Technology Adoption in Standard Setting Organizations: A Model of Exclusion with Complementary Inputs and Hold-up*

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Abstract

I analyze technology adoption in a standardization consortium composed by a majority of vertically-integrated firms and a pure innovator, and its implications for social welfare. Like in most certification bodies, parties negotiate over the royalties after manufacturers' technology adoption, and this generates an hold-up problem. Integrated operators can employ a standard with their inputs and circumvent the hold-up problem, or buy from the specialized firm and enjoy the cost-savings produced by its technology. I show that cross-licensing may lead to the *inefficient* exclusion of the pure innovator and that a policy of early-licensing commitments would result in efficient adoption choices.

JEL codes: K21, L15, L24, L42.
Keywords: Technology Adoption, Technology Standards, Vertical Integration, Licensing Agreements, Cross-licensing, Exclusion.

1 Introduction

Voluntary Standard Setting Organizations (SSOs) are consortia of industry operators devoted to the achievement of an agreement on the rules that define the design of a final product or process. The theoretical literature has recently increased its attention towards the functioning of standard setting

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bodies (see Lerner and Tirole (2006), Choi et al. (2007), and Farrell and Simcoe (2009)), and the empirical work by Rysman and Simcoe (2008) confirms their relevance by showing that they play a crucial role in leading to a bandwagon process among adopters.[1]

The SSOs tend to emphasize the consensus that would characterize their decisions. However, strategic considerations among their participants can be intense and several pieces of evidence show that strong competitive tensions influence the procedure of standard choice, eventually leading to judicial disputes. These disputes mainly arise from the conflicting interests that operators with different business structures try to put forward in the process of standard certification (see Sherry and Teece (2003), DeLacey et al. (2006), Feldman et al. (2009) and Schmalensee (2009)).

This article focuses on the conflict between two categories of firms: vertically integrated operators (like IBM and Nokia), which dominate many standard setting consortia, and pure developers of new technologies (like Rambus and Qualcomm). These firms participate to SSOs with strikingly different objectives. Integrated organizations mostly aim at the important economic benefits that derive from coordination among industry participants. Consequently, they have a clear interest in paying low rates for standard's technologies while competing on the product market. Instead, IPR developers raise most of their revenue from the technology licensing market. They are primarily interested in having a patented technology into a new standard, because this can help them raise a long stream of licensing revenue.

I propose a framework to analyze the incentives that SSOs' firms have to employ patented technologies into their production process. The issue is addressed by studying how market competition and licensing decisions interact with technology adoption. Consequently, the model encompasses two markets: the technology licensing market (or upstream market) and the product market (or downstream market). Moreover, I conduct a welfare analysis to assess the adoption choices that would maximize total welfare.

The game involves two vertically integrated firms and a pure upstream firm. Each firm holds a patented technology; the first vertically integrated firm holds an "essential" technology, whilst the second integrated firm holds a technology that competes with the one of the upstream firm for the employment in the production of a final good. To make the conflict between these two firms more interesting, it is assumed that the technology of the pure innovator is more efficient.

I do not impose that the use of the same bundle of inputs, or technology platform, is mandatory to industry's participants. Thus, two types of scenario can arise from the adoption decision: either operators agree on the employment of the same platform ("technology standard" case), or they decide to use different platforms ("competing platforms" case). The latter outcome captures a situation in which the standardization effort fails and is far from being purely theoretical, because multiple technologies can coexist, for instance, when users' network externalities are not particularly strong.[2]

[1] Rysman and Simcoe (2008) documents that patents disclosed in SSOs receive up to twice as many citations as other patents in the same sector.

[2] An important example is the wireless telephony, where handsets based on different chips' technologies are marketed

Like in most SSOs, in the model licensing takes place after the adoption of a certain technology by industry's operators in their production process; thus a standard hold-up problem arises. To fix the contractual inefficiency caused by the hold-up problem, vertically integrated firms can exchange respective technologies by signing cross-licensing agreements. However, these deals are not possible with the pure upstream firm, because it is not active on the product market. Accordingly, the results of the welfare analysis are affected by the balance between the efficiency of the upstream firm's technology and the inefficiency that characterizes its licensing contracts.

The trade-off that determines manufacturers' choice to use the technology of the stand-alone firm and the outcome of the welfare analysis is as in what follows. *On the one hand, the employment of the independent upstream firm's input allows integrated companies to use a more efficient technology for the production of the final good. On the other hand, it allows the stand-alone firm to exploit monopoly bargaining power over its patented technology (because of the hold-up problem).*

The model delivers the pattern of integrated firms' *technology adoption* as function of two parameters: the one that measures the efficiency of the independent licensor's technology and the one that captures the cost-savings generated by SSO's support of a unique standard. More specifically, if the benefits generated by standardization are large, then vertically integrated firms cross-license their own patents, adopt a common technology standard and forgo the independent firm's input efficiency. Instead, the smaller are the standardization benefits (and the more is the specialized firm efficient), the more likely is that an equilibrium with competing platforms emerges on the product market.[3]

The intuition is simple and has to do with the balancing of the two forces in the trade-off above: as the advantages from having a standard increase, the integrated companies have a growing interest in signing an agreement that allows them to share respective rents. Instead, as the advantages from having a standard decrease, the benefits of using the specialized firm's technology become relatively more important, up to overcome the hold-up problem.

Under the *welfare* point of view, I show that the trade-off between the productive efficiency of the upstream firm technology and the contractual efficiency of cross-licensing may give rise to an *inefficient market outcome*: this happens when integrated operators choose a standard with their own techs although a social planner would adopt a standard with the vertically-specialized firm technology.

Three main assumptions are made concerning the composition and the functioning of the ideal certification body. The *first assumption* is that two vertically integrated firms and one upstream firm populate the representative organization. A framework with a majority of vertically integrated entities is able to capture the conflict between integrated firms and pure innovators. Moreover, it is able to

(Gandal et al. (2003)).

[3]Also Cabral and Salant (2009) and Farrell and Simcoe (2009) show that a scenario with *competing platforms* can arise at equilibrium, although their analysis is based on different underpinnings. More specifically, Cabral and Salant (2009) argues that a unique standard causes a problem of free-riding that reduces the incentives to invest on R&D with respect to a market structure with competing technologies, whereas in Farrell and Simcoe (2009) competing standards are the outcome of a war of attrition.

replicate SSOs' environment in several situations and in particular in two antitrust cases that have been for a long time under the scrutiny of antitrust authorities in the US and Europe: the FTC v. Rambus case and the EC v. Qualcomm case. In both cases major vertically integrated firms were among the plaintiffs and accused upstream developers of keeping a misleading conduct during the phase of standard definition.

The *second assumption* is that it is vertically integrated firms that decide which technologies are included into the standard. This modeling choice is based on the evidence arising from the SSOs operating in the information and communications technology sector, where vertical integration is a pervasive phenomenon. Standardization bodies in this industry are commonly founded by manufacturers with the intent of controlling the development of a particular technology and avoid mis-coordination among vendors.[4] Clearly, being in the pool of founding members allows these firms to play a crucial role in the phase of standard definition.

Further evidence regarding manufacturers' decision power arises from the two organizations involved in the Qualcomm and Rambus cases mentioned above. Gandal et al. (2003) remarks that in ETSI, the SSO of the Qualcomm case, the voting rule allowed even a small minority of operators to impose the adoption of their favorite standard configuration.[5] JEDEC, the SSO of the Rambus case, was mostly composed by vertically integrated manufacturers that, consequently, could strongly influence the composition of a standard.[6]

The *third assumption* is that licensing negotiations take place after downstream manufacturers choice and adoption of a specific technology, in compliance with most of the standard definition processes undertaken in technology certification consortia.[7] The main implication of this assumption is that licensing firms whose technology has been employed have full monopoly power on the determination of the royalty rate (which gives rise to the hold-up problem).

An important impediment to the implementation of an ex-ante licensing policy is the risk that SSOs' participants undertake anticompetitive coordinated practices, which would be punished by antitrust authorities. In an extension to the basic model, I analyze the optimal technology choice by using a negotiation environment that fulfills with the implementation of FRAND agreements' reasonableness

[4]Updegrove (1993) provides a detailed analysis of the strategic motivations that lead manufacturers to push for the formation of standardization consortia. Blind and Thumm (2004) documents that technology-users, rather than technology-developers, are in the majority in formal standardization processes. Also, Blind and Thumm (2004) provides an empirical analysis of the incentives behind patenting and participation to standardization decisions that confirms the conflict between the business models of large companies and small technology-developers.

[5]Indeed, ETSI rules required a majority of 71 percent for standard approval but with a voting weighting system based on European turnover; this favored European producers, and many of these were vertically integrated (for example, Nokia and Sony-Ericsson were in ETSI).

[6]The evidence gathered by the FTC in the Rambus case bears witness to the vast presence of integrated firms in JEDEC (In the Matter of Rambus Inc., Docket No. 9302).

[7]A remarkable exception is VITA, which switched in 2006 to a policy that requires the owners of patented technologies to disclose the maximum royalty rates and provide binding written license declarations at several specified points during the standard development process.

requirement.[8] In other words, there I assume that the holders of substitute patents compete for the employment by producers and set royalty rates before manufacturers commit to the adoption of a specific technology. The result is that early licensing decisions induce integrated companies to design the standard more efficiently.

The game is solved by assuming that active licensors sell technologies by means of royalty rates. Indeed, Layne-Farrar and Lerner (2008) documents that linear royalties are used by a vast majority of patent pools' members to license-out their technology. Under linear pricing, licensing decisions are influenced by two strategic effects, the *Cournot effect* and the *raising rival's costs effect*,[9] whose impact is discussed in the analysis of the adoption cases.

To assess the robustness of the main results to the assumption on the contractual form, I solve the model under two-part tariffs, in which case manufacturers' technology adoption choices only depend on the hold-up problem. Indeed, two-part tariffs contracts are not affected by the Cournot effect and the double marginalization problem (implying that they are more efficient than royalty rates).[10] In analogy to the setting with linear pricing, the result of the game with two-part tariffs is that if the standardization advantages are large, then integrated firms adopt their technologies into the standard and cross-license respective patents. Otherwise, competing platforms are employed. Finally, the inefficient exclusion of the pure innovator arises also in the framework with two-part tariffs.

2 Policy Implications and Discussion of the Results

The main *policy implication* of the model is that cross-licensing agreements may be inefficient. Scholars in the law and economics literature have often stressed the beneficial role of cross-licensing on the level of royalty rates (e.g., Shapiro (2001)). However, it has been overlooked that cross-licensing may also lead to the exclusion of the enterprises that are not in the position to participate to cooperative licensing agreements (like pure innovators), and such exclusionary practice would be welfare-detrimental if pure innovators are more efficient. The implication is that, if the technology of an excluded upstream firm

[8]The licensors that participate to SSOs are often required to commit to license their technologies on Fair Reasonable And Non-Discriminatory (FRAND) terms in case of adoption by manufacturers. A patent holder commitment to license to any interested party on FRAND terms implies that each licensee can obtain a license at the royalty rate established by the patent holder and is not put in comparative disadvantage with respect to other licensees. Choi et al. (2007) provides a survey of the SSOs that require firms to comply with FRAND agreements.

[9]The former effect is caused by the complementarity between the technologies required to produce the final good. Indeed, when pricing their technology independently licensors do not take into account the negative externality they exert on downstream firms (Cournot (1838)). The latter effect is related to the incentive that the downstream competing vertically integrated firms have to increase their rivals' costs as to push them out of the market (Salop and Scheffman (1983, 1987)).

[10]Wang (1998) compares the profitability of licensing contracts with linear royalties and fixed fees for a monopolist licensor that also competes in a downstream duopoly. Although my work shares some analogies with Wang (1998), I am not interested in the optimality of the type of licensing contracts but rather in whether producers' optimal technology choice changes with the type of licensing contract.

is ascertained to be superior,[11] then antitrust authorities should cautiously assess a defense argument based on the pro-efficient effects of cross-licensing by integrated organizations.

Under the *normative* point of view, the model suggests that standard setting consortia should adopt a policy of early-licensing commitments to kill the hold-up problem and allow integrated companies to design the standard efficiently. This result provides an argument in support of the idea that SSOs' participants should be left free to discuss the royalties on patented technologies before a specific standard configuration has been decided. So far, this kind of policy has received a timid support by SSOs (as well as little attention by the theoretical literature), especially because of members' fear of antitrust authorities' intervention. My model shows that competition agencies should also be concerned by the possibility that late licensing decisions would lead to inefficient market outcomes.

The article also delivers two clear and intuitive *testable predictions* regarding the pattern of SSOs' technology adoption choices. *An SSO dominated by integrated firms is expected to sponsor a technology standard if standardization's benefits are strong.* For example, this result is consistent with the employment of the IEEE 802.11n Wi-Fi protocol as industry standard. The IEEE 802.11n protocol is the standard for wireless communications among electronic devices (like laptops, smart-phones and PDAs); clearly, had conflicting protocols emerged on the marketplace, the important network externalities generated by a standardized technology for wireless communications would have not been exploited and the diffusion of the same technology would have been seriously inhibited. This clearly provided manufacturers with the right incentives to achieve coordination.

If standardization is less effective in terms of scale economies, either in production or in demand, then the model predicts that manufacturers' standardization effort is more likely to fail, leading to competing technology platforms. This result is consistent with the evidence in the telecommunications industry, where, as documented by Gandal et al. (2003), the CDMA2000 and the WCDMA (or UMTS) technologies, two incompatible platforms, do coexist on the market.

The CDMA2000 is employed on the US market and is an upgrade of the CDMA technology; moreover, both the CDMA and the CDMA2000 have been developed by Qualcomm (a pure innovator). The WCDMA was adopted by ETSI, an SSO dominated by integrated companies that decides on technology standardization in the European telecommunications industry. The WCDMA is a *variation* of the CDMA2000 platform that is largely incompatible with it. As clarified by Cabral and Salant (2009), the incompatibility between CDMA2000 and WCDMA implies that chipsets meant to work on one platform would not easily work on the other one. However, from the point of view of a user in this industry the costs of multiple incompatible standards are insignificant, because universal access to each other handset is not threatened by incompatibility; this implies that network effects (if any) are not hindered by manufacturers' mis-coordination.

The article proceeds as follows. Section 3 compares my findings with those established in related works. Section 4 presents the model, Section 5 solves the game under contracts with linear royalties

[11] The technical studies carried out by the FTC in the Rambus case provide a clear example of the techniques that can be used to establish technological efficiency.

and Section 6 studies the impact of a policy of early-licensing commitments on adoption choices. In Section 7, I analyze technology adoption under different specifications of model's framework and in Section 8, I test the robustness of the results by employing two-part tariffs contracts. Finally, Section 9 concludes.

3 Related Literature

This article analyzes the scope for "exclusionary effects" in the choice of a technology platform by looking at how *technology adoption interacts with licensing decisions and product market competition.* In Schmidt (2008) and Schmalensee (2009) it is investigated the interdependence of pricing decisions between upstream innovators, downstream producers and integrated entities, however they do not analyze technology adoption and do not study the extent to which cross-licensing can lead to upstream (inefficient) exclusion.[12]

The mechanism for which the stand-alone firm is excluded from the standard shares some analogies with the one in Bernheim and Whinston (1998) and Segal and Whinston (2000), where contracting externalities may give rise to anticompetitive outcomes. Indeed, in my article, *the independent firm's tech is not employed because of the externality exerted on the holder of the essential technology (firm 1 in the model) by the bias in favor of cross-licensing of the other integrated firm (firm 2)*, and by the fact that the upstream firm does not participate to the adoption decision.[13]

Bloch (1995) studies a problem of coalition formation by using a model in which the initiator of an association proposes a cooperative agreement to his product-market competitors. The equilibrium of the model is one where coordination efforts fail, because competing associations always form. My model differs from Bloch (1995) insofar as I provide an analysis of the technology choice adopted by a given organization and the welfare consequences associated with it.

The article is also related to the literature on *patent pools' formation.* Lerner and Tirole (2004) studies an all-or-nothing patent pool formation problem. In that paper, it is developed a framework in which the degree of patents' complementarity is the equilibrium outcome of a game in which licensing decisions are constrained either by demand forces or strategic forces. Instead, I am interested in the analysis of the conflicts between holders of competing technologies for a given degree of complementarity, to understand whether *inefficient holdouts* may arise at equilibrium.

Finally, the contribution of the article to the literature on *vertical integration* is twofold: the first

[12] Schmalensee (2009) focuses on the analysis of the strategic pricing decisions taken by integrated firms and vertically-specialized operators, and then on the pricing schemes that may solve the hold-up problem. Schmidt (2008) proves that, compared to a situation in which only vertically integrated firms are active, the presence of pure upstream innovators triggers royalty rates' and final output's decrease: this result is driven by the incentive that vertically integrated firms have to raise the cost of the inputs sold to downstream rivals (the "raising rival's cots" problem). Schmidt (2008) concludes that cross-licensing agreements between vertically integrated firms can alleviate this problem.

[13] Indeed, could the upstream firm compensate firm 2 for the profit loss suffered when the latter does not cross-license with firm 1, then the adoption of the stand-alone firm's technology would emerge as technology standard.

consists in analyzing the incentive that vertically integrated firms have to exclude an independent firm that operates on the upstream market if inputs are complementary and because of the danger of hold-up, instead the received literature has typically focused on settings with substitute intermediate goods (see Rey and Tirole (2007)). The second consists in investigating whether *cross-licensing can cause inefficient exclusion on the upstream market.*[14]

4 The Model

There are 3 firms: firm 1 and firm 2 are vertically integrated, firm 3 is a stand-alone upstream firm. Each firm owns a patented technology, indexed by τ: two of them are substitute, namely technologies τ_2 and τ_3, the third, τ_1, is perfect complement to the other two.

Upstream firms bear a nil marginal cost and can choose among two pricing schemes to license out their technology: independent licensing or cross-licensing. Cross-licensing is modeled by assuming that active licensors maximize joint profits, moreover cross-licensing can only take place between vertically integrated firms because firm 3 does not operate downstream.

To produce the final good each manufacturer needs τ_1 and only one between τ_2 and τ_3. This assumption limits the scope of the analysis to two alternative platforms, $\mathcal{P}(\tau_1, \tau_2)$ and $\mathcal{P}(\tau_1, \tau_3)$, and makes the conflict between τ_2 and τ_3 more compelling. The framework of the model is given in Figure 1.

[FIGURE 1 ABOUT HERE]

Downstream, vertically integrated firms compete in quantities and produce an homogeneous good. The choice between $\mathcal{P}(\tau_1, \tau_2)$ and $\mathcal{P}(\tau_1, \tau_3)$ is taken by manufacturers in a non-cooperative manner, by comparing own profits under different platform specifications. More specifically, four cases are possible: two in which both integrated firms employ the same inputs, so that a technology *standard (S)* arises, and two in which they employ different inputs, so that two *competing platforms (CP)* coexist on the marketplace.

The technology adoption choice affects the value of the marginal cost of production. Indeed, final good's production process requires the payment of a marginal cost $c \in (0, 1)$ on top of the fees paid to acquire upstream inputs. However, if manufacturers adopt the same platform, or standard, then they pay a marginal cost equal to σc, with $\sigma \in (0, 1)$.[15] Furthermore, technology 3 is superior to technology 2; indeed, if a firm uses τ_3 instead of τ_2, then its marginal cost is discounted by $\epsilon \in (0, 1)$.

[14]Most of the economic literature on licensing has studied the anticompetitive effects imparted by upstream pricing decisions on the downstream market. More specifically, Rey and Salant (2009) analyzes the impact of alternative licensing policies by owners of essential IPRs on downstream competition. Lin (1996) shows that firms can use fixed fee licensing agreements to collude on the product market. Analogously, Eswaran (1994) proves that cross-licensing constitutes a device that facilitates collusion among downstream horizontal competitors.

[15]This formalization can be interpreted as a reduced form of a richer model where joint adoption leads to scale economies, either in production or in demand.

Summarizing, the value of firm i's marginal cost of production is equal to:

$$c_i = \begin{cases} \mathbf{1}\sigma c + (1-\mathbf{1})c & \textit{if firm } i \textit{ adopts } \mathcal{P}(\tau_1, \tau_2) \\ \\ \mathbf{1}\sigma\epsilon c + (1-\mathbf{1})\epsilon c & \textit{if firm } i \textit{ adopts } \mathcal{P}(\tau_1, \tau_3) \end{cases}$$

With $i = 1, 2$ and $\mathbf{1}$ being an indicator function given by:

$$\mathbf{1} = \begin{cases} 1 & \textit{if a standard (S) is chosen} \\ 0 & \textit{if two competing platforms (CP) are chosen} \end{cases}$$

[TABLE 1 ABOUT HERE]

Consumers have inverse demand $P(Q)$, where Q is the total industry output. Assume for simplicity that $P(Q)$ is linear and given by $\max\{0, 1-Q\}$. Demand linearity makes sure that the Cournot-Nash equilibrium of the game exists and is unique.

Finally, side payments are not allowed in this model. Side payments would take the form of conditional contracts in which parties specify before the adoption of a technology what type of transfers they would carry out depending on the same choice. Agreements of this sort can be ruled out invoking the following sorts of argument. First of all, having a contingent nature the parties may be tempted to renegotiate them ex post. Secondly, rational agents may design them to collude on the product market, so that, like other forms of horizontal agreements, they are typically treated as per se unlawful by antitrust authorities.

5 Linear Pricing: Equilibrium analysis

In this section, the results of the analysis carried out assuming that firms set licensing agreements by means of linear pricing and public contracts are presented.

In what follows, w_{jk} indicates the royalty rate set by firm j to firm k, with $j, k = 1, 2$ and $j \neq k$. Instead, $w_{31} = w_{32} = w_3$ is the fee set by firm 3 to both 1 and 2; in other words, firm 3 cannot discriminate among downstream firms.[16] Finally, firm 1 (firm 2) internalizes the cost of using τ_1 (τ_2) in the production process.

The timing of the game follows.

1. *Technology Choice Stage*: downstream firms choose a production technology and sink a fixed investment cost equal to I.
2. *Pricing Scheme and Royalty Setting Stage*: upstream firms whose technology is adopted downstream choose the pricing scheme (independent licensing/cross-licensing) and the royalty rate. Consequently, each downstream firm decides whether to pay the royalty rate (and produce) or give up production.

[16] This hypothesis is consistent with the non-discriminatory requirement that firms in SSOs must comply with when agreeing on FRAND commitments. In Section 7, I show that if one would relax this assumption the main results of the model still go through.

3. *Product Market Competition Stage*: active firms set quantities.

By sinking I, the downstream units commit to firm-specific investments and set up the equipment necessary to carry out final good's production. In what follows, it is assumed that the fixed cost I is big enough to make the technology choice irreversible once the licensing stage is reached and let the hold-up problem arise.

The model is solved by backward induction and the equilibrium concept employed is the Sub-game Perfect Nash Equilibrium (SPE). I first present the two frameworks in which vertically integrated firms jointly employ $\mathcal{P}(\tau_1, \tau_2)$ or $\mathcal{P}(\tau_1, \tau_3)$, i.e. the cases in which a standard arises as outcome of the technology adoption phase. I denote these two cases as $S2$ and $S3$, respectively. Then, I discuss the scenarios that feature the adoption of two competing platforms: the one in which firm 1 adopts $\mathcal{P}(\tau_1, \tau_3)$ and firm 2 adopts $\mathcal{P}(\tau_1, \tau_2)$, which is denoted by $CP32$, and the one in which firm 1 adopts $\mathcal{P}(\tau_1, \tau_2)$ and firm 2 adopts $\mathcal{P}(\tau_1, \tau_3)$, denoted by $CP23$.[17]

The analysis will be conducted under the following parametric assumption:

Assumption 1.
$\epsilon > \bar{\epsilon}(c) \equiv \max\{0, (7c - 3)/4c\}$.

Assumption 1 implies that in the cases with competing platforms the difference between the marginal costs borne by producers is small enough. Consequently, if market monopolization arises at equilibrium it is not due to the cost savings generated by the employment of τ_3, the pure upstream firm's technology.

5.1 Adoption of $\mathcal{P}(\tau_1, \tau_2)$ as Technology Standard- "$S2$"

To begin with, I derive the optimal quantities set by firm 1 and firm 2 for given royalties, then I compute the equilibrium royalty rates.

At the competition stage, each downstream firm maximizes:

$$\max_{q_j \geq 0} \Pi_j = [1 - q_j - q_k - w_{kj} - \sigma c]q_j + q_k w_{jk}$$

With j, k=1,2, $j \neq k$. The equilibrium is characterized by:

$$\begin{cases} q_j^{S2}(w_{12}, w_{21}) = \frac{1-\sigma c - 2w_{kj} + w_{jk}}{3} \\ \\ Q^{S2}(w_{12}, w_{21}) = \frac{2(1-\sigma c) - (w_{jk} + w_{kj})}{3} \\ \\ P(Q^{S2}(w_{12}, w_{21})) = \frac{1 + 2\sigma c + w_{jk} + w_{kj}}{3} \end{cases} \tag{1}$$

At this stage, two sub-cases must be distinguished: the one in which firm 1 and firm 2 license their technologies independently (independent licensing) and the one in which licensing decisions are taken cooperatively (cross-licensing).

[17] The analysis of this last case is discussed in appendix A, because it does not arise as an equilibrium of the adoption game.

5.1.1 Independent Licensing

At the royalty setting stage of the game with independent licensing vertically integrated firms maximize:

$$\max_{w_{jk}\geq 0} \Pi_j^{S2} = [P(Q^{S2}(w_{12},w_{21})) - w_{kj} - \sigma c]q_j^{S2}(w_{12},w_{21}) + q_k^{S2}(w_{12},w_{21})w_{jk}.$$

With j,k=1,2 and $j \neq k$. The first-order condition is:

$$\frac{\partial \Pi_j^{S2}}{\partial w_{jk}} = \underbrace{[P(Q^{S2}) - w_{kj} - \sigma c]\frac{\partial q_j^{S2}}{\partial w_{jk}}}_{>0,\text{ raising rival's costs}} + \frac{\partial P(Q^{S2})}{\partial Q}\frac{\partial Q^{S2}}{\partial w_{jk}} q_j^{S2} + q_k^{S2} + \frac{\partial q_k^{S2}}{\partial w_{jk}} w_{jk} = 0. \tag{2}$$

If firm j raises w_{jk} it trades off the higher revenue generated downstream (partly due to the raising rival's costs effect) with the lower upstream revenue caused by firm k's output contraction downstream. Linearity leads to:

$$w_{jk}(w_{kj}) = \frac{5(1-\sigma c) - w_{kj}}{10}$$

With $j,k = 1,2$ and $j \neq k$. By symmetry, equilibrium wholesale prices are:

$$w_{12}^{S2} = w_{21}^{S2} = 5(1-\sigma c)/11.$$

Plugging this value in (1), under the joint employment of $\mathcal{P}(\tau_1,\tau_2)$ and independent licensing one has the results in Table 2. In particular, active firms' profits are equal to $\Pi_1^{S2} = \Pi_2^{S2} = 14(1-\sigma c)^2/121$ and the consumer surplus is given by $CS = Q^2/2 = 8(1-\sigma c)^2/121$.

[TABLE 2 ABOUT HERE]

At the licensing equilibrium of the game in which vertically integrated firms price their technologies non cooperatively, royalties are determined by two effects: the *Cournot effect* and the *raising rival's costs effect*. The former is caused by the complementarity between the technologies in the standard and the latter is due to the fact that both vertically integrated firms act as monopoly inputs' providers to their product market's rival.

5.1.2 Cross-licensing

Cross-licensing is modeled in the following way. Vertically integrated firms maximize joint profits by setting a royalty rate $W_{CL} = w_{12} + w_{21}$ that implements the monopoly outcome on the product market.

Using Q^{S2} from (1), upstream firms solve:[18]

$$Q^{S2}(W_{CL}) = \frac{2(1-\sigma c) - W_{CL}^{S2}}{3} = \frac{1-\sigma c}{2} \iff W_{CL}^{S2} = \frac{1-\sigma c}{2}$$

Then, symmetry leads to $w_{12}^{S2} = w_{21}^{S2} = W_{CL}^{S2}/2 = (1-\sigma c)/4$.

Cross-licensing allows firms to fix the raising rival's costs and double marginalization effects bringing royalties down to the monopoly level ($W_{CL}^{S2}/2 = (1-c_J)/4 < w_{jk}^{S2} = 5(1-c_J)/11$). Downstream firms split the monopoly's profit and raise $\Pi^{S2} = (1-c_J)^2/8$ each. Moreover, the consumer surplus is equal to $CS = Q^2/2 = (1-\sigma c)^2/8 > 8(1-\sigma c)^2/121$, so that cross-licensing is beneficial to consumers as well.

Comparing the results in Table 2, it is clear that the equilibrium licensing scheme when vertically integrated firms jointly adopt a standard with technology 1 and technology 2 is cross-licensing. Indeed, each firm strictly prefers the cooperative agreement to the non-cooperative one, as $\Pi_j^{S2} = 14(1-c_J)^2/121 < (1-c_J)^2/8 = \Pi^{S2}$.

5.2 Adoption of $\mathcal{P}(\tau_1, \tau_3)$ as Technology Standard - "$S3$"

If vertically integrated firms adopt a standard that displays technology 1 and technology 3, then both benefit from the greater efficiency of τ_3. Moreover, firms are asymmetric at the upstream level, because firm 2 does not license its technology downstream and needs to acquire externally τ_1 and τ_3. Finally, licensing firms 1 and 3 cannot cross-license their technologies, because firm 3 does not operate downstream.

At the product market competition stage, firm 1 solves:

$$\max_{q_1 \geq 0} \Pi_1 = [1 - q_1 - q_2 - w_3 - \epsilon\sigma c]q_1 + q_2 w_{12}.$$

Firm 2 solves

$$\max_{q_2 \geq 0} \Pi_2 = [1 - q_1 - q_2 - w_3 - w_{12} - \epsilon\sigma c]q_2.$$

The results at equilibrium are:

$$\begin{cases} q_1^{S3}(w_{12}, w_3) = \frac{1-\epsilon\sigma c - w_3 + w_{12}}{3} \\ q_2^{S3}(w_{12}, w_3) = \frac{1-\epsilon\sigma c - w_3 - 2w_{12}}{3} \\ Q^{S3}(w_{12}, w_3) = \frac{2(1-\epsilon\sigma c) - (2w_3 + w_{12})}{3} \\ P(Q^{S3}(w_{12}, w_3)) = \frac{1+2\epsilon\sigma c + 2w_3 + w_{12}}{3} \end{cases} \tag{3}$$

[18] Analogously, one can show that the same result holds by explicitly solving for the maximization problem of vertically integrated firms' joint profits. Indeed,

$$W_{CL}^{S2} = \arg\max_{W_{CL}} \Pi_1^{S2} + \Pi_2^{S2} = [1 - Q^{S2}(W_{CL}) - \sigma c]Q^{S2}.$$

At the royalty setting stage, firm 1 solves the following problem:

$$\max_{w_{12}\geq 0} \Pi_1^{S3} = [P(Q^{S3}(w_{12}, w_3)) - w_3 - \epsilon\sigma c] q_1^{S3}(w_{12}, w_3) + q_2^{S3}(w_{12}, w_3) w_{12}.$$

The first-order condition is:

$$\frac{\partial \Pi_1^{S3}}{\partial w_{12}} = [P(Q^{S3}) - w_3 - \epsilon\sigma c] \frac{\partial q_1^{S3}}{\partial w_{12}} + \frac{\partial P(Q^{S3})}{\partial Q} \frac{\partial Q^{S3}}{\partial w_{12}} q_1^{S3} + q_2^{S3} + \frac{\partial q_2^{S3}}{\partial w_{12}} w_{12} = 0.$$

The optimal value of w_{12} is determined by the tradeoff triggered by an higher royalty rate on downstream and upstream revenues. More specifically, the first term is related to the raising rival's costs effect, it is positive and acts only at the expenses of firm 2.

Invoking linearity, firm 1 upstream reaction function is equal to:

$$w_{12}(w_3) = \frac{1 - w_3 - \epsilon\sigma c}{2}. \tag{4}$$

Firm 3 solves the following problem:

$$\max_{w_3 \geq 0} \Pi_3^{S3} = Q^{S3}(w_{12}, w_3) w_3.$$

The resulting first-order condition is:

$$\frac{\partial \Pi^{S3}}{\partial w_3} = \frac{\partial Q^{S3}}{\partial w_3} w_3 + Q^{S3} = 0.$$

Clearly, the raising rival's costs effect does not play any role for firm 3, because it does not operate on the product market. Using linearity, one finds that the reaction function of firm 3 is given by:

$$w_3(w_{12}) = \frac{2(1 - \epsilon\sigma c) - w_{12}}{4}. \tag{5}$$

Solving for w_{12} and w_3 from (4) and (5), one can derive the following equilibrium expressions:

$$\begin{cases} w_{12}^{S3} = \frac{2(1-\epsilon\sigma c)}{7} \\ w_3^{S3} = \frac{3(1-\epsilon\sigma c)}{7} \end{cases} \tag{6}$$

Table 3 summarizes the results of this section. In particular, $\Pi_3^{S3} = 6(1 - \epsilon\sigma c)^2/49 > \Pi_1^{S3} = 4(1 - \epsilon\sigma c)^2/49 > \Pi_2^{S3} = 0$ and the consumer surplus is equal to $CS = Q^2/2 = 2(1 - \epsilon\sigma c)^2/49$.

[TABLE 3 ABOUT HERE]

The equilibrium of the game in which firm 1 and firm 3 price their technologies non cooperatively features a monopoly of firm 1 downstream. This is because, with respect to the case of joint adoption of $\mathcal{P}(\tau_1, \tau_2)$, firm 2 loses a device to face firm 1 competition on the product market (namely, the possibility to price an input of firm 1).

5.3 Competing Platforms: firm 1 uses $\mathcal{P}(\tau_1, \tau_3)$ and firm 2 uses $\mathcal{P}(\tau_1, \tau_2)$ - "$CP32$"

At the product market competition stage, firm 1 solves:

$$\max_{q_1 \geq 0} \Pi_1 = [1 - q_1 - q_2 - w_3 - \epsilon c]q_1 + q_2 w_{12},$$

Firm 2 solves:

$$\max_{q_2 \geq 0} \Pi_2 = [1 - q_1 - q_2 - w_{12} - c]q_2.$$

Firm 2 employs its own technology, then the marginal cost it pays is equal to c. Instead, Firm 1 employes τ_3, thus the marginal cost c is discounted by the parameter ϵ. The reduced form equilibrium results associated with the maximization problems above are given in the following.

$$\begin{cases} q_1^{CP32}(w_{12}, w_3) = \frac{1-c(2\epsilon-1)-2w_3+w_{12}}{3} \\ \\ q_2^{CP32}(w_{12}, w_3) = \frac{1-c(2-\epsilon)+w_3-2w_{12}}{3} \\ \\ Q^{CP32}(w_{12}, w_3) = \frac{2-c(1+\epsilon)-(w_3+w_{12})]}{3} \\ \\ P(Q^{CP32}(w_{12}, w_3)) = \frac{1+c(1+\epsilon)+(w_3+w_{12})}{3} \end{cases} \tag{7}$$

At the royalty setting stage, firm 1 solves:

$$\max_{w_{12} \geq 0} \Pi_1^{CP32} = [1 - Q^{CP32}(w_{12}, w_3) - w_3 - \epsilon c]q_1^{CP32}(w_{12}, w_3) + q_2^{CP32}(w_{12}, w_3)w_{12}$$

The first-order condition follows:

$$\frac{\partial \Pi_1^{CP32}}{\partial w_{12}} = [1 - Q^{CP32} - w_3 - \epsilon c]\frac{\partial q_1^{CP32}}{\partial w_{12}} - \frac{\partial Q^{CP32}}{\partial w_{12}} q_1^{CP32} + q_2^{CP32} + \frac{\partial q_2^{CP32}}{\partial w_{12}} w_{12} = 0$$

Firm 1 takes into account the fact that by raising the value of w_{12} it can exert a negative externality on firm 2 and reduce its product market share. By linearity, firm 1 upstream reaction function is equal to:

$$w_{12}(w_3) = \frac{5 - c(4 + \epsilon) - w_3}{10}. \tag{8}$$

Firm 3 solves the following problem:

$$\max_{w_3 \geq 0} \Pi_3^{CP32} = q_1^{CP32}(w_{12}, w_3)w_3.$$

The first-order condition is:

$$\frac{\partial \Pi^{CP32}}{\partial w_3} = \frac{\partial q_1^{CP32}}{\partial w_3} w_3 + q_1^{CP32} = 0.$$

In this case, firm 3 can exert its monopoly power only at expenses of firm 1 because firm 2 employs its own technology. Using linearity, one finds that the reaction function of firm 3 is equal to:

$$w_3(w_{12}) = \frac{1 - c(2\epsilon - 1) + w_{12}}{4}. \tag{9}$$

By solving for w_{12} and w_3 from (8) and (9), one can derive the following equilibrium expressions:

$$\begin{cases} w_{12}^{CP32} = \frac{19-c(2\epsilon+17)}{41} \\ \\ w_3^{CP32} = \frac{3[5-c(7\epsilon-2)]}{41} \end{cases} \tag{10}$$

The expressions in (10) must be employed in (7) to compute firms' payoffs. The results of this section are in Table 4 .

[TABLE 4 ABOUT HERE]

Remarkably, under Assumption 1 firm 2 produces a positive amount on the market for the final good; this is because, by using τ_2 instead of τ_3, firm 2 is not stifled by the raising rival's costs effect and it is only firm 1 to be held-up by firm 3. More specifically, if $\epsilon \in [\bar{\epsilon}(c), (9c+2)/11c)$, then $q_1^{CP32} > q_2^{CP32} > 0$, and if $\epsilon \in [(9c+2)/11c, 1)$, then $q_2^{CP32} \geq q_1^{CP32} > 0$.[19]

5.4 Technology Choice

In the first stage of the game, vertically integrated firms choose the technology platform they employ for the production of the final good.

Proposition 1.
Assume that side payments are not allowed and that the choice of the technology is taken by vertically integrated firms, then the unique Nash equilibrium of the adoption game features:

i. The employment of $\mathcal{P}(\tau_1, \tau_2)$ as technology standard ($S2$) if $\sigma \leq \tilde{\sigma}(c, \epsilon)$;

ii. The employment of competing platforms ($CP32$) if $\sigma > \tilde{\sigma}(c, \epsilon)$.

Proof. See appendix A.

The main result of Proposition 1 is that the case with technology τ_3 into the standard ($S3$) is not an equilibrium of the technology adoption game. This outcome is determined by the basic trade-off outlined in the Introduction: from the point of view of firms 1 and 2, cross-licensing preserves rents, instead contracting with pure developers is efficient but leads to rent dissipation (because of hold-up). The result in Proposition 1 shows that if σ is small the former effect prevails and if σ is large the latter effect prevails.

More specifically, on the one hand, if the cost-savings generated by having a technology standard are sufficiently important, then the employment of τ_2 is a dominant strategy to firm 2 and the Nash equilibrium is determined by the choice of firm 1. Firm 1 employs technology τ_2 (and cross licenses with 2) if the value of σ is small, instead, as σ increases, the adoption of competing platforms becomes more profitable for firm 1.

[19]Remind that the case with competing platforms in which firm 1 uses $\mathcal{P}(\tau_1, \tau_2)$ and firm 2 uses $\mathcal{P}(\tau_1, \tau_3)$, indexed by $CP23$, is put in the appendix.

On the other hand, if the cost-savings generated by having a technology standard become less important, then the use of $\mathcal{P}(\tau_1, \tau_3)$ is more attractive to firm 2 and the employment of $\mathcal{P}(\tau_1, \tau_2)$ is not a dominant strategy anymore. However, firm 2 still anticipates that in the case of a joint adoption of $\mathcal{P}(\tau_1, \tau_3)$ it would be stifled by the raising rival's costs and hold-up effects. Consequently, if firm 1 would choose $\mathcal{P}(\tau_1, \tau_3)$ then firm 2 would reply by employing its own technology.

Therefore, at equilibrium, either a standard with $\mathcal{P}(\tau_1, \tau_2)$ is chosen or there are competing platforms, with firm 1 employing $\mathcal{P}(\tau_1, \tau_3)$ and firm 2 employing $\mathcal{P}(\tau_1, \tau_2)$.

5.5 Welfare Analysis

The welfare analysis is conducted by assuming that a benevolent planner decides the technology to be employed by comparing the value of social surplus associated with the four cases of adoption ($S2$,$S3$,$CP32$,$CP23$). Hence, the following game is solved:

1. *Technology Choice Stage*: the benevolent planner chooses a production technology.
2. *Pricing Scheme and Royalty Setting Stage*: upstream firms whose technology is adopted downstream choose the pricing scheme (independent licensing/cross-licensing) and the royalty rate. Consequently, each downstream firm decides whether to pay the royalty rate (and produce) or give up production.
3. *Product Market Competition Stage*: active firms set quantities.

In other words, this analysis provides the outcome of a game in which the technology choice is taken by disregarding the strategic interactions that determine the equilibrium of the adoption game in Proposition 1. However, the planner still takes into account both the impact that the employment of a particular technology has on firms' choices at the licensing and product market stages, and the hold-up problem. The result of the game above is in what follows.

Lemma 1.
Assume that the choice of the technology is taken by a benevolent planner, then at the equilibrium she would employ:

i. $\mathcal{P}(\tau_1, \tau_2)$ *as technology standard (*$S2$*) in:*
$\{(\epsilon, \sigma)| \quad \sigma \in (0, \bar{\sigma}(c, \epsilon))\} \quad \setminus \quad \{(\epsilon, \sigma)| \quad \sigma \in (\bar{\bar{\sigma}}(c, \epsilon), \min\{\bar{\sigma}(c, \epsilon), 1\})\}$;

ii. $\mathcal{P}(\tau_1, \tau_3)$ *as technology standard (*$S3$*) in:*
$\{(\epsilon, \sigma)| \quad \sigma \in (\bar{\sigma}(c, \epsilon), 1)\} \quad \setminus \quad \{(\epsilon, \sigma)| \quad \sigma \in (\max\{\bar{\sigma}(c, \epsilon), \bar{\bar{\bar{\sigma}}}(c, \epsilon)\}, 1)\}$;

iii. Competing platforms ($CP32$*) in:*
$\{(\epsilon, \sigma)| \quad \sigma \in (\bar{\bar{\sigma}}(c, \epsilon), \min\{\bar{\sigma}(c, \epsilon), 1\})\} \quad \cup \quad \{(\epsilon, \sigma)| \quad \sigma \in (\max\{\bar{\sigma}(c, \epsilon), \bar{\bar{\bar{\sigma}}}(c, \epsilon)\}, 1)\}$.

Proof. See appendix A.

There are three relevant areas: the joint adoption of $\mathcal{P}(\tau_1, \tau_2)$ maximizes total welfare for low values of σ and the joint employment of $\mathcal{P}(\tau_1, \tau_3)$ maximizes total welfare for high values of σ. However, if σ is big enough the employment of $\mathcal{P}(\tau_1, \tau_3)$ by firm 1 and $\mathcal{P}(\tau_1, \tau_2)$ by firm 2 ($CP32$) can generate a value of surplus bigger than the cases of standard adoption ($S2$ and $S3$).

Using the results of Proposition 1 and Lemma 1, one can derive the following proposition.

Proposition 2.

There is a wedge between the adoption choice taken by integrated entities and the one of the social planner; in this wedge, the exclusion of firm 3 from the standard employed by vertically integrated organizations is inefficient.

Proof. See appendix A.

Proposition 2 shows that the trade-off between the technological efficiency of the upstream firm input and the contractual efficiency of cross-licensing can lead to a technology choice that is sub-optimal from the total welfare point of view. This is because, when the advantages from adopting a standard and the cost savings due to the employment of the specialized firm are sufficiently large, vertically integrated firms may prefer to cross-license their technologies while a benevolent planner would adopt a standard with τ_3.

5.6 A Numerical Example

Here it is presented a numerical example that illustrates the results above. More specifically, it is assumed that the marginal cost of production c is equal to $1/2$.

[FIGURE 2 ABOUT HERE]

For $c = 1/2$ the value of $\bar{\epsilon}(c)$ in Assumption 1 is equal to $1/4$, hence, in the figure, the relevant range of values of ϵ is given by $(1/4, 1)$.

The panel (a) of Figure 2 presents the outcome of the adoption game and the panel (b) presents the results of the welfare analysis. Panel (c) shows the area of total exclusion of $\mathcal{P}(\tau_1, \tau_3)$ (marked by T) and two areas of partial exclusion, P_3 and P_2. In P_3 the adoption of $\mathcal{P}(\tau_1, \tau_3)$ as technology standard is efficient but an equilibrium with competing platforms arises. Instead, in P_2 the adoption of $\mathcal{P}(\tau_1, \tau_2)$ as technology standard is more efficient than the equilibrium with competing platforms.

6 Ex-ante Licensing Policy

In the time-line of the game with linear pricing, active licensors set royalty rates after being employed by manufacturers; this choice grants monopoly power in the negotiations' phase to the licensors whose technology is adopted. In this section, I study the SPE of a game in which the royalty rate stage precedes technology choice and adoption, and let firm 2 and firm 3 compete for the employment of their technologies.

The timing of the new set-up follows.

1. *Licensing Scheme and Royalty Setting Stage*: upstream firms set the royalty rate and the licensing scheme (independent licensing/cross-licensing).
2. *Technology Choice Stage*: downstream firms choose the technology.
3. *Product Market Competition Stage*: active firms set quantities.

This time-line reproduces the results of an auction carried out between the technologies of firm 2 and firm 3 at the competitive conditions prevailing before the adoption phase. In other words, in this framework it is analyzed what consequences would have the implementation of a policy of early licensing commitments on the choice of the technology, so to replicate the effects of FRAND agreements' reasonableness requirement implementation.[20]

Proposition 3.
Assume that active licensors set royalty rates before their technologies have been employed by manufacturers, then the equilibrium of the adoption game features the employment of $\mathcal{P}(\tau_1, \tau_3)$ as technology standard ($S3$) and is efficient.
Proof. See appendix A.

Proposition 3 shows that the hold-up problem crucially tilts the licensing negotiations between firm 1 and firm 3 (the pure innovator). Indeed, the twist in the timing changes the incentives of firm 3 when pricing its technology, instead, the best agreement that firm 2 can aim at reaching with firm 1 does not depend on the timing of the negotiations and consists in cross-licensing respective patents. However, in the set-up of this extension, firm 3, being more efficient, can match the offer of firm 2 and convince firm 1 to employ τ_3.

The resulting normative policy implication is that SSOs members should be allowed to talk about royalties when they choose among the technologies to include in a standard, because this would solve the hold-up problem and lead to a more efficient decision.

7 Technology Adoption in Alternative Frameworks

The model shows that the adoption of $\mathcal{P}(\tau_1, \tau_2)$ as technology standard depends on the profitability of cross-licensing and the severity of the hold-up problem. Based on this, one can analyze SSO adoption choices in different frameworks.

7.1 N vertically integrated firms

If the set-up would include N vertically integrated firms, then the per-firm profits generated by cross-licensing would decrease as N increases. Therefore, it would be more difficult to sustain an equilibrium featuring the joint employment of $\mathcal{P}(\tau_1, \tau_2)$.

[20]Reasonableness requires that licensing decisions taken before technology adoption must be consistent with those decided after technology's employment by manufacturers, so to avoid excessive royalties due to the lack of competitive alternatives.

7.2 N stand-alone upstream firms

If it were the number of upstream firms endowed with the efficient technology to increase, then the scope for the exclusion of firm 3 would remain because the hold-up problem does not depend on the number of upstream firms but rather on the timing of technology adoption.

7.3 Price competition with differentiated products

In a framework with price competition the main results of the article would stay the same. Indeed, the upstream operations of the integrated firms could keep up the profitability of an agreement featuring the joint adoption of $\mathcal{P}(\tau_1, \tau_2)$ and cross-licensing by setting royalty rates equal to the monopoly price and so implementing the monopoly outcome on the downstream market.

7.4 Set-up with one vertically integrated firm

Assume that the framework would embed integrated firm 2 facing the competition of a stand-alone downstream firm, D_1, and that τ_1 and τ_3 are provided by two upstream stand-alone firms, indexed by U_1 and U_3. In this modified setting, the profitability for D_1 of using the technologies of firm 2 and firm U_1 would greatly reduce.[21] Indeed, now D_1 cannot cross-license with firm 2, moreover it would be subject to the raising-rival's costs incentive of integrated firm 2 and the hold-up of firm U_1. Therefore, it is expectable that the payoff of U_1 when it employs $\mathcal{P}(\tau_1, \tau_2)$ with firm 2 is squeezed by firm 2 and U_1, so to make the employment of τ_2 less profitable to D_1 than in the main model.

7.5 Stand-alone firm 3 can discriminate

In case $S3$, firm 3 cannot discriminate between firm 1 and firm 2, but this assumption is not crucial for the exclusion of firm 3 from the technology standard. Indeed, given that at the licensing stage its technology has already been adopted, were firm 3 free to discriminate it would let firm 1 be monopolist and squeeze as much as possible its downstream rent through the royalty rate. Therefore, the scope for the employment of τ_3 would further shrink.

7.6 Acquisition of firm 3 by integrated operators

Assume a merger stage is introduced into the game at which integrated firms can take over firm 3. There are two cases to be distinguished, depending on whether the merger stage precedes or follows the technology adoption stage.

If firm 3 merges with vertically integrated firms before the production technology is chosen, then firm 3 would join the deciding coalition and, clearly, the adoption of platform $\mathcal{P}(\tau_1, \tau_3)$ would emerge at equilibrium. However, if the merger stage would be the first stage, followed by the technology

[21] Notice that firm 1 would be in a strategic position analogous to the one of firm 2 in case $S3$ of the main model. There, the profit of firm 2 is nil.

choice, the licensing game and the product market stage, then the hold-up problem would still affect the results of the technology adoption stage leading to the same qualitative results as in the main model.

8 Two-part Tariffs

In this extension, upstream firms use two-part tariffs to license-out their technology to downstream firms. It is important to remark that contracts by means of two-part tariffs are more efficient than those with linear pricing because they are not affected by the double marginalization problem. Therefore, if the exclusionary result arises in this setting it is entirely caused by the hold-up effect.

The timing of the game follows:

1. *Technology Choice Stage*: downstream firms choose the technology.
2. *Licensing Scheme and Royalty Setting Stage*: upstream firms whose technology is adopted downstream make a public take-it-or-leave-it offer to downstream firms, consisting of a tariff, indexed by $T_{ij} = w_{ij}q_j + F_{ij}$, and a scheme (independent licensing/cross-licensing) at which they license-out their technologies. Consequently, each downstream firm decides whether to pay the fee (and produce) or give up production.
3. *Product Market Competition Stage*: active firms set quantities.

Firms pay the due tariff after the product market competition stage and under the protection of a limited liability constraint for which they cannot pay more than the profits they raise on the market. Therefore, first firms negotiate over the licensing contracts, then they decide to produce and carry out the payment of the tariffs they agreed upon initially.

Without loss of generality, I assume that upstream firms make sequential offers, so to solve the problems of coordination intrinsic to the settings with complementary inputs; more specifically, this assumption rules out those cases in which the sum of the offers exceeds the profit of a downstream firm.

In what follows, I use π to indicate the rent generated by the product market, as opposed to Π, which indicates total profits.

Like in the model with linear prices, I assume that downstream production requires the payment of a marginal cost $c \in (0, 1)$ and that the employment of a standard generates a cost-saving measured by $\sigma \in (0, 1)$. The adoption of τ_3 reduces the cost borne by downstream manufacturers by $\epsilon \in (0, 1)$. Finally, Assumption 1 holds in this setting as in the model with royalty rates.

8.1 Adoption of $\mathcal{P}(\tau_1, \tau_2)$ as Technology Standard

If integrated firms choose $\mathcal{P}(\tau_1, \tau_2)$ as technology standard, at the product market competition stage the equilibrium values are the same as in equations (1) in the model with linear prices.

In particular, $\pi^c = (1-\sigma c)^2/9$ denotes the value of the per-firm Cournot profit and $\pi^m = (1-\sigma c)^2/4$

the one of the monopoly profit at $w_{12} = w_{21} = 0$.

Lemma 2 presents the equilibria of the licensing game when firm 1 and firm 2 set T_{12} and T_{21} *non-cooperatively.*

Lemma 2.
Under independent licensing and technologies τ_1 and τ_2 in the standard, the Nash equilibria of the licensing game are as in what follows:

i. Firm j offers $w_{jk} = (1 - \sigma c)/2$ and $F_{jk} = 0$, firm k offers $w_{kj} = 0$ and $F_{kj} = \pi^m$. Alternatively, firm j and k offer $w_{jk} = w_{kj} = 0$, $F_{jk} = F_{kj} = \pi^m$: in both cases firm j is in, firm k is out, but extracts all downstream profits from firm j. Moreover, $\Pi_j = 0$, $\Pi_k = \pi^m$.

ii. Firm j and k offer $w_{jk} = w_{kj} = 0$, $F_{jk} = \pi^m$ and $F_{kj} \in [\pi^c, \pi^m)$, at which firm j is out and firm k is in. In this case, $\Pi_j = \pi^m$, $\Pi_k = 0$.

Proof. See appendix B.

At a Nash equilibrium of the non-cooperative licensing game, one of the two firms is out of the market but takes rival's downstream profit through the fixed fee. Unfortunately, though, multiple equilibria imply that it is not possible to determine whether it is firm 1 or firm 2 to get the full monopoly profit. In order to get rid of this limitation, I assume that each upstream firm in the SSO has an equal probability of being first in approaching downstream firms. This implies that, in expected terms, vertically integrated firms share the monopoly profit and get $\Pi_j^{S2} = \Pi_k^{S2} = \pi^m/2$.

8.1.1 Cross-licensing

Under cross-licensing, firms set their fees cooperatively, but behave non-cooperatively at the production stage. The best deal that vertically integrated firms can negotiate upon is one at which they equally share the monopoly rent.

Lemma 3.
Under cross-licensing and technologies τ_1 and τ_2 in the standard, at equilibrium firms write the following agreement: firm j offers $w_{jk} = 0$ and $F_{jk} = \pi^m/2$, whilst firm k offers $w_{kj} = (1 - \sigma c)/2$ and $F_{kj} = 0$. At this agreement, firm k is the monopolist and transfers half of the monopoly rent to firm j.

At the cooperative equilibrium, firm j stays out of the market, firm k is monopolist and transfers half of the downstream rent to firm k at the payment stage. Cross-licensing and independent licensing deliver the same total profit to vertically integrated firms under two-part tariffs. Thus, in this framework, the decision over the standard is not affected by cross-licensing.[22]

[22] Clearly, this holds if in the independent licensing case analyzed above firms have an equal probability of being first in making the offer. Otherwise, in the extreme case in which one firm is always the first, independent licensing and cross-licensing would imply a rather different profits' allocation.

8.2 Adoption of $\mathcal{P}(\tau_1, \tau_3)$ as Technology Standard

In case of joint adoption of platform $\mathcal{P}(\tau_1, \tau_3)$, the product market competition stage equilibrium values are the same as in (3).

Here, $\pi^c = (1 - \epsilon\sigma c)^2/9$ is the per-firm profit under Cournot competition and $\pi^m = (1 - \epsilon\sigma c)^2/4$ is the profit under monopoly at $w_{12} = w_3 = 0$.

Lemma 4 presents the equilibrium tariffs in case $S3$.[23]

Lemma 4.
At a Nash equilibrium, firm 3 offers $w_3 = 0$ and $F_3 = \pi^m$. Firm 1 sets either $w_{12} = 0$ and $F_{12} \geq \pi^m - F_3$ or $w_{12} = (1 - \sigma\epsilon c)/2$ and $F_{12} = 0$. In both cases, $\Pi_j^{S3} = 0$, with $j = 1, 2$, $\Pi_3^{S3} = \pi^m$ and either firm 1 or firm 2 would be the downstream monopolist.
Proof. See appendix B.

Lemma 4 shows that under the adoption of standard $\mathcal{P}(\tau_1, \tau_3)$, if firms license their technologies by means of two-part tariffs then the hold-up problem is so severe that the stand-alone upstream firm is able to fully squeeze integrated firms' profits.

8.3 Competing Platforms: firm 1 uses $\mathcal{P}(\tau_1, \tau_3)$ and firm 2 uses $\mathcal{P}(\tau_1, \tau_2)$

The equilibrium values at the product market competition stage when firm 1 uses $\mathcal{P}(\tau_1, \tau_3)$ and firm 2 uses $\mathcal{P}(\tau_1, \tau_2)$ are the same as in (7).

Therefore, at $w_{12} = w_3 = 0$, if firm 1 would be the monopolist its profit would be equal to $\pi_1^m = (1 - \epsilon c)^2/4$. If firm 2 would be the monopolist, then $\pi_2^m = (1 - c)^2/4$. In the case of duopoly, an asymmetric Cournot would arise on the market, with associated payoffs given by $\pi_1^c = (1 - 2\epsilon c + c)^2/9$ and $\pi_2^c = (1 - 2c + \epsilon c)^2/9$.

Lemma 5 presents the equilibrium license fees in scenario $CP32$.

Lemma 5.
At equilibrium, firm 1 sets $w_{12} = 0$ and firm 3 sets $w_3 = 0$. Moreover, the fee of firm 3 is given by $F_3 = \pi_1^c$ and firm 1 replies by setting F_{12} as to push firm 2 out of the downstream market. Consequently, $\Pi_1^{CP32} = \pi_1^m - \pi_1^c$, $\Pi_2^{CP32} = 0$ and $\Pi_3^{CP32} = \pi_1^c$.
Proof. See appendix B.

Firm 3 anticipates that if the fee it sets is too high then firm 1 would stay inactive. Firm 1 replies foreclosing the downstream market, which yields the surplus between the monopoly rent and the Cournot profit.

[23] In analogy to the model with linear pricing, I am also assuming that firm 3 cannot discriminate between firm 1 and firm 2.

8.4 Competing Platforms: firm 1 uses $\mathcal{P}(\tau_1, \tau_2)$ and firm 2 uses $\mathcal{P}(\tau_1, \tau_3)$

The equilibrium values at the product market competition stage in case $CP23$ are given in (11).

If $w_{12} = w_3 = 0$, were firm 1 to be the monopolist then its profit would be equal to $\pi_1^m = (1-c)^2/4$, instead, if firm 2 would be the monopolist then $\pi_2^m = (1-\epsilon c)^2/4$. The per-firm Cournot profits are given by $\pi_1^c = (1-2c+\epsilon c)^2/9$ and $\pi_2^c = (1-2\epsilon c + c)^2/9$.

8.4.1 Independent Licensing

Lemma 6 presents the Nash equilibrium of the licensing game in which all three firms set their tariffs non-cooperatively.

Lemma 6.
At an equilibrium of the licensing game, firms set $w_{21} = w_l = w_n = 0$, $F_{21} > \pi_1^m$, $F_l + F_n \in [0, \pi_c]$, with $l, n = 3, 12$ and $l \neq n$. Therefore, firm 2 gains $\Pi_2^{CP23} = \pi_2^m - \pi_2^c$, instead firm 1 and firm 3 get $\pi_2^c/2$ each.
Proof. See appendix B.

In this case, like in case $CP32$, firm 1 and firm 3 anticipate that by setting an aggregate fee above the Cournot profit of firm 2, this would have incentive to stay inactive. Therefore, they let 2 operate as monopolist and get its Cournot rent. As in Lemma 2, the problem of coordination between firm 1 and firm 3 is solved by assuming that thay have an equal probability to be the first in contracting with firm 2, so that each gets $\pi_2^c/2$ in expectation.

8.4.2 Cross-licensing

Under cross-licensing, firm 1 and firm 2 set their fees cooperatively but behave non-cooperatively at the production stage. The cooperative agreement is accepted by firms 1 and 2 if both are not made worse-off than in the non-cooperative equilibrium.

The vertically integrated firms could agree on a deal that lets firm 2 be active as monopolist and transfer part of the rents to 1 through the fee. In this case, cross-licensing would generate the same amount of total profit as in the independent licensing equilibrium, the integrated organizations would still be held-up by firm 3 and firms 1 and 2 would not improve with respect to the independent licensing case. Indeed, for the integrated firms to improve with respect to the independent licensing equilibrium it must be that the share of the rent left to firm 3 reduces. However, a profitable reply by firm 3 would be to ask a huge fee and break down the cooperative agreement.

8.5 Technology Choice and Welfare Analysis with Two-part tariffs

Proposition 4 presents the results of the adoption game's equilibrium analysis under public licensing contracts and two-part tariffs.

Proposition 4.

Assume that side payments are not allowed and that the choice of the technology is taken by vertically integrated firms, then the unique Nash equilibrium of the adoption game features:

i. The employment of $\mathcal{P}(\tau_1, \tau_2)$ as technology standard if:

$$\sigma \leq \tilde{\sigma}_{TT}(c, \epsilon).$$

ii. The adoption of competing platforms ($CP23$) if:

$$\sigma > \tilde{\sigma}_{TT}(c, \epsilon).$$

Proof. See appendix B.

With two-part tariffs, vertically integrated firms employ respective technologies if σ is low, otherwise a scenario with competing platforms arises.

Two remarks must be done. The first is that *the adoption of $\mathcal{P}(\tau_1, \tau_3)$ is constrained efficient, so that the inefficient and total exclusion of firm 3 emerges also with two-part tariffs.* The second is that, differently from the game with linear pricing, as σ rises above $\tilde{\sigma}_{TT}(c, \epsilon)$ here the Nash equilibrium of the adoption game features case $CP23$, in which firm 1 uses τ_2 and firm 2 uses τ_3. This happens because for a given adoption of τ_3 by firm 2, firm 1's best reply is to avoid the hold-up effect and squeeze part of firm 2's downstream rent through the fee.

Figure 3 illustrates the results of a numerical example in which it is assumed that the marginal cost of production c is equal to 1/5.

[FIGURE 3 ABOUT HERE]

For $c = 1/5$ the value of $\bar{\epsilon}(c)$ is zero, so that the relevant range of values of ϵ is given by the all unit interval. In Figure 3, the area marked by T is the one in which firms 1 and 2 adopt standard $\mathcal{P}(\tau_1, \tau_2)$ and exclude firm 3's technology. Instead, area CP is the one in which integrated firms adopt competing technology platforms.

9 Conclusions

In this article, I studied the incentives that SSOs' vertically integrated firms have to employ patented technologies into their production process. The model develops on the idea that a pure innovator endowed with market power can hold up vertically integrated firms through the sale of an intermediate good. Integrated organizations can choose between two inputs, among which the one provided by the vertically-specialized firm is superior.

The contracting environment employed resembles the one of SSOs in several aspects and in particular in the assumption for which parties negotiate over the royalty fees after downstream manufacturers' choice and adoption of a certain technology. This timing gives a strong bargaining power to upstream suppliers whose technology is employed for the production of the final good and generates the hold-up problem.

The outcome of the welfare analysis shows that by cross-licensing their patents, integrated organizations may inefficiently exclude the pure innovator's superior technology. Moreover, the model rationalizes the pattern of SSOs' technology adoption in major sectors of the information and communications technology industry.

Finally, an important policy conclusion of the article is that, to kill the hold-up problem, firms in SSOs should be allowed to talk about royalties when they choose among competing technologies. Indeed, as shown in the section where a framework with ex-ante licensing is studied, the resulting choice by manufacturers features standard's efficient design. This supports the initiatives by SSOs like VITA, which recently moved towards a policy that requires the owners of patented technologies to disclose the maximum royalty rates and provide binding written license declarations at several specified points during the standard development process.

APPENDIX A. Linear pricing case.

Competing Platforms: firm 1 uses $\mathcal{P}(\tau_1, \tau_2)$ and firm 2 uses $\mathcal{P}(\tau_1, \tau_3)$ - "$CP23$"

To start with, it is important to stress that this scenario does not emerge as Nash equilibrium of the adoption game in the linear pricing case and is here presented for the sake of completeness.

At the product market competition stage, firm 1 solves:

$$\max_{q_1 \geq 0} \Pi_1 = [1 - q_1 - q_2 - w_{21} - c]q_1 + q_2 w_{12},$$

Firm 2 solves:

$$\max_{q_2 \geq 0} \Pi_2 = [1 - q_1 - q_2 - w_3 - w_{12} - \epsilon c]q_2 + q_1 w_{21}.$$

The reduced form equilibrium results of the maximization problems above are as in what follows:

$$\begin{cases} q_1^{CP23}(w_{12}, w_{21}, w_3) = \frac{1 - c(2-\epsilon) + w_3 - 2w_{21} + w_{12}}{3} \\ q_2^{CP23}(w_{12}, w_{21}, w_3) = \frac{1 - c(2\epsilon - 1) - 2(w_3 + w_{12}) + w_{21}}{3} \\ Q^{CP23}(w_{12}, w_{21}, w_3) = \frac{2 - c(1+\epsilon) - (w_3 + w_{12} + w_{21})}{3} \\ P(Q^{CP23}(w_{12}, w_{21}, w_3)) = \frac{1 + c(1+\epsilon) + (w_3 + w_{12} + w_{21})}{3} \end{cases} \tag{11}$$

At the royalty setting stage, firm 1 solves:

$$\max_{w_{12} \geq 0} \Pi_1^{CP23} = [1 - Q^{CP23}(w_{12}, w_{21}, w_3) - w_{21} - c]q_1^{CP23}(w_{12}, w_{21}, w_3) + q_2^{CP23}(w_{12}, w_{21}, w_3)w_{12}.$$

The resulting first-order condition is:

$$\frac{\partial \Pi_1^{CP23}}{\partial w_{12}} = [1 - Q^{CP23} - w_{21} - c]\frac{\partial q_1^{CP23}}{\partial w_{12}} - \frac{\partial Q^{CP23}}{\partial w_{12}} q_1^{CP23} + q_2^{CP23} + \frac{\partial q_2^{CP23}}{\partial w_{12}} w_{12} = 0.$$

Using linearity, firm 1 upstream reaction function is equal to:

$$w_{12}(w_{21}, w_3) = \frac{5 - c(1 + 4\epsilon) - 4w_3 - w_{21}}{10}. \tag{12}$$

Differently from case $CP32$, in case $CP23$ firm 2 licenses τ_2 to firm 1. In particular, firm 2 solves the following problem:

$$\max_{w_{21} \geq 0} \Pi_2^{CP23} = [1 - Q^{CP23}(w_{12}, w_{21}, w_3) - w_{12} - w_3 - \epsilon c]q_2^{CP23}(w_{12}, w_{21}, w_3) + q_1^{CP23}(w_{12}, w_{21}, w_3)w_{21}.$$

The first-order condition follows:

$$\frac{\partial \Pi_2^{CP23}}{\partial w_{21}} = [1 - Q^{CP23} - w_{12} - w_3 - \epsilon c]\frac{\partial q_2^{CP23}}{\partial w_{21}} - \frac{\partial Q^{CP23}}{\partial w_{21}} q_2^{CP23} + q_1^{CP23} + \frac{\partial q_1^{CP23}}{\partial w_{21}} w_{21} = 0.$$

Thus, in this case the royalty rates of both firm 1 and firm 2 are influenced by the raising rival's costs effect. The reaction function of firm 2 is given by:

$$w_{21}(w_{12}, w_{21}, w_3) = \frac{5 - c(\epsilon + 4) - w_3 - w_{12}}{10}. \tag{13}$$

Finally, firm 3 solves:

$$\max_{w_3 \geq 0} \Pi_3^{CP23} = q_2^{CP23}(w_{12}, w_{21}, w_3)w_3.$$

The first-order condition is:

$$\frac{\partial \Pi^{CP23}}{\partial w_3} = \frac{\partial q_2^{CP23}}{\partial w_3} w_3 + q_2^{CP23} = 0.$$

Firm 3 exerts its monopoly power at expenses of firm 2, because firm 1 employs the technology licensed by 2. The reaction function of firm 3 is equal to:

$$w_3(w_{12}, w_{21}, w_3) = \frac{1 - c(2\epsilon - 1) - 2w_{12} + w_{21}}{4}. \tag{14}$$

Solving for $\{w_{12}, w_{21}, w_3\}$ from (12), (13) and (14), one can derive the following equilibrium expressions:

$$\begin{cases} w_{12}^{CP23} = \frac{21 - c(8+13\epsilon)}{54} \\ w_{21}^{CP23} = \frac{12 - c(\epsilon+11)}{27} \\ w_3^{CP23} = \frac{3 - c(7\epsilon - 4)}{18} \end{cases} \tag{15}$$

The equilibrium expressions in (15) must be employed in (11) to compute firms' payoffs. Table 5 summarizes the results of this section.[24]

[TABLE 5 ABOUT HERE]

Under Assumption 1, firm 1 and firm 2 produce a positive amount on the market for the final good (that is, $q_1^{CP23} > 0$ and $q_2^{CP23} > 0$).

[24] In case $CP23$, firm 1 and firm 2 may cross-license respective technologies, however it turns out that a cooperative agreement cannot be reached if one rules out side payments. First of all, the sum of integrated firms' profits can be rewritten as in the following:

$$\Pi_1^{CP23} + \Pi_2^{CP23} = [1 - Q^{CP23}]Q^{CP23} - cq_1^{CP23} - (w_3 + \epsilon c)q_2^{CP23}$$

Hence, one could rewrite above expression as function of W_{CL} and see that the ideal monopolist would set W_{CL} (and share it between firm 1 and firm 2) as to let the firm with the cheaper technology be active on the product market. In other words, one integrated firm would raise positive profits and the other would be made worse off with respect to independent licensing. Consequently, without side payments, a cooperative agreement cannot be found in case $CP23$.

Proof of Proposition 1

The analysis can be greatly simplified by searching for the dominant strategy of firm 2. More specifically, if one compares Π_2^{S2} with Π_2^{CP23} then it turns out that the adoption of $\mathcal{P}(\tau_1, \tau_2)$ is a dominant strategy for firm 2 if σ is low enough:

$$\Pi_2^{S2} = \frac{(1-\sigma c)^2}{8} \geq \Pi_2^{CP23} = \frac{c^2(5\epsilon^2 - 10\epsilon + 14) + 9(1-2c)}{81} \iff$$

$$\sigma \leq \tilde{\tilde{\sigma}}(c,\epsilon) \equiv \frac{9 - 2\sqrt{2}\sqrt{c^2(5\epsilon^2 - 10\epsilon + 14) + 9(1-2c)}}{9c}.$$

$\tilde{\tilde{\sigma}}(c,\epsilon)$ is decreasing in c and increasing in ϵ, moreover if $c \leq .32$ then $\tilde{\tilde{\sigma}}(c,\epsilon) \geq 1$ independently from the value of ϵ.[25]

If the employment of $\mathcal{P}(\tau_1, \tau_2)$ is a dominant strategy for firm 2, then the Nash equilibirum is found by studying the choice of firm 1. In particular, firm 1 compares Π_1^{S2} with Π_1^{CP32} and it chooses $\mathcal{P}(\tau_1, \tau_2)$ if the following holds:

$$\Pi_1^{S2} = \frac{(1-\sigma c)^2}{8} \geq \Pi_1^{CP32} = 2\frac{c^2(90\epsilon^2 - 110\epsilon + 127) - 2c(35\epsilon + 72) + 107}{1681} \iff$$

$$\sigma \leq \tilde{\sigma}(c,\epsilon) \equiv \frac{41 - 4\sqrt{c^2(90\epsilon^2 - 110\epsilon + 127) - 2c(35\epsilon + 72) + 107}}{41c}.$$

With $\tilde{\sigma}(c,\epsilon) < \tilde{\tilde{\sigma}}(c,\epsilon)$, indeed

$$\tilde{\sigma}(c,\epsilon) - \tilde{\tilde{\sigma}}(c,\epsilon) < 0 \iff \Big[c(8+13\epsilon) - 21\Big]\Big[c(95\epsilon - 74) - 21\Big] > 0$$

holds true for all c and ϵ into the unit interval. Summarizing, if $\sigma \in (\tilde{\sigma}(c,\epsilon), \tilde{\tilde{\sigma}}(c,\epsilon)]$ the Nash equilibrium features the adoption of $\mathcal{P}(\tau_1, \tau_3)$ by firm 1 and $\mathcal{P}(\tau_1, \tau_2)$ by firm 2 ($CP32$). Instead, if $\sigma \in (0, \tilde{\sigma}(c,\epsilon)]$ the Nash equilibrium features the adoption of $\mathcal{P}(\tau_1, \tau_2)$ by firm 1 and firm 2 ($S2$).

[TABLE 6 ABOUT HERE]

For σ above $\tilde{\tilde{\sigma}}(c,\epsilon)$ the adoption of platform $\mathcal{P}(\tau_1, \tau_2)$ is not a dominant strategy to firm 2. More specifically, if $\sigma > \tilde{\tilde{\sigma}}(c,\epsilon)$ then $\Pi_2^{CP23} > \Pi_2^{S2}$ and $\Pi_2^{CP32} > \Pi_2^{S3} = 0$; furthermore, given that $\tilde{\tilde{\sigma}}(c,\epsilon) > \tilde{\sigma}(c,\epsilon)$, one has that $\Pi_1^{CP32} > \Pi_1^{S2}$. Hence, firm 2 employs $\mathcal{P}(\tau_1, \tau_3)$ if firm 1 chooses $\mathcal{P}(\tau_1, \tau_2)$, instead, firm 2 adopts $\mathcal{P}(\tau_1, \tau_2)$ if firm 1 uses $\mathcal{P}(\tau_1, \tau_3)$. At the same time, if firm 2 chooses $\mathcal{P}(\tau_1, \tau_2)$, then firm 1 chooses $\mathcal{P}(\tau_1, \tau_3)$ and if firm 2 chooses $\mathcal{P}(\tau_1, \tau_3)$, then firm 1 decides by comparing Π_1^{S3} and Π_1^{CP23}. In this latter case, it turns out that Π_1^{S3} is bigger than Π_1^{CP23} for $\sigma > \tilde{\tilde{\sigma}}(c,\epsilon)$.[26]

[25] The fact that $\tilde{\tilde{\sigma}}(c,\epsilon) \geq 1$ for $c \leq .32$ implies that the analysis of the Nash equilibrium for σ above $\tilde{\tilde{\sigma}}(c,\epsilon)$ is relevant only if $c > .32$.

[26] The proof of this last step is not presented here because not essential to the result that $S3$ does not emerge as Nash equilibrium of the adoption game, but can be provided by the author if requested.

Summarizing, if $\sigma > \tilde{\tilde{\sigma}}(c,\epsilon)$ the Nash equilibrium of the technology adoption game is at $CP32$, instead case $S3$ does not arise at equilibrium. ■

Proof of Lemma 1

In the following, it is analyzed the choice of the benevolent planner for given results of the second and third stage of the game. In particular, the planner decides by comparing the social surplus generated by the four cases of technology adoption.

First of all, it is useful to establish a result that simplifies the analysis below: the total welfare generated by case $CP23$ is smaller than the one associated with case $CP32$. Indeed, the difference between TS^{CP32} and TS^{CP23} can be rewritten as:

$$TS^{CP32} - TS^{CP23} = \frac{[c(8+13\epsilon)-21][c(1,627\epsilon-1,156)-471]}{272,322} > 0 \quad \forall c, \epsilon \in (0,1).$$

Consequently, in the following I can focus on cases $S2$, $S3$ and $CP32$. $S2$ is more efficient than $S3$ if the following holds:

$$TS^{S2} = \frac{3(1-\sigma c)^2}{8} \geq TS^{S3} = \frac{12(1-\epsilon\sigma c)^2}{49} \iff \sigma \leq \bar{\sigma}(c,\epsilon) \equiv \frac{7-4\sqrt{2}}{c(7-4\sqrt{2}\epsilon)},$$

$\bar{\sigma}(c,\epsilon)$ is decreasing in c and increasing in ϵ, moreover $\bar{\sigma}(c,\epsilon) \geq 1$ for all $c \in (0,.19]$ and $\epsilon \in (0,1)$.

Now I check whether case $CP32$ delivers a bigger total surplus than $S3$ and $S2$ above and below $\bar{\sigma}(c,\epsilon)$, respectively. In particular, by using the standard quadratic formula for σ and taking the root whose value lies into the unit interval, it turns out that $S2$ is more efficient than $CP32$ if the following holds:

$$TS^{S2} = \frac{3(1-\sigma c)^2}{8} \geq TS^{CP32} = 4\frac{c^2(139\epsilon^2-138\epsilon+131)-4c(31+35\epsilon)+132}{1681} \iff$$

$$\sigma \leq \bar{\bar{\sigma}}(c,\epsilon) \equiv \frac{123-4\sqrt{6}\sqrt{c^2(139\epsilon^2-138\epsilon+131)-4c(31+35\epsilon)+132}}{123c}.$$

$\bar{\bar{\sigma}}(c,\epsilon)$ is decreasing in c and increasing in ϵ.

$S3$ is more efficient than $CP32$ if the following holds:

$$TS^{S3} = \frac{12(1-\epsilon\sigma c)^2}{49} \geq TS^{CP32} = 4\frac{c^2(139\epsilon^2-138\epsilon+131)-4c(31+35\epsilon)+132}{1681} \iff$$

$$\sigma \leq \bar{\bar{\bar{\sigma}}}(c,\epsilon) \equiv \frac{123-7\sqrt{3}\sqrt{c^2(139\epsilon^2-138\epsilon+131)-4c(31+35\epsilon)+132}}{123\epsilon c}.$$

$\bar{\bar{\bar{\sigma}}}(c,\epsilon)$ is increasing in c for all $c \in (.43,1)$ and decreasing in ϵ if $1 > \epsilon > \bar{\epsilon}(c) > 0$.

The function that generates the locus of points in which $\bar{\bar{\sigma}}(c,\epsilon)$ and $\bar{\bar{\bar{\sigma}}}$ cross $\bar{\sigma}(c,\epsilon)$ is the same. Indeed, after simple algebra manipulations one finds that:

$$\bar{\bar{\sigma}}(c,\epsilon) = \bar{\sigma}(c,\epsilon) = \bar{\bar{\bar{\sigma}}}(c,\epsilon) \iff$$

$$c^2(139\epsilon^2 - 138\epsilon + 131) - 4c(31 + 35\epsilon) + 132 \quad = \quad \Big[\frac{123(1-\epsilon)}{\sqrt{3}(7-4\sqrt{2}\epsilon)}\Big]^2 \tag{16}$$

The function $c(\epsilon)$ along which $\bar{\bar{\sigma}}(c,\epsilon)$ and $\bar{\bar{\bar{\sigma}}}(c,\epsilon)$ cross $\bar{\sigma}(c,\epsilon)$ is obtained by solving (16), which is a quadratic equation in c whose coefficients are functions of ϵ. Applying the quadratic formula and taking the root that lies below $\bar{c}(\epsilon) = 3/(7-4\epsilon)$,[27] one has that the relevant solution to (16) is given by $c_W(\epsilon)$:

$$c_W(\epsilon) = \frac{0.552 + 0.177\epsilon - 0.504\epsilon^2 - 0.662(1-\epsilon)\sqrt{(0.005+\epsilon)(1.359+\epsilon)}}{(1.237-\epsilon)[0.942-(0.993-\epsilon)\epsilon]}.$$

The function $c_W(\epsilon)$ is convex in ϵ; in particular, it is decreasing in ϵ for all $\epsilon \in (0, .22)$ and increasing for all $\epsilon \in (.22, 1)$. Furthermore, $c_W(0) = \bar{c}(0) = .43$, $c_W(.22) = .33$ and $c_W(1) = \bar{c}(1) = 1$. Hence, $\bar{\bar{\sigma}}(c,\epsilon)$ and $\bar{\bar{\bar{\sigma}}}(c,\epsilon)$ do not cross $\bar{\sigma}(c,\epsilon)$ if $c \leq .33$, they cross $\bar{\sigma}(c,\epsilon)$ twice if $c \in (.33, .43]$ and once if $c \in (.43, 1)$. The graph of $c_W(\epsilon)$ is in Figure 4.

[FIGURE 4 ABOUT HERE]

$\bar{\sigma}(c,\epsilon) \geq \bar{\bar{\sigma}}(c,\epsilon)$ and $\bar{\sigma}(c,\epsilon) \geq \bar{\bar{\bar{\sigma}}}(c,\epsilon)$ if the following holds:

$$c^2(139\epsilon^2 - 138\epsilon + 131) - 4c(31 + 35\epsilon) + 132 \geq \Big[\frac{123(1-\epsilon)}{\sqrt{3}(7-4\sqrt{2}\epsilon)}\Big]^2$$

and above inequality is satisfied for all $c \leq c_W(\epsilon)$.[28] The characterization of the areas of constrained maximum welfare follows. To start with, one has that:

$$\forall c \in (0, .33),\ \epsilon \in (0,1), \quad \bar{\sigma}(c,\epsilon) > \bar{\bar{\sigma}}(c,\epsilon) > \bar{\bar{\bar{\sigma}}}(c,\epsilon)$$

Above $\bar{\sigma}(c,\epsilon)$, $CP32$ is more efficient than $S3$ because $\bar{\sigma}(c,\epsilon)$ lies above $\bar{\bar{\bar{\sigma}}}(c,\epsilon)$.[29] Below $\bar{\sigma}(c,\epsilon)$, $S2$ is more efficient than $S3$, however $CP32$ is more efficient than $S2$ into the interval $(\bar{\bar{\sigma}}(c,\epsilon), \bar{\sigma}(c,\epsilon))$. Thus, the planner would decide as in what follows:

i. If $\sigma \in (0, \bar{\bar{\sigma}}(c,\epsilon)]$, then TS^{S2} is bigger than TS^{S3} and TS^{CP32} and the planner would adopt $\mathcal{P}(\tau_1,\tau_2)$ as technology standard;

ii. If $\sigma \in [\bar{\bar{\sigma}}(c,\epsilon), 1)$, then TS^{CP32} is bigger than TS^{S2} and TS^{S3} and the planner would adopt competing platforms.

Instead, for $c \in [.33, 1)$ the three functions of interest $(\bar{\sigma}(c,\epsilon), \bar{\bar{\sigma}}(c,\epsilon), \bar{\bar{\bar{\sigma}}}(c,\epsilon))$ cross each other at least once. In particular, one has that,

$$\forall c \in [.33, .43) \qquad \exists \qquad (\hat{\epsilon}_1, \hat{\epsilon}_2) \in (0,1) \quad s.t.$$
$$\bar{\bar{\sigma}}(c,\epsilon) < \bar{\sigma}(c,\epsilon) \quad \forall \epsilon \in (0,\hat{\epsilon}_1) \cup (\hat{\epsilon}_2, 1) \quad and \quad \bar{\bar{\bar{\sigma}}}(c,\epsilon) > \bar{\sigma}(c,\epsilon) \quad \forall \epsilon \in (\hat{\epsilon}_1, \hat{\epsilon}_2),$$
$$\forall c \in (.43, 1) \quad \exists \quad \hat{\epsilon} \in (0,1) \quad s.t.$$
$$\bar{\bar{\sigma}}(c,\epsilon) < \bar{\sigma}(c,\epsilon) \quad \forall \epsilon \in (\hat{\epsilon}, 1) \quad and \quad \bar{\bar{\bar{\sigma}}}(c,\epsilon) > \bar{\sigma}(c,\epsilon) \quad \forall \epsilon \in (\bar{\epsilon}, \hat{\epsilon}).$$

[27] $\bar{c}(\epsilon)$ is the inverse of $\bar{\epsilon}(c)$ and $\bar{\epsilon}(c)$ is the lower bound of ϵ specified in Assumption 1.

[28] Notice that the coefficient attached to the squared term, equal to $(139\epsilon^2 - 138\epsilon + 131)$, is positive for $\epsilon \in (0,1)$.

[29] Remind that case $S3$ is more efficient than $CP32$ only if σ lies below $\bar{\bar{\bar{\sigma}}}(c,\epsilon)$.

Hence, for $c \in [.33, 1)$ the areas of (constrained) maximum welfare are as in what follows:

i. TS^{S2} is bigger than TS^{S3} and TS^{CP32} in:

$$\{(\epsilon,\sigma)| \quad \sigma \in (0, \bar{\sigma}(c,\epsilon)]\} \quad \setminus \quad \{(\epsilon,\sigma)| \quad \sigma \in (\bar{\bar{\sigma}}(c,\epsilon), \min\{\bar{\sigma}(c,\epsilon), 1\})\};$$

ii. TS^{S3} is bigger than TS^{S2} and TS^{CP32} in:

$$\{(\epsilon,\sigma)| \quad \sigma \in [\bar{\sigma}(c,\epsilon), 1)\} \quad \setminus \quad \{(\epsilon,\sigma)| \quad \sigma \in (\max\{\bar{\sigma}(c,\epsilon), \bar{\bar{\bar{\sigma}}}(c,\epsilon)\}, 1)\};$$

iii. TS^{CP32} is bigger than TS^{S2} and TS^{S3} in:

$$\{(\epsilon,\sigma)| \quad \sigma \in (\bar{\bar{\sigma}}(c,\epsilon), \min\{\bar{\sigma}(c,\epsilon), 1\})\} \quad \cup \quad \{(\epsilon,\sigma)| \quad \sigma \in (\max\{\bar{\sigma}(c,\epsilon), \bar{\bar{\bar{\sigma}}}(c,\epsilon)\}, 1)\}.$$

The characterization of the efficient cases above determines the choice of the benevolent planner, moreover it embeds the case with c smaller than .33 as a special case, in which $\bar{\sigma}(c,\epsilon)$ is bigger than $\bar{\bar{\bar{\sigma}}}(c,\epsilon)$ and $\bar{\bar{\sigma}}(c,\epsilon)$, and the set in which $S3$ is more efficient than $CP32$ is empty.■

Proof of Proposition 2

To prove that the total exclusion of τ_3 from the standard can be inefficient, it has to be shown that the area in which $\bar{\sigma}(c,\epsilon)$ lies below $\tilde{\sigma}(c,\epsilon)$ is not empty for some values of c and ϵ. If this is the case, the adoption of $\mathcal{P}(\tau_1,\tau_3)$ as technology standard ($S3$) is more efficient than the Nash equilibrium featuring the joint employment of $\mathcal{P}(\tau_1,\tau_2)$ ($S2$). More specifically,

$$\tilde{\sigma}(c,\epsilon) \leq \bar{\sigma}(c,\epsilon) \iff$$

$$c^2(90\epsilon^2 - 110\epsilon + 127) - 2c(72 + 35\epsilon) + 107 \quad \geq \quad \left[\frac{41\sqrt{2}(1-\epsilon)}{(7 - 4\sqrt{2}\epsilon)}\right]^2 \tag{17}$$

Like in the proof of Lemma 1, the function $c(\epsilon)$ along which $\bar{\sigma}(c,\epsilon)$ crosses $\tilde{\sigma}(c,\epsilon)$ is obtained by solving a quadratic equation in c whose coefficients are functions of ϵ. Applying the quadratic formula and taking the root that lies below $\bar{c}(\epsilon) = 3/(7-4\epsilon)$ one has that the function that solves (17) with an equality is given by $c_N(\epsilon)$:

$$c_N(\epsilon) = \frac{0.9899 - (0.3188 + 0.3889\epsilon)\epsilon - 0.3602(1-\epsilon)\sqrt{(0.0514+\epsilon)(8.7475+\epsilon)}}{(1.2374-\epsilon)[1.4111 - (1.2222-\epsilon)\epsilon]}.$$

Moreover, (17) is satisfied for all $c \leq c_N(\epsilon)$.[30] The function $c_N(\epsilon)$ is convex in ϵ; in particular, it is decreasing in ϵ for all $\epsilon \in (0, .22)$ and increasing for all $\epsilon \in (.22, 1)$. Also, $c_N(0) = \bar{c}(0) = c_W(0) = .43$, $c_N(.22) = .38 > c_W(.22) = .33$ and $c_N(1) = \bar{c}(1) = c_W(1) = 1$. Hence, $c_N(\epsilon)$ lies above $c_W(\epsilon)$. The graphs of $c_N(\epsilon)$ and $c_W(\epsilon)$ are in Figure 5.

[FIGURE 5 ABOUT HERE]

Summarizing, there is a wedge between the area in which $S3$ is more efficient than $S2$ and the one in which $S2$ is employed by vertically integrated firms; more specifically, such wedge arises for $c > .38$. Also, the fact that $c_N(\epsilon)$ lies above $c_W(\epsilon)$ implies that this wedge lies (at least partly) in the area in

[30]Indeed, the coefficient attached to the squared term, given by $(90\epsilon^2 - 110\epsilon + 127)$, is positive for $\epsilon \in (0,1)$.

which $S3$ is more efficient than $CP32$. Indeed, for any $c > .38$, the value of ϵ in which $\bar{\sigma}(c, \epsilon)$ crosses $\bar{\bar{\bar{\sigma}}}(c, \epsilon)$ and $\bar{\bar{\sigma}}(c, \epsilon)$ is different than the one in which $\bar{\sigma}(c, \epsilon)$ crosses $\tilde{\sigma}(c, \epsilon)$ (in particular, it is strictly bigger if $c > .43$). All this implies that the area of inefficient total exclusion of $\mathcal{P}(\tau_1, \tau_3)$ is not empty. ■

Proof of Proposition 3

Solving the game backwards, the equilibrium values at the product market competition stage when integrated firms choose $\mathcal{P}(\tau_1, \tau_2)$ are the same as in (1), those in case of joint adoption of $\mathcal{P}(\tau_1, \tau_3)$ are given in (3) and those related to the cases with competing platforms are in (7) and (11).

At the royalty setting stage, firm 1 sets a monopoly royalty rate, to push firm 2 out of the market. Instead, firms 2 and 3 compete for the adoption by manufacturers. Firm 2 can offer to firm 1 to cross-license their technologies, however, in this case firms 1 and 2 are not symmetric; firm 2 is constrained by the offer that firm 3 can make to 1 for the employment of τ_3. Consequently, the agreement in this case cannot consist of equally sharing the monopoly profit, instead firm 2 accepts to let firm 1 squeeze all the rents from using technology standard $\mathcal{P}(\tau_1, \tau_2)$, so to increase the chances for the adoption of its technology. Analogously, in all other cases perfect competition between 2 and 3 leads to an equilibrium in which firm 3 leaves manufacturers just indifferent between using τ_2 and τ_3.

[TABLE 7 ABOUT HERE]

In all cases, firm 1 would be the monopolist and firm 2 would be left with a nil payoff. In particular, if $\mathcal{P}(\tau_1, \tau_3)$ would be the technology standard then firm 1 would raise $(1 - \epsilon\sigma c)^2/4$ and if $\mathcal{P}(\tau_1, \tau_2)$ would be the technology standard then firm 1 would raise $(1 - \sigma c)^2/4$. In the case with competing platforms $CP32$ firm 1 would obtain a payoff equal to $(1 - \epsilon c)^2/4$, and in the case with competing platforms $CP23$ firm 1 would gain $(1 - c)^2/4$.

Finally, by assuming that indifference is broken in favor of the more efficient technology one has the result in the proposition, i.e., $\mathcal{P}(\tau_1, \tau_3)$ is adopted as technology standard at equilibrium. ■

APPENDIX B. Two-part tariffs case.

Proof of Lemma 2

To start with, notice that were firm j to set $w_{jk} > 0$ it would raise the royalty rate to kick k out of the market and be monopolist. Then, the best reply by k would be to set $F_{kj} = \pi^m$ and get firm j's downstream rent.

Instead, were $w_{jk} = w_{kj} = 0$, in order to determine the equilibria of the licensing game, I analyze firm k's best response to the fixed fee F_{jk} set by firm j.[31] There are two relevant thresholds: the Cournot profit, indexed by π^c, and the monopoly profit, indexed by π^m. Consequently, three cases must be considered.

[31] Due to symmetry, the firm j's best response will be analogous.

1. If $0 < F_{jk} < \pi^c$ firm k would always be active. More specifically, were it to set $F_{kj} > \pi^c$, it would be a monopolist and attain profit equal to $\pi^m - F_{jk} > 0$. Instead, were k to set $F_{kj} = \pi^c$, it would be duopolist and obtain profit equal to $2\pi^c - F_{jk} > 0$. Therefore, the best response by k to $F_{jk} < \pi^c$ is to set $F_{kj} > \pi^c$, at which k would earn $\Pi_k = \pi^m - F_{jk}$. This is optimal because $\pi^m > 2\pi^c$. If $F_{kj} > \pi^c$, firm j would stay out of the market and earn $\Pi_j = F_{jk}$.
2. If $\pi^c \leq F_{jk} < \pi^m$ firm k would be active only if monopolist, instead it would not find it profitable to produce if duopolist. In particular, were firm k to set $F_{kj} > \pi^m$, it would be a monopolist and gain $\pi^m - F_{jk}$. If k would set $F_{kj} = \pi^m$, it would stay out of the market, but it would fully extract j's monopoly profit. Finally, k may set $F_{kj} < \pi^m$, at which it would be out and have incentive to raise its fee further. Therefore, the best response by k to $\pi^c \leq F_{jk} < \pi^m$ is to set $F_{kj} = \pi^m$, at which j would be the monopolist and k would squeeze all its profit, gaining $\Pi_k = F_{kj}$.
3. If $F_{jk} \geq \pi^m$, firm k is out of the market, independently from the fee it sets. Therefore, k's optimal response is to set $F_{kj} = \pi^m$, stay out, but extract all downstream revenue from the rival.

Equilibrium. Under independent licensing and technologies τ_1 and τ_2 in the standard the Nash equilibria of the licensing game are given by:

i. $w_{jk} = (1 - \sigma c)/2, w_{kj} = 0$, $F_{jk} = 0, F_{kj} = \pi^m$ and $w_{jk} = w_{kj} = 0$, $F_{jk} = F_{kj} = \pi^m$: at these equilibria firm j is in, firm k is out, but extracts all downstream profits from firm j. Moreover, $\Pi_j = 0, \Pi_k = \pi^m$.

ii. $w_{jk} = w_{kj} = 0$ and $F_{jk} = \pi^m, F_{kj} \in [\pi^c, \pi^m)$, at which firm j is out and firm k is in. At this equilibrium, $\Pi_j = \pi^m$, $\Pi_k = 0$. However, k does not have any incentive to deviate if and only if when it sets $F_{kj} = \pi^m$ it anticipates that the continuation equilibrium is such that $\Pi_k^{S2} = 0$.

Finally, notice that there does not exist any equilibrium where $w_{kj} = w_{jk} = 0$ and $F_{jk} < \pi^c$, $F_{kj} > \pi^c$, as the best reply to $F_{kj} > \pi^c$ would be to set $F_{jk} = \pi^m$. ∎

Proof of Lemma 4

First of all, notice that by a standard argument, firm 3 sets $w_3 = 0$ not to distort firm 1's production decisions and tamper downstream rent. Now, if firm 1 sets w_{12} as to monopolize the downstream market it would have all its downstream rent extracted by 3 through the fixed fee.

Instead, if $w_{12} = w_3 = 0$, firm 1 and firm 3 would compete over the fixed fee. In the following, I present the best responses of firm 1 to the fee set by firm 3.

1. If $0 < F_3 < \pi^c$, firm 1 would always be active. The royalty setting game sees firm 1 competing with firm 3. Two responses are possible by 1: the first would be to set $F_{12} > \pi^c - F_3$, at which firm 2 would not operate, the second would be to set $F_{12} \leq \pi^c - F_3$, at which both firms would be active. In the former case firm 1 would gain $\pi^m - F_3$, firm 2's payoff would be nil and firm 3 would extract F_3 from 1. In the latter case, the profit of firm 1 would be equal to

$\pi^c - F_3 + F_{12} = 2\pi^c - 2F_3 = 0$, those of firm 2 would be given by $\pi^c - F_3 - F_{12} = 0$, instead firm 3 would extract $2\pi^c$. Clearly, 1's best response is to set $F_{12} > \pi^c - F_3$, operate as monopolist and gain profit $\Pi_1 = \pi^m - F_3 > 0$.

2. If $\pi^c \leq F_3 < \pi^m$, firm 1 would be active only if monopolist. Setting $F_{12} > \pi^m - F_3$, firm 1 would force firm 2 to stay out of the market and gain $\pi^m - F_3$, instead firm 3 would extract F_3 from 1. Otherwise, setting $F_{12} = \pi^m - F_3$, firm 1 would stay out and extract 2's profit, firm 2, although monopolist, would be left with zero profits, firm 3 would gain F_3 from 2. Therefore, firm 1 optimal response is to fix $F_{12} \geq \pi^m - F_3$, at which either 1 or 2 would be monopolist, but firm 2 would make zero profit in any case, firm 3 would get $\Pi_3 = F_3$ and firm 1's payoff would be equal to $\Pi_1 = \pi^m - F_3$.
3. If $F_3 \geq \pi^m$, firm 1 and firm 2 stay out of the market. Therefore, all firms would earn zero profit.

Equilibrium. First notice that it is a dominant strategy for firm 3 to set $F_3 = \pi^m - \eta$, with $\eta > 0$, small. Consequently, it is an equilibrium for firm 1 to set either $w_{12} = 0$ and $F_{12} = \pi^m - F_3$ or $w_{12} = 0$ and $F_{12} > \pi^m - F_3$ or $w_{12} = (1 - \sigma\epsilon c)/2$ and $F_{12} = 0$: in the first case, 1 would let 2 be a monopolist, but extract all 2's profit (net of F_3, of course), in the second and third cases, 1 would be a monopolist. However, in all three cases the payoff of 1 would be given by $\Pi_1^{S3} = \eta$, instead $\Pi_2^{S3} = 0$ and $\Pi_3^{S3} = F_3 = \pi^m - \eta$. Finally, by focusing on η equal to zero one has the results in the Lemma.

Remark. One may find counterintuitive that firm 3 takes all the industry profit and firm 1, which has a complementary technology, takes none, and also wonder whether there exist other equilibria where firm 1 is able to extract a part of the industry surplus. In fact, this never occurs. Suppose there is a candidate equilibrium where firm 1 sets $F_{12} = k\pi^m$ and firm 3 sets $F_3 = (1-k)\pi^m$, with $k \in (0, 1]$.[32] At this equilibrium, firm 3 would obtain a payoff equal to $F_3 = (1-k)\pi^m$, but it would have an incentive to deviate and set $F_3^{'} = \pi^m$. If $F_{12} = k\pi^m, F_3^{'} = \pi^m$, firm 2 would never produce because it would not be able to recover the cost of the fees, even if firm 1 does not produce. Instead, if firm 1 produces it will not have to pay the fee for the use of technology 1, so there is a continuation equilibrium where firm 1 sells and firm 2 does not and firm 1 transfers all the monopoly profit to firm 3 through the fee. This shows that the unique equilibrium consists in the one identified above where firm 3 extracts all the monopolistic rents from the industry. ■

Proof of Lemma 5

Like in case $S3$ (see Lemma 4), the royalty setting game sees firm 1 competing with firm 3. However, firm 2 now does not employ technology 3.

Firm 3 sets $w_3 = 0$ at equilibrium, not to distort firm 1's operations downstream. If firm 1 replies by setting w_{12} as to monopolize the downstream market it would have all its rent extracted by 3 through the fixed fee.

[32] In the continuation equilibria, either firm 1 is the monopolistic supplier, gaining $\pi^m - (1-k)\pi_m = k\pi^m$, or firm 2 is the monopolistic supplier, with firm 1 gaining $k\pi^m$. In both cases $\pi_3 = (1-k)\pi^m$.

Instead, if $w_{12} = w_3 = 0$, then firm 1 and firm 3 would compete over the fixed fee. In the following, I present the best responses of firm 1 to the fee set by firm 3 at $w_{12} = w_3 = 0$.

1. If $0 < F_3 \leq \pi_1^c$, firm 1 would always be active. The possible responses by 1 follow. The first would be to set $F_{12} > \pi_2^c$, at which firm 2 would not operate. The second would be to shed π_2^c by η, positive and negligible, be active with 2 on the product market and squeeze its Cournot profit.[33] In the former case firm 1 would gain $\pi_1^m - F_3 = \pi_1^m - \pi_1^c$, firm 2's payoff would be nil and firm 3 would extract F_3 from 1. In the latter case, the profit of firm 1 would be equal to $\pi_1^c - F_3 + F_{12} = \pi_1^c + \pi_2^c - \pi_1^c$, the one of firm 2 would be given by $\pi_2^c - F_{12} = 0$, instead firm 3 would get F_3. The best response of 1 is to set $F_{12} > \pi_2^c$, operate as monopolist and gain $\Pi_1 = \pi_1^m - \pi_1^c$. Indeed, $\pi_1^m - \pi_1^c > \pi_2^c$ under Assumption 1.
2. If $\pi_1^c < F_3 \leq \pi_1^m$, firm 1 would be active only if monopolist. Setting $F_{12} > \pi_2^m$, firm 1 would force firm 2 to stay out of the market and gain $\pi_1^m - F_3$, instead firm 3 would extract F_3 from 1. Otherwise, setting $F_{12} = \pi_2^m - \eta$, firm 1 would stay out and extract 2's profit, and firm 2, although monopolist, would be left with a zero payoff. Therefore, firm 1 optimal response is to fix $F_{12} = \pi_2^m - \eta$, at which firms' payoffs are $\Pi_1 = \pi_2^m - \eta$, $\Pi_2 = \eta$ and $\Pi_3 = 0$.
3. If $F_3 > \pi_1^m$, firm 1 would always stay out of the market. If firm 1 would set $F_{12} > \pi_2^m$, then firm 1 and firm 2 would be out of the market. Instead, if 1 would set $F_{12} = \pi_2^m - \eta$, 1 would stay out and extract firm 2's profit thorough the fee. Clearly, 1's best response is to set $F_{12} = \pi_2^m - \eta$, at which 3 and 2 would be left with nothing.

Equilibrium. At equilibrium, firm 1 sets $w_{12} = 0$ and firm 3 sets $w_3 = 0$. Moreover, the fee of firm 3 is given by $F_3 = \pi_1^c$ and firm 1 replies by setting F_{12} as to push firm 2 out of the downstream market. Consequently, $\Pi_1^{CP32} = \pi_1^m - \pi_1^c$, $\Pi_2^{CP32} = 0$ and $\Pi_3^{CP32} = \pi_1^c$.■

Proof of Lemma 6

In case $CP23$, all three firms are active upstream: firm 1 licenses τ_1 to firm 2, firm 2 licenses τ_2 to firm 1 and firm 3 licenses τ_3 to firm 2.

Like in Lemmata 4 and 5, firm 3 sets $w_3 = 0$. If $w_{12} = 0$, were firm 2 to set a positive value of w_{21} then it would try to monopolize the downstream market. In this case, firms 1 and 3 would equally share π_2^m.[34]

If w_{21} were nil and firm 1 would reply by setting a positive value of w_{12}, then it would be firm 1 that tries to monopolize the downstream market. However, in this case it is firm 2 that gets the entire rent from 1, equal to π_1^m.

Now, if $w_{21} = w_{12} = 0$, firms 1, 2 and 3 would compete over the value of the fixed fee. Below, I analyze firm 2 best response to the fixed fees F_n and F_l set by 3 and 1, with $l, n = 3, 12$ and $l \neq n$.

[33] Notice that a third one would be to set $F_{12} < \pi_2^c$, but then 1 would have incentive to raise the fee further.

[34] Here, I am using the assumption for which firm 1 and firm 3 have equal probability of being first in approaching firm 2, as in case $S2$.

1. If $0 \le F_l \le \pi_2^c$ and $0 \le F_n \le \pi_2^c - F_l$, then 2 is always active. Firm 2 can reply setting $F_{21} = \pi_1^c$, then both vertically integrated firms would be active downstream and firm 2 would gain $\Pi_2 = \pi_2^c - F_l - F_n + F_{21} = \pi_1^c$.[35] If firm 2 would set $F_{21} > \pi_1^c$, then it would be monopolist and get $\pi_2^m - \pi_2^c$. Thus, the best response of firm 2 is to set $F_{21} > \pi_1^c$. Indeed, it can be shown that $\pi_2^m > \pi_2^c + \pi_1^c$ under Assumption 1. At this response, the firm that sets F_l gets $\Pi_l \in [0, \pi_2^c]$ and the firm that sets F_n gets $\Pi_n \in [0, \pi_2^c - F_l]$. The coordination problem that arises in this case is again solved by assuming that firm 1 and firm 3 have an equal probability to be the first in contracting with firm 2, like in Lemma 2, so that each firm gets $\pi_2^c/2$ in expectation.
2. If $0 \le F_l \le \pi_2^c$ and $\pi_2^c < F_n \le \pi_2^m - F_l$, then 2 is active only if monopolist. Thus, firm 2 can reply setting $F_{21} = \pi_1^m - \eta$, with η positive and negligible, let firm 1 be monopolist and get π_1^m. Instead, if firm 2 would set $F_{21} > \pi_1^m$, it would be monopolist and gain $\pi_2^m - F_l - F_n = 0$. Hence, the best response of firm 2 is to set $F_{21} = \pi_1^m - \eta$, let 1 be the monopolist and squeeze its downstream profit.
3. If $\pi_2^c < F_l \le \pi_2^m$ and $\pi_2^c < F_n \le \pi_2^m - F_l$, then 2 is active only if monopolist. The analysis carries over as in the previous case, thus firm 2's best response is to set $F_{21} = \pi_1^m - \eta$ and let 1 be the monopolist.
4. If $\pi_2^c < F_l \le \pi_2^m$ and $F_n > \pi_2^m - F_l$, then firm 2 is always out. Consequently, firm 2 would let firm 1 be active as monopolist and squeeze its downstream profit.

Therefore, under $w_{21} = w_{12} = 0$, it is a dominant strategy to firm 1 and firm 3 to set F_l and F_n such that $F_l + F_n \in [0, \pi_c]$, because for a bigger aggregate fee the best response of firm 2 would be to stay inactive and get firm 1 profit by setting $F_{21} = \pi_1^m - \eta$. Consequently, at an equilibrium with $w_{21} = w_{12} = 0$, firm 2 is monopolist and gains $\pi_2^m - \pi_2^c$, instead firm 1 and firm 3 equally share the Cournot profit of firm 2.

The last case to consider is the one at which $w_{21} > 0$ and $w_{12} > 0$. In this case, by using the results in Appendix A of the model with linear price and using $w_3 = 0$, one has that:

$$w_{12}(w_{21}) = \frac{5 - c(4\epsilon + 1) - w_{21}}{10}, \quad w_{21}(w_{21}) = \frac{5 - c(4 + \epsilon) - w_{12}}{10}.$$

Then,

$$\begin{cases} w_{12}^{CP23} = \frac{15 - c(2 + 13\epsilon)}{33} \\ w_{21}^{CP23} = \frac{15 - c(2\epsilon + 13)}{33} \end{cases} \tag{18}$$

and

[35] Notice that if firm 2 would set a fee smaller than the Cournot rent, it would have incentive to raise it further.

$$\begin{cases} q_1^{CP23} = \frac{2[3-c(7-4\epsilon)]}{33} \\ \\ q_2^{CP23} = \frac{2[3-c(7\epsilon-4)]}{33}. \end{cases} \tag{19}$$

With q_1^{CP23} positive under Assumption 1. Consequently, the profits of firm 1 and firm 2 (gross of the fixed fees) are:

$$\begin{cases} \Pi_1^w = (q_2^{CP23})^2 + q_2^{CP23} w_{12}^{CP23} = \frac{2[21-2c(16\epsilon+5)+c^2(41\epsilon^2-50\epsilon+30)]}{363} \\ \\ \Pi_2^w = (q_1^{CP23})^2 + q_1^{CP23} w_{21}^{CP23} = \frac{2[21-2c(16+5\epsilon)+c^2(41-50\epsilon+30\epsilon^2)]}{363} \end{cases} \tag{20}$$

Finally, by following the same procedure as in the case with $w_{21} = w_{12} = 0$, one would find that here the fixed fees would be such that firm 2 gets Π_1^w instead firm 1 and firm 3 equally share Π_2^w. Indeed, either firm 1 or firm 3 do not have incentive to deviate because by setting a higher fee firm 2 would stay inactive, let firm 1 be the monopolist and squeeze its profit. At the same time, firm 2 does not deviate and sets a higher fee on firm 1 because, given $w_{12} = w_{12}^{CP23} > 0$, its profit under monopoly is smaller than the sum of Π_1^w and Π_2^w.[36]

Equilibrium. The equilibrium in case $CP23$ is one at which $w_{21} = w_{12} = w_3 = 0$, $F_{21} > \pi_1^m$, and F_l and F_n are such that $F_l + F_n \in [0, \pi_c]$. Therefore, firm 2 gains $\Pi_2^{CP23} = \pi_2^m - \pi_2^c$, instead firm 1 and firm 3 get $\pi_2^c/2$ each. Notice that firms 1 and 2 do not have incentive to unilaterally deviate and set $w_{ij} > 0$ (with $i \neq j$ and $i, j = 1, 2$) because they would be left with a nil payoff. Also, the case in which both w_{21} and w_{12} are positive is not an equilibrium because firm 1 would have incentive to deviate, set $w_{12} = 0$ and gain $\pi_2^m/2 > \Pi_2^w/2$. ■

Proof of Proposition 4

The adoption of $\mathcal{P}(\tau_1, \tau_2)$ as technology standard emerges at equilibrium if the following condition holds (see Table 8):

[TABLE 8 ABOUT HERE]

$$\begin{aligned} \frac{(1-\sigma c)^2}{8} &\geq \frac{(1-\epsilon c)^2}{4} - \frac{[1-c(2\epsilon-1)]^2}{9} \iff \\ \frac{(1-\sigma c)^2}{8} &\geq \frac{[1-c(2-\epsilon)][5-c(7\epsilon-2)]}{36} \iff \\ \sigma &\leq \tilde{\sigma}_{TT}(c,\epsilon) \equiv \frac{3-\sqrt{2}\sqrt{[1-c(2-\epsilon)][5-c(7\epsilon-2)]}}{3c}. \end{aligned}$$

Otherwise, both firms have incentive to deviate from an equilibrium featuring the joint employment of τ_2. In particular, if $\sigma > \tilde{\sigma}_{TT}(c,\epsilon)$ strategy $\mathcal{P}(\tau_1, \tau_3)$ becomes weakly dominant to firm 2 and case $(CP23)$ emerges as Nash equilibrium of the adoption game. ■

[36] The profit of a monopolist firm 2 at $w_{12} = w_{12}^{CP23}$ is equal to $[(9 + c - 10c\epsilon)/33]^2$.

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Table 1: Manufacturers' Marginal Cost of Production

		Firm 2	
		$\mathcal{P}(\tau_1, \tau_2)$	$\mathcal{P}(\tau_1, \tau_3)$
Firm 1	$\mathcal{P}(\tau_1, \tau_2)$	$\sigma c, \sigma c$	$c, \epsilon c$
	$\mathcal{P}(\tau_1, \tau_3)$	$\epsilon c, c$	$\epsilon\sigma c, \epsilon\sigma c$

Table 2: Results under the joint adoption of $\mathcal{P}(\tau_1, \tau_2)$

	Independent Licensing	**Cross-licensing**
w_{jk}	$5(1-\sigma c)/11$	$(1-\sigma c)/4$
q_j	$2(1-\sigma c)/11$	$(1-\sigma c)/4$
$Q, P(Q)$	$4(1-\sigma c)/11, (7+4\sigma c)/11$	$(1-\sigma c)/2, (1+\sigma c)/2$
CS	$8(1-\sigma c)^2/121$	$(1-\sigma c)^2/8$
Π_1, Π_2, Π_3	$14(1-\sigma c)^2/121, 14(1-\sigma c)^2/121, 0$	$(1-\sigma c)^2/8$, $(1-\sigma c)^2/8$, 0
$Total\ Welfare,\ TS$	$36(1-\sigma c)^2/121$	$3(1-\sigma c)^2/8$

Table 3: Results under the joint adoption of $\mathcal{P}(\tau_1, \tau_3)$

w_{12}^{S3}, w_3^{S3}	$2(1-\epsilon\sigma c)/7, 3(1-\epsilon\sigma c)/7$
q_1^{S3}, q_2^{S3}	$2(1-\epsilon\sigma c)/7, 0$
$Q^{S3}, P(Q^{S3})$	$2(1-\epsilon\sigma c)/7, (5+2\epsilon\sigma c)/7$
CS^{S3}	$2(1-\epsilon\sigma c)^2/49$
$\Pi_1^{S3}, \Pi_2^{S3}, \Pi_3^{S3}$	$4(1-\epsilon\sigma c)^2/49, 0, 6(1-\epsilon\sigma c)^2/49$
$Total\ Welfare,\ TS^{S3}$	$12(1-\epsilon\sigma c)^2/49$

Table 4: Results under the adoption of $\mathcal{P}(\tau_1, \tau_3)$ by firm 1 and $\mathcal{P}(\tau_1, \tau_2)$ by firm 2

$w_{12}^{CP32}, w_3^{CP32}$	$\frac{19-c(2\epsilon+17)}{41}, \frac{3[5-c(7\epsilon-2)]}{41}$
q_1^{CP32}, q_2^{CP32}	$\frac{2[5-c(7\epsilon-2)]}{41}, \frac{2[3-c(7-4\epsilon)]}{41}$
$Q^{CP32}, P(Q^{CP32})$	$2\frac{8-c(3\epsilon+5)}{41}, \frac{25+2c(3\epsilon+5)}{41}$
CS^{CP32}	$2[\frac{8-c(3\epsilon+5)}{41}]^2$
$\Pi_1^{CP32}, \Pi_2^{CP32}, \Pi_3^{CP32}$	$2\frac{c^2(90\epsilon^2-110\epsilon+127)-2c(35\epsilon+72)+107}{1681}, 4\frac{[3-c(7-4\epsilon)]^2}{1681}, 6\frac{[5-c(7\epsilon-2)]^2}{1681}$
$Total\ Welfare,\ TS^{CP32}$	$4\frac{c^2(139\epsilon^2-138\epsilon+131)-4c(31+35\epsilon)+132}{1681}$

Table 5: Results under the adoption of $\mathcal{P}(\tau_1, \tau_2)$ by firm 1 and $\mathcal{P}(\tau_1, \tau_3)$ by firm 2

$w_{12}^{CP23}, w_{21}^{CP23}, w_3^{CP23}$	$\frac{21-c(8+13\epsilon)}{54}, \frac{12-c(\epsilon+11)}{27}, \frac{3-c(7\epsilon-4)}{18}$
q_1^{CP23}, q_2^{CP23}	$\frac{2[3-c(5-2\epsilon)]}{27}, \frac{3-c(7\epsilon-4)}{27}$
$Q^{CP23}, P(Q^{CP23})$	$\frac{3-c(2+\epsilon)}{9}, \frac{6+c(2+\epsilon)}{9}$
CS^{CP23}	$\frac{[3-c(2+\epsilon)]^2}{162}$
$\Pi_1^{CP23}, \Pi_2^{CP23}, \Pi_3^{CP23}$	$\frac{c^2(41\epsilon^2-52\epsilon+56)-30c(2+\epsilon)+45}{486}, \frac{c^2(5\epsilon^2-10\epsilon+14)+9(1-2c)}{81}, \frac{[3-c(7\epsilon-4)]^2}{486}$
$Total\ Welfare,\ TS^{CP23}$	$\frac{c^2(41\epsilon^2-52\epsilon+56)-30c(2+\epsilon)+45}{162}$

Table 6: Adoption Game Nash Equilibrium, $\sigma > \tilde{\tilde{\sigma}}(c, \epsilon)$

		Firm 2	
		$\mathcal{P}(\tau_1, \tau_2)$	$\mathcal{P}(\tau_1, \tau_3)$
Firm 1	$\mathcal{P}(\tau_1, \tau_2)$	Π_1^{S2}, Π_2^{S2}	$\Pi_1^{CP23}, \Pi_2^{CP23}$
	$\mathcal{P}(\tau_1, \tau_3)$	$\underline{\Pi_1}^{CP32}, \underline{\Pi_2}^{CP32}$	Π_1^{S3}, Π_2^{S3}

Table 7: Adoption game under FRAND reasonableness requirement

		Firm 2	
		$\mathcal{P}(\tau_1, \tau_2)$	$\mathcal{P}(\tau_1, \tau_3)$
Firm 1	$\mathcal{P}(\tau_1, \tau_2)$	$(1-\sigma c)^2/4, 0$	$(1-c)^2/4, 0$
	$\mathcal{P}(\tau_1, \tau_3)$	$(1-\epsilon c)^2/4, 0$	$(1-\epsilon\sigma c)^2/4, 0$

Table 8: Adoption game with Two-part tariffs

		Firm 2	
		$\mathcal{P}(\tau_1, \tau_2)$	$\mathcal{P}(\tau_1, \tau_3)$
Firm 1	$\mathcal{P}(\tau_1, \tau_2)$	$(1-\sigma c)^2/8, (1-\sigma c)^2/8$	$[1-c(2\epsilon-1)]^2/18, (1-\epsilon c)^2/4-(1-2\epsilon c+c)^2/9$
	$\mathcal{P}(\tau_1, \tau_3)$	$(1-\epsilon c)^2/4-(1-2\epsilon c+c)^2/9, 0$	$0, 0$

Figure 1: Framework

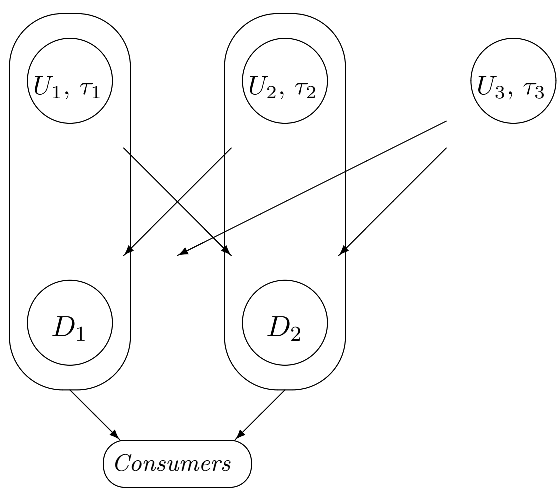
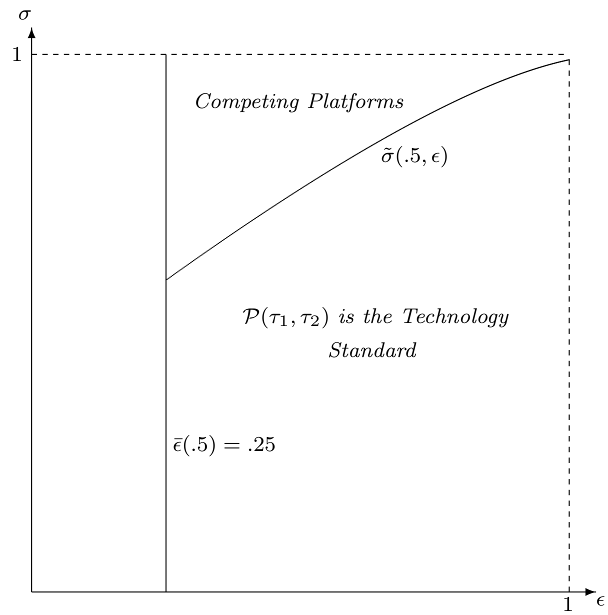
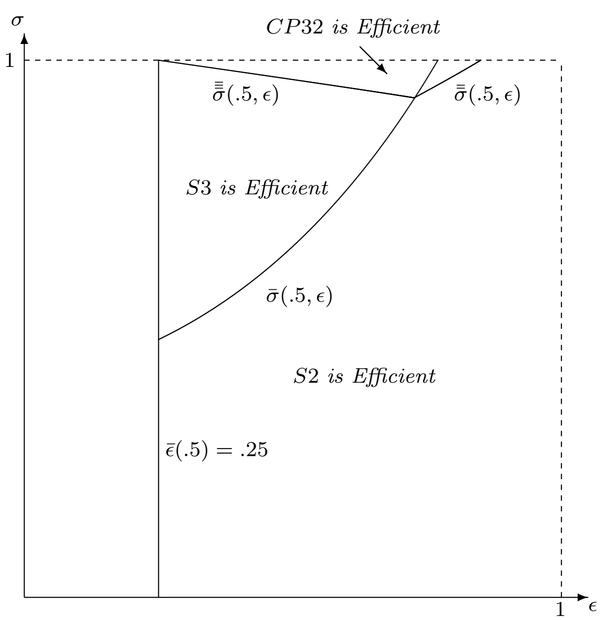


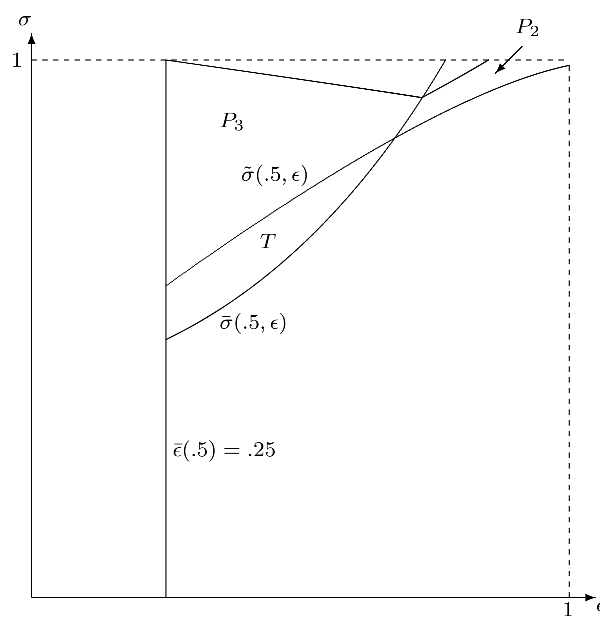
Figure 2: Linear Pricing - Numerical Example, $c = 1/2$



(a) **- Technology Adoption Nash Equilibria**



(b) **- Welfare Analysis**



(c) **- Adoption Equilibria and Inefficient Exclusion**

Figure 3: Two-part tariffs - Numerical Example, $c = 1/5$

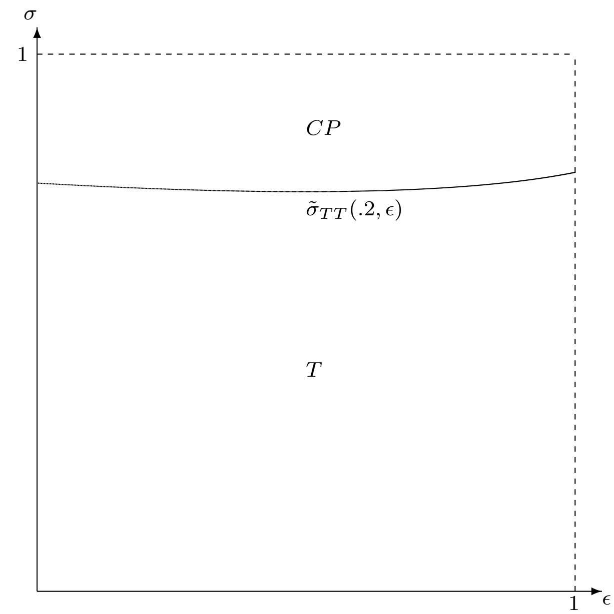


Figure 4: Graph of $c_W(\epsilon)$

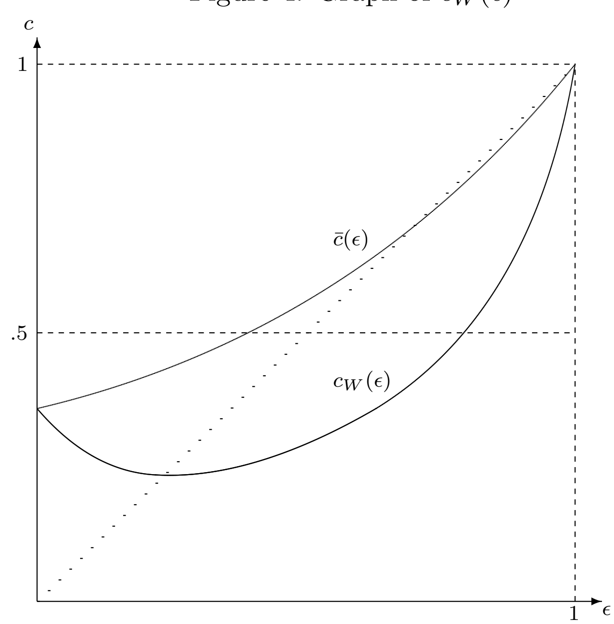


Figure 5: Graph of $c_N(\epsilon)$ and $c_W(\epsilon)$