Financial crisis resolution¹

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Abstract

This paper studies an infinite horizon production economy where financial intermediaries allow final investors to invest in profitable projects. To protect final investors, financial intermediaries are subject to endogenous capital requirements when providing credit to final borrowers competitively. The economy experiences a financial crisis if the intermediary cannot meet demand for credit due to insufficient intermediary capital. A distinctive feature of such an economy is that intermediaries affect capital requirements via their activities on the market for credit. The resulting pecuniary externality severely limits the flow of funds to profitable projects during financial crises. The constrained efficient regulation turns out to be simple: a second best features credit rationing in steady state.

1 Introduction

When the value of intermediary assets falls suddenly, then investment of borrowers that depend on external financing tends to fall as well (see Ivashina and Scharfstein (2010) for recent evidence). The recent financial crisis, 2008-2009, is a reminder that regulation cannot always prevent a large crisis, nor do we know whether prevention can be achieved at acceptable cost in terms of market distortions. Much recent research has been focussing

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on the causes of financial crises - however, considerably less attention has been paid to optimal crisis resolution.

In this paper, I analyze optimal regulation during financial crisis. To this end, I develop a model of an infinite horizon production economy where financial intermediaries (banks) have the special ability to mitigate an agency problem between final borrowers (firms) and final lenders (consumers). I assume that banks cannot commit to use this special ability, which gives rise to endogenous bank capital requirements, very similar to the scenario described in Holmstrom and Tirole (1997). A financial crisis in my model is caused by an unexpected decrease in bank capital which leads to a drop in credit supply and hence lower aggregate investment.

In my model, banks and consumers can trade a complete set of contingent claims, or Arrow-Debreu securities. It is assumed that when a bank extorts payments (e.g. via a buyout) from its creditors, by threatening to withhold its special intermediation ability, it will be excluded from intermediation in the future. Thus, banks' minimum capital requirements are implicitly given by a sequence of participation constraints, as described in Kehoe and Levine (1993). These participation constraints limit the bank's dividend policy and credit supply to ensure bank solvency in all states of the world. Equivalently, one can say banks face endogenous debt constraints, as described in Alvarez and Jermann (2000). These debt constraints limit bank short-selling of available assets to ensure bank solvency in all states of the world. Since banks' endogenous debt constraints determine firms' investment, the mechanics of my model look similar to an economy where each firm is debt-constrained itself, as described in Albuquerque and Hopenhayn (2004).

What distinguishes my model from existing models of firm dynamics is the fact that individual allocations depend on aggregate allocations via a perfectly competitive market for bank loans to firms. During a banking crisis bank capital is scarce such that bank credit to firms is rationed. Banks can raise funds to lend to firms by selling claims to future bank profits, subject to endogenous short-sale constraints (capital requirements). However, perfect competition on the market for loans to firms implies that future bank profits will be low, and eventually zero. In that sense, bank lending in steady state of the laissez-faire competitive equilibrium is excessive. It acts as a negative pecuniary externality leading to an inefficient amount of bank lending during the banking crisis, when bank loans are more scarce compared to the steady state.

How can a regulator improve upon such a laissez-faire competitive equilibrium allocation? To find a non-trivial answer (such as lump sum transfer to banks) to this question one has to ask what a regulator can do. Since banks cannot commit to repay their debt beyond a certain point (they might prefer to extort payments from its creditors and exit), there exists some kind of credit agency in the economy that keeps track of what banks do. In fact, the credit agency records banks' equity, loans to firms, and state-contingent debt, and has the authority to exclude banks.²I define the constrained-efficient allocation as the consumer-welfare maximizing allocation that a regulator can obtain by taking over the credit agency. I show that the regulator will choose to coordinate banks to ration credit when the economy is in steady state. During banking crises banks can then raise additional funds by borrowing against steady state bank profits. A regulator thus has to trade-off distorting economic activity in steady state and alleviating the current credit crunch. Intuitively, a small redistribution of income from workers to bank shareholders in the steady state leads to an increase in wages during the banking crisis, which ex ante more than compensates consumers for lower steady state wages.

In the general version of the model the amount that banks can extort from consumers per unit of loans to firms is higher when aggregate credit supply is low. That is, banks' intermediation ability is more valuable when firms are more productive. As a result, bank capital requirements become tighter when credit is already scarce, leading to a larger output loss during a banking crisis. In an extension, I show that this channel reduces the cost of a banking crisis to bank shareholders. However, the social cost of the banking crisis increases since the increase in lost wages more than offsets reduced losses for shareholders. I then discuss how bank creditor preferences for the composition of banks' loan portfolio may lead to more severe banking crises. In another extension, I argue that the need for bank capital requirements that aim at reducing systemic risk (in the sense of Lorenzoni (2008); such that a reduction in credit supply leads to fire sales of physical assets) would not arise in my model. In particular, I show that during the recovery from an initial

²See Kehoe and Levine (2001) for a brief discussion of such a credit agency.

credit crunch credit supply increases gradually in my model. However, credit supply can be very unstable over time in a finite horizon version of my model. Intuitively, with an infinite horizon and complete markets, banks' shareholder value only depends on how long it takes to reach their steady state value. Thus reduces temptation to react excessively to short term profitable opportunities.

1.1 Related literature

Abraham and Carceles-Poveda (2006), Abraham and Cárceles-Poveda (2008), and Kehoe and Perri (2004) study the effect of aggregate capital on agents' incentive to default. Davila, Hong, Krusell, and Rios-Rull (2005) discuss the redistributional role of the aggregate capital stock in a model with uninsurable idiosyncratic risk.

Jeanne and Korinek (2010) study an economy with a tree and a non-contingent bond. Fire sales of the tree occur since agents cannot insure sufficiently against small fruit and use the value of the tree as collateral: small fruit make the consumption profile steeper thus reducing the price of the tree which in turns makes the consumption profile steeper. They propose to subsidize precautionary savings in times of large fruit (Pigou tax on good-time borrowing). Brunnermeier and Sannikov (2010) study a production economy with a non-contingent bond and adjustment costs for physical capital. They show how an increase in future volatility of productivity reduces current investment due to precaution. The ensuing fall in the price for capital leads to a fall in net worth, and to a further reduction in investment. In my model an increase in future volatility of productivity would mean that bank default values would become more volatile. This will reduce overall debt capacity in my model and thus reduce current credit supply. However, my assumption of complete markets rules out a role for either excessive of insufficient precautionary savings.

Cooley, Marimon, and Quadrini (2004) study an economy where debt constrained producers can choose to default and reenter. They show that innovations at the productivity frontier create investment booms at existing firms as a result of optimal dynamic contracting. Rampini and Viswanathan (2009) consider a two-period version of a model similar to Albuquerque and Hopenhayn (2004). Since time ends before the stochastic steady state is reached the firm will not retain earnings in the last period and will in general not hedge financing risk sufficiently (I will define below what I mean by hedging financial risk). Lorenzoni (2008) shows how initial overinvestment can lead to an inefficiently high volume of fire sales when markets are complete and time finite.

Kashyap, Rajan, and Stein (2008) argue that more attention should be paid to reducing the costs of a financial crisis as it is unlikely that crises can be prevented altogether. They argue that higher bank capital requirements can lead to increased agency costs due to managers pursuing perks near the steady state. They suggest that regulators hold the capital instead, thus insuring banks, similar to a complete market for aggregate risk as considered in my model.Kaminsky and Reinhart (1999) document how attempts to use extensive transfers to troubled banks can lead to fiscal crises that are often followed by currency crises. In particular, domestic banks might have negative equity which may cause sovereign default, especially at smaller countries that backed domestic bank debt implicitly. In my model, there is no government that could issue debt. Peek and Rosengren (2000) discuss how during the Latin American bank crises governments agreed to let foreign multinational banks enter under the condition that they absorb troubled domestic banks. Diamond and Rajan (2000) point out that the possibility of a bank run eliminates the bank's commitment problem: consumers can commit not to renegotiate bank debt due to a collective action problem that leads to a bank run once a bank is insolvent.

2 Example

Consider a representative bank with zero initial net assets that can borrow at gross interest rate $\frac{1}{\beta} > 1$. There is no uncertainty. Banks are owned by consumers and maximize the present value of dividend payments to consumers, or share holder value $V_0 = \sum_{t=0}^{\infty} \beta^t d_t$. Dividend payments d_t must be non-negative at all times. The bank can lend to a representative firm at gross rate R(K), where K is the aggregate capital stock equal to aggregate loans to firms. Intermediaries and firms act competitively on the market for capital loans. Firms also hire labor from consumers on a competitive labor market. Firms have access to a constant returns technology that turns K units of the consumption good and L units of

labor into Y units of the consumption good,

$$Y = F(K, L) = K^{\alpha} L^{1-\alpha} + (1-\delta)K.$$

In equilibrium, L = 1 is inelastically supplied and loan return R(K) and wage w(K) satisfy

$$R(K) = \alpha K^{\alpha - 1} + 1 - \delta$$
$$w(K) = (1 - \alpha) K^{\alpha}.$$

In the absence of financial frictions the bank borrows funds to finance loans to firms k^* solving

$$\max_k R(K)k - \frac{1}{\beta}k$$

where $K = K^* = k^*$ in equilibrium. Note that $\beta R(K^*) = 1$. I.e., when the bank is not borrowing-constrained it will borrow until its profit margin is zero. In a First Best, the bank can always borrow and supply credit to firms such that $K_t = K^*$ at all times. When banks are borrowing-constrained for some reason then credit to firms might be low such that $K_t < K^*$. To study such a case, suppose firms can hide the entire capital unless the bank prevents it. If the bank cannot commit to prevent embezzlement by firms then the bank will be borrowing-constrained in general. The reason is that the bank might be tempted to extort payments from its own creditors by threatening to let firms embezzle part of the loan. Endogenous bank capital requirements arise such that banks are not tempted to extort payments from its creditors. Let $V_t = \sum_{\tau=t} \beta^{\tau-t} d_{\tau}$ be the shareholder value of the bank at period t, then the participation constraint of the bank is

$$V_t \geq k_t$$

When $k_t < K^*$, this constraint will be binding strictly.³ Then the bank will find it optimal to postpone dividend payments such that $V_{t-1} = d_{t-1} + \beta V_t = \beta V_t$ for t = 1, 2, ..., T,

³To see this note that increasing k_t by dk in t - 1 using debt when $V_t > k_t$ leads to period t additional profits of $dk(\beta R(k_t) - 1) > 0$ from the viewpoint of the intermediary. No following participation constraints are adversely affected.

where T is the number of periods to achieve $V_T = K^* \equiv V^*$. We also have

$$K_t = V_t = \beta^{T-t} V^*,$$

implying that the aggregate capital stock in this economy grows at gross rate $\frac{1}{\beta}$ until it reaches its steady state value K^* . Note that the competitive equilibrium has the same capital stock in steady state as the First Best.

Now that we know what happens in competitive equilibrium let us study if it is constrained efficient. Define welfare as the date zero present-value life-time income of consumers

$$W_0 = \sum_{t=0} \beta^t (w(K_t) + d_t),$$

where K_0 is given. Can a constrained regulator achieve a higher level of welfare? Proposition 1 shows that the answer is yes.

Proposition 1. A constrained-efficient allocation features credit rationing. That is, the second best steady state capital stock is less than in competitive equilibrium, $K_{SB} < K^*$.

Proof. A reduction in K^* corresponds to rationing credit in steady state and increases steady state shareholder value while reducing wages. Since the steady state capital stock in laissez-faire competitive equilibrium is the same as in the unconstrained First Best, a marginal income redistribution from consumer to bank at the unconstrained optimum does not affect consumer welfare in steady state. However, from a date zero perspective, the marginal social value of bank income and of labor income differ. The bank has a higher social marginal value of income at date zero: an increase in V^* also increases every $K_t = \beta^{T-t}V^*$ along the transition. This alleviates the capital scarcity in the economy during the transition and increases welfare unambiguously from a date zero perspective.

Another way to see how initial credit supply depends on future bank rents is to write bank shareholder value as the sum of current net assets and future income from rents

$$V_t = R_t k_t - b_t + \sum_{\iota=t+1}^{\infty} \beta^{\iota-t} \left(R_\iota - \frac{1}{\beta} \right) k_\iota,$$

where b_t is bank debt repayable in period t (the bank borrowed βb_t in t - 1). The bank's participation constraint can also be written as a debt constraint

$$b_t \leq (R_t-1)k_t + \sum_{\iota=t+1}^{\infty} \beta^{\iota-t} \left(R_\iota - \frac{1}{\beta}\right)k_\iota.$$

Define $\xi_t = \frac{b_t}{k_t}$ as a measure of leverage then

$$\xi_t = (R_t - 1) + \sum_{\iota=t+1}^{T-1} \left(R_\iota - \frac{1}{\beta} \right) + \frac{1}{1-\beta} \left(R_T - \frac{1}{\beta} \right),$$

where T > t is the length of the transition. Hence, rationing of credit in steady state, an increase in R_T , allows for greater bank leverage during the transition. We can also write down endogenous bank capital requirements during the transition as

$$A_t \ge \beta k_{t+1} - \beta \left(R_{t+1} - \frac{1}{\beta} \right) k_{t+1} - \beta \sum_{\iota=t+2}^{\infty} \beta^{\iota-t} \left(R_\iota - \frac{1}{\beta} \right) k_\iota, \tag{1}$$

where I used $A_t = k_{t+1} - \beta b_{t+1}$ for t < T. Future bank profits reduce the amount of net assets the bank is required to hold. The example in figure 1 shows the economy over time, starting from an initial financial crisis with $A_0 = 0$. Let

$$K^*(\tau) = \left(\frac{\alpha(1-\tau)}{\frac{1}{\beta}-1+\delta}\right)^{\frac{1}{1-\alpha}}$$

be the steady state of the economy when the distortion τ is implemented ($\tau > 0$ is a measure of credit rationing). Note that $K^*(0) = K^*$ is the laissez-faire competitive equilibrium steady state capital stock. In the numerical example, the second best features a distortion of $\tau = 0.1714$ and leads to welfare that is higher by 1.26% in terms of Hicksian equivalent compensation.

For comparison, figure 2 shows the same economy where the bank participation constraint is

$$V_t \geq K_t^{\alpha - 1} k_t$$





tortion τ . Welfare is maximized at $\tau^* = 0.1714$.

(a) Ex ante welfare as a function of steady state dis- (b) Aggregate capital stock over time. Note that distorting the steady state by $\tau = 0.1857 > \tau^*$ leads to even higher initial investment and output but lower welfare.

Figure 1: The second best is achieved by distorting the steady state capital stock by $\tau^* = 0.1714$. Note that the sequence of aggregate capital stocks in panel 1(b) denoted by $\tau = 0$ corresponds to the laissez-faire competitive equilibrium.

that is, the firm has the ability to embezzle production rather than undepreciated physical capital.⁴ Implementation of the Second Best now leads to a welfare increase of 3.27% > 1.26%. Note that in this economy the financial crisis has a higher welfare cost in laissez-faire competitive equilibrium, while the second best can be achieved with a smaller distortion τ^* . The reason is that the second example features bank outside values that depend on the aggregate capital stock, leading to more powerful pecuniary externality and making a regulatory intervention more beneficial. The point of this second example is to illustrate that a severe banking crisis does not necessarily imply that the optimal regulatory intervention will be very costly. It is crucial to understand the nature of the intermediation service provided by banks, i.e. the nature of the bank participation constraint.⁵

⁴It is assumed that a firm can borrow from any one bank, such that the size of the loan depends on the level of aggregate loans. I provide micro-foundation for the different kinds of participation constraints below.

 $^{^{5}}$ In practice, we often see much debate over the size of the bank 'bailout' necessary to alleviate a credit crunch. While this may reflect redistributional issues it may also reflect different beliefs about the nature of the bank participation constraint.





(a) Ex ante welfare as a function of steady state dis- (b) Aggregate capital stock over time. Note that distortion τ . Welfare is maximized at $\tau^* = 0.1429$. torting the steady state by $\tau = 0.1571 > \tau^*$ leads to

(b) Aggregate capital stock over time. Note that distorting the steady state by $\tau = 0.1571 > \tau^*$ leads to even higher initial investment and output but lower welfare.

Figure 2: In this example, the righthand side of the bank participation constraint depends on the aggregate capital stock. The second best is achieved by distorting the steady state capital stock by $\tau^* = 0.1429$. Note that the sequence of aggregate capital stocks in panel 2(b) denoted by $\tau = 0$ corresponds to the laissez-faire competitive equilibrium.

3 Model

Consider an infinite-horizon production economy in discrete time with a single nonstorable consumption good, productive capital, and labor.

Uncertainty

For each t, there is a finite set $S^t = \prod_{j=0}^t S$ of date-t events $s^t = (s_0, s_1, s_2, \ldots, s_t)$. Let s_t be generated by a first order Markov process, with initial state s_0 given. Event s^{τ} , with $\tau \ge t$, is said to follow event s^t (denoted $s^{\tau} \succeq s^t$) if $s^{\tau} = (s^t, s_{t+1}, \ldots, s_{\tau})$. Let $S \equiv \bigcup_{t=1,2,\ldots} S^t \bigcup \{s_0\}$ denote the set of all events. At date 0, nature draws a sequence (s_1, s_2, \ldots) given s_0 , and at date t, the event s^t is revealed. The date-zero probability that s^t is observed is denoted by $\pi(s^t|s_0)$.

Agents, endowments, and production

There is a unit measure of identical consumers. The representative consumer supplies one unit of labor inelastically in each event. Preferences over consumption plans of the form $C = \{c(s^t)\}_{s^t \in \mathcal{S}}$ are characterized by numbers

$$U(C) = \sum_{s^t \in \mathcal{S}} \beta^t u(c(s^t)) \pi(s^t),$$
(2)

where $\beta \in (0,1)$, and $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable. There is a unit measure of identical intermediaries.

The aggregate production technology requires and investment of $K(s^t)$ units of the consumption good in event s^t , and $L(s^{t+1})$ units of labor in event s^{t+1} to produce

$$F(K(s^{t}), L(s^{t+1}); s^{t+1}) = z(s^{t+1})K(s^{t})^{\alpha}L(s^{t+1})^{1-\alpha} + (1-\delta)K(s^{t})$$

units of the consumption good in event s^{t+1} , where $\alpha \in (0,1)$, and $\delta \in (0,1)$ is the rate of capital good depreciation. The function $z(\cdot)$ from events to aggregate productivity is bounded and non-negative. The aggregate technology is operated by a unit measure of short-lived, ex-ante identical firms.

Markets

Each consumer owns a unit share of the representative intermediary. At each event s^t , individual consumers and intermediaries can trade a complete set of contingent securities with the rest of the world, each security promising to pay one unit of the consumption good in $s^{t+\tau} \succ s^t$. It will turn out that in equilibrium no agent defaults; all securities will therefore trade at a common price $q_t(s^{t+\tau}, s^t)$, irrespective of the agent who issued it.⁶ This common price is assumed to be

$$q_t(s^{t+\tau}, s^t) \equiv \beta^{\tau} \pi(s^{t+\tau} | s^t)$$

This assumption makes sense when thinking about the economy studied here as a small open economy with the rest of the world in steady state. The intermediary ranks dividend payment plans of the form $D = \{d(s^t)\}_{s^t \in S}$ by comparing their expected present value at

⁶If an agent would be expected to default then it can issue marginal claims at a price of at most zero. The reason is that the aggregate state is common knowledge such that agents are expected to default with probability either one or zero.

date zero,

$$V(D) = \sum_{s^t \in \mathcal{S}} q(s^t) d(s^t) = \sum_{s^t \in \mathcal{S}} \beta^t \pi(s^t | s_0) d(s^t).$$
(3)

Consumers can trade their shares in the intermediary, hence it makes sense to call V(D) the shareholder value of the intermediary. Interim share prices can be constructed as follows,

$$V_t(s^t) = \sum_{s^ au \in \mathcal{S}(s^t)} eta^{ au - t} \pi(s^ au | s^t) d(s^ au).$$

In the appendix I show that firms will demand aggregate investment $K(s^{t-1})$ and inelastic labor supply of $L(s^t) = 1$ whenever the gross interest rate on loans, R, and the wage rate, w, satisfy

$$R(s^{t}) = z(s^{t})\alpha K(s^{t-1})^{\alpha-1} + 1 - \delta,$$

$$w(s^{t}) = z(s^{t})(1-\alpha)K(s^{t-1})^{\alpha}.$$
(CFM)

Intermediary admissible allocations and objective

I assume that firms are hit by idiosyncratic shocks after investing the funds borrowed from intermediaries. Firms can lie about their profitability and/or the amount of undepreciated capital after production. However, intermediaries have the ability to privately observe the idiosyncratic state of firms it lends to, such that loan repayments will depend on the firm's idiosyncratic state. Since intermediaries lend to a well-diversified pool of firms, they know that their event s^t lending generates a gross return of $R(s^{t+1})$ in future event s^{t+1} .

An Intermediary will generally fund lending to firms by selling claims on future revenue generated from this lending activity. However, the intermediary's creditors understand that they rely on the intermediary to use its private information about firms: claims backed by loans to firms will be worth less if firms can lie and divert output and machinery. For example, if the intermediary's creditor collects loan repayment (in the case of intermediary default) then each firm would claim the worst idiosyncratic state and divert the remainder for private use. Normalizing creditor bargaining power to zero, the intermediary can extort payments from its creditors (e.g. in form of a buyout) in case of intermediary default by threatening to hold up collection of loan repayment.⁷ In the appendix I show that the condition that prevents intermediary holdup is given by

$$V_t(s^t) \ge \mathcal{O}\left(s^t, K(s^{t-1})\right) k(s^{t-1}), \quad \forall s^{t-1} \text{ and } s^t \succ s^{t-1}, \tag{PC}$$

where $\mathcal{O}(s^t, K(s^{t-1}))$ is the intermediary default value per unit of the loan, and $k(s^{t-1})$ is intermediary lending. Note that $\mathcal{O}(s^t, K(s^{t-1}))$ depends on other intermediaries' lending activity via the aggregate investment $K(s^{t-1})$. The reason is that with decreasing returns to scale each firm's profitability depends on the average amount lend to firms.

Further, it is assumed that intermediary dividend payments have to be non-negative at all times.

$$d(s^t) \ge 0, \quad \forall s^t \in \mathcal{S}.$$
 (DNN)

The intermediary can trade contingent claims, denoted by $b_I(s^t)$, such that sequential budget constraints can be constructed as

$$k(s^{t}) + d(s^{t}) = \sum_{s^{t+1} \succ s^{t}} \beta \pi(s^{t+1} | s^{t}) b_{I}(s^{t+1}) + a_{I}(s^{t}),$$
(4)

where net assets evolve according to

$$a_I(s^{t+1}) = R(s^{t+1})k(s^t) - b_I(s^{t+1}).$$

Combining sequential budget constraints leads to a date zero budget constraint

$$\sum_{s^t \in \mathcal{S}} \beta^t \pi(s^t) \left[R(s^t) k(s^{t-1}) - k(s^t) - d(s^t) \right] \ge b_{I,0},\tag{IBC}$$

where $k(s^{-1}) = k_0$ and $b_{I,0}$ are given. An intermediary allocation $\{d(s^t), k(s^t)\}_{s^t \in S}$ that satisfies (PC), (DNN), and (IBC) is called *admissible*. The intermediary's objective, given $\{R(s^t), \mathcal{O}(s^t, K(s^{t-1}))\}_{s^t \in S}$, as well as initial conditions $k_0, b_{I,0}$, is to choose an admissible

⁷As an example, creditors could be asked to buy up the shares of the insolvent intermediary for some strictly positive price.

allocation of dividends and loans to firms that maximize shareholder value (3).

Consumer admissible allocations and objective

Any consumer allocation $\{c(s^t)\}_{s^t \in S}$ that satisfies

$$\sum_{s^t \in \mathcal{S}} \beta^t \pi(s^t | s_0) \left(w(s^t) + d(s^t) - c(s^t) \right) \ge 0 \tag{CBC}$$

is admissible for the consumer. The consumer problem is choose and admissible allocation for consumption that satisfies (CBC) for given $\{w(s^t)\}_{s^t \in S}$ and $\{d(s^t)\}_{s^t \in S}$.

Aggregate resource constraint

At each point in time s^t the country's resource constraint is given by

$$c(s^{t}) + K(s^{t}) - (1 - \delta)K(s^{t-1}) + NX(s^{t}) = z(s_{t})K(s^{t-1})^{\alpha},$$
(5)

where net exports are given by

$$NX(s^{t}) = w(s^{t}) + d(s^{t}) - c(s^{t}) + b_{I}(s^{t}) - \sum_{s^{t+1} \succ s^{t}} \beta \pi(s^{t+1} | s^{t}) b_{I}(s^{t+1}).$$

Definition of a First Best

Below we will compare the laissez-faire competitive equilibrium and the constrained Second Best to the First Best (when there are no frictions) with respect to the dynamics of the aggregate capital stock.

Definition 1. (First Best Investment) We say that the capital stock in the economy follows First Best dynamics at s^t if

$$\sum_{s^{\tau+1}\succ s^{\tau}}\beta\pi(s^{\tau+1}|s^{\tau})R(s^{\tau+1})-1=0,$$

for all $s^{\tau} \succeq s^{t}$.

4 Laissez-faire competitive equilibrium

I assume that, for given $k(s_0)$, initial financial liabilities of the intermediary, $b_I(s_0)$, are large enough such that the intermediary is strictly borrowing constrained. One could think of a large fraction of loans to firms suddenly losing value, reducing bank net assets.⁸ Then the economy experiences a financial crisis and aggregate investment will be below its First Best level temporarily.

Definition 2. (*Competitive Equilibrium*) *A competitive equilibrium for this economy is characterized by (i) an intermediary allocation, (ii) a consumer allocation, (iii) prices for labor and loans, intermediary default values, such that*

- (i) is admissible given (iii) and solves the intermediary problem
- (ii) is admissible for the consumer given (i) and (iii) and solves the consumer problem
- the market for capital loans clears: $K(s^t) = k(s^t)$ for all s^t .

Define intermediary rents as the net present value of a capital loan per unit of loaned capital.

Definition 3. (Intermediary Rents) Intermediary rents from capital loans at s^t are defined as

$$\sum_{s^{t+1}\succ s^t} \beta \pi(s^{t+1}|s^t) R(s^{t+1}) - 1.$$

Proposition 2 states the intuitive result that intermediaries will retain earnings long as they can earn rents on capital loans.

Proposition 2. (*Dividend policy during financial crisis*) *At any event* s^t, the intermediary earns strictly positive rents if and only if (DNN) binds strictly.

⁸Sudden, and apparently unexpected, decreases in the expected repayment of loans are generally associated with real estate lending. See Hoshi and Kashyap (2010) for a comparison between the recent experiences in Japan and the US. The question why the contingency of a sudden drop in borrower quality has not been traded at an arbitrage-free price a priori is beyond the scope of this paper. It is plausible that this misperception was shared by regulators (in the model: credit agency). For example, **?** argues this is likely for the case of the Irish banking crisis 2008. In that case, borrower quality turned out to be low as many debtors were actually property developers eager to invoke limited liability. Also, in contract to the Irish 1970s property bust, the recent crisis was deepened by macroeconomic consequences of debt-deflation (the 1970s were an inflationary environment).

Note that (DNN) binds strictly if and only if (PC) is strictly binding. Hence the intermediary can only earn rents as long as credit to firms is rationed due to intermediaries being constrained. Over time, due to decreasing returns in investment of the aggregate production technology, it would make sense if intermediaries trade contingent claims in such a way as to smooth rents on capital loans. Proposition 3 confirms that this will indeed be the case once rents are zero.

Proposition 3. If rents are zero in s^t , they will be zero in any $s^{\tau} \succ s^t$.

Combining definition 1 and proposition 3 shows that once the intermediary ceases to earn rents on loans to firms, the aggregate capital stock in the economy will be the same as in the unconstrained First Best. Then the intermediary will have saved itself out of being "borrowing-constrained".

Corollary 1. (Competitive equilibrium capital stock in steady state) Once rents on loans to firms are zero, the aggregate capital stock in the economy follows First Best dynamics.

The intermediary will trade contingent claims to ensure that (PC) holds in all events s^t . This results in a significant volume of financial transactions not directly related to intermediary lending to firms. Intuitively, financial assets/ debt capacity should be conserved in states where (PC) is slack, while lending to firms should be expanded in states where (PC) binds strictly. The reason is that the intermediary must provide higher shareholder value along paths that follow a binding participation constraint, which is best achieved by earning rents.

It will be helpful to make the following assumption which guarantees that financial assets of the intermediary are well defined and do not grow without bound.

Assumption 1. (Financial assets in steady state) Let $\{b(s^t; \tau)\}_{s^t \in S}$ be the set of contingent claims issued by the intermediary in equilibrium in a version of the above economy where there is a tax of τ on the absolute value of each claim, $\tau \beta^t \pi(s^t) |b(s^t; \tau)|$. Assume that $\{b(s^t; \tau)\}_{s^t \in S}$ is continuous in τ on \mathbb{R}_+ . Denote the limit of $\{b(s^t; \tau)\}_{s^t \in S}$ as $\tau \searrow 0$ by $\{b_I(s^t)\}_{s^t \in S}$. Assume that the intermediary issues $\{b_I(s^t)\}_{s^t \in S}$ in equilibrium in the above economy.

5 Constrained Second Best

I assume that a constrained social planner (regulator) can coordinate agents in the economy. The regulator then realizes that $K(s^t) = k(s^t)$ such that the effect of each intermediary's credit supply to firms on wages, loan returns, and intermediary default values is internalized. I will provide further discussion of what the regulator can do in section 6.

Since prices of contingent claims are exogenously given and the implied risk free rate equals the inverse consumer subjective discount factor, the SB will feature a constant level of consumption \bar{c} . By (CBC) this constant level of consumption will be an annuity of the present value of wages and dividends,

$$\bar{c} = (1 - \beta) \sum_{s^t \in \mathcal{S}} \beta^t \pi(s^t | s_0) \left(d(s^t) + w(s^t) \right).$$

Hence, consumer life time utility (2) is maximized whenever the welfare measure *W* is maximized, where

$$W(s_0) = \sum_{s^t \in S} \beta^t \pi(s^t | s_0) \left(d(s^t) + w(s^t) \right).$$
(6)

For the social planner it is assumed that admissibility means the same as in section 4: any SB allocation must satisfy (DNN), (PC), and (IBC). We are now in a position to define what is meant by Second Best.

Definition 4. (Second Best) A Second Best is defined as an admissible allocation of capital loans and dividends $\{k_{SB}(s^t), d_{SB}(s^t)\}_{s^t \in S}$ that, given initial conditions $k_{0,SB} = k_{SB}(s^{-1}), b_{I,0}$, maximizes (6) subject to

$$\mathcal{O}(s^{t}, k_{SB}(s^{t-1})) = z(s^{t}) \left[\theta_{1} k_{SB}(s^{t-1})^{\alpha - 1} + \theta_{2} \right]$$
$$R(s^{t}) = z(s^{t}) \alpha k_{SB}(s^{t-1})^{\alpha - 1} + 1 - \delta,$$
$$w(s^{t}) = z(s^{t})(1 - \alpha) k_{SB}(s^{t-1})^{\alpha}.$$

Proposition 4 shows that part of proposition 3 still applies in the SB.

Proposition 4. In Second Best, if (DNN) does not bind in s^t, then (DNN) and (PC) will not bind

in any $s^{\tau} \succ s^{t}$.

On the other hand, proposition 5 states that the Second Best capital stock does not follow First Best dynamics in steady state.

Proposition 5. In SB, if (DNN) does not bind in s^t , then intermediary rents will be constant and strictly positive for all $s^{\tau} \succeq s^t$.

The example in section 2 gives intuition for this result: a reduction in the long run capital stock allows intermediaries to earn rents by rationing credit to firms. Intermediaries can borrow against these rents early on to alleviate the initial credit scarcity. Intuitively, the constrained social planner prefers to smooth out capital loan scarcity over time. Hence, the capital stock will be increased early on when the marginal product of capital is high, and decreased in later periods when the marginal product of capital is low. Note that the Second Best involves strictly higher shareholder value at s_0 , i.e., it corresponds to a 'bailout'.

6 Implementing the Second Best

Let us briefly lay out what a regulator can do. In the laissez-faire competitive equilibrium of the economy studied, there exists a credit agency that records and publishes banks' financial assets, and lending to firms. This agency can also close down banks.⁹ The second best is implemented by a regulator that takes over the credit agency. Proposition 6 states that the second best can be decentralized by imposing an upper bound on steady state loans, k_{SB}^* .

Proposition 6. (Optimal financial crisis resolution) The second best can be decentralized by an upper bound on the bank loan portfolio size $k_{SB}^*(s^t)$. This upper bound is state dependent as aggregate productivity may be persistent.

$$k_{SB}^*(s^t) = \left(\frac{\alpha E(z(s_{t+1})|s_t) \left(1 - (1 - \alpha) \left(1 - \frac{1}{\lambda_0}\right)\right)}{\frac{1}{\beta} - 1 + \delta}\right)^{\frac{1}{1 - \alpha}}$$

⁹In particular, I rule out that the agency can punish shareholders for bank failure, for example via double liability. See Macey and Miller (1992) for a discussion, and more recently Admati and Pfleiderer (2010).

where $\lambda_0 > 1$ is the social return on bank net assets at date zero.

Note that this is equivalent to a Pigou tax of $\tau = (1 - \alpha) \left(1 - \frac{1}{\lambda_0}\right)$ on production. This tax is constant since markets are complete: the social return on bank net assets is smoothed. λ_0 measures the scarcity of bank net assets and determines the size of the distortion that transfers income from workers to shareholders. The assumption of an infinite horizon is crucial: in a three period version of the above economy it can be shown that $\tau < 0$, even though $\lambda_0 > 1$, can be optimal.

6.1 Entry of multinational banks

Entry of foreign banks can help to sustain the flow of credit to domestic firms even as domestic banks face tight borrowing constraints. Peek and Rosengren (2000) discuss this for Argentina, Brazil, and Mexico, and argue that foreign banks also bring expertise to the domestic banking system. The problem is that severe banking crises may lead to not just insufficient, but actually negative bank net assets. If the country's government lets foreign banks enter then competition for loans drives down the value of struggling domestic banks to zero. Governments most likely will not allow all struggling domestic banks to file bankruptcy at the same time. In fact, during the Brazilian banking crisis, Brazil made it a condition for entering multinational banks to absorb struggling domestic banks. But then the entering bank requires the government to restrict further entry: there need to be sufficient rents from loans to be earned during the transition to earn back the cost of absorbing negative equity of troubled domestic banks. Further, if domestic bank shareholders have a stronger lobby than domestic workers, then a government is unlikely to allow foreign bank capital as it will dilute down domestically held equity.

6.2 Recapitalization

In general, banks might fear to identify themselves as lemons when asking for fresh equity during a financial crisis. But special regulatory circumstances also play an important role in making a timely recapitalization difficult. Swire (1992) documents how, for the US, regulatory powers have been expanded significantly in the wake of banking crises. He argues that this might lead to a time inconsistency problem. 'Superpowers' granted to the FDIC include determining when a bank is insolvent, and subordinating claims of insiders and outsiders to the deposit insurance fund's claims. In particular, informal agreements will not be honored by the FDIC, which acts as a receiver. Swire (1992) argues that the specialness of bank, compared to nonfinancial corporate, insolvency law leads to a different kind of bank run. Bank creditors as well as debtors will cease business relations with the bank once it has low equity, as the point of insolvency is unclear due to FDIC discretion in that matter. Hence, FDIC's ex-post toughness on third parties may lead to excessive bank insolvencies ex ante. In particular, recapitalization of banks may become more difficult: potential investors would prefer to wait until after the bank went through an FDIC-orchestrated insolvency as this can eliminate hidden liabilities.¹⁰ Hoshi and Kashyap (2010) describe how uncertainty over regulators' intentions slowed down recovery from the Japanese banking crisis of the 1990s.

6.3 Collusion or concentration

The constrained-efficient banking crisis resolution proposed above involves collusion rather than concentration on the market for bank loans. However, both are ways to recapitalize banks. Consistent with my analysis, Cetorelli and Gambera (2001) find that bank concentration is positively correlated with growth in fast growing, underdeveloped sectors, while negatively correlated with growth in general. In particular, it might be beneficial if the banks serving an industry that experiences a scarcity of investment have some market power. However, bank concentration differs from collusion in that it may also affect otherwise perfectly competitive product markets on which borrowers are active. Cartelization of firms as a result of bank concentration around 1900 has been discussed by Simon (1998). For a recent example of how debt dependence may increase margins on the product market see Chevalier (1995). Rajan and Zingales (2003) argue that incumbent banks influence regulators to hinder financial reform, and thus keep bank industry concentrated, unless

¹⁰Coates and Scharfstein (2009) argue that attempts to recapitalize banks should involve forgiving debt partially. The idea is to reduce the amount of new equity needed to avoid a de facto nationalization of banks, given that it is often unlikely to raise very large amounts of equity from private sources.

pressures from trade and capital flow liberalization are strong.¹¹

While bank concentration, as opposed to bank collusion, may be be interpreted as a possible 'third best' response to a bank crisis, it cannot be cleanly separated from political economy issues. For example, the 1923 Tokyo earthquake cost 38% of Japanese GDP at that time and arguably also represented a large shock to bank net assets. In fact, the number of banks dropped from 2000 before the disaster to about 65 after, while the fraction of total deposits held by the five largest banks increased from 20.5% to 45.7%. In addition, banks became to head bond committees which may have allowed them to exert power over borrowers that had access to direct finance. However, these measures cannot be interpreted solely in the light of optimal regulation, as the Japanese government at that time was also in need of a strong and willing banking sector to finance two wars (1937 war against China, and the second world war).

7 Numerical exercise

As described in section 6, decentralizing the constrained efficient allocation as a competitive equilibrium requires solving for λ_0 , that is, we have to solve for the entire constrained efficient allocation. In this section I compare laissez faire competitive equilibrium (CE) to the constrained efficient allocation (SB). I do this numerically for both as I cannot solve for the SB by hand. Note that the exercise does not feature a realistic calibration and is thus of a qualitative nature. The reason is that the parameters θ_1 , θ_2 are poorly identified and likely vary a lot across different kinds of intermediaries in the economy. Here we assume there is only one kind of intermediary, banks, and assume the economy is in steady state at the time of the unexpected shock that initiates the banking crisis.¹²

First, I solve for the recursive competitive equilibrium with endogenous debt con-

¹¹In that sense regulators may be forced to renege on an earlier promise to grant rents, if international financial integration arrives suddenly and unexpectedly. In that case banks will suddenly be severely underfunded as the loss of future rents would lead to increased capital requirements.

¹²These assumptions of course do not make sense: financial crisis often occur when the economy experiences a large change in in the relative evaluation of a particular asset class, for example real estate. Also, there are many different intermediaries such as commercial banks, investment banks, and large non-financial companies. However, given the difficulties real-world regulators have in designing 'stress tests', a good guess seems appropriate to me at this stage.

straints. I propose aggregate bank net assets and aggregate productivity as sufficient to describe the aggregate state of the economy. I propose bank net assets to be sufficient in characterizing the individual bank. For details, see appendix.

Second, I solve for the constrained Second Best in the context of a dynamic game between regulator and banks, taking the behavior of consumers and outside lenders to the bank as given. I use the recursive formulation developed by Abreu, Pearce, and Stacchetti (1990) where the player that cannot commit (bank) is offered continuation values from staying in the contract. The constrained regulator must grant banks sufficient future shareholder value to satisfy endogenous debt limits. There is a natural bound on shareholder value, consistent with monopolistic lending to firms. I propose aggregate bank net assets, average bank shareholder value, and aggregate productivity as sufficient to describe the aggregate state of the economy.

I solve for the competitive equilibrium (CE) and constrained Second Best (SB) using policy function iteration with cubic spline interpolation, as described in Judd, Kubler, and Schmedders (2002). In particular, I deal with occasionally binding constraints by including penalty functions. I find that when comparing CE and SB the fact that I need an additional penalty function (promise keeping constraint) in SB is not quantitatively important as penalties are relatively small throughout. Details are provided in the appendix.

Table 1 gives a sample calibration where one period corresponds to roughly three years. It is assumed that the economy is in stochastic steady state initially when a sudden, unexpected shock reduces equity of each bank (banking crisis). Figure 3 shows the economy over time for initial bank equity of 0.05. This initially low level of bank capital, together with endogenous bank capital requirements, causes a severe credit crunch: banks increase average loan interest rates until credit demand by firms meets banks' limited credit supply. In fact, the CE capital stock is initially less than one half of its steady state value. Over time, banks build up capital and are able to expand lending to firms. In the numerical example it takes seven periods until the bank balance sheet reaches its pre-crisis size. At that point the aggregate capital stock is not distorted anymore.

The most apparent difference in the SB is that it features a distorted steady state capital stock. The regulator finds it optimal to grant long term excess rents to banks, around 100

Table 1: Calibration		
parameter	value	
subjective discount factor β	0.94	
capital income share α	0.4	
capital depreciation rate δ	0.12	
aggregate productivity z	(0.9,1.1)	
firm cash flow heterogeneity θ_1	0.5	
firm depreciation heterogeneity θ_2	0.15	

basis points in the example. Banks anticipate that they become more valuable in the future. This increases initial shareholder value and thus bank leverage. In fact, endogenous bank capital requirements are relaxed substantially as banks can borrow against future rents. This is why in the SB debt is higher while bank equity is lower. High initial bank leverage means that the initial drop in credit is less severe; in the example the aggregate capital stock falls by much less. While the ex ante welfare loss due to the banking crisis is around 40% smaller in the SB, the CE features higher ex post welfare as its steady state capital stock is undistorted. Recall that welfare is defined as consumers present value life time income from dividends and labor. Hence implementing the constrained second best at date zero increases welfare by 5%, from 15 to 15.75, in terms of Hicksian equivalent variation.

Note that there is a possible time inconsistency issue here: the regulator could ex post decide to not distort the capital stock. This offers a new explanation for financial crises, one where regulators unexpectedly default on their promises to grant rents to banks. In such a case bank capital will be insufficient to satisfy endogenous capital requirements such that aggregate credit supply decreases.

8 Extensions

8.1 Transition dynamics in laissez-faire competitive equilibrium

The purpose of this section is to better understand the qualitative properties of our economy. Let us assume that the productivity shock is i.i.d. with equal probabilities and $z(s_t) \in \{z_L, z_H\}$. Let $0 < z_L < \beta < 1 < z_H < 2$ such that $z_L + z_H = 2$. To better differentiate the effects of intermediary competition on welfare inside and outside of the stochastic



Figure 3: The green dotted line denotes the constrained second best. A regulator smoothes the distortion of the aggregate capital stock over time. As a result, the initial credit crunch is alleviated substantially while the steady state capital stock is distorted indefinitely.

steady state (section 8.3), let us consider two cases: in case one $\theta_1 > 0 = \theta_2$, and in case two $\theta_2 > 0 = \theta_1$. In case one, physical capital cannot be diverted while output can be diverted by firms unless the intermediary uses its proprietary information. In case two, only physical capital can be diverted by firms. To compare the two cases it will be helpful to assume that they are identical in the stochastic steady state. Let $K^* = \left(\alpha/(\frac{1}{\beta} - 1 + \delta)\right)^{\frac{1}{1-\alpha}}$ be the steady state capital stock, then $\theta_1 K^{*\alpha-1} = \theta_2$ is required to lead to identical leverage and dividend policy in steady state.

Assumption 1 ensures that there is a unique stochastic steady state. It is again possible to work backwards since the assumption $z_L < \beta$ guarantees that the intermediary's no-holdup constraint will bind if and only if productivity is high (weakly in the stochastic steady state). The length of the transition is then not measured in periods but in occurrences of the high shock. As we saw, the intermediary will not pay out dividends before reaching the stochastic steady state, and hence is only concerned with reducing the number of steps necessary to reach it.

Proposition 7. During the transition, if $z(s^t) = z_L$ then all variables in the economy remain unchanged in s^t . If $z(s^t) = z_H$ then the growth rate of intermediary shareholder value in s^t is $\frac{2-\beta}{\beta}$, and the capital growth rate in s^t is $\left(\frac{2-\beta}{\beta}\right)^{\frac{1}{\alpha}}$ in case one and $\frac{2-\beta}{\beta}$ in case two.

The average growth rate of shareholder value is the same in both cases as it only depends on the risk free interest rate $\frac{1}{\beta}$. The growth rate of capital (loans) is higher in case one as each additional unit relaxes the holdup constraint via the effect on the marginal product of capital: if firms can abscond with part of output unless the intermediary prevents it, then this amount per unit of the loan decreases with the total size of the loan. This channel implies that the credit crunch is more severe initially in case one but becomes less severe over time as aggregate capital grows. Note that the fact that intermediaries do not internalize this channel has no consequence here: the channel only matters when the intermediary's participation constraint binds, but then lending is determined by this constraint.

Proposition 7 shows that the transition can be characterized by focussing on what happens in event $z(s^t) = z_H$ only. Let K_j denote the aggregate capital stock after z_H has been realized for the j - th time, and similar for the other variables. Consider a period t event s^t and let $j = \sum_{s^\tau \leq s^t} \mathbb{1}(z(s^\tau) = z_H)$, where $\mathbb{1}$ is an indicator function. Then newly issued debt at step j can be expressed as

$$D_j = \frac{1}{2}\beta(B_{H,j} + B_{L,j}),$$

where $B_{i,j}$ is what the bank promises to repay in the period following step *j* if the state turns out to be $z(s^{t+1}) = z_i$. Steady state debt is given by

$$D^* = \left(1 - \theta_1 z_H \frac{\beta}{2 - \beta} K^{*\alpha - 1}\right) K^* = \left(1 - \theta_2 z_H \frac{\beta}{2 - \beta}\right) K^*.$$

Debt dynamics, for either case, can be expressed recursively as

$$D_{j} = \frac{2 - \beta}{\beta} D_{j-1} - 2 \left(\alpha K_{j}^{\alpha - 1} - \delta \right) K_{j} + (K_{j+1} - K_{j})$$

and net asset dynamics are given by

$$A_j = K_{j+1} - D_j.$$

This last equation merely states that during the transition banks finance new loans K_{j+1} using equity and debt. Proposition 8 verifies that the CE capital stock follows FB dynamics "eventually".

Proposition 8. For each of the two cases, for a given $\epsilon > 0$, there exists a $T < \infty$ such that (DNN) does not bind after T periods with probability larger than $1 - \epsilon$.

8.2 Hedging of financial risk

Intermediaries trade contingent claims to finance investment and to preserve debt capacity in the future. If an intermediary is too highly levered then it may deplete debt capacity in some future event where the return on loans is high. In particular, one would expect that the intermediary spreads out borrowing capacity such that the conditional expected return on investment is non-increasing.

<i>t</i> , <i>z</i>	$k_{t,z}$	$b_{t,z}$	$\psi_{t,z}$
0	1.7660	-	-
1,L	1.7119	1.6441	2.0725
1,H	1.8922	1.6539	2.1697

Table 2: A three-period model, competitive equilibrium.

Definition 5. The intermediary is hedging financing risk sufficiently at s^t if

$$E(R(s^{t+1})|s^t) \ge E(R(s^{t+2})|s^{t+1})$$

for all $s^{t+1} \succ s^t$.

Consider an example similar to case one in section 8.1 but with only three periods. The intermediary can invest in loans in periods t = 0, 1 and pays out as dividends all accumulated equity in t = 2. A single productivity shock $z \in \{z_L, z_H\}$ is drawn with equal probability at the beginning of period t = 1. In period t = 2 productivity is z = 1 for sure, independent of period t = 1 productivity. Hence only loans made in period 0 generate a stochastic return. The intermediary faces two possible paths with loan returns of $R_{1,L}$, $R_{2,L}$ (when $z = z_L$) and $R_{1,H}$, $R_{2,H}$ (when $z = z_H$) respectively. Note that $R_{2,L} \neq R_{2,H}$ in general because investment may be path-dependent even though productivity is the same along each path in period 2. If intermediary equity is sufficiently scarce initially, the intermediary will choose to forego hedging financial risk sufficiently such that credit will be scarcer after the economy had been hit with a non-persistent low shock. (Rampini and Viswanathan (2009) also point this out.) In that case, $R_{2,L} > E_0(R_1) > R_{2,H}$. To see this consider the numerical example given by table 2, where $b_{t,z}$ is the amount the intermediary borrowed against period t if z occurred and $\psi_{t,z}$ is the Lagrange multiplier on the nodefault constraint in period t if z occurred. Note that credit supply drops by roughly 0.054 after the low shock occurred. In the example, $E_0(R_1) = 1.2044$ and $R_{2,L} = 1.2097$, $R_{2,H} = 1.1928.$

Policy makers are usually concerned about aggregate asset sales (loan portfolio shrinks after $z = z_L$ is realized) as they can create a negative externality, for example via fire sales of physical firm capital when the intermediary needs to cut lending to firms. This destruction

of wealth at the firm level reduces the loan repayment to intermediaries, leading to a reduction in lending.¹³ Each intermediary would fail to internalize its contribution to reducing the return on loans and hence intermediary wealth (feedback loop). In such a scenario regulators would try to curb initial leverage of intermediaries and hence initial credit supply to avoid a severe reduction in credit supply in some future states.

In my model I do not consider the possibility of a firm asset fire sales, since lending to firms will be non-decreasing throughout (see proposition 7). What is different to the case with only three periods? With a finite horizon the return on equity is always unity in the final period as all equity will then simply be distributed as dividends. When there is an open horizon dividends will only be paid in the stochastic steady state and maximizing date zero present value of dividends just means minimizing the expected number of periods necessary to reach the steady state. Over time the return on equity will thus be non-increasing which also gives a non-increasing return on investment.

Proposition 9. Consider the two cases in section 8.1. The intermediary hedges financing risk sufficiently at all times.

8.3 Different lending standards

Consider again the two cases in section 8.1. Proposition 10 shows that, for a given drop in intermediary net assets, in case one the resulting recession is more severe while intermediary shareholder value decreases by less. The reason is that in case one initial bank leverage is constrained more due to higher endogenous bank capital requirements such that intermediaries earn higher rents on a reduced loan portfolio. Figure 4 compares the two cases. Note that the two economies look identical in stochastic steady state but they exhibit very different transitions once a financial crisis occurs.

Proposition 10. *Consider the two cases in section 8.1. For a given initial loss of intermediary net assets, economic activity and welfare decrease by more in case one, while shareholder value falls by less.*

¹³Lorenzoni (2008) studies a case where debt-financed investment can be inefficiently high initially. In my three-period example, lending to firms may also be inefficiently low initially depending on parameters.



Figure 4: Two cases, both with initial net assets of 0.25. Credit supply is much lower initially in case one but recovers faster as well. The endogenous tightening of capital requirements during the credit crunch in case one implies scarcer credit supply, a higher economic cost of the credit crunch, and a higher initial bank shareholder value.

We could interpret θ_1 , θ_2 as different sets of lending standard, each θ_i measuring intermediary proprietary information, with respect to either cash flow from production or resale value of physical capital, about ex-post heterogenous firms. For example, the idiosyncratic state $\varepsilon(s^t)$ of a firm in s^t (see appendix for more on how firm heterogeneity maps into bank default values) lives on

$$\Omega = \left[-\theta_1 z(s^t), \theta_1 z(s^t)\right] \times \left[-\theta_2 z(s^t), \theta_2 z(s^t)\right].$$

For a constrained bank it would be beneficial to subdivide this space into

$$\hat{\Omega} = \left[-\hat{\theta}_1 z(s^t), \hat{\theta}_1 z(s^t)\right] \times \left[-\hat{\theta}_2 z(s^t), \hat{\theta}_2 z(s^t)\right],$$

where $\hat{\theta}_i \leq \theta_i$ and its complement $\Omega \setminus \hat{\Omega}$. A borrower in $\hat{\Omega}$ will borrow the same amount of capital as idiosyncratic shocks are distributed symmetrically around zero in both sets. The advantage for the bank is that now

$$\widehat{\mathcal{O}} = z \left[\widehat{ heta}_1 K^{lpha - 1} + \widehat{ heta}_2
ight]$$
,

is the unit default value on loans made to borrowers in $\hat{\Omega}$ and

$$\mathcal{O}=z\left[heta_1K^{lpha-1}+ heta_2
ight]$$
 ,

is the unit default value of loans made to the remaining borrowers. The bank could now obtain higher leverage as the default value for a subset of the loan portfolio decreased. Assume the bank can subdivide borrowers into groups at zero cost. However, assume that consumers (e.g. a pension fund) have a nominal cost of verifying that the intermediary actually performed the subdivision. Letting θ_1 refer to soft evidence about firm productivity and θ_2 to hard evidence about firm capital liquidation value. Then consumers likely prefer it if the bank lowers θ_2 , for example, by selecting according to a credit rating. It is likely less costly for consumers to verify if borrowers meet some minimum credit rating rather than some minimum "entrepreneurial productivity" criterion. Consumers cannot coordinate to pay the higher nominal cost each to avoid a more severe drop in credit during a financial crisis, i.e. switching from case one to case two.

9 Conclusion

The goal of this paper is to find the simplest way to embed Holmstrom and Tirole's concern about the necessity of intermediary capital for economic activity in a real business cycle model. In doing so I assume that intermediaries obtain proprietary information about its debtors. In equilibrium, they will need to hold own capital as a way to commit themselves to use this information to protect the interests of its creditors. The economy experiences a financial crisis when insufficient intermediary capital constrains credit supply. Positive insights gained from this exercise are that during financial crises (i) pro-cyclical debtor heterogeneity makes intermediaries hesitant to expand lending in bad times, (ii) differences in lending standards determine how painful a financial crisis is for workers and intermediary shareholders respectively, (iii) individually rational behavior of intermediaries is sufficient to rule out fire sales at the debtor level due to a failure to extend credit. The central normative insight gained from the above analysis is that the social cost of financial crises it too high: intense competition on the market for loans in steady state reduces intermediary debt capacity during the financial crisis. A constrained regulator will find it optimal to ration credit in a steady state to increase the intermediary's shareholder value and debt capacity during the financial crisis. The numerical exercises provided find significant positive welfare effects from implementing this regulation, even for perfectly coinsured consumers.

References

- Abraham, A. and E. Carceles-Poveda (2006). Endogenous incomplete markets, enforcement constraints, and intermediation. *Theoretical Economics* 1(4), 439–459.
- Abraham, A. and E. Cárceles-Poveda (2008). Competitive equilibria with production and limited commitment. *Unpublished Manuscript, University of Rochester and SUNY Stony Brook*.
- Abreu, D., D. Pearce, and E. Stacchetti (1990). Toward a theory of discounted repeated games with imperfect monitoring. *Econometrica: Journal of the Econometric Society*, 1041–1063.
- Albuquerque, R. and H. A. Hopenhayn (2004, 04). Optimal lending contracts and firm dynamics. *Review of Economic Studies* 71(2), 285–315.
- Alvarez, F. and U. J. Jermann (2000). Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica 68*(4), 775–798.
- Brunnermeier, M. and Y. Sannikov (2010). A macroeconomic model with a financial sector. *Princeton University, May, mimeo*.

- Cetorelli, N. and M. Gambera (2001). Banking market structure, financial dependence and growth: International evidence from industry data. *The Journal of Finance* 56(2), 617–648.
- Chevalier, J. (1995). Do LBO supermarkets charge more? An empirical analysis of the effects of LBOs on supermarket pricing. *Journal of finance* 50(4), 1095–1112.
- Coates, J. and D. Scharfstein (2009). Lowering the Cost of Bank Recapitalization. *Yale J. on Reg.* 26, 373.
- Cooley, T., R. Marimon, and V. Quadrini (2004). Aggregate consequences of limited contract enforceability. *Journal of Political Economy* 112(4).
- Davila, J., J. Hong, P. Krusell, and J. Rios-Rull (2005). Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks.
- Diamond, D. and R. Rajan (2000). A theory of bank capital. *The Journal of Finance* 55(6), 2431–2465.
- Holmstrom, B. and J. Tirole (1997, August). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics* 112(3), 663–91.
- Hoshi, T. and A. Kashyap (2010). Will the US bank recapitalization succeed? Eight lessons from Japan. *Journal of Financial Economics* 97(3), 398–417.
- Imbs, J. (2007). Growth and volatility. *Journal of Monetary Economics* 54(7), 1848–1862.
- Ivashina, V. and D. Scharfstein (2010). Bank lending during the financial crisis of 2008. *Journal of Financial Economics* 97(3), 319–338.
- Jeanne, O. and A. Korinek (2010). Excessive volatility in capital flows: A pigouvian taxation approach. *NBER Working Paper*.
- Judd, K., F. Kubler, and K. Schmedders (2002). A solution method for incomplete asset markets with heterogeneous agents. *working paper*.
- Kaminsky, G. and C. Reinhart (1999). The twin crises: the causes of banking and balanceof-payments problems. *American Economic Review* 89(3), 473–500.

- Kashyap, A., R. Rajan, and J. Stein (2008). Rethinking capital regulation. In *Federal Reserve Bank of Kansas City Symposium at Jackson Hole*.
- Kehoe, P. and F. Perri (2004). Competitive equilibria with limited enforcement. *Journal of Economic Theory* 119(1), 184–206.
- Kehoe, T. and D. Levine (2001). Liquidity constrained markets versus debt constrained markets. *Econometrica* 69(3), 575–598.
- Kehoe, T. J. and D. K. Levine (1993). Debt-constrained asset markets. *Review of Economic Studies* 60(4), 865–88.
- Lorenzoni, G. (2008). Inefficient credit booms. *Review of Economic Studies* 75(3), 809–833.
- Peek, J. and E. Rosengren (2000). Implications of the globalization of the banking sector: the Latin American experience. In CONFERENCE SERIES-FEDERAL RESERVE BANK OF BOSTON, Volume 44, pp. 145–170. Federal Reserve Bank of Boston; 1998.
- Rajan, R. and L. Zingales (2003). The great reversals: the politics of financial development in the twentieth century* 1. *Journal of Financial Economics* 69(1), 5–50.
- Rampini, A. and S. Viswanathan (2009). Collateral, risk management, and the distribution of debt capacity. Technical report, Citeseer.
- Simon, M. (1998). The rise and fall of bank control in the United States: 1890-1939. *American Economic Review* 88(5), 1077–1093.
- Swire, P. (1992). Bank Insolvency Law Now That It Matters Again. *Duke Law Journal* 42(3), 469–556.

10 Appendix

10.1 Firm decisions and intermediary hold up values

The "representative firm" consists of a continuum of firms with i.i.d. idiosyncratic characteristics. Since firms choose all actions before they observe their type it makes still sense to speak to the representative firm.

Firms live for two periods and the life of a firm that produces in period t + 1 evolves as follows: (i) at the end of period t the firm borrows capital $k(s^t)$ from the intermediary, (ii) at the beginning of period t + 1 the firm observes aggregate productivity $z(s^{t+1})$ and rents labor $l(s^{t+1})$, (iii) at the end of period t + 1 the firm produces output subject to some idiosyncratic shock $\varepsilon(s^{t+1}, s_i)$, (iv) the firm makes factor payments and is dissolved. Only the intermediary that made the loan in t can observe the firm's idiosyncratic state s_i . Hence only intermediaries will make loans to firms. Otherwise, intermediaries and firms transact under perfect competition as neither one of them has any market power on the market for capital. Each firm borrows from only one intermediary.

Firm idiosyncratic shocks $\varepsilon = (\varepsilon_1, \varepsilon_2)$ are i.i.d. with a zero mean. They affect firm productivity as well as capital depreciation. Firm i hires labor *l* and capital *k* to produce output

$$F(k,l,s^{t+1},s_i) = [z(s^{t+1}) + \varepsilon_1(s^{t+1},s_i)]k(s^t)^{\alpha}l(s^{t+1})^{1-\alpha} + (1-\delta + \varepsilon_2(s^{t+1},s_i))k(s^{t+1}).$$

Following Imbs (2007) I assume that firm idiosyncratic shocks have procyclical variance,

$$\varepsilon_1(s^{t+1}, s_i) \in \left[-\theta_1 z(s^{t+1}), \theta_1 z(s^{t+1})\right]$$

$$\varepsilon_2(s^{t+1}, s_i) \in \left[-\theta_2 z(s^{t+1}), \theta_2 z(s^{t+1})\right],$$

where either shock is uniformly distributed and $\theta_1, \theta_2 > 0.^{14}$ Since only intermediaries can observe $\varepsilon(s^{t+1}, s_i)$, factor payments are $w(s^{t+1})l$ for labor and $\tilde{R}(s^{t+1}, s_i)k$ for capital. That is, firms can write contracts conditional on $\varepsilon(s^{t+1}, s_i)$ only with intermediaries. Firms choose capital and labor to maximize their expected profit

$$\beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) E\left(\left\{ [z(s^{t+1}) + \varepsilon_1(s^{t+1}, s_i)]k(s^t)^{\alpha} l(s^{t+1})^{1-\alpha} + (1 - \delta + \varepsilon_2(s^{t+1}, s_i))k(s^t) - w(s^{t+1}) l(s^{t+1}) - \tilde{R}(s^{t+1}, s_i)k(s^t) \right\} |s^{t+1}\right),$$

subject to solvency in each (s^{t+1}, s_i)

$$[z(s^{t+1}) + \varepsilon_1(s^{t+1}, s_i)]k(s^t)^{\alpha}l(s^{t+1})^{1-\alpha} + (1 - \delta + \varepsilon_2(s^{t+1}, s_i))k(s^t) - w(s^{t+1})l(s^{t+1}) - \tilde{R}(s^{t+1}, s_i)k(s^t) \ge 0.$$

Note that firm profit is just linear in physical capital once labor has been chosen. $\tilde{R}(s^{t+1}, s_i)$ then just clear the market for loans while satisfying firm profit non-negativity as well.

Assumption 2. Firms cannot produce negative output, $\theta_1 \leq 1$. A firm's depreciation rate cannot be negative, $\theta_2 \leq \frac{\delta}{\bar{z}}$ where $\bar{z} = \max_{s \in S} z(s)$.

¹⁴Cyclical idiosyncratic risk does not matter for my main results. However, it allows me to discuss bank risk management in section 8.2.

Assumption 2 also ensures that the bank is not required to insure the firm. The firm's profit maximizing choice is characterized by

$$w(s^{t+1}) = z(s^{t+1})(1-\alpha)k(s^t)^{\alpha}l(s^{t+1})^{-\alpha},$$

$$\tilde{R}(s^{t+1},s_i) = [z(s^{t+1})\alpha + \varepsilon_1(s^{t+1},s_i)]k(s^t)^{\alpha-1}l(s^{t+1})^{1-\alpha} + (1-\delta + \varepsilon_2(s^{t+1},s_i)).$$

Since intermediaries hold a diversified loan portfolio, they are only concerned with the average return on loans for a given s^{t+1} ,

$$R(s^{t+1}) \equiv E\tilde{R}(s^{t+1}, s_i) = z(s^{t+1})\alpha k(s^t)^{\alpha - 1} l(s^{t+1})^{1 - \alpha} + (1 - \delta).$$

If the intermediary is withholding the intermediation service, then its creditors will seize a fraction of the intermediary's loan portfolio and attempt to collect repayments themselves. Suppose the loan portfolio contains loans of *k* and the aggregate state is s^t . Then firms will exploit the fact that creditors do not know the firm-specific state: they will claim $\varepsilon(s^t, \{L, L\}) = (-\theta_1 z(s^t), -\theta_2 z(s^t))$ and simply abscond with any remaining profit. The liquidation value to the creditors is hence $\tilde{R}(s^t, \{L, L\})k$, while it is $R(s^t)k$ for the intermediary. Assuming that creditors have infinitesimal bargaining power, the intermediary can hold its creditor up for at most

$$\mathcal{O}(s^t, K(s^{t-1}))k = \left[R(s^t) - \tilde{R}(s^t, \{L, L\})\right]k = \left[\theta_1 K(s^{t-1})^{\alpha-1} + \theta_2\right] z(s^t)k.$$

Final investors will not engage in intertemporal trade of liquid assets with the intermediary if they expect to be held up. Hence, intermediaries can only trade contingent claims $b^{I}(s^{t})$ such that $d(s^{t}) + p(s^{t}) \ge O(s_{t})k(s^{t-1})$ for all s^{t} . Note that creditors could be able to achieve a liquidation value that exceeds the claims they hold against the intermediary. But then this constraint is slack for sure: hence we call this constraint the no-holdup constraint but realize that we should not take it literally in all cases where it does not bind. The no-holdup constraint can be written as

$$\sum_{s^{\tau} \in \mathcal{S}(s^t)} \beta^{\tau-t} \pi(s^{\tau} | s^t) d(s^{\tau}) \geq \mathcal{O}(s_t) k(s^{t-1}), \quad \forall s^{t-1} \text{ and } s^t \succ s^{t-1}.$$

10.2 Recursive formulation

The recursive formulation is for computational purposes. In what follows I focus on partial equilibria, taking consumer and firm behavior as given.

Intermediary problem

The aggregate state is (A, z) where A is average industry cash. I let *a* denote the representative intermediary's cash or equity. Intermediaries chose dividends *d*, loans *k*, and claims *b* in order to maximize shareholder

value V(A, a, z). An intermediary faces the following budget constraint

$$k' + d = a + \beta \sum_{z'} b'(z') \pi(z'|z)$$
(7)

where expenditures on dividends, new loans, and claim repayment have to equal equity and the proceeds from issuing new claims. The intermediary faces a non-negativity constraint on dividends,

$$d \ge 0. \tag{8}$$

The contract requires that intermediary value in the contract is as least as high as autarky value,

$$V(A',a',z') \ge \mathcal{O}(A,z,z')k',\tag{9}$$

where

$$\mathcal{O}(A, z, z') = \left[\theta_1 K'(A, z)^{\alpha - 1} + \theta_2\right] z'.$$

The law of motion for the intermediary's state is

$$a'(z') = R(A, z, z')k' - b'(z')$$
(10)

where

$$R(A, z, z') = z' \alpha K'(A, z)^{\alpha - 1} + 1 - \delta$$

The perceived aggregate law of motion is given by

$$[K'(A,z), A'(A,z,z')] = \Psi(A,z,z'), \quad z'|z \sim P,$$
(11)

where *P* is the Markov transition matrix for *z*, and Ψ follows from rational expectations.

The intermediary problem is to maximize shareholder value and policy functions are generated by the following Bellman equation

$$V(A, a, z) = \max_{d, (b'), k'} d + \sum_{z'} \beta \pi(z'|z) V(A', a', z')$$
(12)

subject to (7), (8), (9), (10), (11).

Recursive competitive equilibrium

A recursive competitive equilibrium for this economy is defined by

1. a value function V(A, a, z), and intermediary policy functions d(A, a, z), b'(A, a, z; z'), k'(A, a, z)

- 2. a price function R(A, z; z') and a unit default value $\mathcal{O}(A, z, z')$
- 3. an aggregate law of motion that the intermediary perceives, $\Psi(A, z, z')$, $z'|z \sim P$

such that

- given price and default functions, and the perceived aggregate law of motion the intermediary's value function and policy functions solve solve its dynamic problem
- markets clear *A* = *a*
- perceptions are correct, $\Psi(A, z, z') = [k'(A, A, z), a'(A, A, z, z')].$

Social planner problem

The aggregate state is (A, v, z) where A is bank equity and v the value promised to each bank. The planner chooses the aggregate level of new loans k', intermediary claims $\{b'(z')\}_{z' \in S}$, and dividends d. Planner choices have to satisfy bank budget constraints

$$k' + d = A + \beta \sum_{z'} b'(z') \pi(z'|z).$$
(13)

The planner has to set intermediary continuation values that satisfy intermediary participation constraints,

$$v'(z') \ge \left[\theta_1 k'^{\alpha-1} + \theta_2\right] z'k'.$$
(14)

The planner is also obligated to keep any promises, the promise keeping constraint is

$$d + \beta \sum_{z'} \pi(z'|z)v'(z') \ge v.$$
(15)

The law of motion for aggregate intermediary equity is

$$A'(z') = (1 + z'\alpha(k')^{\alpha - 1} - \delta)k' - b'(z').$$
(16)

The planner Bellman equation is

$$W(A, v, z) = \max_{d, (b'), k', v'(z')} d + \beta (1 - \alpha) (k')^{\alpha} E(z'|z) + \beta \sum_{z'} W(A', v', z') \pi(z'|z)$$
(17)

subject to (7), (13), (14), (15), (16). Note that the timing with respect to wages is not relevant because of no aggregate labor risk and the infinite horizon.

10.3 Proof of proposition 3

Letting λ_0 , $\mu_t(s^t)\beta^t\pi(s^t|s_0)$, and $\psi_t(s^t)\beta^t\pi(s^t|s_0)$ being the Lagrange multipliers on constraints (IBC), (DNN), and (PC) respectively, the first order condition for dividends is given by

$$\lambda_0 = 1 + \mu_t(s^t) + \sum_{\tau=0}^t \psi_\tau(s^\tau)$$
(18)

(18) implies that $\mu_t(s^t)$ is monotonically decreasing along a given branch of the event tree. But then $\mu_t(s^t) = 0$ implies $\psi_{\tau}(s^{\tau}) = 0$ and $\mu_{\tau}(s^{\tau}) = 0$ for all s^{τ} such that $s^{\tau} \succ s^t$. Intuitively, the intermediary will always retain earnings as long as it is debt constrained.