Incentives to Invest and to Give Access to New Technologies*

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Abstract

We analyze the incentives of a vertically integrated firm, which is a regulated monopolist in the wholesale market and competes with an entrant in the retail market, to invest and to give access to a new wholesale technology. The new technology is unregulated and produces retail products of a higher quality than the old technology. If the innovation is non-drastic, the vertically integrated firm may be induced to give access to the entrant. Furthermore, if the innovation is non-drastic and small, a duopoly in the retail market is socially optimal, whereas if the innovation is non-drastic but large, a monopoly in the retail market is socially optimal. If the innovation is drastic, the vertically integrated firm does not give access to the entrant. If both firms can invest, but only one does, it is more likely that it is the entrant who invests. The impact on social welfare of the two firms investing, instead of just the vertically integrated firm, is potentially ambiguous.

Keywords: New technology, Investment, Access, Regulation, Next Generation Telecommunications Networks.

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1 Introduction

Consider an industry where a vertically integrated firm is a regulated monopolist in the wholesale market and competes with an entrant in the retail market. The vertically integrated firm can invest in a new technology for the wholesale market. The new technology is unregulated, and allows supplying retail products of a higher quality than those offered by the old technology. If investment occurs, this scenario lays between the case where a vertically integrated firm, which is a monopolist in the wholesale market and competes in the retail market with an entrant, and the case where several vertically integrated firms compete among themselves and with an entrant in the retail market, analyzed, e.g., by Brito and Pereira (2009, 2010) and Ordover and Shaffer (2007). After the investment there will be two alternative wholesale technologies available, which belong to the same entity, are of different qualities, and one is regulated and the other not. Our scenario, motivated by several examples discussed in section 2, such as the investment in next generation fixed telecommunications networks, raises several policy questions regarding the vertically integrated firm's incentives: **(i)** to invest in the new technology and, **(ii)** to give access to the retail market entrant to the new technology.

Regarding the first issue, given that the vertically integrated firm must have incentives to invest, is it socially preferable to have a monopoly or a duopoly in the retail market? In some circumstances, only the vertically integrated firm can invest in the new technology, but in others, perhaps due to public policies, both firms can. Given that if both firms invest, double-marginalization is eliminated, but investment costs are duplicated, is it socially preferable to have only the vertically integrated firm invest, or to have both firms invest?

Regarding the second issue, will the vertically integrated firm voluntarily give access to the entrant, or should open access obligations be extended to the new technology? There is a concern that if the new technology is left unregulated, the industry will be monopolized. However, the vertically integrated firm has conflicting incentives with respect to giving access to the entrant. Conceding access allows the entrant to sell a higher quality product. This, on the one hand, reduces the vertically integrated firm's retail profits, but, on the other hand, increases the vertically integrated firm's wholesale profits. Furthermore, if the entrant, using the old technology, is unable to compete with the vertically integrated firm, using the new technology, denying access to the entrant might benefit the vertically integrated firm, since it allows expelling the entrant from the industry. If, however, the entrant, using the old technology, is able to compete with the vertically integrated firm, using the new technology, giving access to the entrant might benefit the vertically integrated firm, since it allows increasing the wholesale profits.

To analyze these issues, we developed a model that includes the elements described in the scenario presented above.

We distinguish two cases: **(i)** *drastic* innovation, and **(ii)** *non-drastic* innovation. In the former case, the entrant, using the old technology, cannot compete against the vertically integrated firm, using the new technology, even if the access price for the old technology is at marginal cost.[1] In the latter case it can, if the access price for the old technology is low enough.

When the innovation is *non-drastic* and only the vertically integrated firm can invest in the new technology, if the quality improvement enabled by the new technology is small, the regulator sets the access price for the old technology at marginal cost. Since the vertically integrated firm cannot foreclose the market by denying access to the new technology, it voluntarily gives access to the entrant, leading to a duopoly with the new technology. Interestingly, if the quality improvement enabled by the new technology is large, a monopoly is socially preferable to a duopoly. Hence, the regulator sets a high access price for the old technology, such that the entrant, using the old technology, cannot compete against the vertically integrated firm, using the new technology. The vertically integrated firm takes advantage of this and denies access to the entrant to the new technology, becoming a monopolist in the retail market.

When the innovation is *non-drastic* and both the vertically integrated firm and the entrant can invest in the new technology, both firms invest, if the investment cost is low, and only one firm invests, either the vertically integrated firm or the entrant, if the investment cost is high. When only one firm invests, it is more likely that it is the entrant who does so. This happens because the vertically integrated firm pays a lower access price than the entrant when it uses the rival's technology, since it has the outside option of using the old technology. If the investment cost is low, the possibility of both firms investing, instead of just the vertically integrated firm, may increase or decrease welfare, due to the trade-off between the elimination of double marginalization and the duplication of the investment cost. If the investment cost is high, the possibility of both firms investing, instead of just the vertically integrated firm, either leaves unchanged or decreases welfare, due to the presence of an excessive number of firms in the industry.

When the innovation is *drastic* and only the vertically integrated firm can invest in the new technology, it does not give access to the entrant, which is forced out of the industry.

[1] *Access price* is the per unit price the entrant must pay to the incumbent to use the wholesale technology.

When the innovation is *drastic* and both the vertically integrated firm and the entrant can invest in the new technology, again both firms invest, if the investment cost is low, and only one firm invests, if the investment cost is high. The possibility of both firms investing, instead of just the vertically integrated firm, may increase or decrease welfare, if the investment cost is low, and leaves welfare unchanged, if the investment cost is high.

These results have several policy implications, whose discussion we defer until section 7.

Most of the literature on the relationship between optimal access regulation and investment, surveyed by Cambini and Jiang (2009),[2] considered only the case of one regulated technology, and focused either on investment in quality upgrades of an existing technology, e.g., Caillaud and Tirole (2004), Klumpp and Su (2009) and Vareda (2007), or, on the timing of the investment in a new technology, e.g., Gans (2001), Hori and Mizuno (2006) and (2009). We contribute to this literature by analyzing the case where one vertically integrated firm operates a regulated technology and can invest in a superior and unregulated technology, and after the investment occurs both technologies may operate simultaneously.

Some articles on this literature find that a regulatory moratorium may be socially optimal. Gans and King (2004) shows that when investment returns are uncertain and the regulator is unable to commit to an access price, welfare increases if the regulator commits to a regulatory moratorium. Vareda and Hoernig (2007) studies the investment of two firms in a new technology and show that a regulatory moratorium may be required to give the leader the correct incentives to invest, at the same time that it allows charging a lower access price later on.

Other articles compare investment incentives under regulation and under no regulation. Foros (2004) shows, in the context of investment in the quality of an existing technology, that an unregulated incumbent may have incentives to give access to its technology, if the entrant has the ability to produce high quality retail products. Otherwise, the incumbent forecloses the market. Kotakorpi (2006) finds that an unregulated incumbent may under-invest in the quality of its infrastructure and foreclose the market, while a regulated incumbent is most likely to foreclose the market when rivals offer the highest benefits to consumers. Bourreau and Dogan (2005) shows that an incumbent may have incentives to give voluntarily access to an entrant at a very low price to delay its investment in an alternative technology.

Brito et al. (2010) considers a model similar to ours. However, the purpose of this article is to analyze if two-part access tariffs solve the dynamic consistency problem of

[2] See also Guthrie (2006) for a survey of the literature on the relationship between regulation and investment.

the regulation of next generation networks. If the regulator can commit to a policy, a regulatory moratorium may be socially optimal. If the regulator cannot commit to a policy, it can induce investment only when the investment cost is low, and the entrant makes large payments to the incumbent.

The remainder of the article is organized as follows. In section 2, we discuss three motivating cases. In section 3, we describe the model. In section 4, we analyze the case where innovation is *non-drastic* and only the vertically integrated firm can invest. In section 5, we analyze the case where innovation is *non-drastic* and both the vertically integrated firm and the entrant can invest, and compare, from a welfare perspective, the cases where only the vertically integrated firm can invest and the case where both firms can invest. In section 6, we analyze the case where innovation is *drastic*. Finally, in Section 7, we discuss the policy implications and conclude. All proofs are in the Appendix.

2 Motivating Cases

In this section, we present three cases that motivate our analysis.

2.1 Next Generation Fixed Telecommunications Networks

The deployment of next generation fixed telecommunications networks, leading to a multi-service infrastructure for audio, video, and data services, sets the telecommunications sector on the verge of a new era.[3] To give firms the right incentives to invest, and to promote an efficient use of these infrastructures, sectoral regulators must set an adequate regulatory framework for these new telecommunications networks.

Sectoral regulators are considering three main regulatory approaches: **(i)** the *continuity approach*, which consists on maintaining the current regulatory system; **(ii)** the *equality of access approach*, which consists of adding to the provisions of the continuity approach the obligation of the functional separation of the incumbent's wholesale and retail operations; and **(iii)** the *forbearance approach*, which consists on the abstention, permanent or temporary, from regulatory intervention.

The UK was the first country to apply the equality of access approach. The sectoral regulator, *OFCOM*, imposed functional separation to the telecommunications incumbent,

[3] A *Next Generation Network* is a "(...) packet-based network able to provide telecommunication services and able to make use of multiple broadband, QoS-enabled transport technologies and in which service-related functions are independent from underlying transport related technologies." See ITU (2001).

BT. The continuity and equality of access approaches have been criticized with the argument that they reduce the incentives to invest. The obligation to share its next generation network with rivals reduces the incumbent's incentives to invest, and the possibility of using the incumbent's network reduces the entrants incentives to invest in their own networks.

The forbearance approach was followed in US. This approach was justified with the argument that cable television and telecommunications firms, incumbents and entrants, are in an equal footing to deploy their own networks, and that competition among them to do so is welcomed.[4] Telecommunications firms, like *Verizon*, are deploying next generation networks, and they are only obliged to offer to entrants wholesale services equivalent to those they already offered through the old network. Cable television firms, like *Time Warner*, are also deploying their next generation networks, with no open access obligations. The forbearance approach was criticized on the basis that, when there are no alternative networks, it could allow the telecommunications incumbent to re-monopolise the market. In Germany, the sectoral regulator, *AGCOM*, conceded a regulatory moratorium to the next generation network of the telecommunications incumbent, *Deutsche Telekom*. However, the European Commission objected and forced the cancellation of the regulatory moratorium. The Commission argued that the existing ex-ante regulation had to be extended also to this network, since the lack of competition in the German market could lead to the re-emergence of monopoly. When there is a vigorous facilities-based competition between firms using different technologies, such as telecommunications and cable television firms, the concern that the industry may be re-monopolized is less justifiable. In addition, in some markets, mostly urban, entrants are also deploying their own next generation networks, particularly when they already own some parts of an infrastructure that allows them similar investment costs to those of the incumbent. For instance, in Switzerland the energy incumbent is investing in fiber-to-the-home, using its own ducts.

In the short-run, next generation telecommunications networks can probably be thought of as *non-drastic* innovations. However, over the long-run they will most likely become *drastic* innovations.

[4] In the US, telecommunications networks are regulated, whereas cable television networks are not. In addition, cable television firms offered broadband access to the internet first, and still have larger market shares than telecommunications firms.

2.2 Ethernet

Typically, telecommunication incumbents have the regulatory obligation of having a wholesale reference offer for leased lines. As telecommunication incumbents converted their networks from circuited switched to IP packet switched, some of them in the EU, started offering Ethernet connectivity, as an alternative to leased lines. Ethernet connectivity allows point-to-point communication between terminal points of a network at very high debits, from 10 Mbps to 1 Gbps, at a low cost. This allows telecommunications incumbents to under-cut the regulated prices of leased lines and still improve their wholesale margins. Until recently, Ethernet connectivity was not regulated, and these offers were made on a voluntary basis.

Given that retail entrants have the outside option of using leased lines, and given that it allows incumbents to increase their wholesale profits, it is no surprise that they offered wholesale Ethernet connectivity on a voluntary basis. However, as telecommunications incumbents start to deploy their next generation networks and phase-out their traditional telecommunications network, will they still continue to voluntarily offer wholesale Ethernet connectivity? In the EU sectoral regulators seem to think that they will not, and started to regulate Ethernet connectivity market, on the basis that now incumbents have significant market power on this market.

Ethernet connectivity can be seen as a *non-drastic* innovation.

2.3 Fourth Generation Mobile Telecommunications Networks

Several mobile telecommunications firms have announced plans to start deploying fourth generation mobile telecommunications networks. The main advantage of these networks, compared with third generation networks, is that they are IP packet switched and enable broadband access to the internet at gigabit speeds.

In several countries, sectoral regulators imposed to mobile network operators the obligation of having reference offers for retail entrants, known as mobile virtual network operators, for second generation and third generation mobile telecommunications networks.

Will mobile network operators voluntarily give access to their new networks to mobile virtual network operators? Should the reference offers for mobile virtual network operators be extended to fourth generation mobile networks?

In the short-run, fourth generation mobile telecommunications networks can probably be though of as *non-drastic* innovations. However, over the long-run they will most likely become *drastic* innovations.

3 Model

3.1 Environment

Consider an industry that consists of two overlapping markets: the wholesale market and the retail market. The wholesale market produces an input that is indispensable to supply services in the retail market. We refer to the price of the wholesale market as the *access price*. Two firms operate in the industry: the incumbent and the entrant. The *incumbent*, firm i, is vertically integrated, i.e., operates both in the wholesale and the retail markets. In the wholesale market the incumbent is a monopolist. The *entrant*, firm e, operates in the retail market. In the retail market, the incumbent and the entrant sell horizontally differentiated products. We index firms with subscript $j = i, e$. A *sectoral regulator* oversees the industry.

There is available a new technology for the wholesale market. The *new* technology, technology n, produces an input of higher quality than the *old* technology, technology o. In turn, this higher quality input allows supplying retail products of a higher quality than those supplied using the input produced by the old technology. We index the technology used by firm j with subscript $\tau_j = o, n$. If $\tau_i = \tau_e$, we let $\tau_i = \tau_e = \tau$.

We assume first that only the incumbent can invest in the deployment of the new technology. For a number of reasons, the entrant has some disadvantage relative to the incumbent, which precludes it from making this investment. The entrant might not have access to financing, whereas the incumbent does. The entrant's investment cost might be higher than the incumbent's investment cost, such that the investment is profitable for the latter but not for the former. Environmental or municipal regulation might prevent, or make too costly, the deployment by the entrant of the infrastructure required to support the new technology, whereas the incumbent can use the infrastructure that supports the old technology to deploy the new technology. Later we will allow both firms to invest in the deployment of the new technology.

Costs and demand are common knowledge.

To simplify the exposition we will refer to "access to the input produced by the old technology" by "access to the old technology". In addition, we will refer to "using the input produced by the old technology" by "using the old technology". Similarly for the new technology.

The game has four stages which unfold as follows. In stage 1, the sectoral regulator sets the access price for the old technology. In stage 2, the incumbent decides whether to invest and, if investment takes place, the incumbent offers the entrant an access price for the new technology. In stage 3, the entrant chooses which type of technology to use, if any. In stage 4, the incumbent and the entrant compete on retail tariffs.

3.2 Sectoral Regulator

The regulator sets the access price for the old technology, denoted by α_o on $[0, +\infty)$. When the new technology is deployed, the incumbent must: **(i)** offer access to the old technology at access price α_o, if the old technology is not discontinued, or, **(ii)** offer wholesale services of a quality equivalent to those enabled by the old technology at access price α_o, if the old technology is discontinued.

Since we want to investigate the incumbent's incentives to concede access to the new technology, we assume that access to this technology is not mandatory. However, the incumbent may voluntarily sell access to the rival. The access price for the new technology, which is set by the incumbent, is denoted by α_n on $[0, +\infty)$.

The regulator maximizes social welfare, i.e., the sum of the firms' profits and the consumer surplus, denoted by W.

3.3 Consumers

There is a large number of consumers, formally a continuum, whose measure we normalize to 1. Consumers are uniformly distributed along a Hotelling line segment of length 1 (Hotelling, 1929), facing transportation costs tx to travel distance x, with t on $(0, +\infty)$. Consumers are otherwise homogeneous. As in Biglaiser and DeGraba (2001), we assume each consumer has a demand function for retail services given by $y_j = \left(z + v_{\tau_j}\right) - p_j$, where: **(i)** y_j on $\left[0, z + v_{\tau_j}\right]$ is the number of units of retail services purchased from firm j, **(ii)** p_j on $\left[0, z + v_{\tau_j}\right]$ is the per unit price of retail services of firm j, **(iii)** z is a parameter on $(\underline{z}, \overline{z})$,[5] and **(iv)** v_{τ_j} is a parameter that takes value 0 for products supplied using the old technology, i.e., for $\tau_j = o$, and takes value v on $(0, +\infty)$ for products supplied through the new technology, i.e., for $\tau_j = n$.

The lower limit on z implies that the market is always covered, both for a monopoly and a duopoly, while the upper limit implies that the incumbent does not want to invest for every regulated access price and investment cost, when the entrant, using the old technology, can compete against the incumbent, using the new technology. The assumption on v_{τ_j} means that consumers are willing to pay a premium for services produced using the new technology.

[5] More specifically: $\underline{z} := \frac{4}{3}\sqrt{6t}$; for the expression that defines $\overline{z}$ see the proof of Lemma 6.

Let $\chi := v\,(2z + v)$. For $p_j = 0$, the incremental consumer surplus from the investment is $\frac{1}{2}\chi$. We take χ as a measure of *quality improvement* enabled by the new technology.

3.4 Firms

All of the incumbent's marginal costs are constant and equal to zero. The entrant has marginal cost α_{τ_e} on $\{\alpha_o, \alpha_n\}$.

The incumbent is located at point 0 and the entrant at point 1 of the line segment where consumers are distributed.

Firms charge consumers two-part retail tariffs, denoted by $T_j(y_j) = F_j + p_j y_j$, $j = i, e$, where F_j on $[0, +\infty)$ is the fixed fee of firm j.

The incumbent can deploy the new technology at a fixed investment cost of I on $\left[0, \frac{1}{2}\chi\right)$.[6] The upper limit on I ensures that the investment on the new technology increases welfare, both for a monopoly and a duopoly, keeping the access price at the pre-investment level.

Regarding the quality improvement enabled by the new technology, we distinguish two cases: **(i)** if v is on $\left(0, \sqrt{z^2 + 6t} - z\right)$, we say that the investment generates a *non-drastic* innovation; **(ii)** if v is on $\left[\sqrt{z^2 + 6t} - z, +\infty\right)$, we say that the investment generates a *drastic* innovation. The relevance of this definition will become clear in section 6. We borrow this definition from the research and development literature.[7] For notational convenience, we use the equivalent condition that χ is on $(0, 6t)$ for *non-drastic* innovation, and that χ is on $[6t, +\infty)$ for *drastic* innovation.

Given (α_o, α_n), the entrant can either: **(i)** accept α_n, and use the new technology, **(ii)** accept α_o, and use the old technology, or **(iii)** reject α_n and α_o, and exit the industry.

Denote by σ_j, the consumer share of firm $j = i, e$. The profits of firm $j = i, e$ for the whole game, gross of the investment cost, are:

$$\pi_i = [p_i y_i + F_i]\,\sigma_i + \alpha_{\tau_e} y_e \sigma_e$$
$$\pi_e = [(p_e - \alpha_{\tau_e})\,y_e + F_e]\,\sigma_e.$$

3.5 Equilibrium Concept

The sub-game perfect Nash equilibrium is: **(i)** an access price for the old technology, **(ii)** an investment decision, **(iii)** an access price offer for the new technology, **(iv)** a decision of which technology to use, and **(v)** a set of retail tariffs, such that:

[6] For a justification of why this assumption fits well, e.g., investment in next generation networks see Brito et al. (2010).

[7] See Tirole (2003) pg. 391.

(E1) the retail tariffs maximize the firms' profits, given the access prices, the investment decision and the decision of which technology to use;
(E2) the decision of which technology to use maximizes the entrant's profits, given the access prices, the incumbent's investment decision, and the optimal retail tariffs function;
(E3) the investment decision and the access price offer for the new technology maximize the incumbent's profits, given the access price for the old technology, the optimal decision of which technology to use and the optimal retail tariffs function;
(E4) the access price for the old technology maximizes social welfare, given the optimal investment decision, the optimal access price offer for the new technology, the optimal decision of which technology to use and the optimal retail tariffs function.

4 Only the Incumbent Deploys the New Technology

In this section, we characterize the equilibrium of the game for the case where only the incumbent can invest in the new technology and innovation is *non-drastic*. The case where only the incumbent can invest and innovation is *drastic* is analyzed in section 6. We construct the equilibrium by backward induction.

4.1 Stage 4: Retail Prices Game

We characterize the equilibrium of the retail price game for five cases: **(i)** the incumbent does not invest in the new technology, and the entrant exits the industry, **(ii)** the incumbent invests in the new technology, and the entrant exits the industry **(iii)** the incumbent does not invest in the new technology, and the entrant stays in the industry, **(iv)** the incumbent invests in the new technology, the entrant stays in the industry, and selects the old technology, and **(v)** the incumbent invests in the new technology, the entrant stays in the industry, and selects the new technology. In cases (i)-(ii) the retail market is a monopoly. In cases (iii)-(v) the retail market is a duopoly. We use superscripts m_o, m_n, d_o, d_b, d_n to denote variables or functions associated with cases (i)-(v), respectively.

We start with the following Lemma.

Lemma 1: *In equilibrium, firms set the marginal price of the two-part retail tariff at marginal cost, i.e.,* $p_i = 0$ *and* $p_e = \alpha_{\tau_e}$, $\tau_e = o, n$. ■

As usual with two-part tariffs, firms set the marginal price of the retail tariff at marginal cost to maximize the gross consumer surplus, and then try to extract this surplus using the fixed part.

Given Lemma 1, from now on we only discuss the determination of the fixed fees.

4.1.1 Monopoly

Next, we present the equilibrium of the retail game for the two cases where the retail market is a monopoly, which is given by the next Lemma.

Lemma 2: *If the retail market is a monopoly, in equilibrium, the incumbent charges the fixed fee, for $\tau_i = n, o$:*

$$F_i^{m_{\tau_i}} = \frac{(z + v_{\tau_i})^2}{2} - t.$$

■

The profits of the incumbent, gross of the investment cost, for $\tau_i = n, o$, are:

$$\pi_i^{m_{\tau_i}} = \frac{(z + v_{\tau_i})^2}{2} - t.$$

4.1.2 Duopoly

Next, we characterize the equilibrium of the retail price game for the three cases where the retail market is a duopoly.

We use index $k = o, n, b$ to denote, respectively, the case where the two retailers: use the old technology, use the new technology, and use different technologies. Let $D_k := (v_{\tau_i} - v_{\tau_e})(2z + v_{\tau_i} + v_{\tau_e})$. Parameter D_k measures the incumbent's quality advantage relative to the entrant. In a duopoly where both firms use the same technology, i.e., for $k = o, n$, we have $D_k = 0$, while in a duopoly where the entrant uses the old technology and the incumbent uses the new technology, i.e., for $k = b$, we have $D_k = \chi$.

The next Lemma gives the equilibrium retail fixed fees.

Lemma 3: *If the retail market is a duopoly, in equilibrium, the incumbent and the entrant charge fixed fees, for $k = o, b, n$:*

$$F_i^{d_k}(\alpha_{\tau_e}) = \begin{cases} t + \frac{1}{6}\alpha_{\tau_e}\left[6(z + v_{\tau_e}) - 5\alpha_{\tau_e}\right] + \frac{1}{6}D_k & \text{for } \alpha_{\tau_e} \text{ on } \left[0, \sqrt{6t - D_k}\right) \\ \alpha_{\tau_e}(z + v_{\tau_e}) - \frac{1}{2}\alpha_{\tau_e}^2 - t + \frac{1}{2}D_k & \text{for } \alpha_{\tau_e} \text{ on } \left[\sqrt{6t - D_k}, z + v_{\tau_e}\right] \end{cases}$$

$$F_e^{d_k}(\alpha_{\tau_e}) = \begin{cases} t - \frac{1}{6}\alpha_{\tau_e}^2 - \frac{1}{6}D_k & \text{for } \alpha_{\tau_e} \text{ on } \left[0, \sqrt{6t - D_k}\right) \\ 0 & \text{for } \alpha_{\tau_e} \text{ on } \left[\sqrt{6t - D_k}, z + v_{\tau_e}\right]. \end{cases}$$

■

The profit of the incumbent and the entrant, gross of the investment cost, for $k = o, b, n$, are, respectively:[8]

$$\pi_i^{d_k}(\alpha_{\tau_e}) = \begin{cases} \frac{\left(36t^2+\alpha_{\tau_e}^4-60t\alpha_{\tau_e}^2\right)+72\alpha_{\tau_e}t(z+v_{\tau_e})+D_k\left(12t+D_k+2\alpha_{\tau_e}^2\right)}{72t} & \text{for } \alpha_{\tau_e} \text{ on } \left[0, \sqrt{6t-D_k}\right) \\ \alpha_{\tau_e}(z+v_{\tau_e}) - \frac{1}{2}\alpha_{\tau_e}^2 - t + \frac{1}{2}D_k & \text{for } \alpha_{\tau_e} \text{ on } \left[\sqrt{6t-D_k}, z+v_{\tau_e}\right] \end{cases}$$

and

$$\pi_e^{d_k}(\alpha_{\tau_e}) = \begin{cases} \frac{\left[6t-\left(D_k+\alpha_{\tau_e}^2\right)\right]^2}{72t} & \text{for } \alpha_{\tau_e} \text{ on } \left[0, \sqrt{6t-D_k}\right) \\ 0 & \text{for } \alpha_{\tau_e} \text{ on } \left[\sqrt{6t-D_k}, z+v_{\tau_e}\right] \end{cases}$$

In a duopoly, the profit of the incumbent is non-decreasing in the access price, while the profit of the entrant is non-increasing in the access price.[9] If the access price increases, the marginal cost of the entrant increases relative to that of the incumbent. As a consequence, the market share, and thereby the profit of the incumbent, increases, while the entrant's profit decreases.

4.2 Stage 3: Technology Choice

Next, we analyze the entrant's decision of which technology to use.

When indifferent between staying or exiting the industry, the entrant chooses the latter, and when indifferent between asking for access to the old or to the new technology, the entrant chooses the latter.

The next Lemma presents the optimal technology choice.

Lemma 4: *Assume that only the incumbent can invest and that the innovation is non-drastic, i.e., χ is on $(0, 6t)$.*

(i) *Let there be no investment. The entrant:*

$$\begin{cases} \text{accepts } \alpha_o & \text{for } \alpha_o \text{ on } \left[0, \sqrt{6t}\right) \\ \text{exits} & \text{for } \alpha_o \text{ on } \left[\sqrt{6t}, +\infty\right). \end{cases}$$

[8] For α_{τ_e} on $\left[\sqrt{6t-D_k}, z+v_{\tau_e}\right]$, the entrant is present in the industry and constrains the incumbent's pricing behavior. However, it has a 0 market share.

[9] The first part follows from the assumption that z belongs to $(\underline{z}, \overline{z})$.

(ii) *Let there be investment. The entrant:*

$$\begin{cases} \text{accepts } \alpha_n & \text{for } (\alpha_o, \alpha_n) \text{ on } \left[0, \sqrt{6t-\chi}\right) \times \left[0, \sqrt{\alpha_o^2+\chi}\right] \cup \\ & \qquad\qquad\qquad\;\; \left[\sqrt{6t-\chi}, +\infty\right) \times \left[0, \sqrt{6t}\right) \\ \text{accepts } \alpha_o & \text{for } (\alpha_o, \alpha_n) \text{ on } \left[0, \sqrt{6t-\chi}\right) \times \left(\sqrt{\alpha_o^2+\chi}, +\infty\right) \\ \text{exits} & \text{for } (\alpha_o, \alpha_n) \text{ on } \left[\sqrt{6t-\chi}, +\infty\right) \times \left[\sqrt{6t}, +\infty\right). \end{cases}$$

■

Figure 1 illustrates the entrant's equilibrium technology choice for the case where only the incumbent invests and the innovation is *non-drastic*.

[*Figure* 1]

In area A, the access price for the new technology is low and the entrant accepts it. In area B, the access price for the old technology is low and the entrant accepts it. As expected, the higher α_n is, the larger the set of values of α_o for which the entrant selects the old technology. Finally, in area C, both access prices are high and the entrant exits the industry.

4.3 Stage 2: Access Price Offer and Investment Decision

Next, we characterize the incumbent's equilibrium access price offer and investment decision.

Denote by $\alpha_n^*(\alpha_o; \chi)$, the incumbent's optimal access price offer, given the access price for the old technology, α_o, and the quality improvement enabled by the new technology, χ.

The next Lemma characterizes the incumbent's equilibrium access price offer.

Lemma 5: *Assume that only the incumbent can invest and that the innovation is non-drastic, i.e.,* χ *is on* $(0, 6t)$. *In equilibrium, the incumbent offers:*

$$\alpha_n^*(\alpha_o; \chi) = \begin{cases} \sqrt{\alpha_o^2+\chi} & \text{for } \alpha_o \text{ on } \left[0, \sqrt{6t-\chi}\right) \\ \left[\sqrt{6t}, +\infty\right) & \text{for } \alpha_o \text{ on } \left[\sqrt{6t-\chi}, +\infty\right). \end{cases}$$

■

If α_o is high, the entrant, using the old technology, cannot compete against the incumbent, using the new technology. This happens because the entrant sells a lower quality service and has a marginal cost disadvantage. Thus, the incumbent offers an unacceptably high $\alpha_n^*(\alpha_o; \chi)$ to induce the entrant to exit the industry, and thereby become a monopolist.

If α_o is low, the entrant, using the old technology, can compete against the incumbent, using the new technology. Thus, since the incumbent cannot avoid competition from the entrant, it chooses to offer the highest access price for which the entrant selects the new technology. Conceding access to the new technology has two opposing effects on the incumbent's profit. If the entrant uses the new technology, it produces a higher quality product, with which it can compete more effectively with the incumbent. This is the *retail effect*, which has the negative impact of reducing the incumbent's retail profits. In addition, since the entrant earns higher profits, the incumbent can charge a higher access price. This is the *wholesale effect*, which has the positive impact of increasing the incumbent's wholesale profits. The latter effect dominates.[10]

Figure 1 represents, in bold, the incumbent's access price offers as a function of α_o, given χ: $\alpha_n^*(\cdot)$.[11] For low values of α_o, such that the entrant finds it profitable to compete using the old technology, function $\alpha_n^*(\cdot)$ is increasing in both α_o and χ. For high values of α_o, setting $\alpha_n^*(\cdot)$ on $\left[\sqrt{6t}, +\infty\right)$ induces the entrant to exit the industry.

Next, we analyze the incumbent's decision to invest in the new technology, assuming it charges $\alpha_n^*(\cdot)$.

For α_o on $\left[0, \sqrt{6t-\chi}\right)$, denote by $\Delta\Pi_i(\alpha_o) := \pi_i^{d_n}\left(\alpha_n^*(\alpha_o;\chi)\right) - \pi_i^{d_o}\left(\alpha_o\right)$, the incumbent's incremental profit from the investment, excluding investment costs, given that it offers an access price $\alpha_n^*(\alpha_o;\chi)$. Function $\Delta\Pi_i(\cdot)$ is quasi-convex, and takes values lower than $\frac{1}{2}\chi$ for a sufficiently high α_o.[12]

When indifferent between investing and not investing, the incumbent chooses the latter.

The following Lemma presents the optimal investment decision.

Lemma 6: *Assume that only the incumbent can invest and that the innovation is non-drastic, i.e., χ is on $(0, 6t)$. The incumbent:*

$$\begin{cases} \text{invests} & \text{for } (\alpha_o, I) \text{ on } \left[\sqrt{6t-\chi}, +\infty\right) \times \left[0, \frac{1}{2}\chi\right) \cup \\ & \qquad \left[0, \sqrt{6t-\chi}\right) \times \left[0, \min\left\{\Delta\Pi_i(\alpha_o), \frac{1}{2}\chi\right\}\right) \\ \text{does not invest} & \text{for } (\alpha_o, I) \text{ on } \left[0, \sqrt{6t-\chi}\right) \times \left[\Delta\Pi^i(\alpha_o), \frac{1}{2}\chi\right). \end{cases}$$

■

[10] This happens even when the market is covered, as in our case, and therefore, all consumers that the entrant captures are lost by the incumbent. If the market was partially covered, the incumbent would benefit additionally from the entrant's consumers that would otherwise be out of the market.

[11] Actually, in bold we represent the lower boundary of the incumbent's offers.

[12] We present this function and its characteristics in the proof of Lemma 6, in the appendix.

An interesting implication of the properties of $\Delta\Pi_i(\cdot)$ refers to the relationship between α_o and the incentives to invest. Arguably, one could expect that the lower α_o, the larger the incentives to invest. However, unless I is so low that the incumbent invests for any α_o, the decision to invest is non-monotonic in α_o. When $\Delta\Pi_i(\cdot)$ is decreasing in α_o, raising α_o on $\left[0, \sqrt{6t-\chi}\right)$ increases the wholesale profit of the old technology, and hence discourages investment.[13] For higher values of α_o, the incumbent does not invest for some values of I. However, when α_o raises above $\sqrt{6t-\chi}$, the entrant's marginal cost is so large that it is unable to compete with the incumbent, if the incumbent invests. For the same values of I, the incumbent invests and becomes a monopolist. When $\Delta\Pi_i(\cdot)$ is quasi-convex, the decision to invest may even be non-monotonic in α_o for values of α_o for which investing does not foreclose the market. Increasing α_o may make the incumbent switch from investing to not investing, as seen above, but the opposite may also occur.[14]

4.4 Stage 1: Regulation of the Old Technology

Next, we discuss the regulator's choice of the access price for the old technology.

The regulator's only instrument, α_o, impacts welfare directly and indirectly. First, α_o impacts welfare directly through $\alpha_n^*(\cdot)$. Second, α_o impacts welfare indirectly through the investment decision.

When indifferent between a duopoly with $\alpha_n^*(\cdot)$ and a monopoly, the regulator chooses the latter.

The next Remark presents the regulator's objective function, gross of investment costs.

Remark 1:

[13]This case occurs for z on $[z_1, \overline{z})$. A higher α_o also results in a higher α_n and, hence, in larger wholesale profit after investment. However, in this interval for z, the impact on wholesale profit is stronger without investment. Take, for instance, the limit case of $\alpha_o = 0$. Then, as $\left.\frac{\partial \alpha_n^*}{\partial \alpha_o}\right|_{\alpha_o=0} = 0$, a small increase in α_o does not change the incumbent's profit when there is investment and access is granted at $\alpha_n^*(\alpha_o;\chi)$. In contrast, the incumbent's profit when there is no-investment increases with α_o, given our assumption on z. Therefore, a small increase in the regulated access price decreases the incentive to invest.

[14]This case occurs for z on $(\underline{z}, z_1)$. As $\left.\frac{\partial \alpha_n^*}{\partial \alpha_o}\right|_{\alpha_o>0} < 1$, an increase in α_o leads to a smaller increase in $\alpha_n^*(\alpha_o;\chi)$. But, since $\alpha_n^*(\cdot)$ is convex in α_o, the increase in $\alpha_n^*(\cdot)$ gets closer to the increase in α_o the higher α_o is. Additionally, as the investment increases the number of units purchased by each consumer, an increase in $\alpha_n^*(\cdot)$ affects a larger number of units than the increase in α_o. This may increase the incentives to invest provided that α_o is high and that the increase in consumer's demand is large when compared to the initial demand, i.e, that for a given χ, z is sufficiently low.

(i) *For $\tau = o, n$, welfare is given by:*

$$W^s(\alpha_\tau) = \begin{cases} \frac{(z+v_\tau)^2}{2} - \frac{1}{2}t & s = m_\tau \\ \begin{cases} \frac{72t(z+v_\tau)^2 + 5\alpha_\tau^4 - 36t\left(t+\alpha_\tau^2\right)}{144t} & \text{for } \alpha_\tau \text{ on } \left[0, \sqrt{6t}\right) \\ \frac{(z+v_\tau)^2}{2} - \frac{1}{2}t & \text{for } \alpha_\tau \text{ on } \left[\sqrt{6t}, z+v_\tau\right] \end{cases} & s = d_\tau \end{cases}$$

with $W^{d_\tau}(0) > W^{m_\tau}$.

(ii) *Function $W^{d_\tau}(\alpha_\tau)$ is decreasing in α_τ for α_τ on $\left(0, \sqrt{\frac{18}{5}t}\right)$, and increasing in α_τ for α_τ on $\left(\sqrt{\frac{18}{5}t}, \sqrt{6t}\right)$.* ■

Figure 2 illustrates the welfare function, $W^{d_\tau}(\alpha_\tau)$.

$[Figure\ 2]$

Function $W^{d_\tau}(\cdot)$ is quasi-convex because the direct impact of increasing α_τ can be decomposed into the three following opposing effects. First, it has the negative effect of increasing transportation costs, because it moves the indifferent consumer towards the entrant. Second, it has the negative effect of leading the entrant to set a higher marginal retail price. Third, it has the positive effect of making some consumers shift from the entrant to the incumbent, where they face a lower marginal retail price. If $\alpha_\tau = 0$, the third effect is absent because both firms set the same marginal retail price. Thus, increasing α_τ unambiguously lowers welfare. If α_τ is sufficiently high, the third effect may more than compensate the other two.

From Figure 2, if both firms use the same technology and the entrant pays access price α_τ, duopoly is socially preferable to monopoly, $W^{d_\tau}(\alpha_\tau) > W^{m_\tau}$, if and only if, α_τ is on $\left[0, \sqrt{6t/5}\right)$. If the technology used is the new technology, the condition that $\alpha_n^*(\cdot)$ is on $\left[0, \sqrt{6t/5}\right)$ is equivalent to the condition that α_o is on $\left[0, \sqrt{6t/5 - \chi}\right)$. The intuition is straightforward. With a duopoly, compared with a monopoly, on the one hand, average transportation costs are lower, on the other hand, the consumers served by the entrant purchase a sub-optimal quantity. The reason is that since the access price is above marginal cost, the consumers served by the entrant patronize a higher priced firm than the consumers served by the incumbent. Hence, whether duopoly fares better or worse than monopoly, from a social point of view, depends on the value of α_τ. If there is investment, the relevant access price is $\alpha_n^*(\cdot)$, which in turn depends on (α_o, χ).

Given our assumption on I, and keeping α_τ constant, investment increases welfare.[15] However, the optimal α_n offered by the incumbent is different from the α_o set by the reg-

[15]Note that: $W^{d_n}(\alpha_\tau) - I - W^{d_o}(\alpha_\tau) = W^{m_n} - I - W^{m_o} = \chi/2 - I > 0$.

ulator. Thus, a duopoly with the old technology may be socially preferable to a duopoly with the new technology.

The next Lemma characterizes the socially optimal access price for the old technology.

Lemma 7: *Assume that only the incumbent can invest and that the innovation is non-drastic, i.e.,* χ *is on* $(0, 6t)$*. In equilibrium, the regulator sets:*

$$\alpha_o = \begin{cases} 0 & \text{for } \chi \text{ on } \left(0, \frac{6}{5}t\right) \\ \text{on } \left[\sqrt{6t - \chi}, +\infty\right) & \text{for } \chi \text{ on } \left[\frac{6}{5}t, 6t\right). \end{cases}$$

■

For low values of χ, and therefore for low values of $\alpha_n^*(\cdot)$, the regulator sets $\alpha_o = 0$, which leads to a duopoly, while for high values of χ, and therefore for high values of $\alpha_n^*(\cdot)$, the regulator sets an α_o on $\left[\sqrt{6t - \chi}, +\infty\right)$, which leads to a monopoly with the new technology.

The regulator setting a high α_o, or equivalently inducing a monopoly, when χ is high is somewhat counter-intuitive. The reason is, however, straightforward. If χ is high, and therefore if $\alpha_n^*(\cdot)$ is also high, the welfare loss caused by the exercise of market power by a monopolist incumbent is smaller than the welfare loss caused by the entrant having a high marginal cost.

The assumption that firms charge two-part retail tariffs plays an important role in the result that monopoly fares better than duopoly in terms of welfare. With two-part retail tariffs a monopolist sells the socially optimal number of units and extracts all consumer surplus through the fixed fee. Hence, monopoly involves no deadweight loss.

It is never optimal for the regulator to induce no-investment, and thereby a duopoly with the old technology. That is only possible with a high α_o, in which case a duopoly with the new technology and $\alpha_n^*(0; \chi)$, or, a monopoly with the new technology, are both socially preferable to no-investment.

4.5 Equilibrium of the Whole Game

The next Proposition summarizes, for further reference, the equilibrium of the whole game, excluding stage 4.

Proposition 1a: *Assume that only the incumbent can invest and that the innovation is non-drastic, i.e.,* χ *is on* $(0, 6t)$*.*

(I) *If* χ *is on* $\left(0, \frac{6}{5}t\right)$*:* ***(i)*** *the regulator sets* $\alpha_o = 0$*,* ***(ii)*** *the incumbent invests and offers* $\alpha_n^*(\cdot) = \sqrt{\chi}$*, and* ***(iii)*** *the entrant uses the new technology.*

(II) *If* χ *is on* $\left[\frac{6}{5}t, 6t\right)$*:* ***(i)*** *the regulator sets* α_o *on* $\left[\sqrt{6t-\chi}, +\infty\right)$*,* ***(ii)*** *the incumbent invests and offers* $\alpha_n^*(\cdot)$ *on* $\left[\sqrt{6t}, +\infty\right)$*, and (iii) the entrant exits the industry.* ■

There are two types of equilibria, depending on the value of the quality improvement enabled by the new technology, χ. If the innovation is *non-drastic*, the entrant, using the old technology, can compete with the incumbent, using the new technology, if α_o is low. Hence, the regulator can influence both whether the incumbent invests, and the industry is a monopoly or a duopoly. For low values of χ, the regulator sets $\alpha_o = 0$, which leads to a duopoly with the new technology, while for high values of χ, the regulator sets an α_o on $\left[\sqrt{6t-\chi}, +\infty\right)$, which leads to a monopoly with the new technology.

5 Two Firms Can Deploy the New Technology

In this section, we analyze the case where both the incumbent and the entrant can invest in the new technology and innovation is *non-drastic*. The case where both the incumbent and the entrant can invest and innovation is *drastic* is analyzed in section 6

5.1 Preliminaries

Consider the model of section 3, except that both the incumbent and the entrant can invest in the new technology. In particular: **(i)** in stage 2, both firms decide whether to invest; if only one firm invests, it makes an access price offer to the rival; and, **(ii)** in stage 3, if one of the firms did not invest, it chooses which technology to use, if any. We assume that the entrant and the incumbent have the same investment cost. To avoid the proliferation of cases, with no added economic insights, we restrict our attention to the case where t is on $(0, \chi)$ and $\Delta\Pi_i(\alpha_o) > \frac{1}{2}t$. At the end of section 5.3, we explain briefly what changes if we do not assume this. As before, we solve the game by backward induction.

If only the incumbent invests, the game unfolds as in section 4.

If only the entrant invests, or if both firms invest, Lemmas 1 to 3 continue to apply, with the obvious changes.[16]

[16] If only the entrant invests and innovation is *non-drastic*, the gross profits of the entrant and the incumbent are, respectively: $\pi_i^{d_n}(\alpha_n)$ and $\pi_e^{d_n}(\alpha_n)$. If both firms invest, the gross profits of either firm are $\pi_i^{d_n}(0)$.

The reasoning of Lemmas 4 and 5 also continues to apply. Consider the case where only the entrant invests. For all α_o on $[0, +\infty)$, the entrant offers $\alpha_n^*(0; \chi) = \sqrt{\chi}$, and the incumbent uses the new technology. Now consider the case where both firms invest. For all α_o on $[0, +\infty)$, each firm uses its technology and no access prices offers are made.

5.2 Stage 2: Investment Game

Next, we characterize the equilibrium investment.

Denote by

$$\Delta\Pi_{e|I}(\alpha_o) := \begin{cases} \pi_i^{d_n}(0) - \pi_e^{d_n}(\alpha_n^*(\alpha_o; \chi)) & \text{if } \alpha_o \text{ on } \left[0, \sqrt{6t - \chi}\right) \\ \pi_i^{d_n}(0) & \text{if } \alpha_o \text{ on } \left[\sqrt{6t - \chi}, +\infty\right), \end{cases}$$

the entrant's incremental profit from the investment, given that the incumbent invested.

The following Lemma presents the equilibrium of the investment game.

Lemma 6′: *Assume that both the incumbent and the entrant can invest and that the innovation is non-drastic, i.e.,* χ *is on* $(0, 6t)$*. In equilibrium:*
(i) *both firms invest in the new technology, if and only if,* (α_o, I) *is on* $[0, +\infty) \times \left[0, \Delta\Pi_{e|I}(0)\right)$*;*
(ii) *only the entrant invests in the new technology, if and only if,* (α_o, I) *is on* $[0, +\infty) \times \left[\Delta\Pi_{e|I}(0), \Delta\Pi_{e|I}(\alpha_o)\right) \cup \left[0, \sqrt{6t - \chi}\right) \times \left[\Delta\Pi_i(\alpha_o), \frac{1}{2}\chi\right)$*;*
(iii) *either only the entrant invests or only the incumbent invests, if and only if,* (α_o, I) *is on* $\left[0, \sqrt{6t - \chi}\right) \times \left[\Delta\Pi_{e|I}(\alpha_o),\ \min\left\{\Delta\Pi_i(\alpha_o), \frac{1}{2}\chi\right\}\right) \cup \left[\sqrt{6t - \chi}, +\infty\right) \times \left[\Delta\Pi_{e|I}(\alpha_o), \frac{1}{2}\chi\right)$*.* ■

Figure 3 represents Lemma 6′ in the $(\alpha_o, I)-$space.

[*Figure* 3]

If the investment cost is low, there is a unique equilibrium where both firms invest. If the investment cost is high, either there is a unique equilibrium where only the entrant invests, or two equilibria coexist: in one only the incumbent invests, and in the other only the entrant invests. In either case, investment always occurs in equilibrium.

The case where only the entrant invests occurs for a larger set of parameter values than the case where only the incumbent invests. This happens because the incumbent pays a lower access price when it asks for access to the rival's technology than the entrant, given that it has an outside option of using the old technology at a lower price than the entrant. The comparison of Lemmas 6 and 6′ shows that when both firms can invest, the set of parameter values for which investment occurs is slightly larger than when only the incumbent can invest.

5.3 Stage 1: Regulation of the Old Technology

Next, we characterize the socially optimal access price for the old technology.

Denote by $\alpha_o^{**}(I)$, the highest access price for the old technology for which the equilibrium where only the incumbent invests and the equilibrium where only the entrant invests coexist, i.e., $\Delta\Pi_{e|I}(\alpha_o^{**}) \equiv I$, and denote by I^* the investment cost level for which the regulator is indifferent between setting $\alpha_o = 0$ or $\alpha_o = \alpha_o^{**}(I^*)$ i.e., $W^{d_n}(0) \equiv W^{d_n}(\alpha_o^{**}(I^*))$.[17]

Lemma 7′: *Assume that both the incumbent and the entrant can invest and that the innovation is non-drastic, i.e., χ is on $(0, 6t)$. In equilibrium, the regulator sets:*

$$\alpha_o = \begin{cases} \text{on } [0, +\infty) & \text{for } (\chi, I) \text{ on } (0, 6t) \times \left[0, \Delta\Pi_{e|I}(0)\right) \\ 0 \text{ or on } (\alpha_o^{**}(I), +\infty) & \text{for } (\chi, I) \text{ on } \left(0, \frac{6}{5}t\right) \times \left[\Delta\Pi_{e|I}(0), \frac{1}{2}\chi\right) \cup \\ & \qquad \left[\frac{6}{5}t, \frac{18}{5}t\right) \times \left[\Delta\Pi_{e|I}(0), I^*\right) \\ \alpha_o^{**}(I) & \text{for } (\chi, I) \text{ on } \left[\frac{6}{5}t, \frac{18}{5}t\right) \times \left[I^*, \pi_i^{d_n}(0)\right) \cup \\ & \qquad \left[\frac{18}{5}t, 6t\right) \times \left[\Delta\Pi_{e|I}(0), \pi_i^{d_n}(0)\right) \\ \text{on } \left[\sqrt{6t - \chi}, +\infty\right) & \text{for } (\chi, I) \text{ on } \left[\frac{6}{5}t, 6t\right) \times \left[\pi_i^{d_n}(0), \frac{1}{2}\chi\right). \end{cases}$$

■

Let the investment cost be low, i.e., let I be on $\left[0, \Delta\Pi_{e|I}(0)\right)$, since both firms invest, α_o is irrelevant. Otherwise, the regulator's choice of α_o determines not only which firm invests, but also whether the market is a monopoly or a duopoly.

Let I take intermediate values, i.e., let I be on $\left[\Delta\Pi_{e|I}(0), \pi_i^{d_n}(0)\right)$. Keep figure 3 in mind. By setting a price above $\alpha_o^{**}(I)$, the regulator induces an equilibrium with investment by the entrant, whereas by setting access price below $\alpha_o^{**}(I)$ the regulator induces one of two equilibria, where either the entrant or the incumbent invest. Investment by the entrant leads to a duopoly with the new technology with $\alpha_n^*(0; \chi) = \sqrt{\chi}$ and welfare level $W^{d_n}(\alpha_n^*(0; \chi))$, while investment by the incumbent leads to a duopoly with the new technology with a higher access price, $\alpha_n^*(\alpha_o; \chi) = \sqrt{\alpha_o^2 + \chi}$, and welfare level $W^{d_n}(\alpha_n^*(\alpha_o; \chi))$. However, recall from figure 2 that welfare is concave in the access price. Hence, depending on the values of $\alpha_n^*(0; \chi)$ and $\alpha_n^*(\alpha_o; \chi)$, the regulator may prefer to have investment by the entrant or investment

[17] The expression for I^* is presented in the appendix, and $\alpha_o^{**}(I)$ is defined implicitly by $\Delta\Pi_{e|I}(\alpha_o^{**}) \equiv I$.

by the incumbent, as $W^{d_n}\left(\alpha_n^*(\alpha_o;\chi)\right)$ can be higher or lower than $W^{d_n}\left(\alpha_n^*(0;\chi)\right)$. If χ is low, by the same reason as in Lemma 5, duopoly with the new technology with a low access price, $\alpha_n^*(0;\chi) = \sqrt{\chi}$, is socially preferable to a duopoly with any higher access price. Thus, the regulator sets $\alpha_o = 0$, and one of the firms invests and offers access to the rival at $\alpha_n^*(0;\chi) = \sqrt{\chi}$ or sets any α_o on $(\alpha_o^{**}(I), +\infty)$ and only the entrant invests and offers access to the rival at $\alpha_n^*(0;\chi) = \sqrt{\chi}$. If χ takes intermediate values, welfare is initially decreasing in α_o, and then increasing in α_o. Hence, it is possible that welfare with access price $\alpha_n^*(\alpha_o;\chi) = \sqrt{\alpha_o^2 + \chi}$ exceeds welfare with access price $\alpha_n^*(0;\chi) = \sqrt{\chi}$, provided that α_o is large enough. However, to induce $\alpha_n^*(\alpha_o;\chi)$ the regulator must set α_o no higher than $\alpha_o^{**}(I)$ because, otherwise, the only equilibrium is investment by the entrant. Hence, the regulator will either choose an access price of 0 or $\alpha_o^{**}(I)$. If $\alpha_o^{**}(I)$ is small, then $W^{d_n}\left(\alpha_n^*(0;\chi)\right) > W^{d_n}\left(\alpha_n^*(\alpha_o^{**}(I);\chi)\right)$, and if $\alpha_o^{**}(I)$ is large enough then $W^{d_n}\left(\alpha_n^*(0;\chi)\right) < W^{d_n}\left(\alpha_n^*(\alpha_o^{**}(I);\chi)\right)$. Since $\alpha_o^{**}(I)$ is increasing in I, the regulator chooses $\alpha_o^{**}(I)$, if and only if, I is sufficiently large, i.e., if I is on $\left[I^*, \pi_i^{d_n}(0)\right)$. For lower values of I, i.e., if I is on $\left[\Delta\Pi_{e|I}(0), I^*\right)$, the regulator should set $\alpha_o = 0$ or any α_o on $(\alpha_o^{**}(I), +\infty)$, and one of the firms invests, and offers access to the rival at $\alpha_n^*(0;\chi) = \sqrt{\chi}$. Finally, if χ is high, welfare under duopoly with the new technology with $\alpha_n^*(\alpha_o;\chi)$ is always increasing in α_o. Therefore, the regulator should set $\alpha_o = \alpha_o^{**}(I)$ for all I.

Let I be high, i.e., let I be on $\left[\pi_i^{d_n}(0), \frac{1}{2}\chi\right)$. If χ is low, the optimal access price is as in the previous paragraph. If χ is large, monopoly is socially preferable to a duopoly. However, since there are multiple equilibria, the regulator cannot impose this outcome. If one assumes that all equilibria are equally probable, the regulator should set α_o on $\left[\sqrt{6t-\chi}, +\infty\right)$, so that monopoly occurs with some probability.

[*Table* 1]

In the appendix, we present Proposition $1a'$ that summarizes, for the case where both the incumbent and the entrant can invest in the new technology and the innovation is *non-drastic*, the equilibrium of the whole game, excluding stage 4. The information with respect to the equilibrium access price, investment, and welfare is presented in table 1. Recall that the superscript in the welfare function indicates the corresponding market structure. We also present the results for the case where only one firm can invest to make the welfare comparison in section 5.4 easier.[18]

[18] The case where each firm operates the old technology and can deploy a new technology, i.e., the case where there are two incumbents, can be seen as a particular case where an incumbent and an entrant can deploy new technologies and the regulator sets $\alpha_o = 0$. There are only two types of equilibria. If the

Note that if t is on $[\chi, +\infty)$, the equilibrium where the incumbent invests and becomes a monopolist disappears. This happens because the entrant always invests. Since I is on $[0, \chi/2)$, it follows that $\pi_i^{d_n}(0) - I > 0$, or equivalently, that $I < \pi_i^{d_n}(0) = \frac{1}{2}t$. If $\Delta\Pi_i(\alpha_o) < \frac{1}{2}t$, function $\alpha_o^{**}(I)$ is defined by two branches and is no longer increasing in I. This would increase the complexity of Lemma 7 without any additional economic insights.

5.4 Welfare

Next, we compare the welfare levels of the case where only the incumbent can invest and the case where both the incumbent and the entrant can invest, assuming that the innovation is *non-drastic.*

The next Proposition presents the welfare comparison.

Proposition 2a: *Let the innovation be non-drastic, i.e., let χ be on $(0, 6t)$.*

(i) *If (χ, I) is on $(0, 6t) \times \left[0, \min\left\{\Delta\Pi_{e|I}(0), \frac{1}{4}t\right\}\right)$, then, when both firms can invest in the new technology, compared with the case where only the incumbent can invest, welfare increases.*

(ii) *If (χ, I) is on $\left[\frac{6}{5}t, 6t\right) \times \left[\min\left\{\Delta\Pi_{e|I}(0), \frac{1}{4}t\right\}, \frac{1}{2}\chi\right)$, then, when both firms can invest in the new technology, compared with the case where only the incumbent can invest, welfare decreases.*

(iii) *If (χ, I) is on $\left(0, \frac{6}{5}t\right) \times \left[\Delta\Pi_{e|I}(0), \frac{1}{2}\chi\right)$, then, when both firms can invest in the new technology, compared with the case where only the incumbent can invest, expected welfare is the same.* ■

The impact on welfare of both firms being able to invest, instead of just the incumbent, involves three effects. If, in equilibrium, both firms invest, there is a trade-off between the positive effect of the marginal retail prices being set at marginal cost, i.e., double-marginalization is eliminated, and the negative effect of the investment cost being duplicated.[19] If, in equilibrium, only one firm invests, there may be the negative effect of an

investment cost is low both firms invest. If the investment cost is high only one firm invests and offers access at $\alpha_n^*(0; \chi) = \sqrt{\chi}$.

[19] With linear tariffs, a wholesaler with market power sells its product to a retailer with a mark-up above marginal cost. If the retailer also has market power it then adds another mark-up to its own marginal cost and the retail price includes two mark-ups. This is called double-marginalization. In our model since there are two-part retail tariffs and in equilibrium firms charge a marginal retail price equal to marginal cost, in a strict sense there is no double-marginalization. However, given its widespread use, we stick to the

excessive number of firms being present in the industry.

Figure 4 illustrates, in the $(\chi, I)-$space, the change in expected welfare when both firms are able to invest, compared with the case where only the incumbent can invest.

$[Figure\ 4]$

If I is low, both firms invest. When χ is also low, if both firms can invest, instead of just the incumbent, welfare increases, if and only if, $W^{d_n}(0) - W^{d_n}(\alpha_n^*(0;\chi)) > I$, which holds for all parameter values. However, when χ is high, welfare increases, if and only if, $W^{d_n}(0) - W^{m_n} > I$, or equivalently, if and only if, I is on $\left[0, \frac{1}{4}t\right)$, and decreases otherwise.

If I is high, only one firm invests. When χ is low, then, from Propositions 1 and 1′, one firm invests, offers $\alpha_n^*(0;\chi) = \sqrt{\chi}$, and the entrant uses the new technology. In either case, welfare remains the unchanged: $W^{d_n}(\sqrt{\chi})$. When χ is high, and if only the incumbent can invest, then, from Proposition 1a, it invests, the entrant exits the industry and welfare is given by W^{m_n}. However, if both firms can invest, from Proposition 1a', an equilibrium exists such that one firm invests and the rival uses the new technology. As welfare under duopoly at the equilibrium access prices is lower than W^{m_n}, if both firms are able to invest, instead of just the incumbent, the expected welfare decreases.

The case where (χ, I) is on $\left[\frac{6}{5}t, 6t\right) \times \left[\Delta\Pi_{e|I}(0), \frac{1}{2}\chi\right)$, discussed in the previous paragraph, is interesting because the possibility of investment by two firms, instead of only one, lowers welfare for reasons other than the duplication of the investment cost. Value $\alpha_n^*(0;\chi)$ is increasing in χ. From figure 2, if $\alpha_n^*(0;\chi)$ is high enough, monopoly is socially preferable to a duopoly. Hence, if χ is high, a monopoly with the new technology is socially preferable to a duopoly. If only the incumbent can invest, the regulator can induce a monopoly with the new technology by setting a high α_o. However, if the two firms can invest, but only one does in equilibrium, setting a high α_o does not necessarily induce a monopoly. The equilibrium where only the incumbent invests coexists with the equilibrium where only the entrant invests. In that latter case, the incumbent cannot be expelled from the industry. In fact, since it has the option of paying $\alpha_o = 0$ for the old technology, the entrant prefers to offer access to the incumbent to the new technology, as discussed in section 4.3. Hence, both firms may be active when a monopoly is socially preferable to a duopoly.[20]

term double-marginalization to refer to the existence of a wholesale mark-up that will lead to suboptimal consumption.

[20]Given footnote 18, welfare when each firm is vertically integrated and can deploy a new technology is no higher than when an incumbent and an entrant can deploy new technology. This happens because the case of two incumbents is formally equivalent to forcing the regulator to set $\alpha_o = 0$. As seen in Proposition 1a' this is not always optimal.

6 Drastic Innovation

In this section, we analyze the case where the innovation is *drastic*.

If only the entrant invests, or if both firms invest, Lemmas 1 to 3 continue to apply, with the obvious changes. For example, when the firms use different technologies: $6t - D_b < 0$. Hence, value $\sqrt{6t - D_b}$ should be replaced by 0.

The following Proposition summarizes the equilibrium of the game for the case where only the incumbent can invest.

Proposition 1b: *Assume that only the incumbent can invest and that the innovation is drastic, i.e., χ is on $[6t, +\infty)$. In equilibrium:* ***(i)*** *the regulator sets any α_o on $[0, +\infty)$,* ***(ii)*** *the incumbent invests and offers $\alpha_n^*(\cdot)$ on $\left[\sqrt{6t}, +\infty\right)$, and* ***(iii)*** *the entrant exits the industry.* ■

If the innovation is *drastic*, the entrant, using the old technology, cannot compete against the incumbent, using the new technology, even when access to the old technology is priced at marginal cost. Thus, for all α_o, the incumbent offers an unacceptably high access price, which induces the entrant to exit the industry, and becomes a monopolist. Hence, the regulator is unable to influence the market outcome.

We now turn to the case where both firms can invest, whose equilibrium is summarized in the following Proposition.[21]

Proposition 1b': *Assume that both the incumbent and the entrant can invest and that the innovation is drastic, i.e., χ is on $[6t, +\infty)$. In equilibrium:*

(I) *If I is on $\left[0, \Delta\Pi_{e|I}(0)\right)$:* ***(i)*** *the regulator sets any α_o, and* ***(ii)*** *both firms invest.*

(II) *If I is on $\left[\Delta\Pi_{e|I}(0), \frac{1}{2}\chi\right)$:* ***(i)*** *the regulator sets any α_o,* ***(ii)*** *one of the firms invests and offers $\alpha_n^*(\cdot)$ on $\left[\sqrt{6t}, +\infty\right)$, and* ***(iii)*** *the rival exits the industry.* ■

If the investment cost is low, both firms invest. If the investment cost is high, only one firm invests, it offers $\alpha_n^*(\cdot)$ on $\left[\sqrt{6t}, +\infty\right)$, for all α_o on $[0, +\infty)$, and the rival exits the industry. Hence, α_o is again irrelevant. Hence, if both firms can invest, instead of just the incumbent, *drastic* innovation need not lead to monopoly.

Next we compare the welfare level for the case where only the incumbent can deploy a

[21] Note that: $\Delta\Pi_{e|I}(0) = \pi_i^{d_n}(0)$, for χ on $[6t, +\infty)$.

new technology and the game where both firms can deploy the new technology.

Proposition 2b**:** *Let the innovation be drastic, i.e., let* χ *be on* $[6t, +\infty)$.

(i) *If* I *is on* $\left[0, \frac{1}{4}t\right)$*, then, when both firms can invest in the new technology, compared with the case where only the incumbent can invest, welfare increases.*

(ii) *If* I *is on* $\left[\frac{1}{4}t, \Delta\Pi_{e|I}(0)\right)$*, then, when both firms can invest in the new technology, compared with the case where only the incumbent can invest, welfare decreases.*

(iii) *If* I *is on* $\left[\Delta\Pi_{e|I}(0), \frac{1}{2}\chi\right)$*, then, when both firms can invest in new technology, compared with the case where only the incumbent can invest, expected welfare is the same.*■

If I is low, both firms invest. Welfare increases, if and only if, $W^{d_n}(0) - W^{m_n} > I$, or equivalently, if I is on $\left[0, \frac{1}{4}t\right)$ and decreases otherwise. Again, this results from the trade-off between the positive effect of the elimination of double-marginalization, and the negative effect of the duplication of the investment costs.

If the investment cost is high, only one firm invests. If only the incumbent can invest, then, from Proposition 1b, it invests and becomes a monopolist. If both firms can invest, from Proposition 1b', only one of the firms invests and also becomes a monopolist. In either case welfare is W^{m_n}. Hence, if both firms can invest, instead of just the incumbent, welfare does not change.[22]

7 Concluding Remarks and Policy Implications

In this article, we analyzed the incentives of a vertically integrated firm, which is a regulated monopolist in the wholesale market and competes with an entrant in the retail market: **(i)** to invest in a new unregulated technology, and **(ii)** to give access to the entrant to the new technology. For both of these issues the analysis depends on whether the innovation is *drastic* or *non-drastic*.

If the innovation is *non-drastic*, the concern that the industry might be monopolized is not justified. By setting a low enough access price for the new technology, the regulator can ensure that the entrant, using the old technology, can compete against the vertically integrated firm, using the new technology. Since the vertically integrated firm cannot avoid

[22] When each firm operates the old technology and can deploy the new technology, for a drastic innovation, and if I is on $\left[0, \pi_i^{d_n}(0)\right)$, both firms invest and welfare is equal to $W^{d_n}(0)$. Otherwise, only one firm invests and becomes a monopolist with welfare given by W^{m_n}. Thus, the welfare level is the same as in the case where only one firm operates the old technology but both can invest.

competition from the entrant, it concedes access to the new technology. Interestingly, if the quality improvement enabled by the new technology is *non-drastic* and low, a duopoly with the new technology is socially optimal, whereas if the quality improvement enabled by the new technology is *non-drastic* but high, a monopoly with the new technology is socially optimal. This latter policy can be loosely interpreted as a regulatory *moratorium*.

We justified the assumption that only the vertically integrated firm can invest in the deployment of the new technology by some disadvantage of the entrant. In some circumstances the disadvantage can be overcome by public policies. For example, the entrant can be given access to credit, or the regulation that hinders the deployment of the infrastructure required to support the new technology can be softened. Implementing such policies poses at least two types of practical problems. First, sectoral regulators typically do not have the instruments required. Second, these policies could be perceived as state aid, which is restricted in some jurisdictions, like the EU. Furthermore, if the innovation is *non-drastic*, the case for such public policies is not very strong, since the concern that the industry might be monopolized is not justified. Nevertheless, such policies could still be justified if the possibility that both firms invest, instead of just the vertically integrated firm, increases welfare.

When the innovation is *non-drastic*, the possibility of both firms investing, instead of just the vertically integrated firm, may or may not increase welfare, if the investment cost is low, and at best leaves welfare unchanged, if the investment cost is high.

If the innovation is *drastic*, the concern that the industry might be monopolized by the vertically integrated firm is justified. Since the entrant, using the old technology, cannot compete against the vertically integrated firm, using the new technology, even when the access price for the old technology is set at marginal cost, by denying access to the new technology, the vertically integrated firm can expel the entrant from the industry. Two types of policies could be used to remedy this situation. First, the regulator could promote investment by the entrant. However, the possibility of both firms investing, instead of just the vertically integrated firm, solves the monopolization problem, but only if the investment cost is low. Furthermore, welfare is not guaranteed to increase. Second, open access obligations could be extended to the new technology. However, as shown by Brito et al. (2010), unless the regulator can commit to a regulatory policy, which is problematic if the investment cycle is long, open access obligations can reduce, or even eliminate, the vertically integrated firm's incentives to invest. Hence, when the innovation is *drastic*, policy makers face difficult choices.

Appendix

In the appendix we present the proof of all the results. For convenience we present the results for a non-drastic innovation together with those for a drastic innovation.

Lemma 1: See Biglaiser and DeGraba (2001). ■

Lemma 2: A consumer located at distance x from a monopolist using technology $v_{\tau_i} = o, n$ purchases if and only if

$$\frac{\left(z + v_{\tau_i}\right)^2}{2} - tx - F_i > 0 \Leftrightarrow x < \widehat{x}(F_i) := \frac{1}{t}\left[\frac{1}{2}\left(z + v_{\tau_i}\right)^2 - F_i\right].$$

with $\widehat{x}(F_i)$ on $[0,1]$, i.e., F_i on $\left[\frac{1}{2}\left(z + v_{\tau_i}\right)^2 - t, \frac{1}{2}\left(z + v_{\tau_i}\right)^2\right]$. The monopolist's profits are given by $\pi_i^{m_{\tau_i}} = F_i\widehat{x}(F_i)$ with

$$\frac{\partial \pi_i^{m_{\tau_i}}}{\partial F_i} = \widehat{x}(F_i) - \frac{F_i}{t} \text{ and } \frac{\partial^2 \pi_i^{m_{\tau_i}}}{\partial F_i^2} = \frac{-2}{t} < 0.$$

As, given our assumption on z,

$$\left.\frac{\partial \pi_i^{m_{\tau_i}}}{\partial F_i}\right|_{F_i = \frac{1}{2}\left(z+v_{\tau_i}\right)^2 - t} = 2 - \frac{1}{2}\left(z + v_{\tau_i}\right)^2 < 0$$

the optimal fixed charge and profits are:

$$F_i^{m_{\tau_i}} = \pi_i^{m_{\tau_i}} = \frac{1}{2}\left(z + v_{\tau_i}\right)^2 - t.$$

■

Lemma 3: We start by finding the consumer who is indifferent between buying from the incumbent or from the entrant:

$$\begin{aligned}
\frac{\left(z + v_{\tau_i}\right)^2}{2} - tx - F_i &= \frac{\left(z + v_{\tau_e} - \alpha_{\tau_e}\right)^2}{2} - t(1-x) - F_e \Leftrightarrow \\
x(\alpha) &= \frac{1}{2} - \frac{F_i - F_e}{2t} - \frac{\left(z + v_{\tau_e} - \alpha_{\tau_e}\right)^2 - \left(z + v_{\tau_i}\right)^2}{4t}.
\end{aligned}$$

The demand function, in terms of consumers, is for $k = o, n, b$:

$$\begin{aligned}
\sigma_i^k &= \begin{cases} 0 & F_i > F_e + \frac{\alpha\left(2\left(z+v_{\tau_e}\right)-\alpha_{\tau_e}\right)+D_k}{2} + t \\ 1 & F_i < F_e + \frac{\alpha\left(2\left(z+v_{\tau_e}\right)-\alpha_{\tau_e}\right)+D_k}{2} - t \\ x(\alpha_{\tau_e}) & \text{else} \end{cases} \\
\sigma_e^k &= 1 - D_i.
\end{aligned}$$

Given this indifferent consumer, and the fact that $p_i = 0$ and $p_e = \alpha_{\tau_e}$, profit functions, excluding investment costs, become:

$$\begin{aligned}\pi_i &= F_i x(\alpha) + \alpha\left(z + v_{\tau_e} - \alpha_{\tau_e}\right)\left(1 - x(\alpha_{\tau_e})\right)\\ \pi_e &= F_e(1 - x(\alpha_{\tau_e})).\end{aligned}$$

Maximizing each profit function with respect to the fixed fee, we find:

$$\begin{aligned}F_i^{d_k} &= t + \frac{1}{6}\alpha_{\tau_e}\left(6\left(z + v_{\tau_e}\right) - 5\alpha_{\tau_e}\right) + \frac{1}{6}D_k\\ F_e^{d_k} &= t - \frac{1}{6}\alpha_{\tau_e}^2 - \frac{1}{6}D_k.\end{aligned}$$

The indifferent consumer is given by

$$x^* = \frac{1}{2} + \frac{1}{12t}\left(D_k + \alpha_{\tau_e}^2\right),$$

for $\alpha_{\tau_e} < \sqrt{6t - D_k}$.

We now have to ensure that all consumers have a positive surplus, independently of the market structure considered:

$$\begin{aligned}\frac{\left(z + v_{\tau_i}\right)^2}{2} - tx^* - F_i^{d_k} &> 0 \Leftrightarrow\\ 6t + D_k + 4\alpha_{\tau_e}\left(z + v_{\tau_e}\right) - 2\left(z + v_{\tau_i}\right)^2 - 3\alpha_{\tau_e}^2 &< 0.\end{aligned}$$

When both firms use the same technology it must be that $6t + 4\left(z + v_{\tau_i}\right)\alpha - 2\left(z + v_{\tau_i}\right)^2 - 3\alpha^2 < 0$ for all $\alpha < \sqrt{6t}$. This expression is maximized when $\alpha = \frac{2}{3}\left(z + v_{\tau_i}\right)$ at $6t - \frac{2}{3}\left(z + v_{\tau_i}\right)^2$ if $\left(z + v_{\tau_i}\right) < \frac{3}{2}\sqrt{6t}$. If $\left(z + v_{\tau_i}\right) > \frac{3}{2}\sqrt{6t}$ the maximum is $-2((z + v_{\tau_i}) - \sqrt{6t})^2 < 0$ obtained at $\alpha = \sqrt{6t}$. Hence, we must have $6t - \frac{2}{3}\left(z + v_{\tau_i}\right)^2 < 0 \Leftrightarrow \left(z + v_{\tau_i}\right) > 3\sqrt{t}$ if $\left(z + v_{\tau_i}\right) < \frac{3}{2}\sqrt{6t}$.

When the entrant uses the old and the incumbent uses the new technology we must have $6t + 4\alpha_o z - 2z^2 - 3\alpha_o^2 - \chi < 0$ for all $\alpha_o < \sqrt{6t - \chi}$. This function takes maximum value, $6t - \frac{2}{3}z^2 - \chi$, at $\alpha_o = \frac{2}{3}z$ if $z < \frac{3}{2}\sqrt{6t}$. If $z > \frac{3}{2}\sqrt{6t}$, the maximum is $-2(z - \sqrt{6t})^2 - \chi < 0$. Hence, it must be that $6t - \frac{2}{3}z^2 - \chi < 0 \Leftrightarrow z > \frac{1}{2}\sqrt{3}\sqrt{12t + v^2} - \frac{3}{2}v$ for $z < \frac{3}{2}\sqrt{6t}$. As $\sqrt{9t + \frac{3}{4}v^2} - \frac{3}{2}v < 3\sqrt{t} - v < 3\sqrt{t}$ all restrictions are verified for $z > 3\sqrt{t}$.

When $\alpha_{\tau_e} \in \left[\sqrt{6t - D_k}, z + v_{\tau_e}\right]$ the entrant will set $F_e^{d_k} = 0$. In this case, the incumbent's demand is

$$\sigma_i^k = \begin{cases} 0 & F_i > \frac{\alpha_{\tau_e}(2(z + v_{\tau_e}) - \alpha_{\tau_e}) + D_k}{2} + t\\ 1 & F_i < \frac{\alpha_{\tau_e}(2(z + v_{\tau_e}) - \alpha_{\tau_e}) + D_k}{2} - t\\ x(\alpha_{\tau_e}) & \text{else}\end{cases}.$$

The incumbent's best response is to set $F_i^d = \alpha_{\tau_e}(z + v_{\tau_e}) - \frac{1}{2}\alpha_{\tau_e}^2 - t + \frac{1}{2}D_k$. This F_i is set in order to induce the consumer located at 1 to choose the incumbent. If $\alpha_{\tau_e} \geq z + v_{\tau_e}$, no consumer will ever choose the entrant and the incumbent is effectively a monopolist. ■

Lemma 4: Consider initially that there is no investment in the new technology: If $\alpha_o < \sqrt{6t}$, the entrant accepts α_o; otherwise, it exits the industry. Consider now that there is investment in the new technology:

(i) Assume $\alpha_o < \sqrt{6t - \chi}$. This means that accepting α_o results in a positive market share for the entrant. Then, if

$$\pi_e^{d_n}(\alpha_n) \geq \pi_e^{d_b}(\alpha_o) \Leftrightarrow \alpha_n \leq \sqrt{\alpha_o^2 + \chi} < \sqrt{6t},$$

the entrant accepts α_n; if $\alpha_n > \sqrt{\alpha_o^2 + \chi}$, it accepts α_o.

(ii) Assume that $\alpha_o \geq \sqrt{6t - \chi}$. This means that accepting α_o does not result in a positive market share for the entrant. Then, if $\alpha_n < \sqrt{6t}$, the entrant accepts α_n; if $\alpha_n \geq \sqrt{6t}$, it exits the industry.

If $6t \leq \chi$ the entrant would have a non-positive market for any $\alpha_o \geq 0$. ■

Lemma 5: We start by showing that the incumbent's profit function, $\pi_i^{d_k}(\alpha_{\tau_e})$, increases in α_{τ_e} for all $\alpha_{\tau_e} < \sqrt{6t - D_k}$.

First note that $\frac{\partial \pi_i^{d_k}(\alpha_{\tau_e})}{\partial \alpha_{\tau_e}} = \frac{1}{18t}\left(18t(z + v_{\tau_e}) + \alpha_{\tau_e}(D_k - 30t) + \alpha_{\tau_e}^3\right)$, $\left.\frac{\partial \pi_i^{d_k}(\alpha_{\tau_e})}{\partial \alpha_{\tau_e}}\right|_{\alpha_{\tau_e}=0} = (z + v_{\tau_e}) > 0$ and $\frac{\partial^2 \pi_i^{d_k}(\alpha_{\tau_e})}{\partial \alpha_{\tau_e}^2} = \frac{1}{18t}\left(D_k + 3\alpha_{\tau_e}^2 - 30t\right) < 0$. Thus, the incumbent's profit increases with α_{τ_e} if $\left.\frac{\partial \pi_i^{d_k}(\alpha_{\tau_e})}{\partial \alpha_{\tau_e}}\right|_{\alpha_{\tau_e}=\sqrt{6t-D_k}} > 0$.

Additionally, $\frac{\partial\left(\left.\frac{\partial \pi_i^{d_k}(\alpha_{\tau_e})}{\partial \alpha_{\tau_e}}\right|_{\alpha_{\tau_e}=\sqrt{6t-D_k}}\right)}{\partial D_k} = \frac{2}{3}\frac{1}{\sqrt{6t-D_k}} > 0$. Hence, $\frac{\partial \pi_i^{d_k}(\alpha_{\tau_e})}{\partial \alpha_{\tau_e}} > 0$ for all $\alpha_{\tau_e} < \sqrt{6t - D_k}$ if $\frac{1}{18t}\left(18t(z + v_{\tau_e}) + \sqrt{6t - 0}\,(0 - 30t) + (6t - 0)^{\frac{3}{2}}\right) > 0 \Leftrightarrow z + v_{\tau_e} > \frac{4}{3}\sqrt{6t}$, which is true given our assumption on z.

Assume initially that $\alpha_o < \sqrt{6t - \chi}$. We start by finding out the best offer for α_n in the incumbents perspective that is accepted by the entrant. This is the solution to

$$\max_{\alpha_n} \pi_i^{d_n}(\alpha_n),$$

subject to $\alpha_n \leq \sqrt{\alpha_o^2 + \chi}$. Therefore, the optimal access price is $\alpha_n^*(\alpha_o; \chi) = \sqrt{\alpha_o^2 + \chi}$.

We now show that the incumbent will always give access to the new technology to the entrant. This happens because for any $\alpha_o < z$ we have $f(\alpha_o) > 0$, where

$$f(\alpha_o) := \pi_i^{d_n}\left(\sqrt{\chi + \alpha_o^2}\right) - \pi_i^{d_b}(\alpha_o) = -\left(z\alpha_o + \chi - \left(\sqrt{\chi + \alpha_o^2}\right)(z + v)\right).$$

Function $f(\alpha_o)$ is decreasing in α_o because $\left.\frac{\partial f(\alpha_o)}{\partial \alpha_o}\right|_{v=0} = 0$ and $\frac{\partial^2 f(\alpha_o)}{\partial \alpha_o \partial v} < 0$.

Additionally, $f(z) = 0$. Hence, for all $\alpha_o < z$ we have that $f(\alpha_o) > 0$.

Assume now that $\alpha_o \geq \sqrt{6t - \chi}$. Let us start by finding out the best offer α_n, in the incumbents perspective, that is accepted by the entrant. This is the solution to

$$\max_{\alpha_n} \pi_i^{d_n}(\alpha_n),$$

subject to $\alpha_n < \sqrt{6t}$. Therefore, the optimal access price is $\alpha_n = \sqrt{6t} - \varepsilon$.

The incumbent will then prefer that the entrant stays out of the industry if and only if

$$\lim_{\varepsilon \to 0} \pi_i^{d_n}\left(\sqrt{6t} - \varepsilon\right) < \pi_i^{m_n} \Leftrightarrow 6t + (z+v)\left(z + v - 2\sqrt{6t}\right) > 0.$$

But, as $6t + (z+v)\left(z+v-2\sqrt{6t}\right) > 6t + \frac{4}{3}\sqrt{6t}\left(\frac{4}{3}\sqrt{6t} - 2\sqrt{6t}\right) = \frac{2}{3}t > 0$ this is always true. For the same reason, whenever $6t - \chi \leq 0$ the incumbent will prefer that the entrant stays out of the industry. ■

Lemma 6:

(i) Assume that $\chi \geq 6t$. As seen in Lemma 4, the incumbent will not give access to the new technology whatever the access price for the old technology, and thus it obtains monopoly profit in case of investment.

If $\alpha_o < \sqrt{6t}$, in the absence of investment there will be entry. The incumbent will invest if and only if:

$$\pi_i^{m_n} - I > \pi_i^{d_o}(\alpha_o) \Leftrightarrow \pi_i^{m_n} - \pi_i^{d_o}(\alpha_o) > I.$$

As $\pi_i^{m_n} - \pi_i^{d_o}(\alpha_o)$ is decreasing in α_o, we have that $\pi_i^{m_n} - \pi_i^{d_o}(\alpha_o) > \pi_i^{m_n} - \pi_i^{d_o}\left(\sqrt{6t}\right) = \frac{\chi}{2} + \frac{\left(z-\sqrt{6t}\right)^2}{2} > \frac{\chi}{2} > I$: the incumbent will always invest.

If $\alpha_o > \sqrt{6t}$, there will be no entry, independently of the investment decision. The incumbent will invest if and only if:

$$\pi_i^{m_n} - I > \pi_i^{m_o} \Leftrightarrow \frac{\chi}{2} > I,$$

which is always true by our assumption on I. Thus, the incumbent will always invest.

(ii) Assume that $\chi < 6t$. Assume that $\alpha_o < \sqrt{6t - \chi}$. As seen in Lemma 4, the incumbent will always give access to the new technology to the entrant if it invests and it will give access to the old technology if it does not invest. It will be profitable to invest if and only if:

$$\pi_i^{d_n}(\alpha_n^*(\alpha_o; \chi)) - I > \pi_i^{d_o}(\alpha_o) \Leftrightarrow \Delta\Pi^i(\alpha_o) > I.$$

We now characterize $\Delta\Pi_i(\alpha_o)$. By definition,

$$\Delta\Pi_i(\alpha_o) = (z+v)\sqrt{\alpha_o^2 + \chi} - z\alpha_o - \frac{\chi}{72t}\left(60t - \chi - 2\alpha_o^2\right),$$

and: **(i)** $\frac{\partial^2 \Delta\Pi_i(\alpha_o)}{\partial\alpha_o^2} > 0$, **(ii)** $\left.\frac{\partial \Delta\Pi_i(\alpha_o)}{\partial\alpha_o}\right|_{\alpha_o=0} = -z < 0$, **(iii)** $\left.\frac{\partial \Delta\Pi_i(\alpha_o)}{\partial\alpha_o}\right|_{\alpha_o=\sqrt{6t-\chi}} = \frac{\sqrt{6t-\chi}\sqrt{\chi+z^2}}{\sqrt{6t}} + \frac{\chi\sqrt{6t-\chi}-18tz}{18t}$. This is positive, if and only if, $z < z_1(\chi,t) := \frac{\sqrt{t}}{3}\left(\sqrt{6-\chi/t}+\sqrt{(\chi/t)^2/6-11\chi/t+60}\right)$. Additionally, $\Delta\Pi_i(\sqrt{6t-\chi}) < \frac{1}{2}\chi$, if and only if, $z < \overline{z}(\chi,t) := \frac{\sqrt{t}}{72}\left(\sqrt{6-\chi/t}\,(\chi/t+84)+\sqrt{6(\chi/t+156)(\chi/t+12)}\right)$, with $\overline{z}(\chi,t) > z_1(\chi,t)$, for all χ on $(0,6t)$. Hence, for $z > \overline{z}$, $\Delta\Pi^i(\cdot)$ would always be above $\frac{1}{2}\chi$ and the incumbent would invest for every (α_o, I).

Assume that $\alpha_o \geq \sqrt{6t-\chi}$. Then, the incumbent will prefer that the entrant exits if investment has taken place and this case is equal to (i). ■

Remark 1: Note that $W^{d_\tau}(\alpha_\tau) - W^{m_\tau} = 5\left(\frac{6}{5}t-\alpha_\tau^2\right)\left(6t-\alpha_\tau^2\right)/144t$, that $\frac{\partial W^{d_\tau}(\alpha_\tau)}{\partial\alpha_\tau} = \frac{1}{36t}\left(5\alpha_\tau^2-18t\right)\alpha_\tau = 0$ for $\alpha_\tau = 0$ or $\alpha_\tau = \sqrt{\frac{18}{5}t}$ and that $\frac{\partial^2 W^{d_\tau}(\alpha_\tau)}{\partial\alpha_\tau^2} = \frac{1}{12t}\left(5\alpha_\tau^2-6t\right)$. The second derivative at $\alpha_\tau = 0$ is $-\frac{1}{2} < 0$ and at $\alpha_\tau = \sqrt{\frac{18}{5}t} < \sqrt{6t}$ is $1 > 0$. The third derivative is always non negative. ■

Lemma 7: We start by showing that (i) $\Delta\Pi^i(\alpha_o)$ is decreasing until $\alpha_o = \sqrt{\frac{6t}{5}}$ and (ii) $\Delta\Pi^i(\sqrt{\frac{6t}{5}}) > \frac{\chi}{2}$.

(i) As $\frac{\partial^2 \Delta\Pi_i(\alpha_o)}{\partial\alpha_o^2} > 0$, we need to show that

$$\left.\frac{\partial \Delta\Pi^i(\alpha_o)}{\partial\alpha_o}\right|_{\alpha_o=\sqrt{\frac{6t}{5}}} = \frac{\chi}{18t}\sqrt{\frac{6t}{5}} - z + \sqrt{(\chi+z^2)}\frac{\sqrt{\frac{6t}{5}}}{\sqrt{\left(\chi+\frac{6}{5}t\right)}} < 0.$$

This is positive if $\sqrt{(\chi+z^2)}\frac{\sqrt{\frac{6t}{5}}}{\sqrt{\left(\chi+\frac{6}{5}t\right)}} > z - \frac{\chi}{18t}\sqrt{\frac{6t}{5}}$. As both terms in the inequality are positive, this implies that $\left(\sqrt{(\chi+z^2)}\frac{\sqrt{\frac{6t}{5}}}{\sqrt{\left(\chi+\frac{6}{5}t\right)}}\right)^2 > \left(z - \frac{\chi}{18t}\sqrt{\frac{6t}{5}}\right)^2$ which is equivalent to $\frac{6\sqrt{30}+5\frac{\chi}{t}\sqrt{30}-6\sqrt{5}\sqrt{5\frac{\chi}{t}+1356}}{450} < \frac{z}{\sqrt{t}} < \frac{6\sqrt{30}+5\frac{\chi}{t}\sqrt{30}+6\sqrt{5}\sqrt{5\frac{\chi}{t}+1356}}{450}$.

As $\frac{6\sqrt{30}+5\frac{\chi}{t}\sqrt{30}+6\sqrt{5}\sqrt{5\frac{\chi}{t}+1356}}{450} < \frac{6\sqrt{30}+30\sqrt{30}+6\sqrt{5}\sqrt{30+1356}}{450} < \frac{4}{3}\sqrt{6}$, it is impossible to have $\left.\frac{\partial \Delta\Pi^i(\alpha_o)}{\partial\alpha_o}\right|_{\alpha_o=\sqrt{\frac{6t}{5}}} > 0$.

(ii) We now show that $\Delta\Pi^i(\sqrt{\frac{6t}{5}}) > \frac{\chi}{2}$. Let

$$g(\chi) := \Delta\Pi^i\left(\sqrt{\frac{6t}{5}}\right) - \frac{\chi}{2} = \frac{(5\chi-468t)\,\chi}{360t} - z\sqrt{\frac{6}{5}t} + \left(\sqrt{\frac{6}{5}t+\chi}\right)(z+v).$$

Assume that $g(\chi) < 0 \Leftrightarrow \sqrt{\left(\frac{6}{5}+\frac{\chi}{t}\right)\left(\frac{\chi}{t}+\left(\frac{z}{\sqrt{t}}\right)^2\right)} < \frac{\left(468-5\frac{\chi}{t}\right)\frac{\chi}{t}}{360} + \frac{z}{\sqrt{t}}\sqrt{\frac{6}{5}}$. As both sides in the inequality are positive this implies that $\left(\frac{6}{5}+\frac{\chi}{t}\right)\left(\frac{\chi}{t}+\left(\frac{z}{\sqrt{t}}\right)^2\right) < \left(\frac{\left(468-5\frac{\chi}{t}\right)\frac{\chi}{t}}{360} + \frac{z}{\sqrt{t}}\sqrt{\frac{6}{5}}\right)^2 \Leftrightarrow$ $180\left(\frac{z}{\sqrt{t}}\right)^2 + \left(\sqrt{30}\frac{\chi}{t} - \frac{468}{5}\sqrt{30}\right)\frac{z}{\sqrt{t}} + \frac{13\left(\frac{\chi}{t}\right)^2}{2} - \frac{621\frac{\chi}{t}}{5} - \frac{5\left(\frac{\chi}{t}\right)^3}{144} + 216 < 0.$

Therefore we should have $\frac{\sqrt{30}\left(468-5\frac{\chi}{t}\right)}{1800}-\frac{\sqrt{5\left(5\frac{\chi}{t}-108\right)\left(5\frac{\chi}{t}+6\right)\left(5\frac{\chi}{t}-828\right)}}{1800}<\frac{z}{\sqrt{t}}<\frac{\sqrt{30}\left(468-5\frac{\chi}{t}\right)}{1800}+\frac{\sqrt{5\left(5\frac{\chi}{t}-108\right)\left(5\frac{\chi}{t}+6\right)\left(5\frac{\chi}{t}-828\right)}}{1800}$.

By plotting these roots as a function of $\frac{\chi}{t}\in(0,6)$, we observe that $\frac{\sqrt{30}\left(468-5\frac{\chi}{t}\right)}{1800}+\frac{\sqrt{5\left(5\frac{\chi}{t}-108\right)\left(5\frac{\chi}{t}+6\right)\left(5\frac{\chi}{t}-828\right)}}{1800}<\frac{4}{3}\sqrt{6}$, meaning that it is impossible to have $g(\chi)<0$.

This implies that $I<\frac{\chi}{2}<\Delta\Pi^i(\sqrt{\frac{6t}{5}})$, and thus, duopoly with the old technology with $\alpha_o=\left(\Delta\Pi^i\right)^{-1}(I)>\sqrt{\frac{6}{5}t}$ is worse than monopoly on the old technology, which is worst than a monopoly on the new technology.

The regulator's choice is thus between duopoly and monopoly on the new technology. To maximize welfare in the case of duopoly, the regulator will set $\alpha_o=0$ leading to $\alpha_n^*(0;\chi)=\sqrt{\chi}$. If $\chi<\frac{6}{5}t$ this results in higher welfare than the case of monopoly. However, when $\chi>\frac{6}{5}t$, monopoly on the new technology is better than duopoly with the new technology, and thus the regulator sets any α_o larger than $\sqrt{6t-\chi}$. ■

Proposition 1a and 1b: This follows from the Lemmas above. ■

Lemma 6': Let (Y,N) denote investment by the incumbent and no investment by the entrant and let $(N,N),(Y,Y)$ and (N,Y) denote the remaining possibilities for stage 2 of the game where both the incumbent and the entrant can invest in the new technology.

Consider first the case of a *non-drastic* innovation.

(i) (N,N) is never an equilibrium. (N,N) is not an equilibrium if the entrant prefers to invest given that the incumbent does not invest. For the case of $\alpha_o<\sqrt{6t}$ this corresponds to:

$$\pi_i^{d_n}\left(\alpha_n^*(0;\chi)\right)-I>\pi_e^{d_o}\left(\alpha_o\right)\Leftrightarrow I<\pi_i^{d_n}\left(\alpha_n^*(0;\chi)\right)-\pi_e^{d_o}\left(\alpha_o\right).$$

The minimum in α_o of $\pi_i^{d_n}\left(\alpha_n^*(0;\chi)\right)-\pi_e^{d_o}\left(\alpha_o\right)$ occurs at $\pi_i^{d_n}\left(\alpha_n^*(0;\chi)\right)-\pi_e^{d_o}(0)=\frac{36t^2+\chi^2-60t\chi}{72t}+\sqrt{\chi}t\left(z+v\right)-\frac{1}{2}t$ which can be shown to be larger than $\frac{\chi}{2}$ and hence larger than I. For the case of $\alpha_o\geq\sqrt{6t}$ the corresponding condition is $I>\pi_i^{d_n}\left(\alpha_n^*(0;\chi)\right)$, which is stronger than the previous one and thus also impossible to hold.

(ii) (Y,Y) is an equilibrium with $\alpha_o<\sqrt{6t-\chi}$ if and only if:

$$\pi_i^{d_n}(0)-I \quad > \quad \pi_e^{d_n}\left(\alpha_n^*(0;\chi)\right) \tag{1}$$

$$\pi_i^{d_n}(0)-I \quad > \quad \pi_e^{d_n}\left(\alpha_n^*(\alpha_o;\chi)\right). \tag{2}$$

Let $\Delta\Pi_{e|I}(\alpha_o):=\pi_i^{d_n}(0)-\pi_e^{d_n}\left(\alpha_n^*(\alpha_o;\chi)\right)=\frac{1}{2}t-\frac{\left(6t-\chi-\alpha_o^2\right)^2}{72t}$.[23] Clearly, $\Delta\Pi_{e|I}(0)<\Delta\Pi_{e|I}(\alpha_o)$

[23]This definition is valid for $\alpha_o<\sqrt{6t-\chi}$. Otherwise, $\Delta\Pi_{e|I}(\alpha_o):=\pi_i^{d_n}(0)$. For $\alpha_o\leq\sqrt{6t-\chi}$, inspection of $\Delta\Pi_{e|I}(\alpha_o)$ reveals that this is a continuous increasing function in α_o that takes values on the interval $\left[\Delta\Pi_{e|I}(0),\frac{1}{2}t\right]$.

and $\Delta\Pi_{e|I}(0) < \Delta\Pi^{i}(\alpha_o)$.[24] As $\pi_e^{d_n}(\alpha_n^*(\alpha_o;\chi)) \leq \pi_e^{d_n}(\alpha_n^*(0;\chi))$, the two inequalities hold if and only if:

$$I < \Delta\Pi_{e|I}(0) = \frac{(12t-\chi)\,\chi}{72t} < \frac{\chi}{2}.$$

When $\alpha_o \geq \sqrt{6t-\chi}$, (2) should be replaced by $\pi_i^{d_n}(0) - I \geq 0$. But this is implied by (1). Hence, (Y,Y) is an equilibrium if and only if $I < \Delta\Pi_{e|I}(0)$.

(iii) (N,Y) is an equilibrium if and only if:

$$\pi_i^{d_n}(0) - I \leq \pi_e^{d_n}(\alpha_n^*(0;\chi)) \Leftrightarrow I \geq \Delta\Pi_{e|I}(0).$$

We have already shown in (i) that when the incumbent does not invest the entrant will always prefer to invest.

(iv) (Y,N) is an equilibrium with $\alpha_o < \sqrt{6t-\chi}$ if and only if:

$$\begin{aligned} \pi_i^{d_n}(\alpha_n^*(\alpha_o;\chi)) - I &> \pi_i^{d_o}(\alpha_o) \Leftrightarrow I < \Delta\Pi^{i}(\alpha_o) \\ \pi_e^{d_n}(\alpha_n^*(\alpha_o;\chi)) &\geq \pi_i^{d_n}(0) - I \Leftrightarrow I \geq \Delta\Pi_{e|I}(\alpha_o). \end{aligned}$$

(Y,N) is an equilibrium with $\sqrt{6t-\chi} < \alpha_o < \sqrt{6t}$ if and only if

$$\begin{aligned} \pi_i^{m_n} - I &> \pi_i^{d_o}(\alpha_o) \Leftrightarrow I < \pi_i^{m_n} - \pi_i^{d_o}(\alpha_o) \\ 0 &\geq \pi_i^{d_n}(0) - I \Leftrightarrow I \geq \pi_i^{d_n}(0). \end{aligned}$$

(Y,N) is an equilibrium with $\alpha_o \geq \sqrt{6t}$ if and only if

$$\begin{aligned} \pi_i^{m_n} - I &> \pi_i^{m_o} \Leftrightarrow I < \pi_i^{m_n} - \pi_i^{m_o} \\ 0 &\geq \pi_i^{d_n}(0) - I \Leftrightarrow I \geq \pi_i^{d_n}(0). \end{aligned}$$

Note that, by assumption, $I < \pi_i^{m_n} - \pi_i^{m_o} = \frac{\chi}{2} < \pi_i^{m_n} - \pi_i^{d_o}(\alpha_o)$.

Consider now the case of a *drastic* innovation.

(i) (N,N) is never an equilibrium. (N,N) is not an equilibrium if the entrant prefers to invest given that the incumbent does not invest. For the case of $\alpha_o < \sqrt{6t}$ this corresponds to $\pi_i^{m_n} - I > \pi_e^{d_o}(\alpha_0)$, i.e., $I < \pi_i^{m_n} - \pi_e^{d_o}(\alpha_0)$. For the case of $\alpha_o \geq \sqrt{6t}$ the corresponding condition is $\pi_i^{m_n} - I > 0$.

(ii) (Y,Y) is an equilibrium if and only if $\pi_i^{d_n}(0) - I > 0$.

[24] $\Delta\Pi_{e|I}(0) - \Delta\Pi^{i}(\alpha_o) = -\frac{\chi(\chi+\alpha_o^2)}{36t} + z\alpha_o + \chi - \left(\sqrt{\chi+\alpha_o^2}\right)(z+v)$. It is impossible that $z\alpha_o + \chi - \left(\sqrt{\chi+\alpha_o^2}\right)(z+v) > 0$. This is equivalent to $\frac{z\alpha_o+\chi}{z+v} > \sqrt{\chi+\alpha_o^2}$. This implies, by squaring both sides of the inequality, that $\left(\frac{z\alpha_o+\chi}{z+v}\right)^2 - \left(\chi+\alpha_o^2\right) > 0$. Simplifying we obtain $-(z+v)^{-2}(2z+v)(\alpha_o-z)^2 v > 0$ which is false. Hence, $\Delta\Pi_{e|I}(0) - \Delta\Pi^{i}(\alpha_o) < 0$.

(iii) (N, Y) is an equilibrium if and only if $\pi_i^{d_n}(0) - I \leq 0$.

(iv) (Y, N) is an equilibrium with $\alpha_o < \sqrt{6t}$ if and only if $\pi_i^{d_n}(0) - I \leq 0$ and $\pi_i^{m_n} - I > \pi_i^{d_o}(\alpha_0)$. (Y, N) is an equilibrium with $\alpha_o > \sqrt{6t}$ if and only if $\pi_i^{d_n}(0) - I \leq 0$ and $\pi_i^{m_n} - I > \pi_i^{m_o}$. ■

Lemma 7': Consider first the case of a drastic innovation or of a non-drastic innovation and I on $\left[0, \Delta\Pi_{e|I}(0)\right)$. In both these cases, the regulator cannot influence the outcome of the game, and thus he can set any α_o on $[0, +\infty)$.

Consider, now the case of a non-drastic innovation.

If the investment cost I is on $\left[\Delta\Pi_{e|I}(0), \pi_i^{d_n}(0)\right)$, by setting α_o on $(\alpha_o^{**}(I), +\infty)$, the regulator induces an equilibrium with investment by the entrant, whereas by setting α_o on $[0, \alpha_o^{**}(I)]$ the regulator induces one of two equilibria, where either the entrant or the incumbent invest. Investment by the entrant leads to a duopoly with the new technology with $\alpha_n^*(0;\chi) = \sqrt{\chi}$ and welfare level $W^{d_n}(\alpha_n^*(0;\chi))$, while investment by the incumbent leads to a duopoly with the new technology with $\alpha_n^*(\alpha_o;\chi) = \sqrt{\alpha_o^2 + \chi}$ and welfare level $W^{d_n}(\alpha_n^*(\alpha_o;\chi))$. If χ is on $\left(0, \frac{6}{5}t\right)$, $W^{d_n}(\alpha_n^*(0;\chi)) > W^{d_n}(\alpha_n^*(\alpha_o;\chi))$. Thus, the regulator sets $\alpha_o = 0$, and one of the firms invests and offers access to the rival at $\alpha_n^*(0;\chi) = \sqrt{\chi}$.[25] If χ is on $\left[\frac{6}{5}t, \frac{18}{5}t\right)$, $W^{d_n}(\alpha_n^*(\cdot))$ is initially decreasing in α_o, and then increasing in α_o. To induce welfare level $W^{d_n}(\alpha_n^*(\cdot))$, the regulator must set α_o on $[0, \alpha_o^{**}(I)]$ and, hence, it will either choose $\alpha_o = 0$, or, $\alpha_o = \alpha_o^{**}(I)$. From figure 2, there is a $\alpha_o' := \sqrt{\frac{36}{5}t - 2\chi}$ on $(0, +\infty)$ such that $W^{d_n}(\alpha_n^*(0;\chi)) \equiv W^{d_n}(\alpha_n^*(\alpha_o';\chi))$. Thus, if $\alpha_o^{**}(I) < \alpha_o'$, then $W^{d_n}(\alpha_n^*(0;\chi)) > W^{d_n}(\alpha_n^*(\alpha_o^{**}(I);\chi))$, and if $\alpha_o^{**}(I) > \alpha_o'$, then $W^{d_n}(\alpha_n^*(0;\chi)) < W^{d_n}(\alpha_n^*(\alpha_o^{**}(I);\chi))$. Since $\Delta\Pi_{e|I}(\alpha_o)$ is increasing in α_o, $\alpha_o^{**}(I) > \alpha_o'$, if and only if, I is on $\left(I^*, \pi_i^{d_n}(0)\right)$, with $I^* := \frac{1}{2}t - \frac{1}{72t}\left(\chi - \frac{6}{5}t\right)^2$. Hence, if I is on $\left[I^*, \pi_i^{d_n}(0)\right)$, welfare is higher with investment by the incumbent, than by the entrant, and the regulator should set $\alpha_o = \alpha_o^{**}(I)$. If I is on $\left[\Delta\Pi_{e|I}(0), I^*\right)$, the regulator should set $\alpha_o = 0$, and one of the firms invests, and offers access to the rival at $\alpha_n^*(0;\chi) = \sqrt{\chi}$.[26] Finally, if χ is on $\left[\frac{18}{5}t, 6t\right)$, welfare under duopoly with the new technology with $\alpha_n^*(\alpha_o;\chi)$ is always increasing in α_o. Therefore, the regulator should set $\alpha_o = \alpha_o^{**}(I)$.

Let I be high, i.e., let I be on $\left[\pi_i^{d_n}(0), \frac{1}{2}\chi\right)$. If χ is on $\left(0, \frac{6}{5}t\right)$, the optimal access price is as in the case where (χ, I) is on $\left(0, \frac{6}{5}t\right) \times \left[\Delta\Pi_{e|I}(0), \pi_i^{d_n}(0)\right)$. If χ is on $\left[\frac{6}{5}t, 6t\right)$, monopoly is socially preferable to a duopoly. However, since there are multiple equilibria, the regulator cannot impose this outcome. If one assumes that all equilibria are equally probable, the

[25] The same outcome occurs if the regulator sets any α_o on $(\alpha_o^{**}(I), +\infty)$.

[26] Again, the same outcome occurs if the regulator sets any α_o on $(\alpha_o^{**}(I), +\infty)$.

regulator sets α_o on $\left[\sqrt{6t-\chi}, +\infty\right)$. ■

Proposition $1a'$ **and** $1b'$**:**

(I) *If* (χ, I) *is on* $[6t, +\infty) \times \left[0, \pi_i^{d_n}(0)\right)$*:* ***(i)*** *the regulator sets* α_o *on* $[0, +\infty)$*, and* ***(ii)*** *both firms invest.*

(II) *If* (χ, I) *is on* $[6t, +\infty) \times \left[\pi_i^{d_n}(0), \frac{1}{2}\chi\right)$*:* ***(i)*** *the regulator sets* α_o *on* $[0, +\infty)$*,* ***(ii)*** *only one firm invests and offers* $\alpha_n^*(\cdot)$ *on* $\left[\sqrt{6t}, +\infty\right)$*, and* ***(iii)*** *the rival exits the industry.*

(III) *If* (χ, I) *is on* $(0, 6t) \times \left[0, \Delta\Pi_{e|I}(0)\right)$*:* ***(i)*** *the regulator sets* α_o *on* $[0, +\infty)$*, and* ***(ii)*** *both firms invest.*

(IV) *If* (χ, I) *is on* $\left(0, \frac{6}{5}t\right) \times \left[\Delta\Pi_{e|I}(0), \frac{1}{2}\chi\right)$*:* ***(i)*** *the regulator sets* $\alpha_o = 0$ *or* α_o *on* $(\alpha_o^{**}, +\infty)$*,* ***(ii)****, and either* ***(iia)*** *only the incumbent invests and offers* $\alpha_n^*(0;\chi) = \sqrt{\chi}$ *and* ***(iiia)*** *the entrant uses the new technology, or,* ***(iib)*** *only the entrant invests and offers* $\alpha_n^*(0;\chi) = \sqrt{\chi}$*, and* ***(iiib)*** *the incumbent uses the new technology.*

(V) *If* (χ, I) *is on* $\left[\frac{6}{5}t, \frac{18}{5}t\right) \times \left[\Delta\Pi_{e|I}(0), I^*\right)$*:* ***(i)*** *the regulator sets* $\alpha_o = 0$ *or* α_o *on* $(\alpha_o^{**}, +\infty)$*,* ***(ii)****, and either* ***(iia)*** *only the incumbent invests and offers* $\alpha_n^*(0;\chi) = \sqrt{\chi}$ *and* ***(iiia)*** *the entrant uses the new technology, or,* ***(iib)*** *only the entrant invests and offers* $\alpha_n^*(0;\chi) = \sqrt{\chi}$*, and* ***(iiib)*** *the incumbent uses the new technology.*

(VI) *If* (χ, I) *is on* $\left[\frac{6}{5}t, \frac{18}{5}t\right) \times \left[I^*, \pi_i^{d_n}(0)\right)$*: the regulator sets* $\alpha_o = \alpha_o^{**}$*, and either* ***(iia)*** *only the incumbent invests and offers* $\alpha_n^*(\alpha_o^{**};\chi) = \sqrt{\chi + (\alpha_o^{**})^2}$ *and* ***(iiia)*** *the entrant uses the new technology, or,* ***(iib)*** *only the entrant invests and offers* $\alpha_n^*(0;\chi) = \sqrt{\chi}$*, and* ***(iiib)*** *the incumbent uses the new technology.*

(VII) *If* (χ, I) *is on* $\left[\frac{18}{5}t, 6t\right) \times \left[\Delta\Pi_{e|I}(0), \pi_i^{d_n}(0)\right)$*: the regulator sets* $\alpha_o = \alpha_o^{**}$*, and either* ***(iia)*** *only the incumbent invests and offers* $\alpha_n^*(\alpha_o^{**};\chi) = \sqrt{\chi + (\alpha_o^{**})^2}$ *and* ***(iiia)*** *the entrant uses the new technology, or,* ***(iib)*** *only the entrant invests and offers* $\alpha_n^*(0;\chi) = \sqrt{\chi}$*, and* ***(iiib)*** *the incumbent uses the new technology.*

(VIII) *If* (χ, I) *is on* $\left[\frac{6}{5}t, 6t\right) \times \left[\pi_i^{d_n}(0), \frac{1}{2}\chi\right)$*:* ***(i)*** *the regulator sets* α_o on $\left[\sqrt{6t-\chi}, +\infty\right)$*, and either* ***(iia)*** *only the incumbent invests and offers* $\alpha_n^*(\cdot)$ *on* $\left[\sqrt{6t}, +\infty\right)$ *and* ***(iiia)*** *the entrant exits the industry, or,* ***(iib)*** *only the entrant invests and offers* $\alpha_n^*(0;\chi) = \sqrt{\chi}$*, and* ***(iiib)*** *the incumbent uses the new technology.* ■

Follows from Lemmas 1 to 5, with the obvious changes, and from Lemmas 6'and 7'. ■

Proposition 2a and 2b: Follows from Propositions $1a$ and $1a'$, and $1b$ and $1b'$. Note that $\Delta\Pi_{e|I}(0) > t/4$, if and only if, χ is on $\left(\left(6 - 3\sqrt{2}\right)t, 6t\right)$. ■

8 Figures

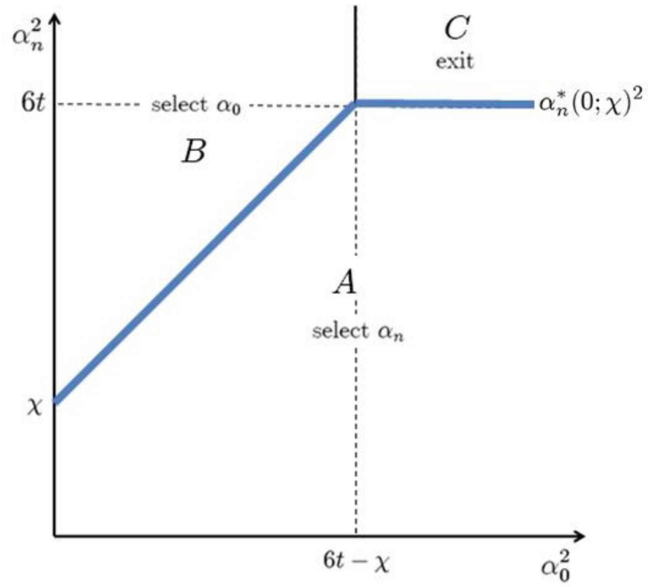

Figure1: The entrant's decision to select the old or new network and the incumbent's choice of α_o for the case of a non drastic innovation.

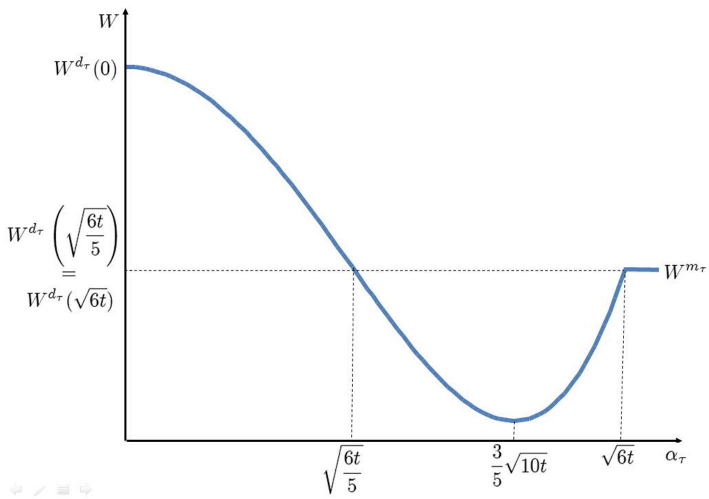

Figure 2: Welfare as a function of the access price

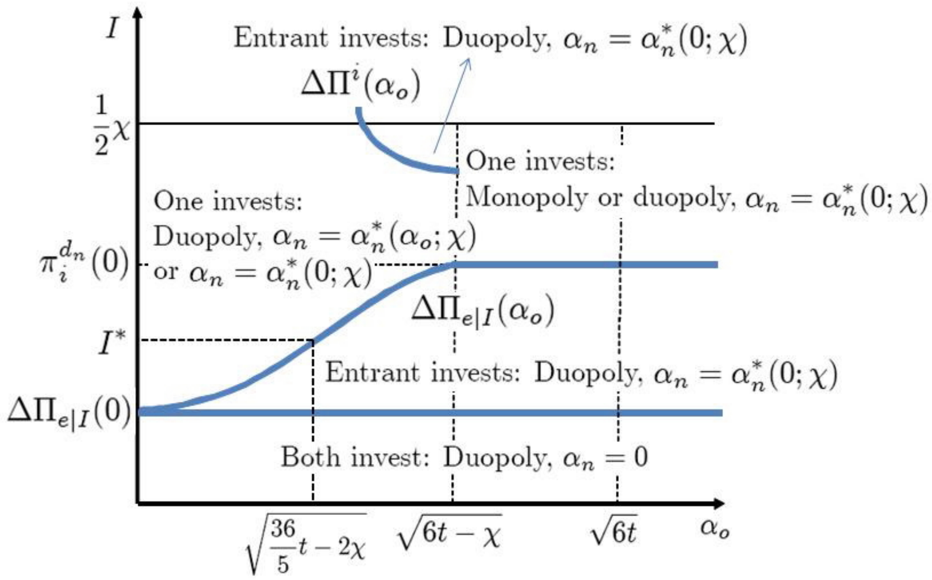

Figure 3: Equilibrium investment decision and resulting market structure for the case of a non drastic innovation.

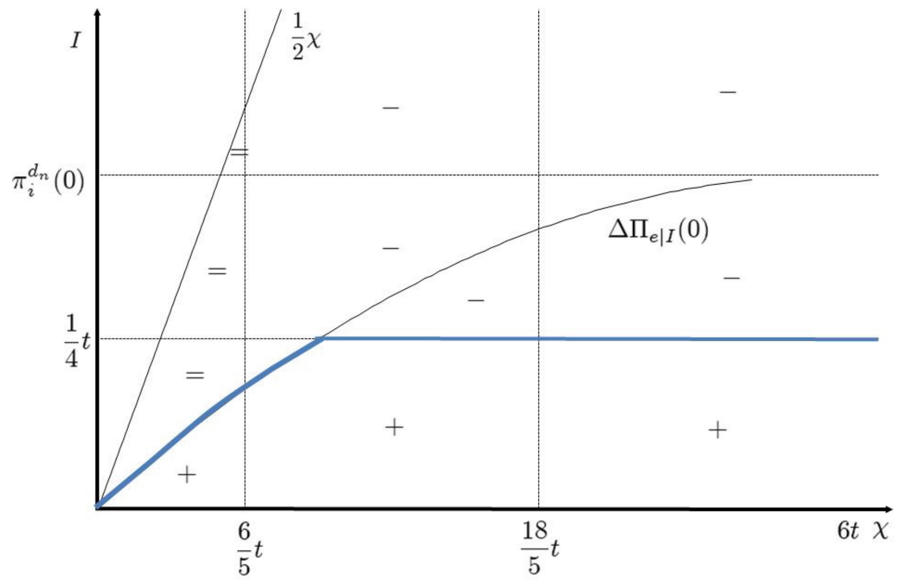

Figure 4: Sign of the change in expected welfare in equilibrium when both firms can invest when compared to the case in which only the incumbent can invest.

9 Tables

		χ		
		$(0, \frac{6}{5}t)$	$[\frac{6}{5}t, \frac{18}{5}t)$	$[\frac{18}{5}t, 6t)$
	Both Firms can Invest			
I	$[0, \Delta\Pi_{e\|I}(0))$	*any* α_o *both invest* $W^{d_n}(0)$	*any* α_o *both invest* $W^{d_n}(0)$	*any* α_o *both invest* $W^{d_n}(0)$
I	$[\Delta\Pi_{e\|I}(0), I^*)$	$\alpha_o = 0$ or on $(\alpha_o^{**}, +\infty)$ *i or e invest* $W^{d_n}(\alpha_n^*(0))$	$\alpha_o = 0$ or on $(\alpha_o^{**}, +\infty)$ *i or e invest* $W^{d_n}(\alpha_n^*(0))$	$\alpha_o = \alpha_o^{**}$ *i or e invest* $W^{d_n}(\alpha_n^*(\alpha_o^{**}))$ or $W^{d_n}(\alpha_n^*(0))$
I	$[I^*, \pi_i^{d_n}(0))$	$\alpha_o = 0$ or on $(\alpha_o^{**}, +\infty)$ *i or e invest* $W^{d_n}(\alpha_n^*(0))$	$\alpha_o = \alpha_o^{**}$ *i or e invest* $W^{d_n}(\alpha_n^*(\alpha_o^{**}))$ or $W^{d_n}(\alpha_n(0))$	$\alpha_o = \alpha_o^{**}$ *i or e invest* $W^{d_n}(\alpha_n^*(\alpha_o^{**}))$ or $W^{d_n}(\alpha_n^*(0))$
I	$[\pi_i^{d_n}(0), \frac{1}{2}\chi)$	$\alpha_o = 0$ *i or e invests* $W^{d_n}(\alpha_n^*(0))$	α_o on $[\sqrt{6t-\chi}, +\infty)$ *i or e invests* W^{m_n} or $W^{d_n}(\alpha_n^*(0))$	α_o on $[\sqrt{6t-\chi}, +\infty)$ *i or e invests* W^{m_n} or $W^{d_n}(\alpha_n^*(0))$
I	Only the Incumbent can Invest			
I	$[0, \frac{1}{2}\chi)$	$\alpha_o = 0$ *i invests* $W^{d_n}(\alpha_n^*(0))$	α_o on $[\sqrt{6t-\chi}, +\infty)$ *i invests* W^{m_n}	α_o on $[\sqrt{6t-\chi}, +\infty)$ *i invests* W^{m_n}

References

BIGLAISER, G., and DEGRABA, P., 2001, "Downstream integration by a bottleneck input supplier whose regulated wholesale prices are above costs", *Rand Journal of Economics*, 32, 2, 302-315.

BOURREAU, M., and DOGAN, P., 2005, "Unbundling the Local Loop", *European Economic Review* 49, 173-199.

BRITO, D. and PEREIRA, P., 2010, "Incentives to Concede Access to Bottleneck Inputs under Oligopoly: A Prisoners' Dilemma?", *Southern Economic Journal*, 76(3), 660-77.

BRITO, D. and PEREIRA, P., 2009, "Product Differentiation when Competing with the Supplier of a Bottleneck Input", *Regional Science and Urban Economics*, 19(1), 43-53.

BRITO, D., PEREIRA, P., and VAREDA, J., 2010, "Can Two-Part Tariffs Promote Efficient Investment on Next Generation technologies?", *International Journal of Industrial Organization*, 28(3), 323-333.

CAMBINI, C., and JIANG, Y., 2009, "Broadband Investment and Regulation: A Literature Review", *Telecommunications Policy,* 33, 10-11, 559-574.

CAILLAUD, B. and TIROLE, J., 2004, "Essential facility financing and market structure", *Journal of Public Economics*, 88, 667– 694.

FOROS, O., 2004, "Strategic Investments with Spillovers, Vertical Integration and Foreclosure in the Broadband Access Market", *International Journal of Industrial Organization,* 22, 1-24.

GANS, J., 2001, "Regulating Private technology Investment: Optimal Pricing for Access to Essential Facilities", *Journal of Regulatory Economics*, 20, 2, 167-189.

GANS, J., and KING, S., 2004, "Access Holidays and the Timing of technology Investment", *Economic Record,* 80, 89-100.

GUTHRIE, G., 2006, "Regulating technology: The Impact on Risk and Investment", *Journal of Economic Literature*, 44, 925-972.

HORI, K. and MIZUNO, K., 2006. "Access pricing and investment with stochastically growing demand," *International Journal of Industrial Organization*, 24(4), 795-808.

HORI, K. and MIZUNO, K., 2009. "Competition schemes and investment in technology under uncertainty," *Journal of Regulatory Economics*, 35(2), 179–200.

HOTELLING, H., 1929, "Stability in Competition", *Economic Journal*, 29, 41-57.

ITU, 2001. "General overview of NGN", ITU-T Recommendation Y.2001.

Klumpp, T. and Su, X., 2007, “Open Access and Dynamic Efficiency”, forthcoming in *American Economic Journal - Microeoconomics.*

Kotakorpi, K., 2006, "Access Price Regulation, Investment and Entry in Telecommunications", *International Journal of Industrial Organization,* 24, 1013-1020.

Lewin, D. ,Williamson, B., Cave, M., 2009, "Regulating Next-Generation Fixed Access to Telecommunications Services", Available at SSRN: http://ssrn.com/abstract=1313704.

Tirole, J., 2003, "The Theory of Industrial Organization", The MIT Press.

Vareda, J., and Hoernig, S., 2007, "The race for telecoms technology investment with bypass: Can access regulation achieve the first best?", CEPR Discussion Paper 6012.

Vareda, J., 2007, "Unbundling and the Incumbent Investment in Quality Upgrades and Cost Reduction", FEUNL working article 526.

WIK, 2008, "Leading European study shows business case for Next Generation Broadband favours former incumbents", WIK.