Patents in a Model of Growth with Persistent Leadership

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Abstract

This paper analyzes the effects of patent policies in a quality - ladder model of growth where incumbent firms preemptively innovate in order to keep their position of leadership. The amount of R&D undertaken by leaders increases if an innovation becomes more valuable to an entrant and policies that make it easier to replace incumbents and to obtain considerable market power right upon entry increase growth. I show that making patent policies conditional on whether an innovation is made by an entrant or an incumbent can increase growth and also analyze the effects of conditioning the strength of patent protection on the size of the lead. In certain cases, an intermediate probability of patent enforcement leads to the highest average rate of growth.

1 Introduction

In many industries innovation continuously improves the quality or reduces the costs of existing goods, implying a process of creative destruction where old innovations are displaced by new ones. Given that patents are used to stimulate innovation, the specific design of the patent system can have important consequences

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for the incentives to innovate in such a context. O ´Donoghue and Zweimüller (2004) study the effects of patent policies in a quality-ladder model of growth, assuming that innovations are always carried out by entrants. In reality, innovations are however often carried out by incumbent firms implying that these incumbents stay the industry leaders for a sustained period of time[1]. In this paper I analyze the effects of patent policies in a quality-ladder model with persistent leadership where incumbents innovate preemptively. The basic model that I use is a simplified version of Denicolò (2001) who again builds on the seminal work of Gilbert and Newbery (1982) but does not analyze patent policy. I assume that an incumbent firm that has done two successive innovations (is two steps ahead) can earn larger profits than an entrant who has to compete with the previous incumbent in the product market. However, the incumbent has less stand - alone innovation incentives than the entrant due to the Arrow replacement effect. I model this in the simplest possible way by assuming that increasing the lead beyond two steps does not lead to larger profits so that a monopolistic R&D firm does not have any incentives to continue innovating after it has reached a two-step lead (in *Appendix 2* I look at a more general case and find that the main results are robust). All firms have access to the same R&D technology, which is characterized by decreasing returns at the industry level. There is free entry into the R&D sector and the condition that expected profits of entrants must be zero pins down the equilibrium rate of innovation. As entrants value entry (which brings a one-step lead) less than leaders value not being replaced and keeping their (two-step) lead, all R&D is carried out by leaders (who are assumed to move first). However, the amount of R&D that leaders undertake to prevent replacement depends on the value of an innovation for an entrant expecting to become the new leader upon entry. Because of that, the effects of patent policy on growth mainly stem from the effect they have on the value of an innovation for an entrant (which is hypothetical, as entry does not occur in equilibrium).

If entrants have to pay fees upon entry in order to compensate the previous innovator that they displace and on the technology of whom their innovation builds,

[1]Malerba and Orsenigo (1999) find that between 1978 and 1991, the percentage of patents granted to firms that had already innovated within their sector was 70% in Germany, 60% in France, 57% in the UK, 39% in Italy and 68% in the USA.

this reduces the value of an innovation for them and therefore entry pressure and growth as the R&D effort that the incumbent needs to exert in order to prevent entry is reduced. On the contrary, allowing voluntary deals like price collusion between entrants and incumbents that allow them to avoid the phase of competition increases the value of an innovation for entrants and growth. Equilibrium growth is maximal if a "patent transfer scheme" is implemented which forces incumbents to hand all their patents over to an entrant who has improved upon their technology. While all these results also hold in a context where innovations are carried out by entrants and where there is leapfrogging, there are also differences between the cases of persistent leadership and leapfrogging:

O ´Donoghue and Zweimüller (2004) show that making new innovations infringe on the patents of previous innovators ("leading breadth") can stimulate growth by allowing firms to consolidate market power (if that is not possible without leading breadth) in the case of leapfrogging. In the case of persistent leadership, the incumbent however already enjoys the maximal possible market power so that leading breadth mainly acts as a barrier to entry unless it allows both firms to considerably reduce total R&D costs.

In the leapfrogging model, introducing a patentability requirement (a minimal inventive step) can increase growth and welfare by avoiding an excessive rate of turnover, low markups and inefficiently small innovative steps. In the case of persistent leadership, a patentability requirement can still be useful if it is imposed on the incumbent, but if it is also imposed on entrants it decreases entry pressure and the amount of R&D undertaken by incumbents.

In the case where in order to make an innovation, two R&D stages have to be completed where the first consists in discovering an intermediate R&D input, growth is maximal if entrants are allowed to patent the intermediate R&D input while incumbents are not. The reason for this is that such a policy prevents incumbents from blocking follow-on R&D by entrants but still makes them race to invent the R&D input in order to prevent that an entrant patents it.

If entrants have to incur some fixed (catch-up) costs before they can use the state of the art R&D technology, no entrant ever enters the R&D sector as they expect that upon entry the incumbent will undertake the preemptive amount of

R&D so that they cannot earn any profits by doing R&D themselves anymore. In this case, incumbents need not do any R&D in order to preempt entry and equilibrium growth is zero in the case of full patent protection. If patents however expire with a positive probability so that incumbents are replaced from time to time, firms again have incentives to innovate in order to obtain a lead over their rivals. Therefore, average growth is maximized for an intermediate probability of patent protection. This is also the case in other scenarios where incumbents can prevent entry without innovating themselves, like for example if they can make ex ante agreements with potential entrants in order to reduce R&D spending or if they can increase the R&D costs of entrants by hiring a certain number of researchers but can make these researchers do other things than R&D instead.

While in the main case the level of preemptive R&D exerted by leaders does not depend on their lead, this might not be the case if the probability of patent protection is state dependent. If patents expire sufficiently more quickly in the case of a one-step than in the case of a two-step lead, firms with a one-step lead do more R&D than firms with a two-step lead as being two steps ahead becomes relatively more attractive. While increasing the probability of patent expiration reduces growth in the main case, increasing the probability of expiration for firms with a one-step lead can increase average growth for a certain range of parameters. However, given firms with a one-step lead do more R&D than firms with a two-step lead, increasing the rate of patent expiration for firms with a two-step lead cannot increase average growth by making a one-step lead more likely. The reason for this is that increasing the probability of expiration decreases the value of an innovation for an entrant and therefore entry pressure and the incentives for firms with a two (or zero –) step lead to innovate and this negative effect on R&D overcompensates the positive effect stemming from a higher probability of being in the state with a one-step lead.

The analysis is similar in the case of a fixed entry fee: while increasing such a fee (that has to be paid by entrants once they enter the product market) decreases entry pressure and R&D effort of firms with a two-step lead, it can induce firms with a one-step lead to race faster as obtaining a two-step lead becomes relatively more attractive.

In *Appendix 2* I look at a more general model where even incumbents can increase their profit flows by innovating and where incumbents with a two-step lead charge the unconstrained monopoly price while firms with a one-step lead have to engage in limit pricing (this is the setup of Denicolò (2001)). I find that the effects of patent policies are mainly the same and moreover show that in the case where equilibrium growth is excessive, it is better to reduce innovation incentives by introducing a price cap than by making profit flows more backloaded because monopoly distortions are only reduced in the first case.

2 Related literature

In most quality-ladder growth models (like Aghion and Howitt (1992)) the case of leapfrogging is analyzed although in the case of competitive R&D with Walrasian markets, incumbents are actually indifferent about their share in total R&D so that there might as well be (some) persistence in leadership (see Cozzi (2007)). In many continuous time patent race models (like Reinganum (1983 and 1985)) where innovation occurs with a Poisson arrival rate so that there is no duplication and where marginal R&D costs are increasing at the firm - but not at the industry level, preemption is not possible and in the standard case with simultaneous moves and drastic innovations, incumbents invest less in R&D than challengers. In a similar setup with fixed costs of entering the R&D sector, Etro (2004) finds that in the case where there is free entry and industry leaders move first in the R&D game (are Stackelberg leaders), they do more R&D than entrants so that there is some persistence in leadership. Denicolò (2001) analyzes the case where preemption is possible due to decreasing R&D productivity at the industry level and finds that there is persistent leadership if innovations are nondrastic and incumbents move first. Fudenberg, Gilbert, Stiglitz and Tirole (1983) analyze under which conditions preemption is possible and when there can be competition in patent races.

The question of how antitrust policies should be designed in innovative industries where entrants expect to become the next incumbents is analyzed by Segal and Whinston (2007) who also find that entrants should in most cases be well protected against incumbents in order to guarantee that profit flows for successful innovators do not become too backloaded. They also study the case of innovation by leaders but do not analyze patent policies in this context.

State - dependent intellectual property rights have first been introduced by Acemoglu and Akcigit (2008) who argue that patent protection should be stronger for firms that have a larger technological lead over their rivals. They analyze a model of step - by - step innovation in which there is a race between two firms in each sector and where the laggard first has to catch up through duplicative (but noninfringing) R&D before he can undertake frontier R&D (unless there is compulsory licensing of the leading edge technology). R&D productivity is assumed to be decreasing at the firm - but not at the industry level so that patent policy can also have an effect on innovation by affecting the division of total R&D between firms. An important difference in my model is therefore that there is free entry (without the need of catchup - R&D) and that there is the possibility to preempt entry. In Acemoglu and Akcigit (2008) innovation incentives are largest if firms are in the neck - and - neck state and decrease if the lead of one firm over the other becomes larger while in my model they are either the same in all states or larger in the case of a one-step lead than in the case of a two-step lead and the neck - and - neck case. In order to induce firms that are one step ahead to do more R&D than needed to prevent entry, expiration rates have to be sufficiently state dependent in my model, so that there are threshold effects that do not exist in Acemoglu and Akcigit (2008).

Bessen and Maskin (2009) analyze the role of patents in a model of sequential innovation with imperfect (inefficient) licensing markets and find that innovation and welfare (even for innovators) can be larger in the case without than in the case with patent protection if firms can appropriate some surplus even without patents. They assume that the same two firms stay in the market forever and that there is no entry and that even imitating firms can make positive profits. A firm that suffers from not having patent protection today might therefore gain from it tomorrow. In my model, innovators cannot make any profits without patent protection and due to the assumption of free entry, entrants and imitators earn zero profits so that an innovator whose patent expires cannot benefit from having free access to the innovations of others in the future. But even under these assumptions that might be expected to favour stronger patent protection and without assuming inefficient licensing, I also find that innovation incentives and (average) growth can in some cases decrease if patent protection is increased beyond a certain level. While I distinguish between forward protection and the protection against imitation, Bessen and Maskin analyze a specific patent policy which protects a certain innovation against imitation and at the same time gives the inventor blocking power (forward protection) over follow-on innovations. And it is mainly this forward protection in combination with the malfunctioning licensing markets that makes it possible that patents inhibit innovation in their model.

Horowitz and Lai (1996) analyze a quality-ladder model in which only the leading firm is able to do R&D and in which the length of patents determines after which date a competitive fringe is allowed to copy a previously invented version of a good. There is limit pricing and per period profits of the leading firm are larger the farther it has "escaped" the competitive fringe through innovation. However, due to the Arrow replacement effect, innovation incentives decrease in the size of the lead. The authors show that increasing the length of patent protection has two effects: it on the one hand increases the size of innovations but also reduces the frequency of innovating as the reduced speed of catch-up by rivals allows the leading firm to keep its lead even if it innovates less frequently. The main difference in my model is therefore that a similar pattern can arise in equilibrium under preemptive patenting of leading firms even though entrants can potentially also do R&D. However, in the section in which I analyze expiring patents I do not at the same time allow for a varying size of the inventive step so that my model is less general in this respect.

Llobet and Suarez (2010) analyze the case where protection against imitation and forward protection have to be set at a similar level because the patent office or the antitrust authorities cannot distinguish between knock-off imitations and new innovations that pass a certain novelty requirement. While forward protection reduces innovation and protection against imitation increases it if these policies can be set independently, if both have to be set at the same level, the authors find that in many cases an intermediate extent of total protection maximizes growth and also welfare.

Chu (2009a) studies a generalized version of the model of O ´Donoghue and Zweimüller (2004) and quantitatively estimates the effect of blocking patents on R&D using US data. He looks at the case where profit flows are backloaded due to the granting of forward protection which allows previous innovators to block follow-on innovations and compares it to the case where profit flows are completely frontloaded. He finds that eliminating blocking patents can increase R&D by about two to six times. While Chu (2009a) does not describe how exactly blocking patents can be eliminated, that means how in the case with forward protection the bargaining power of previous innovators can be reduced so that profit flows for new innovators become more frontloaded, I show that complete frontloading can be obtained through a patent transfer scheme which forces previous innovators to freely hand their patents over to entrants who improve upon their innovations. Such a patent transfer scheme might therefore be an effective way to eliminate blocking patents and might encourage R&D substantially.

Using a dataset and a survey from the German manufacturing sector, Czarnitzki, Etro and Kraft (2009) find empirical support for the basic claim of this paper that incumbents invest more in R&D if entry pressure increases.

3 The model setup

3.1 Technology and preferences

There is a continuum of individuals of mass one that are all endowed with L units of labour and derive utility from the consumption of a perfectly divisible homogenous good h and of a quality good q of which either one or zero units can be consumed. Intertemporal utility of each agent is given by

$$U(\tau) = \int_{t=\tau}^{\infty} (h(t) + q(t))\, e^{-\rho(t-\tau)} dt.$$

Producing one unit of the homogenous good requires a labour input of one and for simplicity it is assumed that the production costs of the quality good are zero. There exist generations $i \in \{1, ..., k\}$ of the quality good with $q_{i+1} = q_i + \mu$ so that quality increases by the same absolute amount μ from generation to generation.

In a given period, the utility derived from the consumption of quality goods ($q(t)$) is equal to the quality of the newest (highest i) generation that is consumed. Consuming an older generation in addition to a newer one therefore does not create any additional utility. The next generation $k+1$ of the quality good can be invented if R&D is undertaken. Using the amount $n(t)$ of labour generates the instantaneous Poisson arrival rate $\phi(t) = \min\left\{\left(\frac{n(t)}{c}\right)^{\frac{1}{1+\epsilon}}, \phi_m\right\}$ of an innovation. This technology is the same for all firms and if $\epsilon > 0$, marginal and average R&D productivity decrease in the total (industry wide) number of R&D workers that are hired[2]. The arrival rate cannot surpass the level ϕ_m due to technological reasons. As will become clear later on this assumption is made in order to keep the analysis (especially of "state dependent intellectual property rights") simple.

3.2 First best

In each period labour has to be allocated between the production of the homogenous good and R&D so that the resource constraint $L = h(t) + n(t)$ is satisfied. An innovation increases the utility derived from consuming the newest version of the quality good by μ and as this increase is permanent (as it is incorporated in the quality of future generations) the increase in intertemporal utility resulting from an innovation is given by $\frac{\mu}{\rho}$. Maximizing intertemporal utility subject to the resource constraints therefore amounts to maximizing the expected value of undertaking R&D $\phi(t)\frac{\mu}{\rho}$ minus its opportunity cost in terms of the homogenous good $n(t) = c\phi(t)^{1+\epsilon}$. This gives the optimal innovation arrival rate as $\phi^* = \min\left\{\left(\frac{\mu}{\rho c(1+\epsilon)}\right)^{\frac{1}{\epsilon}}; \phi_m; \left(\frac{L}{c}\right)^{\frac{1}{1+\epsilon}}\right\}$. In the case where $\epsilon \to 0$ and $\frac{\mu}{\rho c} > 1$ (**Condition 1**) it is therefore optimal to do the maximal possible amount of R&D which either leads to the maximal arrival rate ϕ_m or the rate $\left(\frac{L}{c}\right)^{\frac{1}{1+\epsilon}}$ that results if the whole labour force does R&D[3]. In the following it is assumed that L

[2] A reason for this might be that it is impossible to perfectly coordinate the R&D activities of all researchers so that the probability of duplicative research increases in the number of workers hired. In order to explicitly model duplication, it would however be more appropriate to use a discrete time version of the model. In such a setup one could assume that an innovator obtains a patent with probability $\frac{1}{N}$ if N innovators have made the same invention in a given period.

[3] Even if the whole labour force does R&D, none of the homogenous good is produced and consumed while (the newest version of) the quality good is still consumed as it is costless to produce.

is large relative to ϕ_m so that the maximal instantaneous arrival rate of an innovation is given by ϕ_m. Under these conditions, increasing the rate of innovation ϕ if it is below the maximal rate in a decentralized equilibrium is therefore welfare increasing. Because of that, I only analyze the effect of policies on the rate of innovation in the following sections (where $\epsilon \rightarrow 0$ will be assumed) knowing that a policy that increases innovation also increases welfare given that it does not imply any other costs. In the *Appendix 2* I look at a more general model and also analyze the optimal policy in the case where equilibrium growth is excessive.

3.3 Market allocation

The price of the homogenous good is normalized to one and both the sector producing the homogenous good and the R&D sector are assumed to be perfectly competitive so that the wage per unit of labour is equal to one. An innovator of a new generation of the quality good gets a patent which allows her to exclude others from producing her generation of the good. There is Bertrand competition between firms producing quality goods so that the maximal price that can be charged by the leading firm is given by the difference between the quality of its good compared to that of the closest competitor. A firm with a one-step lead can therefore charge a price equal to μ but firms with a lead of two or more steps are assumed to be constrained by a competitive fringe which does not allow to charge a price larger than $\pi_2 \leq 2\mu$. This competitive fringe might arise because patent protection is imperfect so that pirating firms can produce the newest generation of the quality good at a marginal cost (which might include fines) equal to π_2 or simply because there is a limited patent breadth allowing entry of this form. Another interpretation would be that there is a price cap imposed by regulators in order to reduce static inefficiencies arising from monopoly pricing (which are however not present in the simple model considered here but arise in the more general setting analyzed in *Appendix 2*). Another interpretation of the feature that profits cannot be increased if the lead of a firm exceeds two steps would be that firms with a two-step lead can charge the unconstrained monopoly price and make profits π_2 while profits of firms with a one-step lead (π_1) are lower because these firms have to engage in limit pricing to keep the closest competitor out of the market. However, such an analysis would require a richer model like the one studied by Denicolò (2001). In *Appendix 2* I look at such a more realistic but also somewhat more complicated model and obtain similar results as in the simple model analyzed here. In the following, I denote prices and flow profits[4] of a firm with a one-step lead by $\pi_1 = \mu$ and those of a firm with a two - or more step lead by $\pi_2 > \pi_1$. The feature that an innovation increases profits for entrants more than for incumbents (that means that $\Delta\pi_1 = \pi_1 > \Delta\pi_2 = \pi_2 - \pi_2 = 0$ if the incumbent has a two-step lead and $\pi_1 \geq \pi_2 - \pi_1$ in the case of a one-step lead) is commonly called the "Arrow replacement effect" and can be obtained in many settings[5].

In order to guarantee that a positive amount of the homogenous good is consumed in equilibrium, the relation $L > \pi_2$ must be satisfied. Due to the linearity of the intertemporal utility function, the rate of interest must be equal to the rate of time preference in equilibrium: $r(t) = r = \rho$.

If more than one firm undertake R&D at the same time, a firm i that hires a fraction β_i of the total R&D labour and that therefore bears the fraction β_i of the total R&D costs (that are given by $n(t) = c\phi(t)^{1+\epsilon}$) is assumed to obtain the innovation in the fraction β_i of cases in which one occurs. As the total innovation probability in interval dt is given by $\phi(t)dt$, the innovation probability for firm i is therefore given by $\beta_i\phi(t)dt$.

3.4 Equilibrium

Given an innovation has the value V_E for a newly entering firm, entry occurs until the average cost of innovating is equal to this value, that means until the zero

[4] R&D costs are not taken into account here and would reduce net profits below the value labeled flow profits here.

[5] Another way to introduce this effect into this model is to assume that utility is given by: $U(\tau) = \int_{t=\tau}^{\infty} (x(t)^{\nu} + q(t))\, e^{-\rho(t-\tau)} dt$. For $\nu < 1$, incremental profits fall with the size of the lead so that entrants have larger stand - alone innovation incentives than incumbents. If $\nu > 1$, the Arrow replacement effect is reversed and monopolists have larger (stand - alone) incentives to innovate. The case where $\nu > 1$ is however less relevant from an economic point of view as it seems more reasonable to assume that marginal utility from consuming the homogenous good is decreasing.

profit condition $V_E \leq c\phi(t)^{\epsilon}$ is satisfied[6]. This condition pins down a lower bound for the equilibrium rate of innovation as an increasing function of V_E. But the fact that this condition depends on the value of an innovation for an entrant does not imply that in equilibrium R&D is actually carried out by entrants. Take the case of an incumbent firm with a two-step lead: without any threat of entry, this firm does not do any R&D as it cannot increase profits above the current level π_2. However, if there is free entry, the incumbent knows that if she does not do any R&D, the instantaneous probability of replacement is given by the zero profit condition as $\widetilde{\phi} = \left(\frac{V_E}{c}\right)^{\frac{1}{\epsilon}}$. Given that entrants can adjust their amount of R&D after observing the level of R&D undertaken by the incumbent (who is assumed to move first), this probability is independent of whether the incumbent does part of the R&D herself, as all what matters for entry are the average costs of R&D that are increasing in the overall level of R&D so that one unit of R&D undertaken by the incumbent crowds out one unit undertaken by entrants. The incumbent can therefore reduce the probability of replacement from $\widetilde{\phi}$ to $(1-\beta)\widetilde{\phi}$ if she bears the costs $\beta c\widetilde{\phi}^{1+\epsilon}$. As successfully innovating entrants have to compete with the previous incumbents and only get profits π_1 upon entry (and maybe later on π_2 if they do a follow-on innovation) the value of an innovation V_E for an entrant is lower than the willingness to pay of an incumbent firm for keeping its two-step lead. Because of that, the incumbent finds it profitable to increase its R&D effort as long as $\beta < 1$ as the marginal value of reducing the probability of losing the two-step lead exceeds the marginal costs that are always equal to the average costs $c\widetilde{\phi}^{\epsilon} = V_E$ under free entry[7]. Therefore, an incumbent with a two-step lead

[6] Take the case where R&D productivity is decreasing at the industry level due to duplication and where, if two firms are successful at the same time, each gets the patent with probability one half. Then, an entering firm that contributes to an overall increase in R&D effort does not take into account that it increases the risk of duplication for all other firms as well and therefore still finds it profitable to enter if the average costs are lower than V_E even if the marginal costs are higher. As total R&D costs are given by $n(t) = c\phi(t)^{1+\epsilon}$, average costs are given by $a(t) = c\phi(t)^{\epsilon}$.

[7] The value of incumbency is given by $V_I = \frac{\pi_2 - \beta c\widetilde{\phi}^{1+\epsilon}}{r+(1-\beta)\widetilde{\phi}}$. Inserting $\widetilde{\phi} = \left(\frac{V_E}{c}\right)^{\frac{1}{\epsilon}}$ and deriving with respect to β, we obtain that $sign \frac{\partial V_I}{\partial \beta} > 0$ if $V_E\left(\left(\frac{V_E}{c}\right)^{\epsilon} + r\right) < \pi_2$. This condition is satisfied given that $V_E < V_I = \frac{\pi_2 - \beta c\left(\frac{V_E}{c}\right)^{\frac{1+\epsilon}{\epsilon}}}{r+(1-\beta)\left(\frac{V_E}{c}\right)^{\frac{1}{\epsilon}}}$.

finds it profitable to preempts entry by doing exactly as much R&D as needed to push average R&D costs up to (or slightly above) the value of an innovation for an entrant (so that the zero profit condition is satisfied with equality)[8]. This is exactly the argument first brought forward by Gilbert and Newbery (1982). The same reasoning can be applied to an incumbent with a one-step lead who is willing to pay more for the next innovation (which guarantees a two-step lead) than an entering firm (which only gets a one-step lead). However, as it will become clear later on, an incumbent with a one-step lead might under certain circumstances even do more R&D than necessary to prevent entry.

We can therefore conclude that there is persistent leadership and that the rate of innovation depends (through the zero profit condition) positively on the value of an innovation V_E of an entrant. To solve for the equilibrium we therefore need to determine V_E given that an entering firm expects to expand its lead to two steps in the future and then to stay the leader and to do only as much R&D as needed to prevent entry. Denoting the value of being two (one) steps ahead by V_2 (V_1) and the R&D effort undertaken by a firm with a two-step (one-step) lead by ϕ_2 (ϕ_1), the following arbitrage conditions must be satisfied:

$$rV_2 = \pi_2 - c\phi_2^{1+\epsilon} \qquad \text{and} \qquad rV_1 = \pi_1 - c\phi_1^{1+\epsilon} - \phi_1 V_1 + \phi_1 V_2.$$

The right hand side in the first equation indicates the per period profits derived from selling the quality good minus the costs of conducting the level of R&D ϕ_2 that is needed to prevent entry. In the second equation there are two additional terms because in the case of an innovation (which occurs with arrival rate ϕ_1) the firm gains a two-step lead and looses its one-step lead. As the value of an innovation for an entrant is exactly the value of getting a one-step lead, we can

[8] The equilibrium analyzed here can either be seen as one where the incumbent is a Stackelberg leader in the R&D game or as a Walrasian equilibrium where the total demand for R&D labour (which is a decreasing function of the wage) is equal to the supply (which is perfectly elastic for the wage of one) and in which the auctioneer allocates all R&D labour to the incumbent who is willing to pay at least as much for it as the entrants. As the entrants get zero profits in equilibrium, they are indifferent about the quantity of R&D that they undertake and, in case an incumbent would do less R&D than the equilibrium level, the Walrasian auctioneer would just assign a larger amount of R&D to entrants in order to obtain the equilibrium (see Cozzi (2008) for a more detailed discussion).

A similar equilibrium might also be obtained as the outcome of a second price auction in which incumbents and entrants simultaneously bid for R&D labour.

solve the two equations for V_E:

$$V_E = V_1 = \frac{\pi_1 - c\phi_1^{1+\epsilon}}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2^{1+\epsilon}}{r}$$

The zero profit condition therefore becomes:

$$\frac{\pi_1 - c\phi_1^{1+\epsilon}}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2^{1+\epsilon}}{r} = c{\phi_2}^{\epsilon} \leq c{\phi_1}^{\epsilon}.$$

While the zero profit condition is for sure binding in the case where the incumbent firm has a two-step lead, it might be the case that a firm with a one-step lead wants to get a two-step lead so quickly that it does more R&D than required to increase average R&D costs to a level that prevents entry.

In order to simplify the analysis, the case where $\epsilon \to 0$ is considered in the following. As the value of an innovation for an entrant (V_E) depends on the amount of R&D ϕ_2 that entrants expect to do in the future to prevent entry (and not on current R&D), in the case where $\epsilon = 0$ and for V_E given, the equilibrium amount of R&D undertaken by the incumbent or by entrants is undetermined if the ZP condition is satisfied with equality ($V_E = c$). However, for ϵ slightly positive, the preemption equilibrium results and the R&D rate is well specified in each period and equal to the one that entrants expect to choose themselves once they are leaders. In the following it is therefore assumed that in the case where $\epsilon = 0$ the equilibrium is selected that results as the limit if $\epsilon \to 0$. For $\epsilon = 0$, marginal and average R&D costs are equal to c and the zero profit condition is given by:

$$\text{(ZP)} \quad V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r} = c$$

In order to determine the equilibrium value of ϕ_2 we need to know the level of ϕ_1 chosen by a firm with a one-step lead. Such a firm takes V_2 and therefore ϕ_2 as given and maximizes V_1 ($= V_E$) with respect to ϕ_1. We get: $\frac{\partial V_E}{\partial \phi_1} = \frac{\pi_2 - \pi_1 - cr - c\phi_2}{(r+\phi_1)^2}$ so that $\phi_1 = \phi_m$ if $\pi_2 - \pi_1 - cr - c\phi_2 > 0$ and $\phi_1 = \phi_2$ if $\pi_2 - \pi_1 - cr - c\phi_2 \leq 0$. If ϕ_2 is low enough, firms with a one-step lead chose the maximal possible R&D effort ϕ_m while, if ϕ_2 is large, firms with a one-step lead have no incentives to conduct more R&D than needed to preempt entry and they set $\phi_1 = \phi_2$[9]. As $V_2 = \frac{\pi_2 - c\phi_2}{r}$

[9] Note that the amount of R&D needed to preempt entry is always given by ϕ_2 as the value of an innovation for an entrant does not depend on the size of the lead of the current leader.

The conditions under which $\phi_1 = \phi_m$ can actually result in equilibrium are analyzed in *Sections 1* and *12*.

decreases in ϕ_2, V_E also decreases in ϕ_2 if ϕ_1 is set to maximize V_E given ϕ_2[10]. By implicitly differentiating the zero profit condition, we can state the following:

Proposition 1 *The equilibrium growth rate ϕ_2 is increasing in π_1 and π_2 and decreasing in r $(= \rho)$ and c.*

The intuition for these results is straightforward: as all the R&D is carried out by the leading firm, there is persistent leadership and given the leading firm already has a lead of two steps, it does just as much R&D as needed to preempt entry. And this preemptive R&D level depends positively on the value of an innovation for an entrant who anticipates to become the next leader[11] which again increases in the profit flows π_1 and π_2 and decreases in the rate of interest r due to discounting. Entry pressure also increases if R&D costs are lower, that means if c decreases.

In the following sections, the effects of different patent policies are analyzed.

4 Compensating previous innovators

As each innovation improves upon the last one, innovators use the knowledge accumulated by previous innovators as an R&D input. Let us now assume that there is a policy requiring a successful innovator to pay a fixed fee F upon entry into the product market in order to compensate previous innovators. Even in the case where the entire fee has to be given to the previous incumbent, there is still persistent leadership and preemption. The reason for this is that the incumbent firm values not being replaced more than the entrant values entering so that there is no value of F for which at the same time the entrant is willing to enter and the incumbent is willing to permit entry by not doing any preemptive R&D. If nevertheless entry occurs (out of equilibrium), entrants therefore expect to become the next leaders and to do all the follow-on R&D themselves and to

[10] If ϕ_2 is small, $\phi_1 = \phi_m$ which is independent of ϕ_2 so that V_E clearly decreases in ϕ_2. At the critical level $\phi_2 = \frac{\pi_2 - \pi_1}{c} - r$ firms with a one - step lead are indifferent between setting $\phi_1 = \phi_m$ or $\phi_1 = \phi_2$ and if ϕ_2 increases more, we have $\frac{\partial V_E}{\partial \phi_1} < 0$ so that $\frac{\partial V_E}{\partial \phi_2} < 0$ for sure holds if firms select the preemptive R&D level $\phi_1 = \phi_2$.

[11] Note however that this value is hypothetical as there is no entry in equilibrium

never receive any licensing fees from others. The value of an innovation for an entrant is therefore given as:

$V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r} - F$

The zero profit condition is now $\frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r} = c + F$ so that an increase in F unambiguously reduces equilibrium growth ϕ_2. The intuition for that is simply that any kind of licensing fee or other fixed fee that has to be paid upon entry decreases the value of an innovation for an entrant and therefore entry pressure and the R&D effort of the incumbent that is required to deter entry[12].

If it is possible to subsidize entry (impose a negative F) this increases the rate of R&D ϕ_2 although the subsidy never has to be paid in equilibrium (given it is not too large)[13].

5 Voluntary deals

In order to reduce the value of an innovation for entrants and entry pressure, incumbents would clearly like to commit to compete with entrants in the product market once entry occurs[14]. However, if they cannot commit to compete and voluntary deals are allowed, they have incentives to negotiate with entrants once entry occurs in order to consolidate market power (e. g. to collude) and to obtain the maximal possible joint profits. This can for example be achieved if one of the firms sells its patent (or an exclusive license) to the other. Without any voluntary deal the value of the entrant firm is given by $V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$ while the value of the former incumbent drops to zero. Consolidating market power allows to increase joint profits by charging the higher price π_2 instead of π_1 and also makes it possible to reduce total R&D expenditures in the case where the entrant would undertake R&D effort $\phi_1 = \phi_m > \phi_2$ if there was competition. The reason for the latter is that in the case of a voluntary deal profit flows are already maximal

[12] While the effect of an increase in F on ϕ_2 is clearly negative, increasing F might however induce a higher R&D effort of firms with a one - step lead - as will be discussed later on in *Section 12*.

[13] Forcing the incumbent firm to pay this entry subsidy would push it even more to innovate preemptively in order to avoid entry.

[14] In the case of software an easy way to compete would be to put the software in the public domain (make it downloadable free of charge).

so that there is no reason to do more R&D than the level ϕ_2 which is necessary to prevent further entry. Therefore, the value $V_2 = \frac{\pi_2 - c\phi_2}{r}$ can be shared among the two parties who engage in a voluntary deal and each has to get as least as much as the outside option without such a deal. The greater the bargaining power of the entrant, the greater is therefore the value of an innovation to an entrant and the larger is the equilibrium rate of growth[15]. However, it is not clear anymore whether there is persistent leadership in this case or whether entrants undertake some (or all) R&D. The reason for this is that the entrant now values entering as much as the incumbent values not being replaced as a replaced incumbent still gets a share of the surplus and because entry does not lead to a reduction in the total surplus like in the case of competition. The equilibrium rate of growth that is determined by the zero profit condition is however independent of whether R&D is carried out by the incumbent or by entrants. Allowing voluntary deals therefore increases the equilibrium rate of growth ϕ_2 (unless entrants have no bargaining power at all) and only leads to a reduced R&D effort in the out of equilibrium case where a firm enters and would choose the high R&D effort $\phi_1 = \phi_m$ if it had to compete with the previous incumbent.

6 Patent transfer scheme

In the case of voluntary deals equilibrium growth ϕ_2 is maximal if the entrant has all the bargaining power and gets V_2 right upon entry. If entrants tend to be weak bargainers, an interesting question is whether patent policy can be designed in a way to increase the value of an innovation for an entrant and to reduce the bargaining power of incumbents. Incumbents have bargaining power because they can reduce profits of entrants by competing with them in the product market and therefore, the simplest way to reduce their bargaining power is to prevent them from competing once entry has occurred. This can for example be achieved by forcing holders of patents of previous generations of the good to freely give

[15] If the entrant gets the fraction $\alpha\frac{\pi_2 - c\phi_2}{r} > \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$ of the "profit pie" that is shared with the incumbent (where α indicates the bargaining power of the entrant) the zero profit condition is given by $\alpha\frac{\pi_2 - c\phi_2}{r} = c$ so that ϕ_2 increases in α and is larger than in the case without a voluntary deal.

their patents to an entrant firm if it has improved upon their innovations. Such a "patent transfer scheme" therefore implies an expropriation of previous patent holders in order to give the maximal (or desired level of) market power to entrants. If entrants get profit streams π_2 right upon entry there can again be either persistent leadership or leapfrogging (or a mixture of both) as incumbents and entrants have the same incentives to innovate (the incumbents value not being replaced the same as entrants value entering). The equilibrium rate of growth can now be obtained from the zero profit condition $V_2 = c$, implying that $\phi_2 = \frac{\pi_2}{c} - r$.

7 Forward protection

Now, let us assume that the patent of an entrant infringes on the patent of the previous generation of the quality good, so that the newest generation can only be produced with the consent of the holder of the patent of the previous generation. O ´Donoghue and Zweimüller (2004) call such an arrangement "forward protection" and assume that this is the only possibility to consolidate market power between entrants and incumbents so that voluntary deals are only possible or allowed if the incumbent has blocking power over the innovation of an entrant.

Given there is an incumbent (she) with a two-step lead and an entrant (he) has made an invention and expects to become the new leader and to do all the follow-on R&D in order to preempt further entry. If he does not reach any agreement with the previous incumbent he will not be able to produce his newly invented good and will only start making profits once he has done a second innovation that does not infringe on the patent of the previous incumbent anymore. The value of the entrant´s position is then given as $V_E = \frac{-c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$. Given entry has occurred, the previous incumbent knows that she will be replaced with probability ϕ_1 and loose all her profits to the entrant who will then be the next leader and get profit flows π_2. The value of being the previous incumbent after entry has occurred and without any voluntary deal is therefore given as $V_I = \frac{\pi_2}{r+\phi_1}$. If an incumbent can commit not to make any deals with entrants after entry occurs she will do so as engaging in a voluntary deal can only increase the value of an innovation for an entrant and therefore entry pressure and the R&D costs that have to be born by the incumbent to preempt entry ex ante. In the case without any voluntary deal forward protection therefore clearly decreases the value of an innovation for an entrant because it prevents him from producing his invention while in the case without forward protection (and without voluntary deal) he can at least earn profits π_1 upon entry.

Given incumbents cannot commit, is there any gain to be made from negotiating with entrants after entry occurs? As in the case without voluntary deal the incumbent keeps her two-step lead until she is replaced and the entrant directly gets a two-step lead when replacement occurs, a voluntary deal cannot increase joint profits by increasing market power and prices (as these are always kept at the maximal level)[16]. Therefore, the only case in which the joint surplus can be increased through a voluntary deal is the case where without such a deal the entrant chooses the high R&D effort $\phi_1 = \phi_m$ which is larger than the effort $\phi_1 = \phi_2$ required to prevent entry and therefore leads to a lower aggregate payoff[17]. But even in this case the value of an innovation for an entrant is lower than in the case without forward protection (and without voluntary deal) if his bargaining power is low. But if the bargaining power of the entrant is large it is possible that the value of an innovation for an entrant is greater in the case with forward protection and voluntary deal than in the case without forward protection and without voluntary deal. For this to be possible it has to be the case that the entrant can credibly threaten to set $\phi_1 = \phi_m$ (and therefore to replace the incumbent quickly) if no agreement is reached so that the incumbent is willing to pay a lot to the entrant

[16] This is however not the case in the more general model in *Appendix 2* where the innovation of the entrant allows to increase joint profits above the level that the incumbent can obtain alone.

[17] The analysis here focuses on the case where there is persistent leadership even if the incumbent might in some cases be indifferent between doing the R&D herself and allowing entry. As already mentioned in the section on voluntary deals, these make leaders indifferent between preemptions and allowing entry. However, there is a new case here in which there might be some turnover even without voluntary deals: if entrants with a one - step lead (whose innovations are blocked by the previous incumbent due to forward protection) find it optimal to choose the R&D effort $\phi_1 = \phi_2$, incumbents with a two - step lead are also indifferent between preemption and allowing entry as the costs of preempting are the same as the costs of entering and as profits are not eroded by entry (if however $\phi_1 = \phi_m$, preemption is cheaper than entering and there is persistent leadership). Therefore, it is possible that a firm enters, does follow - on R&D to obtain a two - step lead, stays the leader for some time and then allows entry of a firm that becomes the next leader. However, equilibrium growth is the same in these cases as in the case of persistent leadership.

if he chooses the low R&D effort $\phi_1 = \phi_2$. In *Appendix A* (*15.2*) I show that equilibrium growth is larger in the case with forward protection and voluntary deal than in the case without forward protection and without voluntary deal if the entrant has all the bargaining power, if ϕ_m is large enough and if $\pi_2 > 2cr$.

The effects of forward protection analyzed here in a setup with persistent leadership differ from those in models with leapfrogging as analyzed by O ´Donoghue and Zweimüller (2004) and O ´Donoghue, Scotchmer and Thisse (1998). In these models firms only get profits π_1 without a voluntary deal and forward protection (which is assumed to be the only way to make voluntary deals possible) allows to increase joint market power and profits to π_2. If the bargaining power of incumbents is not too large so that entrants get a large enough share of the increased profit pie right upon entry (that means if profit flows are not too backloaded), forward protection therefore increases the value of an innovation and equilibrium growth. The mechanism is therefore different than in the case of persistent leadership where forward protection cannot increase joint market power and mainly acts as a barrier to entry unless it allows to considerably reduce joint R&D costs.

It should be noted that in both the case of leapfrogging and the case of persistent leadership simply permitting voluntary deals without granting forward protection unambiguously increases the value of an innovation (for an entrant) and equilibrium growth. The reason for this is that forward protection greatly increases the bargaining power of the incumbent which has already been shown to decrease the value of an innovation (for an entrant) and equilibrium growth[18]. And again, introducing a patent transfer scheme which makes private negotiations unnecessary and gives all the market power to entrants right upon entry leads to even larger innovation incentives in both cases. (For a more detailed discussion of the case of leapfrogging see the *Appendix 15.1*).

[18]If forward protection was only granted to firms with a one - step lead it might however increase the value of an innovation for an entrant and entry pressure so that incumbents with a two - step lead would have to undertake more R&D in order to prevent entry.

8 Patentability requirement

So far it has been assumed that each innovation increases the quality of the good by the fixed amount μ. However, it might be possible for R&D firms to target different innovation sizes at different costs. In this case, there is another instrument that patent policy can use: a patentability requirement that sets a lower bound on the innovative step below which an innovator cannot obtain a patent. Let us assume that the amount of R&D labour n_i that a firm i needs to employ in order to target innovation size μ and to obtain the arrival rate ϕ is given by $n_i = c\phi\lambda(\mu)(n_{tot})^{\epsilon}$ with $\frac{\partial\lambda(\mu)}{\partial\mu} > 0$ and with $n_{tot} = n_i + n_{-i}$ indicating the overall (industry - wide) amount of R&D labour used. For a given amount of R&D labour, targeting larger innovative steps μ therefore implies a lower hazard rate ϕ, and given that $\epsilon > 0$, R&D productivity decreases in the total amount of R&D labour that is employed at the industry level. Due to this assumption, there is again the possibility for an incumbent firm to preempt entry by undertaking a large enough amount of R&D in order to increase the R&D costs of entrants.

Expected profits of entrants can be written as $E\Pi_E = \phi V_E - c\phi\lambda(\mu)(n_{tot})^{\epsilon}$ so that the zero profit condition that must be satisfied in equilibrium is given by $V_E = c\lambda(\mu)(n_{tot})^{\epsilon}$. This condition pins down the total amount of R&D labour n_{tot} hired and as incumbents and entrants pay the same wage for this R&D labour but incumbents value not being replaced more than entrants value entry ($V_I > V_E$, that means that the value of incumbency is larger than the value of entering), incumbents again find it profitable to do all the R&D in equilibrium and to preempt entry. It should be noted that it is assumed here that only the total amount of R&D labour hired by incumbents matters for the R&D costs of entrants so that the costs that incumbents need to incur to preempt entry do not depend on the size of the inventive step and the hazard rate which they choose. An incumbent whose lead is large enough to allow her to charge the maximal price π_2 therefore does not care about the size of her innovations but only about the total number of R&D workers she needs to hire in order to make entry unprofitable. The incentives to innovate of an entrant however depend on the sequence of innovative steps chosen to reach a lead that is large enough to charge price π_2. Without any patentability requirement, entrants chose the sequence and sizes of inventive steps that maximize the present discounted value of their R&D activity. Therefore, any binding restrictions imposed by patent policy on the R&D decisions of entrants and incumbents with a one-step lead necessarily decrease the value entrants derive from undertaking R&D and therefore reduce entry pressure. And if entry pressure is reduced, incumbents with a two-step lead need to do less R&D in order to discourage entry so that the number of researchers $n = n_{tot}$ that is hired in the preemption equilibrium decreases.

Taking the simple example where entrants find it optimal to do two steps in order to get the maximal profits and again looking at the case where $\epsilon \to 0$, the expected profits of a potential entrant are given by $E\Pi_E = \phi_0 \left(\frac{\pi_1 - c\phi_1\lambda(\mu_2)}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - n}{r} \right) - c\phi_0\lambda(\mu_1)$ where ϕ_0 and ϕ_1 indicate the arrival rates chosen if the entrant has not made any innovation and if he is one step ahead and n the amount of R&D labour the incumbent with a two-step lead has to hire in order to preempt entry. Note that $\pi_1 = \mu_1$ and $\pi_2 = \mu_1 + \mu_2$ if the entrant can freely chose the inventive steps μ_i but that in the case of a patentability requirement that imposes $\mu_i > \overline{\mu}$ (with $\overline{\mu} < \pi_2$ [19]) it might well be that $\pi_2 < 2\overline{\mu}$, especially if there is a price cap implying that $\pi_2 < 2\pi_1$. For a given value n of the R&D costs of a firm with a two-step lead, $E\Pi_E$ decreases if patent policy forces entrants to select different innovation sizes μ_i than their preferred ones. As the condition $E\Pi_E = 0$ must hold in equilibrium, n therefore decreases if a binding patentability requirement for entrants is imposed. The same reasoning holds if entrants find it optimal to undertake more than two innovative steps in order to obtain the maximal lead. Imposing a patentability requirement on entrants and incumbents with a one-step lead therefore reduces equilibrium R&D in the preemption equilibrium for any given size of the inventive step targeted by the incumbent (who has a two-step lead). However, imposing a patentability requirement solely on incumbents with a two-step lead has no effect on their overall R&D spending and can moreover prevent that

[19] If $\overline{\mu} \geq \pi_2$ the patentability requirement is so strict that it requires innovations to be drastic. In this case there is however no phase of competition and the innovation incentices of entrants and incumbents are the same so that there won ´t be the preemption equilibrium anymore. If $\overline{\mu}$ is continuously increased and passes the threshold π_2 the equilibrium rate of growth also changes continuously. A small change in the patentability requirement can therefore not lead to a discontinuous change in the rate of growth by inducing a switch from an equilibrium with persistent leadership to one with leapfrogging.

they target inefficiently little inventive steps. In the simple model analyzed here, incumbents are indifferent with respect to the inventive step chosen so that they might not select the socially optimal step size. And in a more general model, they might strictly prefer to select a size of the inventive step which is inferior to the socially optimal one so that imposing a patentability requirement on incumbents is likely to be even more beneficial[20].

If entrants can freely choose the size of the inventive step and incumbents cannot commit to compete in the product market once entry occurs, allowing voluntary deals is again growth enhancing if there is persistent leadership as it increases the value of an innovation for an entrant and entry pressure. Entry pressure is maximal if entrants are allowed to patent arbitrarily small quality improvements and if they are allowed to consolidate market power with incumbents after entering and to reduce their R&D effort below the one they would undertake if there was competition. However, if voluntary deals are allowed, incumbents are indifferent between preempting and allowing entry so that inefficiently little inventive steps might be realized in equilibrium if entry occurs. In order to avoid that and to give incentives for incumbents to preempt entry, it might be optimal to restrict voluntary deals by for example imposing a price cap in the case of price collusion so that entrants again value entry less than incumbents value not being replaced. Then, voluntary deals increase entry pressure and the R&D effort undertaken by incumbents without leading to inefficiently small innovative steps in equilibrium (at least as long as there is a patentability requirement for incumbents with a two-step lead).

To sum up, imposing a patentability requirement on entrants and incumbents with a one-step lead reduces entry pressure and growth while only imposing it on incumbents with a two-step lead might be useful to avoid inefficiently small inventive steps. This result differs from the ones of O ´Donoghue (1998), O ´Donoghue, Scotchmer and Thisse (1998) and O ´Donoghue and Zweimüller (2004) who show that in the case of leapfrogging a minimal patentability requirement is not only welfare -, but also growth - enhancing. The reason for their finding is that in the

[20] In a model where incumbents can increase their profit flows through R&D Denicolo (2001) shows that incumbents tend to pursue too little innovative steps.

case of leapfrogging inefficiently small innovative steps are targeted and profit flows are reduced due to competition while the rate of turnover (creative destruction) increases, which leads to lower innovation incentives. Contrary to that, incumbents always have the maximal market power in the case of persistent leadership and by relaxing the patentability requirement for entrants but not for incumbents, entry pressure and equilibrium growth can be increased, without leading to inefficiently small inventive steps in equilibrium.

While it might be difficult to impose different patentability requirements on incumbents with unconstrained monopoly power (that have a two-step lead) and incumbents that are constrained by competition (that have a one-step lead), making the patentability requirement conditional on whether an innovation is realized by an entrant or an incumbent should be easy to implement as long as the patent office can observe whether the firm who is filing for the patent already holds a patent for the previous generation of the good. And simply relaxing the patentability requirement of entrants compared to that of both types of incumbents already increases entry pressure and equilibrium growth.

8.1 Imperfect preemption

The result that there is always full preemption even if differing patentability requirements are imposed on entrants and incumbents depends on the way how R&D undertaken by incumbents affects R&D costs of entrants. In the analysis above, all what matters for the R&D costs of entrants is the total amount of R&D labour hired by incumbents. This specification for example reflects the case where the wage of researchers increases in the number of researchers that are hired to do R&D and where incumbents preempt entry by pushing this wage up. One way of modeling this would be to assume that some workers like doing research (relative to the production of the homogenous good) more than others so that increasing the total number of workers employed in the R&D sector is only possible if the wage is increased as workers who do not like research that much have to be compensated accordingly to become researchers[21]. The analysis might however

[21] The difference of this specification relative to that in *Section 10.3* is that here it is not possible to increase wages and to preempt entry if the hired workers are used to produce the homogenous good are not to do R&D. The reason for this is that firms who do not do R&D

change if wages are constant and if the reason for decreasing R&D productivity is that the risk of duplicative R&D increases in the number of researchers hired. If entrants and incumbents pursue different sizes of innovative steps it is still possible for incumbents to preempt entry by undertaking a sufficient amount of R&D in order to increase the probability that research undertaken by entrants turns out to be duplicative (and not awarded a full patent), however, it might become prohibitively costly to do so. The reason for this is that if incumbents pursue larger inventive steps, the arrival rate of an innovation is lower for a given amount of R&D labour hired. And this might considerably reduce the threat of duplication for entrants and encourage entry even if in the case where both the incumbent and an entrant innovate at the same time the entrant does not make any profits as the innovation of the incumbent is larger and receives the patent. Therefore, the stricter the patentability requirement on incumbents is compared to that on entrants, the more costly it gets for incumbents to preempt entry and at a certain point they might not want to preempt entry anymore[22]. In such a case there would be a leapfrogging equilibrium in which the patentability requirement for entrants (if it is binding) determines the size of inventive steps carried out in equilibrium. If preemption works through the channel of duplication and not the channel of increasing wages for R&D labour, the patentability requirements on entrants and incumbents should therefore not differ too much in order to avoid getting back into a leapfrogging equilibrium in which inefficiently small inventive steps are realized and innovation incentives are lower as profits are reduced due to competition in the product market.

Another reason why there might not be perfect preemption is the presence of asymmetric information. A strong assumption of the analysis so far has been that incumbents are perfectly informed about the R&D costs of entrants and the extent to which R&D productivity decreases (ϵ) and can therefore exactly compute the level of R&D they need to undertake to make entry unattractive. If incumbents

cannot benefit from wage discounts.

[22] If it was however possible to accumulate some innovation - step specific capital in order to increase the chance of discovering the next step preemption would become easier. In this case the innovation - step specific capital depreciates once the next step is invented and an increase in the innovation probability of a rival discourages R&D not only due to an increased risk of duplicative R&D but also due to the risk of loosing the own capital more quickly.

however only know the distribution of possible R&D costs c of entrants or of ϵ, they might find it profitable to allow entry with a positive probability if this saves a lot of R&D costs. In such a case there would therefore not be perfect preemption and from time to time entrants would do R&D and replace leaders[23]. In order to avoid that inefficiently small innovations are actually patented by entrants, imposing a patentability requirement on them might be useful in this case[24]. However, as long as entrants do not do much R&D in equilibrium, the patentability requirement for them should still be weaker than that imposed on incumbents in order to keep entry pressure high.

Preemption might also be difficult if it works though the channel of duplication and if the R&D effort undertaken by incumbents is not directly observable for entrants. However, incumbents have incentives to make their R&D programs public in order to deter entry and even if entrants cannot verify how much incumbents spend on R&D, they might make inferences by observing how frequently incumbents innovated (patented) in the past. And incumbents might opt to keep innovating even if this is costly in order to build a reputation as active innovators so that potential entrants stay out of the market because they perceive that the risk of duplicative R&D is too high.

9 Patent buyout schemes

Hopenhayn, Llobet and Mitchell (2006) study the optimal patent system in a quality ladder model where the patent office cannot observe the size of the inven-

[23] Incumbents face the trade - off that if they want to reduce the probability of being replaced, they have to do more R&D, which is costly and does not increase their own profits. If the distribution of c is such that low values of c are realized with a low probability, incumbents therefore select a threshold $\widetilde{c}$ and only undertake as much R&D as needed to deter entry if $c > \widetilde{c}$ so that there is entry in the case where $c < \widetilde{c}$.

[24] In the case where an innovation that does not pass the patentability requirement for entrants occurs (for example because someone made the invention by chance, that means without having invested in it) it should infringe on the patent of the incumbent and not be put in the public domain as this would destroy the monopoly profits of the incumbent and decrease the incentives to make innovations that pass the patentability requirement. The incumbent should however be allowed to use the innovation as it is of higher quality than her own good.

tive step and therefore cannot impose a patentability requirement[25]. Under the assumption that innovations are always realized by entrants they find that the optimal R&D incentives can be obtained through a patent buyout scheme which requires that an innovator pays a certain fee to the previous patent holder in order to be allowed to replace her, and that in addition gives a menu of contracts indicating the level of the fee an innovator has to pay to the patent office now in order to fix the level of the fee that future innovators are required to pay to him in order to replace him in the future. As buying stronger protection against future innovations is more costly, only firms with a large innovative step that allows to charge a larger price and to earn larger profits find it profitable to buy strong protection and this encourages large innovative steps. Now the question is whether such a buyout scheme would work in the setup of preemptive patenting by leading firms (that have access to the same R&D technology as entrants). Given an incumbent can buy protection against future entry by paying a fee today, this just gives an additional instrument to prevent entry which is likely to reduce innovation incentives as it can be used as a substitute for incumbent R&D.

If the incumbent does all the R&D and charges the unconstrained monopoly price π_2 and if the patent office cannot observe the size of the inventive steps undertaken by the incumbent it is in fact not possible to give incentives to pursue larger inventive steps with such a buyout scheme (where the incumbent would pay a fee to herself if she innovates).

10 Potential perils of strong patents

10.1 Fixed costs of entering the R&D sector

So far it has been assumed that there is free entry into the R&D sector and that there are only variable costs of undertaking R&D. Let us now look at the case where a firm which has not itself invented the currently newest quality good has to pay a fixed cost before being able to do R&D targeted at improving the quality of the good by one more step. Given entry into the R&D sector has occurred,

[25] They use a specification where R&D productivity $\lambda(\mu)$ is randomly changing over time so that large innovations should only be targeted if this productivity is large.

the analysis is similar as above and an incumbent firm with a lead of two steps does exactly as much R&D as needed in order to make entrants get (slightly less than) zero expected profits if they do R&D[26]. Entrants therefore do not do any R&D in equilibrium even if they have paid the fixed costs of entering the R&D sector. Expecting this outcome, there are therefore no incentives for entrants to pay the fixed costs of entering as they know that given they enter, the incumbent will undertake so much R&D that they cannot make any profits. Therefore, there is no entry in equilibrium and an incumbent with a two-step lead does no R&D at all (as this would only increase costs without rising profits above the current level π_2). Equilibrium growth ϕ_2 is therefore zero[27] if there are (arbitrarily small) positive fixed costs of entering the R&D sector and if the incumbent can preempt entry without ever doing R&D herself.

The result that equilibrium growth is zero if there are fixed costs of entering the R&D sector was derived under the assumption that patents never expire. However, if patents expire and if in the case of expiration the newest available quality of the good falls in the public domain, firms might again have incentives to undertake R&D in order to become the next leader by inventing a new patentable version of the quality good. Once a firm has obtained a sufficient lead, there is again no entry pressure and no R&D is undertaken until the patent expires. If patents do not expire too quickly and if the fixed costs of entering the R&D sector are not too high, average growth can therefore be positive as incumbents regularly lose their lead which gives incentives for new entrants to innovate in order to become the next incumbents. If patents however expire too quickly, no firm finds it profitable to do R&D so that equilibrium (average) growth is again zero. Average growth is therefore maximal for an intermediate strength of patent protection (or a more detailed analysis see *Appendix B (15.3)*).

If patent protection was weakened by allowing entry of a competitive fringe capable of producing the good with the currently highest quality but at higher marginal costs, this would reduce the maximal price that innovators (with a two-

[26] Again, it is assumed that incumbents move first in the R&D game.

[27] In a model where R&D increases profit flows, equilibrium growth would depend on the incentives of a monopolist to undertake R&D, and these are generally insufficiently small compared to the social optimum if there are problems of appropriability.

step lead) can charge but would lead to the same equilibrium in which there is no entry pressure as entrants do not find it profitable to pay the fixed costs of entering the R&D sector. If patents are infinitely lived, such a policy therefore cannot increase the average rate of growth. Short (that means quickly expiring) and broad (that means allowing for a large maximal markup) patents are therefore better in stimulating innovation than long and narrow ones.

If if it was possible to subsidize entry by fully reimbursing the fixed costs of entering the R&D sector incurred by entrants, such a policy would however be preferable to a reduction of patent protection as it would increase entry pressure and growth while the subsidies never have to be paid in equilibrium.

10.1.1 Application: Trade Secrecy

If innovations can be patented without the obligation to make the knowledge freely available to competitors, or if innovators completely rely on trade secrecy as a means of appropriation, competing firms first have to engage in duplicative catch-up R&D before they can start to conduct frontier R&D. And the expenses for the catch-up R&D play the same role as the fixed costs of entering the R&D sector discussed above. Therefore, entrants have no incentives to engage in catch-up R&D as they anticipate that they cannot make any profits once they get access to the same research technology as the incumbent. If catch-up R&D only allows to get access to the state of the art technology which is needed to conduct further R&D but does not allow to compete with the incumbent in the product market (due to patent protection), the incumbent does exactly as much R&D as needed to discourage R&D by the entrant who has caught up. And even if catch-up R&D allows the entrant to compete with the incumbent in the product market[28] (which is the case if there is only trade secrecy and no patent protection) this will not yield any profits if there is Bertrand competition and - once the catch-up occurred - the two firms might engage in such a fierce patent race that expected profits for entrants are zero or very small. Anticipating this outcome, entrants therefore never engage in costly catch-up R&D and due to the lack of entry pressure, incumbents with a two-step lead do not innovate. In order for

[28] This is usually assumed in "step - by - step" growth models like the one of Acemoglu and Akcigit (2008).

patents to stimulate growth it is therefore important that patented technologies are required to be made public so that potential competitors do not have to engage in catch-up R&D before they can access the state of the art technology. If it is hard to enforce patents if the state of the art technology is freely accessible to competitors, a solution might be to grant patents of shorter duration to firms that do not want to fully release their knowledge, but to oblige these firms to release their technological knowledge after patent expiration. Then, there are again incentives to undertake R&D once patents have expired. In the case where only trade secrecy can be used as a means of appropriation, average growth is maximal for an intermediate extent of trade secret protection, as in this case incumbents loose their lead from time to time so that there are incentives to race for the next innovation.

10.2 Ex ante agreements

Assume now that there is just a limited number of firms which are able to do R&D and that they can make ex ante agreements about their joint R&D effort and about how to split profits. Given one of these firms already has a lead of two steps and can charge price π_2, they clearly do not have any incentive to do further R&D (as they cannot get larger profits than π_2) and growth is zero. If it is difficult to prevent such ex ante agreements and to make firms compete, another possibility to increase growth above zero is to make patents expire with a positive probability (like in the case of fixed costs of entering the R&D sector): If no firm owns a patent and if the probability of patent expiration is not too large, firms capable of doing R&D find it profitable to innovate (even if there are ex ante agreements) as they can only make profits if they possess at least one patent on a version of the quality good that is newer than those that are in the public domain.

Here again the main assumption of the model that, once a two-step lead is obtained, nothing of the increase in consumer welfare arising from an innovation can be appropriated by innovators, is clearly unrealistic. Moreover, one can argue that ex ante agreements might improve R&D efficiency by exploiting synergies and avoiding costly duplication. But even in a setup where monopolists do some R&D in order to increase their profit flows, these R&D incentives tend to be low as a monopolist takes into account that she replaces her old innovation by introducing a new one. If the monopolist does not hold a patent on the old innovation anymore, this Arrow replacement effect disappears so that average growth might be increased if patents expire with a positive probability. And given that a monopolistic firm can only appropriate part of the social surplus created by an innovation and there are no deadweight losses of monopoly pricing, it does less R&D than socially optimal (if patents never expire), so that in the case where R&D and growth can be increased by making patents expire more quickly this also increases social welfare.

10.3 Heterogeneous R&D labour

If labour is heterogenous in the sense that some workers are better researchers than others but if all are equally good in producing the homogenous good, the wage per effective unit of R&D labour increases in the total number of researchers hired as less and less able people have to be recruited. In this case, preemption is not only possible if a certain amount of R&D is actually undertaken by the incumbent (like in the case with a perfectly elastic R&D - labour supply but decreasing R&D productivity) but also if the incumbent hires the most able researchers and makes them produce the homogenous good instead. Given that the pool of workers from which entrants can hire only contains sufficiently untalented researchers, they do not find it profitable to engage in R&D and there is again no growth in equilibrium if patents are fully protected. In the case where the R&D process requires different complementary inputs, it might even be possible to preempt entry by buying up a sufficient amount of a single R&D input which is in scarce supply. And even if such an input can only be used for research, the resulting R&D productivity will be very low as incumbents mainly focus on preempting entry and not on innovating. In these cases, the average rate of growth is again maximal for an intermediate strength of patent enforcement.

An interesting point in this and the last two sections is that, although there are no deadweight losses due to intellectual property rights, the government still has incentives to make patents expire more quickly ex post then promised ex ante as this encourages follow-on R&D and growth (as long as future innovators do not fear that the government will not enforce their patents either).

10.4 Intermediate R&D inputs

Let us now look at the case where in order to improve the quality of the good by one step, two R&D stages have to be completed. In the first stage, an intermediate R&D input (which might be thought of as an idea) has to be invented and this input is used in the second stage to invent an improved version of the quality good. The two stages could as well be interpreted as a research and a development phase. The R&D technology at each stage is again stochastic and assumed to be of the same form as in the main model, allowing for preemption due to decreasing R&D productivity at the industry level[29]. In the following I analyze the implications of granting patents on intermediate R&D inputs on the incentives to innovate.

If an incumbent with a two-step lead has patented the intermediate good required to invent the next generation of the quality good she can block further entry as in order to replace her as the leader, entrants need to have access to (a license for) the intermediate R&D input which can be denied if it is protected by a patent. As the incumbent cannot increase her profits through further innovation and as the value of an innovation for an entrant is lower than the value the incumbent attaches to keeping her leadership position, she never agrees to license the intermediate R&D input so that there is no entry. If an entrant has a patent on the intermediate good he can either license it to the incumbent or use if to proceed with the second R&D stage. Both firms clearly have incentives to license the input because the incumbent values the possibility to block entry more than the entrant values the possibility to undertake follow-on R&D. As a result, there is again no follow-on research in equilibrium. If it is however not possible or not allowed to license to the incumbent, the entrant uses the input in order to start the second stage of R&D aimed at inventing a new generation of the quality good which allows him to earn profit flows π_1 (until he advances to a two-step lead). Again, the incumbent values a patent on the intermediate good more than an entrant and if no firm has yet invented the intermediate good she will preempt

[29] The results would not change if at the second stage there was a deterministic technology specifying adoption costs as a decreasing function of adoption time and allowing for preemption.

entry (into the intermediate R&D goods sector) by undertaking the level of R&D needed to discourage R&D of entrants. Once the incumbent has invented the intermediate good, she can block further entry and equilibrium growth is zero.

In the case where no patents are granted on intermediate R&D inputs, no firm has an incentive to invent such a good: an entrant who invents it cannot make any profits out of it as it would fall in the public domain and would be supplied at marginal cost which also implies that there is free entry into the second stage R&D race which pushes expected profits in this stage down to zero. Also an incumbent with a two-step lead has no incentive to invent the intermediate good if no entrant does, as this does not allow to increase her profits. Therefore, equilibrium growth in this case without patents is zero as well. However, it is possible to have sustained growth if the leader is not allowed to patent intermediate R&D inputs but if the entrants are, with the restriction that they are not allowed to license to the leader. In this case, the incumbent has incentives to preempt entrant R&D at each stage without ever being able to block future entry completely: the reason for this is that the incumbent still finds it worthwhile to invent the intermediate good (even though it does not prevent entrants from participating in the race for the second R&D stage) as this prevents that entrants invent the intermediate good which would allow them to replace the incumbent in the future. And once the intermediate good is invented by the incumbent and freely accessible to entrants, the incumbent has again incentives to preempt entry in the race for the second R&D step (which leads to an improvement in the quality of the good) due to the standard reasoning in the main model. Once the next version of the quality good is invented, the whole process starts again.

To sum up, granting patents on intermediate goods only to entrants does not reduce the incentives for leaders to invent these goods and furthermore guarantees entry pressure and sustained R&D by leaders also in the second R&D stage. In order for this mechanism to work, it is however important that incumbents disclose the blueprints of intermediate goods that they discover, as keeping them secret would prevent entrants from competing in the second stage. While collusion in the intermediate good stage is bad for R&D incentives as it allows incumbents to block entry, allowing collusion (or licensing) in the product market if entrants have invented a new version of the quality good again increases entry pressure and innovation incentives due to the same mechanism as discussed above[30].

11 State dependent intellectual property protection

In the following I analyze the effects of patent expiration in the basic model. Like Acemoglu and Akcigit (2008) I allow for "state dependent intellectual property protection", meaning that the probability of patent expiration can depend on whether the firm has a one-step or a two - (or more -) step lead over its rivals. The patents of an incumbent with a one-step lead expire with the instantaneous Poisson arrival rate γ_1 and and those of an incumbent with a two-step lead with the instantaneous arrival rate γ_2. In the case of patent expiration the newest innovation of the incumbent falls in the public domain allowing competitors to fully catch up. This specification implicitly assumes that patents on second - newest goods never expire and the case in which firms with a two-step lead can lose this lead for a one-step lead (because their patent on the second newest good expires) is not considered.

The value of an innovation for an entrant V_E (that means the value of being one step ahead) and the value V_2 of being two (or more) steps ahead can now be derived from the following arbitrage conditions (again looking at the case where $\epsilon \to 0$):

$$rV_2 = \pi_2 - c\phi_2 - \gamma_2 V_2$$

$$rV_E = \pi_1 - c\phi_1 - \gamma_1 V_E - \phi_1 V_E + \phi_1 V_2$$

from which we get:

[30] Again, there might not be persistent leadership in this case anymore as entrants value entry into the product market in the same way as incumbents value not being replaced, so that incumbents might not preempt entry (that means they might "allow" an entrant to develop an intermediate R&D input and then do follow - on R&D in order to invent the next version of the quality good).

In the case where $\epsilon > 0$, an entrant who has invented the intermediate R&D input and patented it would invest less in R&D to find the new version of the quality good than an incumbent would need to invest to preempt entry in this case (as entry is only discouraged if average and not marginal R&D costs are equal to the marginal benefits). Therefore, preemption would be more expensive for incumbents than entry is for entrants and incumbents might prefer not to undertake any preemptive R&D.

$V_E = \frac{\pi_1 - c\phi_1}{r+\gamma_1+\phi_1} + \frac{\phi_1}{r+\gamma_1+\phi_1}\frac{\pi_2 - c\phi_2}{r+\gamma_2}$

This value decreases in both expiration rates, also if ϕ_1 is chosen as an optimal response to γ_1 and γ_2. The equilibrium arrival rate of an innovation might now depend on the state in which the economy is. If there is an incumbent with a two-step lead the arrival rate is ϕ_2 and it is determined by the zero profit condition $V_E = c$. Again looking at the limit case where $\epsilon \to 0$ in the more general model with decreasing R&D productivity, the arrival rate in the case where the newest generation of the good is in the public domain is also given by $\phi_0 = \phi_2$. While R&D is now undertaken by entrants and not by the incumbent (as there is none) the arrival rate is the same because the value of an innovation for an entrant is independent of whether he replaces an incumbent or not (if there is no collusion) and because the same zero profit condition holds in both cases which pins down the same equilibrium value of ϕ[31]. In the case where there is an incumbent with a one-step lead, the arrival rate is either given by the minimal amount necessary to preempt entry ($\phi_1 = \phi_2$) or by the maximally feasible rate $\phi_1 = \phi_m$. In the first case, the arrival rate is therefore the same in all possible states. As V_E decreases in the arrival rate if $\phi_1 = \phi_2$, the zero profit condition indicates that in this case an increase in any of the probabilities of patent expiration reduces the equilibrium arrival rate as long as the change in patent policy does not induce firms to chose $\phi_1 = \phi_m$ instead of $\phi_1 = \phi_2$.

In the case where $\phi_1 = \phi_m$, the arrival rate depends on the state of the economy and is higher in the case where the leader has a one-step lead than in the cases where no firm has a lead or where the leader has a two-step lead (in which it is given by $\phi_0 = \phi_2$). Now the question is whether patent policy can affect the choice of ϕ_1 and whether, given $\phi_1 = \phi_m$, increasing any of the rates of patent expiration can increase the probability of being in the state of high growth and even increase average growth through such a composition effect. In order to analyze this question, I first derive conditions under which firms with a one-step lead actually choose the high R&D effort $\phi_1 = \phi_m$ in equilibrium:

Lemma 2 *A firm with a one-step lead selects R&D effort* $\phi_1 = \phi_m$ *if* ϕ_m *is large*

[31] If $\epsilon = 0$, firms are actually indifferent with respect to the optimal value of ϕ but it is again assumed that they pick the value that results as the limit when $\epsilon \to 0$.

enough and if $\frac{\pi_2}{2c} - \gamma_2 > r > \frac{\pi_1}{c} - \gamma_1$ ***(Condition 2)***
Proof. *see Appendix C* ■

As **Condition 2** implies that $\gamma_1 > \gamma_2$ must hold, firms that are one step ahead therefore only have incentives to do more R&D than necessary to prevent entry if patents expire sufficiently more quickly in the case of a one-step lead then in the case of a two-step lead (and if collusion is not possible). Like in Acemoglu and Akcigit (2008), granting stronger patent protection to firms with a bigger lead induces firms with a smaller lead to race faster (increase ϕ_1) in order to obtain a bigger lead and stronger protection more quickly.

11.1 Average arrival rate

In the case where $\phi_1 = \phi_m > \phi_2 = \phi_0$, the arrival rate depends on the size of the lead, which itself changes stochastically over time. In order to calculate the average rate of growth (arrival rate) we need to compute for which fraction of the time the economy is in which state (on average). To simplify the intuition one can also think about a slightly modified version of the model in which there is a continuum of symmetric quality good sectors of mass one[32] and compute in which fraction of the sectors the lead is equal to 0, 1 or 2 steps. Denoting the proportion of time or the fraction of sectors in which the lead is equal to k steps by σ_k, the following conditions need to be satisfied in order to guarantee that the average entry into the state k equals the average exit to other states:

$$\sigma_o\phi_2 = \gamma_1\sigma_1 + \gamma_2\sigma_2 \quad (k = 0)$$
$$\sigma_1(\phi_m + \gamma_1) = \sigma_0\phi_2 \quad (k = 1)$$
$$\sigma_2\gamma_2 = \sigma_1\phi_m \quad (k = 2)$$

The left hand sides stand for the exit from the corresponding states k and the right hand sides for the entry into these states. Taking as an example the case where $k = 0$, the average fraction of sectors leaving this state is given by the probability of an innovation in this state ($\phi_2 = \phi_0$) times the fraction of sectors

[32] Utility in this case is given by: $U(\tau) = \int\limits_{t=\tau}^{\infty} \left(x(t) + \int\limits_{j=0}^{1} q_j(t)dj \right) e^{-\rho(t-\tau)}dt$ and the production technologies are the same for each sector.

where the state is given by a lead of zero (σ_0). Entry into this state occurs due to the expiration of patents in sectors with a lead of one or two steps and is given by the arrival rate of patent expiration in the case of a one-step lead times the fraction of sectors with a one-step lead ($\gamma_1\sigma_1$) plus the corresponding expression for $k = 2$ ($\gamma_2\sigma_2$). Using the condition $\sigma_0 + \sigma_1 + \sigma_2 = 1$ and these three equations we can compute:

$$\sigma_0 = \frac{(\phi_m+\gamma_1)\gamma_2}{(\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m)}$$

$$\sigma_1 = \frac{\phi_2\gamma_2}{(\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m)}$$

$$\sigma_2 = \frac{\phi_2\phi_m(\phi_m+\gamma_1)\gamma_2}{(\phi_m+\gamma_1)\gamma_2((\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m))}$$

The average arrival rate $\widehat{\phi}$ is now given by the weighted sum of arrival rates in the different states with the weights given by σ_k:

$$\widehat{\phi} = \phi_m\sigma_1 + \phi_2(\sigma_0 + \sigma_2) = \frac{\phi_2(\gamma_2(\phi_m+\gamma_1)+\phi_m(\phi_2+\gamma_2))}{(\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m)}$$

11.2 The effect of expiring patents on average growth

Proposition 3 *Given that ϕ_m is large enough and **Condition 2** holds so that $\phi_1 = \phi_m$, increasing any of the rates of patent expiration γ_1 or γ_2 decreases the average arrival rate $\widehat{\phi}$ (that means $\frac{\partial\widehat{\phi}}{\partial\gamma_1} < 0$ and $\frac{\partial\widehat{\phi}}{\partial\gamma_2} < 0$).*

Proof. *See Appendix D* ■

Increasing any of the instantaneous expiration rates decreases the value of an innovation for an entrant and therefore the arrival rate $\phi_2 = \phi_0$ in the states where no firm has a lead and where the leading firm has a two-step lead and preemptively innovates. Increasing γ_1 decreases the probability σ_1 that the economy is in the state with a one-step lead where the arrival rate is at its maximum $\phi_1 = \phi_m$ (see *Appendix D*). Therefore, increasing γ_1 unambiguously reduces the average arrival rate. Increasing γ_2 can however increase σ_1 so that there is a composition effect: reducing patent protection for firms with a two-step lead can increase the probability that the economy is in the state where there is a one-step lead and where the arrival rate is maximal. However, this composition effect is not strong enough to compensate the negative effect that an increase in γ_2 has on the arrival rate ϕ_2 in the other states so that an increase in γ_2 still leads to a reduction in the average arrival rate $\widehat{\phi}$[33].

For γ_1 given and large enough so that $\phi_1 = \phi_m$ initially holds, increasing γ_2 first continuously decreases the average arrival rate and at a certain threshold induces firm to choose $\phi_1 = \phi_2$ so that $\widehat{\phi}$ falls discontinuously[34]. After this threshold is passed, increasing γ_2 again continuously decreases $\widehat{\phi}$ and if γ_2 is sufficiently larger than γ_1 there is a (continuous) switch to a leapfrogging equilibrium as incumbents with a one-step lead do not find it profitable anymore to obtain a two-step lead and prefer not to do any R&D and to be replaced by entrants.

For γ_2 given and starting from γ_1 low enough so that $\phi_1 = \phi_2$, increasing γ_1 first reduces the average arrival rate, but when a certain threshold of γ_1 is reached, an upward jump of $\widehat{\phi}$ occurs while $\widehat{\phi}$ again decreases if γ_1 is increased beyond the threshold. There is therefore a non-monotonic relation between the average arrival rate and the probability of patent expiration in the state of a one-step lead.

It is important to note that using state dependent intellectual property rights is only possible if the patent office can observe the size of a firm ´s lead, which is even more difficult than to determine whether an innovation meets a certain patentability requirement. And if so much information is available it might actually be feasible and optimal to use prices and not patents to encourage innovation[35].

12 Entry costs and expiring patents

In the previous section it was shown that state - dependent intellectual property protection can induce firms with a one-step lead to exert the maximal R&D effort. However, this is not the only possibility to get such a situation. In fact, we can

[33] In a more general model where $\epsilon > 0$ and where there is no upper bound ϕ_m on the innovation rate, firms with a one - step lead would in addition reduce their R&D effort ϕ_1 if obtaining a two - step lead becomes less profitable (due to a reduction in γ_2) so that the composition effect would be even weaker.

[34] While the arrival rate ϕ_1 jumps discontinuously at this threshold, this is not the case for the value of an innovation and ϕ_2 as firms are indifferent between choosing $\phi_1 = \phi_m$ and $\phi_1 = \phi_2$ at the threshold level of γ_2.

[35] A prespecified price could be paid to an innovator depending on how much his innovation improved upon the previous generation of the good. In order to avoid deadweight losses, all innovations would be put in the public domain.

also have $\phi_1 = \phi_m$ in the case where there are sufficiently high fixed costs of entering the production sector (in the form of licensing or patent litigation fees for example). In the following, I analyze the effect of such entry costs on the average arrival rate if patents of firms with a two-step lead (but not those of firms with a one-step lead) expire regularly:

The value of an innovation for an entrant is given by: $V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r+\gamma} - F$ where γ stands for the instantaneous expiration rate of patents of firms with a two (or more) - step lead[36] and F for the fixed entry cost or licensing fee that even has to be paid (also by a previous incumbent) if no firm has a lead. The zero profit condition is given by: $V_E = F + c$ and for simplicity it is assumed that $\pi_2 = 2\pi_1$.

Proposition 4 *$\phi_1 = \phi_m$ holds in equilibrium if $F > \frac{\pi_1}{r} - c$ and $\phi_m > \frac{F}{c}(r-\gamma) - 2\gamma > 0$.*

Proof. *See Appendix E* ■

The fixed fee F therefore needs to exceed a certain threshold in order to induce firms with a one-step lead to exert the maximal R&D effort. The intuition for this is that increasing F reduces the value of an innovation for an entrant and therefore the amount of R&D that an incumbent with a two-step lead needs to undertake in order to preempt entry. This again increases the value of being two steps ahead and induces firms with a one-step lead to try harder to reach a two-step lead.

As derived in the section on state dependent intellectual property rights (but taking the case where $\gamma_1 = 0$ and $\gamma_2 = \gamma$) the average arrival rate is given by $\widehat{\phi} = \frac{\phi_2(\gamma\phi_m + \phi_m(\phi_2+\gamma))}{\phi_m\gamma + \phi_2(\gamma+\phi_m)}$. Given that $\phi_1 = \phi_m$ this rate decreases in F as ϕ_2 decreases in F (and $\widehat{\phi}$ increases in ϕ_2). Therefore, starting from $F = 0$ so that initially $\phi_1 = \phi_2$, increasing F first reduces the average arrival rate, but at a critical level, induces firms with a one-step lead to choose $\phi_1 = \phi_m$, which makes $\widehat{\phi}$ jump upward. Once this threshold level of F is surpassed, $\widehat{\phi}$ again decreases in F while firms keep their choice $\phi_1 = \phi_m$. In the case where $\phi_1 = \phi_m$, increasing the rate of patent expiration γ unambiguously reduces $\widehat{\phi}$ due to the same reasons due to

[36] This rate cannot be too large, as there would then be a leapfrogging equilibrium where no firm has incentives to ever reach a two- step lead where patents expire very quickly.

which in the case of state dependent intellectual property protection an increase in γ_2 led to a decrease in $\widehat{\phi}$ (Proof: see *Appendix F*).

13 Concluding remarks

While most of the literature analyzing the role of patents in models of cumulative innovation focuses on the case of leapfrogging, this paper looks at the other extreme of persistent leadership and preemptive patenting and comes to different conclusions in some points. While there are certainly cases where the assumption of leapfrogging makes sense and where preemption is not possible due to technological or informational reasons (or where full preemption is not desirable, like in *Section 8.1*), it still seems to be a relevant case and is furthermore consistent with the evidence that many incumbent firms do innovate in reality. And in the cases where patents expire, where collusion is possible between entrants and incumbents or where innovations are drastic, the model can also explain some turnover.

It is important to note that the result that innovation incentives are increased if a larger share of total profits is allocated to entrants at the expense of previous innovators depends on the assumption made in quality ladder models that each innovation builds on a previous innovation. If there is however an initial product innovation on which all following innovations build, the incentives to come up with this initial innovation clearly decrease if follow-on innovators can easily replace the initial innovator without any compensation. In such a case there is therefore a trade - off for patent policy and encouraging initial R&D (by requiring entrants to compensate previous inventors with licensing fees, by granting forward protection or by forbidding ex post collusion) comes at the cost of reducing follow-on R&D (see Chu, Cozzi and Galli (2010) and also Denicolò (2002)). Also in the case without an initial innovation but where innovating requires access to the knowledge base of previous innovators and in which this knowledge depreciates without costly maintenance effort, some compensation for earlier innovators implying a more backloaded profit stream for an entrant might be required in order to give incentives to invest in the conservation of previous knowledge on which future innovations build.

As this paper only looks at quality improving innovations for a given good (or different independent goods), it does not address the question of optimal patent breadth if substitutable but differentiated goods can be invented. While granting patents for such goods tends to increase the incentives to invent them or to increase their quality, it creates direct competitors for the initial good and reduces the profits that inventors can earn if they improve the quality of this good. If two differentiated goods are very substitutable so that consumers do not benefit much from consuming both of them instead of only one, it is clearly more efficient if R&D is only undertaken to (continuously) increase the quality of one of the two goods and not of both goods at the same time. In a decentralized equilibrium, the costly invention of closely substitutable goods that do not have a higher quality than an existing good could be prevented by making these inventions infringe on the patent of the newest generation of the existing good or by directly transferring them to the holder of this patent through a patent transfer scheme[37].

[37] In some cases, granting patents that however infringe on the patent of the good with the currently highest quality might be preferable to a patent transfer scheme as it would allow entrants to obtain some compensation if their innovation allows the industry leader to increase its total profits. This could be the case if entrants are able to develop goods that do not improve upon the maximal existing quality but are substantially differentiated so that they permit to extract more surplus from consumers if they are supplied simultaneously with the good that has the highest overall quality.

Like in the main model, noninfringing patents should only be granted to (or a patent transfer scheme only applied to) innovations that improve upon the highest of all existing qualities among closely substitutable goods. This would stimulate cumulative innovation by making profit flows frontloaded and encourage entrants to improve upon the good with the highest existing quality and not to engage in wasteful duplication in order to steal some of the market of previous innovators.

14 References

Acemoglu, Daron and Akcigit, Ufuk (2008); "State - Dependent Intellectual Property Rights Policy"; working paper

Aghion, Philippe and Peter Howitt (1992); "A model of Growth through Creative Destruction", Econometrica 60: 323- 51

Barro, R.J. and Sala- i- Martin, X. (1995); "Economic Growth" McGraw - Hill, New York

Bessen, James and Eric Maskin (forthcoming); "Sequential Innovation, Patents, and Imitation", Rand Journal of Economics, vol. 40, issue 4, pp. 611 - 635

Chu, Angus C. (2009); "Effects of Blocking Patents on R&D: A Quantitative DGE Analysis", Journal of Economic Growth, 2009, vol. 14, pp. 55-78.

Chu, Angus C., Cozzi, Guido and Silvia Galli (April 2010); "Innovation-Specific Patent Protection", IEAS Working Paper No. 09-A010

Cozzi, Guido (2007); "The Arrow Effect under Competitive R&D"; The B.E. Journal of Macroeconomics, Volume 7, Issue 1

Czarnitzki, Dirk and Federico Etro, and Kornelius Kraft (2008); "The Effect of Entry on R&D Investment of Leaders: Theory and Empirical Evidence", ZEW Discussion Paper No. 08-078

Denicolò, Vincenzo (2001); "Growth with non - drastic innovations and the persistence of leadership"; European Economic Review, vol. 45(8), pp. 1399-1413

Denicolò, Vincenzo (2002); "Sequential innovation and the patent-antitrust conflict", Oxford Economic Papers, pp. 1-20

Etro, F. (2004); "Innovation by Leaders", The Economic Journal, Vol. 114, 495 (April), pp. 281-310Fudenberg, Drew, Richard Gilbert, Joseph Stiglitz and Jean Tirole (1983) "Preemption, leapfrogging and competition in patent races," European Economic Review 22, 3-31.

Gilbert, Richard J & Newbery, David M G, (1982). "Preemptive Patenting and the Persistence of Monopoly," American Economic Review, vol. 72(3), pages 514-26

Hopenhayn, Hugo & Gerard Llobet & Matthew Mitchell, (2006); "Rewarding Sequential Innovators: Prizes, Patents, and Buyouts," Journal of Political Economy, vol. 114(6), pages 1041-1068

Horowitz, Andrew W. and Lai, Edwin L.- C. (1996); "Patent Length and the Rate of Innovation", International Economic Review vol. 37 (4), pages 785-801

Llobet, Gerard and Javier Suarez (February 2010); "Entrepreneurial Innovation, Patent Protection and Industry Dynamics"; CEMFI Working Paper No. 1001

Malerba, F. and L. Orsenigo (1999); "Technological Entry, Exit and Survival: and Empirical Analysis of Patent Data", Research Policy, 28, 643 - 660

O'Donoghue, Ted, Suzanne Scotchmer and Jacques François Thisse (1998) "Patent Breadth, Patent Life and the Pace of Technological Progress" Journal of Economics and Management Strategy, 7, 1-32.

O'Donoghue, Ted and Joseph Zweimüller (2004) "Patents in a Model of Endogenous Growth" Journal of Economic Growth, 9, 81-123.

O´Donoghue, Ted (1998); "A patentability requirement for sequential innovation"; RAND Journal of Economics Vol. 29, No. 4, pp. 654-679

Reinganum, Jennifer (1983); "Uncertain Innovation and the Persistence of Monopoly"; American Economic Review, Vol. 73, No. 4

Reinganum, Jennifer (1985); "Innovation and Industry Evolution"; The Quarterly Journal of Economics, Vol. 100, No. 1

Segal, Ilya, and Michael D. Whinston (2007). "Antitrust in Innovative Industries." American Economic Review, 97(5): 1703–1730.

15 Appendix 1

15.1 The case of Leapfrogging

If R&D can only be undertaken by firms that have ideas and if incumbents happen to never have ideas for follow-on R&D but if there are enough entrants who do, there is leapfrogging and R&D is only carried out by entrants. Therefore, no firm ever gets a lead of more than one step and profit flows are given by π_1. If the (correctly anticipated) rate of arrival of an innovation is given by ϕ, the following arbitrage condition determines the value of an innovation in this setup: $rV = \pi_1 - \phi V$ so that $V = \frac{\pi_1}{r+\phi}$. If the costs of R&D are again given by $n(t) = c\phi(t)^{1+\epsilon}$, entry occurs until average costs are equal to V (which only depends on the future realization of ϕ) so that the zero profit condition is given by: $V = c\phi(t)^{\epsilon}$. Taking again the limit where $\epsilon \to 0$ this condition can be solved for the equilibrium rate of R&D: $\phi = \frac{\pi_1}{c} - r$.

15.1.1 Compensating previous innovators

If a licensing fee F has to be paid to the previous innovator, the value of an innovation becomes $V = \frac{\pi_1}{r+\phi} - F(1 - \frac{\phi}{r+\phi})$. It decreases in F as the present discounted value of paying the licensing fee upon entry and receiving it back in the future if another firm enters is negative if the interest rate r is positive[38]. V also decreases in ϕ (given that $F < \frac{\pi_1}{r}$, which needs to hold for V to be positive) so that the zero profit condition $V = c$ implies that the equilibrium rate of innovation ϕ decreases in F. Introducing fees to compensate previous innovators therefore reduces growth like in the case of persistent leadership.

15.1.2 Voluntary deals and patent transfer scheme

Permitting voluntary deals between an entrant and the previous incumbent allows them to charge price π_2, so that they can share the value $V_c = \frac{\pi_2}{r+\phi}$. As the entrant can get $V_E = \frac{\pi_1}{r+\phi}$ without a voluntary deal he must at least receive the same

[38] Here it is assumed that the licensing fees don ´t depend on the level of quality realized through the innovation. If they would rise sufficiently with each innovation there could be a positive present discounted value due to these licensing fees. However, in our model where profits don ´t increase over time such a scheme has to be ruled out as it would resemble a Ponzi game.

amount with a deal while the previous incumbents´ outside option is zero (the profits derived if there is competition). Let us assume that the relative bargaining power between incumbents and entrants is always the same so that entrants get the share α of total profits and incumbents the share $1-\alpha$. In order to guarantee that the entrant gets more than his outside option the condition $\alpha > \frac{1}{2}$ needs to hold (as under this condition, $\frac{\alpha\pi_2}{r+\phi} > \frac{\pi_1}{r+\phi}$ is satisfied because $\pi_2 \leq 2\pi_1$). The value of an innovation is now given by $V_E = \frac{\alpha\pi_2}{r+\phi} + \frac{\phi(1-\alpha)\pi_2}{(r+\phi)^2}$ and it increases in α and decreases in ϕ if $\alpha \geq \frac{1}{2}$. The zero profit condition $V_E = c$ therefore implies that the equilibrium rate of innovation ϕ increases in the bargaining power α of the entrants. The intuition for this result is that profit flows for an entering firm become more frontloaded and have a higher present discounted value (for a given replacement rate ϕ) if the bargaining power of entrants is increased (this is basically the result of O ´Donoughue and Zweimüller (2004)). Again, growth is maximal if entrants have all the bargaining power and get the full profit flows π_2 right upon entry. This maximal rate of growth can again be implemented through a patent transfer scheme which forces previous inventors to hand their patents over to an entrant who successfully improves upon their innovation.

We can therefore conclude that also in the case of leapfrogging, imposing fees to compensate previous innovators is bad for growth, but that allowing voluntary deals (collusion) is growth enhancing and that the maximal rate of growth can be obtained with a patent transfer scheme.

15.2 A

In the case without forward protection the value of an innovation for an entrant is given by $V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$ and in the case with forward protection but without voluntary deal by $V_E = \frac{-c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$. In the latter case, π_1 is therefore replaced by zero. A firm with a one-step lead selects R&D effort $\phi_1 = \phi_m$ if $\frac{\partial V_E}{\partial \phi_1} > 0$, which is satisfied if $\phi_2 < \frac{\pi_2 - \pi_1}{c} - r$ (**Condition 3**). Given $\phi_1 = \phi_m$, we can solve the zero profit condition $V_E = c$ for ϕ_2 to get: $\phi_2 = \frac{\pi_2}{c} - \frac{r}{\phi_m c}(2c\phi_m + cr - \pi_1)$. Plugging this expression into **Condition 3** gives the condition $\phi_m(\pi_1 - cr) < -r(\pi_1 - cr)$. If $\pi_1 > cr$, which is the case if there is no forward protection and if **Condition 1** from *Section 3.2* is satisfied[39], there is a contradiction so that we get $\phi_1 = \phi_2$. In the case where $\pi_1 < cr$, which holds if there is forward protection but no voluntary deal, we obtain that $\phi_m > r > \frac{\pi_1}{c}$ (**Condition 4**) must hold in order to get $\phi_1 = \phi_m$. So, the question is whether **Condition 4** is compatible with the condition $\phi_m > \phi_2 > 0$ that must be be satisfied in order to have an equilibrium. $\phi_m > \phi_2$ is satisfied if ϕ_m is large enough and for ϕ_m large the condition $\phi_2 > 0$ holds if $\pi_2 > 2cr$ (**Condition 5**). Taken together, **Conditions 4** and **5** therefore require that $\pi_2 > 2cr > 2\pi_1$ which cannot hold in the case without forward protection where $\pi_2 \leq 2\pi_1$.

Summing up, a firm with a one-step lead selects R&D effort $\phi_1 = \phi_2$ in the case without forward protection but if there is forward protection without a voluntary deal $\pi_1 = 0$, so that firms with a one step lead select the high R&D effort $\phi_1 = \phi_m$ if $\pi_2 > 2cr$. If there is forward protection and $\pi_2 > 2cr$, an incumbent will only consider cooperating with the entrant after entry occurs if she gets at least the value $V_I(entry) = \frac{\pi_2}{r+\phi_m}$ which she can obtain without a voluntary deal. Given that the entrant has all the bargaining power, a voluntary deal under forward protection therefore allows him to obtain the value of an innovation $V_E = \frac{\pi_2 - c\phi_2}{r} - \frac{\pi_2}{r+\phi_m}$. Using the zero profit condition $V_E = c$, the arrival rate ϕ_2 in the case of forward protection and voluntary deal can now be computed as $\phi_2^F = \frac{(\pi_2 - cr)\phi_m - cr^2}{c(r+\phi_m)}$ (A). In the case without forward protection and voluntary deal the zero profit condition is given by $\frac{\pi_1 - c\phi_2^{NF}}{r+\phi_2^{NF}} + \frac{\phi_2^{NF}}{r+\phi_2^{NF}} \frac{\pi_2 - c\phi_2^{NF}}{r} = c$ (B), which determines the arrival rate ϕ_2^{NF} in the case without forward protection. In order to determine in which case the arrival rate is larger, we replace ϕ_2^{NF} in B by ϕ_2^F from A and - taking into consideration that the left hand side in (B) is decreasing in the preemptive R&D effort ϕ_2^{NF} - we find that $\phi_2^F > \phi_2^{NF}$ if $(\pi_2 - \pi_1)\left(rc\phi_m^2 + 2r\phi_m + cr^2\right) - \phi_m r \pi_2^2 > 0$. As this condition is satisfied it ϕ_m is large enough, we can conclude that the rate of growth is larger in the case with forward protection and voluntary deal than in the case without forward protection and without voluntary deal if the entrant has all the bargaining power, if $\pi_2 > 2cr$ and if ϕ_m is large enough.

[39] Inserting the equilibrium conditions $\pi_1 = \mu$ and $r = \rho$ into **Condition 1** ($\mu > \rho c$) gives $\pi_1 > rc$.

15.3 B

In the following, I look at the case where patents expire with a constant (flow) probability γ and where in the case of expiration, the newest available quality of the good falls in the public domain and is supplied at the marginal cost of zero[40]. In this case the incumbent loses all profits and becomes equal to all other potential entrants.

In the case where the currently newest good is in the public domain, the R&D incentives depend on the value of getting a one-step lead which is given by $V_1 = \frac{\pi_1 - c\phi_1}{r+\gamma+\phi_1} + \frac{\phi_1 \frac{\pi_2}{r+\gamma}}{r+\gamma+\phi_1}$. A firm with a one-step lead that faces no entry pressure sets it ´s R&D level equal to $\phi_1 = \phi_m$ if $\gamma \leq \frac{\pi_2 - \pi_1}{c} - r$ (as then $\frac{\partial V_1}{\partial \phi_1} \geq 0$) and does not do any R&D ($\phi_1 = 0$) if $\gamma > \frac{\pi_2-\pi_1}{c} - r$. If there is no incumbent monopolist and if at least one firm has sunk the fixed cost of entering the R&D sector, it chooses the R&D level $\phi_0 = \phi_m$ if $V_1 > c$ (as then the marginal benefit of R&D is larger than the marginal cost c). If $\gamma \leq \frac{\pi_2-\pi_1}{c} - r$, this condition is satisfied and we have $\phi_0 = \phi_1 = \phi_m$. If $\frac{\pi_1}{c} - r > \gamma > \frac{\pi_2-\pi_1}{c} - r$, we get $\phi_1 = 0$ and $\phi_0 = \phi_m$ and if $\gamma > \frac{\pi_1}{c} - r$ we get $\phi_0 = \phi_1 = 0$. While a firm with a two -step lead does not do any R&D ($\phi_2 = 0$), a firm with a one-step lead therefore does the maximal amount of R&D ($\phi_1 = \phi_m$) if $\gamma \leq \frac{\pi_2-\pi_1}{c} - r$ and in the case where no firm has a lead, we get the maximal R&D effort ($\phi_0 = \phi_m$) if $\gamma < \frac{\pi_1}{c} - r$. However, in the latter case there is only entry into the R&D sector if the expected value of entering is larger than the fixed costs R of entering, that means if $V_0 = \frac{\phi_m(V_1-c)}{r} > R$ which is more likely to be satisfied if the probability of patent expiration γ is low and if the fixed costs R are low[41]. From the results

[40] The specification is chosen for reasons of simplicity and neglects the case where firms with a two - step lead lose this lead for a one - step lead because their patent on the second newest good expires. It is therefore implicitly assumed that patents on second - newest goods never expire.

[41] If entry into the R&D sector is unprofitable if already one firm has entered and undertakes the maximal amount of R&D, there is clearly the issue of which firm can enter first and get some rents. For simplicity it could be assumed that one of the potential entrants (not including the previous incumbent) is drawn randomly to get the right to move first. Another way of modeling would be to assume that there are no fixed costs of entering the R&D sector if the currently newest good is in the public domain. Even though there is the upper bound on the total arrival rate ϕ_m, the zero profit condition can be satisfied with equality if firms in the aggregate undertake more R&D than necessary to obtain ϕ_m so that average costs increase above c. This can happen if the individual probability of obtaining a patent depends on the

above we can calculate the long run average rate of growth g. If patents never expire ($\gamma = 0$), the incumbent firm does not do any R&D once it has reached a lead of two steps as there is no entry pressure and average growth is zero. If patents expire too quickly ($\gamma > \frac{\pi_1}{c} - r$), firms do not find it profitable to race for the next innovation once old patents have expired so that average growth is again zero. For an intermediate probability of patent expiration (and low enough fixed costs of entering the R&D sector) average growth is however positive as incumbents regularly lose their lead which gives incentives for new entrants to innovate in order to become the next incumbents. The average arrival rate of an innovation (average growth) is given by[42] $g = \frac{\gamma^3+2\gamma^2\phi_m}{(\phi_m+\gamma)^2}$ if $\gamma \leq \frac{\pi_2-\pi_1}{c} - r$ and by $g = \frac{\phi_m\gamma}{\phi_m+\gamma}$ in the case where $\frac{\pi_1}{c} - r > \gamma > \frac{\pi_2-\pi_1}{c} - r$. In both cases g can be shown to increase in γ.

15.4 C

A firm with a one-step lead selects R&D effort $\phi_1 = \phi_m$ if $\frac{\partial V_E}{\partial \phi_1} > 0$, which is satisfied if $\phi_2 < \frac{\pi_2}{c} - r - \gamma_2 - \frac{\pi_1(r+\gamma_2)}{c(r+\gamma_1)}$ (**Condition 6**). Given $\phi_1 = \phi_m$, we can solve the zero profit condition $V_E = c$ for ϕ_2 to get: $\phi_2 = \frac{\pi_2}{c} - \frac{r+\gamma_2}{\phi_m c}(2c\phi_m + cr + c\gamma_1 - \pi_1)$. Plugging this expression into **Condition 6** gives the condition $\phi_m(\pi_1 - cr - c\gamma_1) < -(r+\gamma_1)(\pi_1 - cr - c\gamma_1)$. In the case where $\pi_1 - cr - c\gamma_1 < 0$, this implies that $\phi_m > r + \gamma_1 > \frac{\pi_1}{c}$ (**Condition 7**) must hold in order to get $\phi_1 = \phi_m$. So the question is whether **Condition 7** is compatible with the condition $\phi_m > \phi_2 > 0$ that must be be satisfied in order to have an equilibrium. $\phi_m > \phi_2$ is satisfied if ϕ_m is large enough and for ϕ_m large the condition $\phi_2 > 0$ holds if $\frac{\pi_2}{c} - 2(r+\gamma_2) > 0$ (*Condition 8*). Taken together, **Conditions 7** and **8** therefore require that $\frac{\pi_2}{2c} - \gamma_2 > r > \frac{\pi_1}{c} - \gamma_1$. As $\pi_2 \leq 2\pi_1$, this condition implies that $\gamma_1 > \gamma_2$ must hold.

Summing up, a firm with a one-step lead selects R&D effort $\phi_1 = \phi_m$ if ϕ_m is

ratio between individual and total R&D spending.

[42] The average arrival rate is calculated in the following way (see also section *11.1*): the proportions of time σ_k in which the lead is equal to k steps can be derived by setting the probability of losing a k - step lead due to patent expiration or R&D equal to the probability of obtaining such a lead coming from states with different sizes of the lead. The average arrival rate is then simply given as $g = \sum_{i=1}^{k} \sigma_i \phi_i$.

large enough and if $\frac{\pi_2}{2c} - \gamma_2 > r > \frac{\pi_1}{c} - \gamma_1$, which implies that $\gamma_1 > \gamma_2$ must hold.

15.5 D

Given $\phi_1 = \phi_m$, we can solve the zero profit condition $V_E = c$ for ϕ_2 to get: $\phi_2 = \frac{\pi_2}{c} - \frac{r+\gamma_2}{\phi_m c}(2c\phi_m + cr + c\gamma_1 - \pi_1)$. From this we obtain $\frac{\partial \phi_2}{\partial \gamma_1} < 0$ and $\frac{\partial \phi_2}{\partial \gamma_2} < 0$ given that $r > \frac{\pi_1}{c} - \gamma_1$ (**Condition 2** from *Lemma 2*) holds. The average arrival rate is given by: $\widehat{\phi} = \phi_m \sigma_1 + \phi_2(\sigma_0 + \sigma_2) = \phi_m \sigma_1 + \phi_2(1 - \sigma_1)$. Using $\sigma_1 = \frac{\phi_2 \gamma_2}{(\phi_m + \gamma_1)\gamma_2 + \phi_2(\gamma_2 + \phi_m)}$, we obtain $sign \frac{\partial \sigma_1}{\partial \gamma_1} = sign \left\{ \frac{\partial \phi_2}{\partial \gamma_1} (\phi_m + \gamma_1) \gamma_2 - \phi_2 \gamma_2 \right\} < 0$, so that we get $\frac{\partial \widehat{\phi}}{\partial \gamma_1} = \frac{\partial \sigma_1}{\partial \gamma_1}(\phi_m - \phi_2) + \frac{\partial \phi_2}{\partial \gamma_1}(1 - \sigma_1) < 0$.

Deriving the average arrival rate $\widehat{\phi} = \frac{\phi_2(\gamma_2(\phi_m+\gamma_1)+\phi_m(\phi_2+\gamma_2))}{(\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m)}$ with respect to γ_2 gives $sign \frac{\partial \widehat{\phi}}{\partial \gamma_2} =$
$sign \frac{\partial \phi_2}{\partial \gamma_2} \left[(\gamma_2 (\phi_m + \gamma_1) + \phi_m(\phi_2 + \gamma_2)) \gamma_2(\phi_m + \gamma_1) \ + \phi_2 \phi_m (\gamma_2(\phi_m + \gamma_1) + \phi_2(\gamma_2 + \phi_m)) \right] +$ $\phi_m \phi_2^2(\phi_m - \phi_2) < 0$. The derivative is negative as $\frac{\partial \phi_2}{\partial \gamma_2} = -2 - \frac{r + \gamma_1 - \frac{\pi_1}{c}}{\phi_m} < -2$ under **Condition 2**■

15.6 E

We have $\phi_1 = \phi_m$ if $\frac{\partial V_E}{\partial \phi_1} > 0$, which holds if $\phi_2 \leq \frac{\pi_2}{c} - \frac{(\pi_1 + cr)(r+\gamma)}{rc}$ (**Condition 9**). Inserting the equilibrium value $\phi_2 = \frac{\pi_2}{c} - 2r - 2\gamma - \frac{r+\gamma}{c\phi_m}(cr + Fr + F\phi_m - \pi_1)$ obtained from the zero profit condition into **Condition 9** implies that $r(\pi_1 - cr - rF) \leq -\phi_m(\pi_1 - cr - rF)$, which can only hold if $\pi_1 - cr - rF < 0$ (**Condition 10**). Taking the case where $\pi_2 = 2\pi_1$ we get $\phi_2 > 0$ if $\frac{F}{c}(r - \gamma) - 2\gamma > 0$. $\phi_m > \phi_2$ holds if $\phi_m^2 > \phi_m \left(\frac{\pi_2}{c} - 2r - 2\gamma - \frac{F(r+\gamma)}{c} \right) - \frac{r+\gamma}{c}(cr + Fr - \pi_1)$, which is satisfied even for the maximum values of π_1 and π_2 (given by **Condition 10**) if $\phi_m > \frac{F}{c}(r - \gamma) - 2\gamma$.

Summing up, $\phi_1 = \phi_m$ holds in equilibrium if $\pi_1 - cr - rF < 0$ and if $\phi_m >$

$\frac{F}{c}(r - \gamma) - 2\gamma > 0$■

15.7 F

Using the same analysis as in *Appendix D* but with $\gamma_1 = 0$ and $\gamma_2 = \gamma$, we can derive: $sign \frac{\partial \widehat{\phi}}{\partial \gamma} =$

$sign\left\{\frac{\partial\phi_2}{\partial\gamma}\left[\gamma\phi_m+\phi_m^2(\phi_2+\gamma)\gamma+\phi_2\phi_m(\gamma\phi_m+\phi_2(\gamma+\phi_m))\right]+\phi_m\phi_2^2(\phi_m-\phi_2)\right\}<$ 0 as $\frac{\partial\phi_2}{\partial\gamma}=-2-\frac{cr+Fr+F\phi_m-\pi_1}{c\phi_m}<-2$ due to **Condition 10** from *Appendix E*■

16 Appendix 2: Increasing profit flows

This section studies a more general setup in which even firms with a two-step lead can increase their profits by conducting R&D. Like Denicolò (2001), I look at a one - sector version of the growth model of Barro and Sala - i - Martin (1995, Chapter 7).

16.1 Model setup and equilibrium

The economy is populated by identical individuals of mass one who inelastically supply one unit of labour and intertemporal preferences are given by $U(\tau)=\int\limits_{t=\tau}^{\infty}c\,(t)\,e^{-\rho(t-\tau)}dt$. There is a final good y which can be consumed, used for research or used one - for - one to produce intermediate goods x_i, of which there exist generations $s\in\{1,...,k\}$. The final good is produced using labour (which is in fixed supply) and intermediate goods according to the following production function: $y_k=X_k^{\alpha}$, with $X_k=\sum\limits_{s=0}^{k}q^s x_i$ and $0<\alpha<1$. q^s indicates the quality of the intermediate good of generation s and innovation allows to introduce new intermediate goods the quality of which is increased by the factor $q>1$ compared to the previous generation.

The final good sector is assumed to be competitive while intermediate goods can be protected by patents. Since different generations of intermediate goods are perfect substitutes, only the best quality (the newest generation) is used in equilibrium so that the final good production function reduces to $y_k=q^{k\alpha}x_k^{\alpha}$. In equilibrium, the rate of interest r coincides with the rate of time preference ρ. Normalizing the price of the final good to one, the demand for the latest generation of the intermediate good as a function of its price p_k can be derived as

(1) $x_k=\alpha^{\frac{1}{1-\alpha}}q^{\frac{k\alpha}{1-\alpha}}p_k^{-\frac{1}{1-\alpha}}$

In a stationary equilibrium p_k is constant and from (1) and the production function it follows that the growth factor between two innovations in terms of the final good is given by $g \equiv \frac{y_{k+1}}{y_k} = q^{\frac{\alpha}{1-\alpha}}$.

Given the newest innovation (generation k of the intermediate good) is protected by a patent, its price is set in order to maximize profits $\pi_k = (p_k - 1)\, x_k$. If innovations are drastic or if the lead of the leading firm is so large (due to successive innovations or collusive agreements with the closest competitors) that no competitor can profitably underprice her, the (unconstrained) monopoly price, which is given by $p^M = \frac{1}{\alpha}$, is charged. Monopoly profits are then given as:

(2) $\quad \pi_k^M = \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) q^{\frac{k\alpha}{1-\alpha}}$

which can also be written as $\pi_k^M = \pi^M g^k$ with $\pi^M \equiv \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha)$.

Innovations are therefore drastic if $q \geq \frac{1}{\alpha}$. In the case where innovations are non - drastic ($q < \frac{1}{\alpha}$) and the last generation of the quality good is available to a competitor, there is limit pricing and the leader charges a price equal to its quality advantage $p^c = q < p^M$ in order to keep competitors out of the market. In this case profits are given by

(3) $\quad \pi_k^C = \alpha^{\frac{1}{1-\alpha}} (q-1)\, q^{\frac{k\alpha}{1-\alpha}} q^{-\frac{1}{1-\alpha}}$

which can also be written as $\pi_k^C = \pi^C g^k$ where $\pi^C \equiv \alpha^{\frac{1}{1-\alpha}} (q-1)\, q^{-\frac{1}{1-\alpha}}$.

In the following it is assumed that $\alpha q^2 \geq 1 \geq \alpha q$, which implies that each single innovation is non - drastic, but that a two-step lead allows the leader to charge the unconstrained monopoly price. π_k^C and π_k^M can therefore be reinterpreted as profits of firms with a one-step and a two (or more) - step lead[43]. Compared to the simple model used in the analyses above, the main difference is therefore that these profits grow by the factor g when an innovation takes place.

R&D can be undertaken by using the final good as an input. The arrival rate of the $k+1$ th innovation is given by $\phi(k+1) = \min\left\{ \left(\frac{n}{cg^k}\right)^{\frac{1}{1+\epsilon}}, \phi_m \right\}$ where n denotes the total amount of the final good used in the R&D sector and $\epsilon \geq 0$. ϕ_m again sets an upper bound which the arrival rate cannot surpass due to technological restrictions. The difference of this specification compared to that in the simple model is that R&D costs $n(\phi)$ increase from generation to generation. This assumption is needed to offset the effect of increasing profit flows and to obtain a balanced growth path with a constant innovation arrival rate. It is again

[43] If a price cap $q < \overline{p} < \frac{1}{\alpha}$ is imposed, profits of firms with a two - (or more -) step lead are decreased, but $\pi_k^C < \pi_k^M$ still holds.

assumed that all firms have access to the same R&D technology and that there is free entry into the R&D sector. In the following part of the analysis it is assumed that an innovator obtains an infinitely lived patent that prevents others from producing her generation of the good and that there is no forward protection and that voluntary deals are not permitted.

As profit flows grow with each innovation, even incumbents might gain from innovating in this setup and it is not as clear as in the simple model whether entrants or incumbents have the larger stand - alone innovation incentives. If an incumbent with a two-step lead innovates, she obtains profits $\pi^M g^{k+1}$ instead of the previous profits that were given by $\pi^M g^k$. Incumbents with a one-step lead also obtain profits $\pi^M g^{k+1}$ if they innovate, but these replace the lower one-step lead profits that were given by $\pi^C g^k$. Therefore, incumbents with a one-step lead have larger stand - alone innovation incentives than incumbents with a two-step lead (as $\pi^M(g^{k+1} - g^k) < \pi^M g^{k+1} - \pi^C g^k$ if $\pi^M > \pi^C$). Entrants gain $\pi^C g^{k+1}$ if they innovate (and do not lose any previous profits) and their stand - alone innovation incentives are larger than those of incumbents with a two-step lead if $\pi^C g^{k+1} > \pi^M(g^{k+1} - g^k)$, that means if $\frac{\pi^M}{\pi^C} < \frac{g}{g-1}$. In *Appendix X* (*Section 16.4*) I show that this condition holds if $\alpha q^2 \geq 1 \geq \alpha q$, so that there is again the well - known Arrow replacement effect.

As R&D productivity is decreasing at the industry level ($\epsilon \geq 0$), incumbents can again preempt entry and as they value not being replaced more than entrants value entry, they do all the R&D so that there is again persistent leadership. Due to the Arrow replacement effect, incumbents with a two-step lead only conduct as much R&D as necessary to prevent entry, so that the equilibrium innovation rate is again determined by a zero profit condition and depends on the value of an innovation for an entrant who expects to become the new incumbent after entry.

In order to keep the analysis simple, I again focus on the limit case where $\epsilon \rightarrow 0$ in the following. The expected value $V_2(k)$ of having a two-step lead and supplying generation k of the intermediate good can now be calculated from the following arbitrage condition:

$$rV_2(k) = \pi^M g^k - \phi_2 c g^k - \phi_2 V_2(k) + \phi_2 V_2(k+1)$$

The last terms on the right hand side indicate that if an innovation occurs with the arrival rate ϕ_2, the firm stops supplying generation k and starts supplying generation $k+1$ of the good. Taking into account that $V_2(k+1) = gV_2(k)$ along a balanced growth path, this condition can be rewritten as

(4) $V_2(k) = g^k \frac{\pi^M - \phi_2 c}{r - \phi_2(g-1)}$

A balanced growth path however only exists if the denominator is positive, which is the case if $r > \phi_2(g-1)$. The expected value $V_1(k)$ of getting a one-step lead and supplying generation k of the intermediate good (which coincides with the value of an innovation for an entrant $V_E(k)$) can be derived from the arbitrage condition:

$rV_1(k) = \pi^C g^k - \phi_1 c g^k - \phi_1 V_1(k) + \phi_1 V_2(k+1)$

Inserting $V_2(k+1) = g^{k+1} \frac{\pi^M - \phi_2 c}{r - \phi_2(g-1)}$ from (4) we can solve for

(5) $V_1(k) = g^k \left(\frac{\pi^C - \phi_1 c}{r + \phi_1} + \frac{\phi_1}{r + \phi_1} \frac{g\left(\pi^M - \phi_2 c\right)}{r - \phi_2(g-1)} \right)$

ϕ_1 and ϕ_2 again denote the innovation arrival rates chosen by firms with a one - and a two-step lead, which turn out to be independent of k. As the value that the invention of the k th generation of the intermediate good has for an entrant is given by $V_1(k)$ and the innovation costs when the current version of the intermediate good has quality $k-1$ are given by $n = cg^{k-1}\phi$, the zero profit condition is given by $V_1(k) = cg^{k-1}$, which can be written as:

(6) $\frac{\pi^C - \phi_1 c}{r + \phi_1} + \frac{\phi_1}{r + \phi_1} \frac{g\left(\pi^M - \phi_2 c\right)}{r - \phi_2(g-1)} = \frac{c}{g}$

This condition is similar to the one obtained in the simple model, with the difference that there is the growth factor g (an increase in g increases equilibrium growth) and that π_1 and π_2 are replaced by π^C and π^M. In order to determine the value of ϕ_1, we again need to determine how the value of an innovation for an entrant $V_1(k)$ depends on ϕ_1. Deriving (5) we find that $sign \frac{\partial V_1(k)}{\partial \phi_1} = sign \left\{ \phi_2 \pi^C (g-1) - \pi^C r + g r \pi^M - c r^2 - c r \phi_2 \right\}$ which is independent of ϕ_1. As in the simple model we therefore have $\phi_1 = \phi_m$ if $sign \frac{\partial V_1(k)}{\partial \phi_1} > 0$ and $\phi_1 = \phi_2$ if $sign \frac{\partial V_1(k)}{\partial \phi_1} \leq 0$. The equilibrium rate of growth ϕ_2 is determined by the zero profit condition in which the appropriate value of ϕ_1 is inserted.

16.1.1 The "chicken - egg" problem

The setup of the model analyzed here can be considered to be a bit ad - hoc or lacking microeconomic foundations as there is the "chicken - egg" problem that the final good is produced using intermediate goods that are again produced using the final good.

Thinking about micro - foundations, one might want to imposes the constraint that final goods can only be produced out of intermediate goods that already exist and assume that there is a (tiny) amount of the intermediate good to start with and that it is possible to repeat the process of producing final goods out of intermediate goods ad infinitum in a given period (using the same production function). However, this would allow to produce an infinity of the final good if the labour that is used to produce the final good would not wear out if the production process is repeated. And even if each unit of labour could only be used once to transform intermediate into final goods, the economy would not converge to the fixed point derived in the model above. It would therefore be necessary to specify how much of the intermediate good is available at the start, how often the production process can be repeated at a given point in time and what the shape of the production function is for each repetition.

All these problems can be avoided if a model without intermediate goods (in the line of the simple model studied in the main part of the paper) is used:

Utility is given by $U(\tau) = \int\limits_{t=\tau}^{\infty} c\,(t)\, e^{-\rho(t-\tau)}dt$ with $c(t) = \left(q^k \cdot x(q^k)\right)^{\alpha} + h$ where $x(q^k)$ indicates the quantity of the good of quality q^k that is consumed (in equilibrium, only the highest quality is consumed) and h stands for the consumption of a homogenous good. The total labour endowment is given by L and can be used one - for - one to produce the homogenous good or the quality good or to do R&D (using the same R&D production function as in the intermediate goods model). This model yields identical solutions for prices, profits and the consumption of the quality good and also the welfare analysis is (qualitatively) the same. However, as the consumption of the quality good and profits grow over time, the consumption of the homogenous good declines and shrinks to zero if labour supply does not increase over time. Therefore, the model only yields an interior solution if either the labour endowment is very large to start with or if it grows at a sufficient rate so that the consumption of the homogenous good never drops to zero.

16.2 The effects of patent policies

As there is persistent leadership and incumbents only do as much R&D as needed to prevent entry, growth depends positively on the extent of entry pressure, that means on the value that an innovation has for an entrant. Like in the simple model, the value of an innovation for an entrant increases if the profits of firms with a one-step lead ($\pi^C g^k$) increase. Therefore, allowing **voluntary deals** that permit entrants to consolidate market power with previous incumbents and to avoid the phase of competition again increases growth.

Requiring entrants to pay **licensing fees** upon entry in order to compensate previous incumbents reduces the value of an innovation for an entrant and growth like in the simple model as entrants expect to become the next leaders and to never receive any licensing payments from others in the future[44].

Also the analysis of **forward protection** in this setup is very similar, but now a voluntary deal allows to increase joint profits by using the superior production technology of entrants: if an entrant has invented generation $k+1$ of the quality good, the incumbent can still obtain the previous profit flows $\pi^M g^k$ if she does not permit the entrant to produce, but if she negotiates with the entrant in order to obtain access to the new technology, joint profit flows can be increased to $\pi^M g^{k+1}$. If a voluntary deal however does not allow to decrease joint R&D expenditures, forward protection still reduces growth compared to the case without forward protection and without a voluntary deal. The reason for this is that the entrant can maximally (if he has all the bargaining power) get the additional profits $(g^{k+1} - g^k)\pi^M$ resulting from his innovation until he advances to a 2 - step lead. But these additional profits are - due to the Arrow replacement effect - lower than the profit $\pi^C g^{k+1}$ which he would get without forward protection in the phase of competition. Therefore, forward protection decreases the value of an innovation

[44] Even if there was leapfrogging (because incumbents don ´t have ideas for R&D) it is not possible to increase innovation incentives by making licensing fees grow sufficiently over time. The reason for this is that licensing fees can maximally grow in line with profits (otherwise there would be a Ponzi game), but that in equilibrium the interest rate has to be at least as large as this expected rate of profit growth so that discounted profits cannot increase if licensing fees are introduced.

for an entrant and equilibrium growth if it does not allow for a considerable reduction in joint R&D costs.

As increasing π^C increases the value of an innovation for an entrant, entry pressure and growth are maximal if entrants get the maximal possible profit flows $\pi^M g^k$ right upon entry. And this can again be obtained through a **patent transfer scheme**. Therefore, growth is again maximized if a patent transfer scheme is implemented. And such a scheme would prove particularly beneficial in the case where private negotiations (e.g. voluntary deals and in particular licensing agreements) are inefficient due to asymmetries of information (like for example in Bessen and Maskin (forthcoming)).

In quality ladder models like the one studied here, an increase in the rate of growth might however not always be **welfare** improving, as equilibrium growth might be excessive for certain parameter values[45]. However, this does not imply that a good way to reduce growth is to make profit flows more backloaded or to discourage entry by reducing π^C. In fact, it is always preferable to impose a price cap equal to $\overline{p} < \frac{1}{\alpha}$ on incumbents if this is possible[46]. Such a price cap decreases profits $\pi^M g^k$ of firms with a two-step lead and therefore also the value of an innovation for an entrant and the rate of growth, but at the same time it reduces monopoly distortions stemming from patent protection, which is not the case if entry pressure is reduced by reducing π^C or in another way which does not affect π^M. As there is persistent leadership, quality goods are only

[45] If an increase in growth is obtained at the cost of a decrease in current consumption arising from larger monopoly distortions, welfare might rise or fall. Denicolo (2001) gives numerical examples for both cases.

In the case where $\epsilon > 0$ there is a further effect (beside the "business stealing effect" that can arise in quality ladder models) through which overall innovation incentives might become excessive: if R&D productivity decreases at the industry level and there is free entry, firms don ´t take into account that their R&D activity also increases the costs of other firms, so that more R&D is undertaken than in the case where entrants maximize their joint profits. Put differently, entry occurs until the value of an innovation is equal to the average and not the marginal costs of innovating and this condition is satisfied for a larger overall amount of R&D. This effect is the same as the one analyzed in the well known problem of the "tragedy of the commons".

[46] This could also be done by patent policy by allowing a competitive fringe to produce copied versions of the good that have higher production costs or a lower quality. Then, incumbents would have to cut their price in order to prevent entry.

supplied by incumbents with a two - (or more -) step lead so that monopoly distortions only depend on the price charged by firms with a two-step lead and not on the hypothetical price that would arise in a phase of competition where a firm has a one-step lead. Reducing the price and profits π^C of firms with a one-step lead therefore only reduces entry pressure and growth without reducing monopoly distortions. Because of that, the optimal patent policy should always make profit flows as frontloaded as possible (that means set π^C equal to π^M like in the case of a patent transfer scheme) and reduce innovation incentives in the case where they are excessive by limiting the maximal price $\overline{p}$ and profits $(\pi^M g^k)$ that firms with the maximal lead can charge (for a more formal analysis of this argument see *Appendix Y* (*Section 16.5*).

16.3 Patentability requirement

Let us now assume that the amount of the final good n_i that a firm i needs to use as a research input in order to improve the quality $\overline{q}$ of the current generation of the quality good by the factor μ (> 1) and to obtain the arrival rate ϕ is given by $n_i = c\overline{q}^{\frac{\alpha}{1-\alpha}}\phi\lambda(\mu)(n_{tot})^{\epsilon}$ with $\frac{\partial\lambda(\mu)}{\partial\mu} > 0$ and with $n_{tot} = n_i + n_{-i}$ indicating the overall (industry - wide) amount of R&D labour used. The only difference between this specification and that in the simple model is that R&D costs are assumed to grow at the same rate as output (note that $y = K\overline{q}^{\frac{\alpha}{1-\alpha}}$ with K constant) in order to obtain a balanced growth path. Again, all what matters for preemption is therefore the total amount of resources used for R&D. As long as incumbents value not being replaced more than entrants value entry, they therefore preempt entry and if their stand - alone innovation incentives are lower than that of entrants, they just do as much R&D as needed to prevent entry. In this case, entry pressure and growth can again be increased if the patentability requirement for entrants and incumbents with a one-step lead is relaxed (if it was binding before). Given the R&D technology specified above and the previous result about the existence of the Arrow replacement effect if innovative steps are equal for entrants and incumbents (with the notation $\mu = q$), there clearly is preemption if both entrants and incumbents (with a two-step lead) face the same binding patentability requirement. And in this case, slightly relaxing the patentability requirement for entrants therefore increases entry pressure and growth without leading to a leapfrogging equilibrium.

It is however not so clear whether there is still persistent leadership if a much stronger patentability requirement is imposed on incumbents than on entrants as this might render preemption too costly (especially if voluntary deals are possible after entry occurred). If incumbents tend to pursue smaller inventive steps than entrants it is also possible that a (weak) patentability requirement only binds for incumbents but not for entrants so that relaxing it for entrants would not lead to increased entry pressure.

16.4 Appendix X

Replacing π^M, π^C and g by their values, the inequality $\frac{\pi^M}{\pi^C} < \frac{g}{g-1}$ implies that $\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) < \frac{q-1}{q\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}$ (i). This inequality is satisfied for the largest feasible value of q, which is given by $q_{\max} = \frac{1}{\alpha}$ (note that $\alpha q^2 \geq 1 \geq \alpha q$ must hold). If the right hand side (RHS) of (i) decreases in q, (i) therefore always holds. $\frac{\partial RHS}{\partial q} < 0$ holds if $1-\alpha > q^{\frac{\alpha}{1-\alpha}} - \alpha q^{\frac{1}{1-\alpha}} \equiv R$ (ii). We have $\frac{\partial R}{\partial q} < 0$ so that (ii) always holds if it holds for the smallest feasible value of q which is given by $q_{\min} = \frac{1}{\sqrt{\alpha}}$. Inserting $q = q_{\min}$ into (ii) gives the inequality $\alpha + \alpha^{-\frac{\alpha}{2(1-\alpha)}} - \alpha^{\frac{1-2\alpha}{2(1-\alpha)}} < 1$ (iii). This condition (iii) is satisfied with equality if $\alpha = 1$ and using simulations it can be shown that (iii) is strictly satisfied if $0 < \alpha < 1$. Therefore, $\frac{\pi^M}{\pi^C} < \frac{g}{g-1}$ holds given that $\alpha q^2 \geq 1 \geq \alpha q$ and $0 < \alpha < 1\blacksquare$

16.5 Appendix Y

Denoting the current generation of the quality good by k, consumption is given by $c(k) = y_k - x_k - cg^k\phi$ where the last term stands for the R&D spending in terms of the final good. Inserting $y_k = q^{k\alpha}x_k^\alpha$ and $x_k = \alpha^{\frac{1}{1-\alpha}}q^{\frac{k\alpha}{1-\alpha}}p^{-\frac{1}{1-\alpha}}$, we obtain $c(k) = g^k\left(\alpha^{\frac{\alpha}{1-\alpha}}p^{-\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}p^{-\frac{1}{1-\alpha}} - c\phi\right)$ which can be shown to decrease in p (given that $p > 1$).

Along a balanced growth path[47], consumption, the input of intermediate goods and R&D investment all grow at the rate g between periods (a period is the

[47] In this model there are no transition dynamics and the economy jumps to a new steady state immediately so that the welfare analysis boils down to a comparison of steady state utilities.

random time interval between two innovations) and expected intertemporal utility is given by $W(k) = g^k \frac{c(k)}{\rho - \phi(g-1)}$. The equilibrium rate of growth ϕ can be increased if either π^c or π^M - which is a positive function of p - are increased. If a given rate of growth can be obtained with different combinations of π^c and π^M, it is therefore always welfare improving to choose the combination in which π^C is maximal and π^M and therefore p as low as possible as this minimizes the deadweight losses associated with patent policy and maximizes consumption and intertemporal utility given the targeted rate of innovation. Entry pressure should therefore be kept as high as possible to push incumbents to innovate and if the equilibrium rate of growth turns out to be excessive, it should be reduced by decreasing the market power of incumbents but not by making entry more difficult.