Patent Races with Dynamic Complementarity

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Abstract

Recent models of multi-stage R&D have shown that a system of weak intellectual property rights may lead to faster innovation by inducing firms to share their intermediate technological knowledge. In this article I argue that this literature may have over-estimated the potential benefits from a leaky system. I introduce a distinction between *plain* and *sophisticated* technological knowledge, which has not been noticed so far but plays a crucial role in determining how different appropriability rules map into incentives to innovate.

I argue that the positive effect of weak intellectual property regimes on the sharing of intermediate technological knowledge vanishes when technological knowledge is sophisticated, as is likely to be the case in many high tech industries.

Keywords: multi-stage patent race, innovation, imitation, technological knowledge.

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1 Introduction

In the presence of technological spillovers, a profit-maximizing firm may decide to voluntary reveal its intermediate innovative knowledge to rivals so as to benefit from their "cooperation" in the subsequent stages of the research (De Fraja (1993)). Building on this insight, Bessen and Maskin (2009) and Fershtman and Markovich (2009) have developed models of multi-stage R&D in which a system of weak intellectual property rights may foster innovation by inducing firms to share their intermediate technological knowledge[1].

In this article I argue that this literature may have over-estimated the potential benefits from a leaky system. I introduce a distinction between two kinds of technological knowledge: *plain* and *sophisticated*. Where technological knowledge is *sophisticated* if it requires a voluntary (even if cost-less) act to be gathered; by contrast, knowledge is *plain* when it is apparent to everybody once it has been disclosed[2].
In order to better understand this distinction, let us consider a few examples. A correctly decoded *rebus*[3] is plain because its solution is apparent to everybody once explained. Similarly, the improved graphical user interface of new operating system softwares, or particular functional tools incorporated in a word processor, are plain technological knowledge. On the other hand, consider a problem such as the search for a complex mathematical proof, say a

[1]The role of disclosure in ensuring cumulative progress has also been studied, among others by Scotchmer and Green (1990), Scotchmer (1991), Gallini (1992), Anton and Yao (2004), Bessen (2005) and Denicolò and Franzoni (2004) in different frameworks.

[2] More formally, it is as if an inventor could send a simple message to the public: "to progress in research you shall use technology z" and this is a *sufficient* condition to let "z" become common knowledge to receivers. On the other hand, when technology is sophisticated, this message is complex and can be disregarded by receivers. Therefore sending the message is only a *necessary* condition to let "z" become common knowledge to others.

[3]A *rebus* is a word-image riddle by which the only correct reading is veiled through figures (where, for example, a picture of an eye stands for "I").

proof of Fermat's last theorem. Here the argument is very complex and must follow a precise sequence of steps. Thus, even if the inventor discloses the innovative knowledge, he or she cannot take it for granted that those exposed to the new ideas have in fact understood them. Here, we say, technology is "sophisticated" as agents can remain ignorant even after disclosure.

This simple distinction plays a crucial role in how different appropriability systems shape the incentives to innovate in patent races with dynamic technological complementarity.
To see this point, consider the reason why an inventor may want to disclose its superior intermediate technological knowledge. This is to enable the rival to conduct research on equal footing in the next stages of the race, and hence to increase his research effort. Since under a weak property regime the final innovation is not fully appropriable, the leader may then benefit from the rival's eventual success.
However, I contend that when innovative knowledge is sophisticated the rival may prefer to remain ignorant and free-ride on the leader's R&D effort in the last stages of the patent race. Only if he can be "forced" to acquire the innovative knowledge (as in the plain knowledge case) he is prevented from free-riding. In the sophisticated case, by contrast, free-riding is a feasible strategy, so knowledge acquisition must be *incentive compatible*.
To study in depth how this insight can be applied to a multi-stage patent race, I develop a simple model in the next sections. I compare two different patent regimes: one with *strong* protection, where the first inventor alone can utilize the invention, and one with *weak* protection, in which both firms can utilize the new technology irrespective of who achieved it.
I show that in the plain technology case, weak protection can be socially desirable in terms of both the pace of innovation and expected consumer surplus. However, in the case of sophisticated knowledge, this result is reversed and a strong patent protection is typically socially desirable.

2 The Model

Consider a patent race with two[4] risk-neutral firms $i = 1, 2$ conducting research on a multi-stage invention.
The acquisition of innovative knowledge is sequential, and each stage of research, when completed, produces intermediate technological knowledge that is necessary to proceed to the the next stage. After the final stage is completed, the innovation can eventually be commercialized.
For simplicity, the demand function for the new product is assumed to be linear

$$P = \alpha - Q \tag{1}$$

where $\alpha \in \mathbb{R}^+$ denotes the size of the market.

Firm's payoffs depend on patent policy, which determines whether imitation is lawful or not. As in Fershtman and Markovich (2009), I assume that only the final innovation is patentable, so intermediate discoveries have no legal protection during the race[5].
As for the final innovation, I study two alternative regimes. The first one, called "strong", prevents imitation, thereby creating a barrier to enter in the final market. This guarantees monopoly profits to the first inventor

$$\Pi^m = \Pi^m(\alpha) \ , \tag{2}$$

whereas the laggard obtains nothing.
The alternative regime, called "weak", allows perfect, cost-less imitation. In this case, as soon as the new good is developed by one firm, it can be produced and commercialized by both. Thus the market is always a duopoly, and firms

[4] This assumption is for tractability purposes and convenience only. In case of three or more firms, indeed, similar results could be obtained numerically.

[5] In other words, only the new good as a sum of all previous inventions reflects a general patentability requirement, present in most patent laws, according to which an invention should be sufficiently inventive, i.e. non-obvious, in order to be patented.

equally share profits

$$\Pi^d = \Pi^d(\alpha, \delta) \tag{3}$$

where the parameter $\delta \in [0,1]$ captures the intensity of product market competition. This allows me to study in a reduced form all possible competitive configurations ranging from Bertrand to perfect collusion.

For simplicity, I suppose there are two stages of research ($s = 1, 2$), and three periods ($t = 0, 1, 2$). In earlier periods, i.e., $t = 0, 1$, firms produce research outcomes that are uncertain and depend on the effort exerted in R&D by each contender. More specifically, each innovation requires one unit of time to be developed, and any level of R&D investment is mapped into a probability to successfully complete the current stage at the end of each period. In the last period, i.e., $t = 2$, the innovation process is over and, if all steps have been developed, profits are realized otherwise firms earn zero payoffs.

As a consequence of the described staggered research process, firms obtain zero payoffs when there is even a single period without improvements in research. Hence they have incentives to speed up the process even without the push of competition, i.e. waiting one period is very costly[6].

At each step of the race firms select simultaneously and non-cooperatively a level of effort $x_t^i \in [0, \overline{x}_t]$. The upper-limit $\overline{x}_t$ is uncertain. It is drawn from a uniform distribution on the interval $(0, 1)$, independently and identically at the beginning of each period[7].

Let s_t^i denote firm i's level of technological knowledge in period t, with the initial level of knowledge standardized to zero, i.e., $s_0^i = 0$. R& D effort

[6]This feature can be seen as assuming a positive firm's time discount factor.

[7]Independence across periods simplifies much computations and it can be seen as a specific case of a more general model in which at the opposite side of the spectrum values are perfectly correlated.

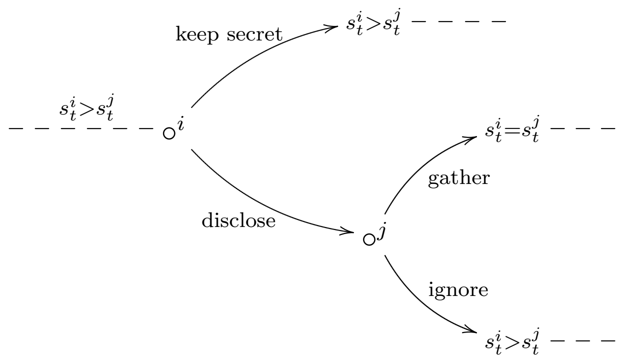

Figure 1: Cooperation and knowledge sharing at any period t when technological knowledge is sophisticated.

makes s^i_t increase of one unit with probability x^i_t. That is,

$$s^i_{t+1} = \begin{cases} s^i_t + 1 & \text{with probability} \quad x^i_t \\ s^i_t & \text{otherwise} \quad . \end{cases} \tag{4}$$

Hence, even if both firms are initially symmetric, over time their level of knowledge can differ. With a higher level of technological knowledge, any firm that is ahead in the race is called "leader" while the laggard is the "follower". More specifically, a technological leader belongs to $\mathcal{L}_t = \{i : s^i_t = t\}$ the set of those L_t firms able to develop $s = t$ research steps at time t.

Importantly, the only effective way to protect inventions from imitation during the race is to keep it secret to rivals. Therefore, as shown in Figure 1, a leader must decide to *disclose* its knowledge or to *keep* it *secret*.
The effects of disclosure on followers depends on the nature of technological knowledge. When technology is plain, the follower reaches automatically the leader's technological knowledge. By contrast, in the sophisticated technology case, there is one further decisional node that let the follower decide whether to *ignore* or *gather* the disclosed intermediate knowledge.

Finally, to include the possibility of competition in the product market, it is assumed that secrecy is no longer feasible when the good is commercialized. One can imagine that when the innovation is eventually brought to the market, a cost-less process of reverse-engineering takes place.

To summarize, the timing of the game is as follows:

- At $t = 0$, a $\bar{x}_0$ is drawn, firms are symmetric and set their R&D efforts simultaneously. Nature then determines which firm succeeds.

- At $t = 1$, a $\bar{x}_1$ is drawn, firms observe the progress in research of each contender and the leader may decide to disclose or keep secret its superior knowledge. If the technology is sophisticated, the follower decide either to ignore or acquire the new technology when it is available. Finally, R&D efforts are again chosen simultaneously. Nature determines which firm succeeds.

- If at $t = 1$ at least one firm succeeded, at $t = 2$, the good is commercialized and firms earn profits according to the patent regime in place, to the market size α and, if the system of protection is weak, to the degree of competition δ.

In the following sections I solve the game under both policy regimes, consider separately the effects of technological knowledge being plain or sophisticated. I then analyze the overall probability of innovation and some broader concept of social welfare. I then study the effects due of the introduction of licensing contracts between firms in the strong regime. Last section summarizes the main results and concludes the paper.

3 Strong Patent Regime

To study the (sub-game perfect) equilibria of the model, I proceed backwards starting from the situation in the product market.
When imitation is prohibited by law, inventors can patent the new product and exclude competitors in the final market. Hence, at $t = 2$, there are three

possible alternative scenarios. First, innovation has not been attained, the retail market is left uncovered and both firms obtain zero profits.
In the second scenario, only one firm has developed and patented the good, i.e., $L_2 = 1$, so monopoly prevails in the product market.
In the third and last case, both firms have innovated all stages and both applied for a patent, i.e., $L_2 = 2$. However, because only one patent can be granted, one applicant is chosen randomly. Thus, the product market is again a monopoly.

Going back to period $t = 1$, three cases arise once more. However, differently from before, inventors can not apply for exclusive patent protection on the intermediate technology. Thus, when $L_1 = 1$, the only effective protection tool is secrecy.
To understand leader's incentives to keep the intermediate technological knowledge secret or to disclose it, suppose that at least one firm grabbed intermediate technological knowledge so that $\mathcal{L}_1 = \{i : s_1^i = 1\}$ is non-empty. After having observed the realization of $\overline{x}_1$, each firm $l \in \mathcal{L}_1$ selects a level of R&D expenditures so as to maximize the following expected payoff function

$$U_1(L_1) \quad = \quad x_1^l\left[(1-y)^{L_1-1}\cdot\Pi^m + y^{L_1-1}\cdot\frac{\Pi^m}{2}\right] - c\cdot x_1^l \tag{5}$$

where y denotes the opponent's expected intensity of research.

Notice that (5) is decreasing in the number of firms active in research, i.e., L_1 when $y > 0$. Every intermediate leader, therefore, will keep its superior technological knowledge secret and thereby force the rival to quit competition, setting R&D investments to zero.
In this case, due to linearity of payoffs, leader's equilibrium R&D investment is either $x_1^{l*} = \bar{x}_1$ or zero.
To avoid proliferation of cases, however, we focus on the set of parameter values where research is always pursued when there is one leader in the

market. To be more precise, the assumption is $\Pi^m \geq c$, which, given the linearity of the demand function (1), can be better expressed in terms of market size,

$$\alpha \in \mathcal{A} = \{a : a \geq 2\sqrt{c}, a \in \mathbb{R}^+\} \ . \tag{6}$$

In this model, hence, broad patents imply that competition in research can occur only when both firms have improved, i.e., $L_1 = 2$. The following lemma characterizes the form of the resulting equilibria when there is competition in research.

Lemma 1. *At the second period, when both firms have succeeded in period 1, there exists a threshold $\hat{x}_1(\alpha)$ such that*

(i) if $\bar{x}_1 < \hat{x}_1(\alpha)$, the unique equilibrium investment in R&D is $x_1^{l*} = \bar{x}_1$, $\forall l \in \mathcal{L}_1$, (maximum effort),

(ii) if $\bar{x}_1 \geq \hat{x}_1(\alpha)$, the unique (symmetric) equilibrium investment in R&D is $x_1^{l*} = \hat{x}_1(\alpha)$, $\forall l \in \mathcal{L}_1$, (limited effort). □

The intuition behind this result is simple. If there is no competition in research, each firm finds always profitable to invest in R&D to achieve the last innovation and, due to the linearity of payoff function, the investment level is set up to the upper limit. With competition, on the other hand, expected profits are reduced proportionally to rival's effort in R&D that again depends on the realized $\bar{x}_1$. If this value is "low", i.e., below the threshold, both firms invest up to the maximum level. This is because, conditional on being successful, each firm has high probability to exclude the rival through patents[8]. By contrast, when the upper limit is "high", i.e., above the threshold, expected profits are not enough to reward both contenders. Here, firms need to moderate individual R&D investments so as to keep break-even expectations.

[8] The probability that both firms will be jointly developing the next stage is small and vanishes when the upper limit is small, i.e. $\bar{x}_1^2$.

Note that $\hat{x}_1(\alpha)$ is increasing in α, therefore the probability that the realized upper limit will be "low" or "high" depends directly on the market size. More specifically, the larger the market size the higher the probability that both firms will exert the maximum effort in R&D.

Proceeding backwards to the first period, all firms are symmetric and simultaneously decide how much to invest in research. Each firm i solves the following problem,

$$\max_{x_0 \in [0,\bar{x}_0]} U_0 = x_0^i \cdot \sum_{k=0}^{1} y^k (1-y)^{1-k} \cdot E_0[U_1^{i*}(k+1)] - c \cdot x_0^i \ , \tag{7}$$

where $E_0[U_1^{i*}(k)] \geq 0$ denotes the average payoff that the firm is expecting to get with $L_1 = k+1$ leaders including the firm itself.
Notice that this payoff function is again linear and decreasing in y. However the slope can now be negative even without competition from the other firm, i.e., $y = 0$. In other words, there are innovations whose size would allow investments in the second stage, i.e., $\alpha \in \mathcal{A}$, but that, in expectations, are not profitable at the first stage even in the absence of competition in research. The next lemma characterizes all firms equilibria.

Lemma 2. *At the first period, there exists a threshold $\hat{x}_0(\alpha)$ such that*

(i) if $\hat{x}_0(\alpha) < 0$ both firms do not invest in R&D,

(ii) if $\bar{x}_0 < \hat{x}_0(\alpha)$, the unique equilibrium R&D level is $x_0^{l*} = \bar{x}_0$ $\forall l \in \mathcal{L}_0$, (maximum effort),

(iii) if $\bar{x}_0 \geq \hat{x}_0(\alpha)$, the unique (symmetric) equilibrium R&D level is $x_0^{l*} = \hat{x}_0$ $\forall l \in \mathcal{L}_0$, (limited-effort). □

Of course, given the recursive structure of the race, the equilibria described above are analogous to the previous lemma. Again, the risk of facing tough competition in the future can reduce investments at the current stage.

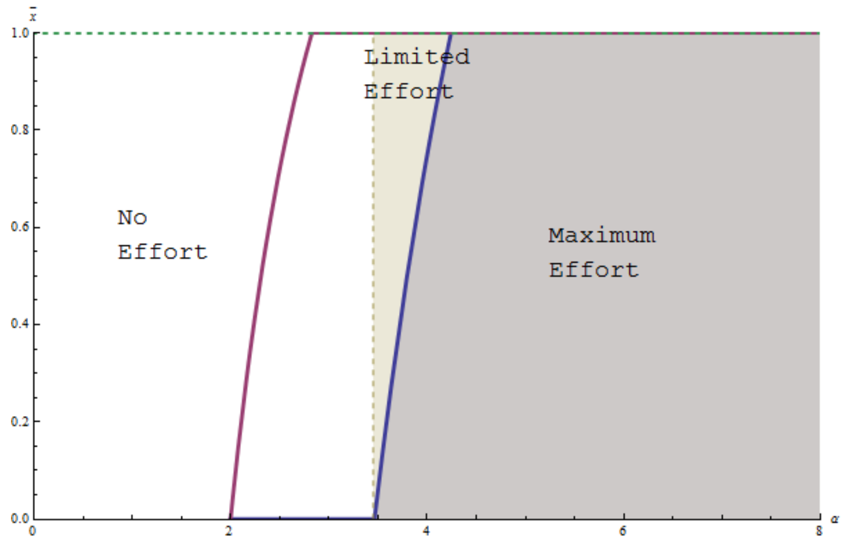

Figure 2: Strong patent protection regime, equilibrium cut-off functions: $\hat{x}_1(\alpha)$ red curve and $\hat{x}_0(\alpha)$ blue curve, for $c = 1$.

Furthermore, condition i) of the lemma tells us how large should be the demand parameter to let firms start investing in research. To be precise, the race starts if

$$\alpha \in \mathcal{A}_0 = \{a : a \geq 2 \cdot \sqrt{3c}, a \in \mathbb{R}^+\} \quad . \tag{8}$$

In order to fix ideas on all the results obtained, Figure 2 depicts the two thresholds $\hat{x}_0(\alpha)$ and $\hat{x}_1(\alpha)$ as a function of market size. By considering the vertical axis as the set of all achievable upper-limits, we observe various patterns of R&D corresponding to three different parameter regions: *no effort, limited effort,* and *maximum effort* region.

Specifically, in the no effort set of values, i.e., the white area, because $\alpha \notin \mathcal{A}_0$ all firms do not invest in R&D. For intermediate values of market size, all firms invest at the first stage and, depending on the realized upper limit, individual R&D expenditure can be lower than the realized upper limit, i.e., limited effort, or exactly the upper limit, i.e., maximum effort. In all cases, furthermore, the intensity of research is maximum at every time that one firm reaches the second stage of innovation.

4 Weak Patent regime

In a regime of "weak" protection, there is no legal shield to first inventors. Imitation is cost-less and trade secret protection is possible only at the steps of the race. Hence, irrespective of who invented first, both rivals will be competing on equal footing in the product market.

In this setting, firms are effectively playing a game of private provision of a public (from the viewpoint of the firms) good, i.e., the innovative technological knowledge. In this game, the leader has always an incentive to communicate its innovative knowledge to the laggard. However, there naturally arises an incentive to free-ride on the rival's effort. Furthermore, since R&D efforts are chosen independently and simultaneously, the scope for free-riding depends on whether the laggard can commit to apprehend innovative knowledge (sophisticated technological knowledge) or not (plain technological knowledge).

4.1 Plain Technological Knowledge

Let us first consider the case of plain technological knowledge by which a disclosed intermediate technology becomes common knowledge for all players. As before, I solve the game proceeding backwards.

Recall that at the end of the race, i.e., $t = 2$, neither secrecy nor intellectual property law permit the leader to exclude its rivals, therefore firms equally split duopoly profits

$$\Pi^d(\alpha, \delta) = \frac{(1-\delta)}{2} \cdot \Pi^m \quad . \tag{9}$$

Going back to $t = 1$, suppose that at least one firm improved its position in the race, i.e., $L_1 > 0$. Now every technological leader $l \in \mathcal{L}_1$ sets a level of

R&D expenditure so as to maximize the following expected payoff function

$$U_1^w(L_1) = \left[1 - (1 - x_1^l)(1 - y)^{L_1 - 1}\right]\Pi^d - c \cdot x_1^l \tag{10}$$

where y is the equilibrium opponent's R&D expenditure.
In this case, and differently with respect to the strong regime, the payoff function (10) is increasing in L_1 for $y > 0$.
Therefore if one firm goes ahead in the race, the leader will always prefer to make its superior technological knowledge freely accessible rather than to keep it secret. Thus, a weak regime of patent protection allows a shift from a strategy of secrecy to a spontaneous emergence of cooperation.

At this point, it is important to examine the role of plain technological knowledge. First recall that under this assumption a simple public message is a sufficient condition to pool contenders abilities. Therefore, whereas with broad patents competition in research occurs only when firms improve jointly at the first stage, here, whatever firm progresses in the race the other advances as well. Thus, firms are always symmetric at the second stage and either $\mathcal{L}_1 = \{1, 2\}$ or it is empty. Building on this observation, the following lemma characterizes all equilibrium outcomes at this stage.

Lemma 3. *At the second period, when at least one firm succeeded in period 1, there exists a threshold $\hat{x}_1^w(\alpha, \delta)$ such that*

(i) if $\hat{x}_1^w(\alpha, \delta) < 0$ both firms do not invest in R&D,

(ii) if $\bar{x}_1 < \hat{x}_1^w(\alpha, \delta)$, the unique equilibrium R&D expenditure level is $x_1^{l*} = \bar{x}_1 \ \forall l \in \mathcal{L}_1$ (maximum effort),

(iii) if $\bar{x}_1 \geq \hat{x}_1^w(\alpha, \delta)$ the unique (symmetric) equilibrium R&D expenditure level is $x_1^{l*} = \hat{x}_1^w(\alpha, \delta) \ \forall l \in \mathcal{L}_1$ (limited effort). □

As for the strong patent case, the equilibrium R&D investments depend on a cut-off value $\hat{x}_1^w(\alpha, \delta)$. If the realized upper limit is above this value, firms jointly moderate their R&D efforts, or else both exert the maximum

effort in research. Although similar, this equilibrium behavior differs from before because it is driven by the underlying public good game between firms. In this case, due to the lack of patent protection, each firm wants to free ride on the rival's effort but both end up sharing costs of research as the only symmetric non-cooperative equilibrium in this game.
Again the threshold is a function of market size, and it allows us to pin down the value of market size needed to start research at this step of the race

$$\alpha \in \mathcal{A}^w = \{a : a \geq 2\sqrt{(2c/(1-\delta)}, a \in \mathbb{R}^+\} \ . \tag{11}$$

By contrasting this condition with the one corresponding to the strong patent protection regime (6), we observe an important drawback of a system with narrow patents in a multi-stage innovation race[9]. That is, because the lack of legal barriers in the product market reduces revenues to first inventors, inventions need an higher final demand to be pursued at this stage. More specifically, the set of parameter values $\mathcal{A}^w$ is a proper subset of the corresponding set $\mathcal{A}$ derived for the strong patent protection regime.
Moreover, this difference can be larger if the the degree of competition in the final market δ is higher.

Going back to $t = 0$, each firm l will solve the following problem

$$\max_{x_0^l \in [0, \bar{x}_0]} \ U_0^w = \left[x_0^l + y \cdot (1 - x_0^l)\right] E_0[U_1^{w^*}(2)] - c \cdot x_0^l \ \ , \tag{12}$$

where $E_0[U_1^{w^*}(2)]$ is the average equilibrium payoff when all firms are active in the next stage.

Lemma 4. *At the first period there exists a threshold $\hat{x}_0^w(\alpha, \delta)$ such that*

(i) if $\hat{x}_0^w(\alpha, \delta) < 0$ both firms do not invest in R&D,

(ii) if $\bar{x}_0 < \hat{x}_0^w(\alpha, \delta)$, the unique equilibrium R&D expenditure level is $x_0^{l^*} = \bar{x}_0 \ \forall l \in \mathcal{L}_0$, (maximum effort),

[9] As emphasized among others by, Green and Scotchmer (1995) and Denicolò (2000).

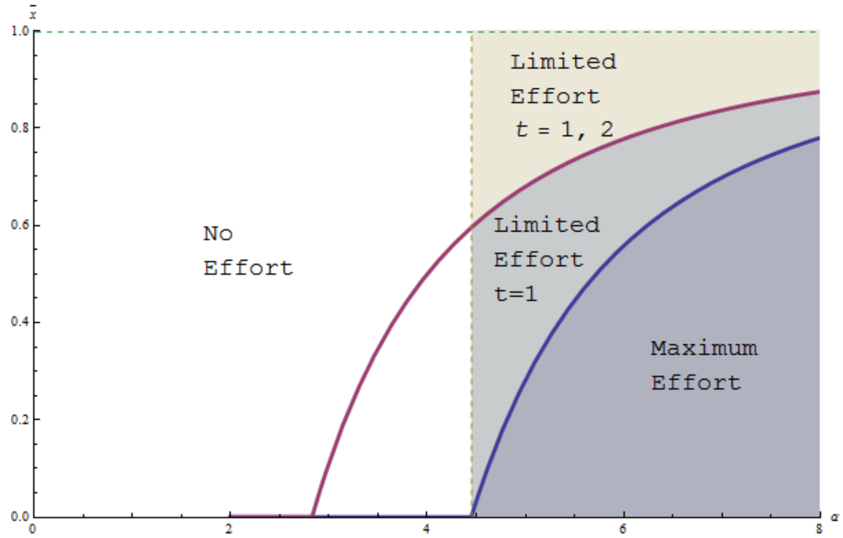

Figure 3: Weak patent protection regime, equilibrium cut-off functions: $\hat{x}_1^w(\alpha,\delta)$ purple curve and $\hat{x}_0^w(\alpha,\delta)$ blue curve for $c=1$ and $\delta=0$.

(iii) if $\bar{x}_0 \geq \hat{x}_0^w(\alpha,\delta)$, the unique (symmetric) equilibrium R&D expenditure level is $x_0^{l*} = \hat{x}_0^w(\alpha,\delta)\ \forall l \in \mathcal{L}_0$, (limited effort). □

Again it is better to express the condition under which the research process can start in terms of market size

$$\alpha \in \mathcal{A}_0^w = \{a : E_0[U_1^{w^*}(2)] \geq c, a \in \mathbb{R}^+\} \quad . \tag{13}$$

Fixing $\delta = 0$, Figure 3 shows the two thresholds $\hat{x}_1^w(\alpha,\delta)$ and $\hat{x}_0^w(\alpha,\delta)$ for the case of perfect collusion in the product market. Because collusion ensures the highest possible reward to inventors, this constitutes the "upper bound" case.

After $\bar{x}_1$ or $\bar{x}_0$ are drawn, as we may conclude from the comparison with Figure 2, now firms are more likely to exert lower individual levels of effort in R&D than in the strong regime. Broad patents, however, do not necessarily produce higher aggregate levels of R&D. The reason is that, in a strong patent protection regime, a leader does not share technological knowledge with rivals. By contrast in a weak regime, firms are eagerly revealing information and cooperating in research. Thus, narrow patents will ensure an higher overall number of firms active in research at the intermediate stage.

4.2 Sophisticated Technological Knowledge

I next consider the case of sophisticated technological knowledge. Now, the follower can pretend it has not acquired the intermediate technology even when the leader made it available for free and imitation is cost-less. This gives the follower the opportunity to commit not to conduct any second-stage research if this commitment is profitable.
In this model, in particular, I added a new decision node in the tree of the game (as displayed in Figure 1). Now the follower can decide either to *ignore* or *gather* intermediate knowledge once disclosed by the leader.

To better understand the motives behind this decision, suppose that at $t = 1$ only one firm innovated, i.e., $L_1 = 1$. First, recall that the leader firm is better off when it transfers knowledge to the rival, even when it happens for free. Then, imagine that the follower, denoted by $f \equiv i \notin \mathcal{L}_1$, can credibly ignore the new technology and quit research. In this case, the leader updates beliefs on rival's R&D investment, i.e., setting $y = 0$. Thus, whereas leader's payoff function (10) reduces to

$$U_1^w(1) \;\; = \;\; x_1^l \cdot (\Pi^d - c) \;\; , \tag{14}$$

the follower's payoff function becomes

$$U_1^f(1) \;\; = \;\; y^l \cdot \Pi^d \tag{15}$$

where y^l denotes the leader's R&D effort. Because of (6), furthermore, the leader chooses always $x_1^{l^*} \equiv y^{l^*} = \bar{x}_1$.
Therefore, such commitment strategy is a (sub-game perfect) equilibrium as long as the follower's expected profit when it ignores the new technology, i.e.,

$\bar{x}_1 \cdot \Pi^d$, is lower than the payoff expected from being active in research, i.e., $U_1^{w^*}(2)$. This leads to the following result.

Proposition 1. *In a weak regime of patent protection with sophisticated technological knowledge, when only one firm succeeded in period 1, there exists a threshold $\hat{x}_1(\alpha, \delta)$ such that*

- *if $\bar{x}_1 \leq \hat{x}_1^w(\alpha, \delta)$, the follower gathers the free technological knowledge and sets $x_1^{f^*} = \bar{x}_1$ (maximum effort),*
- *if $\bar{x}_1 > \hat{x}_1^w(\alpha, \delta)$ the follower ignores the free technological knowledge setting $x_1^{f^*} = 0$, (no effort).* □

This result, somewhat surprisingly, has a close similarity with the equilibrium R&D actions when technological knowledge is plain (Lemma 3).
The intuition is simple, the reason why a technological knowledge matters is that firms are willing to commit in order to better free ride on the rival's effort. However, when the market size is high, i.e., the upper limit is "low", the positive effects of complementarity in research are stronger than the benefits from free riding. Thus, facing innovations with sophisticated technological knowledge does not alter the equilibrium R&D.
By contrast, for lower values of the market seize, i.e., when the upper limit is "high", free riding is a more salient problem. That is, whereas in the plain technological case firms are "forced" to be symmetric and both moderate their investments in R&D as a solution to a public good game, here the follower may instead quit research and yet put the rival in the position to increase its R&D investments as the only firm active in research. Hence, as long as effort's savings are large enough to cover the decreased probability of reaching the next stage, it is an equilibrium for the follower to ignore the new technology.

Consider now the first period, each firm l solves the following problem

$$\max_{x_0^l \in [0,\bar{x}_0]} U_0 = x_0^l \Big[(1-y) \cdot E_0[U_1^{l^*}] + y \cdot E_0[U_1^{w^*}(2)] \Big] + (1-x_0^l) \cdot y \cdot E_0[U_1^{f^*}] - c \cdot x_0^l \tag{16}$$

The next lemma characterizes the equilibria at this stage.

Proposition 2. *At $t = 0$ in a weak regime of patent protection with sophisticated technological knowledge, firms behave in the same way as in the plain technological knowledge case but with a threshold $\hat{x}_0^{ws}(\alpha, \delta)$ that is non-greater than the corresponding cut-off value $\hat{x}_0^{w}(\alpha, \delta)$ for every α and δ.*

The above proposition tells the consequences of a sophisticated technological knowledge in terms of R&D investments at the first period. This comes from the payoff function (16) which captures the augmented incentives for firms to diminish R&D expenditures at this stage. Given the structure of the model, this reduction is captured by a cut-off value function $\hat{x}_0^{ws}(\alpha, \delta)$ that is smaller with respect to case of plain technological knowledge. Moreover, the difference between the two thresholds is non-increasing in the market size α and in δ.

Further, there is an explicit condition on the minimum market size above which the research process can start

$$\alpha \in \hat{\mathcal{A}}_0^{ws} = \{a : E_0[U_1^{l^*}] \geq c, a \in \mathbb{R}^+\} \quad . \tag{17}$$

Again, a comparison with previous outcomes leads to the following result:

Corollary 1. *The minimum level of market size required to stimulate research in a regime of weak patent protection is smaller when the technological knowledge is plain rather than when the technological knowledge is sophisticated.*

In summary, when firms are facing innovations with weak patent protection, the nature of technological knowledge produced can strongly alter the equilibrium behavior at every step of the race. The direction of such change is unambiguous, innovations with sophisticated technological knowledge reduce the overall effectiveness of a weak system in developing cumulative inventions. This is because not only the number of firms active in research may result reduced but also because the enhanced free riding opportunity at the second stage provide incentives to lower investments at the first period as well.

5 Expected Probability to Innovate

In this section, I examine some of the implications of different patent protection regimes. First, I look at the *pace of innovation* in a system ensuring strong patent protection. That is, I follow the preceding equilibrium analysis to pin down an average probability that all steps of innovation are accomplished by at least one firm.

This is with the purpose to build a benchmark solution which can provide a basis for a comparison between strong and weak patent protection regimes. From this perspective, this permits also to compare separately the effects of different patent regimes on innovation patterns from those on consumer surplus. Next, I examine how consumer surplus is affected by both regimes.

5.1 Strong Patent Regime

Let examine first the pace of innovation when firms can protect revenues with broad patents. Once a firm succeeds at the first stage, it does not disclose for free the new intermediate knowledge to followers. Thus, only successful innovators can proceed in the race and will invest according to the equilibrium strategy of lemma 1.

When averaged for all the possible values of $\bar{x}_1$, the probability that the second stage invention is developed by at least one firm is

$$\mu_1(L_1) = \begin{cases} 0 & \text{if } L_1 = 0 \\ \int_0^1 \bar{x}_1 d\bar{x}_1 = 1/2 & \text{if } L_1 = 1 \\ \int_0^{\hat{x}_1} \bar{x}_1(2-\bar{x}_1)d\bar{x}_1 + \int_{\hat{x}_1}^1 \hat{x}_1(2-\hat{x}_1)d\bar{x}_1 & \text{if } L_1 = 2 \end{cases} \tag{18}$$

Going back to the previous stage of innovation, the (average) probability that all steps of innovation are accomplished is

$$\mu = \int_0^1 \sum_{k=0}^{2} p_k \cdot \mu_1(k)\ d\bar{x}_0 \tag{19}$$

$$p_k > 0 \quad \forall k \text{ iff } \alpha \in \mathcal{A}_0 \quad ,$$

where p_k denotes the probability that after the first period there are k firms still active in research.

Fixing the cost $c = 1$, Figure 4 depicts μ as a function of market size. As expected, the pace of innovation exhibits a s-shaped curve. This is caused, in the model, by the existence of three different parameter regions in which R&D can occur with various intensities. More specifically, there are zero R&D investments for low-demand inventions, i.e., $\alpha \notin \mathcal{A}_0$. Positive investments, but of limited intensity, arise for inventions of intermediate size. Finally, when α is sufficiently high, investments are always set at the maximum level. Here, the curve touches an upper-bound $\bar{\mu} \approx 0.4$ and is constant.

5.2 Weak Patent Regime

Now, consider a regime of weak patent protection, beginning with the case of plain technological knowledge.

5.2.1 Plain Technological Knowledge

Once patents are too narrow to protect revenues in the final market and imitation is cost-less, leaders disclose voluntarily technological knowledge

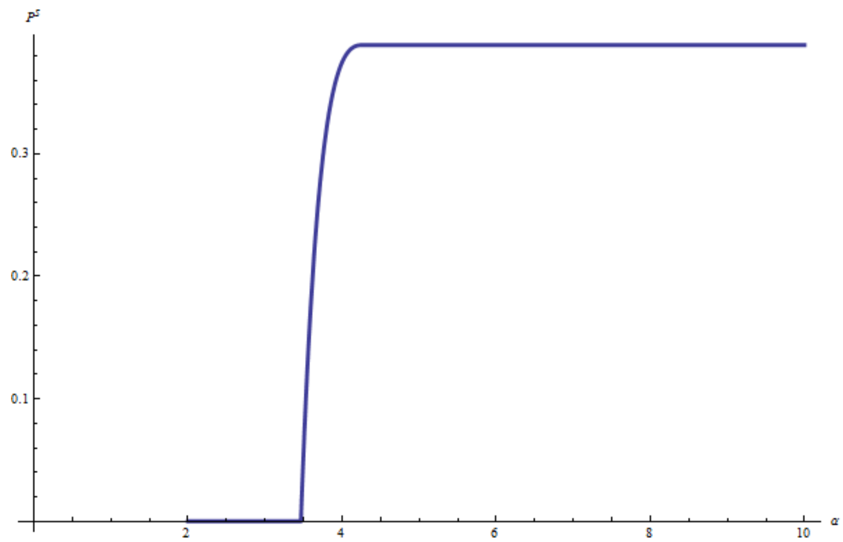

Figure 4: Pace of innovation under a strong patent protection regime ($c = 1$)

at the intermediate stage. In addition, if technological knowledge is plain, followers can not commit to prevent this technological sharing and invest in research (as described in lemma 3).
Thus, the average probability to develop second stage invention is

$$\mu_1^w(L_1) = \begin{cases} 0 & \text{if } L_1 = 0 \\ \int_0^{\hat{x}_1^w} \bar{x}_1(2-\bar{x}_1)d\bar{x}_1 + \int_{\hat{x}_1^w}^1 \hat{x}_1^w(2-\hat{x}_1^w)d\bar{x}_1 & \text{if } L_1 > 0 \end{cases} \tag{20}$$

Therefore, the overall pace of innovation is

$$\mu^w = \int_0^1 \sum_{k=0}^{2} p_k^w \cdot \mu_1^w(k)\ d\bar{x}_0 \tag{21}$$

$$p_k^w > 0 \quad \forall k \ \text{ iff } \ \alpha \in \mathcal{A}_0^w$$

where p_k^w denotes the probability that after the first period there are k firms active in research.

To better understand by how much the strong and the weak regimes are different, Figure 5 plots both probability curves as a function of market size.

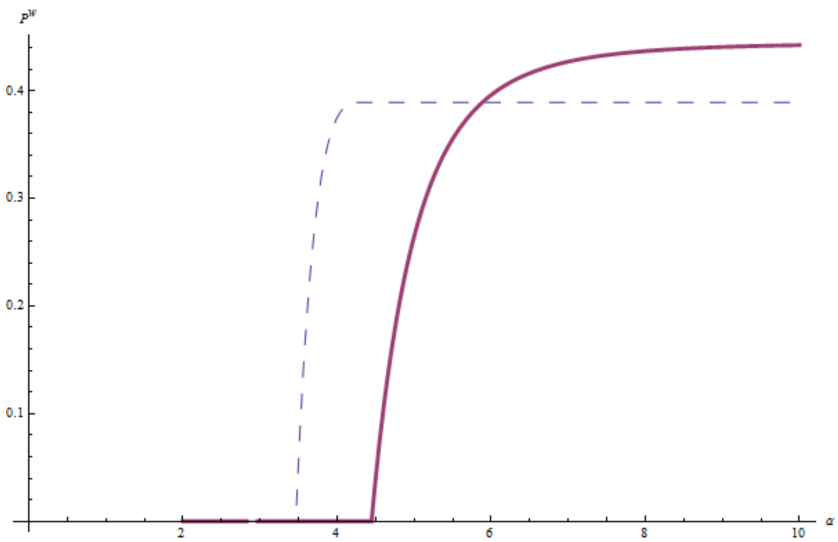

Figure 5: Pace of innovation under a weak patent protection regime and plain technological knowledge ($c = 1$ and $\delta = 0$).

As the figure shows, the (bold) curve representing the probability of success under a weak regime is positive for higher levels of market size and increases at a slower rate with respect to the (dashed) curve representing the same probability under a strong regime.

Note that the two curves are monotonic and cross at $\alpha \approx 5.5$. Hence, although broad patents deliver an higher pace of innovation for intermediate values of market size, there exists a level of α above which the pace of innovation is enhanced by a weak system. And notice that this must be true as long as the probability of success in a weak regime reaches an upper-bound, here $\bar{\mu}^w \approx 0.44$, which is higher than the corresponding value for a strong patent regime.

The following proposition states the general form of this result:

Proposition 3. *For every $\delta < 1$, there exists a threshold value $\hat{\alpha}$ such that*

- *if $\alpha < \hat{\alpha}$ the pace of innovation under weak protection is smaller than under strong protection,*

- *if $\alpha \geq \hat{\alpha}$, instead, the pace of innovation under weak protection is higher than under strong protection.*

Moreover the threshold is non-decreasing in δ. $\square$

The above result can be easily understood when taking into account, as emphasized among others by Bessen and Maskin (2009) and Fershtman and Markovich (2009), the role of technological complementarity in multi-stage innovations. For instance, when complementarity is better exploited by technological transfers, relaxing competition through weak patent protection might increase the pace of innovation. Of course, a weak patent protection regime makes smaller the expected rewards for inventors (but strictly positive for $\delta < 1$). In a strong regime, on the other hand, the possibility to exclude rivals via patents provides high-powered incentives to exert effort in research but kills cooperation. Indeed, knowledge is never voluntarily transfered by leaders[10]. Thus, there exists a tension between inducing cooperation and incentivize research with higher rewards. As the market size increases, however, such trade-off finds a solution thereby a weak patent protection regime should prevail on a strong regime for innovations with larger demand.

5.2.2 Sophisticated Technological Knowledge

Under the assumption of sophisticated technological knowledge, the pace of innovation in a weak system may change. Now when one single firm improves one step in the race and discloses its superior knowledge, the follower can decide to imitate or quit research at its own advantage.

Further making use of the equilibrium described in proposition 1, one can

[10] In a more general case, leaders could ask for money in change of their superior knowledge. For the time being let us assume that any form of licensing is forbidden for anti-trust reasons or, simply, impossible.

derive the (average) probability of second stage invention development,

$$\mu_1^{ws}(L_1) = \begin{cases} \int_0^{\hat{x}_1^w} \bar{x}(2-\bar{x})d\bar{x} + \frac{(\hat{x}_1^w)^2}{2} & \text{if } L_1 = 1 \\ \mu_1^w(L_1) & \text{otherwise} \end{cases} \tag{22}$$

Going back to the first period, the overall pace of innovation is

$$\mu^{ws} = \int_0^1 \sum_{k=0}^{2} p_k^{ws} \cdot \mu_1^{ws}(k) \; d\bar{x}_0 \tag{23}$$

$$p_k^{ws} > 0 \quad \forall k \;\text{ iff }\; \alpha \in \mathcal{A}_0^{ws} \quad ,$$

where p_k^{ws} is the probability that at least k firms will be active in research at the second stage of the race.
Contrasting (21) with (23) ensues the following result:

Proposition 4. *The pace of innovation in a regime of weak patent protection with sophisticated technological knowledge is smaller than the corresponding value in the plain technological knowledge case.* □

Corollary 2. *When $\mu^w > 0$ the difference $(\mu^w - \mu^{ws})$ is non-negative and tends to zero if and only if $\alpha \to \infty$.* □

At this point our simple model demonstrates that inventions with sophisticated technological knowledge might represent a limiting factor in overall system performance with weak protection. The main logic behind this result is that, although a regime encouraging cooperation may generate higher pace of innovation, the free riding problem introduced by a weak patent protection regime could weaken this positive outcome if firms are able to commit and quit research whenever it is convenient.

In summary, a weak system can produce innovation with an higher probability than a strong system when cooperation is assured by high enough final invention's profits. Furthermore, whereas the type of technological knowledge does not alter the results for a strong system, it modifies the incentives associated to a weak patent regime. That is, the probability of innovating under a weak system is higher when the technology is plain rather than when it is sophisticated. Such a difference, however, reduces as long as the prize for the race reaches larger levels and yet it vanishes only in the limit of infinite market size.

6 Consumer Surplus and Welfare

The analysis of the previous section has shown how different patent systems affect the pace of innovation. To better assess all the potential benefits from either systems, I must explore some broader measures of social welfare.
As obvious, a regime of strong patent protection results in a monopoly in the retail market and thus, it carries relevant deadweight-losses and poor consumer surplus[11]. By contrast, a weak system can count on competition to provide a larger range of alternatives more desirable from the point of view of social welfare.

In this section, I study the effects of different patent systems on the expected surplus accruing to consumers. Given the demand function (1), consumer surplus can be easily computed (see the appendix) and represented in a reduced form as a function of δ and the demand parameter α,

$$S(\alpha, \delta) = \frac{\alpha^2}{8} \left(1 + \delta^{\frac{1}{2}}\right)^2 \quad . \tag{24}$$

This function encompasses both cases of duopoly and monopoly in the product market. For instance, in presence of a duopoly with perfect colluding firms, i.e., $\delta = 0$, equation (24) gives a consumer surplus that is the same of a monopolistic market. Hence, let denote this value by $S^m(\alpha) \equiv S(\alpha, 0)$.

[11] I implicitly assume that patents life is infinite. As we will discuss next, an "optimized" patent regime may allow for finite patents life and this may reduce the expected deadweight-loss under a strong patent regime. However, restricting attention to this sub-optimal case will bolster the argument against a weak patent system.

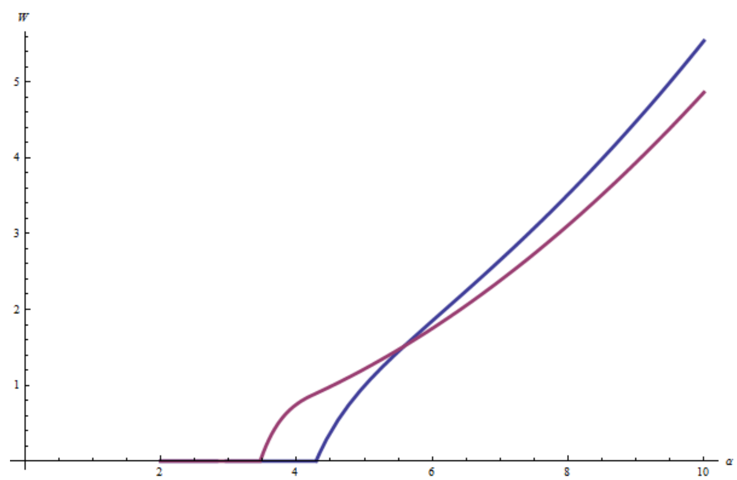

Figure 6: Welfare under the strong (purple curve) and weak patent protection regime with plain technological knowledge when $\delta = 0$ (blue curve).

At this point, I shall define social welfare as the expected consumer surplus delivered by firms under each of the two regimes. Specifically, social welfare is described by two distinct curves. If the system ensues broad patents, this is

$$W^s = \mu \cdot S^m(\alpha) \ . \tag{25}$$

If the system allows for weak patent protection, by contrast, the corresponding curve is

$$W^w = \begin{cases} \mu^w \cdot S(\alpha, \delta) & \text{if TK is plain} \\ \mu^{ws} \cdot S(\alpha, \delta) & \text{if TK is sophisticated} \end{cases} .$$

Fixing perfect collusion in the retail market, i.e., $\delta = 0$, Figure 6 shows both welfare curves when technological knowledge is plain. Recall that, for δ close to zero, both regimes offer roughly the same consumer surplus and the only difference is the pace of innovation. Again, hence, a weak regime performs better in presence of inventions with large market size.

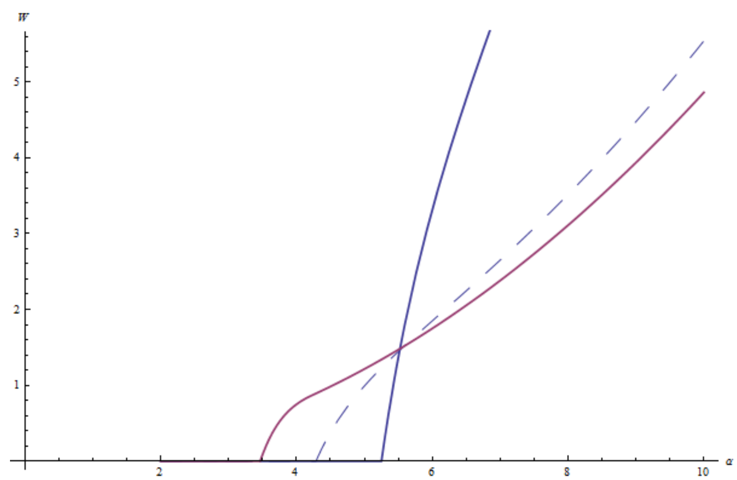

Figure 7: Welfare under the strong (purple curve) and weak patent protection regime with plain technological knowledge when $\delta = 1/3$ (blue curve).

Suppose that there is some mild competition in the product market, i.e., $\delta = 1/3$. By looking now at Figure 7, the curve of social welfare under a weak regime rotates when δ switches to positive values. This movement can be easily explained. On the one hand, firms' incentive to participate in the race are weakened, i.e., the initial value of market size must be larger. On the other hand, the difference between the expected consumer surplus in the two regimes grows exponentially larger as the market size increases[12].
This comparative static result adds to the growing literature and common view that a weak patent system yield substantially larger social benefits than a strong regime. Next I am going to show that, under the assumption of a sophisticated technological knowledge, the positive effects of a weak system are reduced.

Suppose now that a policy maker knows the true demand parameter α or

[12]Notice that the expected consumer surplus (24) increases exponentially in α but it grows faster for $\delta > 0$.

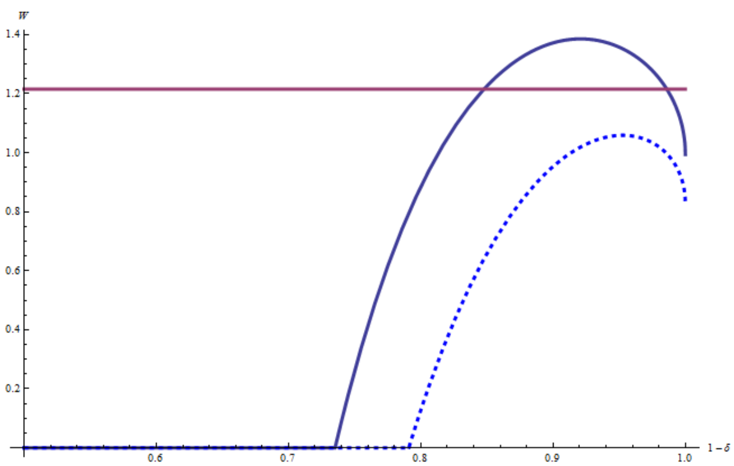

Figure 8: Welfare under the strong protection regime (purple), under the weak protection regime with plain technological knowledge (blue) and the weak regime with sophisticated technological knowledge (dashed blue), ($\alpha =$ 5, $c = 1$).

can estimate the distribution of its value[13]. For example, he can assess the relevance of follow-on inventions for consumers.

Fixing α, Figure 8 depicts again the welfare curves but now as a function of $1-\delta$ the intensity of collusion in the product market. Of course, the welfare function in a strong regime is constant. By contrast, the welfare function in a weak system varies in a non-monotonic way with respect to δ, regardeless of being either with plain technological knowledge or with sophisticated.

We observe, however, that when the technological knowledge is plain there is a set of parameter values of δ such that a weak system performs better than a strong system. On the other hand, if instead the technological knowledge is sophisticated, a weak system results in a reduced social welfare with respect to the broad patents case.

In summary, as this simple example shows, a policy maker should carefully take into account the appropriate definition of technological knowledge before

[13]As it is assumed in Bessen and Maskin (2009).

adopting a weak system of patent protection.

7 Licensing

In the more general case firms are able to contract upon technology sharing. To capture the role of licensing contracts in a simple way, suppose that the follower firm could offer a contingent contract to induce leader's disclosure. As long as offered transfers are such that the amount of leader's expected profits remain untouched after disclosure, technological sharing is achieved[14]. By restricting attention to the upper bound case, i.e. $\delta = 1$, licensing ensues the following result:

Proposition 5. *The expected consumer surplus under a strong protection system enhanced by licensing contracts, is greater or equal than that of a weak patent regime.*

As illustrated by Figure 9, licensing improves substantially social welfare in a strong regime.
Hence, licensing could compensate many of the advantages highlighted for the weak regime.
It is important to point out that this is not true for levels of competition in the product's market that are different from the $\delta = 0$ case. A full comparative statics on such other cases is beyond the scope of this article. This is simply because a full comparison between the two regimes should also consider a strong patent system that is "optimized". In that case, a strong system may compensate the improved performance of weak patent regime by setting, for instance, adequate length and breadth of patents.

[14]For example, it could state that if the licensee develops the new invention, it keeps all profits. By contrast, if both firms develop the last stage invention at the same time, profits accrue to the early innovator only.

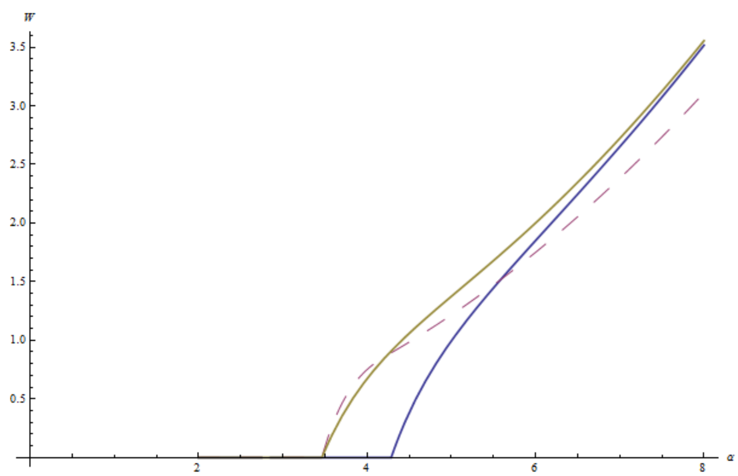

Figure 9: Welfare with licensing. Licensing (green) substantially dominates both regimes, plain strong (dashed) and weak (dark-yellow)

8 Conclusions

According to their influential article, Bessen and Maskin (2009) claim that the reason why stronger intellectual property rights might not promote innovation has to do with sequential complementary innovations. The concern of this article is that this literature may have over-estimated the potential benefits from a weak system.
To address this issue, I developed a simple model of patent race for a two-stage invention. The principal novelty consists of the introduction of alternative definitions of technological knowledge which standard literature has not explored so far. Hence I studied the patent race when technological knowledge is either *plain*, thereby it becomes common knowledge once disclosed, or *sophisticated*, by which even if freely accessible it transmits only when firms decide to gather it.

I show that, although a weak patent protection regime outperforms sometimes a system of broader protection, the benefits from a weak system could vanish under the assumption of sophisticated technological knowledge. More

specifically, since R&D investments in a weak regime can be seen as a public good from the view point of firms, a sophisticated technological knowledge allows lagged behind firms to commit to fully free-ride on leader's effort. Indeed, even when imitation is cost-less, lagged behind firms may have incentives to refrain from cooperation quitting research. This strategy, by contrast, is not possible in the plain case.
The model demonstrates, further, that if firms are allowed to contract licensing fees upon technology sharing, broad patents provide better pace of innovation than a weak system. In this case, therefore, the potential benefits from a weak system are limited to the smaller dead-weight losses generated in the product market. Nevertheless, it is pointed out that, this outcome should be better examined and contrasted with an optimized strong patent regime, by which length and breadth are set to maximize social welfare.

Finally, a straightforward policy implication comes out from this work. That is, the policy maker should carefully look at the technological nature of inventions while deciding to ease protection rules with the scope to foster cooperation.

Appendices

Proof of Lemma 1.
When both firms succeed in the first period, i.e., $L_1 = 2$, the payoff function (5) is linear in x_1^l, and its slope depends on y.
As a first step, I define the level of y such that the slope of (5) vanishes,

$$y = 2 \cdot \left(1 - \frac{c}{\Pi^m}\right) \equiv \hat{x}_1(\alpha) \tag{26}$$

By the lineraty of the demand function, this is equivalent to

$$\hat{x}_1(\alpha) = 2 \cdot \left(1 - \frac{4c}{\alpha^2}\right) \tag{27}$$

Notice that $\hat{x}_1(\alpha)$ is also the level of rival's effort such that leader's expected reward is zero, irrespective of its own effort. Hence, if this threshold exhibits negative values, the optimal action of both firms is to choose zero investments. By restricting attention to $\alpha > 2\sqrt{c}$, however, this case is ruled out. At $t = 1$ an upper limit $\bar{x}_1$ is drawn. Thus, two alternative scenarios are possible. First, consider that $\bar{x}_1 < \hat{x}_1(\alpha)$ and so expected profits are positive for any rival's level of R&D. Thereby a unique equilibrium exists in which both agents exert the highest level of effort, i.e., $x_1^{l^*} = \bar{x}_1 \; \forall l \in \mathcal{L}_1$.
Consider, next, the opposite case in which $\bar{x}_1 \geq \hat{x}_1(\alpha)$. Here, multiple equilibria arise. There are two possible asymmetric equilibria: either $x_1^{l^*} = \overline{x}_1$ and $y = 0$ or vice-versa. However there is also a unique symmetrical equilibrium in which effort is $x_1^{l^*} = y = \hat{x}_1(\alpha)$. In this equilibrium, both firms earn zero expected profits. **Q.E.D.**

Proof of Lemma 2.
Recall first that firms have uniform priors about $\overline{x}_1 \in (0,1)$. Thus, according to the equilibrium payoffs of lemma 1, expected payoffs at the next stage must be averaged over all possible upper-limit values. Thus, if $L_1 = 1$

$$E_0[U_1^*(1)] = \int_0^1 \bar{x}(\Pi^m - c)d\bar{x} = (\Pi^m - c)/2 \quad , \tag{28}$$

and when $L_1 = 2$

$$E_0[U_1^*(2)] \;\; = \;\; \int_0^{\hat{x}_1} \left(\bar{x} \cdot (2 - \bar{x}) \frac{\Pi^m}{2} - c \cdot \bar{x} \right) d\bar{x} \; . \tag{29}$$

As before, the slope of (7) vanishes if

$$y = \frac{E_0[U_1^*(1)] - c}{E_0[U_1^*(1)] - E_0[U_1^*(2)]} \equiv \hat{x}_0(\alpha) \quad . \tag{30}$$

Notice that (30) is a function of market size and, after some algebra, it can be shown that it is non-negative when $\alpha \geq 2 \cdot \sqrt{3c}$.
Thus, when $\alpha \notin \mathcal{A}_0$, the unique equilibrium for all firms is to select zero R&D expenditures.

Finally notice that $\alpha \in \mathcal{A}_0$ implies that the race can start if and only if the equilibrium effort expected at the second stage is maximum. This observation simplifies a lot (30) that now reduces to

$$\hat{x}_0(\alpha) \equiv 3 \cdot \left(1 - \frac{12c}{\alpha^2}\right) \tag{31}$$

The rest of the proof is analogous to the previous lemma. **Q.E.D.**

Proof of Lemma 3.
In a weak patent regime with plain technological knowledge, intermediate inventors disclose for free their technological knowledge and followers cannot prevent sharing.
Therefore, by the linearity of (10), the equilibrium R&D investments depend again on a cut-off function

$$\hat{x}_1^w(\alpha) \equiv 1 - \frac{8c}{(1-\delta)\cdot\alpha^2} \ . \tag{32}$$

And in an analogous manner to the analysis conducted before, we have the reported solutions. **Q.E.D.**

Proof of Lemma 4.
After averaging for all possible upper limit's realizations, I obtain the following expected payoff function

$$E_0[U_1^{w^*}(2)] = \int_0^{\hat{x}_1} \left[1-(1-\bar{x})^2\right]\Pi^d - c\cdot\bar{x}\; d\bar{x} + \int_{\hat{x}_1}^1 (\Pi^d - c)\; d\bar{x} \tag{33}$$

Therefore it can be defined a new threshold $\hat{x}_0^w(\alpha)$. The remaining parts of the proof are analogous to earlier lemmas. **Q.E.D.**.

Proof of Proposition 1.
Suppose initially that the firm chooses to "gather" the available technology. Again the decision is taken after $\bar{x}_1$ is drawn and the realization known. Thus,

two cases are possibile. Suppose first that $\bar{x}_1 \leq \hat{x}_1^w$, recall that gathering the available technological knowledge is an equilibrium if

$$\bar{x}_1\Pi^d \leq \bar{x}_1(2-\bar{x}_1)\Pi^d - c\bar{x}_1$$

But this expression is equivalent to $\bar{x}_1 \leq \hat{x}_1^w$ as supposed at the beginning. Hence, gathering is an equilibrium in this case.
Suppose next that $\bar{x}_1 < \hat{x}_1^w$, now the above condition can be rewritten as $\bar{x}_1 \leq \hat{x}_1^w$ that is against the initial assumption. Hence, gathering is not an equilibrium in this case. **Q.E.D.**

Proof of Proposition 2.
First, notices that a sophisticated invention does not alter competition when both firms succed at the second period. It changes instead incentives at the first stage by allowing lagged behind firms to quit the race depending on the realized $\bar{x}_1$.
Given the recursive linear structure of payoffs, this R&D pattern reflects into a cut-off that is necessarily lower than the corresponding function for the plain case. In fact, after averaging payoffs over all possible realization of $\bar{x}_1$,

$$E_0[\hat{U}_1^{f*}] = \int_0^{\hat{x}_1^w} \bar{x}[(2\Pi^d - c) + \bar{x})]d\bar{x} + \int_{\hat{x}_1^w}^1 \bar{x}\Pi^d d\bar{x} \tag{34}$$

$$E_0[\hat{U}_1^{l*}] = \int_0^{\hat{x}_1^w} \bar{x}[(2\Pi^d - c) + \bar{x}]d\bar{x} + \int_{\hat{x}_1^w}^1 \bar{x}[\Pi^d - c]d\bar{x} \tag{35}$$

Thus, in the same manner as before, I define a new threshold $\hat{x}_0^{ws}(\alpha)$ and by some simple algebra it can be shown that this is non-greater than $\hat{x}_0^w(\alpha)$. The remaining part of the proof is the same as that in previous cases. **Q.E.D.**

Proof of proposition 3. If $\delta < 1$, notice that when $\alpha \to \infty$ then the pace of innovation in a weak regime reaches an upper-bound that is 0.44 which is above the corresponding value in a strong regime, i.e., 0.40.

Recall further that the pace of innovation in a weak regime is continuous and takes positive values for α that are higher than in the case of strong patent protection. Therefore there must be a value of α such that both curves crosses.

Proof of proposition 4. Recall that $\hat{x}_0^{ws}(\alpha) \leq \hat{x}_0^{w}(\alpha)$ and this tells us that the result holds. Further it is derived with some simple algebra.

Proof of Proposition 5.

At the second period there is sharing if and only if $\bar{x} < \hat{x}_1$. Notice, however, that this condition is going to be always verified in equilibrium.

At the first period a new threshold is defined

$$\tilde{x}_0 = \frac{E_0[U_1^{l^*}] - c}{E_0[U_1^{l^*}] + E_0[U_1^{f^*}] - E_0[U_1^*]} \tag{36}$$

and so the expected probability to innovate μ^{sl} is computed and the corresponding welfare depicted in the figure 9 . **Q.E.D.**

Consumer surplus and welfare

Given the linear demand $p = \alpha - q$ and assuming null marginal costs in producing the new good, the equilibrium quantities produced in a monopoly are $q^m = p^m = \frac{\alpha}{2}$. Thus, $\Pi^m = \frac{\alpha^2}{4}$. Hence, the social surplus under monopoly can be computed as $S^m = \Pi^m/2$.

If, by contrast, the product market is a duopoly, let us consider the corresponding equilibrium quantity produced as $q^d = (1+\gamma)q^m$, that is a function of the quantities produced in the product market under monopoly with $\gamma \geq 0$. Therefore, $p^d = \alpha - \frac{\alpha}{2}(1+\gamma)$.

By using the definition of duopoly profits, i.e., $\Pi^d \equiv (1-\delta)\Pi^m/2$, we can substitute values to obtain $\gamma = (1-\delta)^{1/2}$.

Finally, social surplus in duopoly reduces to equation (24).

Contents

References

J.J. Anton and D.A. Yao. Little patents and big secrets: managing intellectual property. *RAND Journal of Economics*, 35(1):1–22, 2004. ISSN 0741-6261.

J. Bessen. Patents and the diffusion of technical information. *Economics Letters*, 86(1):121–128, 2005. ISSN 0165-1765.

J. Bessen and E. Maskin. Sequential innovation, patents, and imitation. *The RAND Journal of Economics*, 40(4):611–635, 2009.

G. De Fraja. Strategic spillovers in patent races. *International Journal of Industrial Organization*, 11(1):139–146, 1993.

V. Denicolò. Two-stage patent races and patent policy. *the RAND Journal of Economics*, 31(3):488–501, 2000. ISSN 0741-6261.

V. Denicolò and L. A. Franzoni. Patents, Secrets, and the First-Inventor Defense. *Journal of Economics & Management Strategy*, 13(3):517–538, 2004. ISSN 1530-9134.

C. Fershtman and S. Markovich. Patents, imitation and licensing in an asymmetric dynamic R&D race. *International Journal of Industrial Organization*, 2009.

N.T. Gallini. Patent policy and costly imitation. *The RAND Journal of Economics*, pages 52–63, 1992. ISSN 0741-6261.

J.R. Green and S. Scotchmer. On the division of profit in sequential innovation. *The RAND Journal of Economics*, 26(1):20–33, 1995.

S. Scotchmer. Standing on the shoulders of giants: cumulative research and the patent law. *The Journal of Economic Perspectives*, 5(1):29–41, 1991. ISSN 0895-3309.

S. Scotchmer and J. Green. Novelty and disclosure in patent law. *The RAND Journal of Economics*, pages 131–146, 1990. ISSN 0741-6261.