Disasters and Recoveries

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Twenty years ago, Thomas A. Rietz (1988) showed that infrequent, large drops in consumption make the theoretical equity premium large. Recent research has resurrected this 'disaster' explanation of the equity premium puzzle. Robert J. Barro (2006) measures disasters during the XXth century, and finds that they are frequent and large enough, and stock returns low enough relative to bond returns during disasters, to make this explanation quantitatively plausible. Xavier Gabaix (2007) extends the model to incorporate a time-varying incidence of disasters, and he argues that this simple feature can resolve many asset pricing puzzles.

These papers make the simplifying assumption that disasters are permanent. Mathematically, they model log consumption per capita as the sum of a unit root process and a Poisson jump. However a casual look at the data suggests that disasters are often followed by recoveries. The first contribution of this paper is to measure recoveries (Section I) and introduce recoveries in the Barro-Rietz model (Section II). I find that the effect of recoveries hinges on the intertemporal elasticity of substitution (IES): when the IES is low, recoveries may increase the equity premium implied by the model; but when it is high, the opposite happens.

Empirical research in finance documents that stock returns and excess stock returns are forecastable. A second contribution of the paper is to examine if the disaster model can also account for these facts (Section III).
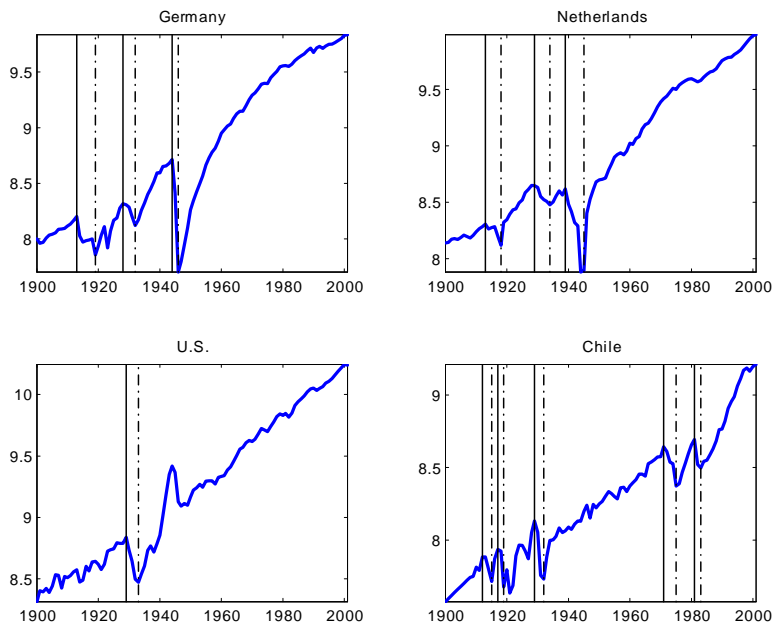
Figure 1: Log GDP per capita (in 1990 dollars) for four countries: Germany, Netherlands, the U.S. and Chile. The disaster start (resp.end) dates are taken from Barro (2006), and are shown with a vertical full (resp. dashed) line.

1   Measuring Recoveries

Figure 1 plots log GDP per capita for four countries (Germany, Netherlands, the U.S., and Chile). The vertical full lines indicate the start of disasters, and the vertical dashed lines the end of disasters, as defined by Barro.[1] In many cases, GDP bounces back just after the end of the disaster, as predicted by the neoclassical growth model following a capital destruction or a temporary decrease in productivity.

To quantify the magnitude of recoveries, Table 1 present some statistics using the entire sample of disasters[2] identified by Barro. Barro measures disasters as the total decline in GDP from peak to through. Using 35 countries, he finds 60 episodes of GDP declines greater than 15% during the XXth century. Because the end of the disaster is the trough, this computation implies that GDP goes up following the disaster. The key question is, How much?

The first column reports the average across countries of the cumulated growth, in each of the

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[1] The data is from Maddison (2003).

[2] Except the most recent episodes in Argentina, Indonesia and Urugay, for which the next five years of data are not yet available.

In percent	All disasters (57 events)		Disaster greater than 25% (27 events)	
Years after Trough	Growth from Trough	Loss from previous Peak	Growth from Trough	Loss from previous Peak
0	0	-29.8	0	-41.5
1	11.1	-22.8	16.1	-32.7
2	20.9	-16.8	31.3	-24.2
3	26.0	-13.7	38.6	-20.4
4	31.5	-10.2	45.5	-16.9
5	37.7	-6.1	52.2	-13.4

Table 1: Measuring Recoveries. The table reports the average of (a) the growth from the trough to 1,2,3,4,5 years after the trough and (b) the difference from the current level of output to the previous peak level, for 0,1,2,3,4,5 years after the trough.

first five years following a disaster. The average growth rate is 11.1% in the first year after a disaster, and the total growth in the first two years amounts to 20.9%. This is of course much higher than the average growth across these countries over the entire sample, which is just 2.0%. The second column computes how much of the 'gap' from peak to trough is resorbed by this growth, i.e. how much lower is GDP per capita compared to the previous peak. At the trough, on average GDP is 29.8% less than at the previous peak. But on average, this gap is reduced after three years to 13.7%.

Of particular interest are the larger disasters, because diminishing marginal utility implies that people care enormously about them. Columns 3 and 4 replicate these computations for the subsample of disasters larger than 25%. These disasters are also substantially reversed, because the average growth in the first two years after the disaster is over 30%. Measuring disasters and recoveries certainly deserves more study, but it seems clear that the iid assumption is incorrect: growth is substantially larger after a disaster than unconditionally. (However, recoveries might be less strong for consumption than for GDP if people are able to smooth consumption during disasters.)

2 A Disaster model with Recoveries and Epstein-Zin utility

How do recoveries affect the predictions of the disaster model? To study this question, I extend the Barro-Rietz model and allow for recoveries. The consumption process in the Barro-Rietz model is:

$$
(1) \qquad \Delta \log C_t \ = \ \mu + \sigma \varepsilon_t, \text{ with probability } 1 - p,
$$

$$
= \ \mu + \sigma \varepsilon_t + \log(1 - b), \text{ with probability } p,
$$

where ε_t is iid $N(0,1)$. Hence, each period, with probability p, consumption drops by a factor b. The realization of the disaster is iid and independent of ε_t at all dates. To allow for recoveries, I modify this process as follows: if there was a disaster in the previous period, then, with probability π, consumption goes back up by an amount $- \log(1-b)$. (In Gourio (2007) I allow for more complex dynamics.) For reasons that will become clear, it is also useful to extend the model and allow for Epstein-Zin preferences. When $\pi = 0$, we obtain the Barro-Rietz model, and the log equity premium is given by the formula:

$$
(2) \qquad \log \frac{ER^e}{R^f} = \sigma^2 \theta + \log \left(\frac{\left(1 - p + p(1-b)^{-\theta}\right)\left(1 - p + p(1-b)\right)}{1 - p + p(1-b)^{1-\theta}} \right),
$$

where θ is the coefficient of risk aversion. When $p = 0$ or $b = 0$, we obtain the well-known formulas of the lognormal iid model, which generate an equity premium puzzle (and a risk-free rate puzzle). When $p > 0$ and $b > 0$, the equity premium is increasing in the probability of disasters p and in their size b (assuming p is small). Because risk-averse agents fear large changes in consumption, a small probability of a large drop of consumption can make the theoretical equity premium large.

When $\pi > 0$, there is no useful closed form solution, but it is easy to solve the model numerically. I use the same parameter values as Barro, except for the intertemporal elasticity of substitution α, for which I consider a range of possible values. In particular, I use the historical

distribution of disasters b instead of a single value. I also follow Barro and assume that government bonds default with probability 0.4 during disasters, and that the recovery rate is $1 - b$. With these parameter values, the equity premium is 0.18% without disasters and 5.6% with disasters and no government defaults, and finally 3.5% with disasters and government defaults. Importantly, this result is influenced by the largest historical disasters: if we use exclude from the distribution of b the ten disasters larger than 40% (which all occurred during World War II), the equity premium is reduced to 0.8%.

Table 2 shows the (log geometric unconditional) equity premium, as a function of the probability of a recovery, for four different elasticities of substitution: 1/4 (Barro's number), 1/2, 1 and 2. In this computation, the risk aversion θ is kept constant equal to 4. Note that the four lines intersect for $\pi = 0$ since in this case, consumption growth is *iid*, and the IES does not affect the equity premium. Perhaps surprisingly, when the IES is low, the equity premium is increased by the possibility of a recovery. To understand this result, it is useful to recall the present-value identity in the case of power utility. The price of a claim to $\{C_t\}$ is

$$(3) \qquad \frac{P_t}{C_t} = E_t \sum_{k \geq 1} \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{1-\alpha},$$

hence the fact that a recovery may arise, i.e. that $C_{t+1}, C_{t+2}, ...$, is higher than would have been expected without a recovery, can increase or decrease the stock price today, depending on whether $\alpha > 1$ or $\alpha < 1$. The intuition is that good news about the future have two effects: on the one hand, they increase future dividends (equal to consumption), which increases the stock price today (the cash-flow effect), but on the other hand they increase interest rates, which lowers the stock price today (the discount-rate effect). The later effect is stronger when interest rates rise more for a given change in consumption, i.e. when the intertemporal elasticity of substitution (IES) is low. Given a low IES, the price-dividend ratio falls more when there is a possible recovery than when there are no recoveries. This in turn means that equities are more risky ex-ante, and as a result

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Probability of a recovery π	0.00	0.30	0.60	0.90	1.00
IES = 0.25	3.31	4.62	5.91	7.19	7.64
IES = 0.50	3.31	3.30	3.03	2.26	1.68
IES = 1	3.31	2.69	1.94	1.00	0.54
IES = 2	3.31	2.42	1.52	0.63	0.30

Table 2: Unconditional log geometric equity premium, as a function of the intertemporal elasticity of substitution (IES) and the probability of a recovery. This table sets risk aversion 4 and the other parameters as in Barro (2006).

the equity premium is larger.

When the IES is not low however, recoveries reduce the equity premium. The intuition is that the decrease in dividends is transitory and thus in disasters stock prices fall by a smaller amount than dividends do, making equities less risky than in the iid case. These results are consistent with the literature on autocorrelated consumption growth and log-normal processes (John Y. Campbell (1999), Ravi Bansal and Amir Yaron (2004)). While Bansal and Yaron emphasize that the combination of *positively* autocorrelated consumption growth and an IES *above* unity can generate large risk premia, Campbell shows that when consumption growth is *negatively* autocorrelated, risk premia are *larger* when the IES is below unity. Recoveries induce negative serial correlation, so even though Campbell's results do not directly apply (because the consumption process is not lognormal), the intuition seems to go through.

Of course, there is no clear agreement on what is the proper value of the IES. The standard view is that it is small (e.g. Robert Hall (1988)), but this has been challenged by several authors (see among others Bansal and Yaron, Casey Mulligan (2004) and Fatih Guvenen (2006)). How then, can we decide which IES is more reasonable for the purpose of studying recoveries? The natural answer is to use data on asset prices during disasters. A low IES implies huge interest rates following a disaster if consumers anticipate a recovery, while a high IES implies moderately high interest rates, and a small increase in the P-D during disasters. In the data, interest rates are not huge, but the P-D ratio tends to fall during disasters, though not necessarily by a large

amount. Hence, it is not clear which model fits the data best. More fundamentally, in the model, disasters are instantaneous while they are more gradual in the data, making it difficult to find an empirical counterpart to asset prices right after the disaster.

3   Return Predictability in the Disaster model

Given the success of the disaster model in accounting for the risk-free rate and equity premium puzzles, it is important to study if the model can also account for additional asset pricing facts. Empirical research documents that both the stock return and the excess stock return is forecastable. The basic regression is

$$(4) \qquad R_{t+1}^e - R_{t+1}^f = \alpha + \beta \frac{D_t}{P_t} + \varepsilon_{t+1},$$

where R_{t+1}^e is the equity return and R_{t+1}^f the risk-free return. As an illustration, John Cochrane (2007) reports for the annual 1926-2004 U.S. sample: $\beta = 3.83$ (t-stat $= 2.61$, $R^2 = 7.4\%$). A key feature of the data is that using as the left-hand side the equity return R_{t+1}^e rather than the excess return $R_{t+1}^e - R_{t+1}^f$ does not change the results markedly: $\beta = 3.39$ (t-stat $= 2.28$, $R^2 = 5.8\%$).

To generate variation in expected returns over time, we need to introduce some variation over time in the riskiness of stocks. The natural idea is to make the probability of disaster-time varying. Hence, consider the following environment: there a representative agent who has CRRA utility with risk aversion α. The disaster probability changes over time according to a monotone first-order Markov process, governed by the transition probabilities $F(p_{t+1}|p_t)$, where p_t is the probability of a disaster at time $t + 1$, which is drawn at time t. Formally, $\Delta \log C_{t+1} = \mu + \sigma \varepsilon_{t+1}$ with probability $1 - p_t$, and $\Delta \log C_{t+1} = \mu + \sigma \varepsilon_{t+1} + \log(1 - b)$ with probability p_t. Assume that the realization of p_{t+1} is independent of the realization of disasters at time $t + 1$, conditional on p_t. (This simplification allows to obtain an analytical solution; it implies that the P-D ratio is conditionally uncorrelated with current dividend growth.) The following result is easy to prove:

Proposition: If the probability p_t is always small, then (1) the risk-free rate and expected equity return are decreasing in p_t, (2) the equity premium is increasing in p_t, (3) the P-D ratio is increasing in p_t if and only if $\alpha > 1$.

This result implies that the correlation between equity risk premia and price-dividend ratio is positive if $\alpha > 1$ and negative (as in the data) if $\alpha < 1$. Intuitively, an increase in p reduces expected growth, which increases the incentives to save, leading to a reduction in the risk-free rate and the expected equity return. Because the risk of disaster is higher, the risk premium increases, but by less than the change in the risk-free rate. The P-D ratio may go up or down, depending on whether the change in expected return is larger than the change in expected dividend growth. This depends on the strength of the interest rate response, and thus on the IES.

This result creates a problem for the simplest model of disasters. First, while most researchers use $\alpha > 1$, this generates a counterintuitive *positive* correlation between the P-D ratio and disaster probability, and this implies that a high P-D ratio forecasts a smaller risk premium, which is the inverse of the data. But using $\alpha < 1$ reduces the risk premium (and also implies that recoveries reduce the equity premium). More fundamentally, there are *no* parameter values which will generate both the stock return *and* excess stock return predictability.

The natural escape route is to separate the IES and risk aversion, and to use the IES to control movements in the risk-free rate. When the disaster probability is *iid*, i.e. $F(p_{t+1}|p_t) = F(p_{t+1})$, and risk aversion θ is greater than unity, it is possible to show that the P-D ratio increasing in the probability of a disaster p if and only if the elasticity of substitution is less than one, i.e. $\alpha > 1$. The risk premium is still increasing in the probability of disaster, but now the expected equity return need not be decreasing in the probability of disaster if $\alpha < 1$. Hence, a calibration with an IES above unity can generate qualitatively both excess return and return predictability. Numerical experiments suggest that, even if one relaxes the iid assumption, it is not easy to match the data quantitatively, however.

It is also possible to extend the result above to the case of a time-varying size of disaster b.

Formally, assume that $\Delta \log C_{t+1} = \mu + \sigma \varepsilon_{t+1}$, with probability $1 - p$, and $\Delta \log C_{t+1} = \mu + \sigma \varepsilon_{t+1} + \log(1 - b_t)$, with probability p; that b_t follows a first-order Markov process and that the realization of $\Delta \log C_{t+1}$ and b_{t+1} is independent conditional on b_t. Then, the proposition above holds where b_t replaces p_t. Gabaix resolves this tension by assuming that the size of dividends disaster changes over time, but not the size of consumption disasters. There may be other resolutions of this conundrum, but they remain to be worked out.

4   Conclusions

The disaster explanation of asset prices is attractive on several grounds: there are reasonable calibrations which generate a sizeable equity premium. Disasters can easily be embedded in standard macroeconomic models. Moreover, the explanation is consistent with the empirical finance literature which documents deviations from log-normality. Inference about extreme events is hard, so it is possible that investors' expectations do not equal an objective probability. But precisely because the disaster explanation is not rejected on a first pass, we should be more demanding, and study if it is robust and if it can account quantitatively for other asset pricing puzzles. The current paper points toward some areas which would benefit from further study.[3]

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[3] Gourio (2007) studies the implications of the disaster model for the cross-section of expected stock returns.

References

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