Price Discrimination and the Hold-Up Problem: A Contribution to the Net-Neutrality Debate*

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1 Introduction

In many situations of economic exchange the value of trade is significantly affected by the respective parties' investments before trade occurs. One of the recently often discussed examples is provided by the debate on *net-neutrality*. Internet service providers (ISPs) argue, that they should be allowed to price discriminate among different content-providers for access to their network and customers so that they have incentives to invest in the quality of their network (*eg.* connection speed). On the other hand, the content-providers also have to invest in cost reduction or the quality of their content (e.g. vis-à-vis final customers or advertisers) increasing the value of getting access to the ISP's platform and users. If this investment takes place before trade occurs and cannot be contracted on ex-ante, a hold-up problem may occur when there is some bargaining power on the ISP's side.

In this paper we consider the decision on allowing the ISPs to charge content-providers as given, and analyze the impact on the content-providers investment behavior. In the existing literature on the hold-up problem the content-providers typically have unit-demand and thus ISPs charge only a fixed access-fee to content-providers. In contrast to this we would like to allow for more general forms of demand and hence tariffs, which seems to be relevant for the case of ISPs selling bandwidth to content-providers. This brings up the scope of non-linear

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tariffs, *i.e.* second-degree price discrimination. Furthermore we compare a standard single-sided setting with a two-sided one, where the ISP might charge both sides of the market: content providers and end-users.

The paper consists of two parts: In the first one (section 2) we study a vertical industry structure: An input provider (*eg.* an Internet backbone provider or a content delivery network) is used by a content-provider to reach end-users. This simple vertical structure allows to abstract away from the downstream market and study a stylized model with one seller (the input provider) and one buyer (the content-provider serving an end-user market), in which the seller engages in second-degree price discrimination ex-post, while the buyer can invest ex-ante to improve her type. The two key assumptions of the paper are: First, the buyer's investment is not-contractible. And second, after investment has been sunk, the seller (*i.e.* the non-investing party) has the full bargaining power. The setting is studied under different observability regimes, in which the seller might or might not observe the investment sunk by the buyer before designing her price schedule. As expected there is under-investment in equilibrium - a hold-up problem arises - in both situations irrespective whether investment is observable by the seller or not.

Under reasonable assumptions (although not in general) the equilibrium investment level is smaller when the seller observes investment compared to when it is not. Rather surprisingly this result does not depend on shape of the buyers surplus and thus demand functions, but only on the impact of investment on the distribution of the buyer's type. As an immediate consequence, but contrary to popular belief we get that the buyer is better off and the seller worse off when investment is observable. The intuition for this result is quite simple: For higher (equilibrium) investment the seller faces a *better* distribution over the buyer's types, where on the other hand the buyer is worse off due to a less favorable tariff charged by the seller. We further show that *fierce* competition between sellers or the possibility for the seller and buyer to contract before investment takes place, solves the hold-up problem.

In appendix A.3 there is a short overview of the case when the seller is restricted to charge a linear price. It turns out that the impact of observability on the equilibrium investment level is a lot more sensitive to the demand structure than under second-degree price discrimination. To answer this question a lot of information on demand together with the distribution of the buyer's type is needed, because the way investment affects the elasticity of expected demand plays the crucial role.

The second part of the paper (section 3) looks at a two-sided market version of the model, in which the seller also engages in the downstream market serving end-users. Then one can think of the buyer again as a content-provider and the seller as an ISP providing the content-provider access to the end-users connected via her platform. We allow the seller to also charge the end-user a fixed fee for access to the seller's platform over which trade between the end-user and the buyer can be carried out. The buyer multi-homes while the seller picks a single seller's platform. This situation can be studied in the framework laid out in the first part of the paper, and therefore most of the reasoning from the first part is applicable also in these settings. However, the big caveat is that the result regarding competition does not hold anymore due to the different nature of the competition in the two-sided setting. In equilibrium sellers do not compete for the buyer at all. They charge buyers the same tariffs (scaled for market size) like under the monopoly. Competition only takes place on the end-user side, who are bribed to join the respective seller's platform via the access-fee. Therefore competition has no effect on investment, and thus does not solve or dampen the hold-up problem.

The result on ex-ante commitment also holds in the two-sided market section. However, it should be taken with a grain of salt when applied to a setting with a multitude of sellers as it is the case for ISPs where there are thousands of ISPs only in the US, because at least for the limiting case of a unit-mass of sellers free-riding renders commitment before investment takes place completely ineffective.

The findings of the paper provide some insights into the net-neutrality debate. Letting ISPs or content delivery networks charge content providers for access to their network will create a hold-up problem. In an one-sided monopoly setting the problem can be dampened or solved by ex-ante commitment or competition between sellers, *i.e.* we may infer that the hold-up problem will likely play a minor role for content providers using content-delivery networks and backbone providers to deliver their content to end-users. When two-sided market structures are present, we see two effects: First, the hold-up problem will be less severe, as the seller (the platform) will not try to extract as much from the buyer (content provider) in the first place in order not to hurt the other side of the market too much: the end-users. But second, competition between sellers does not influence the hold-up problem at all. The sellers constitute a competitive bottleneck; in equilibrium they compete only for end-users and charge buyers the monopoly tariff. This means that competition between ISPs will not be enough to solve the hold-up. Adding to that ex-ante commitment seems to be plagued by a free-riding problem when there are many sellers around. Last but not least in both regimes - single and two-sided - observability of investment might decrease equilibrium investment. These detrimental effect can be get rid off by requiring that the seller may only charge a single non-linear tariff to all buyers, *i.e.* a prohibition to discriminate between different buyers based on investment.

Hermalin and Katz (2009) study a similar setup like in section 2: A seller proposes to sell at a certain price to the buyer who has unit-demand (take it or leave it offer). Ex-ante investment influences the buyer's surplus from trade. Contracting between the seller and buyer before the investment is sunk is not feasible. Like in this paper they find that observability of investment by the seller decreases equilibrium investment under the monotone-hazard rate assumption. They also try to study the intermediate case, where investment is unobservable, but an additional signal of returns depending on investment is present, but the impact of observability is arbitrary, *i.e.* it depends on parameter values of the model.

In the applications outlined above assuming unit-demand is clearly restrictive - especially from the two-sided markets perspective. A straight-forward extension is to allow for a more general demand pattern for the buyer side, but to keep linear prices - this it the exercise carried out in appendix A.3 of this paper. In this extension we get quite arbitrary results on the impact of observability on investment. The introduction of the possibility for the seller to second-degree price discriminate - at first sight a complication of the setting - allows to get similarly strong results than in Hermalin and Katz (2009). It is surprising that in the adverse-selection setting the hazard-rate over the buyer's type plays the key role like in the unit-demand case, and that no further (non-standard) assumptions on the buyer's surplus are needed. In Hermalin and Katz (2009) the distribution over types ties down demand completely, because the buyer's type equal the buyer's valuation, while in our setting each type has her own demand (or rather surplus) function.

The contracting stage between seller and buyer follows the spirit of Maskin and Riley (1984), who analyze the problem of a monopolist with incomplete information over customers' types. To maximize profits the monopolist uses non-linear prices for screening. Guesnerie and Laffont (1984) study in detail assumptions ensuring the existence and uniqueness of solutions in these kinds of adverse-selection problems. While in the monopoly case the downward incentive constraint plays the key role (hindering high valuation types to mimic low valuation ones), Rochet and Stole (2002) find for the duopoly case that also the upward incentive constraint can be binding, which significantly complicates the analysis.

On a broader scope this paper is related in to many contributions in the vertical integration literature. Williamson (1975, 1979) and Klein et al. (1978) were among the first to discuss the hold-up problem. They describe that *opportunistic behavior* in ex-post bargaining over the distribution of joint-surplus may hurt incentives to carry out relationship specific investment ex-ante. They discuss long-term contracts and vertical integration as ways to solve the problem. Grossman and Hart (1986) formalize some of these ideas. In all scenarios (vertical integration versus non-integration) efficient ex-post bargaining takes place, but non-contractible ex-ante actions influence the status-quo of the bargaining process. Hence actions are not chosen to maximize joint gains from trade ex-post, but individual surplus after negotiations (*i.e.* too little weight is put on joint surplus, too much on improving the status-quo). In an alternative model of vertical integration Riordan (1990) uses a hold-up model with adverse-selection to study the non-integration case. Schmidt's (1996) goal is to analyze the trade-off between allocative and productive efficiency in the context of privatization, for which he employs a model with non-contractible ex-ante investment and adverse-selection. Laffont and Martimort (2002) present a similar model, where in certain parameter ranges the investing party mixes over investment levels in equilibrium.

Another interesting connection to the literature on vertical integration is Crémer (1995). He finds that in some settings the principal wants to commit not to acquire information about an agent in order not to discover excuses for bad performance and thus to boost the agent's efforts. This serves as a commitment device in order to be tough on a badly performing agent. In this paper the non-observability helps the seller to not react to deviations from the equilibrium investment level, which leaves the buyer with a higher marginal return from investment.

Tirole (1986) studies ex-ante investment in procurement. Inefficient bargaining between two parties takes place after a non-contractible investment has been sunk. He finds underinvestment independent of whether investment is observed. In case of observability of investment also allows contractability of it (although keeping the assumption that bargaining on trade takes place only after the investment stage), equilibrium will be higher then under unobservable investment and might even exceed the first-best level.

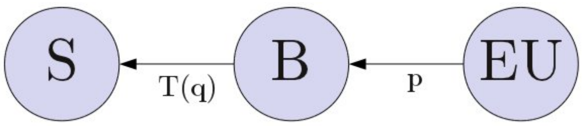
The debate on net-neutrality contains a whole range of more or less connected issues, which includes for example the discussion on product-line restrictions (*i.e.* whether a *discrimination* based on different quality levels should be allowed or prohibited). It is only recently that more and more these questions get an analytical treatment: Schuett (2010) provides a survey of this growing literature.

One of the arguments in the political debate on net-neutrality is that ISPs should be allowed to charge content-providers for access to their customers in order to provide them with better incentives to invest into the quality of their platform. Cañon (2009) and Economides and Tåg (2009) address this issue. They later find in a two-sided market setting that for *reasonable* parameter ranges a net-neutrality regulation (imposing on a platform to not charge content providers) improves total welfare under a monopoly and competition.

Hermalin and Katz (2007) discuss the question whether ISPs should be restricted to only provide a single quality to all users from a traditional and a two-sided markets perspective. They find that putting such restrictions on a platform *often* reduces welfare, because content-providers which would otherwise have bought low-quality access will drop out of the market and those willing to go for the high-quality will have to settle with a mediocre quality level, which is in many configurations not compensated for by relatively higher-quality purchases in the segment of content-providers heading for medium-quality access. Choi and Kim (2010) consider the impact such product-line restrictions on the ISPs and on content-providers incentives to invest. Their results are mixed: The ISP's and the content-providers' investment can be harmed or boosted by the product-line restriction. But when the ISPs bargaining power is high enough the content-providers will invest more under the single quality regulation.

2 Monopoly input provider

In the first part of the paper we study a situation in which a buyer uses a seller's input in order to serve end-users in a downstream market. We assume that a linear price p is used in the downstream market and the buyer is a monopolist in this market. The monopoly input seller only deals with the buyer, and does not engage in any kind in the downstream market, nor does she observe any downstream interaction taking place. Examples are Internet-backbone-providers selling bandwidth connecting a content-provider to a certain geographical area, or Internet content delivery networks (*eg.* Akamai Technologies), which allow content owners to outsource file download, video streaming,... activities to a (more efficient) third party.



We assume that the buyers may be of different type θ. The parameter θ could then by interpreted as the quality of buyer's product perceived by the end-users (more general: θ as a parameter of the end-users' aggregate demand). Different interpretations of θ might be a cost parameter in the buyer's production or the size of the downstream market, which might not - *eg.* in the case of an Internet-backbone-provider - be observable by the seller. We stick to the first interpretation and suppose that end-users' inverse demand depends positively on the buyer's type θ: $p(\theta, q)$. Hence, a buyer θ's profits from trade in the downstream market(s) necessitating the use of q units of the seller's product are given by $V(\theta, q) = qp(\theta, q)$.

2.1 A simple one seller one buyer model

In the outlined model we abstract from the concrete examples originating from the net-neutrality debate and study a more stylized and simplified situation in which a potential downstream market can be summarized by the buyer's profit function $V(\theta, q)$. In this baseline model there is a single buyer of a product or service, who faces a monopoly seller. Before trade occurs the buyer might sink an investment I which increases her valuation of the seller's output. This setting should mimic a situation where a firm needs an input of a monopoly supplier to serve her respective downstream market(s). The value of the downstream product and thus the buyer's surplus (i.e. profits) also depends on an ex-ante investment of the buyer (e.g. in the quality of the downstream product). The details of the downstream market are abstracted away for the moment.

In the baseline setting the game evolves as follows[1]:

1. The buyer chooses an investment level $I \geq 0$ to improve the value of the seller's output.
2. Nature assigns a type $\theta \in [\underline{\theta}, \overline{\theta}]$ to the buyer, where $0 < \underline{\theta} < \overline{\theta} < \infty$, according to the probability distribution function $f(\theta|I)$ (and cumulative distribution function $F(\theta|I)$ respectively)[2].
3. The seller offers a tariff $T(q)$ to the buyer. Two cases are analyzed: In the first case the seller observes neither the investment nor the type of the buyer, while in the second case investment is observed by the seller, but she still does not know the type of the buyer. The production function of the seller exhibits constant marginal cost c.
4. The buyer learns his type and chooses q, the quantity of the good to buy from the monopoly seller. A buyer of type θ will maximize her net surplus given by $V(\theta, q) - T(q)$.

If the seller could observe the buyer's type, she could extract the buyer's joint surplus from trade, which would in turn remove any incentives for the buyer to invest in the first place - This is the well known hold-up problem. Due to the assumption that the buyer's type cannot be observed directly (just the investment or the distribution of types after investment has taken place) the seller cannot perfectly extract the buyer's surplus, which

[1]In terms of notation I borrow from the section on non-linear prices in Tirole (1988).

[2]An extension to the case where $\overline{\theta} = \infty$ is straight-forward, but additional care has to be taken such that all involved integrals converge (*eg.* such that there is a finite socially efficient investment level, exchanging differentiation and integration needs more attention as well).

leaves the possibility of positive investment in step 1. Whether the resulting investment level is distorted compared to an yet to be defined benchmark and in which direction, will be analyzed in different scenarios in the following sections.

To be able to apply standard techniques, we have to put some structure onto the model. The following assumption collects restrictions on the distribution of the buyer's types θ and the impact of investment on it:

Assumption 1 (Monotone hazard-rate & First-order stochastic-dominance)**.** *For any $I > 0$ and $\theta \in [\underline{\theta}, \overline{\theta}]$ the hazard-rate*

$$h(\theta|I) := \frac{f(\theta|I)}{1 - F(\theta|I)}$$

is non-decreasing in θ. Furthermore, the distribution function fulfills the first-order stochastic-dominance property in I, i.e.. $\frac{\partial F}{\partial I} < 0$. For every $I \geq 0$ we assume full support of $f(\theta|I)$.

The assumption that the hazard-rate is non-decreasing in θ is a standard one in the literature on price discrimination. The role of the second part of the assumption is to capture that a higher investment I improves the distribution of θ, *i.e.* that a higher investment level will lead to higher returns. For all occurring functions, distributions etc. we assume that they are smooth (*i.e.* differentiable as many times *as needed*).

The second assumption concerns the buyer's surplus function:

Assumption 2 (Normalization and single-crossing condition)**.** *For any type θ the buyer's surplus from consuming nothing is zero: $V(\theta, 0)$. To fulfill the assumptions of simple models of upstream price discrimination we assume that downstream profits and marginal profits increase in θ:*

$$\frac{\partial V}{\partial \theta}, \frac{\partial^2 V}{\partial q \partial \theta} > 0$$

To rule out free lunch and certain boundary solutions we impose the following:

Assumption 3 (Boundary conditions)**.** *For all types θ of the buyer marginal buyer's surplus is bigger than c, and social surplus*[3] *is bounded from above.*

The assumptions up to now are standard in the literature on adverse-selection. Nevertheless we have to check if the assumptions hold for $V(\theta, q)$ representing the buyer's downstream profits. The following conditions must hold:

$$\frac{\partial V}{\partial \theta} = q\frac{\partial p}{\partial \theta} > 0\,, \qquad \text{and} \qquad \frac{\partial^2 V}{\partial \theta \partial q} = \frac{\partial p}{\partial \theta} + q\frac{\partial^2 p}{\partial \theta \partial q} > 0$$

[3]Social surplus includes seller, buyer, end-user surplus.

The last assumption does not hold for all demand functions, but *eg.* can be easily verified for linear demand functions. Other examples of demand function of this type are $p(\theta, q) = \theta\mathrm{p}(q)$ and $p(\theta, q) = \mathrm{p}(q) + \theta$. Note that for many parametrizations of demand the single-crossing assumption holds in the relevant range, *i.e.* as long as marginal revenue is increasing.

Example. *To illustrate the following results we will employ a numerical example throughout this part of the paper using the following parameters and functional forms, which fulfill all assumptions imposed up to now:*

- *Surplus function:* $V(\theta, q) = 2\theta\sqrt{q}$
- *Types:* $\underline{\theta} = 1$, $\overline{\theta} = 10$
- *Distribution over types*[4]*:* $F(\theta, I) = 1 - \left(\frac{\overline{\theta}-\theta}{\overline{\theta}-\underline{\theta}}\right)^{\frac{1}{\beta(1+I)}}$, $\beta = 1$
- *Marginal cost:* $c = 0.1$

To simplify notation significantly we introduce the function $\alpha(I) := \frac{1}{\beta(1+I)}$. *For the computations we stick to arbitrary parameters* $\underline{\theta}, \overline{\theta}, \beta$, *and* c, *but the following graphs are drawn for the numerical values stated above.*

2.2 Optimizing joint surplus

First we take a look at the optimal joint surplus, by maximizing the sum of the seller's and buyer's surplus when there is no information and contracting problem[5]. Put into other words we look at the buyer's surplus if she had access to the same technology for providing the good as the seller. We will use the results from this discussion as benchmark in the remaining paper.

We determine the optimal investment level I^{fb} and consumption levels $q^{fb}(\theta)$ for all θ such that joint surplus is maximized:

$$\max_{I, \{q(\theta)\}} \int_{\underline{\theta}}^{\overline{\theta}} [V(\theta, q(\theta)) - cq(\theta)]\, f(\theta|I)\, d\theta - I$$

The objective function is continuous in I and all $q(\theta)$, so we might first optimize for a given investment I. The necessary condition for optimality given I is

$$\left[\frac{\partial V}{\partial q}(\theta, q^{fb}(\theta)) - c\right] f(\theta|I) = 0 \quad \text{for (almost) all } \theta \in [\underline{\theta}, \overline{\theta}]$$

[4]See appendix on how this distribution was derived.

[5]It should be pointed out that this formulation does not take into account the downstream end-user surplus in general, only if the buyer is able to fully extract end-user surplus.

or equivalently by

$$\frac{\partial V}{\partial q}(\theta, q^{fb}(\theta)) = c \quad \text{for (almost) all } \theta \in [\underline{\theta}, \overline{\theta}] \,.$$

When the buyer's type is θ, joint-surplus can be written as

$$W(\theta) := V(\theta, q^{fb}(\theta)) - cq^{fb}(\theta) \,.$$

With the help of the envelope theorem

$$\frac{dW}{d\theta}(\theta) = \frac{\partial V}{\partial \theta}(\theta, q^{fb}(\theta))$$

we get

$$W(\theta) = \int_{\underline{\theta}}^{\theta} V(t, q^{fb}(t))\, dt + W(\underline{\theta})$$

which is obviously strictly increasing in θ. Taking the expectation over θ given the investment level I yields the expected gross joint-surplus

$$\mathcal{W}(I) = \int_{\underline{\theta}}^{\overline{\theta}} W(\theta) f(\theta|I)\, d\theta \,.$$

Lemma 1. *Expected gross joint-surplus $\mathcal{W}(I)$ is increasing in I.*

Proof. We know that for $I' > I$ the distribution represented by $f(\theta|I')$ first-order stochastically dominates the one represented by $f(\theta|I)$ and $W(\theta)$ is increasing in θ, hence

$$\mathcal{W}(I) = \int_{\underline{\theta}}^{\overline{\theta}} W(\theta) f(\theta|I)\, d\theta < \int_{\underline{\theta}}^{\overline{\theta}} W(\theta) f(\theta|I')\, d\theta = \mathcal{W}(I') \,.$$

□

Proposition 1. *There exists a maximum of joint surplus for an investment level I^{fb}.*

Proof. $\mathcal{W}(I)$ is bounded from above by $W(\overline{\theta})$. Furthermore due to the smoothness assumptions $\mathcal{W}(I)$ continuous in I. Therefore $\lfloor \mathcal{W}(I) \rfloor = \max\{\mathcal{W}(I) - I, 0\}$ has a compact support and is continuous in I too, which in turn guarantees that $\lfloor \mathcal{W}(I) \rfloor$ (and hence $\mathcal{W}(I)$) attains its supremum for some I^{fb}. □

However, given the current setup of the model it is not guaranteed that investment is socially desirable at all. From now on we consider only the interesting case:

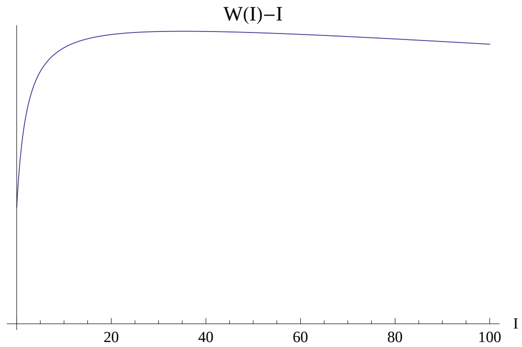
Assumption 4 (Socially desirable investment)**.** *We assume that investing in trade is socially desirable, i.e.*

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[V(\theta, q^{fb}(\theta)) - cq^{fb}(\theta) \right] f(\theta|I^{fb})\, d\theta > I^{fb} \,.$$

Example (Continued). *Using the functional forms from above it is possible to derive a closed form solution of the gross-return of investment:*

$$\mathcal{W}(\alpha) = \frac{1}{c}\left[\underline{\theta}^2 + 2\underline{\theta}\frac{\underline{\theta} - \overline{\theta}}{\alpha + 1} + 2\frac{(\underline{\theta} - \overline{\theta})^2}{(\alpha + 1)(\alpha + 2)}\right]$$

Note that $\mathcal{W}(I) = \mathcal{W}(\alpha(I))$*.*



It is possible to show that $\mathcal{W}(I) - I$ *is concave and thus a unique maximum* $I^{fb} \approx 35.2$ *exists.*

2.3 Unobservable investment

When neither the buyer's investment level from stage 1 nor the result of nature's draw in step 2 are known to the seller in stage 3, the buyer's investment choice and the seller's tariff choice are in fact simultaneous moves. Only after these decisions are made, the buyer chooses on the quantity to buy in step 4 after having learned the realization of θ and the tariff proposed by the seller.

To solve this game, we employ backwards induction starting with the buyer's quantity choice and using the result as an input in the simultaneous move game. We consider pure strategy equilibria in the simultaneous move game[6]. Given the anticipated investment level

[6]This is not to say that mixed strategy equilibria might not exist. Laffont and Martimort (2002) outline a situation with ex-ante non-contractible, non-observable investment by an agent and a follow-up adverse-selection stage. The analysis is carried out under the restriction to two types and only two admissible investment levels (0 or I). Depending on parameters three different equilibria exist:

1. The agent invests (and thus first-best effort) although the principal anticipates this and designs the contract under this premise.
2. The agent does not invest (socially inefficient), which is taken into account by the principal as well.
3. The agent randomizes between investing and not investing. The principal offers contracts based on the induced probability distribution over types, which leaves the agent indifferent whether to invest or not.

Schmidt (1996) shows that in this model under quite strong assumptions and a continuous investment choice, that there exists only a unique pure strategy equilibrium exhibiting underinvestment.

I_a the seller's optimization problem in step 3 has a well known solution (see *eg.* Maskin and Riley, 1984), which is characterized[7] by the following equations:

$$\frac{\partial V}{\partial q}(\theta, q^*(\theta, I_a)) = c + \frac{1}{h(\theta|I_a)} \frac{\partial^2 V}{\partial q \partial \theta}(\theta, q^*(\theta, I_a)) \tag{1}$$

$$\frac{\partial T}{\partial q}(q^*(\theta, I^*), I_a) = \frac{\partial V}{\partial q}(\theta, q^*(\theta, I_a)) \tag{2}$$

The first equation determines quantities bought by each type θ, while the second allows to determine the optimal tariff. There is one caveat however, because there a solution to the first-equation might not exist for certain types, which signifies that the seller does not want to serve them. We get a boundary solution $q^*(\theta, I_a) = 0$ for these types. Due to the fact that incentive compatibility necessitates a non-decreasing quantity schedule in types, there exists a cut-off point for shutdown $\tilde{\theta}(I_a)$ given the level of investment: Below this cut-off the seller does not serve the respective type, above the point quantities are determined by equation (1).

It is easy to see and well known, that quantities traded are below first-best levels, *i.e.* $q^*(\theta, I_a) < q^{fb}(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta})$, because

$$\frac{\partial V}{\partial q}(\theta, q^*(\theta, I_a)) > c\,.$$

Only type $\overline{\theta}$ gets the efficient quantity: *no distortion at the top*. Due to the fact that buyer $\tilde{\theta}(I_a)$'s surplus is zero and the envelope theorem we know that buyer θ's surplus from trade is given by

$$\begin{array}{lcllc} U(\theta, I_a) & := & \int_{\tilde{\theta}(I_a)}^{\theta} \frac{\partial V}{\partial \theta}(t, q^*(t, I_a))\, dt & \text{for } \theta > \tilde{\theta}(I_a) & \text{and} \\ U(\theta, I_a) & := & 0 & \text{for } \theta < \tilde{\theta}(I_a)\,. & \end{array} \tag{3}$$

Trivially $U(\theta, I_a)$ is increasing in θ.

Using these result it is possible to discuss the optimal investment behavior of the buyer in step 1. We know that in equilibrium the seller sets an optimal tariff based on her anticipation I_a of the prevalent investment level, so we can neglect other tariffs without loss of generality. We know that given the seller's anticipated investment level is I_a, the buyer's gross expected surplus of a buyer investing I is given by

$$\mathcal{U}(I, I_a) := \int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I_a) f(\theta|I)\, d\theta\,. \tag{4}$$

The buyer chooses the level of investment to maximize her net expected surplus:

$$\max_{I \geq 0} \mathcal{U}(I, I_a) - I \tag{5}$$

[7] necessary, but not sufficient conditions

Like in the section before discussing the first-best solution for the maximization problem $\mathcal{U}(I, I_a)$ is bounded from above by $W(\overline{\theta})$, continuous in I and non-negative only on a compact set. Thus a maximum exists and is obtained for some level of investment. This is already enough information to show that investment lies below first-best level:

Proposition 2. *In any pure strategy equilibrium the buyer's equilibrium investment, denoted by I^*, is below first-best investment level*[8]*, i.e.* $I^* < I^{fb}$.

Proof. The proof involves a simple revealed preference argument:

$$\begin{aligned} \mathcal{U}(I^*, I^*) - I^* &\geq \mathcal{U}(I^{fb}, I^*) - I^{fb} \\ \mathcal{W}(I^{fb}) - I^{fb} &\geq \mathcal{W}(I^*) - I^* \end{aligned}$$

Adding up both inequalities, plugging in and rearranging yields:

$$\int_{\underline{\theta}}^{\overline{\theta}} [W(\theta) - U(\theta, I^*)] \, [f(\theta|I^{fb}) - f(\theta|I^*)] \, d\theta \geq 0$$

Now observe that $W(\theta) - U(\theta, I^*)$ is strictly increasing in θ (due to the below first-best quantities under price discrimination and single-crossing):

$$\frac{\partial}{\partial \theta} [W(\theta) - U(\theta, I)] = \frac{\partial V}{\partial \theta}(\theta, q^{fb}(\theta)) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta, I)) > 0$$

Thus first-order stochastic-dominance mandates that $I^* \leq I^{fb}$. Knowing this we want to show that the inequality is strict by contradiction: Suppose that $I^* = I^{fb}$. Being an optimum level of investment I^* must fulfill the first-order condition for the maximization problem $\max_I \mathcal{U}(I, I^*) - I$:

$$\frac{\partial \mathcal{U}}{\partial I}(I^*, I^*) = \int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I^*) \frac{\partial f}{\partial I}(\theta|I^*) \, d\theta = 1$$

The same holds for $I^* = I^{fb}$ and first-order condition characterizing the socially optimal level of investment:

$$\frac{\partial \mathcal{W}}{\partial I}(I^*) = \int_{\underline{\theta}}^{\overline{\theta}} W(\theta) \frac{\partial f}{\partial I}(\theta|I^*) \, d\theta = 1$$

We can construct the following contradiction, due to the first-order stochastic-dominance together with the fact, that $W(\theta) - U(\theta, I^*)$ is strictly increasing in θ as discussed in the proof of proposition 2:

$$1 = \int_{\underline{\theta}}^{\overline{\theta}} W(\theta) \frac{\partial f}{\partial I}(\theta|I^*) > \int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I^*) \frac{\partial f}{\partial I}(\theta|I^*) = 1$$

Thus, $I^* < I^{fb}$ must be true. □

[8] This result can be easily extended to all investment levels in the support of the buyer's mixed investment strategy, in the case in which the seller continues to play a pure strategy.

Now that these two results have been established the question arises whether one can expect a pure-strategy equilibrium to exist. It turns out that this requires strong regularity assumptions on the adverse-selection problem and on the distribution over types:

Lemma 2 (Regularity of $\mathcal{I}(I_a)$)**.** *We assume that the first-order condition of the seller's problem has a unique solution for each type θ and that the respective sufficient second-order condition (i.e. the second-order condition corresponding to* (1)*) is fulfilled*[9]*. Furthermore the distribution of types is assumed to be strictly convex in investment, i.e. $F_{II} > 0$. Then for all levels of anticipated investment I_a the investment maximization problem has a unique solution and $\mathcal{I}(I_a)$ is a continuous function.*

Proof. First we show that $\mathcal{U}(I, I_a)$ is also continuous in I_a. We get that $q^*(\theta, I_a)$ is continuous in I_a due to the regularity assumption on the seller's problem and the smoothness of all involved functions, *i.e.* $\frac{\partial V}{\partial q}$ and $\frac{\partial^2 V}{\partial q \partial \theta}$ are continuous in q and $h(\theta|I)$ is continuous in I. Now we have to show that $U(\theta, I_a)$ is continuous in I_a, *i.e.* that if $I_n \to I_a$ then

$$U(\theta, I_a) = \lim_{I_n \to I_a} U(\theta, I_n) = \lim_{I_n \to I_a} \int_{\underline{\theta}}^{\theta} \frac{\partial V}{\partial \theta}(t, q^*(t, I_n))\, dt\,.$$

But this is a direct consequence of the Dominated Convergence Theorem, because $\frac{\partial V}{\partial \theta}(t, q^*(t, I_n)) < \frac{\partial V}{\partial \theta}(t, q(t))$ for all n (the inequality holds because of the single-crossing condition and a downward distortion of the quantities traded due to second-degree price discrimination).

Now we need to prove that $\mathcal{U}(I, I_a)$ is continuous in I_a, which follows again from the Dominated Convergence Theorem, because $U(\theta, I_a) < W(\theta)$ for all I_a. Therefore the correspondence $\mathcal{I}(I_a)$ mapping the seller's anticipation to optimal buyer's investment level is upper-hemicontinuous due to Berge's maximum theorem. So there might be jumps in the buyer's best-response, which may lead to non-existence of a pure-strategy equilibrium.

The convexity of the distribution function renders the buyer's problem in stage 1 concave given I_a:

$$\int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I_a) \frac{\partial^2 f}{\partial I^2}(\theta|I) > 0$$

Therefore the investment choice problem has a unique solution, and thus $\mathcal{I}(I_a)$ is also continuous (upper-hemicontinuous and single-valued). □

[9]Guesnerie and Laffont (1984) study a class of principal-agent problems, for which a solution can be shown to exist and is unique. They assume *concavity and regularity of the surrogate social welfare function*, which boils down in our case to the stated properties. For further details I refer to their exposition.

It should be noted that the implications of the lemma might also hold, when some of the conditions in the Lemma are not met[10]. Even in other cases (like in the numerical example), the implication of the Lemma may hold although some of its conditions are violated. Using these results we can show the following proposition:

Proposition 3. *If the assumptions or even only the implication of Lemma 2 hold, an equilibrium in pure strategies exists. The equilibrium investment level will be denoted by I^*.*

Proof. Finding an equilibrium boils down to finding a fixed point of the function $\mathcal{I}(I_a)$. We know that $\mathcal{I}(0) \geq 0$. If $\mathcal{I}(0) = 0$, we have already found an equilibrium. In the other case $\mathcal{I}(0) > 0$ we certainly know that $\mathcal{I}(W(\overline{\theta})) \leq W(\overline{\theta})$ (because you would never want to invest more than can be recuperated at maximum). With the help of Bolzano's theorem a fixed point exits. □

After having established the existence of a pure-strategy equilibrium, we wonder whether it is unique. In general multiple pure-strategy equilibria cannot be ruled out, but under additional assumptions one can show that the buyer's reaction function $\mathcal{I}(I_a)$ is decreasing, which immediately implies uniqueness. One could interpret this is as a situation where the buyer's decision on investment and the seller's reaction to different anticipated investment levels are strategic substitutes.

Assumption 5 (Decreasing hazard-rate in investment)**.** *For any $I > 0$ and $\theta \in [\underline{\theta}, \overline{\theta}]$ the hazard-rate $h(\theta|I)$ is decreasing in I:*

$$\frac{\partial h}{\partial I}(\theta|I) < 0 \qquad \textit{for all } \theta \in [\underline{\theta}, \overline{\theta}]$$

It is well known that the decreasing hazard-rate assumption implies first-order stochastic-dominance (assumption 1). For a method to construct probability distributions with the properties appearing in this paper see the appendix.

To show the uniqueness of the equilibrium under this assumption, we have to derive some preparative results. Some of these results will be used later on, where the interpretation of I_a is not the seller's anticipation of the buyer's investment, but an actual observation of the investment level sunk. First, using these assumptions allows to pin down the impact of changes in investment on the quantities traded in stage 4:

[10]Especially the assumption on the convexity of the distribution function together with the later imposed monotone hazard-rate assumption 5 is very restrictive. In the appendix I cite two examples for distributions fulfilling all properties imposed at some point in the paper on the distribution of types.

Lemma 3. *When assumption 5 holds, the quantity chosen by any type $\theta > \tilde{\theta}(I_a)$ is decreasing in the level of anticipated investment I_a for all $\theta \in [\tilde{\theta}(I_a), \overline{\theta}]$, i.e.*

$$\frac{\partial q^*}{\partial I_a}(\theta, I_a) < 0\,.$$

Proof. It is well known that the IC constraint of the seller's problem can be backed out and plugged into the seller's objective function:

$$\max_{q(\cdot|I)} \int_{\tilde{\theta}(I_a)}^{\overline{\theta}} \left[\left(V(\theta, q(\theta|I)) - cq(\theta|I)\right) f(\theta|I) - \frac{\partial V}{\partial \theta}(\theta, q(\theta|I))(1 - F(\theta|I)) \right] d\theta$$

This reformulation allows to optimize surplus type by type. Hence it is possible to carry out a revealed preference argument. Suppose $I'_a > I_a$:

$$\begin{aligned} V(\theta, q^*(\theta|I_a)) - cq^*(\theta|I_a) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I_a))\frac{1}{h(\theta|I_a)} &\geq V(\theta, q^*(\theta|I'_a)) - cq^*(\theta|I'_a) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I'_a))\frac{1}{h(\theta|I_a)} \\ V(\theta, q^*(\theta|I'_a)) - cq^*(\theta|I'_a) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I'_a))\frac{1}{h(\theta|I'_a)} &\geq V(\theta, q^*(\theta|I_a)) - cq(\theta|I_a) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I_a))\frac{1}{h(\theta|I'_a)} \end{aligned}$$

Subtracting the second inequality from the first yields:

$$\left[\frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I'_a)) - \frac{\partial V}{\partial \theta}(\theta, q^*(\theta|I_a))\right] \left[\frac{1}{h(\theta|I_a)} - \frac{1}{h(\theta|I'_a)}\right] \geq 0$$

Due to the monotone-hazard rate assumption the right term in brackets is negative, such that the first term in brackets has to be non-positive. But this can only be true if $q^*(\theta|I_a) \geq q^*(\theta|I'_a)$.

The inequality is strict, because if $q^*(\theta|I_a) = q^*(\theta|I'_a)$ held, at least one first-order condition would be violated. □

Lemma 4. *Under assumption 5 buyer θ's surplus from trade is decreasing in anticipated investment levels I_a for all $\theta \in [\tilde{\theta}(I_a), \overline{\theta}]$. Furthermore marginal surplus w.r.t. anticipated investment is decreasing in θ.*

Proof. Straight-forward differentiating buyer θ's surplus with respect to the anticipated level investment and application of Lemma 3 yields the first result:

$$\frac{\partial U}{\partial I_a}(\theta, I_a) = \int_{\tilde{\theta}(I_a)}^{\theta} \left[\frac{\partial^2 V}{\partial \theta \partial q}(t, q^*(t, I_a))\frac{\partial q^*}{\partial I_a}(t, I_a)\right] dt < 0$$

Differentiating once more by θ shows the second part of the result:

$$\frac{\partial^2 U}{\partial I_a \partial \theta}(\theta, I_a) = \frac{\partial^2 V}{\partial \theta \partial q}(\theta, q^*(\theta, I_a))\frac{\partial q^*}{\partial I_a}((\theta, I_a)) < 0$$

□

Lemma 5. *Under assumption 5 the buyer's reaction function is decreasing in the seller's anticipation of the level of investment.*

Proof. By revealed preference we get for $I' > I$

$$\begin{aligned} \mathcal{U}(\mathcal{I}(I), I) - I &\geq \mathcal{U}(\mathcal{I}(I'), I) - I' \\ \mathcal{U}(\mathcal{I}(I'), I') - I' &\geq \mathcal{U}(\mathcal{I}(I), I') - I\,. \end{aligned}$$

Subtracting the second from the first inequality yields

$$\int_{\underline{\theta}}^{\overline{\theta}} [U(\theta, I) - U(\theta, I')]\,[f(\theta|I) - f(\theta|I')]\,d\theta \geq 0\,.$$

We know that the first difference inside the integral is increasing in θ:

$$\frac{\partial}{\partial\theta}[U(\theta, I) - U(\theta, I')] = \frac{\partial V}{\partial\theta}(\theta, q^*(\theta, I)) - \frac{\partial V}{\partial\theta}(\theta, q^*(\theta, I')) > 0$$

This together with the FOSD property shows that $\mathcal{I}(I') > \mathcal{I}(I)$. □

Using these results allows to state and prove the uniqueness of the equilibrium:

Proposition 4. *Under assumptions 2 and 5 the equilibrium (should it exist) is unique.*

Proof. Suppose by contradiction that there are multiple equilibria in pure strategies. Pick two and denote the equilibrium levels of investment with I_1^* and I_2^*, *w.l.o.g.* $I_1^* < I_2^*$. The following inequality chain establishes the desired contradiction and thus establishes the uniqueness of the equilibrium in pure-strategies:

$$I_1^* = \mathcal{I}(I_1^*) > \mathcal{I}(I_2^*) = I_2^*$$

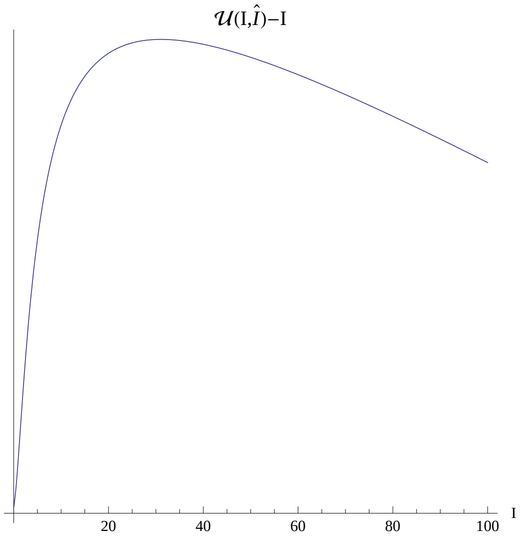
The inequality holds because the best-response function is decreasing due to Lemma 5. □

Summing up, we have seen that price-discrimination by the seller leading to a partial rent extraction from the buyer distorts the investment decision of the buyer, who does not take into account the returns of the investment going to the seller and is harmed by distorted traded quantities. Therefore the investment level is too low compared to the efficient level. Existence of a unique pure-strategy equilibrium hinges on the decreasing hazard-rate assumption (stricter than the first-order stochastic-dominance property) and some regularity assumption on the buyer's best-response function w.r.t. investment, which establishes strategic substitutability across investment and anticipated investment.

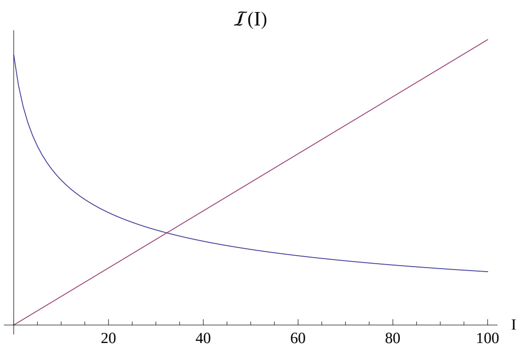
Example (Continued)**.** *Given the functional forms specified in the section before, we can compute closed form solutions for the gross returns to investment:*

$$\begin{aligned}\mathcal{U}(\alpha,\alpha_a) &= \frac{2(\overline{\theta}-\tilde{\theta}(\alpha_a))^{\alpha+1}}{c\alpha_a(\alpha+1)(\overline{\theta}-\underline{\theta})^{\alpha-1}}\left[(\overline{\theta}-(\alpha_a+1)\tilde{\theta}(\alpha_a))+\frac{\alpha_a+1}{\alpha+2}(\overline{\theta}-\tilde{\theta}(\alpha_a))\right]\\ \mathcal{U}(I,I_a) &= \mathcal{U}(\alpha,\alpha_a)\end{aligned}$$

The following graph shows the net return from investment if the seller anticipates the first-best investment level. The optimal buyer's response is $\mathcal{I}(I^{fb})=31.1\neq I^{fb}$, which is thus no equilibrium investment level.



It is possible to show that $\mathcal{U}(I,I_a)-I$ is concave for all I_a, which is why the buyer's reaction function is single valued. Furthermore all sufficient second-order conditions are fulfilled. In the following graph we see the best-response function and the identity function. The crossing defines the unique pure-strategy equilibrium investment $I^*=32.3$.



2.4 Observable investment

Again, as in the previous section, the seller is allowed to propose any tariff $T(q)$ to the buyer in step 3. The difference is that this time we assume that the seller is able to observe the investment in stage 1. As usual the problem is solved backwards by assuming that the buyer has already sunk her investment in step 1. The reasoning of the seller is identical to what was discussed before, with the only difference, that the seller no longer anticipates the buyer's investment, but she directly observes the level of investment which has been sunk before. So we still get that the buyer's surplus after θ has realized when the seller has observed an investment level I is given by $U(\theta, I)$.

The crucial difference from the setting before is, that in step 1 the buyer does not take the tariff charged by the seller but the seller's reaction function as given when deciding on the level of investment, because the seller observes the level of investment after the buyer has already sunk the investment and sets an optimal tariff based on this information. This removes the simultaneous move from the game, and allows for a solution of the game using backwards induction only (*i.e.* without resorting to solving the fixed-point problem like in the previous section). Thus the expected surplus of a buyer sinking an investment I includes the seller's reaction and is therefore given by

$$\mathcal{U}(I, I) = \int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I) f(\theta|I) \, d\theta \, .$$

By the very same argument already used twice before a maximum of $\mathcal{U}(I, I) - I$ is obtained for some I^{**}. Hence, an equilibrium of the game exists, and while it is not necessarily the case that the solution is unique, a multiplicity of equilibria might arise only in exceptional cases (as the optimization problem for choosing I is not necessarily strictly concave, multiple solutions cannot be ruled out). As before there is an inefficiently low level of trade[11].

With the very same revealed preference argument as used in the proof of proposition 2 it is possible to rule out over-investment:

Proposition 5. *Under observable investment the buyer's equilibrium investment does not exceed the first-best investment level, i.e.* $I^{**} \leq I^{fb}$.

One at the first sight surprising result follows without any further assumptions:

Proposition 6. *The buyer is better off, when the seller observes the level of investment compared to the situation where the investment level is not observed.*

[11] again with the exception of type $\overline{\theta}$, who consumes the first-best quantity.

Proof. The proof is simple and short: I^{**} maximizes $\mathcal{U}(I,I) - I$. Thus, for other levels of investment the net-returns are lower - this includes the equilibrium value of investment I^* when investment is not observable by the seller:

$$\mathcal{U}(I^{**}, I^{**}) - I^{**} \geq \mathcal{U}(I^*, I^*) - I^*$$

□

When framed slightly differently the result becomes quite obvious: I^* is in the set of admissible investment levels under observed investment, so the buyer's net surplus must be at least as high as under non-observable investment. Put into other words again: Even under unobservable investment in equilibrium the buyer does not want to fool the seller by picking a non-anticipated investment level. This incentive not to deviate from the equilibrium level of investment is *controlled* by an equilibrium relationship. But when the buyer's investment is observable by the seller, the equilibrium level of investment is controlled directly by the buyer herself, *i.e.* the anticipated level of investment enters directly the objective function instead of being fixed at an arbitrary (from the point of view of the buyer) level.

We would expect that, because of the partial extraction of surplus by the seller, the buyer's investment would be inefficiently low as well. However, this result does not always hold, and can be shown only after invoking additional assumptions on the impact of investment on the capability of the seller to extract rent from the buyer. To be able to characterize the distortion of investment, we need to assume again, that the hazard-rate is decreasing in investment (*i.e.* assumption 5). Then with the help of Lemma 3 and the fact, that $W(\theta) - U(\theta, I^*)$ is strictly increasing in θ, it is straight-forward to compare the equilibrium investment level with the socially optimal one.

Proposition 7. *When the seller engages in second-degree price discrimination and assumption 5 holds, the equilibrium investment is strictly lower than the socially optimal level:* $I^{**} < I^{fb}$.

Proof (by contradiction). Suppose that $I^{**} = I^{fb}$. Being an optimum level of investment I^{**} must fulfill the first-order condition for the maximization problem $\max_I \mathcal{U}(I,I) - I$:

$$\int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I^{**}) \frac{\partial f}{\partial I}(\theta | I^{**})\, d\theta + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U}{\partial I_a}(\theta | I^{**}) f(\theta | I^{**})\, d\theta = 1 \tag{6}$$

Note that the second term on the left hand side is negative, because of Lemma 4. On the other hand the first-order condition characterizing the socially optimal level of investment is given by:

$$\int_{\underline{\theta}}^{\overline{\theta}} W(\theta)\frac{\partial f}{\partial I}(\theta|I^{fb})\,d\theta = 1$$

By using lemma 4 we can establish the following contradiction:

$$\begin{aligned} 1 = \int_{\underline{\theta}}^{\overline{\theta}} W(\theta)\frac{\partial f}{\partial I}(\theta|I^{**}) &> \int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I^{**})\frac{\partial f}{\partial I}(\theta|I^{**}) \\ &> \int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I^{**})\frac{\partial f}{\partial I}(\theta|I^{**}) + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U}{\partial I_a}(\theta|I^{**})f(\theta|I^{**}) = 1 \end{aligned}$$

Thus $I^{**} < I^{fb}$ must hold. □

Contrary to the section before, existence and (in almost all cases) uniqueness follow directly from the sequential structure of the setup. In general the equilibrium investment level is also distorted, but this time the strict downward distortion follows from the decreasing (in investment) hazard-rate assumption. To better understand this, it helps to take a closer look at the respective first-order condition - equation (6). Marginal returns to investment can be split in two components: First on the left there is the rent effect, which comes from the improvement of the distribution of the types due to investment. This part is smaller than under maximization of the first-best, because parts of the rents go to the seller. The second term captures a strategic effect, which comes about by the seller's reaction to the change in investment. While intuitively it might seem compelling at the first sight, that this term should be negative, it is not under all circumstances: The seller's willingness and need to distort the low θ quantities is tightly connected to the shape of the hazard-rate over θ. Depending on how investment affects this shape determines whether the strategic effect is positive or negative. Under the decreasing hazard-rate assumption we have seen that it turns out to be negative and thus the total effect is negative and investment is strictly downward distorted.

Having discussed the distortion of investment under observable and unobservable investment, it would be interesting to compare both kinds of distortions. It turns out that the comparison can be done again with the help of the decreasing hazard-rate assumption.

Proposition 8. *If assumption 5 holds, the equilibrium investment is smaller compared to the case when the seller can observe it before deciding on her choice of the tariff, i.e.* $I^{**} < I^*$.

Proof. Again a revealed preference argument:

$$\begin{aligned} \mathcal{U}(I^*, I^*) - I^* &\geq \mathcal{U}(I^{**}, I^*) - I^{**} \\ \mathcal{U}(I^{**}, I^{**}) - I^{**} &\geq \mathcal{U}(I^*, I^*) - I^* \end{aligned}$$

Adding up both inequalities, plugging in and rearranging yields:

$$\int_{\underline{\theta}}^{\overline{\theta}} [U(\theta, I^{**}) - U(\theta, I^{*})]\, f(\theta|I^{**})\, d\theta \geq 0$$

Due to lemma 4 we get that $I^{**} \leq I^{*}$. Again, to get the strict inequality we suppose by contradiction that $I^{**} = I^{*}$. But then the following inequality chain holds and yields the desired contradiction:

$$1 = \int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I^{**}) \frac{\partial f}{\partial I}(\theta|I^{**})\, d\theta > \int_{\underline{\theta}}^{\overline{\theta}} U(\theta, I^{**}) \frac{\partial f}{\partial I}(\theta|I^{**})\, d\theta + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U}{\partial I_a}(\theta|I^{**}) f(\theta|I^{**})\, d\theta = 1$$

The first equality is the first-order condition for optimal investment under non-observable investment. The following inequality holds because of Lemma 4. The last equation is the first-order condition for optimal investment under observable investment. □

Comparing the results from propositions 6 and 8 is a bit striking: While on the one hand equilibrium investment drops compared to a situation when the seller might not observe investment, the buyer on the other hand benefits from the seller being able to observe investment. As pointed out before this is explained by the fact that in the game where the investment is not observed investment and tariff choice are de-facto simultaneous moves. While the sequential nature of the game with observable investment establishes the buyer (*i.e.* the investor) as the Stackelberg leader and the seller as the Stackelberg follower of the game.

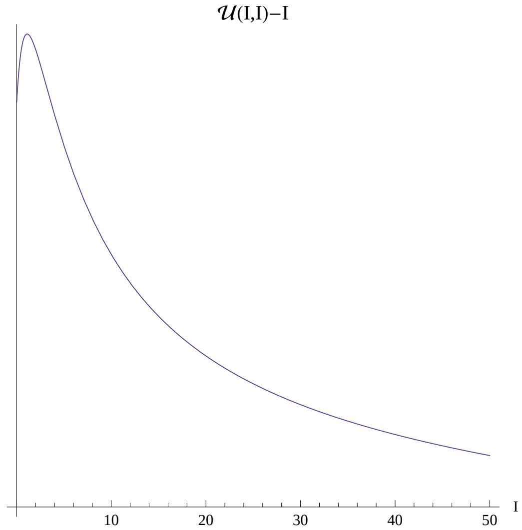
Proposition 9. *If assumption 5 holds, the seller is worse off in equilibrium when investment is observable, because she is always worse off if the prevailing value of investment is smaller.*

It is easy to prove the last result via a simple revealed preference argument again, which is therefore left to the reader. The immediate follow-up question is how the joint buyer and seller surplus behaves in the two different cases. Unfortunately this is not a clear cut case, it rather depends on the relationship between the inefficiency introduced by price discrimination and the level of investment. Joint surplus under price discrimination for a prevailing level of investment I is given by $\int_{\underline{\theta}}^{\overline{\theta}} [V(\theta, q^{*}(\theta, I)) - cq^{*}(\theta, I)]\, f(\theta|I)\, d\theta - I$. The marginal (gross) returns to investment are then

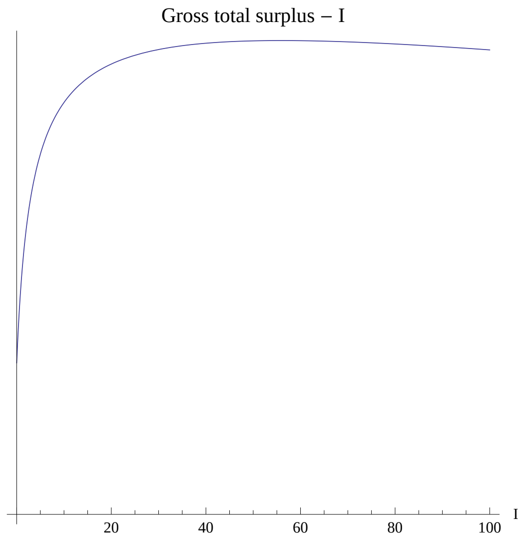
$$\int_{\underline{\theta}}^{\overline{\theta}} [V(\theta, q^{*}(\theta, I)) - cq^{*}(\theta, I)] \frac{\partial f}{\partial I}(\theta|I)\, d\theta + \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{\partial V}{\partial q}(\theta, q^{*}(\theta, I)) - c\right] \frac{\partial q^{*}}{\partial I_a}(\theta, I) f(\theta|I)\, d\theta\,.$$

The first term gives the improvement of the joint surplus to the increase of investment keeping the charged tariff fixed. It is clearly positive as $V(\theta, q^{*}(\theta, I)) - cq^{*}(\theta, I)$ is increasing in θ. The second term results from the dead-weight loss due to price-discrimination, which is - given the decreasing hazard-rate assumption - negative because $\frac{\partial q^*}{\partial I_a} < 0$ and $\frac{\partial V}{\partial q}(\theta, q^*(\theta, I)) - c > 0$. Unfortunately it is not possible to trace out the joint effect, and thus we cannot conclude in general that joint surplus is higher under unobservable investment. Without stating the result formally it is however possible to show with a revealed preference argument that the optimal second-best investment level, which maximizes joint-surplus given the inefficiency introduced by second-degree price discrimination, is larger than the equilibrium investment level under observable and non-observable investment. Thus if joint-surplus is concave in the prevailing investment level - as it is the case in the example below - we indeed get that joint surplus decreases when investment becomes observable.

Example (Continued)**.** *In the example we are able to reproduce the analytic results. Here is a plot of* $\mathcal{U}(I, I) - I$*, which can be shown to be concave, and thus has a unique maximizer* $I^{**} = 1.11$*.*



In general it is not possible to show that joint surplus increases when investment becomes non-observable, but in this example it is the case that joint-surplus increases from 443.40 *when investment is observed to* 855.71 *when it is not observed.*



2.5 Ex-ante commitment/contracting

In the previous two sections we have seen, that only sub-optimal amounts are invested by the buyer, because the seller's decision on the tariff $T(q)$ takes place after the buyer's investment decision, which is why the seller does not take into account her impact on the ex-ante investment decision. To analyze this effect more clearly we will consider a situation in which the seller can commit to a tariff $T(q)$ before the buyer chooses the level of investment or put into other words a situation in which the seller and buyer might write a contract prior to the buyer sinking her investment.

2.5.1 Ex-ante contracting

In this section we allow both parties to sign a contract before investment takes place. This change in the admissible contracts modifies the sequence of events of the game:

1. The seller commits to offer a tariff $T(q)$ to the buyer in the future, and additionally might charge a fixed fee A upfront. The production function of the seller exhibits constant marginal cost c.

2. The buyer observes the seller's tariff $T(q)$ and fixed fee A, chooses whether to accept the seller's offer and pay the fixed fee, and decides on the amount $I \geq 0$ to invest in order to improve the value of the seller's output.

3. Nature assigns a type $\theta \in [\underline{\theta}, \overline{\theta}]$ to the buyer according to the probability distribution function $f(\theta|I)$ (and cumulative distribution function $F(\theta|I)$ respectively), where $0 <$

$\underline{\theta} < \overline{\theta} < \infty$.

4. Given her type the buyer chooses q, the quantity of the good to buy from the monopoly seller.

Proposition 10. *The equilibrium contract proposed by the seller is composed of a fixed fee* $A = \mathcal{W}(I^{fb}) - I^{fb}$ *and a tariff* $T(q) = cq$*. Thus first-best quantities are traded and investment attains the socially optimal level.*

Proof. The seller's profit equals the maximum of joint surplus outlined in section 2.2. What remains to be checked is, if the buyer accepts the contract in stage 2. As a first step we observe that the buyer's quantity choice in step 4 is socially optimal, *i.e.* a type θ chooses to buy $q^{fb}(\theta)$ units from the seller. Thus the expected buyer's profit is $\mathcal{W}(I)$ when investing I units in stage 2. But then the objective function of the buyer is identical to the one in the joint surplus maximization problem, so that the buyer will choose the socially efficient investment level I^{fb}.

This leaves the buyer with a surplus of $\mathcal{W}(I^{fb}) - I^{fb}$ after having accepted the offer of the seller. But this surplus equals the upfront fee A asked for by the seller, which leaves the buyer with zero profits when accepting the seller's offer, *i.e.* she is indifferent whether to accept the contract proposed by the seller or not. □

Unsurprisingly, we have seen that the seller is able to fully extract the buyer's follow-up profit in the first stage, which allows the seller to sell at cost at stage 4, which in turn induces the buyer to choose first-best quantities and investment. It should be pointed out, however, that the buyer breaks even only in expected terms, *i.e.* there are states of nature in which the respective buyer's types make a loss (when taking the fixed fee A into account).

2.5.2 Ex-ante commitment with ex-post participation constraints

Consider the timing from the previous section, but suppose that the seller only can commit to a tariff in stage 1, but that the buyer can only sign the contract after the investment has been sunk and the draw of nature has materialized. The buyer will only consume a positive amount, if she is able to at least break even. This renders the seller's problem more complicated, because she has to take into account an additional incentive compatibility constraint for investment (*i.e.* has to bear in mind the buyer's best response investment).

Proposition 11. *Equilibrium investment under ex-ante commitment with ex-post participation constraints is higher than under non-observable investment.*

Proof. First note, that the seller is at least as well-off as under non-observable investment. She could just pick the equilibrium tariff from that situation, the buyer would invest the same amount as in the non-observable investment case. Furthermore equilibrium investment under commitment can never be below the level under non-observability, because as already pointed out before in the non-commitment case the seller's profits are increasing in equilibrium investment level, which means that the seller even ignoring the incentive compatibility constraint for investment would be worse off inducing less investment compared to the case where she charges the equilibrium tariff from the non-observable investment case.

To show that the inequality is strict, we construct a deviation from the allocation schedule under non-observable investment towards the first-best schedule, which increases investment and the seller's profit. Consider the following allocation schedules for $\delta \in [0,1]$:

$$\bar{q}(\theta,\delta) := (1-\delta)q^*(\theta,I^*) + \delta q(\theta) \qquad \text{for all } \theta > \tilde{\theta}(I^*)$$

First we need to show that investment increases (strictly) in δ (which can be proved via the usual revealed preference argument together with checking the first-order conditions). Now note that the seller's profits depend on δ via two channels: via a change in the allocation schedule per type and via a change in investment. Hence profits can be denoted by $\pi(\bar{q}(\cdot,\delta), I(\delta))$. By differentiating with respect to δ we get

$$\frac{d\pi}{d\delta}(\delta = 0) = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \bar{q}}{\partial \delta}(\theta, 0)\,[\text{f.o.c. at } \delta = 0]\, f(\theta|I)\, d\theta + \frac{\partial \pi}{\partial I}\frac{dI}{d\delta} = \frac{\partial \pi}{\partial I}\frac{dI}{d\delta} > 0\,.$$

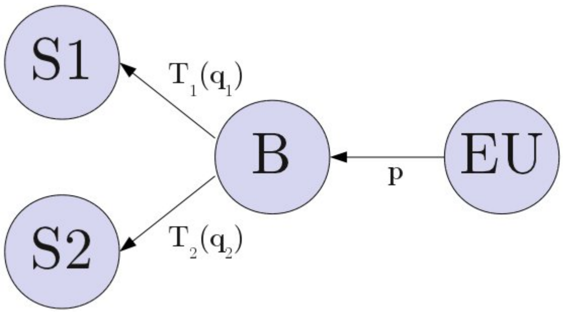
The first term of the expression is zero because of the envelope theorem, while the second part is strictly positive due to investment being strictly increasing in δ and the first-order stochastic dominance property. Hence there exists a small δ such that the corresponding allocation schedule strictly improves the seller's profits under commitment. As discussed before this can only be the case, when equilibrium investment has strictly increased. □

2.6 Competition

In the previous section we have seen that if contracting is not feasible before the investment takes place, investment will typically be distorted compared to the socially optimal level. Under more or less restrictive assumptions it was possible to show that investment is indeed downward distorted irrespective of investment being observable by the seller. One interesting question which remains is whether this effect will go away for a situation where there is competition between different sellers.

We add a second competing seller to the model and give the buyer the choice between buying from one of the sellers (*i.e.* we will assume exclusive dealing with one of the sellers).

This setup will not cover all envisioned real-world scenarios, but for example fits a setting where a content-provider has the choice between different backbone providers in order to serve the content-consumers.



In the literature price differentiation under competition has already been studied for example by Rochet and Stole (2002). In the following we will adopt their approach for our setting: We extend the buyer's type by one additional dimension $x \in [0,1]$, which will be interpreted as the location of the buyer relative to two sellers, called *left* and *right* seller respectively.

Assumption 6. *The distribution of the two dimensions θ and x of the type are assumed to be independent. The marginal distribution over x is log-concave be denoted by the cumulative distribution function L (where $L' = l$).*

Buying from the left seller necessitates a transportation cost of σx irrespective of the quantity, while buying from right seller costs $\sigma(1-x)$, where $\sigma > 0$ is the fixed marginal transportation cost per unit of distance from the respective seller. The sequence of events will therefore look like this:

1. The buyer chooses an investment level I to improve the value of the seller's output.
2. Nature assigns a type $(\theta, x) \in [\underline{\theta}, \overline{\theta}] \times [0,1]$ to the buyer according to the joint probability distribution function $f(\theta|I)l(x)$.
3. The left and the right seller choose simultaneously on which tariff $T_L(q)$ and $T_R(q)$ to offer to the buyer. The production function of both sellers exhibits a constant marginal cost c.
4. The buyer chooses which seller $S \in \{L, R\}$ she wants to buy from and on q, the quantity of the good to buy from the chosen seller.

Proposition 12. *Assume that the buyer's surplus function has the form* $V(\theta, q) = \theta q - \frac{1}{2}q^2$ *and assumption 6 holds. If* σ *is sufficiently small, so that all types of buyers* $((\theta, x) \in [\underline{\theta}, \overline{\theta}] \times [0, 1]$ *choose to buy positive quantities, the sellers offer a* cost plus fixed fee *tariff in the symmetric equilibrium, i.e.* $T_S = c + F_S$ *for* $S \in \{L, R\}$*. The fixed-fee does not depend on the anticipation on the buyer's investment, only on the marginal transportation cost* σ *and the distribution buyer's type on the location* $x \in [0, 1]$*. The marginal consumer is indifferent between buying from the left and the right seller, denoted by* $\hat{x} \in [0, 1]$*, is also independent of the investment level.*

Proof. See Rochet and Stole (2002). The independence of F_L, F_R and $\hat{x}$ from the investment level follows from the fact, that they only depend on the marginal distribution L over the location x of the types. □

This proposition yields a strikingly simple result, which unfortunately does not generalize to cases where competition is weak. Then the equilibrium (conditional on the anticipated investment level) takes a more complex form, which is in combination with ex-ante investment out of the scope of this paper. Another limitation is that due to the complexity of the dynamic programming problem Rochet and Stole (2002) restrict themselves to a simple functional form of the buyer's surplus.

Sticking to the assumptions in the proposition we continue to search for the equilibrium of the outlined game. In proposition 12 we have seen, that the equilibrium tariff charged by the sellers is independent of the investment level, which allows to take the sellers' behavior as given studying the investment decision on the buyer. This also implies that the observability of the investment level does not play a role in determining the equilibrium investment level.

Proposition 13. *If* σ *is sufficiently small as required in proposition 12 (i.e. competition is fierce enough), then the equilibrium exhibits socially optimal quantities traded in stage 4, and the socially optimal level of investment is chosen by the buyer in stage 1.*

Proof. Given σ is sufficiently small proposition 12 applies, the buyer chooses to consume irrespective of her type (θ, x). Irrespective of which seller she buys from, she consumes socially optimal units $q^{fb}(\theta)$, which maximizes $V(\theta, q) - cq$.

The choice which seller to buy from is however in general not efficient, *i.e.* $\hat{x}$ lies somewhere inside the unit interval. But for symmetric distributions on x, efficient choice of the seller arises (*i.e.* $\hat{x} = \frac{1}{2}$) (it might also happen for asymmetric distributions by chance). Due to the fact that investment does not change the equilibrium tariffs of the sellers, it also does not influence the expected equilibrium transportation plus access-cost:

$$C = \int_0^1 \min(F_L + \sigma x, F_R + \sigma(1-x))h(x)dx$$

Using this we easily see that the expected revenue from investing I in stage 1 is given by

$$\mathcal{W}(I) - C \,.$$

But then the socially optimal level I^{fb} will also maximize the buyer's profits, because the buyer's profit are given by joint surplus minus a constant - the expected transportation plus access-cost only, *i.e.*

$$I^{fb} = \arg\max_{I} \mathcal{W}(I) - C - I \,.$$

A border solution can be trivially ruled out, because it would imply that $\mathcal{W}(0) - C \leq \mathcal{W}(I^{fb}) - C - I^{fb} < 0$ which would be a contradiction to full-market coverage. □

2.7 First summary, policy recommendations

We have studied a situation in which a buyer might invest to improve her type before contracting and trade with the seller occurs. It turns out that due to the bargaining power being on the side of the seller the buyer's incentives to invest are lower than would be optimal for joint seller and buyer profits, as parts of the returns to investment are expropriated by the seller in the adverse-selection stage. Thus equilibrium investment in a pure strategy equilibrium is lower than its first-best level. Of course in a situation in which the seller might observe the buyer's type θ perfectly there would be no investment of the buyer at all. In order to show that equilibrium investment is lower when investment becomes observable, we need to put more structure on how investment affects the distribution of the buyer's types (decreasing hazard-rate assumption) and we have to assume that the first-order condition of the seller's problem fully ties down the quantity schedule proposed by the seller.

While the seller benefits from non-observability of investment, the buyer prefers the situation where investment is observable, which is because she might exploit her role of a Stackelberg leader in this game. Thus the seller prefers ex-ante not to be informed about the level of investment chosen by the buyer, which resembles results in Crémer (1995) pointed out in the introduction. The impact on joint surplus cannot be determined in general, but in the example - like in all cases where joint-surplus is concave in investment - there is an improvement in joint-surplus when investment is not observable. Suppose that the seller would be able to commit ex-ante to not observe investment, she would choose to make that commitment and thus bring investment closer to the first-best. If such a commitment is not feasible, one can achieve the same goal by regulation: Differing from our model sellers (ISPs) typically deal with a large number of buyers. Consider the limit case of a continuum of buyers. Then a regulative policy which allows for non-linear prices, but forces the seller to not discriminate between buyers, *i.e.* to propose the same (non-linear) tariff to all buyers, allows to attain the same level of investment as under non-observability. This is because there exists an equilibrium in which all buyers invest I_a and the deviation of a single buyer will not induce the seller to change the tariff.

Trivially ex-ante contracting solves the hold-up problem, but having to respect ex-post participation constraints for the buyer brings it back. Nevertheless it can be shown that the possibility for the seller to commit ex-ante to a certain non-linear tariff increases investment and the buyer's surplus.

The last part of this section hints at the fact that we may rely on competition to solve this hold-up problem irrespective of the observability of investment. However in the case of competing ISPs this result is likely not applicable, because competition mainly takes place only the other side of the market - attracting end-users. But for Internet-backbone-providers or content delivery networks selling their services to content-providers the assumption of fierce competition of the type studied seems very plausible.

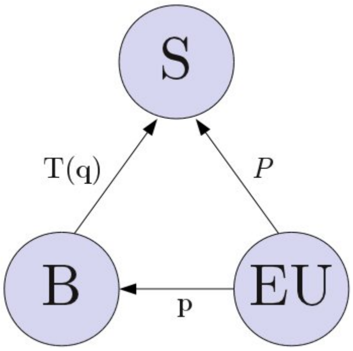
3 Two-sided market

In the discussion so far the relationship between the respective parties was very simple: There is a buyer using input from a monopoly seller. The buyer might consume the seller's product directly, or use it as an input to sell a product in a downstream market. However, in some of the applications mentioned in the introduction, the market structure might exhibit two-sided market features: Suppose that the seller is an access-provider (e.g. an ISP) to end-users, and the buyer (e.g. a content-provider) provides a service (paid or financed via advertisements) to these end-users. Then the seller might not only charge the buyer, but also the end user, and two-sided market effects *might* appear and thus have to be considered. To study situations like these, we extend the model to allow the seller to charge the end-user as well.

3.1 Monopoly

Consider a unit mass of end-users, each has unit demand. An end-user derives surplus from consuming one unit of the buyer's product (*eg.* access to her content library) given by $v + \theta$

where v is an iid draw from the distribution $G(v)$ on $[\underline{v}, \overline{v}]$ which is assumed to have a strictly increasing hazard-rate. Checking the conditions from the previous section 2 shows that $p(\theta, q) = G^{-1}(1-q) + \theta$ indeed fulfills the assumptions of the base model. To be able to get in contact with the buyer and to consume, the end-user has to connect the the seller's platform. We will restrict ourselves to the case where the seller might charge the end-user an access-fee $\mathcal{P}$, before the end-user learns her or the buyer's type. We consider this case for two reasons: On the one hand the description of the seller's problem will be very simple. On the other hand this is exactly what ISPs are doing currently in many cases. They charge monthly fees irrespective of the consumption pattern of the end-user, which may be due to significant risk-aversion of end-users.



These additional features change the timing of events[12]:

1. The buyer chooses an investment level $I \geq 0$.
2. Nature assigns a type θ to the buyer.
3. The seller offers a tariff $T(q)$ to the buyer and access-fee $\mathcal{P}$ to the end-user.
4. Each end-user decides whether to joint the seller's platform.
5. The buyer learns his type θ, and chooses quantity q observing the number of connected end-users, which induces the downstream price $p(\theta, q)$.
6. Nature assigns to each end-users independently a type v.
7. Each end-users learns the buyer's type θ and her own type v. Observing the price $p(\theta, q)$ each end-user decides whether to buy.

[12]shortened version to point out differences with previous setup

This last stage can be viewed as two distinct ones, in which the buyer first sets the downstream price and end-users decide whether to consume after having observed her own and the buyer's type as a second step. The presentation above merges both stages. Like previously the buyer's downstream profits will be denoted by

$$V(\theta, q) = qp(\theta, q) = qG^{-1}(1-q) + q\theta\,.$$

The first new feature to consider when solving the game is the question whether the end-users will join the platform or not. As there is no ex-ante heterogeneity between end-users the seller can extract the whole expected end-user surplus via the fixed fee. The end-users expected surplus given a buyer's type θ is given by

$$u(\theta, q) = \int_{p(\theta,q)-\theta}^{\overline{v}} [v + \theta - p(\theta, q)]\, g(v)\, dv\,.$$

Hence the seller maximizes

$$\int_{\underline{\theta}}^{\overline{\theta}} [T(q(\theta|I)) + u(\theta, q(\theta|I)) - cq(\theta|I)]\, f(\theta|I)\, d\theta$$

subject to the buyer's self-selection constraints. Define a modified surplus function

$$\overset{\triangle}{V}(\theta, q) := V(\theta, q) + u(\theta, q)$$

and observe that

$$\frac{\partial \overset{\triangle}{V}}{\partial \theta}(\theta, q) = \frac{\partial V}{\partial \theta}(\theta, q) \qquad \text{for all } (\theta, q)\,.$$

One can show that all assumptions required from the surplus function in the previous exposition are met by the modified surplus function $\overset{\triangle}{V}$. With this information and the usual trick of plugging the buyer's incentive constraint into the seller's profit function, we get that the seller optimizes the following expression:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\overset{\triangle}{V}(\theta, q(\theta|I)) - cq(\theta|I) \right) f(\theta|I) - \frac{\partial \overset{\triangle}{V}}{\partial \theta}(\theta, q(\theta|I))(1 - F(\theta|I)) \right] d\theta$$

This means that even in this situation the seller is faced with the same standard adverse-selection problem given an agent's surplus function $\overset{\triangle}{V}$. Therefore most of the results from section 2.1 follow by adapting the arguments from the base-model: There is under investment in both observability regimes (here joint surplus also includes end-user surplus, because it is part of the seller's profits). Observability and ex-ante commitment still have the same impact on equilibrium investment. Only the competition results are not applicable, which will be discussed in the following section 3.2.

On top of these results we can go a bit further and compare the situation where the seller may charge the end-users with one where she cannot.

Lemma 6. *Given the same anticipated level of investment I_a the seller offers the buyer a tariff which induces higher quantities compared to a situation in which the seller might not charge end-users. As a direct consequence each type of buyer is better-off given investment.*

Proof. We denote by $\overset{\triangle}{q}(\theta, I_a)$ the quantity schedule prevailing in the current setting given the anticipated level of investment. We carry out the usual revealed preference argument (see the proof of Lemma 3 for a reference) and use $V_\theta = \overset{\triangle}{V}_\theta$ to get:

$$\overset{\triangle}{V}(\theta, \overset{\triangle}{q}(\theta, I_a)) - V(\theta, \overset{\triangle}{q}(\theta, I_a)) \geq \overset{\triangle}{V}(\theta, q^*(\theta, I_a)) - V(\theta, q^*(\theta, I_a))$$

This is equivalent to

$$u(\theta, \overset{\triangle}{q}(\theta, I_a)) \geq u(\theta, q^*(\theta, I_a))\,.$$

But $u(\theta, q)$ is increasing in q because the end-users' inverse demand is falling in q:

$$\frac{\partial u}{\partial q}(\theta, q) = -\frac{\partial p}{\partial q}(\theta, q)\int_{p(\theta,q)-\theta}^{\overline{v}} g(v)dv = -q\frac{\partial p}{\partial q}(\theta, q) > 0$$

Hence $\overset{\triangle}{q}(\theta, I_a) \geq q^*(\theta, I_a)$. The strict inequality can be shown to hold by plugging into both first-order conditions, which leads to a contradiction in case equality would hold.

Let's call $\overset{\triangle}{U}(\theta, I_a)$ a buyer θ's surplus given anticipated investment. It can be computed analogously to the base-model setting with $\overset{\triangle}{q}$ playing the role of q^* in equation (3). From this equation it is straight-forward to see that a higher quantity schedule increases a buyer θ's surplus given the anticipated level of investment. □

Let us take a closer look at the allocation profile in the two-sided market setting given the anticipated level of investment:

$$\frac{\partial V}{\partial q}(\theta, \overset{\triangle}{q}(\theta, I_a)) = c + \frac{1}{h(\theta|I_a)}\frac{\partial^2 V}{\partial q \partial \theta}(\theta, \overset{\triangle}{q}(\theta, I_a)) - \frac{\partial u}{\partial q}(\theta, \overset{\triangle}{q}(\theta, I_a))$$

Compared to the first-order condition of the monopoly input provider case (1) we have an additional negative term on the left-hand side which captures the increase in end-user surplus due to an increase in quantities. As we have seen just before optimal quantities will be higher than under the monopoly input provider case. This is to partially compensate for the downstream inefficiency introduced by the buyer.

Further note that the seller is charging below marginal cost for type $\overline{\theta}$ (and for types close to $\overline{\theta}$), *i.e.* the seller is willing to make a marginal loss on the buyer side in order to extract more profits from the end-user side. For low types there is the conventional rent-extraction motive and the quantity upwards distortion in order to increase end-user surplus, whose net effect depends on parameter values.

Proposition 14. *Suppose assumption 5 holds, then under unobservable investment the buyer will invest more when the seller is able to charge end-users.*

Proof. Denote by $\overset{\triangle}{I}{}^*$ an equilibrium investment level under non-observable investment when the seller is able to charge end-users. Again a simple revealed preference argument like in the proof of Lemma 5 leads to the following inequality:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[U(\theta, I^*) - \overset{\triangle}{U}(\theta, \overset{\triangle}{I}{}^*) \right] \left[f(\theta | I^*) - f(\theta | \overset{\triangle}{I}{}^*) \right] d\theta \geq 0 \tag{7}$$

Consider the first derivative with respect to θ of the first term in square brackets. It can be written like this:

$$\begin{aligned} \frac{\partial}{\partial \theta} \left[U(\theta, I^*) - \overset{\triangle}{U}(\theta, \overset{\triangle}{I}{}^*) \right] &= \frac{\partial V}{\partial \theta}(\theta, q^*(\theta, I^*)) - \frac{\partial V}{\partial \theta}(\theta, \overset{\triangle}{q}(\theta, \overset{\triangle}{I}{}^*)) \\ &= \left[\frac{\partial V}{\partial \theta}(\theta, q^*(\theta, I^*)) - \frac{\partial V}{\partial \theta}(\theta, \overset{\triangle}{q}(\theta, I^*)) \right] + \left[\frac{\partial V}{\partial \theta}(\theta, \overset{\triangle}{q}(\theta, I^*)) - \frac{\partial V}{\partial \theta}(\theta, \overset{\triangle}{q}(\theta, \overset{\triangle}{I}{}^*)) \right] \end{aligned}$$

The first term in square brackets is negative due to the quantity differential given investment. Now suppose by contradiction that $I^* > \overset{\triangle}{I}{}^*$. Hence the second-term is negative as well, which is why the first term in square brackets in inequality (7) is decreasing in θ due to lemma 3. Therefore inequality (7) has to hold strictly in the opposite direction due to the first-order stochastic dominance property. This is the desired contradiction. Thus $I^* \leq \overset{\triangle}{I}{}^*$ has to hold. The strictness of inequality can be shown by the usual first-order condition argument. □

It would be natural to try to find a similar result under observable investment. But although the rent-effect gets stronger due to the seller charging the end-user, the impact on the strategic-effect is not clear. So it cannot be ruled out that investment might indeed fall in this situation compared to the monopoly input seller case.

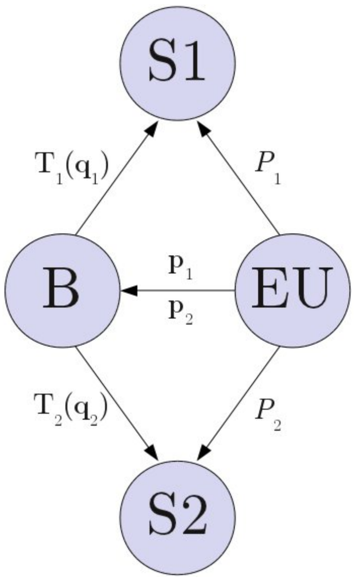
Corollary 15. *Giving the seller the possibility to also charge the end-users improves the seller's surplus under non-observable investment.*

Proof. Not charging the end-users and charging the buyer the original tariff under non-observable investment is still an admissible strategy. A deviation from it will only be carried out if it is strictly beneficial to the seller. □

We have seen that giving the seller the opportunity to also charge the end-users is favorable in order to promote investment. But the underinvestment results from the base-model are still valid. Only ex-ante contracting - if feasible - can solve the hold-up. In the next section we study if competition might be a valuable tool in this setting as well.

3.2 Competitive bottleneck

In the base-model fierce competition allows to get rid of the (downward) distortion of the investment level. In the competition regime of section 2.6 the sellers compete for an input needed to serve the buyer's end-users. In this setting the buyer needs a seller in order to connect to the end-users of the respective seller. Suppose again, that there are two sellers $i = 1, 2$ present (a generalization to n sellers is straight-forward), and end-users single-home, while the buyer multi-homes. Sellers cannot cooperate and the only admissible mechanism is to charge a non-linear tariff to the buyer, exclusive dealing is assumed to be not possible. It is useful to visualize the situation first:



Compared to the situation before the timing of events basically stays the same augmented only for the competition between sellers: In stage 3 both sellers set their tariffs and access-fees simultaneously. In the following step 4 the end-users decide on whether to connect and to which platform and finally in step 5 the buyer decides on the quantities on both seller's platforms. In order to simplify the notation we drop investment from the distribution function and other variables.

Let $x_i \in [0,1]$ be the fraction of end-users joining platform i. First note that end-user inverse demand is - up to scaling for the market share - unchanged from the section before: $p(\theta, \frac{q_i}{x_i})$, where we use a shorthand notation: $\frac{q_i}{x_i} = \frac{q_i(\theta, x_i)}{x_i}$. Given the quantity schedule $q_i(\cdot)$ offered by the seller to the buyer an end-user's expected surplus from joining seller i is denoted by $u_i^e(q_i(\cdot), x_i)$, which is the expectation over

$$u(\theta, \frac{q_i}{x_i}) = \int_{p(\theta, \frac{q}{x_i})-\theta}^{\overline{v}} [v + \theta - p(\theta, \frac{q_i}{x_i})]\, g(v)\, dv\,.$$

We further note that the buyer's surplus from serving end-users on seller i's platform is given by $x_i V(\frac{q_i}{x_i}, \theta)$ because $V(\frac{q_i}{x_i}, \theta) = \frac{q_i}{x_i} p(\frac{q_i}{x_i}, \theta)$. V and p are the buyer's profits and end-users downstream inverse demand from the previous section.

We denote by $\mathcal{P}_i$ the access-fee for end-users charged by the seller joining platform i, and by $\bar{u}_i$ the expected net-surplus from joining seller i's platform. Sellers compete to attract end-users. Based on expected net-surplus from joining the sellers' platforms end-users decide on which platform to join:

$$x_i \in \phi_i(\bar{u}_i, \bar{u}_j) \tag{8}$$

The function ϕ pins down the substitutability of the two seller's platforms. For the case of perfect competition ϕ is given by

$$\begin{aligned} \phi_i &= \{0\} && \text{if } \bar{u}_i < \bar{u}_j \\ \phi_i &= [0,1] && \text{if } \bar{u}_i = \bar{u}_j \\ \phi_i &= \{1\} && \text{otherwise}\,. \end{aligned}$$

Given this we can state seller i's problem. She maximizes

$$x_i \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(V(\theta, \frac{q_i}{x_i}) - c\frac{q_i}{x_i} \right) f(\theta) - \frac{\partial V}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta + \mathcal{P}_i \right\},$$

subject to equation (8), and where expected net end-user surplus is given by

$$\bar{u}_i = u_i^e(q_i(\cdot), x_i) - \mathcal{P}_i\,. \tag{9}$$

The solution strategy is similar to the insulated equilibrium approach proposed in White and Weyl (2010). When thinking about their optimal strategy sellers condition their tariffs charged to the buyers on the number of end-users having joined their respective platform (but note that we do not impose this behavior on sellers). Hence we get a dichotomy: Monopoly behavior on the buyer side, and competition on the end-user side:

Proposition 16. *Under perfect competition the sellers charge the same tariff (only scaled for relative platform size) like under a monopoly as discussed in the previous section 3.1, which is based on the allocations $x_i\overset{\triangle}{q}(\theta)$. Furthermore the sellers do not make any profit and use all profits from trade with the buyer to attract end-users to their respective platform.*

Proof. First we solve equation (9) for $\mathcal{P}_i$, which allows to rewrite seller i's profit as

$$x_i \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(V(\theta, \frac{q_i}{x_i}) + u_i(\theta, \frac{q_i}{x_i}) - c\frac{q_i}{x_i} \right) f(\theta) - \frac{\partial V}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta - \bar{u}_i \right\} .$$

Recalling $\overset{\triangle}{V} = V + u$ and $\overset{\triangle}{V}_\theta = V_\theta$ we can write seller i's profit as

$$x_i \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\overset{\triangle}{V}(\theta, \frac{q_i}{x_i}) - c\frac{q_i}{x_i} \right) f(\theta) - \frac{\partial \overset{\triangle}{V}}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta - \bar{u}_i \right\} .$$

From this expression one can see that the choice of the allocation profile or tariff scaled for market share x_i is independent of the choice of the optimal net surplus to be left to end-users. It is also independent of what the other seller is doing, *i.e.* the surplus $\bar{u}_j$ left to end-users by seller j, and therefore also independent of the actual market share x_i. The seller's profit from trade with the buyer looks the same up to scaling as the seller's profit in the monopoly case. Hence the optimal quantity schedule has to be a scaled version of the optimal quantity schedule from the monopoly case: $x_i\overset{\triangle}{q}(\theta)$. Summing over both sellers it is already clear that the buyer is faced with the same situation as under the monopoly: $\overset{\triangle}{q}(\theta) = x_i\overset{\triangle}{q}(\theta) + x_j\overset{\triangle}{q}(\theta)$.

Given this behavior of the sellers vis-à-vis the buyer we can study the competition in net end-user surplus. First denote by $x_i\pi_B$ the profit each seller makes from trade with the buyer. With this notation and by using equation (8) seller i's profit is given by

$$\phi_i(\bar{u}_i, \bar{u}_j) \left\{ \pi_B + u_i - \bar{u}_i \right\} .$$

Note that $\pi_B + u_i$ are constant with respect to the surplus $\bar{u}_i$ promised to end-users. Thus the only equilibrium of this game can be both sellers offering all the surplus generated to the end-users: $\bar{u}_i = \pi_B + u_i$. Suppose not, then $\pi_B + u_i > \bar{u}_i$ for some i. Promising $\bar{u}_i$ can only be optimal if any end-users are attracted, hence $\pi_B + u_j > \bar{u}_j$ must hold too. But this cannot be optimal for seller $2 - i$, because she could attract all end-users by just undercutting seller i, which is a contradiction to assertion.

Finally, using the definition of $\bar{u}_i$ we can recuperate the access-fee

$$\mathcal{P}_i = u_i - \bar{u}_i = -\pi_B .$$

□

The intuition is quite simple: Due to competition each seller wants to offer maximum net surplus to end-users. Thus she maximizes her profits plus the end-users' surplus and redistributes her profits to the end-user via the access-fee. The access-fee can be positive or negative. In situations where the sellers are willing to make a loss from trade with the buyer in order to boost end-user surplus, access-fees will be positive (in order to recouperate the loss on the buyer side). When sellers make a profit in equilibrium out of dealing with the buyer, they redistribute their profits to the end-users via a negative access-fee (See appendix A.2 for the treatment of the case when negative access-fees are infeasible). In fact the result even holds for other forms of competition, *eg.* for competition à la Hotelling. The only change is that sellers will not transfer all surplus to the end-users and thus will end up with positive profits.

For the discussion on the hold-up problem this result is disappointing. Contrary to the monopoly input provider case, even very fierce competition between sellers (no differentiation between sellers, same cost structure) cannot establish first-best investment. The only effect of competition is that the sellers are not capable anymore to extract end-user surplus, the whole sellers' surplus goes to the end-users instead.

3.3 Ex-ante commitment

Another possibility to soothe the hold-up problem discussed in the monopoly input provider case, was ex-ante commitment by the seller. In the monopoly case we know that the result from the base-model (section 2.5) applies, *i.e.* that commitment will lead to a higher level of investment. Due to the fact that there are many competing ISPs (there are thousands of ISPs worldwide), common agency problems will arise. Studying commitment in this situation with a finite number of competing sellers is quite tricky. For the limit case of an unit mass of sellers competing for the buyer it is however trivial: Commitment by a single seller to a tariff trying to induce more investment is in vain, because she has zero mass in the buyer's investment considerations. A more general analysis of this case is left for future research, but it seems quite safe to conjecture that free-riding between sellers will pose a problem for a finite number of sellers too.

4 Conclusions

What are the implications of the paper for the net-neutrality debate? Content-delivery networks and backbone providers are already free to charge content-providers today. As long there is enough competition amongst these input providers, investment by content-providers will not be harmed due to a hold-up. But this result does not apply in the two-sided market case. The hold-up problem is immune to competition in this setting. This means that even fierce competition between ISPs in order to attract end-users does not influence the severity of the hold-up. Ex-ante commitment of ISPs to charge a certain tariff before investment takes place still works in the monopoly case, but as there are thousands of IPSs worldwide the free-riding problem between ISPs will significantly limit its effectiveness. The punchline is that in a two-sided market of the type outlined in the paper the hold-up problem seems to be pervasive.

Of course this paper only studies a small part of a large set of issues around net-neutrality, and thus does not provide final insights into this debate. But the framework of the model is general enough to be useful in future research, *eg.* to identify regulatory measures to curb the negative impact on investment of access-providers charging content-providers. Furthermore investment incentives for access-providers to invest in the quality of their networks should be brought into the picture as well. Another question which could be studied with only minor modifications to the present model is how the two-sided tariff structure might influence content-providers' efforts to serve their content more efficiently, *i.e.* in a way which uses less bandwidth and hence lowers the access-provider's costs. This question might be very important in the context of mobile access-providers, who are faced by a more or less fixed total bandwidth for all end-users in a certain area given the transmission technology used.

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A Appendix

A.1 Discussion of the assumptions on the distribution over types

Throughout the paper we impose restrictions on the distribution function over the types and the impact of the investment level on it:

1. Monotone hazard-rate (w.r.t. types): Assumption 1
2. First-order stochastic-dominance (w.r.t. investment): Assumption 1
3. Decreasing hazard-rate in investment: Assumption 5

These assumptions are commonly used and have economic justifications. But the question arises how restrictive they are when invoked at the same time. It is well known that assumption 5 about the decreasing hazard-rate in investment implies first-order stochastic-dominance with respect to investment (i.e. assumption 1). One way to proceed in order to see how the other assumptions fit together is to use the relationship between the hazard-rate and distribution function,

$$F(\theta, I) = 1 - e^{-\int_{\underline{\theta}}^{\theta} h(t|I)\, dt},$$

choose a hazard rate function in line with the desired properties and compute the distribution function.

An alternative - probably more intuitive - construction procedure for a rich class of distributions in line with all mentioned assumptions starts from a uni-modal distribution over a finite or infinite interval I with a non-decreasing hazard-rate and call the respective distribution function $\tilde{F}(t)$ and the corresponding hazard-rate by $\tilde{h}(t)$ for $t \in I$. Furthermore we have to choose an (smooth) function $m(\theta, I)$, which is increasing in θ and maps onto I for any $I \geq 0$. Now consider the following distribution function:

$$F(\theta, I) := \tilde{F}\left(m(\theta, I)\right)$$

Denote by f the respective density function and by h its hazard-rate. To find a set of sufficient restrictions on m such that this distribution adheres to the properties above, we compute the relevant terms:

Property	Needed sign	Sufficient condition(s)
$F_I = \tilde{F}'\, m_I$	< 0	$m_I < 0$ (also necessary!)
$h_\theta = \tilde{h}'(m_\theta)^2 + \tilde{h}\, m_{\theta\theta}$	≥ 0	$m_{\theta\theta} \geq 0$
$h_I = \tilde{h}'\, m_\theta\, m_I + \tilde{h}\, m_{I\theta}$	< 0	$m_{I\theta} < 0$

Example. *Take any distribution on $\mathcal{R}^+$ with a non-decreasing hazard-rate, like the exponential or for certain parameter values the Weibull distribution, and consider this function:*

$$m(\theta, I) = \frac{\ln\left(\frac{\overline{\theta}-\theta}{\overline{\theta}-\underline{\theta}}\right)}{I+1}$$

It maps onto $\mathcal{R}^+$ for any non-negative value of I. Furthermore all derivatives fulfill the conditions outlined above:

$$m_I, m_{I\theta} < 0 < m_\theta, m_{\theta\theta}$$

Thus, $F(\theta, I) := \tilde{F}(m(\theta, I))$ is a distribution function with the desired properties. Plugging in for the exponential distribution (for any parameter $\beta > 0$) yields the following result:

$$\begin{aligned} F(\theta, I) &= 1 - \left(\frac{\overline{\theta}-\theta}{\overline{\theta}-\underline{\theta}}\right)^{\frac{1}{\beta(1+I)}} \\ h(\theta, I) &= \frac{1}{\beta(1+I)(\overline{\theta}-\theta)} \end{aligned}$$

It is obvious, that the two monotone-hazard rate assumptions and the first-order stochastic-dominance property are fulfilled.

When we leave the case of a finite interval for θ there are even simpler examples. Start with a random variable t on $[0, \infty]$ with a hazard rate $h(t)$ strictly increasing in t. Then consider $\theta = t + I$. It's hazard rate is $h(\theta - I)$ which is clearly increasing in θ and decreasing in I.

Up to now I neglected the convex distribution function assumption which is used in proofing that the buyer's reaction function is single-valued. Distributions fulfilling this assumption and the two monotone hazard-rate assumptions seem to be quite rare. Two examples for such distributions are given in Spaeter (1998):

$$\begin{aligned} F(\theta, I) &= \left[\frac{\overline{\theta}-\theta}{\beta(1+I)\overline{\theta}} + 1\right]\frac{\theta}{\overline{\theta}} \\ G(\theta, I) &= (I+k)^{\overline{\theta}-\theta}\frac{\theta-\underline{\theta}}{\overline{\theta}-\underline{\theta}} \qquad \text{for } k > 1 \end{aligned}$$

A.2 Competitive bottleneck with non-negative access fees

We have seen in section 3.2 that in equilibrium competition might lead sellers to attract end-users via a negative access-fee. There might be cases however, where negative access-fees may not be feasible, *eg.* because of the presence of end-users who are not interested in trading with the buyer anyway, but just want to cash in the negative access-fee. Consider that such a non-negativity constraint on access-fees is in place and binding. Using the notation from section 3.2 we can write seller i's profits as

$$\phi_i(u_i^e(q_i(\cdot), x_i), u_j^e(q_j(\cdot), x_j)) \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(V(\theta, \frac{q_i}{x_i}) - c\frac{q_i}{x_i} \right) f(\theta) - \frac{\partial V}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta \right\}.$$

We first observe that under perfect competition both sellers make zero profits (from dealing with the buyer). Suppose both make positive profits, then a seller with a market share below 1 could increase the quantity schedule a little bit in order to increase the end-users expected surplus, and therefore attract all end-users which increases profits. Now suppose that only one seller makes positive profits, then the other seller could mimic the profitable seller but increase the quantity schedule a little bit, hence attract all end-users, and thus make a positive profit.

Clearly a seller would just exit the market if she made negative profits. Hence in equilibrium both sellers will maximize end-user surplus under a zero-profit condition:

$$\max \int_{\underline{\theta}}^{\overline{\theta}} u(\theta, \frac{q_i}{x_i}) f(\theta)\, d\theta$$
$$s.t. \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(V(\theta, \frac{q_i}{x_i}) - c\frac{q_i}{x_i} \right) f(\theta) - \frac{\partial V}{\partial \theta}(\theta, \frac{q_i}{x_i})(1 - F(\theta)) \right] d\theta = 0$$

Denoting the Lagrange multiplier of the constraint by λ we get the following necessary first-order condition:

$$\frac{\partial V}{\partial q}(\theta, \frac{q_i}{x_i}) = c + \frac{1}{h(\theta)} \frac{\partial^2 V}{\partial q \partial \theta}(\theta, \frac{q_i}{x_i}) - \frac{1}{\lambda} \frac{\partial u_i}{\partial q}(\theta, \frac{q_i}{x_i})$$

Notice that for $\lambda = 1$ this is the first-order condition of the monopoly seller's problem. By assumption we are in the case where the monopoly seller's profit from trade with the buyer is positive. Hence $\lambda < 1$ has to hold. Suppose not, i.e. $\lambda \geq 1$, then by a revealed preference argument quantities would drop for all θ compared to the monopoly case, and therefore end-user surplus would decrease as well. But this cannot be true in equilibrium, because charging the (scaled) optimal tariff from the monopoly case would clearly be a profitable deviation.

Given that $\lambda < 1$ in equilibrium quantities can be shown to be larger than in the monopoly case by revealed preference. Hence we get the following result:

Lemma 7. *For a given anticipated investment level I_a the seller offers the buyer a tariff inducing higher quantities compared to a situation in which accees-fees are not restricted to be positive.*

The proof uses the same technique as in lemma 6. Using this lemma in another revealed preference argument like in the proof of proposition 14 we get:

Proposition 17. *Suppose assumption 5 holds and that the non-negativity constraint on the access-fee binds, then under unobservable investment the buyer will invest more under competition between sellers than in a monopoly.*

When the non-negativity constraints are binding, sellers compete to attract end-users by offering better expected trades with the buyer by raising the quantity schedules to a point, where the seller's profits get zero. This higher quantity schedule in turn induces buyers to invest more in equilibrium than under the monopoly case (under non-observable investment). This is in contrast to the main result that competition does not change the equilibrium investment behavior of the buyers when there are no restrictions on the access-fee.

A.3 Linear pricing

We have seen in the first part of the paper that the optimal tariff for the seller is non-linear, which is driven by the seller's wish to extract as much surplus from the buyer as possible constraint by incentive compatibility. Now we trace out what happens if the seller is restricted to (*eg.* forced by regulation) charge a linear tariff $T(q) = pq$ in stage 3. It turns out that the problem, while technically simpler at the first sight, is harder to analyze and it is more difficult to draw conclusive results. The impact of observability on investment relies on even stricter assumptions on the demand side of the problem.

Furthermore it would be interesting to see the impact of a potential regulatory constraint on the seller to charge only a linear price on the investment level and surplus. Unfortunately it is feasible only in concrete examples to derive results. This is related to the more general ambiguity of price-discrimination on buyer and total surplus[13], which is only further aggravated by the ex-ante investment stage present in this paper.

A.3.1 Unobservable investment

Let us start again with the case that the seller does not observe the level of investment sunk by the buyer before choosing the optimal price. Again the problem is solved by backwards induction: Given a per-unit price p in step 4 a buyer of type θ maximizes her surplus:

$$\max_q V(\theta, q) - pq$$

[13]Trivially seller surplus will fall due to the additional constraint on its pricing behavior.

A sufficient condition ($V(\theta,q)$ is strictly concave in q) for the optimal $q(\theta,p)$ is given by the first-order condition (to simplify this section, assume that there are no border solutions)

$$\frac{\partial V}{\partial q}(\theta,\overline{q}^*(\theta,p)) = p\,. \tag{10}$$

Using the single-crossing condition we get that chosen quantities are decreasing in price and increasing in type:

$$\frac{\partial \overline{q}^*}{\partial p}(\theta,p) < 0 < \frac{\partial \overline{q}^*}{\partial \theta}(\theta,p)$$

We can compute buyer θ's surplus with the help of the envelope theorem

$$\overline{U}(\theta,p) = \int_{\underline{\theta}}^{\theta} \frac{\partial V}{\partial \theta}(\theta,\overline{q}^*(t,p))\,dt + \overline{U}(\underline{\theta},p) \tag{11}$$

where buyer $\underline{\theta}$'s surplus is given by

$$\overline{U}(\underline{\theta},p) = V(\underline{\theta},\overline{q}^*(\underline{\theta},p)) - p\overline{q}^*(\underline{\theta},p)\,.$$

With the help of these expressions we can write the buyer's expected surplus as follows:

$$\overline{\mathcal{U}}(I,p) = \int_{\underline{\theta}}^{\overline{\theta}} \overline{U}(\theta,p) f(\theta|I)\,d\theta$$

Given the quantity choices of all buyer's types (or all buyer's potential types) the seller maximizes profits in stage 3 given anticipated investment I:

$$\max_p \pi(p|I) = \max_p\,(p-c)\int_{\underline{\theta}}^{\overline{\theta}} \overline{q}^*(\theta,p) f(\theta|I)\,d\theta \tag{12}$$

The optimal price $\mathfrak{p}(I)$ has to fulfill the first-order condition:

$$\frac{\partial \pi}{\partial p}(\mathfrak{p}(I)|I) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\overline{q}^*(\theta,\mathfrak{p}(I)) + (\mathfrak{p}(I)-c)\frac{\partial \overline{q}^*}{\partial p}(\theta,\mathfrak{p}(I))\right] f(\theta|I)\,d\theta = 0 \tag{13}$$

or equivalently

$$\frac{\mathfrak{p}(I)-c}{\mathfrak{p}(I)} = -\frac{\int_{\underline{\theta}}^{\overline{\theta}} \overline{q}^*(\theta,\mathfrak{p}(I)) f(\theta|I)\,d\theta}{\mathfrak{p}(I)\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \overline{q}^*}{\partial p}(\theta,\mathfrak{p}(I)) f(\theta|I)\,d\theta}\,. \tag{14}$$

In general it is not possible to tell how the seller anticipation of the buyer's investment in step 1 influences the price. This depends on how investment affects the elasticity of (expected) demand. This relationship can be very complex, because it depends on the interaction between the demand of single buyer types, the distribution of types and the impact of investment on this distribution. The pure-strategy equilibrium investment level and price (conjecturing that such an equilibrium indeed exists) are called $\overline{I}^*$ and p^*.

A.3.2 Observable investment

Now suppose that the seller observes the buyer's investment when choosing the price level. We first note that the quantity choice of the buyer in step 4 stays the same as in the previous section: The quantity bought still is $q(\theta, p)$ - see equation (10). The seller's reaction function $\mathfrak{p}(I)$ also stays the same, with I being the observed level of investment (instead of the anticipated one). Thus the objective function of the buyer in stage 1 looks like this:

$$\int_{\underline{\theta}}^{\overline{\theta}} \overline{U}(\theta, \mathfrak{p}(I)) f(\theta|I)\, d\theta - I$$

The existence of a pure strategy equilibrium is trivial, like in the case of non-linear tariffs and observable investment. We denote an equilibrium investment level by $\overline{I}^{**}$ and the corresponding price level as $p^{**} = \mathfrak{p}(\overline{I}^{**})$.

If we employ the usual revealed preference argument

$$\begin{aligned} \overline{\mathcal{U}}(\overline{I}^{*}, p^{*}) - \overline{I}^{*} &\geq \overline{\mathcal{U}}(\overline{I}^{**}, p^{*}) - \overline{I}^{**} \\ \overline{\mathcal{U}}(\overline{I}^{**}, p^{**}) - \overline{I}^{**} &\geq \overline{\mathcal{U}}(\overline{I}^{*}, p^{*}) - \overline{I}^{*}, \end{aligned}$$

add up both inequalities, plug in and rearrange, we get:

$$\int_{\underline{\theta}}^{\overline{\theta}} [\overline{U}(\theta, p^{**}) - \overline{U}(\theta, p^{*})]\, f(\theta|\overline{I}^{**})\, d\theta \geq 0$$

This inequality can only hold if $p^{**} \leq p^{*}$, *i.e.* the equilibrium price charged by the seller drops when investment is observable. Whether investment drops or rises depends on the seller's reaction function. If it is increasing for all (anticipated) investment levels, *i.e.* $\mathfrak{p}'(I) > 0$, we get that observability leads to a drop in investment like in the case where second-degree price discrimination was admissible. If not, we may even see investment rise (*eg.* when the seller's reaction function is decreasing in relevant levels of (anticipated) investment).

A.3.3 Summary

This is only a very brief sketch of the linear-prices case (very similar results hold for the two-part tariff case). Irrespective of the observability of investment one can easily show that underinvestment compared to first-best will prevail, because the price chosen by the seller in equilibrium is trivially larger than marginal cost. Thus an inefficiently low quantity is traded, which in turn leads to inefficiently low investment. However, the effect of observability on investment depends on the elasticity of aggregate (or expected) demand, which cannot be trivially be traced back to simple assumptions on buyer's surplus and the distribution of types. This is in stark contrast to the case of second-degree price discrimination, where the influence of observability hinges only on an assumption on the impact of investment on the distribution of the buyer's type (assumption 5). Thus, to assess the impact of observability requires much more detailed information on the demand structure under linear-prices, than under second-degree price discrimination.