L-Performance with an Application to Hedge Funds

Serge Darolles, Christian Gouriéroux, Joann Jasiak

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1. Tail Risk and Financial Markets

- Don't risk using Gaussian Distribution
- The particular case of Hedge Fund investing
- Tail risk beyond financial markets: A transdisciplinary outlook

Don't risk using Gaussian Distribution

- Most of risky decisions are taken using Gaussian Distribution
- *Volatility* is used as the unique measure of risk
  - Volatility is easy to explain: *just the dispersion around the mean*

$$\sigma = E[(X - E(X))^2]^{\frac{1}{2}}$$

  - Volatility determines a gaussian distribution implicitely used in many financial models
- But the Gaussian distribution assumption has *severe drawbacks*
  - Underestimates the likelihood of extreme maket moves
  - Large volatility does not mean large losses

Don't risk using Gaussian Distribution

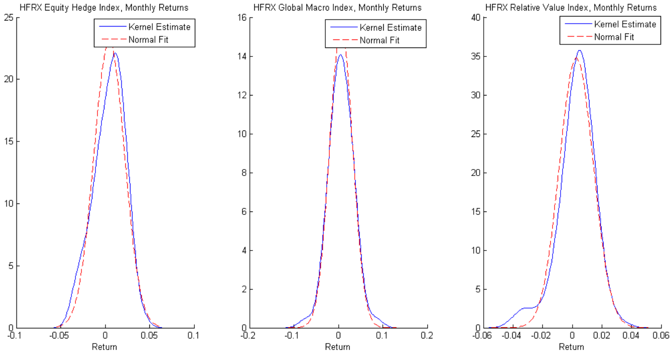
- *Key for measuring extreme risk:* ignores every day's ups and downs to concentrate on large losses
- Example of capital reserves and rainy days accounts
  - Normal distribution forecasts are too low
  - Conditional VAR measures the typical loss to a portfolio in turbulant market
- Today, professional investors rely on non-normal estimates of extreme risk
  - To manage the risk of an existing portfolio ...
  - ... but also to allocate funds, select securities and measure performance
- *This paper* applies extremes theory to the specific field of performance measure

The particular case of Hedge Fund investing

- Hedge Funds are (offshore/not regulated) investment vehicules with
  - *few investment constraints:* leverage and short selling are common practices
  - *low transparency:* we only get historical returns
  - *low liquidity:* size of the historical sample path is low
- Example of Hedge Funds strategies
  - **Equity Hedge** Long/Short Equity, Market Neutral, ...
  - **Relative Value** Fixed Income arb., Volatility arb., Convertible arb., ...
  - **Global Macro** Directional bets on global markets

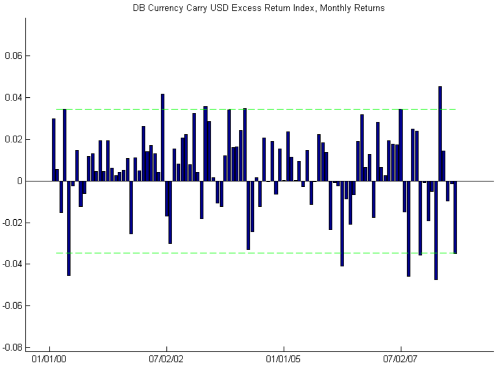
The particular case of Hedge Fund investing

- Returns of Hedge Funds indices

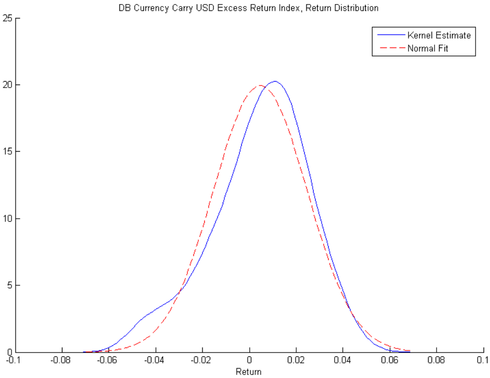


The particular case of Hedge Fund investing

- Returns of a synthetic FX carry trade strategy
  - Long currencies with high interest rate
  - Short currencies with low interest rate
  - Risk is on shocks on the FX rate

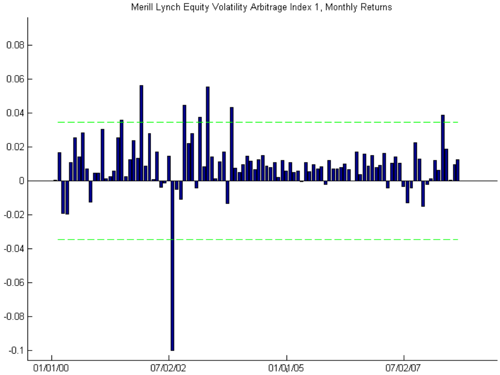


The particular case of Hedge Fund investing

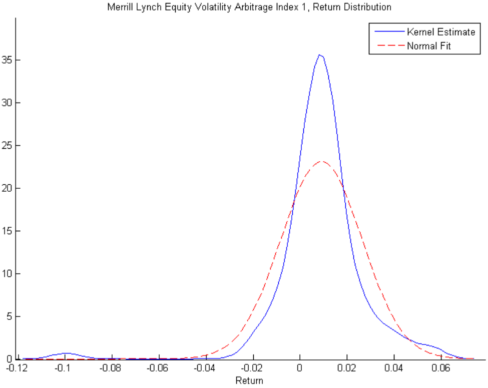


The particular case of Hedge Fund investing

- Returns of a synthetic volatility arbitrage strategy: selling covered call options
  - Long implied volatility (cash in the call premium)
  - Short historical realized volatility
  - Risk is on positive shocks on the historical realized volatility



The particular case of Hedge Fund investing



The particular case of Hedge Fund investing

- *Consequence on performance measures*
  We cannot use the Sharpe ratio and we do not want to use volatility as the risk measure

$$SR = \frac{Mean}{\sigma}$$

- Alternative ratios
  Mean/VaR(5%) [see e.g. Dowd (1999), Gregoriou, Gueyle (2003)]
  TVaR(5%)/VaR (5%) where TVaR denotes the so-called Tail-VaR [see e.g. Gourieroux, Liu (2007)]

- Unified framework for performance measure taking into account extreme risk

Tail risk beyond financial markets: A transdisciplinary outlook

- Tail risk is not specific to finance
- New risks are emerging due to social, environmental and economic development
- Characterized by extreme nature
  Frequency and gravity of these events have increased over recent decades
- Assessment of risk management by insurance companies

Tail risk beyond financial markets: A transdisciplinary outlook

- Beyond finance, a scientific literature exists on the statistical modelling of catastrophic events such
  - **Extreme floods**
    Hosking and Wallis (1987), Parameter and Quantile Estimation for the Generalized Pareto Distribution, Technometrics
  - **Extreme wind speed**
    Whalen et al. (2004), An Evaluation of the Self-Determined Probability Weighted Moment Method for Estimating Extreme Wind Speeds, Journal of Wind Engineering and Industrial Aerodynamics
  - **Extreme traffic volume in computer networks**
    Hosking (2007), Some Theory and Practical Uses of Trimmed L-Moments, Journal of Statistical Planning and Inference

Tail risk beyond financial markets: A transdisciplinary outlook

- Example of extreme floods
  - Flows by locations
  - Few extreme observations for each location
- Can we use homogeneity accross locations to model extreme events ?
  - First, build homogeneous set of locations
  - Second, calibrate a distribution on all observations of this homogeneous set

Tail risk beyond financial markets: A transdisciplinary outlook

- **Solutions proposed:** Use of L moments instead of the conventional moments definition
- We are aware of only two published applications of L-moments in finance
  - *Karvanen (2006), Estimation of Quantile Mixtures via L-moments and Trimmed L-Moments, Computational Statistics and Data Analysis*
  - *Gouriéroux and Jasiak (2008), Dynamic Quantile Model, Journal of Econometrics, forthcoming*

2. L-Moments

- Limits of the conventional moments definition
- L-moments of probability distributions
- L-moments and order statistics
- Trimmed L-moments of order one/two

Limits of the conventional moments definition

- Probability distribution X traditionally described by the moments of the distribution

$$\mu_1 = E[X], \quad \mu_r = E[(X - E(X))^r], r = 2, 3, 4$$

- Volatility is the square root of μ_2
- Skewness and Kurtosis as ratios of conventional moments

$$\gamma = \mu_3/\mu_2^{3/2}, \quad \kappa = \mu_4/\mu_2^2$$

- Simple estimators $\hat{\mu}_r$, $\hat{\gamma}$ and $\hat{\kappa}$ are given by the empirical counterpart of these moments

Limits of the conventional moments definition

- Some bad properties
  - $\hat{\gamma}$ and $\hat{\kappa}$ are severely biased (Wallis and al. (1974))
  - Bounds on $\hat{\gamma}$ and $\hat{\kappa}$ that depends on the sample size

$$\hat{\gamma} \leq n^{1/2} \qquad \hat{\kappa} \leq n+3$$

- For a sufficiently skewed distribution, impossible to get this skew in the sample counterpart
- Example of a 2-parameter lognormal distribution with $\sigma = 1$
  - $\gamma = 6.91$
  - $\hat{\gamma} \leq 4.41$ if $n = 20$

L-moments of probability distributions

- Alternative systems to describe the shape of distributions
- Modification of the *Probability Weighted Moments* (PWM) introduced by Greenwood et al. (1979)

$$M_{p,r,s} = E[X^p(F(X))^r(1 - F(X))^s]$$

- Two useful special cases

$$\alpha_r = \int_0^1 Q(u)(1-u)^r du \quad \beta_r = \int_0^1 Q(u)u^r du$$

where Q is the quantile function of X ($Q = F^{-1}$)

- Can be compared to $E[X^r] = \int_0^1 Q(u)^r du$
- But difficult to interpret as a measure of the scale and shape of a distribution

L-moments of probability distributions

- Only some linear combinations of the PWM carry interesting information
  a. $\alpha_0 - 2\alpha_1$ on the scale of the distribution
  b. $6\beta_2 - 6\beta_1 + \beta_0$ on the skewness of the distribution
- Hosking (1990) defined the L-moments of a distribution X as

$$\lambda_r = \int_0^1 Q(u) P_{r-1}(u) du$$

where $P_r(u)$ is a set of polynomials of degree r in u, satisfying the orthogonality condition (*Shifted Legendre polynomials*)

$$\int_0^1 P_r(u) P_s(u) du = 0 \;\; if \;\; r \neq s$$

L-moments and order statistics

- *An intuitive justification for L-moments* linear combination of the data arranged in ascending order (order statistics)
- Let $X_{i:r}$ refers to **a conceptual random sample**, where r is the sample size and i the rank in the ordered sample
- **Remark 1** No relation a priori between the size of the conceptual random sample and the number of available observations
- **Remark 2** Link between the ordered statistics and the *PWM*

$$E[X_{i:r}] = \frac{n!}{(r-1)!(n-r!)} \int_0^1 Q(u) u^{r-1} (1-u)^{n-r} du$$

L-moments and order statistics

- *Conceptual sample of size 1*
  Distribution shifted toward the right $\Longrightarrow$ larger values of $X_{1:1}$
  $X_{1:1}$ gives information about the location of the distribution

$$\lambda_1 = E[X_{1:1}]$$

- *Conceptual sample of size 2*
  Distribution concentrated around the central observation $\Longrightarrow$ $X_{2:2}$ and $X_{1:2}$ will be close together
  $X_{2:2} - X_{1:2}$ is a measure of the scale of the distribution

$$\lambda_2 = \frac{1}{2}E[X_{2:2} - X_{1:2}]$$

L-moments and order statistics

- *Conceptual sample of size 3*
  Symmetric distribution $\Longrightarrow$ extremes will be equidistant to the central observation

$$X_{3:3} - X_{2:3} \simeq X_{2:3} - X_{1:3} \Longrightarrow X_{3:3} - 2X_{2:3} + X_{1:3} \simeq 0$$

$$\lambda_3 = \frac{1}{3}E[X_{3:3} - 2X_{2:3} + X_{1:3}]$$

- *Conceptual sample of size 4*

$$\lambda_4 = \frac{1}{4}E[X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}]$$

- Linear combinations of order statistics (*L-statistics*)

Trimmed L-moments of order one

- Trimmed L-Moments first introducted by Elamir and Seheult (2003) as a generalization of the L-moments
- Increase the conceptual sample size to trim extreme values
- In the case of the moment of order one, the sample size increases from 1 to $2n+1$
- The definition of the $n-$Trimmed L-moment of order 1 is then

$$\lambda_{1,n} = E(X_{n+1:2n+1}), \ n \geq 0$$

- Equal to the expectation of the **median of the conceptual sample**

Trimmed L-moments of order one

- Some interesting properties
  - More robust than L-moments
  - Exist even if the distribution does not have a mean (*Cauchy distribution*)

$$E(|X|^{1/(n+1)}) < \infty$$

  - Functional expression involving polynomials and the quantile function

$$\lambda_{1,n} = \int_0^1 Q(u)\frac{(2n+1)!}{(n!)^2}u^n(1-u)^n du$$

  - Bridge the mean and the median

$$\lambda_{1,0} = \int_0^1 Q(u)du = EX \qquad \lambda_{1,\infty} = \lim_{n\to\infty}\lambda_{1,n} = Q(0.5)$$

Trimmed L-moments of order two

- We introduce a new definition of the $(r, n)-$Trimmed L-moment of order 2

$$\lambda_{2,r,n} = E(\tilde{X}_{2n-r+1:2n+1} - \tilde{X}_{r+1:2n+1}),\ 0 \leq r \leq n-1.$$

- Expected range of a conceptual sample after deleting the r smallest and r largest conceptual observations
- Extends the usual definition which corresponds to the special case $r = n - 1$
- *Financial applications* Preferable to choose a value of r much smaller than $n - 1$ to better capture extreme risks

Trimmed L-moments of order two

- Analytical expression of the trimmed L-moment of order 2

$$\lambda_{2,r,n} = \int_0^1 Q(u)\frac{(2n+1)!}{r!(2n-r)!}[u^{2n-r}(1-u)^r - u^r(1-u)^{2n-r}]du$$

- When $n \to \infty$, $r \to \infty$, with $r/(2n+1) \to \alpha$,

$$\lambda_{2,r,n} \to Q(1-\alpha) - Q(\alpha)$$

  By decreasing $r/(2n+1)$, we reveal the differences between the right and the left tail extremes

- When $r = n-1$, $r/(2n+1) \to 1/2$ when $n \to \infty$ and

$$\lambda_{2,r,n} \to Q(1/2) - Q(1/2) = 0$$

3. L-performance measure

- Definition
- Estimation of L-moments
- Estimation of L-performance

Definition

- *By analogy with the Sharpe performance ratio* the L-performance is defined as

$$L_{r,n} = \lambda_{1,n} / \lambda_{2,r,n}$$

- Set of L-performances that depends on the shrinkage parameters r and n
- When $n \to \infty$, $r \to \infty$, with $r/(2n+1) \to 5\%$, we get

$$\lim_{r,n \to \infty} L_{r,n} = \frac{Q(50\%)}{Q(95\%) - Q(5\%)} = \frac{VaR(50\%)}{VaR(95\%) - VaR(5\%)}$$

where VaR denotes the Value at Risk computed on historical returns

Definition

- If $X = \mu + \sigma U$, where U has symmetric distribution, L-performances are proportional to μ/σ, up to a scale factor depending on r, n, and U
- If $X_i = \mu_i + \sigma_i U_i$, where U_i have the same symmetric distribution, L-performances are proportional to μ_i/σ_i
- In the particular case of Gaussian distribution, the L-performance is equivalent to the standard Sharpe performance measure

Estimation of L-moments

- Start with the analytical expression of the L-moments

$$\lambda(Q,P)=\int_0^1 Q(u)P(u)du$$

where P is a polynomial and Q the quantile function

- Can be consistently estimated by the sample counterpart

$$\hat{\lambda}_T(Q,P)=\lambda(\hat{Q}_T,P)=\int_0^1 \hat{Q}_T(u)P(u)du$$

where

$$\hat{Q}_T(u)=inf\{x:\frac{1}{T}\sum_{t=1}^{T}\mathbf{I}_{(X_t\leq x)}\geq u\}$$

is the sample quantile function

Estimation of L-moments

- This estimator is also equal to

$$\hat{\lambda}_T(Q, P) = \frac{1}{T} \sum_{t=1}^{T} x_{t:T} P(t/T).$$

- Asymptotic properties are directly obtained from

$$\sqrt{T}[\hat{Q}_T(u) - Q(u)] \Rightarrow -\int_0^1 \frac{1}{f[Q(u)]} B(u) du$$

  where B is a brownian bridge defined on $[0, 1]$

- Asymptotic normality with asymptotic variance

$$\int_0^1 \int_0^1 \frac{\min(u_1, u_2) - u_1 u_2}{f[Q(u_1)] f[Q(u_2)]} P(u_1) P(u_2) du_1 du_2$$

Estimation of L-performance

- We have

$$\hat{L}_{r,n,T} = \hat{\lambda}_{1,n,T}/\hat{\lambda}_{2,r,n,T}$$

- By the delta method

$$\sqrt{T}\left(\hat{L}_{r,n,T} - L_{r,n}\right)$$

$$= \frac{1}{\lambda_{2,r,n}}\sqrt{T}(\hat{\lambda}_{1,n,T} - \lambda_{1,n}) - \frac{\lambda_{1,n}}{\lambda_{2,r,n}^2}\sqrt{T}(\hat{\lambda}_{2,r,n,T} - \lambda_{2,r,n}) + o_P(1)$$

$$= \frac{1}{\lambda_{2,r,n}}\int_0^1 \sqrt{T}[\hat{Q}_T(u) - Q(u)][P_{1,n}(u) - L_{r,n}P_{2,n}(u)]du$$

- Finally

$$\sqrt{T}(\hat{L}_{r,n,T} - L_{r,n}) \xrightarrow{d} N(0, \eta_{r,n}^2)$$

4. Application to Hedge Funds

- The selected funds
- Fund returns distribution
- Comparison of estimated performances

The selected funds

- The Hedge Fund Research Inc. (HFR) database
  - Return data available on the period July 2004-June 2007
  - Only single strategy hedge funds (no fund of hedge funds)
- We get 3654 pure hedge funds managing $ 860 billion
- Self declared strategy describing the type of portfolio management
- We use a random set of single hedge funds in different categories

The selected funds

Ticker	Fund Name	Strategy	Fund Assets
IC EH	Ibis Capital, LP	Equity Hedge	35 000 000
PF EH	Platinum Fund Ltd.	Equity Hedge	1 009 357 000
PM EH	PM Capital Australian Opportunities Fund	Equity Hedge	19 000 000
RA EH	RAB Europe Fund	Equity Hedge	189 000 000
RB EH	Robeco Boston Partners Long/Short Equity	Equity Hedge	45 990 000
SP EH	Sprott Opportunities Hedge Fund	Equity Hedge	313 000 000
RC EH	Robbins Capital Partners	Equity Hedge	25 000 000
FS EM	Fairfield Sentry	Equity Market Neutral	6 800 000 000
I EM	Invesco QLS Equity	Equity Market Neutral	806 461 000
LC EN	Large Cap Core Equity	Equity Non-Hedge	3 063 300 000
TR EN	Thames River European Fund	Equity Non-Hedge	42 888 960
LES SS	Leveraged Short Equity Index Hedge	Short Selling	3 000 000
APM MA	APM Global Fixed Income Composite Fund	Macro	177 000 000
FX MA	FX Concepts Global Currency Program	Macro	3 600 000 000
HC MA	Haidar Jupiter International	Macro	80 387 000
ML MA	Maple Leaf Macro Volatility Fund	Macro	635 000 000
WI MA	Winton Futures Fund	Macro	4 440 000 000
DF MF	Discus Fund Limited	Managed Futures	47 989 382
CG RV	Clinton Multistrategy Fund	Relative Value Arbitrage	685 000 000
EF RV	Endeavour Fund	Relative Value Arbitrage	2 767 000 000
WE RV	Western Investment Institutional Partners	Relative Value Arbitrage	62 000 000
PI MA	Paulson International	Merger Arbitrage	3 373 000 000
CSS ED	Courage Special Situations Offshore Fund	Event-Driven	350 567 000
RO ED	Rosseau Limited Partnership	Event-Driven	157 263 000
BF FI	BlackRock Fixed Income FlobOpp Fund	Fixed Income Arbitrage	1 333 000 000
BR FI	Blue River Advantaged Muni Fund	Fixed Income Arbitrage	1 600 000 000
DM FI	Drake Global Opportunities Fund	Fixed Income Arbitrage	3 896 000 000
RCG DS	RCG Carpathia Overseas Fund	Distressed Securities	498 000 000

Fund returns distribution

- A distribution is symmetric if and only if all L-moments of order 1 are equal

Ticker	$\lambda_{1,n=0}$	$\lambda_{1,n=1}$	$\lambda_{1,n=2}$	$\lambda_{1,n=3}$	$\lambda_{1,n=4}$	$\lambda_{1,n=5}$	$\lambda_{1,n=\infty}$
CSS ED	0.499%	0.436%	0.615%	0.448%	0.450%	0.452%	0.443%
DF MF	2.344%	2.138%	2.994%	2.174%	2.182%	2.187%	2.256%

- For CSS ED and DF MF, the distribution is almost symmetric, but skewed in general
- Comparing mean and median can be misleading (see IC EH and FX MA)

Ticker	$\lambda_{1,n=0}$	$\lambda_{1,n=1}$	$\lambda_{1,n=2}$	$\lambda_{1,n=3}$	$\lambda_{1,n=4}$	$\lambda_{1,n=5}$	$\lambda_{1,n=\infty}$
FX MA	0.859%	1.016%	1.533%	1.128%	1.127%	1.116%	0.883%
IC EH	0.470%	0.187%	0.298%	0.231%	0.241%	0.246%	0.455%

- L-moment for $n = 2$ is very high for FX MA: the skewness concerns rather the central part of the distribution

Fund returns distribution

- Financial interpretation
  Illiquid assets, smoothing techniques used to avoid small negative returns
- These practices create asymmetry in the returns distribution
- $\lambda_{1,2} - \lambda_{1,1}$ is a measure of this effect and can be used as a test statistic for manipulation
- Easy to obtain the law of the test statistic from the asymptotic properties of the sample quantile function

Fund returns distribution

- L-moments of order 2 for n varying provides similar classification of risks

Ticker	Power Moments σ	L-Moments $\lambda_{2,r=0,n=1}$	$\lambda_{2,r=1,n=2}$	$\lambda_{2,r=1,n=3}$	$\lambda_{2,r=1,n=4}$	$\lambda_{2,r=1,n=5}$
RO ED	5.93%	0.883%	0.812%	0.279%	0.069%	0.015%

- Example of RO ED with largest variance and also largest L-moments of order 2 for any value of trimming

Ticker	Power Moments σ	L-Moments $\lambda_{2,r=0,n=1}$	$\lambda_{2,r=1,n=2}$	$\lambda_{2,r=1,n=3}$	$\lambda_{2,r=1,n=4}$	$\lambda_{2,r=1,n=5}$
RC EH	4.88%	0.654%	0.582%	0.195%	0.048%	0. 010%
WI MA	4.21%	0.718%	0.691%	0.244%	0.062%	0.014%

- Example of RC EH and WI MA
  - RC EH is more risky than WI MA in terms of variance
  - But less risky in terms of L-moments

Comparison of estimated performances

	Sharpe Performance				L-Performances					
Ticker	SR	Rank	Skew.	Kurt.	$L_{0,1}$	Rank	$L_{1,2}$	Rank	$L_{1,3}$	Rank
AC CA	0.05	**32**	0.12	1.49	0.18	**32**	0.57	**32**	1.51	**32**
AI CA	0.39	**15**	0.61	0.80	2.09	**19**	3.15	**23**	6.66	**24**
APM MA	0.29	**23**	0.14	-1.12	1.34	**29**	1.81	**30**	3.48	**30**
AS MA	0.22	**30**	-0.55	-0.50	1.54	**28**	2.43	**28**	5.08	**28**
BF FI	-0.24	**35**	-0.49	1.43	-1.61	**35**	-2.10	**34**	-4.12	**34**
BR FI	0.29	**22**	-0.14	0.81	2.08	**21**	3.65	**19**	8.33	**17**
CG RV	0.28	**25**	0.34	1.45	1.79	**25**	3.00	**24**	6.74	**23**
CSS ED	0.35	**18**	-0.04	-0.14	2.09	**20**	3.22	**22**	6.78	**22**
DF MF	0.48	**8**	-0.04	-0.89	2.72	**13**	4.11	**16**	8.72	**16**
DM FI	0.32	**19**	0.38	0.17	1.68	**27**	2.56	**27**	5.48	**26**
EF RV	0.03	**33**	0.11	0.41	-0.14	**33**	-0.16	**33**	-0.27	**33**
FO MA	0.49	**5**	-0.40	0.90	3.36	**8**	5.34	**7**	11.53	**9**
FS EM	0.74	**1**	0.62	0.47	4.63	**1**	7.06	**1**	15.14	**1**
FX MA	0.28	**26**	-1.23	2.03	2.70	**14**	5.00	**11**	11.71	**8**
GU EH	0.28	**27**	-1.16	2.68	2.56	**16**	4.67	**13**	10.81	**11**
HC MA	0.36	**17**	-0.62	0.90	2.96	**11**	5.30	**8**	12.33	**5**
I EM	0.49	**6**	-0.24	-0.81	2.98	**10**	4.75	**12**	10.50	**12**
IC EH	0.14	**31**	0.92	2.82	0.46	**31**	0.84	**31**	1.95	**31**
LC EN	0.39	**16**	-0.39	-0.64	2.53	**17**	4.22	**15**	9.65	**15**
LES SS	-0.33	**36**	0.36	-0.87	-2.73	**36**	-4.34	**36**	-9.56	**36**
ML MA	0.28	**28**	0.30	2.83	1.96	**23**	3.27	**21**	7.15	**21**
OAM EH	0.26	**29**	0.33	0.58	1.34	**30**	2.00	**29**	4.09	**29**
PE MA	0.42	**14**	-0.08	-0.06	2.58	**15**	3.91	**17**	8.09	**18**
PF EH	0.31	**21**	-0.26	-0.24	2.05	**22**	3.49	**20**	7.94	**19**
PI MA	0.48	**7**	2.78	11.03	3.40	**6**	5.22	**9**	11.24	**10**

Comparison of estimated performances

- The values of different performance measures cannot be compared directly, but the rankings are comparable
- Rankings are quite stable
  - LES SS is always the worst performing fund
  - FS EM is always the best performer

Ticker	SR	Rank	Skew.	Kurt.	$L_{0,1}$	Rank	$L_{1,2}$	Rank	$L_{1,3}$	Rank
	Sharpe Performance				L-Performances					
FS EM	0.74	**1**	0.62	0.47	4.63	**1**	7.06	**1**	15.14	**1**
LES SS	-0.33	**36**	0.36	-0.87	-2.73	**36**	-4.34	**36**	-9.56	**36**

- But some funds can fluctuate when we use different performance measures

Ticker	SR	Rank	Skew.	Kurt.	$L_{0,1}$	Rank	$L_{1,2}$	Rank	$L_{1,3}$	Rank
	Sharpe Performance				L-Performances					
SP EH	0.64	**3**	0.66	0.07	3.52	**3**	5.04	**10**	10.35	**13**
FX MA	0.28	**26**	-1.23	2.03	2.70	**14**	5.00	**11**	11.71	**8**
RB EH	0.45	**11**	-0.51	1.07	3.42	**5**	5.73	**3**	12.68	**3**

5. Concluding Remarks

- The objective to improve upon the Sharpe ratio
- The shrinkage parameters allows for fine tunning of the performance that can produce a set of fund performances and ranking
- Alternative definition of efficient portfolios and efficient frontiers
- Extension to conditional L-performance is immediate using the notion of concomitant ranks
- Fitted L-performances are also easily computable