. Comparative Statistics for the Power-Power Case

1	2	3	4	5	6	7	8	9
α	r_f	$\sigma(r_f)$	r_e	$\sigma(r_e)$	μ_{eq}	$\sigma(\mu_{eq})$	$\frac{\mu_{eq}}{\sigma(\mu_{eq})}$	$\frac{\sigma(M)}{E[M]}$
Panel A: Baseline parameter values: $\beta = 0.944$, $\gamma = 0.647$, $\alpha = 48.367$								
	2.660	0.952	8.410	4.581	5.750	4.715	1.219	2.640
Panel B: $\beta = 0.944$, $\gamma = 0.0$								
0.0	5.953	0.000	5.953	4.456	0.000	4.456	0.000	0.000
3.0	5.813	0.189	6.077	4.660	0.265	4.671	0.057	0.096
10.0	4.568	0.845	6.888	5.640	2.320	5.843	0.397	0.711
25.0	-1.736	0.932	10.466	6.037	12.202	6.011	2.030	4.215
60.0	0.004	1.026	13.212	4.927	13.208	4.996	2.643	6.152
Panel C: $\beta = 0.944$, $\gamma = 0.2$								
0.0	6.345	0.160	6.375	4.324	0.030	4.322	0.007	0.007
3.0	6.234	0.308	6.453	4.444	0.220	4.452	0.049	0.078
10.0	5.552	0.786	6.855	4.956	1.303	5.087	0.256	0.447
25.0	0.121	0.674	9.295	5.698	9.175	5.793	1.584	3.265
60.0	-0.970	0.912	11.793	4.865	12.763	4.925	2.592	5.975
Panel D: $\beta = 0.944$, $\gamma = 0.8$								
0.0	7.499	0.648	7.611	3.975	0.112	3.928	0.029	0.029
3.0	7.470	0.686	7.615	3.980	0.145	3.935	0.037	0.040
10.0	7.392	0.778	7.628	3.995	0.236	3.955	0.060	0.082
25.0	7.144	0.999	7.669	4.036	0.526	4.028	0.131	0.225
60.0	5.374	1.348	7.946	4.200	2.572	4.320	0.595	1.265
Panel E: $\beta = 0.944$, $\gamma = 1.5$								
0.0	8.800	1.235	8.986	3.717	0.186	3.516	0.053	0.055
3.0	8.877	1.137	9.013	3.733	0.137	3.546	0.039	0.050
10.0	9.015	0.932	9.059	3.763	0.044	3.604	0.012	0.095
25.0	9.195	0.606	9.114	3.808	-0.081	3.695	-0.022	0.171
60.0	9.360	0.255	9.164	3.858	-0.195	3.807	-0.051	0.233
Panel F: $\beta = 0.944$, $\gamma = 3.0$								
0.0	11.416	2.542	11.647	4.161	0.230	3.213	0.072	0.113
3.0	11.807	2.071	12.069	3.863	0.262	3.248	0.081	0.127
10.0	12.297	1.224	12.416	3.806	0.119	3.507	0.034	0.225
25.0	12.623	0.529	12.611	3.903	-0.012	3.784	-0.003	0.262
60.0	12.776	0.222	12.701	3.983	-0.075	3.936	-0.019	0.263

Notes: Except for numbers in Columns 1, 8 and 9, all numbers are in percentage. The variables in the first row and columns 2-6 are defined as in Table 2. $\sigma(M)/E[M]$ is the ratio of the standard deviation to the mean of the pricing kernel.

Table 5. Decomposition of r_f, r_e and μ_{eq} for the Power-Power Case

1	2	3	4	5	6	7	8	9	10
	$r_f = r_f^* + \Delta r_f^L + \Delta r_f$			$r_e = r_e^* + \Delta r_e^L + \Delta r_e$			$\mu_{eq} = \mu_{eq}^* + \Delta\mu_{eq}^L + \Delta\mu_{eq}$		
α	r_f^*	Δr_f^L	Δr_f	r_e^*	Δr_e^L	Δr_e	μ_{eq}^*	$\Delta\mu_{eq}^L$	$\Delta\mu_{eq}$
Panel A: Baseline parameter values: $\beta = 0.944$, $\gamma = 0.647$, $\alpha = 48.367$									
	7.208	-0.001	-4.547	7.299	0.001	1.111	0.090	0.002	5.657
Panel B: $\beta = 0.944$, $\gamma = 0.0$									
0.0	5.953	0.000	0.000	5.953	0.000	0.000	0.000	0.000	0.000
3.0	5.953	0.000	-0.140	5.953	0.000	0.124	0.000	0.000	0.265
10.0	5.953	0.000	-1.385	5.953	0.000	0.935	0.000	0.000	2.320
25.0	5.953	0.000	-7.689	5.953	0.000	4.513	0.000	0.000	12.202
60.0	5.953	0.000	-5.949	5.953	0.000	7.259	0.000	0.000	13.208
Panel C: $\beta = 0.944$, $\gamma = 0.2$									
0.0	6.345	0.000	0.000	6.375	0.001	0.000	0.030	0.001	0.000
3.0	6.345	0.000	-0.111	6.375	0.001	0.078	0.030	0.001	0.189
10.0	6.345	0.000	-0.793	6.375	0.001	0.479	0.030	0.001	1.273
25.0	6.345	0.000	-6.224	6.375	0.001	2.920	0.030	0.001	9.144
60.0	6.345	0.000	-7.315	6.375	0.001	5.417	0.030	0.001	12.733
Panel D: $\beta = 0.944$, $\gamma = 0.8$									
0.0	7.500	-0.001	0.000	7.610	0.001	0.000	0.110	0.003	0.000
3.0	7.500	-0.001	-0.028	7.610	0.001	0.005	0.110	0.003	0.033
10.0	7.500	-0.001	-0.106	7.610	0.001	0.017	0.110	0.003	0.123
25.0	7.500	-0.001	-0.355	7.610	0.001	0.059	0.110	0.003	0.414
60.0	7.500	-0.001	-2.125	7.610	0.001	0.335	0.110	0.003	2.460
Panel E: $\beta = 0.944$, $\gamma = 1.5$									
0.0	8.806	-0.006	0.000	8.989	-0.003	0.000	0.183	0.003	0.000
3.0	8.806	-0.006	0.077	8.989	-0.003	0.028	0.183	0.003	-0.049
10.0	8.806	-0.006	0.215	8.989	-0.003	0.074	0.183	0.003	-0.141
25.0	8.806	-0.006	0.395	8.989	-0.003	0.128	0.183	0.003	-0.267
60.0	8.806	-0.006	0.559	8.989	-0.003	0.179	0.183	0.003	-0.381
Panel F: $\beta = 0.944$, $\gamma = 3.0$									
0.0	11.444	-0.027	0.000	11.685	-0.038	0.000	0.241	-0.011	0.000
3.0	11.444	-0.027	0.390	11.685	-0.038	0.422	0.241	-0.011	0.032
10.0	11.444	-0.027	0.881	11.685	-0.038	0.769	0.241	-0.011	-0.111
25.0	11.444	-0.027	1.207	11.685	-0.038	0.964	0.241	-0.011	-0.243
60.0	11.444	-0.027	1.359	11.685	-0.038	1.055	0.241	-0.011	-0.305

Notes: Except for the numbers in Column 1, all numbers are in percentage. The variables r_f^*, r_e^*, and μ_{eq}^* are the mean riskfree rate, stock return, and the mean equity premium, respectively, for benchmark model I. The variables r_f^L, r_e^L, and μ_{eq}^L are the mean riskfree rate, stock return, and the mean equity premium, respectively, for benchmark model II. $\Delta r_f^L = r_f^L - r_f^*$ denotes the change of the mean riskfree rate due to learning only. $\Delta r_f = r_f - r_f^L$ denotes change of the riskfree rate due to ambiguity. The other variables Δr_e^L, Δr_e, $\Delta\mu_{eq}^L$, $\Delta\mu_{eq}$, are defined similarly.

Table 6. Predictability and persistence of excess returns

1	2	3	4	5	6	7	8	9	10
	Baseline parameter values			Benchmark model I			Benchmark model II		
Horizon	Slope	R^2	Variance ratio	Slope	R^2	Variance ratio	Slope	R^2	Variance ratio
1	1.040	0.064	1.000	0.443	0.022	1.000	0.776	0.020	1.000
2	1.372	0.074	0.848	0.599	0.025	0.989	1.028	0.022	0.987
3	1.511	0.071	0.769	0.685	0.024	0.979	1.137	0.022	0.975
5	1.635	0.061	0.683	0.780	0.022	0.960	1.346	0.021	0.955
8	1.741	0.053	0.619	0.907	0.020	0.933	1.649	0.020	0.927

Notes: The slope and R^2 are obtained from an OLS regression of the excess returns on the log dividend yield at different horizons. The variance ratio is computed in the same way as Cecchetti (1990, 2000). The reported numbers are the mean values of 10,000 Monte Carlo simulations, each consisting of 123 excess returns and dividend yields.

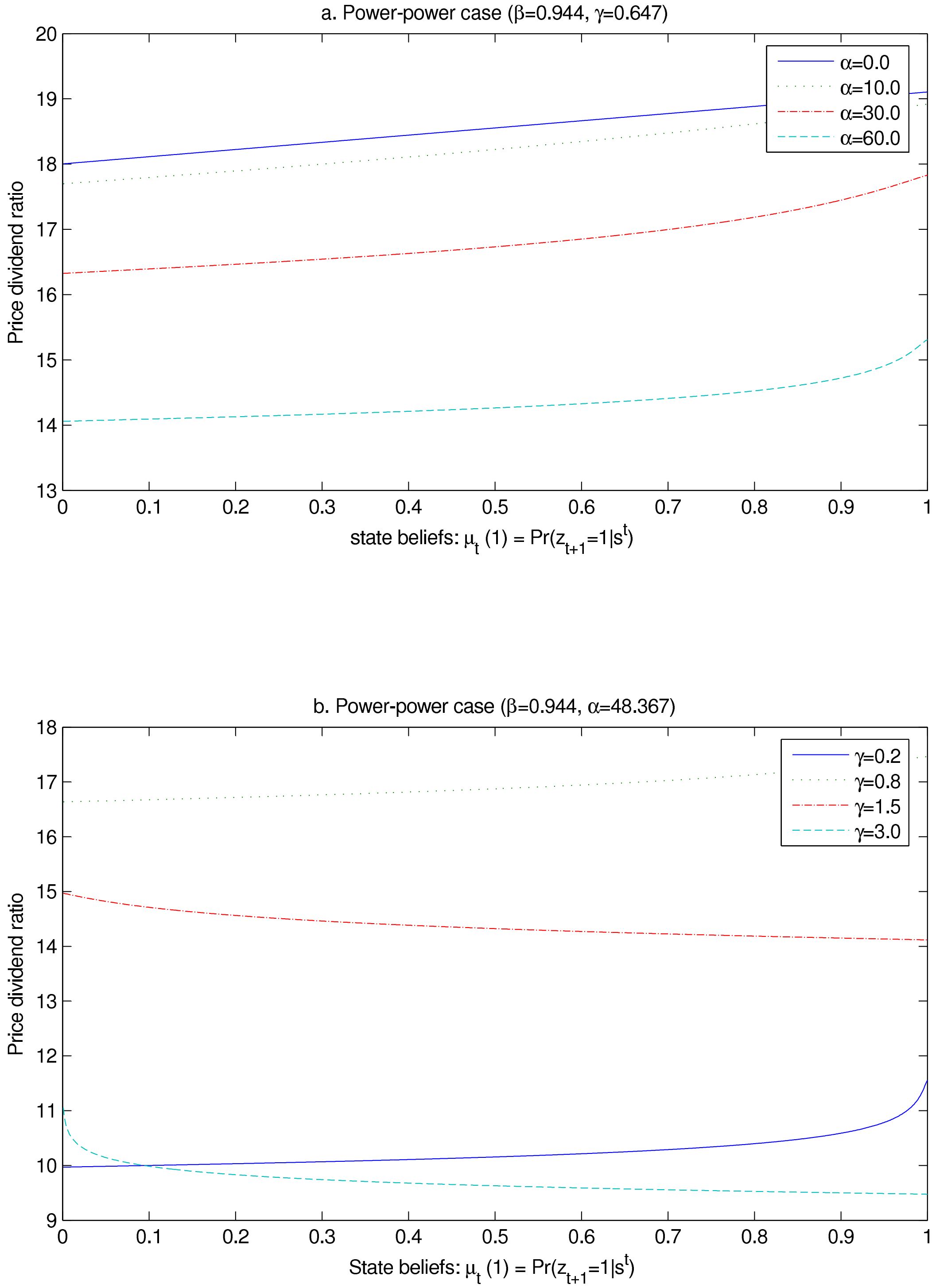


Figure 1: **Price dividend ratio as a function of the posterior probability of the high-growth state.**

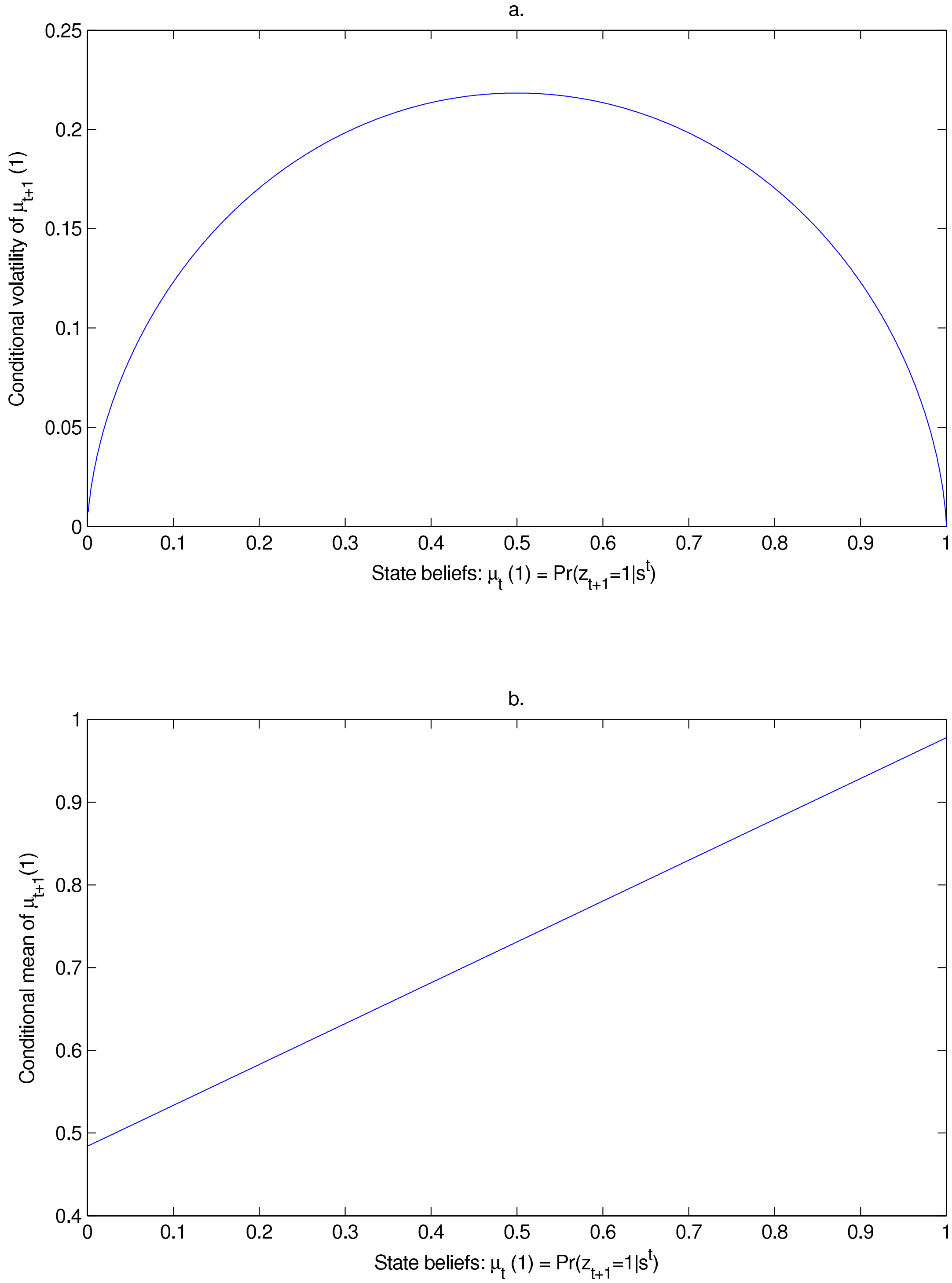
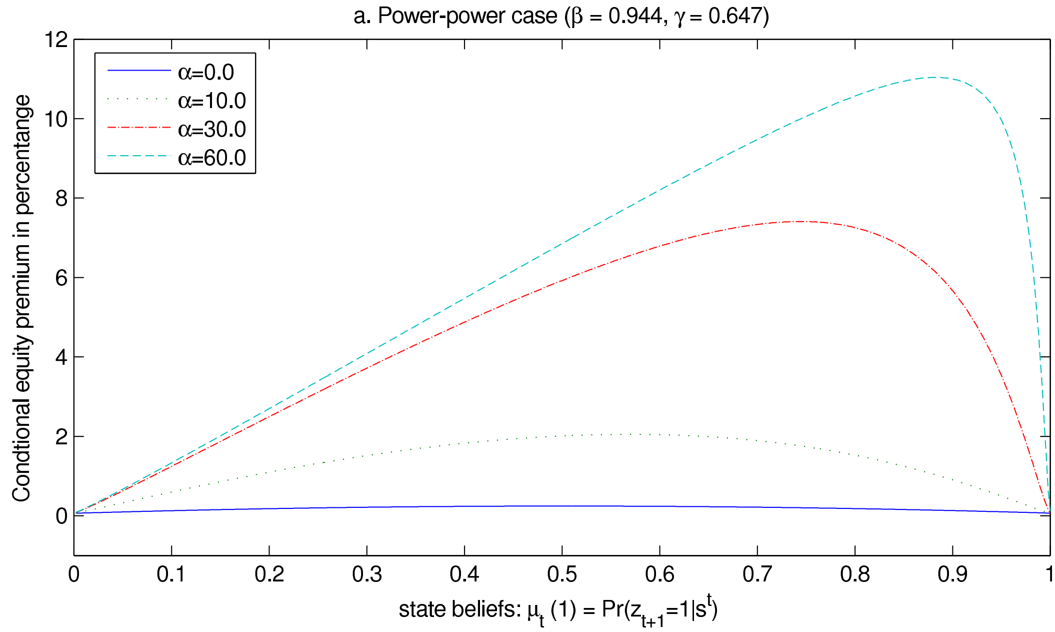


Figure 2: **Conditional mean and volatility of the probability of the high-growth state in the next period as functions of the current state beliefs.**



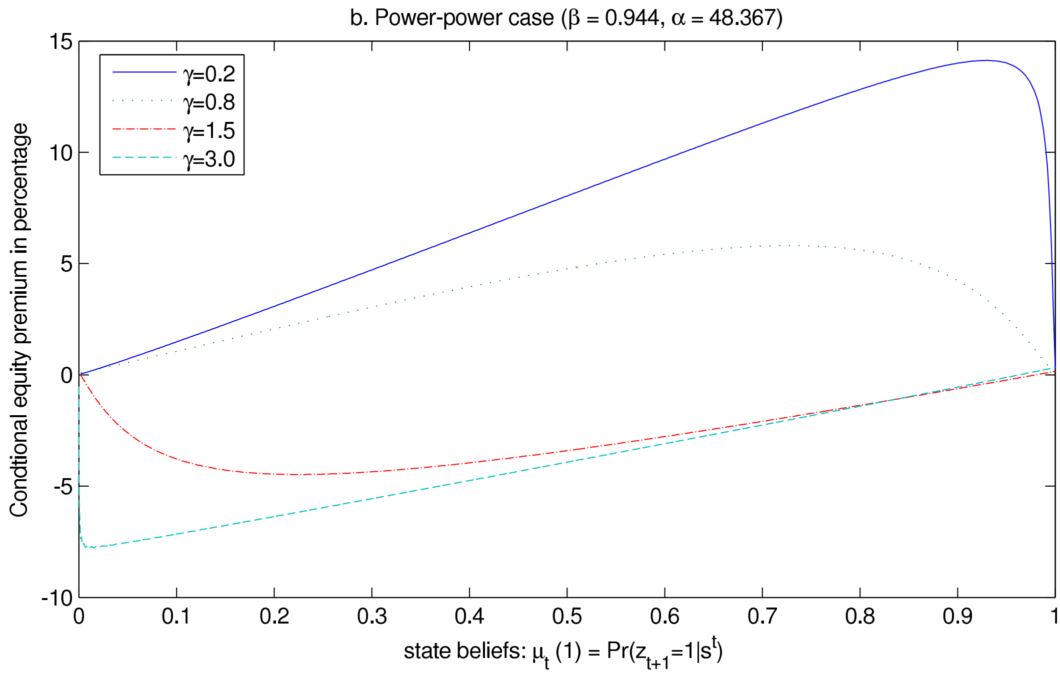
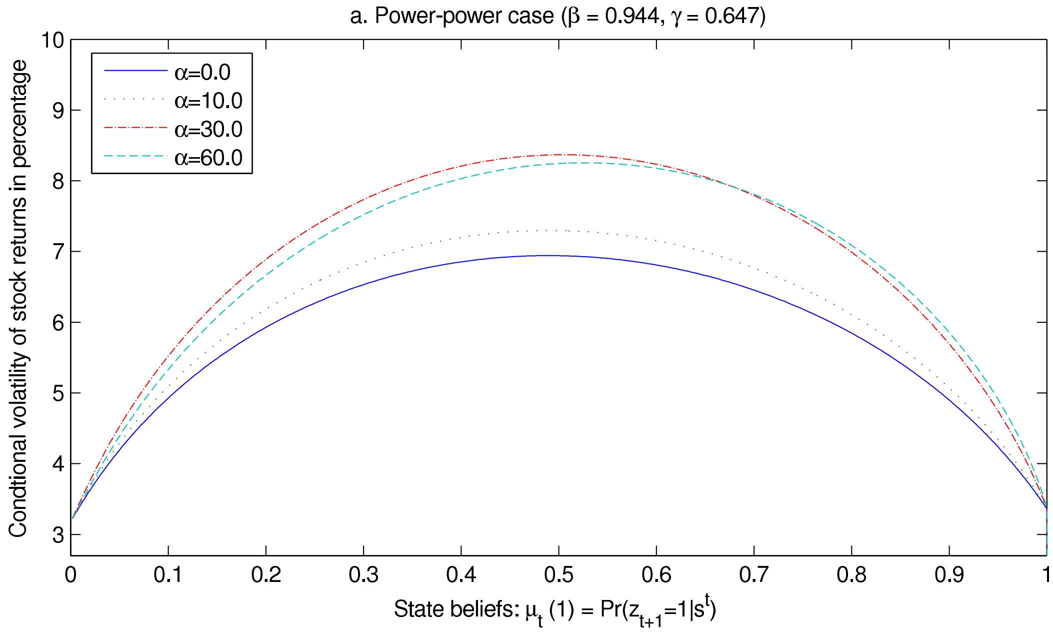


Figure 3: **Conditional expected equity premium as a function of the beliefs about the high-growth state.** Panel a plots this function for different values of the ambiguity aversion parameter α. Panel b plots this function for different values of the risk aversion parameter γ.



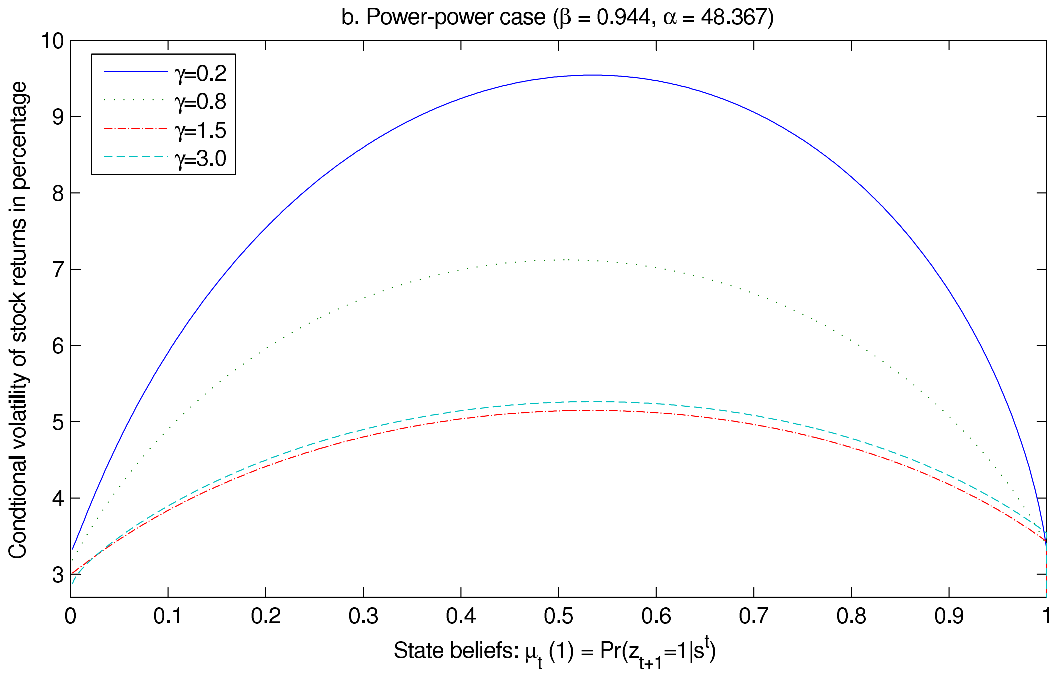


Figure 4: **Conditional volatility of stock returns as a function of the beliefs about the high-growth state.** Panel a plots this function for different values of the ambiguity aversion parameter α. Panel b plots this function for different values of the risk aversion parameter γ.

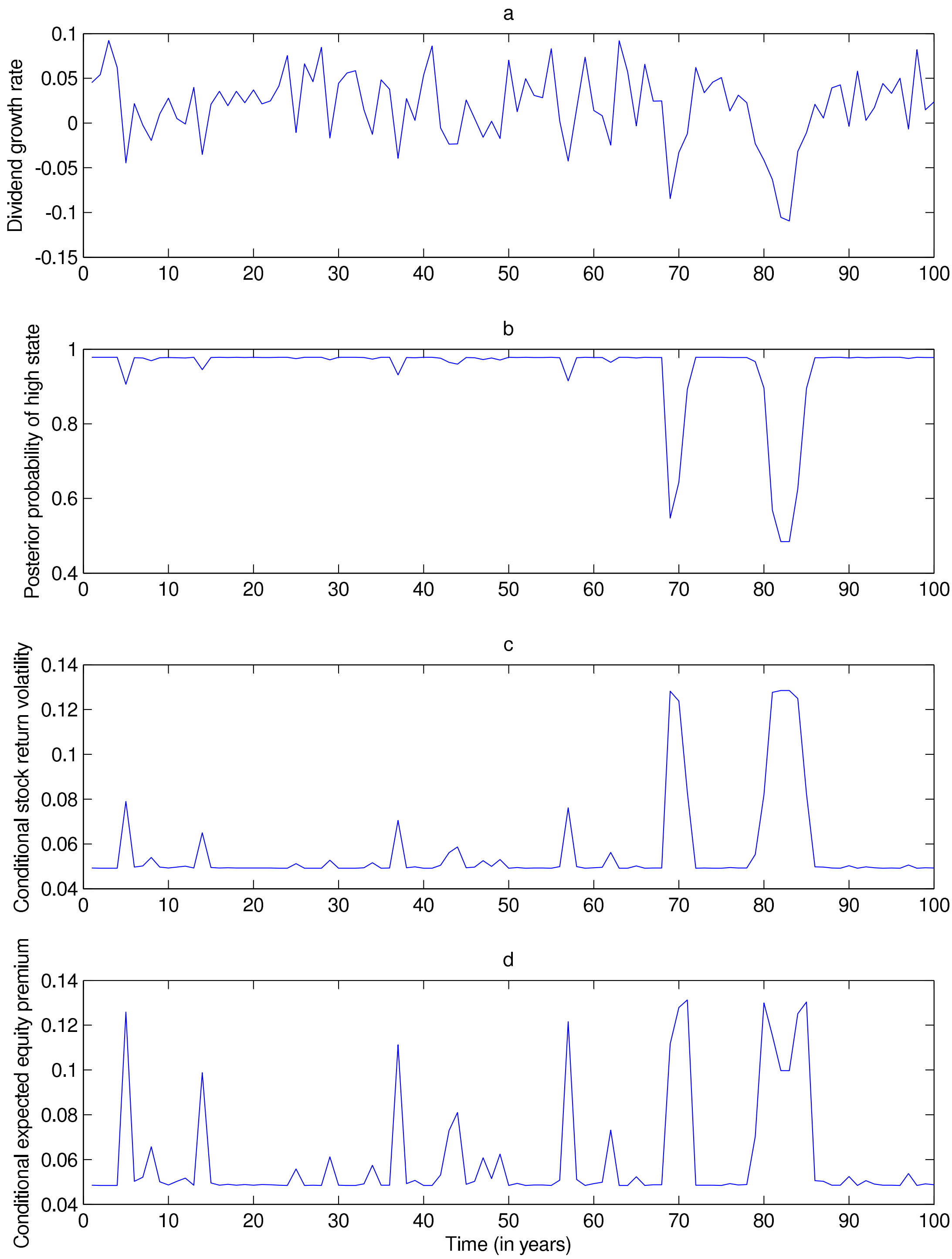


Figure 5: **Simulated time series of dividend (consumption) growth, posterior probability of the high-growth state, conditional volatility of stock returns, and conditional expected equity premium.** Parameter values are set as the baseline values given in Table 4.

Conclusion

- We analyze the quantitative asset pricing implications of a new utility model by studying two tractable parametric specifications
- Ambiguity aversion plays a key role which is different from risk aversion
- Learning under ambiguity generates sizable equity premium and significant cyclical return dynamics

New Developments and Future Research

- How to estimate ambiguity aversion parameter? Miao and Qu (2008)
- Portfolio choice under ambiguous return predictability. Chen, Ju and Miao (2008)
- Asset pricing with ambiguous rare disasters. Chen, Ju, and Miao (2008)
- Production economy? Chen, Ju and Miao (2008)
- Heterogeneity?
- Intertemporal substitution, risk aversion, and ambiguity aversion? Hayashi and Miao (2008)
- Continuous time? Skiadas (2008)