Marketing Strategies in the Presence of Network Effects

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Problem

Aim: maximize revenue when selling to a social network

Setting:

- Positive externalities:
  - a set S_t of buyers have purchased before time t
  - valuation $v_i(S_t)$ of buyer i depends on S_t
  - assume that $v_i(A) \leq v_i(B)$ for $A \subseteq B$
- Myopic: buy as soon as valuation exceeds price $v_i(S_t) \geq p_{i,t}$

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Three Problems

1. Optimal revenue: maximize total revenue

$$\underbrace{\sup_{\pi}}_{\text{policies}} \underbrace{\mathbb{E}_{(S_t)_{t=1}^T|\pi,S_0=\emptyset}}_{\text{over buyer sequence}} \underbrace{\sum_{t=1}^{T}}_{\text{times}} \underbrace{\sum_{i\in S_t\setminus S_{t-1}}}_{\text{new buyers}} \underbrace{p_{i,t}}_{\text{payments}}$$

over all policies π which set price $p_{i,t}$ for each buyer i given set S_t

The authors also restrict the class of policies as follows

2. Optimal seeding:
   - Initially, a seed set of buyers of given size is given the item for free
   - All other sales are at fixed price p^*
3. Optimal non-discriminatory selling:
   - A sequence of prices $(p_t)_{t=1}^T$ is decided in advance

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Contributions

Thm 1. Optimal revenue is NP-hard.

Thm 2. Optimal seeding is NP-hard.
But a greedy method gives a constant factor approximation.

Thm 3. Optimal non-discriminatory selling can be solved by dynamic programming.
The runtime is polynomial in the maximum possible price, assuming prices are integers.

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Results

Seed'n'Sell: *What if you do optimal seeding followed by non-discriminatory selling?*

Answer: Given enough price changes,

$$\text{revenue[non-discriminatory selling]} \geq 0.88 \text{ revenue[seed'n'sell]}$$

Good News:
Simulations suggest that "enough price changes" is a small number.

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   Could you get even better results by running the DP algorithm at each time step to select p_t given S_t?
4. The DP algorithm is only polynomial in $p_{\max}$, the maximum possible price, for integer prices
   Could you reformulate this as a fully-polynomial time approximation scheme (FPTAS)?

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