A Cross-Sectional Test of The Disaster Model

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Issue

- In the Arrow-Debreu framework, stock prices must satisfy:

$$p_t = E\left(\frac{\beta U'(C_{t+1})}{U'(C_t)}(d_t + p_{t+1})\right) \quad (1)$$

- **The risk premium puzzle** (Mehra-Prescott (1985)):
  1. Historically, the average return on the U.S stock market over the risk free rate (T-Bill) has been 7.4%.
  2. In a Lucas economy with CRRA utility, this premium cannot be explained with conventional level of risk aversion (relative risk aversion below 10).
  3. In other words, securities prices or the risk free rate appear are too low in reality to be explained by standard asset pricing models.....
- $\Longrightarrow$ Huge amount of research to explain the puzzle (limited stock market participation, incomplete markets etc...)

The disaster model

- **Rietz (1988):**
  1. The process for aggregate consumption considered in Mehra and Prescott (1988) does not allow for "*disasters*": a rare and sharp fall in aggregate consumption.
  2. The risk premium puzzle disappears if one accounts for such rare events.
- **Barro (2006); Barro and Ursua (2007):**
  1. Calibrate the disaster model using the historical frequency of disasters in the 20th century (disasters=large fall in real consumption).
  2. Show that with this calibration, the model can generate a risk premium in the ballpark of the observed risk premium and low expected bill rates.

Overview of the disaster model

- **Disaster Model means**: The process for aggregate consumption is:

$$\frac{C_{t+1}}{C_t} = \begin{array}{l} e^{\mu+\sigma\epsilon_t} \text{ with probability (1-p)-no disaster} \\ e^{\mu+\sigma\epsilon_t+\log(1-b)} \text{ with probability p-disaster} \end{array} \tag{2}$$

with ϵ_t i.i.d and normally distributed.

- **Two sources of risk**: (i) "normal" (ϵ_t) and (ii) disaster.
- With a power utility function + representative agent, the Euler equation (1) implies:

$$\log\left(\frac{E(R_m)}{R_f}\right) = \underbrace{\gamma\sigma^2}_{\text{Risk Premium Without Disaster}} + \underbrace{f(p, b, \gamma)}_{\text{Disaster Risk Premium}}$$

where R_m is the return on the market portfolio.

Cross-sectional asset pricing implication

- **If the disaster model is correct then cross-sectional variations in assets payoffs come from:**
  1. Variations in exposure to normal risk (λ_i)
  2. Variations in exposure to disaster risk (η_i)
- **BUT** how to model stock dividends in the disaster model?
- Assets with relatively low exposure to disaster risk helps to "hedge" this risk $\Longrightarrow$ They should have a smaller expected return, other things equal. Formally, the risk premium for security i:

$$\log\left(\frac{E(R_i)}{R_f}\right) = \lambda_i \gamma \sigma^2 + f(p, b, \gamma, \eta_i)$$

Measuring a stock exposure to disaster

- **Problem:** how to measure exposure to disaster risk (η_i)?
- Using the theory, Gourio (2007) shows that this exposure can be measured empirically by estimating β_{id} in the following regression:

$$Log(\frac{R_{it+1}}{R_{ft+1}}) = \alpha_i + \beta_{id}(R_{t+1m} - R_{t+1f}) * 1_{\{R_{t+1m}-R_{t+1f}<u\}} + \epsilon_{t+1}$$

- This equation is also a way to test a "disaster CAPM" in which the role of the market portfolio is played by the factor $(R_{t+1m} - R_{t+1f}) * 1_{\{R_{t+1m}-R_{t+1f}<u\}}$:

$$Log(\frac{R_{it+1}}{R_{ft+1}}) = \beta_{id}E(R_{t+1m} - R_{t+1f}) * 1_{\{R_{t+1m}-R_{t+1f}<u\}})$$

Findings

- This model does not explain the cross-sectional variation of stock returns better than the CAPM $\Longrightarrow$ Does not explain the standard asset pricing anomalies (e.g., book-to market or size).
- "Disaster βs" are higly correlated with CAPM betas.
- Same findings with a CCAPM.
- $\Longrightarrow$The disaster model solves the risk premium puzzle but not other asset pricing anomalies...

Questions/Comments 1/2

- The paper shows that β_{id} is a proxy for η_i but it is not entirely clear that it implies the single factor structure implicit in the "disaster CAPM". Can this be shown? **Does the disaster model imply that the stochastic discount factor can be written:**

$$m_t = 1 - b(R_{t+1m} - R_{t+1f}) * 1_{\{R_{t+1m} - R_{t+1f} < u\}}$$

- **The choice of u seems key to measure the exposure to disaster risk?**
- **How should the dividend process for individual securities be specified in the "disaster model"?** In reality the correlation between aggregate consumption and dividends is not too high (cf Campbell (2003) or Bansal and Yaron (2004)) as corporate profits account for a small portion of national income. **Why not use Gabaix (2007)'s specification?**

Questions/Comments 2/2

- **In reality, do growth (resp. large) stocks fare better than value (resp. small) stocks during disasters?**
- **On the "disaster model"**
  1. The disaster model postulates a specific stochastic process for aggregate consumption. Is it consistent with the data? How does it perform compared to other possible processes for aggregate consumption?(e.g. Fisher and Calvet and (JFE-2007, JME-2008))
  2. There are other specifications of the stochastic process for aggregate consumption and dividends that solve the risk premium puzzle (e.g., Bansal and Yaron (2004, JOF). How do they differ from the disaster model? Is the disaster model a better description of the process for aggregate consumption?