

Innovation Uncertainty and Indirect Network Externalities^{*}

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Abstract

In this paper we study the impact of incomplete information and innovation uncertainty on market equilibrium, product variety and welfare, in the presence of indirect network externalities. We evaluate the interaction between information, or the lack of it, and the inherent inefficiencies due to private decision-making in a setting which incorporates indirect network externalities. Such settings often exhibit suboptimal equilibrium. This paper examines the impact of uncertainty, information structure and preferences on a competitive equilibrium and specifies the conditions under which non-biased uncertainty may expand adoption and variety. We show that scenarios where such phenomena occur may be welfare enhancing and, under certain conditions, ex-ante superior compared to equilibrium generated under complete information.

Keywords: Network Externalities, Two-Sided Markets, Indirect Network Effects, Product Variety, Social Efficiency, Innovation Uncertainty

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I. Introduction & Motivation

In this paper we study the impact of uncertainty on innovation adoption, variety and welfare, in the presence of indirect externalities. We evaluate the impact of uncertainty on market equilibrium in a setting which often exhibits suboptimal allocation due to un-internalized externalities. We shall attempt to show that under certain conditions uncertainty may bring about an equilibrium which Pareto-dominates the one generated under full information. In such a setting uncertainty may be welfare enhancing.

The implementation of a new technology or network is almost always associated with uncertainty, even in cases where post adoption benefits are clear and very significant.

"What use could this company make of an electrical toy? "

Commented William Orton, Western Union's president in response to an offer from Alexander Graham Bell to sell his telephone company for \$100,000.....;

"There is no reason for any individual to have a computer in their home ."

Stated Ken Olsen the president and founder of Digital Equipment Corp, in 1977.

Every innovation is ridden by uncertainty, however when this innovation is network centric both uncertainty and its impact may be magnified.

In markets which incorporate externalities, a competitive equilibrium may yield a suboptimal allocation. Competitive agents optimize their private objectives leading to an equilibrium which does not necessarily coincide with optimal social welfare. Even in cases where agents are aware of the externality reaction, it is often difficult to generate a synchronizing mechanism which can bring the market equilibrium closer to social optimum. In this paper we wish to show how incomplete information and some degree of uncertainty may, under certain conditions, partially compensate for this inability of the competitive market equilibrium to fully exploit the potential of an externality intensive innovation.

Nyssen (1993) has elegantly demonstrated that permanent but weak over-optimism can create a sustainable bubble in a Grossman and Helpman endogenous growth model and that the resulting equilibrium exhibits increased growth and welfare.

A framework for expectation based, self-ending, rational bubbles, with uncertainty, has been described by Blanchard (1979). Blanchard pointed out to the potential real effects of expectations and to the consistency of rational expectations with bubbles due to uncertainty. Both of these papers as well as subsequent followers of this line of research rely on over or under-optimism to impact the market. Our evaluation is very different as we assume rational unbiased expectations. We are interested in investigating the potential impact on equilibrium when uncertainty and its interaction with indirect externalities are the primary driver and not self-fulfilling biased expectations.

Our evaluation will focus on those cases where the signals and transitory information states are derived from a symmetric distribution, which does not introduce any additional systematic bias regarding the anticipated innovation, thus the result will be driven by the existence of uncertainty or incomplete information, per-se. The realization in our evaluation will be equal to the expectation mean; nevertheless, the existence of transitory uncertainty will have a longer term impact.

Direct network externalities are associated with the direct utility or interaction benefits generated for existing users on a network due to the addition of a new member (as in a phone or fax network). Indirect externalities come about through the forces of market interaction (as in aggregate demand or spillover effects, as exhibited in hardware platforms and compatible software varieties, or the proliferation of the Iphone usage base and the compatible App-store application variety). The results presented in our paper address the co-interaction between incomplete information and inefficiencies associated with un-internalized externalities and the nature of private optimization in a context which involves such externalities.

Katz and Shapiro (1985) provided the groundwork for the distinction between the potential impact of direct and indirect network externalities. Leibovitz & Margolis (1994, 2002) distinguish between technological and pecuniary externalities and claim that indirect network effects are in most cases welfare neutral as they shift profits when prices react to network changes. Church, Gandal and Krause (2008) address the issue of variety and indirect network externalities and have shown that in a setting which incorporates increasing returns to scale, free entree and a preference for variety indirect network effects may produce adoption externalities. We shall use a similar setup in our model.

Most of the existing literature on network externalities, both direct and indirect, demonstrates that a competitive equilibrium will result in a sub-optimal allocation. Farrell and Saloner (1985), Chou and Shy (1990) and Church et al (2008) are leading examples of such papers. Recent literature on two sided markets addresses similar issues. Hagiu (2009) compares two sided markets with proprietary platforms Vs open platforms. Proprietary platforms act in a strategic manner, setting prices and affecting trade. Typically the proprietary platform will have a target maximizing owner while an open platform will not. The setup depicted in this paper is equivalent to a two sided market with an open platform. The existence of a suboptimal allocation under a competitive equilibrium with non-strategic two sided markets is also demonstrated in Hagiu (2009).

In this paper we study the impact of signal variance, information structure and preferences on market equilibrium and specify the conditions under which uncertainty may expand adoption and potentially improve welfare. The persistence of post realization impact is of specific interest in such settings due to the existence of upfront investments and network externalities. Katz and Shapiro (1986b) refer to uncertainty in a network externality setting; however they address the question of network switching with strategic networks when consumers are homogeneous and their number is determined exogenously. Farrell and Saloner (1985) investigate the role of uncertainty in technology switching scenario where firms do not know the switching preferences of their peers. A recent paper by Halaburda and Yehezkel (2011) deploys a somewhat similar uncertainty setup, comparing the impact of ex-ante uncertainty and ex-post realization on contract decisions of strategic platforms within a two sided market, their evaluation differs from our setting not only in the type of platforms but also in the fact that we are evaluating the size and variety of the resulting multiplayer networks while their focus is on platform competition. The existence of uncertainty in all of these platform selection papers has a potentially negative effect on market efficiency. In contrast, this paper shows that uncertainty may improve market efficiency and welfare. As far as we are aware this is the first paper which undertakes the task of evaluating the impact of uncertainty per-se on market equilibrium with indirect network externalities, endogenous consumer adoption, variety and a non-strategic network.

Indeed the existence of uncertainty followed by a non-biased realization may produce an ex-post decrease in network size and firm profits coupled with initial over-adoption; however, we shall attempt to show that some scenarios where such phenomena occur may be welfare enhancing and, under certain conditions, ex-ante superior compared to an equilibrium generated under complete information. Evaluating the conditions which produce such results may assist us in better understanding the forces at play during innovation related ‘bubbles’ which may manifest in connection with network externality intensive technologies as detailed in Zvilichovsky (2007).

Consumer taste for diversity in consumption is a common mechanism used when researching innovation. Dixit & Stiglitz (1977) formulated one of the early specifications which focused on variety. Such tastes generate the demand for differentiated products which can [imperfectly] substitute for one another. Either (1982) provided an interpretation to this specification which is of interest to us. Consumers under this interpretation consume a final homogeneous good which is assembled from differentiated intermediate inputs. Under this interpretation variety in these imperfectly substitutable inputs has a constant return to scale for a given variety, however total factor productivity rises with the number of available varieties. Grossman and Helpman (1991) evaluate the impact of knowledge spillovers in the context of a Dixit and Stiglitz variety based endogenous growth model. They outline the conditions which may generate ongoing technological progress and show that in the presence of public knowledge externalities market equilibrium provides insufficient incentives for new variety. A preference for variety shall also be the driving force of this paper. Consumers derive positive utility for variety, which they can only consume while being part of the network. Consumer taste for variety, in conjunction with the size of the network, shall drive equilibrium variety.

The remainder of this paper is organized as follows: Section II describes a model of new network formation, subject to upfront investment costs, explicit variety preferences and inherent indirect network externalities. The incomplete information structure and the artifacts of uncertainty are also defined. Section III evaluates market equilibrium under private consumer signals generated from a non-biased symmetric distribution and compares results to a complete information scenario. The interrelationship between signal variance, equilibrium adoption and welfare are evaluated

in detail. Section IV describes the conditions required for the results to hold under a general variety preference specification. Section V evaluates market equilibrium under a common signal and compares it to complete information and private signal scenarios. Section VI concludes.

II. The Model

In this section we present a model which may provide insight into the market and welfare implications of transitory uncertainty in the presence of indirect network externalities. Incomplete information and uncertainty interact with the effects of indirect externalities and may impact both interim and post realization outcome. We evaluate network size, variety and welfare and compare the expected outcome between different information scenarios. We show that non-biased uncertainty may have an expanding effect on both market variety and network size. Under certain conditions, such uncertainty may also enhance the welfare of the information absent market and drive the market equilibrium closer to the social optimum. In some cases welfare shall be increasing with the degree of uncertainty.

The model describes the formation of an open two sided market, which accommodates both consumers and innovating firms. Consumers join the market so that they may consume the variety of products offered on this new network or platform. Producers on their part must pre-decide if they wish to develop specific products that can only be consumed by consumers that have actively decided to join the network. Consumers who wish to join the network must incur a network cost while firms must invest an upfront setup fee in order to implement the innovation. Products are non-divisible (for example a consumer can-not consume half of a software application). This setup is applicable to scenarios where consumers must buy a competitively supplied hardware, select an open source operating system or adopt a specific industry standard. Producers in such scenarios could be the developers of software titles or applications that are specific to this new standard.

Indirect network externalities impact variety and network size under equilibrium as shown by Church et al (2008) which have used a similar setup under certainty. In the model, producers can only sell the new products to consumers on the network, thus the

expected size of the consumer network impacts firm adoption. At the same time, firm adoption determines maximum variety which may impact consumers' network adoption decisions. Network size impacts demand which, in equilibrium, determines the amount of available variety.

Consumers & Variety Producers

Consumers are heterogeneous with regard to their individual network fit. They are distributed according to their virtual distance from the core of the network with an individual type parameter d , $d \sim Uniform[0,1]$. The cost for a user of type- d to join the network is kd , where k is a parameter associated with the specific network technology.

Consumer utility is defined as:

$$U = g(N) + x - kd, \quad (1)$$

Where: N is the number of different varieties consumed,
 x denotes the consumption of the numeraire,
and $g(N)$ is increasing and strictly concave.

The production side of the economy allows for free entree of monopolistically competing firms. Prior to production, firms must invest a fixed upfront investment f . As we have in mind products such as software or Internet services we assume marginal cost is fixed, or even decreasing, thus the total setup results in increasing returns to scale.

Timing, Information and Adoption

Potential customers and firms targeting the early Internet market, prior to the wide-scale adoption of the World Wide Web and its application, could not accurately know the learning costs, adoption tastes or benefits associated with joining such a network or utilizing the various applications. The experience was so different from the previously existing technology that market participants having to make decisions often relied on

partial information. In the model we embody this information gap as uncertainty pertaining to the network cost parameter k .[‡]

This paper shall evaluate and compare three types of information scenarios: (i) full information, (ii) private beliefs based on an unbiased symmetrically distributed private signal and (iii) a common belief based on a public signal drawn from the same symmetric distribution. Following actual consumption, values are fully revealed. So as not to introduce any systematic bias when comparing between scenarios we assume that signal mean is equal to the true, but yet unknown, value. In all of the compared scenarios the actual value, or the realization when applicable, shall be equal to this signal mean. The only difference between the compared scenarios shall thus be the information path. We shall also evaluate the path dependency of the post uncertainty competitive equilibrium. This section deals with information scenarios (i) and (ii), section III evaluates scenarios of the third type.

The market is played according to the following sequence:

- Period 0: Signals observed.
 Initial beliefs formed.
 Adoption decisions taken.
 Investment of fixed cost by firms.
- Period 1: Market play. Cost parameter k revealed post consumption.
- Period 2: Market play with fully known variables.

Each firm's upfront investment f is a one-time cost which should be undertaken prior to production. Consumer network cost kd is a per period cost and is borne by each consumer who wishes to join or stay on the network.

[‡] One may similarly incorporate uncertainty pertaining to the consumer's private attributes, such as the distance parameter d . This alternative setup would produce similar results; however we prefer attaching the uncertainty to a market value as this simplifies the exposition significantly.

Let k^* denote the true but yet unknown cost parameter. We assume that each consumer acts upon a private signal \tilde{k} generated from the following symmetric source distribution:[§]

$$\tilde{k} = \begin{cases} (1+\sigma)k^* & \text{with probability } \lambda \\ (1-\sigma)k^* & \text{with probability } \lambda \\ k^* & \text{with probability } 1-2\lambda \end{cases}$$

where $0 \leq \sigma < 1$ and $0 \leq \lambda < 0.5$.

$$\text{By definition } E(\tilde{k}) = k^* \quad \text{and} \quad V(\tilde{k}) = 2\lambda(k^*)^2 \sigma^2.$$

Signal variance is increasing in both λ and σ . Full certainty exists when λ or σ equal 0.

III. Market Equilibrium under Private Signals

We solve the game under a sub-game perfect Nash equilibrium given the available state of information. As we wish to focus on the impact of uncertainty we will focus on a setup where full certainty market equilibrium results in an internal solution - adoption occurs but is not universal.

We shall initially perform our evaluation using an explicit variety taste function $g(N) = N^\beta$ with $0 < \beta < \frac{1}{2}$.^{**} In section IV we perform a formal evaluation of the model under non-specific variety preferences and evaluate the conditions which are required for the results to hold under general variety driven utility functions.

The explicit utility function used in this section implies an inelastic demand of up to one unit of each product variety. In the case of a non-divisible product and a symmetric equilibrium this utility function is equivalent, in our model, to the classic Dixit

[§] For ease of exposition we limit our explicit analysis to a tri-value discrete distribution, however our qualitative results hold for any truncated symmetric distribution.

^{**} This restriction on β is placed to guarantee an internal solution, in this specific setup a value of β higher than 0.5 will result in an incentive for firms to self-expand the market by price reductions yielding universal adoption. A formal proof of this is not included, as in section IV we prove the conditions required for uncertainty to expand consumer and firm adoption for any $g(N)$, applying the result of section IV to the specific function $g(N) = N^\beta$ yield this explicit condition $\beta < \frac{1}{2}$.

and Stiglitz (1977)^{††} variety specification; under the condition that the resulting function should yield a decreasing marginal utility from variety. As we shall later demonstrate, our qualitative results hold for a wide range of variety driven utility specifications, provided certain conditions apply to the ratio between average and marginal utility.

Without loss of generality we shall assume marginal production costs are 0, however all of the described results qualitatively hold under any fixed marginal production cost.

Consumers will join the network only if they receive a benefit greater than their network participation cost kd . Once on the network they will consume all products with a price lower than marginal utility. After ranking the products by increasing price p_i a consumer will consume N products such that $p_N \leq \beta N^{\beta-1}$. The price of the marginal product consumed is equal or lower than the marginal utility from consuming it. A consumer's adoption decision does not impact his exogenous income.

Given N and a symmetric equilibrium, the price of all products actually consumed shall be $p = \beta N^{\beta-1}$ the marginal utility from the N^{th} product consumed. The symmetric price will not be lower than this marginal utility as no single firm has an incentive to offer a price lower than its competitors due to the restriction on β which guarantees that the added value derived from expanding the market via a further price reduction yields a return which is smaller than the forgone revenue on the quantity sold at the above specified price. The price will not be higher because the consumers will not purchase it as it will cost more than the utility it generates.

Consumer's aggregate reaction function, given N , is derived by utility maximization performed by each consumer based on his private belief and type, aggregated across the signal and type distributions. Each consumer acts upon the received signal and his belief regarding the signal distribution. A consumer shall join the network when his utility from joining the network is equal to his personal expected network 'cost' (i.e. a consumer of type d shall join iff $\leq E \left[\frac{(1-\beta)N^\beta}{k} | \tilde{k} \right]$). Aggregating across the three

^{††} $V([\int_0^N x(j)^\alpha dj]^{1/\alpha})$ with $0 < \alpha < 1$ and V given a CES form.

signal type subgroups generates the result. See Appendix A for proof of equilibrium equations.

Let S denote the portion of consumers which have decided to join the network. Equation (2) describes the relation between the consumer network size S and known variety N , subject to the assumption that all solution are internal^{**}:

$$S(N) = \frac{(1-\beta)N^\beta[1-(1-2\lambda)\sigma^2]}{k^*(1-\sigma^2)} \quad (2)$$

The number of firms who invest and enter the network is determined by their expectation regarding the size of the consumer network. Firms are risk neutral and profit maximizing. Prior to realization, firms anticipate future network size based on the described signal distribution and consumer beliefs. Due to free entry, the number of the firms is determined by equating profits to 0. Firms internalize the reaction function of the consumers to the available variety and the expected equilibrium price.

Equation (3) details the equilibrium number of firms entering the market. This is derived by equating future sales revenue of a typical firm to the upfront investment cost f while simultaneously assuming the size of the network is defined by equation (2), Appendix A provides a formal proof.

$$N = \left[\frac{2\beta(1-\beta)[1-(1-2\lambda)\sigma^2]}{fk^*(1-\sigma^2)} \right]^{\frac{1}{1-2\beta}} \quad (3)$$

The equilibrium network size in period 1 is given by the simultaneous solution to equations (2) & (3) as described in equation (4):

$$S_1 = \left[\frac{2\beta}{f} \right]^{\frac{\beta}{1-2\beta}} \left[\frac{(1-\beta)[1-(1-2\lambda)\sigma^2]}{k^*(1-\sigma^2)} \right]^{\frac{1-\beta}{1-2\beta}} \quad (4)$$

Following consumption and revelation of k (denoted as \hat{k}), period-2 consumers react and adjust their participation decisions. The network size during period-2 is derived by using the same number of firms detailed in equation (3) which have joined the network during period-1 and applying reaction function (2). All firms who have joined the network continue to produce as the fixed investment cost has already been invested

^{**} Equation (2) holds for an internal solution provided the favorable signal $(1-\sigma)k^*$ does not induce universal adoption for all of the consumers who have received it. See Appendix A for the calculation of equilibrium when the expanding nature of the favorable signal is saturated.

and the market price is higher than marginal cost. Consumer's network participation in period-2 is determined under conditions of certainty with a known value for k as given by the following equation:

$$S_2 = \frac{1}{\hat{k}} \left[\frac{2\beta[1-(1-2\lambda)\sigma^2]}{fk^*(1-\sigma^2)} \right]^{\frac{\beta}{1-2\beta}} [(1-\beta)]^{\frac{1-\beta}{1-2\beta}} . \quad (5)$$

When the realization is k^* , second period network size is detailed by equation (6):

$$S_2 = \left[\frac{2\beta[1-(1-2\lambda)\sigma^2]}{f(1-\sigma^2)} \right]^{\frac{\beta}{1-2\beta}} \left[\frac{(1-\beta)}{k^*} \right]^{\frac{1-\beta}{1-2\beta}} . \quad (6)$$

Proposition 1

Under an internal solution:

- a) Unbiased symmetric uncertainty with private consumer signals expands both network size and variety
- b) Post-realization adoption will also exhibit an increased network size and variety

Proof: Differentiating equations (3) , (4) and (5) with respect to σ and λ produces the following inequalities: $\frac{\partial S_1}{\partial \sigma} > 0$, $\frac{\partial S_1}{\partial \lambda} > 0$, $\frac{\partial S_2}{\partial \sigma} > 0$, $\frac{\partial S_2}{\partial \lambda} > 0$, $\frac{\partial N}{\partial \sigma} > 0$, $\frac{\partial N}{\partial \lambda} > 0$, Thus we get the result that any decrease in the amount of information, as embodied in the increase in the values of λ or σ is network expanding. The variance of the distribution of signal \tilde{k} is $2\lambda(k^*)^2\sigma^2$ which is increasing in both λ and σ .

Let us denote N_c and S_c as the internal equilibrium values under certainty. The explicit relationship between the certainty equilibrium and general equilibrium under uncertainty, for the case where the revealed value is k^* , is derived by dividing each of equations (4), (5) and (6) by a similar equation with σ and λ held at 0. Results are detailed in equations (7) through (9):

$$N = N_c \left[\frac{1-(1-2\lambda)\sigma^2}{1-\sigma^2} \right]^{\frac{1}{1-2\beta}} \quad (7)$$

$$S_1 = S_c \left[\frac{1 - (1 - 2\lambda)\sigma^2}{1 - \sigma^2} \right]^{\frac{1-\beta}{1-2\beta}} \quad (8)$$

$$S_2 = S_c \left[\frac{1 - (1 - 2\lambda)\sigma^2}{1 - \sigma^2} \right]^{\frac{\beta}{1-2\beta}} \quad (9)$$

Comparing equations (7), (8) and (9) reveals that under the specified internal conditions the ranking of the consumer participation, subject to the described limitations is $S_1 > S_2 > S_c$.

The network and variety expansion is generated by the introduction of symmetric unbiased uncertainty. The number of consumers who are induced to join the network by receiving a favorable signal is higher than the number of consumers inhibited from joining when they received a non-favorable signal. This is true although the signal distribution is symmetric and the number of consumers receiving a favorable signal is equal to those receiving a non-favorable one, thus expansion is generated although the signal distribution is unbiased. §§

Adoption shall be increasing in the level of variance as long as market equilibrium is internal for all signal subgroups, i.e. adoption shall increase with the increase in variance as long as there exist consumers which have received signal $(1 - \sigma)k^*$ and have not yet decided joined the network. When uncertainty reaches the point where all of the λ consumers receiving this signal have decided to join, expansion will no longer happen. A further increase in variance beyond this point can only deter some additional consumers to join the network as all λ consumers which have received signal $(1 - \sigma)k^*$ have already joined. For degrees of uncertainty where the favorable signal group is saturated, the reaction function shall be:

$$S(N) = \lambda + \frac{(1 - \beta)N^\beta [(1 - \lambda) + (1 - 2\lambda)\sigma]}{k^*(1 + \sigma)}, \quad (10)$$

which is decreasing as variance increases beyond the saturation threshold.

§§ A detailed evaluation of the different consumer subgroups impacted by uncertainty is provided in the following section which quantifies the welfare impact across periods and signal groups.

Welfare

In the previous sections we have evaluated the impact of uncertainty on adoption. We shall now turn our focus to the evaluation of equilibrium welfare under changing degrees of uncertainty. Under certainty $k = k^*$ is assumed to be fully known while under uncertainty k is revealed only at the end of period-1. Welfare comparison is performed under the assumption that true k is equal to k^* in all scenarios.

Market welfare is defined as the aggregate of consumer utility and firm profits. Under certainty, single period welfare assuming equal allocation of the fixed costs to the two periods is defined as

$$W = \int_0^S [(1-\beta)N^\beta - k^*s + y]ds + \int_S^1 yds + \beta N^\beta S_c - N \frac{f}{2} . \quad (11)$$

Following integration and substitution we get

$$W = N^\beta S + y - \frac{k^* S^2}{2} - N \frac{f}{2} . \quad (12)$$

Under certainty both periods are identical; however we define welfare per period in order to facilitate an easier comparison to scenarios under uncertainty. Total welfare shall be defined as the sum of welfare across these two periods $\hat{W} = 2W$.

Socially optimal allocation under a known k^* is derived by maximizing \hat{W} . This is achieved by differentiating welfare with respect to N and S and equating to 0. An internal solution generates the socially optimal allocation defined by equations (13) & (14):

$$N_o = \left[\frac{2\beta}{fk^*} \right]^{\frac{1}{1-2\beta}} , \quad (13)$$

$$S_o = \left[\frac{1}{k^*} \right]^{\frac{1-\beta}{1-2\beta}} \left[\frac{2\beta}{f} \right]^{\frac{\beta}{1-2\beta}} . \quad (14)$$

Except for cases where market equilibrium results in a universal adoption, the Nash market equilibrium, under certainty, results in a sub optimal number of firms and sub optimal consumer network size. This is evident by comparing equations (3) and (4) to

(13) and (14) when λ or σ are set to 0 and is similar to results obtained elsewhere^{***}. Due to un-internalized network externalities internal market equilibrium is suboptimal.

It is interesting to note that the wedge between market equilibrium and social optimum is increasing in β , thus a higher degree of preference for variety induces a larger gap between social optimum and market equilibrium. We shall attempt to show that under certain conditions uncertainty not only expands adoption but can also produce welfare results that are higher and closer to market equilibrium.

The Impact of Incomplete Information on Welfare

We shall now compare between different scenarios and evaluate the welfare impact of changing degrees of uncertainty. All of the compared scenarios shall have the same ex-ante mean, $E[\tilde{k}] = k^*$, and the same ex-post realization k^* . The information difference between the described scenarios shall be the information variance under ex-ante uncertainty. When evaluating explicit equations we focus on parameters, and world states, where all possible states result in an internal solution and the expanding nature of the uncertainty variance has not been saturated (i.e. not all of the λ consumers which have received signal $(1 - \sigma)k^*$ have decided to join the network).

Proposition 2

Under an internal solution, there exists a range of unbiased symmetric uncertainty, where the resulting welfare is higher than that under certainty & welfare is increasing in the degree of uncertainty.

Proof: We shall evaluate the welfare impact on each of the periods independently. Period-1 Firm profits do not generate a welfare difference across scenarios as firm profits during this period are 0 under both certainty and uncertainty. This is due to the fact that in equilibrium the number of consumers purchasing variety equals the expectations of the firms.

^{***} This sub-optimal interior market equilibrium under certainty is the same result as that obtained in Church, Gandal & Kraus (2008).

Consumer welfare is a more challenging evaluation. The availability of additional variety under a bounded level of uncertainty increases the welfare of all consumers on the network. In addition the size of the network increases thus generating a further increase in welfare. This increase may be offset by the non-optimal composition of the S_1 consumer types who participate in the network during period-1. Under certainty all of the consumers who have decided to join the network have an individual type which is lower than S_c . When uncertainty is introduced, the total number of consumers who join the network during period-1 increases, however they do not all have a personal type that is lower than S_1 . Some consumers who would have joined the network under certainty receive a non-favorable signal and decide not to join, they are replaced by other consumers who would not have joined under certainty but have received a favorable signal which has induced them to join.

The number of consumers who are induced to join by a favorable signal is

$\lambda \frac{(1-\beta)N^\beta}{(1-\sigma)k^*} - \lambda \frac{(1-\beta)N^\beta}{k^*}$ which is equal to:

$$\delta S^+ \equiv \frac{\lambda \sigma (1-\beta) N^\beta}{(1-\sigma) k^*}.^{\dagger\dagger\dagger} \quad (15)$$

The number of consumers who are deterred from joining by a non-favorable signal is

$\lambda \frac{(1-\beta)N^\beta}{k^*} - \lambda \frac{(1-\beta)N^\beta}{(1+\sigma)k^*}$ which is equal to:

$$\delta S^- \equiv \frac{\lambda \sigma (1-\beta) N^\beta}{(1+\sigma) k^*}. \quad (16)$$

One can verify that $\delta S^+ > \delta S^-$ thus the net number of additional consumers who join the network under an uncertainty induced expansion is $\delta S^+ - \delta S^-$ which is equal to:

$$\Delta S_1 \equiv \frac{2\lambda \sigma^2 (1-\beta) N^\beta}{(1-\sigma^2) k^*}. \quad (17)$$

Welfare equation (12) incorporates the total network joining ‘cost’ incurred by all of the consumers that have joined the network, which under full certainty is $\frac{k^* S_t^2}{2}$. Under period-1 uncertainty this simple formulation cannot be used as some of the consumers

^{†††} The maximum number of consumers which may be induced to join the network upon receiving a favorable signal is $\delta S^+ \equiv \lambda - \frac{\lambda(1-\beta)N^\beta}{k^*}$.

which have joined the network in period-1 have a d -type which is higher than S_1 while some lower d -type consumers have not joined the network as they receive a non-favorable signal. The additional cost of the δS^- consumers which have been replaced by higher d -types should be accounted for.

The reduction in welfare due to the suboptimal composition of the S_1 consumers is calculated by aggregating the total kd cost for all those consumers which have joined the network under uncertainty but have a d -type that is higher than S_1 (their number is equal to $(\delta S^+ - \Delta S)$) and subtracting the cost that would have been incurred by the same number of consumers (these are the δS^- consumers) of lower d -type, that would have joined the network of size S_1 under certainty. This difference in ‘cost’ between these groups is calculated to be $k^* \delta S^+ \delta S^-$, as further detailed in Appendix A.

When substituting the explicit values for an internal solution as detailed in equations (15) and (16) we get the following explicit term which represents the additional cost:

$$\frac{[\lambda \sigma (1-\beta) N^\beta]^2}{(1-\sigma^2) k^*}. \quad (18)$$

Period-1 welfare under uncertainty with an aggregate consumer adoption of S_1 and equilibrium N as detailed in equation (3) is thus equal to:

$$W_1 = N^\beta S_1 + y - \frac{k^* s_1^2}{2} - \frac{[\lambda \sigma (1-\beta) N^\beta]^2}{(1-\sigma^2) k^*} - N \frac{f}{2}. \quad (19)$$

Calculating $\Delta W_1 = W_1 - W_c$ generates the following condition which determines the sign of ΔW_1 :

$$v^{\frac{1}{1-2\beta}} \left[\left(1 - \frac{v}{2} \right) - \frac{\lambda^2 \sigma^2}{(1-\beta)(1-(1-2\lambda)\sigma^2)} \right] - 0.5 > 0 \quad (20)$$

$$\text{where } v = \frac{[1 - (1-2\lambda)\sigma^2]}{1 - \sigma^2}.$$

An interesting attribute of this condition is that it is solely determined by signal variance and variety preference. This condition is satisfied for a robust support, as can be shown in Figure 1 which plots the value of the LHS of equation (20). This plot spans the full range of σ and β for a given value of $\lambda=0.33$. Plot results are only applicable for an internal solution.

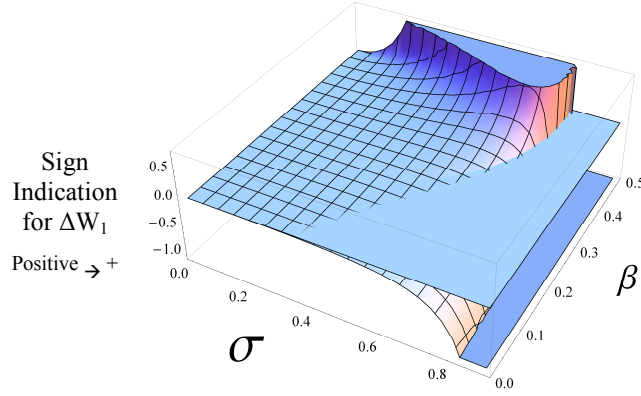


Figure 1: The sign indication for ΔW_1 under varying degrees of uncertainty and variety preference. A positive value guaranties that $W_1 > W_c$.
Calculated with $\lambda = 0.33$

Thus we have shown that that period-1 welfare under uncertainty is superior to that obtained under full certainty for a bounded range of σ . The range of σ which guarantees a welfare increase in period-1 is increasing with the level of variety preference β .

Let us now evaluate period-2 welfare under uncertainty. During this period consumers re-evaluate their adoption decision based on the newly revealed k^* , thus all adopting consumers will have a personal type $d < S_2$ and equation (12) can be used to calculate period-2 welfare. The internal market equilibrium, under certainty is sub optimal with a lower than optimal number of firms and a lower than optimal number of consumers. In the vicinity of an internal market equilibrium the welfare function is increasing in both the number of consumers and variety. From proposition 1 we know that adoption is increasing in the degree of uncertainty, thus it flows directly from this result that period-2 consumer welfare following an uncertain period-1 will be higher than period-2 welfare under complete certainty, at least for the range of uncertainty which generates an increased variety which is lower than social optimum. On the other hand firm profits during period-2 will be lower than those generated under certainty as the number of consumers on the network in period-2 shall be lower than the expectation formed ex-ante by firms. Under certainty firm profits are 0 while following uncertainty period-2 firms shall incur a loss equal to $N(S_1 - S_2)\frac{f}{2}$, which accounts for the excess

upfront investment above what would have been justified for a post-realization network size of S_2 .

Calculating $\Delta W_2 = W_2 - W_c$ generates the following condition which determines the sign of ΔW_2 :

$$(1 - \beta)(v^{\frac{2\beta}{1-2\beta}} - 1) - v^{\frac{1}{1-2\beta}}(S_1 - S_2) > 0 \quad (21)$$

$$\text{where } v = \frac{[1 - (1 - 2\lambda)\sigma^2]}{1 - \sigma^2}.$$

This condition, which unlike condition (20) depends on equilibrium adoption, has a robust support as will be demonstrated in the simulations which follow.

Under an internal equilibrium, the explicit welfare impact across both periods is detailed in equation (21):

$$N_c^\beta S_c \eta - \frac{K^* S_c^2}{2} \tilde{\eta} - N_c f \hat{\eta} - \frac{\left[\lambda \sigma (1 - \beta) N_c^\beta v^{\frac{\beta}{1-2\beta}} \right]^2}{(1 - \sigma^2) k^*}, \quad (21)$$

$$\text{where } v = \frac{[1 - (1 - 2\lambda)\sigma^2]}{1 - \sigma^2}$$

$$\eta = v^{\frac{1}{1-2\beta}} + v^{\frac{2\beta}{1-2\beta}} - 2$$

$$\tilde{\eta} = v^{\frac{1-2\beta}{1-2\beta}} + v^{\frac{2\beta}{1-2\beta}} - 2$$

$$\hat{\eta} = v^{\frac{1}{1-2\beta}} - 1$$

Proposition 3

There may exist a signal variance that will produce market equilibrium with a variety equal to social optimum; however total welfare for such uncertainty will remain lower than social optimum.

Proof: A value $\bar{\sigma}$ such that $0 < \bar{\sigma} < 1$ and $\bar{\sigma}$ satisfies $\frac{(1 - \beta)[1 - (1 - 2\lambda)\bar{\sigma}^2]}{1 - \bar{\sigma}^2} = 1$ would

result in a variety equal to socially optimal market variety equilibrium. This may be verified by inserting this value into equations (3) and (4) which then become identical to equations (15) and (16) which define an internal maximum welfare equilibrium. This result would not yield a welfare value which equals full social welfare as one has to

account for the decrease in welfare due to the consumer composition of S_1 and the firm loss in period-2, generated by uncertainty, as discussed in the previous section.

Simulated Examples – Private Signals

Figure 2 depicts a simulation of equilibrium variety, network size and welfare under different degrees of uncertainty. The top portion of the figure displays variety (N), while the lower portion displays network size (S). Relative welfare change is depicted in green between these two graphs. In each of these sections the black dashed lines depict socially optimal values (N_o & S_o respectively) while the yellow dashed lines depict market equilibrium values under certainty (N_c & S_c respectively). The blue ascending lines plot the reaction of market equilibrium variety N_1 and network size S_1 to varying degrees of uncertainty, while the light blue lines plots the period-2 post realization network size S_2 .

In the simulation graphs one can study the adoption expanding features of uncertainty. For example, when $\sigma = 0.5$, variety increases from 1.1 under certainty to 1.7 under uncertainty while network size increases from 30% to 41% and 34% for period-1 and period-2 respectively. Welfare peaks at about this level of uncertainty and begins to drop even before the values equal to social optimum are reached. This is due to the consumer composition and over investment as detailed in the previous sections. At a level of about $\sigma = 0.6$ equilibrium values are close to the optimal allocation however welfare is already decreasing as discussed. Uncertainty can induce further over adoption as can be seen for values of $\sigma > 0.6$. The expanding forces are reversed and network size and variety begin to gradually decrease once all of the consumers who received a favorable signal have decided to join the network.

*** Simulations were performed given a fixed λ , thus σ solely determines signal variance and is referred to as the degree of signal variance.

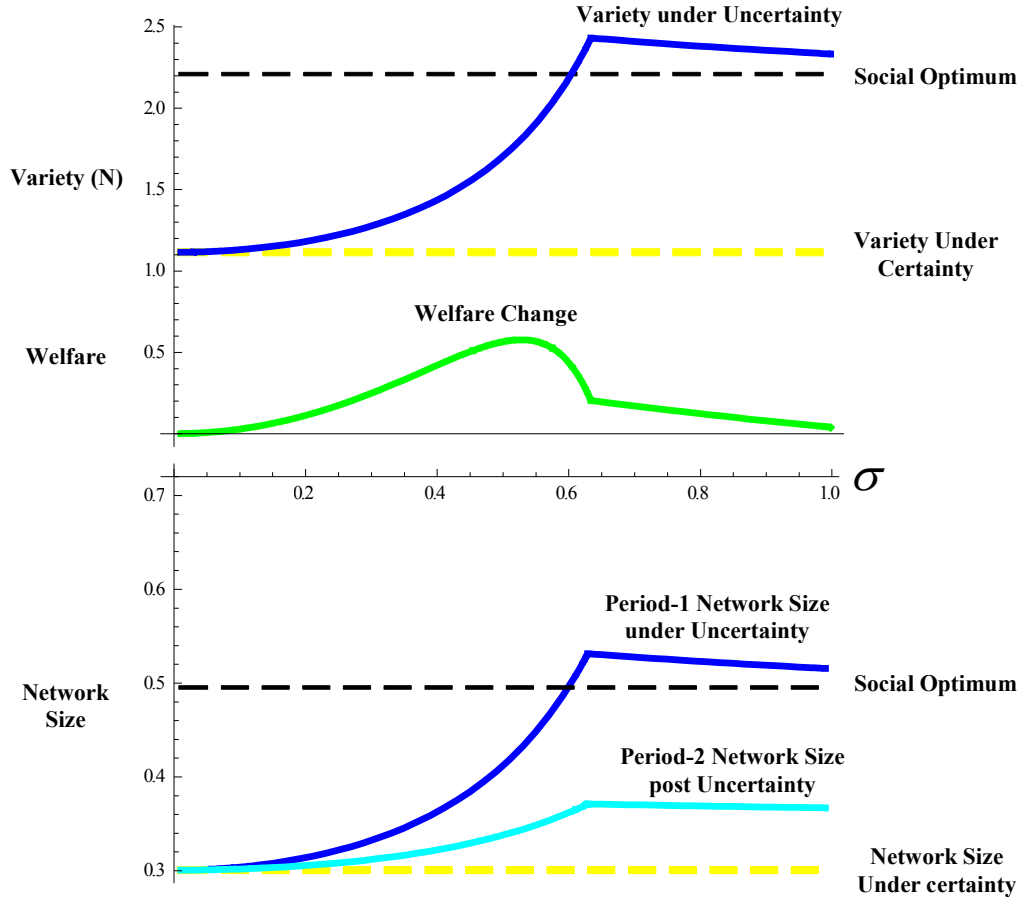


Figure 2: Equilibrium reaction (Variety, Network size and Welfare) to varying degrees of signal variance as represented by the value of σ . ($\beta = 0.27, f = 0.15, k^* = 2.5, \lambda = 0.33$)

Figure 3 provides a closer look at the welfare changes induced by uncertainty. The solid green line shows the overall welfare change while the dashed darker blue line depicts period-1 welfare changes. The third dashed line depicts welfare changes for period-2. It is interesting to note that the ranking of the welfare impact may change as the degree of uncertainty increases. In this example, for $\sigma > 0.6$ period-1 welfare change drops below period-2 and eventually becomes negative due to the composition of consumers and the rate of consumer adoption which at this level of uncertainty is already on the decline. Period-2 welfare change is stable and even slightly increasing in higher σ values as the slight decrease in variety (which has already been expanded too much) reduces period-2 firm loss.

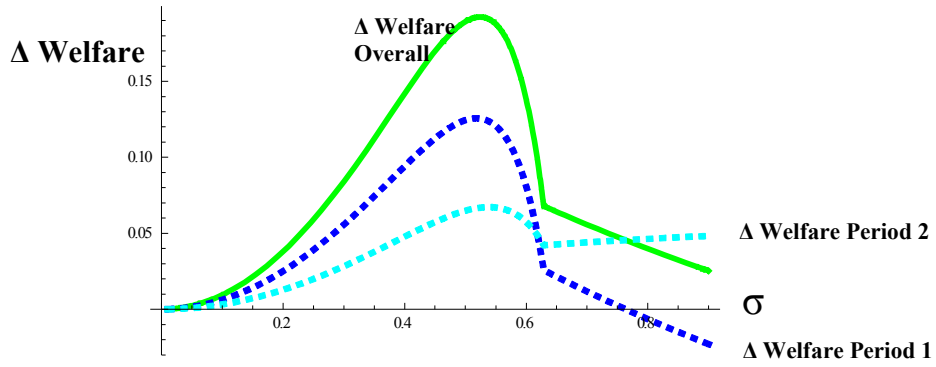


Figure 3: Welfare change under varying degrees of signal variance
 $(\beta = 0.27, f = 0.15, k^* = 2.5, \lambda = 0.33)$

Figure 4 depicts simulated welfare change under varying degrees of variety preference and signal variance. Within the simulated parameters, for every taste of variety there exists a bounded range of uncertainty, as represented by signal variance, which is welfare enhancing. Furthermore, as long as the market equilibrium is fully internal (values of β smaller than 0.42 in this specific simulation), an increasing level of variety preference expands the degree of uncertainty support which is welfare enhancing.

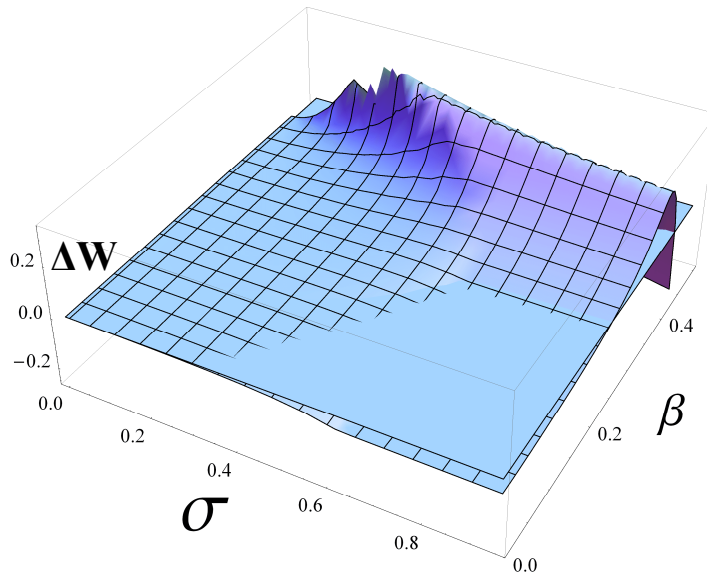


Figure 4: Welfare impact of changing degrees of signal variance and variety preference under a private signal. $(f = 0.15, k^* = 2.5, \lambda = 0.33)$

IV. Non Explicit Variety Preference

We shall now return to our basic specification and evaluate results under a general taste for variety as specified by a continuous increasing strictly concave function $g(N)$. Our aim is to demonstrate that the results described so far are not limited to the specific utility specification used in the previous sections.

Proposition 4

The network and variety expanding features as detailed in Proposition 1 hold for any general taste for variety $g(N)$; provided the following condition is satisfied:

$$\frac{g(N)}{N} > 2g'(N) .$$

i.e. Marginal variety preference should be significantly lower than average variety preference.

Proof: Once on the network a consumer purchase all varieties such that $p_N < g'(N)$. Following the same reasoning detailed in the explicit analysis, Given N and a symmetric equilibrium, the price of all products actually consumed shall be exactly $p_N = g'(N)$.

Under certainty, a known variety N and an internal solution; consumer network size shall be defined by:

$$S(N) = \frac{g(N) - NP_N}{k} . \quad (22)$$

Evaluating equilibrium across the three signal type groups under a private signal is similar to that which was performed in section II and appendix A for the explicit function. Thus the network size reaction function to a given N , under equilibrium prices shall be:

$$S(N) = \frac{(g(N) - Ng'(N))[1 - (1 - 2\lambda)\sigma^2]}{k^*(1 - \sigma^2)} . \quad (23)$$

Under free entry and a sub game perfect Nash equilibrium firms enter the market until expected equilibrium profits are equal to 0. Under certainty equation (24) describes the entry condition for the marginal firm:

$$2P_N S(N) - f = 0 \quad . \quad (24)$$

Thus the value of N which satisfies the equilibrium free entree condition, when firms know the consumer reaction function as detailed in (22) is defined in equilibrium by the following equation:

$$g'(N)(g(N) - Ng'(N)) = \frac{fk}{2} \quad . \quad (25)$$

Under private signals using the reaction function detailed in equation (23) yields the following equation defining equilibrium variety N :

$$g'(N)(g(N) - Ng'(N)) = \frac{fk}{2} \frac{1 - \sigma^2}{(1 - (1 - 2\lambda)\sigma^2)} \quad . \quad (26)$$

What are the conditions which are required for uncertainty to be network and variety expanding?

From equation (23) it is clear that under a given variety the size of the network expands under uncertainty. To evaluate the equilibrium reaction of variety we can note that the right side of equation (26) decreases as the uncertainty variance or magnitude increases. Thus the required condition for uncertainty to be variety increasing (and network increasing) is that the derivative of the left hand side of equation (26) with respect to N is negative. Calculating the derivative and using the fact that $g'' < 0$ we get the condition specified in Proposition 4: §§§

$$\frac{g(N)}{N} > 2g'(N) \quad . \quad (27)$$

As long as the average utility is greater than twice the marginal utility Propositions 1 and 2 hold for any function $g(N)$.

§§§ Applying the general condition detailed in equation (23) to the specific function $g(N) = N^\beta$ used in the explicit model analysis of section III, imposes the condition $0 < \beta < 0.5$ which was used throughout the analysis.

Solving for the social optimal allocation under known parameters we get the following relationships:

$$S(N) = \frac{g(N)}{k} , \quad (28)$$

$$g'(N)g(N) = \frac{fk}{2} . \quad (29)$$

When comparing the socially optimal allocation to the market equilibrium as defined by equations (23) and (26) one can verify that market equilibrium (for an internal solution) is always sub optimal with less than optimal variety and a network that is smaller than optimal allocation. From Propositions 4 we know that under an internal solution uncertainty is network expanding for any variety taste $g(N)$ provided $g(N)$ satisfied condition (27); thus subject to the specific parameters, uncertainty expands network size and variety and may also be welfare enhancing for a bounded range of uncertainty.

The described results are consistent with a diversified set of commonly used variety preference. For example, the Dixit & Stiglitz (1977) specification coupled with a log utility function as used by Grossman and Helpman is consistent with our results as described in equation (30).

$$g(N) = \ln\left(\left[\int_0^N x(j)^\alpha dj\right]^{1/\alpha}\right) \quad \text{with } 0 < \alpha < 1 , \quad (30)$$

Where $x(j)$ denotes the quantity consumed from variety j .

When firms face the same cost structure and a non-devisable good, as in our model, this specification reduces to equation (31):

$$g(N) = \frac{1}{\alpha} \ln(N) . \quad (31)$$

Applying condition (27) to this utility specification yields the explicit condition for proposition 4 to hold: $\ln(N) > 2$. Thus the described results hold when using log utility provided equilibrium baseline variety under certainty is high enough.

V. Equilibrium under a Common Signal

In this section we extend the model and evaluate results under the assumption that the market follows a common signal generated from the previously described distribution. Prior to gaining experience with the new innovation, market participants will set their beliefs based on this common signal. Under such a signal, market reaction may be driven by a strong interim bias (positive or negative) towards the new technology based on the specific signal realization. A common signal generates a market bias even when the signal distribution is symmetric with a mean equal to the realized value.

Period-2 equilibrium depends on the specific signal received during period-1. Under signal $(1 - \sigma)k^*$ period-2 variety shall remain the same as period-1 and consumer's participation will adapt to the revealed k^* which is a path similar to that observed under private signals. Under a non-favorable signal $(1 + \sigma)k^*$ period-2 variety may increase as the revealed k^* induces additional consumers to join the network. The additional number of firms deciding to join shall not generate variety equal to a certainty scenario under k^* as there is only one period remaining to sell the product and f is unchanged. In the specific setup described, with 2 periods and $\sigma < 1$, the number of firms will not increase in period-2 due to the symmetric impact of f and k on the adoption decision of firms.

As in section III, evaluation is performed under the specification $g(N) = N^\beta$. We evaluate welfare under a common signal and compare explicit results to both the full certainty and the private consumer signal scenarios previously described. Ex-ante there exist three separate scenarios which may unfold. The mean welfare result across these 3 scenarios will determine the ex-ante welfare under a common signal. Each of these scenarios is solved using the same procedure discussed previously, however during period-1 all participants follow the same signal.

Equation (33) details the explicit formulation, under an internal solution, of the mean of welfare difference between the common signal scenario and the one with full certainty.

$$E[\Delta \hat{W}] = E[W] - W_c = N_c^\beta S_c \theta - \frac{k^* S_c^2}{2} \tilde{\theta} - N_c f \hat{\theta} \quad (33)$$

Where

$$\begin{aligned} \hat{\varphi} &\equiv \frac{1}{1+\sigma} & \tilde{\varphi} &\equiv \frac{1}{1-\sigma} \\ \theta &\equiv \frac{1}{\tilde{\varphi}^{1-2\beta}} + \frac{2\beta}{\tilde{\varphi}^{1-2\beta}} + \frac{1}{\hat{\varphi}^{1-2\beta}} + \frac{2\beta}{\hat{\varphi}^{1-2\beta}} - 4 \\ \tilde{\theta} &\equiv \frac{2\beta}{\tilde{\varphi}^{1-2\beta}} + \frac{2-2\beta}{\tilde{\varphi}^{1-2\beta}} + \frac{2\beta}{\hat{\varphi}^{1-2\beta}} + \frac{2-2\beta}{\hat{\varphi}^{1-2\beta}} - 4 \\ \hat{\theta} &\equiv \frac{1}{\tilde{\varphi}^{1-2\beta}} + \frac{1}{\hat{\varphi}^{1-2\beta}} - 2 \end{aligned}$$

The explicit condition for $E[\Delta \hat{W}]$ to be positive is:

$$\theta > \frac{(1-\beta)}{2} \tilde{\theta} + 2\beta \hat{\theta} . \quad (34)$$

Equation (33) provides the result for the mean expected welfare change under a common signal structure; however with probability one this will not be a scenario that can materialize. It is only an indication of the mean expected result prior to receiving a common signal. This is different from the private signal scenarios where the described results are the actual expected equilibrium values. Figure 5b depicts a simulated example comparing between the mean expected welfare of a common signal world, prior to signal generation, to a certainty scenario.

As can also be confirmed from condition (34) there may exist a support in which the ex-ante mean welfare of the common signal-tracking world is superior to the one with complete information; However when evaluating this scenario one must also take into consideration that under this setup the signal conditioned results can also be very damaging. The negative welfare impact that will occur when the common signal is a non-favorable one can be understood by reviewing Figure 5c which plots the welfare change under a non-favorable, $(1+\sigma)k^*$ signal, compared to the full certainty scenario; under all parameter settings welfare will decrease. Figure 5a shows results under favorable signal $(1-\sigma)k^*$; for very high variety preferences the impact is always positive while for lower preferences the welfare impact becomes negative under higher degrees of uncertainty.

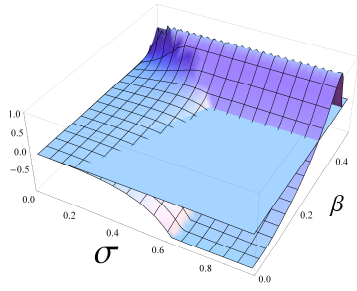


Figure 5a
Welfare change following a
favorable common signal

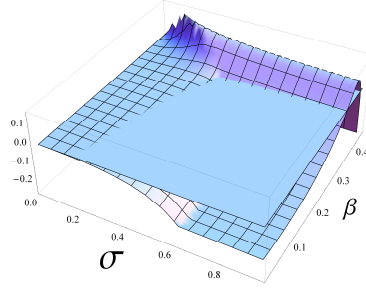


Figure 5b
Mean Welfare change $E[\Delta \widehat{W}]$
under a common signal

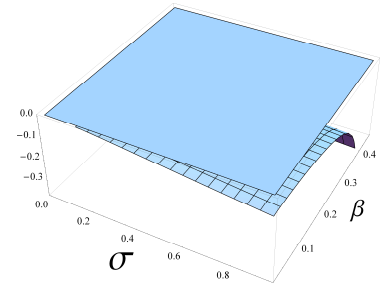


Figure 5c
Welfare change following a
Non-favorable common signal

Figure 5: Welfare impact of changing degrees of Signal Variance and Variety Preference, under a Common Signal information structure. ($f = 0.15, k^* = 2.5, \lambda = 0.33$)

Comparing Equilibrium under Private and Public Signals

The common signal structure generates inferior results, when compared to the private signal. It is inferior due to its potential to reduce overall welfare under a non-favorable common signal but also due to the fact the under most parameters, even if one were to ignore this possible outcome under a non-favorable signal, this scenario generates a smaller mean welfare increase (if any) under high variety preferences and a larger mean welfare decrease under low variety preferences.

Figure 6 compares between the mean expected common signal welfare results and those reached under the private signal scenario. The graph plots the difference between the welfare impact of the private signal scenario (i.e. the welfare impact of a private signal scenario over full certainty) and the mean expected change in welfare under a common signal over full certainty. We can notice that throughout most of the simulated range, the private signal scenario is ex-ante welfare superior.

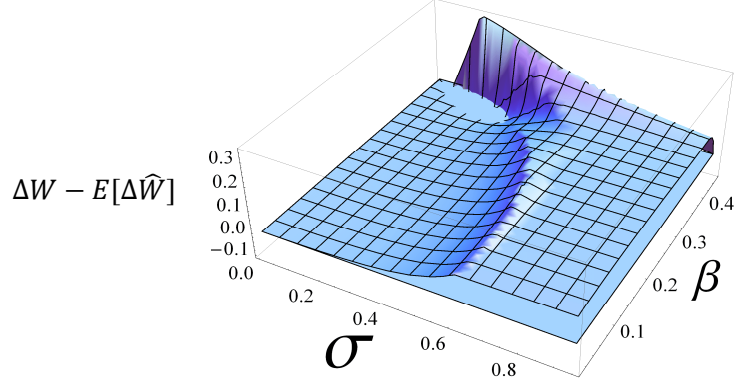


Figure 6: Comparing consumer Private signal tracking with Common signal tracking.
Welfare change under a private signal subtracted by
the mean expected welfare change under a common signal
($f = 0.15, k^* = 2.5, \lambda = 0.33$)

In cases where a policy maker may be able to impact the quality of information, or possibly reduce signal variance, are we able to provide clear guidance? Under a common signal scenario a policy maker will have to take into account the possibility of a non-favorable signal which will significantly harm both adoption and welfare even when the mean result prior to signal realization may be welfare enhancing, thus it may prove difficult to support such a policy. Under private signals, a policy maker may find it easier to support a laissez faire approach provided one assumes that signal variance is bound by a certain level or that variety preference is high enough. Under these assumptions both adoption and welfare are increasing in the degree of uncertainty for a robust set of cases.

VI. Conclusion

In this paper we studied the potential impact of uncertain innovation on market equilibrium, variety and welfare, in the presence of indirect externalities. We evaluated the interaction between information, or the lack of it, and the inherent inefficiencies due to private decision-making under externalities. Our results show that unbiased uncertainty may expand network adoption and product variety. In such settings, uncertainty may also be welfare enhancing.

We evaluated the interaction between the degree of innovation uncertainty and consumer preferences and categorized the various conditions impacting overall welfare.

Uncertainty may often produce an ex-post decrease in firm profits coupled with over adoption; however, we show that some scenarios where such phenomena occur may be welfare enhancing and, under certain conditions, ex-ante superior to an equilibrium generated under complete information. In such cases uncertainty partially compensates for the inability of the competitive market equilibrium to fully exploit the potential of an externality intensive innovation. This result holds even when the signals and transitory information states are derived from an unbiased symmetric distribution and may be generated by the existence of uncertainty per-se and not over optimism.

From a policy perspective, this paper has demonstrated that the often-assumed link between complete information and a better decision process in the initial phases of externality intensive innovations warrants a refined review as to the underlying market structure and outcome, especially when the technology at play exhibits strong positive network externalities. In such cases, uncertainty driven market experimentation, which fosters some over adoption and seemingly ‘wasted’ resources, may nevertheless be a preferred path.

Appendix A

Calculating an internal equilibrium under a private signal.

Aggregate consumer reaction to a given variety N: The marginal d -type consumer to join the network is defined such that the mean of his expected cost adjusted utility from variety is equal to the expected network cost. As discussed in section III the price of all varieties on the network shall be symmetric and equal to marginal utility, $P_N = p = \beta N^{\beta-1}$. The total cost for N varieties is βN^β and utility from variety is N^β . Each consumer knows his personal type thus the indifference equation defining the marginal consumer(s) shall be:

$$d = E \left[\frac{(1-\beta)N^\beta}{k} | \tilde{k} \right].$$

Under the signal structure defined we have three sub-groups of consumers based on the signal received. Each consumer who receives a signal \tilde{k} shall apply his knowledge regarding the symmetric distribution and decide based on the mean of network value subject to the possible states of the world based on his signal.

$$E\left[\frac{(1-\beta)N^\beta}{k}|\tilde{k}\right] = (1-\beta)N^\beta E\left[\frac{1}{k}|\tilde{k}\right] = (1-\beta)N^\beta \left(\lambda \frac{1}{\frac{k}{1+\sigma}} + \lambda \frac{1}{\frac{k}{1-\sigma}} + (1-2\lambda) \frac{1}{k}\right) = \frac{(1-\beta)N^\beta}{\tilde{k}},$$

thus consumers use the explicit signal received to drive their adoption decision even when they are aware of the signal distribution. We shall now aggregate across all consumers:

A mass of λ consumers have received signal $(1+\sigma)k^*$ and their marginal d is $\frac{(1-\beta)N^\beta}{(1+\sigma)k^*}$.

A mass of λ consumers have received signal $(1-\sigma)k^*$ and their marginal d is $\frac{(1-\beta)N^\beta}{(1-\sigma)k^*}$.

A mass of $(1-2\lambda)$ consumers have received signal k^* and their marginal d is $\frac{(1-\beta)N^\beta}{k^*}$.

Assuming an interior solution for all signal subgroups, we sum over these groups and receive the aggregate number of consumers:

$$S(N) = \lambda \frac{(1-\beta)N^\beta}{(1+\sigma)k^*} + \lambda \frac{(1-\beta)N^\beta}{(1-\sigma)k^*} + (1-2\lambda) \frac{(1-\beta)N^\beta}{k^*}. \quad (A1)$$

After rearranging we get the result:

$$S(N) = \frac{(1-\beta)N^\beta[1-(1-2\lambda)\sigma^2]}{k^*(1-\sigma^2)}. \quad (A2)$$

Adoption criteria and equilibrium: Firms decide to join the network based on the known R&D cost f and their expected aggregate revenue from selling the product on the network in periods 1 and 2. Due to free entry firms will join the network until profits are equal to 0. Thus the indifference equation defining firm entry is:

$$2\beta N^{\beta-1}S(N) = f \quad (A3)$$

Under a Nash equilibrium and an internal solution firms correctly anticipate the reaction function of consumers to variety, subject to the existing information.

Substituting $S(N)$ from equation (A2) into the indifference equation (A3) for the fully interior case and rearranging to extract the value of N we get :

$$N = \left[\frac{2\beta(1-\beta)[1-(1-2\lambda)\sigma^2]}{fk^*(1-\sigma^2)} \right]^{\frac{1}{1-2\beta}}. \quad (\text{A4})$$

Substituting this value of N back into the consumer reaction function will derive the equilibrium value of $S(N)$. Period-1 equilibrium network size is derived by substituting (A4) back into (A2) which yields the following result:

$$S_1 = \left[\frac{2\beta}{f} \right]^{\frac{\beta}{1-2\beta}} \left[\frac{(1-\beta)[1-(1-2\lambda)\sigma^2]}{k^*(1-\sigma^2)} \right]^{\frac{1-\beta}{1-2\beta}}.$$

The consumer participation decision during period-2 shall be based on the true value of k^* which has been revealed after consumption in period-1. Firms have already invested their fixed cost and are selling at a price which is above marginal cost thus all will continue and produce during period-2. The variety available during period-2 is thus derived by the number of firms which have entered the market in period-1. Substituting the value of N received under uncertainty during period-1 into the period-2 reaction function under certainty (which under the realization k^* is $S(N) = \frac{(1-\beta)N^\beta}{k^*}$) and rearranging will produce period-2 network size:

$$S_2 = \left[\frac{2\beta[1-(1-2\lambda)\sigma^2]}{f(1-\sigma^2)} \right]^{\frac{\beta}{1-2\beta}} \left[\frac{(1-\beta)}{k^*} \right]^{\frac{1-\beta}{1-2\beta}}.$$

Equilibrium under private signal uncertainty when the favorable signal reaction is saturated:

For those values of signal variance where the expanding forces of the increasing variance have been exhausted (i.e. all λ consumers which have received signal $(1 - \sigma)k^*$ have decided to join the network) the aggregate reaction function across signal subgroups is:

$$S(N) = \lambda + \lambda \frac{(1-\beta)N^\beta}{(1+\sigma)k^*} + (1 - 2\lambda) \frac{(1-\beta)N^\beta}{k^*}. \quad (A5)$$

Which is similar to (A1) but with the first term replaced by λ . Rearranging yields the following solution:

$$S(N) = \lambda + \frac{(1 - \beta)N^\beta [(1 - \lambda) + (1 - 2\lambda)\sigma]}{k^*(1 + \sigma)}.$$

Substituting the result of equation (A5) into the indifference equation (A3) will generate equation (A6) which should hold under equilibrium:

$$\lambda N^{\beta-1} + \frac{(1 - \beta)N^{2\beta-1} [(1 - \lambda) + (1 - 2\lambda)\sigma]}{k^*(1 + \sigma)} = \frac{f}{2\beta}. \quad (A6)$$

N cannot be algebraically extracted from this equation, however this is a solvable polynomial equation and the value of N which satisfies this equation is the equilibrium value, which can then be used to calculate equilibrium consumer adoption.

Calculating the additional cost exerted by period-1 consumers under a private signal.

The number of period-1 consumers under uncertainty with a d -types higher than S_1 is equal to $(\delta S^+ - \Delta S)$ which in turn is equal to δS^- which is the number of consumers with lower d -types which have been deterred from joining by a non-favorable signal. The additional cost incurred by the less than optimal composition of S_1 is:

$$\int_{S_1}^{S_1 + \delta S^-} k^* s ds - \int_{S_1 - \Delta S - \delta S^-}^{S_1 - \Delta S} k^* s ds, \quad ,$$

which after integration , substitution and summation equals $k^* \delta S^- (\delta S^- - \Delta S)$. Using the fact that $(\delta S^- - \Delta S) = \delta S^+$ we can generate the final equation which summarizes the additional cost incurred from the non-optimal composition of S_1 to be: $k^* \delta S^+ \delta S^-$.

Equilibrium under a public signal \tilde{k} :

Under a public signal the market receives a common signal \tilde{k} . Evaluating the conditional reaction of each consumer is identical to the results outlined under a private signal. The difference in reaction relies on the fact that this structure may create an interim bias towards the new technology as all of the market follows the same signal which may be over or under optimistic. Prior to the signal being delivered a bias does not exist as ex-ante the market may be conditioned in both directions, however after signal delivery the market is biased with a probability of 2λ

Ex-ante there exist three separate scenarios which may unfold. For each signal there exists a period-1 market equilibrium that is solved as if \tilde{k} is the true value using the same set of equations previously discussed. Period-2 equilibrium depends on the specific signal received during period-1. When the signal received was k^* then period-2 equilibrium values are equal to those of period-1. When the signal is $k^*(1 - \sigma)$ then period-2 variety shall remain the same as period-1 and consumer's participation will adapt to the revealed k^* which is a path similar to that observed under private signals. Under a non-favorable signal $k^*(1 - \sigma)$ period-2 variety may increase as the revealed k^* , which is higher than that which was assumed during period-1, induces additional consumers to join the network, potentially inducing additional variety as well. The additional number of number of firms deciding to join shall not generate variety equal to a certainty scenario under k^* as there is only one period remaining to sell the product and f is unchanged. In our specific setup with 2 periods and $\sigma < 1$ the number of firms will not increase in period 2.

The mean welfare result across these 3 scenarios will determine the ex-ante welfare of such a scenario.

Following is a detailed proof for equilibrium under a common signal $(1-\sigma)k^*$.

Under this signal the marginal consumer *d-type* for joining the network is $\frac{(1-\beta)N^\beta}{(1-\sigma)k^*}$, thus the reaction function given signal $(1-\sigma)k^*$ is:

$$S(N) = \frac{(1-\beta)N^\beta}{k^*(1-\sigma)}. \quad (A7)$$

Under Nash equilibrium and an internal solution firms correctly anticipate the reaction function of consumers to variety subject to the existing information. Substituting equation (A7) into the indifference function for the fully interior case (A3) and rearranging to extract the value of N we get:

$$N = \left[\frac{2\beta(1-\beta)}{fk^*(1-\sigma)} \right]^{\frac{1}{1-2\beta}}.$$

Substituting this value of N back into the consumer reaction function (A7) will derive the period-1 equilibrium value under this signal:

$$S_1 = \left[\frac{2\beta}{f} \right]^{\frac{\beta}{1-2\beta}} \left[\frac{(1-\beta)}{k^*(1-\sigma)} \right]^{\frac{1-\beta}{1-2\beta}}.$$

The consumer participation decision during period-2 shall be based on the true value of k^* which has been revealed after consumption in period-1. The variety available during period-2 is derived by the number of firms which have entered the market in period-1. Firms have already invested their fixed cost and are selling at a price which is above marginal cost thus all will continue and produce during period-2. Substituting the value of N received under uncertainty during period-1 into the reaction function under certainty and rearranging yields the following period-2 equilibrium value under this signal:

$$S_2 = \left[\frac{2\beta}{f(1-\sigma)} \right]^{\frac{\beta}{1-2\beta}} \left[\frac{(1-\beta)}{k^*} \right]^{\frac{1-\beta}{1-2\beta}}.$$

Deriving the equivalent solution under common signal $(1+\sigma)k^*$ is performed in an equivalent manner subject to the appropriate substitutions while the equilibrium under a common signal k^* is equal the full certainty one.

References

- Blanchard O.J., 1979. "Speculative Bubbles ,Crashes and Rational Expectations", *Economics Letters* 3, 387-389.
- Church J. , N. Gandal and D. Krause, 2008. "Indirect Network Effects and Adoption externalities". *Review of Network Economics* 7, 337-357.
- Dixit A. and Stiglitz J.E., 1977. "Monopolistic competition and optimum product diversity ", *American Economic Review* 67, 297-308.
- Either W.J., 1982. "National and International Returns to scale in the modern theory of international trade", *American Economic Review* 72, 389-405.
- Farrell, J. and G. Saloner ,1985. "Standardization, Compatibility, and Innovation", *RAND Journal of Economics* 16, 70-83.
- Farrell J and Saloner G, 1986. "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation", *American Economic Review* 76, 940-955.
- Farrell, J. and G. Saloner , 1986a. "Standardization and Variety", *Economic Letters* 20, 71-74.
- Grossman G.M. and N. Yanagawa, 1993. "Asset Bubbles and Endogenous Growth", *Journal of Monetary Economics* 31, 3-19.
- Grossman G.M. and E. Helpman, 1991. "Innovation and Growth in the Global Economy", MIT Press,Cambridge Massachusetts .
- Hagiu A. ,2009. "Proprietary vs. Open Two Sided Platforms and Social Efficiency", Working Paper 06-12, AEI-Brookings Joint Center.
- Halaburda H. and Yehezkel Y.,2011 . "Platform Competition under Asymmetric Information Harvard Business School working paper 11-080.
- Katz M.L. and C. Shapiro , 1985. "Network Externalities, Competition and Compatibility", *American Economic Review* , 75 , 424-440.
- Katz, M. and C. Shapiro, 1986a. "Technology Adoption in the Presence of Network Externalities" , *Journal of Political Economy* 9, 822-41.
- Katz, M. and C. Shapiro , 1986b . Product Compatibility Choice in a Market with Technological Progress, Oxford Economic Papers, New Series, Vol. 38, Supplement: Strategic Behavior and Industrial Competition, 146-165.

Katz, M. and C. Shapiro, 1994. "Systems Competition and Network Effects", *The Journal of Economic Perspectives* 8, 93-115.

Kiyotaki N., 1988. "Multiple Expectational Equilibria Under Monopolistic Competition", *QJE*, 695-713.

Leibowitz S.J. and S.E. Margolis, 1994. "Network Externalities an Uncommon Tragedy", *The Journal of Economic Perspective* 8 , 133-150.

Leibowitz S.J. and S.E. Margolis, 2002. "Network Effects", *Handbook of Telecommunication Economics* Vol 1 editors, Cave M. , S Majumdar and I. Vogelsang, Amsterdam Elsevier , 76-94.

Nyssen J., 1993. "Social Efficiency of Bubbles in the Grossman and Helpman Endogenous Growth Model", *Economics Letters* 45 , 197-202.

Zvilichovsky D., 2007. "Technology Adoption, Bubbles and Productivity", Foerder Institute for Economic Research, Working Paper, 3-07.