A Mixed Integer Linear Programming Model For Solving Large-Scale Integrated Location-Routing Problems For Urban Logistics Applications at Groupe La Poste

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Abstract

In this work we present a large scale static, deterministic mixed inter linear programming model (MIP) solving an integrated location-routing problem (LRP) in the context of urban logistics services (ULS). We aim at determining a cost optimal infrastructure and fleet design for an urban consolidation and transportation network. The model is meant to quantitatively evaluate potential strategic moves of postal operators (POs) into the domain of urban logistics services. It was developed to support the decision making of POs such as French Groupe La Poste in designing an optimal facility network and fleet composition for the centralized consolidation and transportation of freight flows within urban centers. Based on comprehensive analyses with operating data obtained from La Poste, this paper aims at identifying the key determinants of the characteristics of an optimal infrastructure and fleet design to support the development of profitable ULS. Further, we discuss the sensitivity of the optimal design to changes in the input data of the LRP model.

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1 Introduction

Modeling motivation and objective The principal objective of this research was to examine the possibilities for postal operators (POs) to harvest potential synergies between postal operations and capabilities and local delivery markets in urban areas. These synergies arise from the fact that fleet operations with the vehicular scale of a major postal operator would allow improved utilization of vehicles, better decision support software, and reaping thereby economies of scale and scope for collection and delivery services within and between urban regions. This could simultaneously improve the efficiency of the local logistics economy as well as providing additional profits for the PO.

Urban Logistics Services (ULS) are representative of a number of key issues currently faced by POs. Given the financial challenges of continuing to meet the postal USO in the face of declining volumes occasioned by electronic substitution, ULS represents a possible profitable opportunity for diversification building on existing competencies of POs. PO-based ULS may also contribute to more sustainable local transport, to the extent that the vehicular fleet of POs employs less polluting vehicles and more efficient scheduling of collection and delivery operations. Since e-commerce is expected to grow significantly over the next decade, improving the efficiency of local logistics markets will be an area of growing importance. Thus, entering the market for ULS as an overarching consolidator of item flows into and out of urban centers is a potential strategic move for La Poste and other major POs offering both additional revenues for the company and welfare improvements for society as a whole.

Increasing public awareness of the importance of sustainability and environmental protection has led to the introduction of tighter emissions regulations, emissions trading schemes and other pollution reducing measures by the European authorities. In parallel, advances in e-commerce, both B2B and B2C, have led to significant growth in parcel deliveries. These trends, coupled with increasingly congested urban areas, have given rise to the need for more rational planning of final mile logistics. The fast developing field of Urban Logistics encompasses the modeling and empirical research focused on these issues. In particular, the task of designing an optimal transportation infrastructure to provide urban logistics services at a high quality of service and in the most cost-effective way has become a key challenge for both logistics service providers as well as urban planners. Given the central role of postal operators in the parcel market, it is natural to investigate how a major PO like La Poste could contribute to a more efficient and sustainable Urban Logistics infrastructure. A fundamental research challenge in this regard concerns the modeling to support the planning for such PO-led urban logistics services. Addressing this challenge was the principal objective of this research.

The work presented here had the dual purpose of understanding efficient provision of ULS as well as supporting La Poste's decision making in designing an optimal infrastructure and fleet for its own potential strategic initiative in developing ULS for major French metropolitan areas. The complexities in launching a major initiative in ULS are summarized in Figure 1.

Model functionality and features Designing an optimal transportation infrastructure and fleet for the provision of ULS is equivalent to a large-scale integrated location-routing problem (LRP). The aim of the optimization model developed in this work is to minimize the total cost of providing a given urban logistics service at a given quality while respecting a broad set of constraints.

The optimization model accommodates two parallel network architectures for different item classes: 1) a transportation network with only one level of consolidation that allows for a direct shipment of large items between the central city hub and the actual demand occurrences; 2) a transportation network with two levels of consolidation for small items that are shipped between the actual demand occurrences and the city hub through intermediate depots.

Given a certain demand pattern whose corresponding urban freight flows have to be accommodated in a consolidated manner, the key questions to be answered by the model are:

- Where should the central consolidation center (i.e. the city hub) for the considered city be located?
- How many intermediary depots should be in operation as a second level of consolidation and deconsolidation between the city hub and the actual demand for collection and delivery services?
- Where should these intermediary depots be located?

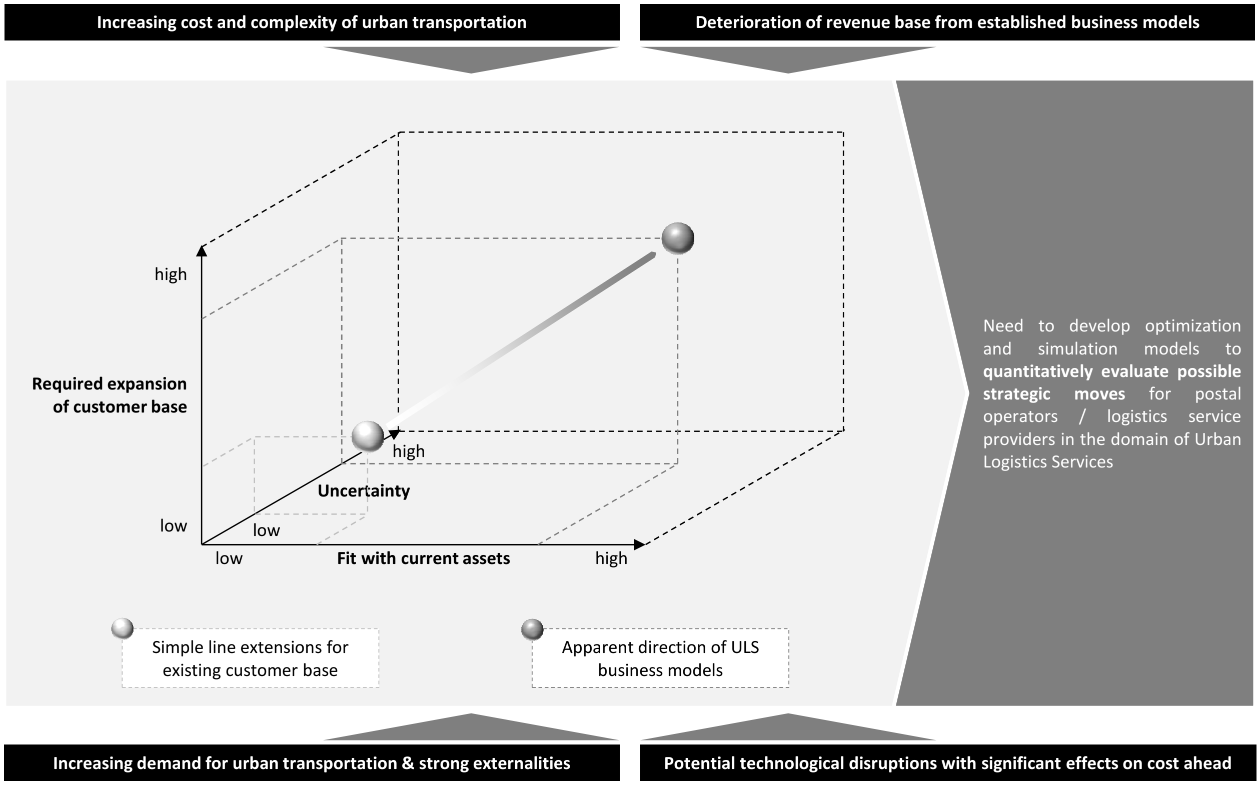


Figure 1: Motivation for research in the field of urban logistics

- Which city areas should be served from which facility using which kind of vehicle?
- How many vehicles of which type should be located at which facility?

These key questions correspond to the decision variables of the optimization model. It allows for a joint consideration of downstream and upstream freight flows when determining the optimal solution. The model incorporates a number of constraints in order to assure the practical feasibility of the solution obtained. Among these are individual capacity restrictions at the level of the de/consolidation facilities. These include the capacity of each facility in terms of available space for the processing of shipments and the loading and parking of vehicles. Similarly, the model accounts for capacity restrictions of the various vehicles under consideration. In addition, the model can account for access restrictions to certain city areas for specific vehicle types as well as positioning restrictions at certain facility locations for specific vehicle types. These constraints can, for example, be used to depict reduced pollution zones in cities. A further key constraint is on the level of quality of service. To this end, the user can specify specific maximum allowed service times that may not be exceeded for certain parts of the transportation service.

Mixed integer linear programming Technically, the model presented here makes use of various numerical optimization techniques to find an optimal solution to the optimization problem described above within a reasonable time. It can accommodate up to 1,600 city segments with non-zero demand, 400 potential facility locations, four vehicle types and two levels of item consolidation. Currently, the model is designed as a single-stage model which optimizes facility locations, facility-city segment assignments, and vehicle type selection and routing. The model assumes static input data and parameters focused on a typical day. Using appropriate linear approximations of nonlinear relationships within the model, it was implemented as a large scale Mixed Integer Linear Programming Model (MILP) that can be solved by high-end simplex based industrial solvers such as IBM ILOG CPLEX or GUROBI.

The model is able to process complex input data as the basis of optimization. This includes non-trivial demand characteristics of independent city segments. On an individual city segment level, the user can provide the model with absolute demand figures for pickup and delivery of items as well as with general item size distributions.

Model outputs The outputs of the optimization model presented here give answers to the key questions mentioned above. They can be used to determine to optimal number and location of consolidation facilities, the optimal number, type and positioning of vehicles within the transportation network, the optimal item size related threshold between the two parallel transportation networks for small and large item transportation as well as the impact of changes in the demand characteristics on the overall cost and the optimal infrastructure design. The model output can be used to construct a detailed breakdown of the total cost of ULS into its most important subcomponents, allowing for a detailed analysis of the sensitivities of the main cost drivers of the model to changes in the input data and parameters of the optimization.

2 Location-Routing Problems

2.1 Relevance of Location-Routing Problems

Academic relevance As opposed to the more widely discussed location-allocation problem, an LRP requires that demand nodes are serviced along optimal tours once they have been allocated to facilities which, in turn, have been optimally located. Thus, LRPs can actually be interpreted as a combination of the classical location-allocation problem and the vehicle routing problem (VRP) [Min et al., 1998]. Nagy and Salhi [2007] define an LRP as the combination of a facility location problem being the "master problem" and a vehicle routing problem with actual tour planning for less than full truck load (LTL) customer demands being the "subproblem" that has to be solved simultaneously. They emphasize the importance of both problems being solved in an integrated fashion. They point out that even though many practitioners are aware of the fact that separating depot location and vehicle routing decisions will lead to suboptimal outcomes [Rand, 1976], the interrelation between the two is often ignored by both academics and practitioners. Two obvious potential reasons for this mentioned by them are that many locational problems do not have an actual routing aspect and that LRPs are conceptually more difficult than classical location problems. However, Nagy and Salhi [2007] also refer to a perceived inconsistency in the planning horizons for location problems and VRPs that lead some researchers to be cautious about the LRP concept. These authors claim that location decisions are of a strategic nature due to the long-term horizon of facility investments, while routing is a tactical problem as routes can easily be changed over time. This criticism, however, is rejected by a study of Salhi and Nagy [1999] who show that by the use of an integrated location-routing approach cost can be decreased over a long planning horizon during which routes are allowed to change.

Relevance in practice Operational Research is a research discipline that is heavily driven by the practical applicability of academic theories and findings [Nagy and Salhi, 2007]. In Table 1, Nagy and Salhi [2007] have summarized a number of exemplary research contributions that had significant impact on real-world problem settings. The table confirms that there is not only an academic dimension to the concept of LRP but that it is indeed relevant and applicable to practice in a business environment, but also in other fields such as health care or military operations. It also sheds some light on the size of the largest LRP instances examined today, in terms of the number of facilities and demand nodes considered.

One of our collaboration partners for this research project is the French *Groupe La Poste*, one of Europe's largest POs. La Poste is considering to expand their customer base and product portfolio by acting as a central consolidator of inbound and outbound freight flows for the largest French cities. The results of this research project shall help La Poste in designing the required transportation infrastructure and fleet composition in a cost optimal way while ensuring a high level of service quality at the same time. From an academic perspective, the participation of a large national PO such as La Poste in this research project is of very high value as it provides us with valuable insights into daily real-life operations and the specificities that an optimization model has to respect in order to generate outputs that are applicable to practice. Moreover, we have access to large scale real-world data as a basis for our model runs and analyses. This guarantees a high practical relevance and real economic impact of our findings and the managerial implications derived from them.

Paper	Application area	Country	Facilities	Customers
Watson-Gandy and Dohrn (1973)	Food and drink distribution	United Kingdom	40	300
Bednar and Strohmeier (1979)	Consumer goods distribution	Austria	3	50
Or and Pierskalla (1979)	Blood bank location	United States	3	117
Jacobsen and Madsen (1980)	Newspaper distribution	Denmark	42	4510
Nambiar et al. (1981)	Rubber plant location	Malaysia	15	300
Perl and Daskin (1984, 1985)	Goods distribution	United States	4	318
Labbé and Laporte (1986)	Postbox location	Belgium	Not given	Not given
Nambiar et al. (1989)	Rubber plant location	Malaysia	10	47
Semet and Taillard (1993)	Grocery distribution	Switzerland	9	90
Kulcar (1996)	Waste collection	Belgium	13	260
Murty and Djang (1999)	Military equipment location	United States	29	331
Bruns et al. (2000)	Parcel delivery	Switzerland	200	3200
Chan et al. (2001)	Medical evacuation	United States	9	52
Lin et al. (2002)	Bill delivery	Hong Kong	4	27
Lee et al. (2003)	Optical network design	Korea	50	50
Wasner and Zäpfel (2004)	Parcel delivery	Austria	10	2042
Billionnet et al. (2005)	Telecom network design	France	6	70
Gunnarsson et al. (2006)	Shipping industry	Europe	24	300
Lischak and Triesch (2007)	Parcel delivery	Poland	22	750

Table 1: A summary of LRP applications [Nagy and Salhi, 2007]

2.2 Exact Algorithms And Heuristics For Solving an LRP

There are comprehensive surveys of the existing LRP literature by Balakrishnan et al. [1987], Laporte [1987, 1989], Berman et al. [1995], Min et al. [1998] and Nagy and Salhi [2007]. For the following, we mainly want to build on the two most recent works which also summarize those of their predecessors to a large extent.

Types of location-routing problems Both Min et al. [1998], Nagy and Salhi [2007] generally classify LRP models according to the following key aspects.

1. Hierarchical LRP structure.
   Most of the existing LRP models expose a classical hierarchical LRP structure where a number of demand nodes is connected to their respective facility through vehicle tours with multiple stops. There is no connection between the individual facilities. However, there are also more complex extensions to the classical LRP structure.

2. Nature of input data.
   The input data may be either deterministic or stochastic. To date, the majority of the existing literature is based on deterministic models and thus neglects uncertainty.

3. Planning period.
   One can distinguish static LRP models with only a single planning period and dynamic LRP models that consider multiple planning periods. The current LRP literature is still dominated by static models.

4. Objective function.
   Here, the distinction is between single objective and multiple objective models. Further, the type of objective variable is of interest. Most of the existing LRP models have total cost minimization as a single objective. Only very few models have a different objective such as on-time delivery. Even fewer authors construct multi-objective models.

5. Solution space.
   An LRP model can have a discrete, network or continuous solution space. Most LRP models to date deal with discrete facility locations.

6. Number of facilities.
   In an LRP there may be a single or multiple facilities to be located. The majority of existing LRP models to date deals with the multiple facility case.

7. Number and type of vehicles.
   Most of the existing LRP models consider homogeneous fleets (i.e. only a single vehicle type is available) and the number of vehicles to be used is not fixed in advance. However, there may be also applications of LRPs with heterogeneous fleets and / or a fixed number of vehicles to be used.

8. Facility capacities.
   Facilities may or may not be restricted in their capacity to handle goods and accommodate vehicles.

9. Vehicle capacities.
   Vehicles may or may not be restricted in their carrying capacity.

10. Route structure.
    The route structure can be characterized along various dimensions. First, one can distinguish node routing from arc routing techniques.[1] Further, vehicles may be allowed to perform only a single or multiple tours. Also, vehicle tours may include pickup nodes, delivery nodes, or both.

11. Time windows.
    The transportation task may be required to be completed in an unspecified time without a deadline, within soft time windows and loose deadlines or within hard time windows and strict deadlines.

[1] see e.g. Levy and Bodin [1989]

12. Source of model data.
    LRP models may be fed with hypothetical or real-world data.

13. Solution method.
    The reviews mentioned above generally distinguish two types of solution methods for solving LRPs: exact algorithms and heuristics. Both will be covered in more detail below.

Exact algorithms In analogy to the classification scheme suggested by Laporte [1992], Min et al. [1998] classify the exact algorithms for solving LRPs into four categories:

1. direct tree search / branch-and-bound;
2. dynamic programming;
3. integer programming;
4. non-linear programing.

A very commonly cited example of an exact algorithm for solving LRPs is the work by Laporte et al. [1986]. They develop a formulation and an exact algorithm based on integer programming that can be applied to generic capacitated location-routing problems.

Heuristics For large scale LRPs, the use of exact algorithms is often prohibited by the fact that both the facility-allocation problem and the VRP are so-called NP-hard problems. Thus, the task of solving the LRP being a combination of the two with an exact algorithm is often too complex and time consuming to be a feasible option. To overcome these difficulties, there are numerous heuristics decomposing the entire LRP into multiple phases to reduce complexity while not giving up too much on the accuracy of the obtained solutions.

Min et al. [1998] classify the relevant heuristics into four groups, according to their mode of decomposition:

1. location-allocation-first, route-second;
2. route-first, location-allocation-second;
3. savings / insertion;
4. tour improvement / exchange.

Most of the savings / insertion heuristics reviewed by Min et al. [1998] are based on the famous works of Clarke and Wright [1964] and Rosenkrantz et al. [1977]. These heuristics are very efficient in forming reasonably good clusters of demand nodes that are allocated to a specific route and facility. Further they allow for incorporating practical requirements such as hard time windows into the LRP. However, for an increasing number of demand nodes, they as well encounter severe computational limitations [Laporte, 1992].

2.3 Research Gaps in The LRP Literature

Based on their exhaustive reviews of existing LRP literature, Min et al. [1998] and Nagy and Salhi [2007] identify a number of research gaps that require more detailed attention. Part of these shall be addressed in the course of this paper.

Use of route length approximation formulae Nagy and Salhi [2007] explicitly state that, as routing plays a subservient role in LRP, approximation formulae for tour lengths and thus for the tour cost can often be used instead of explicit vehicle routing algorithms within the solution process of the LRP to speed up the process. They suggest that these formulae should be extended in order to allow for e.g. heterogeneous fleets and a joint consideration of pickup and delivery.

Uncertainty Most of the available LRP literature focuses on the development of purely deterministic models. For real-world applications, however, various sources of uncertainty, e.g. related to demand, cost, travel times and other crucial elements of the decision problem, have to be considered. It would thus be of high practical and academic relevance to consider stochasticity to a greater extent with respect to LRP models. Nagy and Salhi [2007] suggest to employ *robustness analysis* to cater for stochastic (dynamic) LRPs.

Time windows and multiple optimization objectives Only very few publications on LRPs account for specific time window requirements with regards to the provision of logistics services. Closely related to this is the observation that most existing LRP models focus on cost as their sole optimization objective. In practice, however, cost minimization is only part of the objective of POs and LSPs. In fact, they also have to ensure that they will meet certain customer requirements, such as on-time delivery, while providing their service at the least possible cost. Thus, the incorporation of multiple optimization objectives and time windows in particular would be a fruitful extension to existing literature.

Multiple periods Almost all LRP related articles consider static models. It would however offer interesting insights to develop dynamic LRP models that account for multiple time steps and changing model parameters over time.

Vertical supply chain integration While topics like closed-loop supply chains and reverse logistics are constantly gaining in relevance and public attention, the majority of the existing LRP literature still considers inbound pickup operations and outbound delivery operations as two separate optimization problems. Thus, multi-stage LRP models that consider inbound and outbound item flows simultaneously would offer a great research potential.

Application to real-world decision problems Min et al. [1998] highlight the importance of actually applying LRP models and solution algorithms to real-world decision problems in order to broaden the spectrum of considered location-routing options but also to provide evidence of their efficacy and practicality. While LRPs are generally a very well-researched class of decision problems, the academic contributions to this stream of research are too rarely applied and adapted to actual real-world problem settings. Closing the gap between theory and practice offers many appealing research opportunities.

3 Tour Length And Cost Approximations

As pointed out by Nagy and Salhi [2007], the use of route length estimation (RLE) formulae in the context of integrated location-routing problems may be the key to speed up the solution process for LRPs significantly, which in turn is crucial for their applicability to large-scale real-life problem settings. The general idea behind the use of RLE formulae in LRPs is that once we have a closed form expression that approximates the expected length of a vehicle tour originating from a given facility to service demand in a given influence area with given demand characteristics and geographical properties sufficiently well, we can find an optimal solution to the LRP without having to solve the vehicle routing part of the problem explicitly by complex algorithms or heuristics.

Many RLE formulae make use of regression analysis to establish a closed form expression that links demand characteristics and geographical properties of demand areas to the expected route length and thus the total routing cost. Among the first to develop a regression based closed form approximation formula for route lengths were Christofides and Eilon [1969] as well as Eilon et al. [1971]. Their works were later extended by Daganzo [1984] and Stokx and Tilanus [1991]. Following Nagy and Salhi [1996], the approximation formulae developed by the above mentioned authors take the following general form:

$$T = aD/c + bD/\sqrt{n} \tag{1}$$

where T denotes the total routing cost per depot, D denotes the sum of the direct distances from the depot to all its associated demand nodes, n is the number of demand nodes served by the depot, c is the average number of demand nodes (stops) served on a single feasible vehicle tour, and a and b are regression coefficients.

As pointed out by Nagy and Salhi [1996], a major drawback of such a formulation is that one is required to know the average number of demand nodes (stops), c, served on a feasible vehicle tour beforehand in order to apply it. However, c turns out to be heavily dependent on the location of the depot and the definition of its influence area (i.e. the area in which demand nodes are served from this depot). Thus, a usable value for c is generally not available which leads Nagy and Salhi [1996] to developing a formula that does not rely on this measure. They make use of data and constraints that are clearly defined in advance, such as maximum vehicle capacity constraints, maximum travel distance constraints, known demand levels and known drop times, to avoid the explicit use of c in their formulation.

A general drawback of empirical formulas derived on the basis of regression analysis is that they only describe historically observed relationships between the determinants of the routing problem and the observable route lengths and routing cost. As such, they are unable to explain causal relations between changes in individual determinants of the routing problem and changes in the resulting route lengths and routing cost. Moreover, to obtain a valid route length and routing cost approximation formula for a specific problem setting, one requires historical data for the regression analysis that is compatible to this specific problem setting. Thus, if one wants to extend the simple routing problem captured by Equation (1) by introducing additional determinants of and constraints to the routing problem, it becomes a challenging task to find appropriate data that can be used to determine the exact form and regression coefficients of a suitable tour length and routing cost approximation formula.

In order to obtain a closed form approximation of routing cost that is easily applicable and adaptable to arbitrary configurations of the routing problem, it is thus worthwhile to set up an analytical formulation that aims at explaining routing cost explicitly on the basis of their (supposed) causal relationship to the determinants of the routing problem, which are given as fixed model parameters. Bruns et al. [2000] provide an example of an LRP model used to optimize the transportation network design for Swiss parcel delivery services that makes use of such an analytical routing cost approximation formula. They identify a number of key cost components that are relevant for this specific transportation problem and use them as a basis to derive an analytical approximation formula for the total cost of transportation incurred at the level of individual network facilities depending on the chosen network design.

Smilowitz and Daganzo [2005] present an optimization model solving the network design problem for integrated package distribution systems. Following Daganzo [1999], their model is based on a very general analytical approximation formula for the average unit cost of transportation from a facility to a specific area with positive demand:

$$f(r, v, n, \delta) \approx c_d^{\prime u} + \frac{r c_d^u + c_q^u}{nv} + \left(\frac{n-1}{n}\right)\left(\frac{c_d^u k (\delta)^{-\frac{1}{2}} + c_q^u}{v}\right) \tag{2}$$

where v describes the batch size of items delivered, n denotes the number of demand nodes (stops) served along a single VRP route within the considered area, δ denotes the density of demand nodes (stops) within this area, r denotes the average distance between the demand nodes of this area and the facility, $c_d^{\prime u}$ denotes the handling cost per item for a vehicle of type u, c_d^u denotes the cost of overcoming distance for a vehicle of type u, and c_q^u denotes the cost of stopping a vehicle of type u at a demand node or facility. The value of the constant k depends on the distance metric used for the model and can be estimated through simulation.[2] For a Manhattan grid metric, assuming that demand is random and follows a Poisson process, following Smilowitz and Daganzo [2005], one can assume a value of $k \approx 0.8$.

This formulation of an approximation of transportation cost will also form the basis of the cost formulation underlying the LRP model presented in this work. We compared the results of the approximation by Daganzo [1999] to the ones obtained from applying the Clarke-Wright Savings Algorithm for a couple of exemplary routing problems of limited size and concluded that the closed form expression by Daganzo [1999] approximates the per unit cost of transportation sufficiently well.[3]

[2]See, e.g., Daganzo [1999]
[3]See Appendix A

4 Location-Routing at La Poste

4.1 Problem Setting

Groupe La Poste, the national PO of France, is currently considering to move into the field of urban logistics services to broaden their customer base and product portfolio and compensate some of the decline in revenue and profits from their traditional business fields such as mail. *La Poste* is likely to focus its primary urban logistics activity on parcel transportation, where they have the most experience. A potential strategic move for *La Poste* could thus be to position themselves as a central consolidator for urban inbound and outbound parcel flows in the largest French cities. Based on existing facilities but also new infrastructure investments, they might consider creating a dedicated transportation network infrastructure for such services.

Most probably, such an infrastructure would comprise two levels of consolidation. The first level of consolidation consists of a single large consolidation center per city, the so-called “city hub”. All parcel flows, regardless of whether they are inbound or outbound, large or small parcels, will be processed through this facility. For parcels below a certain threshold level with regards to their physical size there is a second level of consolidation below the city hub consisting of multiple so-called “intermediate depots”. At each of the intermediate depots, only the small parcel flows related to the surrounding influence area of this particular intermediate depot will be processed. The so-called “last mile” of transportation of both inbound and outbound small parcel flows related to this area will originate from the depot, where these flows are, in turn, consolidated and shipped to the city hub for further processing in larger, aggregated shipments.

To accomplish the transportation of parcels between demand nodes and the two different consolidation facility levels, *La Poste* could rely on a total of four different vehicle types in the following example. First, pedestrians with trolleys are characterized by relatively low speed and small carrying capacity. However, this mode of transportation exposes very little hourly cost of operation and their service time per stop is relatively low. Fixed cost incurred from a pedestrian equipment are also very low. Further, this mode of transportation is very agile, it can use even small roads, alleyways or sidewalks and is not restricted by one-way roads, traffic lights, etc. that would apply to motorized modes of transportation. Thus, the average distance pedestrians have to travel between two locations is relatively low compared to other modes of transportation. Second, bikes are considerably faster than pedestrians but differ only marginally in their carrying capacity. They are still relatively agile and flexible in the way they can move within the city and also expose low hourly cost of operation and equipment fixed cost. Their required service time per stop is a bit longer than for pedestrians, as a bike has to decelerate and accelerate, the driver has to get off and onto the bike, walk to the door, etc. Third, vans are significantly faster than bikes or pedestrians and expose a much larger carrying capacity. However, their fixed cost and hourly cost of operation are significantly higher, too. Moreover, they require much more space for parking, loading and sorting at the facility they are positioned at, which again incurs cost for the rent of this space. Also, vans are much more bound to traffic rules and are only able and allowed to drive on roads of a specific size and quality. Thus, the average distance travelled by a van between two locations is considerably longer than for the non-motorized modes of transportation. Finally, trucks are by far the largest and fastest vehicle type considered in this context, however they are also the most expansive ones in terms of fixed cost, hourly cost of operation and space requirements. Further, due to their large size, trucks have to make the most detours when travelling between two locations.

The use of trucks is supposed to be restricted to the consolidated shipment of small parcels between the city hub and the intermediate depots. Due to the large distances that have to be travelled and the considerable size of large parcels, the direct shipment of parcels whose size exceeds the predefined threshold between the city hub and the actual demand nodes should only be accomplished by vans. For the transportation of small parcels between the intermediate depots and the demand nodes, all vehicle types but trucks should be available.

By applying an LRP model logic to this problem setting we aim at determining the cost optimal configuration of the transportation infrastructure and fleet described above. More precisely, we aim at answering the question how many and which potential locations should accommodate an intermediate depot, where the city hub should be located, how the influence areas of the individual intermediate depots should be defined and how many vehicles of which type should be positioned at which facility.

4.2 Classification of The Intended LRP Model

With regards to the LRP classification scheme presented in Section 2.2, our model is supposed to have the following properties:

- classical hierarchical LRP structure;
- deterministic input data;
- static model;
- single objective of total cost minimization;
- discrete solution space;
- up to 400 facility locations;
- no restrictions on the number of vehicles, up to four different vehicle types, heterogeneous fleets;
- capacitated facilities;
- capacitated vehicles;
- node routing with multiple tours per vehicle allowed and joint consideration of pickup and delivery nodes along a single tour;
- hard time windows with strict deadlines;
- real-world model data;
- heuristic used to solve LRP.

4.3 Model Input

4.3.1 Complex Geographies And Demand Distributions

Arbitrary city geographies and spatial distributions of absolute demand In large scale real-world ULS applications, simulation and optimization models need to consider complex geographical constraints to the design of an optimal transportation network infrastructure as well as complex, non-trivial demand distributions across the urban area that serves as the basis of simulation and optimization. Such geographical constraints and demand characteristics can hardly be described in continuous closed-form expressions. Furthermore, there is no closed-form expression to calculate transportation cost in a single step for non-trivial geographies across which demand is heterogeneously distributed.

To overcome these difficulties without excessive loss in precision and granularity of the obtained simulation and optimization results we apply a segmentation approach that can be used to depict arbitrary city geographies and demand structures at almost arbitrary levels of precision and granularity. We divide the considered city area into a large number of rectangular city segments which are later on considered individually to calculate the cost of serving demand nodes within them. Each city segment is individually described by a set of parameters defining its geographic shape and demand characteristics. First, its geographic attributes are described by its specific two dimensional coordinates within the overall model geography as well as its extent along both geographical axes. Second, the specific demand characteristics of a segment are described by the absolute density of demand nodes (stops) to be served in this segment, by the average number of pickups and deliveries per stop in this segment as well as by discrete probability distribution functions for the physical size of the items to be delivered and picked up in this segment. Within a given segment, demand is assumed to be uniformly distributed which allows for the application of fast and reliable closed-form approximations for the cost of transportation incurred from serving the segment such as the approximation formula used by Smilowitz and Daganzo [2005].

Distinction of pickup and delivery demand One of the key features of our model is that it considers pickup and delivery item flows in an integrated solution approach. The impact of serving pickup and delivery demand jointly on a single vehicle tour mainly affects the effective carrying capacity of a vehicle. As long as a vehicle serves only pickup demand nodes or only delivery demand nodes along a single tour, the calculation of its effective carrying capacity in terms of the number of nodes that can be served on a single tour is straightforward:

$$\xi^n = \frac{\xi^v}{\rho E\left[V\right]} \tag{3}$$

where ξ^n denotes the number of demand nodes that can be served, ξ^v denotes the carrying capacity of the vehicle in terms of physical volume that can be filled with items, ρ denotes the average number of pickup or delivery items per demand node, and $E\left[V\right]$ denotes the average physical volume required by an item delivered or picked up within this city segment. $E\left[V\right]$ can be determined based on the probability distribution functions for the physical size of the items to be delivered and picked up in this segment.

Once we introduce the possibility of delivering and picking up items along one and the same tour, this calculation becomes more complex. One has to distinguish whether vehicles optimize their routing only with respect to the tour length, which we will call "blind routing" in the following, or whether vehicles can actually take into account the trade-off between tour length and vehicle space utilization, which we will call "smart routing" in the following.

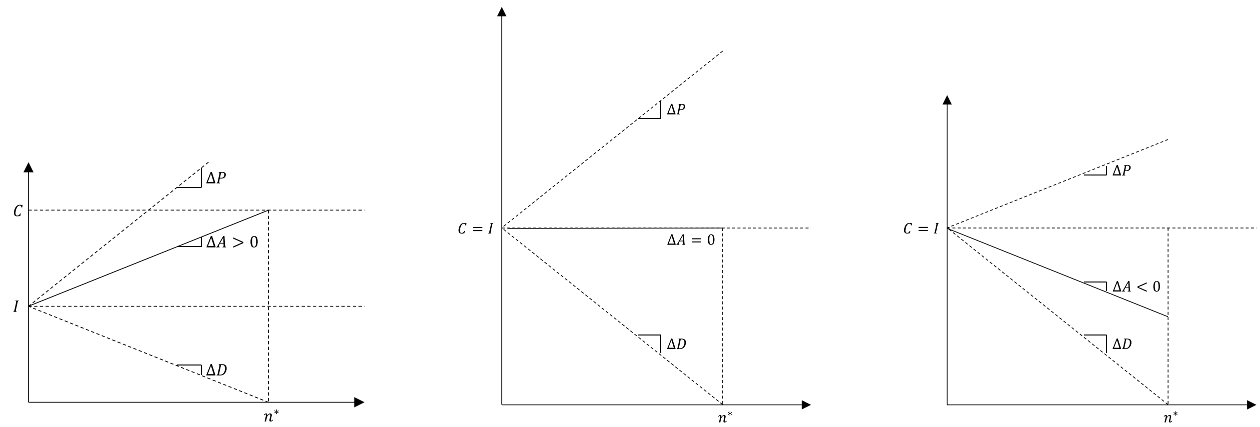
Blind routing In the case of blind routing, vehicle tours are only optimized with respect to the total distance travelled. Vehicles are blind with respect to what type of demand actually awaits them at the individual demand nodes. The vehicle tours are optimized assuming averaged demand characteristics at each demand node. To illustrate this, consider an exemplary city segment for which the average number of pickup items per demand node is 0.6 and the average number of delivery items per demand node is 0.5. Given this, blind routing assumes that at each demand node, exactly 0.6 items will indeed be picked up and 0.5 items will be delivered. Thus, it is optimal for each vehicle to choose the shortest route because demand nodes are identical so that there is no reason to prefer a node that would lead to a detour over the one that would be next on a distance optimized route. It ignores the fact that, in a realistic environment, it would rather be the case that there will be one item for pickup at 60% of all demand nodes and one item for delivery at 50% of all demand nodes and vehicles would know ahead of time which node bears what kind of demand. In this case, the number of demand nodes that a vehicle can serve on average depends on the incremental volume contributions per demand node for pickup and delivery items. Given the blind routing assumption, if, on average, at each demand node less space is freed up by delivery items than is additionally required by pickup items ($\Delta D < \Delta P$), it is optimal to choose the initial vehicle load such that at the same time when all initially loaded delivery items are gone, the vehicle is completely filled by pickup items. Otherwise ($\Delta D \geq \Delta P$), it is optimal to start with a fully loaded vehicle. This is illustrated in Figure 2. Thus, following the logic described in Figure 2, the number of demand nodes that can be effectively served by a single vehicle on a single tour can be determined as follows:

$$\xi^n = \frac{\xi^v}{\max\left[\rho^D E\left[V^D\right], \rho^P E\left[V^P\right]\right]} \tag{4}$$

where ξ^v denotes the carrying capacity of the vehicle in terms of physical volume, ρ^P denotes the average number of pickup items per demand node, ρ^D denotes the average number of delivery items per demand node, $E\left[V^P\right]$ denotes the expected physical size of a pickup item, $E\left[V^D\right]$ denotes the expected physical size of a delivery item and ξ^n denotes the corresponding optimal number of demand nodes serviced per tour.

It would not be optimal to continue visiting more demand nodes and only servicing pickup demand if there is still some space left in the vehicle after all delivery items are gone because these demand nodes would have to be serviced a second time to care for their delivery demand.

Blind routing, of course, is an unrealistic assumption. However, we will use this approach within our model for simplification and leave the case of smart routing subject to future extensions to this model.



a) $\Delta P > \Delta D$ b) $\Delta P = \Delta D$ c) $\Delta P < \Delta D$

Figure 2: Illustration of the blind routing logic

Smart routing Smart routing refers to the more realistic case that demand characteristics at the individual demand nodes are not homogeneous and that there is no fractional demand, i.e. that demand can only occur in terms of full items. Further, smart routing implies that it is known ahead of time, i.e. at the start of the considered time period, which demand node bears which kind of demand (pickup or delivery) and what quantity of demand. One could even assume that the exact physical size of the items to be picked up or delivered is known ahead of time. If all this information is available before the vehicles actually start their tours, one could imagine intelligent computerized routing algorithms that would create vehicle tours that may be no longer optimal in terms of the total distance travelled but may however lead to lower total cost of transportation because the utilization of the vehicle carrying capacity is improved so that more demand nodes can be served per tour. For example, one could think of a vehicle stopping only at demand nodes with pickup demand after having delivered all its delivery items.

Incorporating smart routing into this LRP model would have meant a tremendous amount of additional complexity and is currently out of the scope of this work. However, smart routing constitutes an interesting field for future research and extensions to this model.

4.3.2 Multiple Parallel Network Architectures

As mentioned above, our LRP model accommodates for two parallel network architectures for different item classes, i.e. a transportation network with only one level of consolidation that allows for a direct shipment of large items between the central city hub and the demand nodes and a transportation network with two levels of consolidation for small items that are shipped between the demand nodes and the city hub through intermediate depots. The only criterion that is relevant for the decision whether an item is shipped through the first or the latter network is its physical size. A threshold value α^{ν} is defined. Items smaller or equal to α^{ν} will be shipped through the indirect network, larger items will be shipped directly from the city hub. The parameter α^{ν} is later on subject to sensitivity analyses that aim at identifying a cost-optimal threshold between the two parallel network infrastructures.[4]

4.3.3 Heterogeneous Vehicle Portfolio

The LRP model presented here allows for multiple vehicle types to choose from and for mixed fleets. The vehicle types differ in both their physical and economic properties. The following vehicle characteristics can be defined:

- Average speed within pickup and delivery tours;
- Average speed outside of tours, i.e. on line-haul distances between tours and facilities;

[4]See Section 4.6.3

- Space requirements for parking, sorting and loading;
- Carrying capacity in terms of physical volume;
- Time required for tour setup;
- Time required for serving a demand node (stop);
- Time required to load / unload a small item;
- Time required to load / unload a large item;
- Detour factor, determining to what extent the actual distance a vehicle has to drive between two locations differs from their straight line distance, e.g. due to turns that have to be made. This is directly related to the vehicle size and its ability to use only large streets or also small roads and alleyways;
- Hourly wage of the driver;
- Hourly cost of operation;
- Daily fixed cost.

4.3.4 Accounting For Existing Infrastructure

Our LRP model allows for greenfield / brownfield optimizations as well as for optimization scenarios where an existing facility infrastructure, or part of it, is taken as given. It allows for certain facility locations to be forced to accommodate an active facility. Moreover, it allows for certain facility locations only to be allowed to accommodate a certain kind of facility. These infrastructure constraints are considered in more detail in Section 4.5.

4.4 Cost Formulation

Given the definitions in Appendix B, the total average daily cost of transportation for the described urban logistics network sums up to (see Equation 124 on page 39):

$$K^T = K^h + K^f + K^{a,S} + K^{b,S} + K^{a,L} + K^{S,a,\zeta} + K^{S,b,\zeta} + K^{L,\zeta}$$

where K^h denotes handling cost (see Equation 54 on page 32), K^f denotes facility fixed cost (see Equation 56 on page 32), $K^{a,S}$ denotes the cost of small parcel transportation between demand nodes and intermediate depots (see Equation 57 on page 32), $K^{b,S}$ denotes the small parcel transportation cost between the intermediate depots and the city hub (see Equation 77 on page 34), $K^{a,L}$ denotes large parcel transportation cost (see Equation 98 on page 36), $K^{S,a,\zeta}$ denotes the cost of space required at the intermediate depots for sorting, vehicle parking, and loading (see Equation 118 on page 38), $K^{S,b,\zeta}$ denotes the cost of space required at the city hub for sorting, vehicle parking, and loading related to small parcel transportation (see Equation 120 on page 38), and $K^{L,\zeta}$ denotes the cost of space required at the city hub for sorting, vehicle parking, and loading related to large parcel transportation (see Equation 122 on page 39).

4.5 The Optimization Problem

4.5.1 Definition of The Single-Stage Full Flexibility Optimization Problem

The optimization problem is to minimize the total daily cost of transportation:

$$min_{S^S_{d,i,s}, S^L_{c,i,l}, a^D_d, a^C_c, a^{C,H}_{c,b}} \; K^T \tag{5}$$

subject to

$$\sum_d \sum_s S^S_{d,i,s} = 1 \qquad \forall i \tag{6}$$

$$\sum_c \sum_l S^L_{c,i,l} = 1 \qquad \forall i \tag{7}$$

$$S^S_{d,i,s} \leq a^D_d \qquad \forall d, i, s \tag{8}$$

$$S^L_{c,i,l} \leq a^C_c \qquad \forall c, i, l \tag{9}$$

$$\sum_b a^{C,H}_{c,b} = a^C_c \qquad \forall c \tag{10}$$

$$a^D_d \geq z^{a,D}_d \qquad \forall d \tag{11}$$

$$a^C_c \geq z^{a,C}_c \qquad \forall c \tag{12}$$

$$\sum_c a^C_c = 1 \tag{13}$$

$$\sum_d a^D_d \geq b^{D,l} \tag{14}$$

$$\sum_d a^D_d \leq b^{D,u} \tag{15}$$

$$\sum_i \left[\sum_s \left[S^S_{d,i,s} q^S_{d,i,s} \zeta^v_s \right] \right] \leq \zeta^D_d \qquad \forall d \tag{16}$$

$$\sum_i \left[\sum_l \left[S^L_{c,i,l} q^L_{c,i,l} \zeta^v_l \right] \right] + \sum_b \left[a^{C,H}_{c,b} \cdot \max \left[q^{b,S,D}_{c,b}, q^{b,S,P}_{c,b} \right] \cdot \zeta^v_b \right] \leq \zeta^C_c \qquad \forall c \tag{17}$$

Equation (6) ensures that each city segment is served from exactly one intermediate depot using exactly one type of vehicle with regards to small item transportation. Equation (7) ensures that each city segment is served from exactly one city hub location using exactly one type of vehicle with regards to large item transportation. Equation (8) ensures that city segments are only served from active intermediate depot locations with regards to small item transportation. Equation (9) ensures that city segments are only served from the active city hub location with regards to large item transportation. Equation (10) ensures that the transportation of small items between the city hub and the intermediate depots is only performed from the active city hub using only a single vehicle type. Equations (11) and (12) allow for forcing certain prespecified facility locations for intermediate depots and the city hub to be active if required. Equation (13) ensures that only a single city hub location is activated. Equations (14) and (15) define a lower and an upper bound to the number of active intermediate depots that may be located. Equation (16) ensures that the maximum available physical space for sorting, vehicle loading and parking is not exceeded at any intermediate depot location. Equation (17) ensures that the maximum available physical space for sorting, vehicle loading and parking is not exceeded at the city hub location.

An overview of the notation used here can be found in Appendices B and C.

4.5.2 A Two-Stage Optimization Approach to Overcome Computational Infeasibility

Computational complexity The model we presented so far is theoretically able to optimize the transportation network over various dimensions. More precisely, the solver algorithm applied here will be looking for answers to the following questions when determining the optimal network configuration:

- How many intermediate depots should be installed?
- Where should these intermediate depots be installed?

- Which city segment should be served from which intermediate depot using which type of vehicle?
- Where should the city hub be located?

At the same time, any potential solution to the optimization problem has to respect the following constraints:

- Is there sufficient parking space at the intermediate depot chosen to serve a city segment for the vehicle type chosen to serve this segment?
- Is every city segment served from exactly one intermediate depot using exactly one vehicle type?

Together, the above stated decisions and constraints lead to highly complex trade-offs in finding the optimal solution to the problem:

- If there is not enough parking space available at a given intermediate depot to serve a given city segment with a given vehicle type from this intermediate depot, should this segment still be served from this depot using a different vehicle type or should it be served from a different intermediate depot?
- If we decide to serve a city segment from another intermediate depot than would have been our first choice, which vehicle type should be used? How does this re-allocation affect the optimality of the initial allocation of city segments to the intermediate depot which shall now serve the additional city segment?
- If we re-allocate certain city segments to other intermediate depots due to intermediate depot capacity constraints, how does this affect the optimality of the initially chosen set of active intermediate depots? Should they be located differently due to the changes?

This list of potential trade-offs is by far not exhaustive, but the examples given here already illustrate that the complexity of the solution process with regards to finding the optimal transportation network design is strongly driven by the combination of capacity constraints in terms of parking space at the intermediate depot level and the flexibility to choose among various types of vehicles with heterogeneous cost characteristics and heterogeneous capacity requirements at the intermediate depot level. Thus, for the model dimensions required to capture real-life decision problems faced by a PO in terms of the number of available facility locations, city segments and available vehicle types, solving the optimization model presented so far requires a tremendous amount of physical resources and time. In practice, this would make scenario specific optimizations and large scale sensitivity analyses based on this model impossible. However, given the considerations above, relaxing the capacity constraints at the intermediate depot level and eliminating the flexibility to actively choose among various types of vehicles to serve a given city segment can drastically simplify the problem and speed up the solution process.

The general simplification approach In the following, we take advantage of this potential reduction in problem complexity and move from our initial model setup which is based on a single-stage optimization under full flexibility and respecting all model constraints simultaneously towards a two-stage iterative model setup. This new model setup is iterative in the sense that it is executed multiple times, each time with a different given and fixed number of active intermediate depots to be located, in order to determine the optimal number of intermediate depots to activate. It is a two-stage model in the sense that first, an approximate interim solution to the integrated LRP is determined, which is then used as a starting point for a more precise routing decision which is however separated from the facility location decision at this stage.

In the first stage, the model assumes infinite capacities at the intermediate depot level. Given this assumption, each and every city segment can actually be served at optimal cost, i.e. by the combination of intermediate depot and vehicle type that results in the least cost to serve this city segment. Thus, the actual optimization process is no longer required to distinguish between different options to serve a city segment in terms of the vehicle type used because the optimal vehicle type for serving a given segment from a given intermediate depot can already be determined ex ante. The actual optimization problem reduces to finding the optimal locations for the given number of active intermediate depots. The allocation of city segments to intermediate depots immediately and unambiguously follows from the current choice of active intermediate depot locations as each city segment is by definition assigned to the intermediate depot among the set of active intermediate depots that can serve the city segment at minimum cost. The information on the cost to serve a given city segment from a given intermediate depot location is already known before the optimization starts.

The locations of the active intermediate depots that result from the first optimization stage are then taken as a fixed input to the second stage of the optimization process. Here the routing part of the optimization problem is addressed for a given set of active intermediate depot locations. Unlike the first stage of the optimization process, all model constraints are imposed on the optimization, i.e. capacities at the intermediate depot level are no longer infinite but strictly limited. Furthermore, there is full flexibility to choose among various vehicle types to serve a city segment from a given intermediate depot location. The optimal solution to the second stage of the optimization process may deviate from the cost optimal solution to the uncapacitated city segment allocation performed in the first stage of the process. In order to satisfy the newly imposed capacity restrictions on the intermediate depot level it may become necessary to serve certain city segments from different active intermediate depot locations that have more capacity left and / or to change the vehicle type used to serve the city segment even though these re-allocations may lead to higher cost.

If the number of active intermediate depots chosen is too low, such that the entire demand cannot be served while at the same time respecting all capacity constraints on the intermediate depot level, the second stage of the optimization process will be infeasible and the chosen solver algorithm will interrupt without a result. In this case, the number of active intermediate depots has to be increased until the problem becomes feasible also in the second stage of the optimization process.

Numerous comparative simulations with smaller scale models have shown that the final network design obtained from this two-stage optimization approach are sufficiently close and often even identical to the results obtained from the full scale single-stage model.

Changes to the definition of the cost model In order to allow for the above described two-stage optimization procedure to produce valid results, we have to modify our cost model and assume homogeneous facility characteristics for the two-stage optimization problem. More precisely, fixed cost are assumed to be identical across all intermediate depot locations. Furthermore, all intermediate depot locations are assumed to offer the same maximum available physical space for sorting, vehicle loading and parking.

$$f_d^D = \bar{f}^D \qquad \forall d \tag{18}$$

$$\zeta_d^D = \bar{\zeta}^D \qquad \forall d \tag{19}$$

In the same way, city hub locations are assumed to have homogeneous characteristics.

$$f_c^C = \bar{f}^C \qquad \forall c \tag{20}$$

$$\zeta_c^C = \bar{\zeta}^C \qquad \forall c \tag{21}$$

Reduced Optimization Problem For Solving The Facility Location Sub-Problem In the first stage of our two-stage optimization procedure, the optimization problem is still to minimize the total daily cost of transportation. However, in this case, facility capacity constraints are not considered and the number of active intermediate depots to be located is given as a fixed exogenous input parameter denoted by $\overline{N}^d$. There is no choice with regards to the vehicle types employed. Given this and given the above described assumption of homogeneous facility capacities and fixed cost, the minimum total daily cost of transportation can be defined as (see Equation 128 on page 40):

$$\tilde{K}^T = \sum_d \sum_i \tilde{S}_{d,i}^S \tilde{k}_{d,i}^{S,a} + \sum_c a_c^C \left(\tilde{k}_c^{S,b} + \tilde{k}_c^{L,a} \right)$$

Thus, the reduced optimization problem can be formulated as follows:

$$min_{\tilde{S}_{d,i}^S, a_d^D, a_c^C} \; \tilde{K}^T \tag{22}$$

subject to

$$\sum_d \tilde{S}^S_{d,i} = 1 \qquad \forall i \tag{23}$$

$$\tilde{S}^S_{d,i} \leq a^D_d \qquad \forall d, i \tag{24}$$

$$a^D_d \geq z^{a,D}_d \qquad \forall d \tag{25}$$

$$a^C_c \geq z^{a,C}_c \qquad \forall c \tag{26}$$

$$\sum_c a^C_c = 1 \tag{27}$$

$$\sum_d a^D_d = \overline{N}^d \tag{28}$$

Reduced Optimization Problem For Solving The Vehicle Routing Sub-Problem The optimal values to the decision variables a^D_d and a^C_c from the first reduced optimization model are now taken as fixed exogenous input parameters. They will be denoted as $\overline{a}^D_d$ and $\overline{a}^C_c$ in the following.

The optimization problem is again to minimize the total daily cost of transportation. This time, however, facility capacities have to be respected and there is full flexibility with regards to the choice of vehicle type to serve a given segment from a given location.

$$min_{S^S_{d,i,s}, S^L_{c,i,l}, a^{C,H}_{c,b}} \; K^T \tag{29}$$

subject to

$$\sum_d \sum_s S^S_{d,i,s} = 1 \qquad \forall i \tag{30}$$

$$\sum_c \sum_l S^L_{c,i,l} = 1 \qquad \forall i \tag{31}$$

$$S^S_{d,i,s} \leq \overline{a}^D_d \qquad \forall d, i, s \tag{32}$$

$$S^L_{c,i,l} \leq \overline{a}^C_c \qquad \forall c, i, l \tag{33}$$

$$\sum_b a^{C,H}_{c,b} = \overline{a}^C_c \qquad \forall c \tag{34}$$

$$\sum_i \left[\sum_s \left[S^S_{d,i,s} q^S_{d,i,s} \zeta^v_s \right] \right] \leq \zeta^D_d \qquad \forall d \tag{35}$$

$$\sum_i \left[\sum_l \left[S^L_{c,i,l} q^L_{c,i,l} \zeta^v_l \right] \right] + \sum_b \left[a^{C,H}_{c,b} \cdot \max \left[q^{b,S,D}_{c,b}, q^{b,S,P}_{c,b} \right] \cdot \zeta^v_b \right] \leq \zeta^C_c \qquad \forall c \tag{36}$$

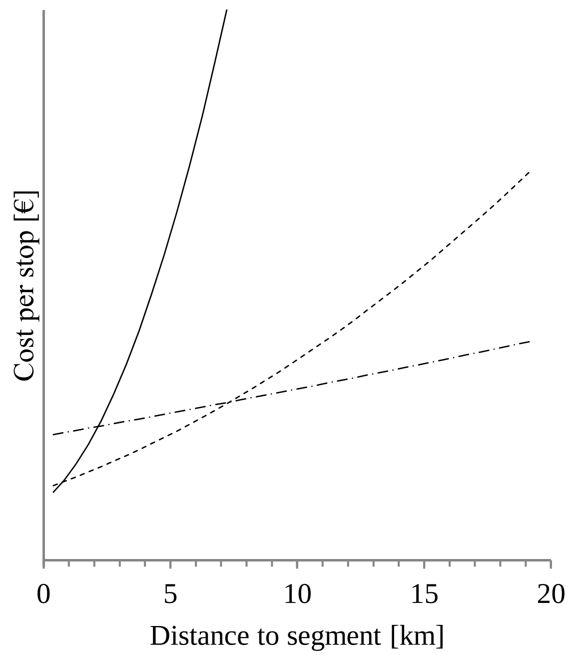


Figure 3: Cost per stop as a function of vehicle type and line haul distance for a given stop density and average item size

4.6 Top Level Analyses

4.6.1 Vehicle Competitiveness

Based on the vehicle and cost parameters provided by *La Poste*, we first want to analyze the general competitive position of the different vehicle types relative to each other. We exclude trucks from this analysis as these are set as the vehicle type providing the second-tier transportation between the city hub and the intermediate depots only. To compare the competitive position of the three remaining vehicle types, we analyze the cost of serving a single city segment with a size of 500×500 meters with small parcel transportation depending on the stop density within the segment and the line-haul distance between the intermediate depot and the segment.

As expected, Figure 3 shows that for each vehicle type the cost to provide the city segment with small parcel delivery and pickup increases in the line-haul distance between the intermediate depot and the city segment and decreases in the stop density within the city segment. Further, cost slightly increase in the average parcel size.

Interestingly, our analysis shows that pedestrians are almost strictly inferior to the other vehicle types. Even for very short line-haul distances, bikes offer lower cost than pedestrians. Only for very high stop densities, pedestrians turn out to be the cost optimal mode of transportation within a very short distance from the intermediate depot. This competitive advantage of pedestrians increases when the average parcel size decreases. Moreover, vans only become the cost optimal mode of transportation for very long line-haul distances between the intermediate depot and the city segment. These findings suggest that the vast majority of small parcel pickup and delivery demand will be served by bikes.

4.6.2 Optimal Number And Location of Facilities

Subsequently, we widen the scope of our analysis from the individual city segment level to the entire city level. Based on a stylized city layout with a quadratic city area of 20×20 kilometers that is equally split into 1600 quadratic city segments with a size of 500×500 meters each, we analyze the total cost of transportation of both small and large parcels, the optimal positioning of consolidation facilities, and the total number of required vehicles as a function of the number of intermediate depots to be located and the demand characteristics of the city.

First, we consider the case of a uniform stop density across the entire city area with mixed fleets being allowed (see Figure 4). We plot the total cost of transportation and the total number of vehicles used (i.e. the total number of employees required for transportation) over the number of intermediate depots to be located for a uniform stop density of 20 and 50 stops/km^2, respectively. We find that the cost optimal number of facilities increases less than proportionally in the stop density. As expected, the total number of vehicles employed decreases in the number of intermediate depots to be positioned as the line-haul distances for those vehicles become shorter the more intermediate depots there are.

Second, we look at a non-uniform demand distribution where the stop density is gradually increasing towards the city center. While the inner 10% of the city area expose a stop density of 200 stops/km^2, the outer 40% of the city area only have a stop density of 20 stops/km^2. The remaining city area in between has a stop density of 50 stops/km^2. The rather erratic development of the total number of vehicles employed in this case can be explained by significant shifts in the fleet composition with changing numbers of intermediate depots to be located. While in the case of uniform stop densities throughout the city a single vehicle type is used almost exclusively (bikes), the split between bikes and pedestrians can change significantly with an increasing number of intermediate depots for the case of non-uniform stop densities.

Third, we consider the case were no mixed fleets are allowed but all small parcel demand has to be served using only a single vehicle type. We restrict this analysis to the case of a uniform stop density of 20 stops/km^2 throughout the entire city area. The optimal number of facilities to be located for only bikes being allowed is only about half the number of facilities required for only pedestrians being allowed. This number decreases even further if we only allow for vans (see Figure 5). Overall, however, despite the higher number of facilities required, the lowest total cost of transportation materialize for the bike-only case in this analysis.

The graphs in Figures 4 and 5 already point towards the central cost trade-off in designing an optimal transportation network for *La Poste*'s parcel transportation and consolidation service: At first, total cost of transportation decrease significantly in the number of intermediate depots to be located as the number of vehicles required for transportation decreases significantly since line-haul distances quickly shorten the more facilities are in place. However, with the number of intermediate depots increasing further, the second-tier transportation of small items between the city hub and the increasing number of intermediate depots becomes increasingly complex, fragmented and costly. Thus, at some point, total cost of transportation start increasing again.

At the same time, it has to be noted that the total cost curves in all of the analyses presented above are extremely flat around the optimum. This implies that the additional cost incurred from deviating from the optimal number of intermediate depots (i.e. from the optimal level of consolidation of small parcel transportation) are very limited. Moreover, the cost curves are generally even more flat to the right side than they are to the left side. This implies that it is financially speaking even less risky to locate more intermediate depots than might actually be optimal instead of locating fewer intermediate depots. This aspect becomes even more interesting once the currently static and deterministic LRP model is extended to the dynamic and stochastic case, which we will suggest as a promising area for future research in Section 5.

4.6.3 Optimal Threshold Between Parallel Network Architectures

Based on our model, an optimal threshold value in terms of parcel size between the two parallel network infrastructures can be determined. This threshold value defines the degree of shipment consolidation that is cost optimal. Some preliminary analyses based on our model have shown that the majority of item size classes should rather be shipped directly between the city hub and the actual demand nodes.

5 Future Research

There are a number of open issues which warrant further study in the context of urban logistics. Building on the research questions identified in Section 2.3, we consider the following to be especially fruitful paths for further research.

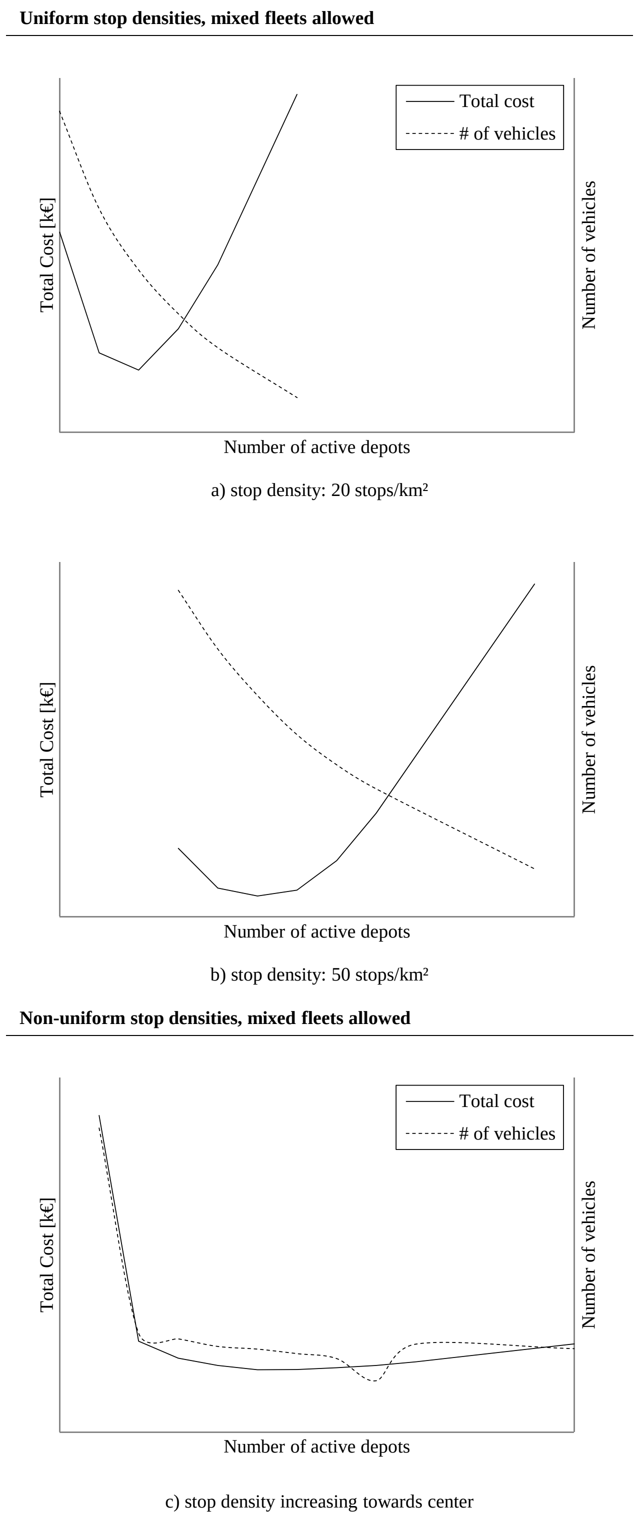


Figure 4: Cost optimal number of facilities for various demand distributions, mixed fleets

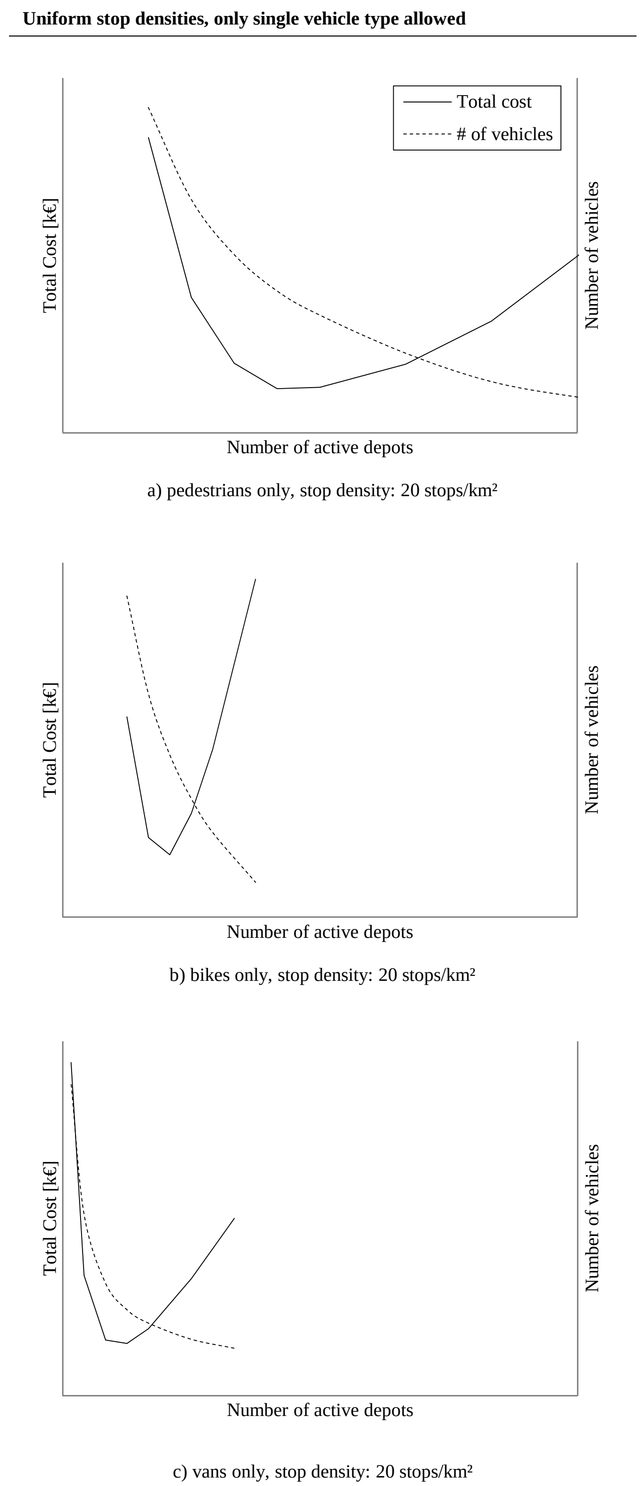


Figure 5: Cost optimal number of facilities, uniform demand distribution, single vehicle type only

Improving the integrated optimization of upstream and downstream item flows

Smart routing: As discussed in Section 4.3.1, introducing smart routing behavior with regards to the joint consideration of pickup and delivery demand would be a useful extension to the current model.

Introducing dynamic considerations and stochasticity

Robust optimization: As indicated in Figure 1, the dimension of uncertainty or risk which naturally accompanies the extension of service offerings has not been explicitly included in the analyses of this paper. Since uncertainties can be attributed both to future changes in the market place as well as to tightening of regulatory standards, such as emission allowances, a particularly promising way of incorporating uncertainty would be robust optimization where no prior probability distribution is required as an input but rather uncertainty sets. The objective of robust optimization is not to obtain a solution which works best under a particular scenario but rather a solution which works reasonably well over a broad set of uncertainty realizations [Cornuejols and Tütüncü, 2007]. Robust optimization is used to determine an optimal solution to a decision problem given that there is uncertainty and / or variability in one or more of the parameters that define the problem [Ben-Tal and Nemirovski, 1998]. It is thus an appropriate methodology if:

- some of the problem parameters are estimates and carry estimation risk;
- there are constraints with uncertain parameters that must be satisfied regardless of the values of these parameters;
- the objective function or the optimal solutions are particularly sensitive to perturbations;
- the decision maker cannot afford to take low-probability but high-magnitude risks.

We generally distinguish two types of robustness, constraint robustness and objective robustness. While constraint robustness is concerned with the fact that uncertainty may put the feasibility of potential solutions at risk, objective robustness deals with the fact that uncertainty may affect the proximity of generated solutions to optimality.

There are multiple orientations to robust optimization. Worst case robust optimization aims at finding a solution that is optimal under the most adverse conditions (absolute robustness). Relative robust optimization aims at finding a solution that e.g. avoids falling severely behind competition under all possible scenarios (relative robustness). Adjustable robust optimization allows for adjustments to the orientation of the optimization over time in multi-period models.

Including the dynamics of market demand into the analyses

Pricing: For successful entry into the domain of ULS as a central consolidator, the actions and reactions of all relevant stakeholders have to be considered and if possible included into the quantitative study summarized above. This could be accomplished by developing a discrete-choice model which explains and predicts modal choice in the urban logistics context. This study could be based on a multinomial logit[5] or generalized extreme value (GEV)[6] model with choice data provided by either revealed or stated preferences from, among others, stakeholder interviews. The additional expected output would be information for POs and LSPs on how to price their new types of services by means of the marginal rates of substitution.

Incentive system design: Another topic of relevancy in the present context is the provision of appropriate incentives such that smaller logistics service providers and/or larger competitors trade their capacities and demands on an urban logistics IT platform with the goal of an overall optimized match of capacity demand and supply. A simulation approach based on electronic agents seems particularly fruitful, building on previous research on capacity options and collaborative game theory.

[5] For an introduction to multinomial logit models see e.g. McFadden [1973], McKelvey and Zavoina [1975]
[6] The theory of GEV models has been derived by McFadden [1978]

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Appendix

Appendix A: Approximation Performance of Smilowitz and Daganzo [2005]

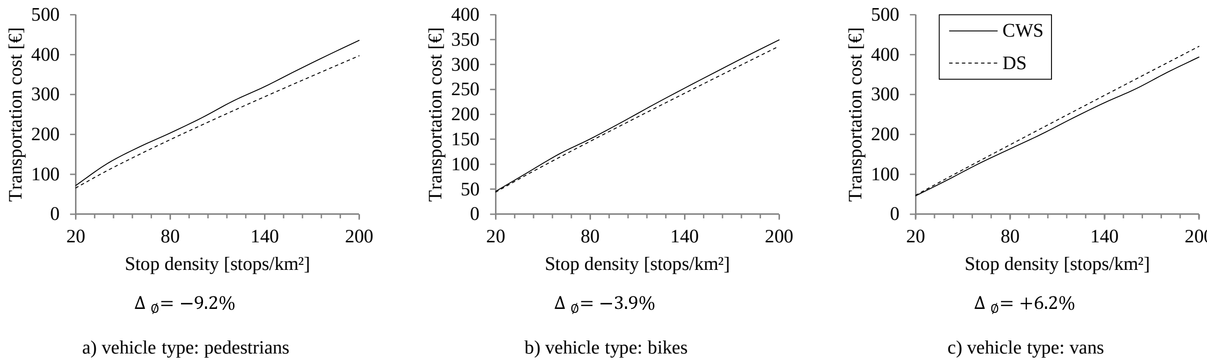

Figure 6: Comparison of transportation cost (excluding line-haul) between explicit application of Clarke Wright Savings Algorithm and closed-form approximation of Smilowitz and Daganzo [2005]

Appendix B: Cost Model Formulation

Model Variables And Parameters

The following variables and parameters will be used in our cost model formulation.

Decision variables

$S^S_{d,i,s}$: Binary variables defining whether city segment u_i is served from depot location l_d using vehicle type g_s for small item delivery.

$S^L_{c,i,l}$: Binary variables defining whether city segment u_i is served from city hub location l_c using vehicle type g_l for large item delivery.

a^D_d: Binary variables defining whether an active depot is positioned at depot location l_d.

a^C_c: Binary variables defining whether an active city hub is positioned at city hub location l_c.

$a^{C,H}_{c,b}$: Binary variables defining whether second-tier item transportation from city hub location l_c is performed using vehicle type g_b.

Exogenously given parameters

$s^{v,o}_j$: Average travelling speed of vehicle type g_j outside tours, i.e. during line-haul [km/h].

$s^{v,i}_j$: Average travelling speed of vehicle type g_j within tours, i.e. during transfer from one stop to the next stop [km/h].

t^a_j: Service time per stop for vehicle type g_j [h/stop].

t^s_j: Setup time per tour for vehicle type g_j [h/tour].

$t^{b,S}_j$: Average time to load or unload a small item to or from a vehicle of type g_j [h/item].

$t_j^{b,L}$: Average time to load or unload a large item to or from a vehicle of type g_j [h/item].

$T_{max}^{S,a}$: Maximum allowed service time for small item pickup and delivery between intermediate depots and demand nodes [h].

$T_{max}^{S,b}$: Maximum allowed service time for small item transfer between city hub and intermediate depots [h].

T_{max}^{L}: Maximum allowed service time for large item delivery [h].

x_p: X-coordinate of facility location l_p [km].

y_p: Y-coordinate of facility location l_p [km].

x_i: X-coordinate of lower left corner of city segment u_i [km].

y_i: Y-coordinate of lower left corner of city segment u_i [km].

Δ_i^x: Dimension of city segment u_i along the x-axis [km].

Δ_i^y: Dimension of city segment u_i along the y-axis [km].

$\vartheta_{p,i}$: Detour factor defining the shortest possible distance between facility location l_p and city segment u_i relative to a straight line between the two [].

$\vartheta_{c,d}$: Detour factor defining the shortest possible distance between facility location l_c and facility location l_d relative to a straight line between the two [].

c_D^h: Handling cost per item at intermediate depot level [€/item].

c_{CC}^h: Handling cost per item at the city hub level [€/item].

w_j: Hourly wage for driver of vehicle type g_j [€/h].

c_j^v: Cost of vehicle operation for vehicle type g_j [€/h].

γ_i: Absolute stop density in city segment u_i [stops/km^2].

ρ_i^P: Average number of items to be picked up per stop in city segment u_i [items/stop].

ρ_i^D: Average number of items to be delivered per stop in city segment u_i [items/stop].

f_d^D: Facility fixed cost for an intermediate depot activated at facility location l_d [€].

f_c^C: Facility fixed cost for a city hub activated at facility location l_c [€].

f_j^v: Vehicle fixed cost for vehicle type g_j [€].

$\kappa_{i,j}$: Distance metric factor determining the expected traveling distance between two demand nodes in city segment u_i for vehicle type g_j [].

κ_b: Distance metric factor determining the expected traveling distance between two demand nodes (here: intermediate depots) in the entire city area for vehicle type g_b [].

κ_j^H: Detour factor determining the expected line-haul distance between two deterministic locations for a vehicle of type g_j relative to their straight line distance [].

ξ_j^ν: Carrying capacity of vehicle type g_j in terms of physical volume [cm^3].

$p_{i,h}^P$: Probability of a pickup item in city segment u_i belonging to the item size class ν_h [].

$p_{i,h}^D$: Probability of a delivery item in city segment u_i belonging to the item size class ν_h [].

ζ_j^v: Physical space requirement of vehicle type g_j for sorting, loading and parking [m^2].

c_p^{ζ}: Cost of physical space at facility location l_p [€/m²]

$z_p^{i,D}$: Binary variable defining whether facility location l_p should be excluded from the set of available locations for intermediate depots.

$z_p^{i,C}$: Binary variable defining whether facility location l_p should be excluded from the set of available locations for the city hub.

α^{ν}: Threshold value for physical item size, determining boundaries of parallel network architectures.

N^D: Number of potentially available intermediate depot locations [].

The Urban Logistics Transportation Cost Model

Defining Model Dimensions

Facility locations The set of potential facility locations for the city hub and / or intermediate depots is defined as:

$$L := \{l_p\} \tag{37}$$

The set of facility locations that are allowed for the positioning of intermediate depots is a subset of L:

$$L_D := \{l_d\} = \left\{l_p | z_p^{i,D} < 1\right\} \subseteqq L \tag{38}$$

The set of facility locations that are allowed for the positioning of the city hub is a subset of L:

$$L_C := \{l_c\} = \left\{l_p | z_p^{i,C} < 1\right\} \subseteqq L \tag{39}$$

City segments The set of city segments with non-zero demand is defined as:

$$U := \{u_i\} \tag{40}$$

Vehicle types The set of overall available vehicle types is defined as:

$$G := \{g_j\} \tag{41}$$

The set of vehicle types that are allowed for the transportation of small items between intermediate depots and customers is a subset of G:

$$G_{S,a} := \{g_s\} \subseteqq G \tag{42}$$

The set of vehicle types that are allowed for the transportation of large items between intermediate depots and customers is a subset of G:

$$G_L := \{g_l\} \subseteqq G \tag{43}$$

The set of vehicle types that are allowed for the transportation of small items between the city hub and the intermediate depots is a subset of G:

$$G_{S,b} := \{g_b\} \subseteqq G \tag{44}$$

The set of vehicle types that are allowed to be positioned at an intermediate depot location l_d is a subset of $G_{S,a}$:

$$G_d^S \subseteqq G_{S,a} \tag{45}$$

The set of vehicle types that are allowed to be positioned at city hub location l_c for the purpose of large item transportation is a subset of G_L:

$$G_c^L \subseteqq G_L \tag{46}$$

The set of vehicle types that are allowed to be positioned at city hub location l_c for the purpose of small item transportation to the intermediate depots is a subset of $G_{S,b}$:

$$G_c^H \subseteqq G_{S,b} \tag{47}$$

The set of vehicle types that are allowed to travel within city segment u_i is a subset of G:

$$G_i^U \subseteqq G \tag{48}$$

Item sizes The set of possible item sizes is defined as:

$$V := \{\nu_h\} \tag{49}$$

Reducing Model Size And Complexity

Omitting infeasible item flows Reducing the number of potential item flows to be considered is crucial in reducing model complexity and improving computation time. Thus it is required to exclude those item flows that are infeasible. An item flow may be infeasible for various reasons:

- The time required for the line haul between the corresponding facility location and the corresponding city segment using the corresponding vehicle type may exceed the total service time allowed, i.e. the time span in which all demand has to be served.
- The corresponding vehicle type may not be positioned at the corresponding facility location.
- The corresponding vehicle type may not travel within the corresponding city segment.

The set of feasible item flows for small items between intermediate depots and city segments is defined as:

$$F_S := \left\{(l_d, u_i, g_s) \,|t_{d,i,s}^r + t_s^a \leq T_{max}^s \wedge g_s \in G_d^S \wedge g_s \in G_i^U\right\} \tag{50}$$

The set of feasible item flows for large items between the city hub and city segments is defined as:

$$F_L := \left\{(l_c, u_i, g_l) \,|t_{c,i,l}^r + t_l^a \leq T_{max}^L \wedge g_l \in G_c^L \wedge g_l \in G_i^U\right\} \tag{51}$$

Here, the time required for line haul from intermediate depot or city hub location l_p to city segment u_i using vehicle type g_j is defined as:

$$t_{p,i,j}^r = \frac{r_{p,i}}{s_j^{v,o}} \tag{52}$$

The distance between facility location l_p and city segment u_i depends on the distance metric used as well as potential physical obstacles that need to be circumvented. The latter can be captured by a detour factor for each pair (l_p, u_i). Using an Euclidian metric, the distance of interest can be defined as:

$$r_{p,i} = \frac{\int_0^{\Delta_i^y} \int_0^{\Delta_i^x} \sqrt{\left(x_p - (x_i + x)\right)^2 + \left(y_p - (y_i + y)\right)^2} dxdy}{A_i} \vartheta_{p,i} \tag{53}$$

where $\vartheta_{p,i} \geq 1$.

Calculating The Total Cost of Transportation

Handling cost Handling cost occur at the level of the intermediate depots, where only small items have to be received, sorted, stored and loaded, as well as at the level of the city hub where both large and small items have to be processed. Total handling cost can thus be defined as:

$$K^h = c_D^h \sum_i \left[A_i \gamma_i \left(\rho_i^P \sum_{h|\nu_h \leq \alpha^\nu} [p_{i,h}^P] + \rho_i^D \sum_{h|\nu_h \leq \alpha^\nu} [p_{i,h}^D] \right) \right] + c_{CC}^h \sum_i \left[A_i \gamma_i \left(\rho_i^P + \rho_i^D \right) \right] \tag{54}$$

Here, the physical area of city segment u_i is defined as:

$$A_i = \Delta_i^x \Delta_i^y \tag{55}$$

Facility fixed cost Total facility fixed cost are defined as the sum of the fixed cost of all active intermediate depots and the fixed cost of the active city hub.

$$K^f = \sum_d \left[a_d^D f_d^D \right] + \sum_c \left[a_c^C f_c^C \right] \tag{56}$$

First-tier transportation cost contributions for small item pickup and delivery The cost of transportation of small items between intermediate depots and demand nodes can be defined as:

$$K^{a,S} = \sum_d \sum_i \sum_s S_{d,i,s}^S k_{d,i,s}^{a,S} \tag{57}$$

Where the cost contribution of serving city segment u_i from a given depot location l_d using a given vehicle type g_s for small item pickup and delivery is defined as:

$$k_{d,i,s}^{a,S} = q_{d,i,s}^S \left[m_{d,i,s}^S t_s^s w_s + 2 m_{d,i,s}^S r_{d,i} \frac{w_s + c_s^v}{s_s^{v,o}} + m_{d,i,s}^S n_{d,i,s}^S \Psi_{i,s} \left(w_s + c_s^v \right) + f_s^v \right] \tag{58}$$

In the expression above, the number of vehicles of type g_s that is required to serve a given city segment u_i from a given depot location l_d for small item pickup and delivery is defined as:

$$q_{d,i,s}^S = \frac{A_i \gamma_i \left(\rho_i^P p_i^{P,S} + \rho_i^D p_i^{D,S} \right)}{\xi_{d,i,s}^{r,S}} \tag{59}$$

Here, the real vehicle capacity of a vehicle of type g_s in terms of average number of stops that can be performed within the maximum allowed service time when serving city segment u_i from depot location l_d for small item pickup and delivery is defined as:

$$\xi_{d,i,s}^{r,S} = \xi_{i,s}^{n,S} \delta_{d,i,s}^S \tag{60}$$

Here, the average number of full vehicle capacity utilization tours that a single vehicle of type g_s can perform within the maximum allowed service time when serving city segment u_i from intermediate depot location l_d for small item pickup and delivery is defined as:

$$\delta_{d,i,s}^S = \frac{T_{max}^S}{t_{i,s}^{\xi,S} + t_s^s + 2 t_{d,i,s}^r + t_{i,s}^{b,S}} \tag{61}$$

With the time required to perform a single full vehicle capacity tour with a vehicle of type g_s given the stop density of city segment u_i for small item pickup and delivery being defined as:

$$t_{i,s}^{\xi,S} = \xi_{i,s}^{n,S} \Psi_{i,s} \tag{62}$$

With the average time required by a vehicle of type g_s to serve a single stop and move on to the next stop given the stop density of city segment u_i being defined as:

$$\Psi_{i,s} = t_s^a + \frac{\kappa_{i,s}}{s_s^{v,i}\sqrt{\gamma_i}} \tag{63}$$

With the average time required by a vehicle of type g_s to overcome the linehaul distance between intermediate depot location l_d and city segment u_i being defined as:

$$t_{d,i,s}^r = \frac{r_{d,i}\kappa_s^H}{s_s^{v,o}} \tag{64}$$

With the cummulative time required to unload the full vehicle capacity along the tour for a vehicle of type g_s given the stop density of city segment u_i for small item pickup and delivery being defined as:

$$t_{i,s}^{b,S} = \xi_{i,s}^{n,S} t_s^{b,S} \tag{65}$$

With the nominal full vehicle capacity in terms of the average number of stops that can be performed on a single tour of a vehicle of type g_s when jointly considering small item pickup and delivery for a given stop density of city segment u_i being defined as:

$$\xi_{i,s}^{n,S} = \frac{\xi_s^\nu}{\max\left[\Delta D_i^S, \Delta P_i^S\right]} \tag{66}$$

Here the average vehicle capacity in terms of volume that is filled up by the average number of small items that are picked up at a single stop in city segment u_i is defined as:

$$\Delta P_i^S = E_i^{\nu,P,S} \rho_i^P p_i^{P,S} \tag{67}$$

Accordingly, the average vehicle capacity in terms of volume that is freed by the average number of small items that are delivered at a single stop in city segment u_i is defined as:

$$\Delta D_i^S = E_i^{\nu,D,S} \rho_i^D p_i^{D,S} \tag{68}$$

Here, the expected size in terms of volume of a small item to be picked up in city segment u_i is defined as:

$$E_i^{\nu,P,S} = \sum_h \left[\nu_h p_{i,h}^{P,S}\right] \tag{69}$$

Accordingly, the expected size in terms of volume of a small item to be delivered in city segment u_i is defined as:

$$E_i^{\nu,D,S} = \sum_h \left[\nu_h p_{i,h}^{D,S}\right] \tag{70}$$

With the probability of a small item that is to be picked up in city segment u_i belonging to item size class ν_h being defined as:

$$p_{i,h}^{P,S} = \begin{cases} \frac{p_{i,h}^P}{p_i^{P,S}} & \forall h \mid \nu_h \leq \alpha^\nu \\ 0 & \forall h \mid \nu_h > \alpha^\nu \end{cases} \tag{71}$$

With the probability of a small item that is to be delivered in city segment u_i belonging to item size class ν_h being defined as:

$$p_{i,h}^{D,S} = \begin{cases} \frac{p_{i,h}^D}{p_i^{D,S}} & \forall h \mid \nu_h \leq \alpha^\nu \\ 0 & \forall h \mid \nu_h > \alpha^\nu \end{cases} \tag{72}$$

With the probability of an item to be picked up in city segment u_i being a small item:

$$p_i^{P,S} = \sum_{h|\nu_h \leq \alpha^{size}} \left[p_{i,h}^P\right] \tag{73}$$

With the probability of an item to be delivered in city segment u_i being a small item:

$$p_i^{D,S} = \sum_{h|\nu_h \leq \alpha^{size}} \left[p_{i,h}^D\right] \tag{74}$$

The actual number of stops per tour that a vehicle of type g_s can perform when serving city segment u_i from depot location l_d for small item pickup and delivery is defined as:

$$\begin{aligned} n_{d,i,s}^S &= \min\left[\xi_{d,i,s}^{r,S}, \xi_{i,s}^{n,S}\right] \\ &= \begin{cases} \xi_{d,i,s}^{r,S} & \delta_{d,i,s}^S < 1 \\ \xi_{i,s}^{n,S} & \delta_{d,i,s}^S \geq 1 \end{cases} \end{aligned} \tag{75}$$

The average number of tours that a vehicle of type g_s starts within the maximum allowed service time when serving city segment u_i from depot location l_d for small item pickup and delivery is defined as:

$$m_{d,i,s}^S = \max\left[\delta_{d,i,s}^S, 1\right] \tag{76}$$

Second-tier transportation cost contributions for small item pickup and delivery For the second-tier item transportation between intermediate depots and teh city hub, pickup and delivery item flows cannot be considered jointly, as items to be delivered have to be shipped to the intermediate depots at the beginning of the business day while items that have been picked up during the business day need to be shipped from the intermediate depots to the city hub at the very end of the business day. Thus, the following formula applies for the calculation of second-tier transporation cost for small item pickup and delivery:

$$K^{b,S} = \sum_c \sum_b a_{c,b}^{C,H} k_{c,b}^{b,S} \tag{77}$$

Where the cost contribution of serving the intermediate depots from a given city hub location l_c using a given vehicle type g_b for small items is defined as:

$$k_{c,b}^{b,S} = \left(q_{c,b}^{b,S,D} + q_{c,b}^{b,S,P}\right)(w_b + c_b^v)\, T_{max}^{S,b} + \max\left[q_{c,b}^{b,S,D}, q_{c,b}^{b,S,P}\right] f_b^v \tag{78}$$

With the number of second-tier transporation vehicles required for item delivery to the intermediate depots, $q_{c,b}^{b,S,D}$, being defined as:

$$q_{c,b}^{b,S,D} = \frac{N^d}{\xi_{c,b}^{r,S,b,D}} \tag{79}$$

With the number of active intermediate depots being defined as:

$$N^d = \sum_d a_d^D \tag{80}$$

With the real vehicle capacity of a vehicle of type g_b in terms of average number of intermediate depots that can be served with items to be delivered within the maximum allowed service time from city hub location l_c is defined as:

$$\xi_{c,b}^{r,S,b,D} = \xi_b^{n,S,b,D} \delta_{c,b}^{S,b,D} \tag{81}$$

Here, the average number of full vehicle capacity utilization tours that a single vehicle of type g_b can perform within the maximum allowed service time when serving intermediate depots with items to be delivered from city hub location l_c is defined as:

$$\delta_{c,b}^{S,b,D} = \frac{T_{max}^{S,b}}{t_b^{\xi,S,b,D} + t_b^s + 2t_{c,b}^r + t_b^{b,S,b,D}} \tag{82}$$

With the time required to perform a single full vehicle capacity tour with items to be delivered with a vehicle of type g_b given the density of intermediate depots across the city area being defined as:

$$t_b^{\xi,S,b,D} = \xi_b^{n,S,b,D} \Psi_b \tag{83}$$

With the average time required by a vehicle of type g_b to serve a single intermediate depot and move on to the next one given the density of intermediate depots across the city area being defined as:

$$\Psi_b = t_b^a + \frac{\kappa_b}{s_b^{v,i} \sqrt{\frac{N^d}{A^T}}} \tag{84}$$

With the average time required by a vehicle of type g_b to overcome the average line-haul distance between city hub location l_c and the potential intermediate depot locations being defined as:

$$t_{c,b}^r = \frac{\frac{\sum_d [r_{c,d}]}{N^D} \kappa_b^H}{s_b^{v,o}} \tag{85}$$

With the line-haul distance between city hub location l_c and intermediate depot location l_d being defined as:

$$r_{c,d} = \sqrt{(x_c - x_d)^2 + (y_c - y_d)^2} \vartheta_{c,d} \tag{86}$$

With the physical area of the entire city being defined as:

$$A^T = \sum_i A_i \tag{87}$$

With the cummulative time required to unload the full vehicle capacity along the tour for a vehicle of type g_b given the density of intermediate depots across the city area for items to be delivered being defined as:

$$t_b^{b,S,b,D} = \xi_b^{n,S,b,D} t_b^{b,S} \tag{88}$$

With the nominal full vehicle capacity in terms of the average number of intermediate depots that can be served with items to be delivered on a single tour of a vehicle of type g_b a given density of intermediate depots across the city area being defined as:

$$\xi_b^{n,S,b,D} = \frac{\xi_b^\nu}{\frac{\Delta D^{S,b}}{N^d}} \tag{89}$$

With the average physical volume of all small items to be delivered across the entire city area being defined as:

$$\Delta D^{S,b} = \sum_i \left[A_i \gamma_i \rho_i^D p_i^{D,S} E_i^{\nu,D,S} \right] \tag{90}$$

In the same way, the number of second-tier transporation vehicles required for item pickup from the intermediate depots, $q_{c,b}^{b,S,P}$, is defined as:

$$q_{c,b}^{b,S,P} = \frac{N^d}{\xi_{c,b}^{r,S,b,P}} \tag{91}$$

Here, the real vehicle capacity of a vehicle of type g_b in terms of average number of intermediate depots that can be served with items to be picked up within the maximum allowed service time from city hub location l_c is defined as:

$$\xi_{c,b}^{r,S,b,P} = \xi_b^{n,S,b,P} \delta_{c,b}^{S,b,P} \tag{92}$$

Here, the average number of full vehicle capacity utilization tours that a single vehicle of type g_b can perform within the maximum allowed service time when serving intermediate depots with items to be picked up from city hub location l_c is defined as:

$$\delta_{c,b}^{S,b,P} = \frac{T_{max}^{S,b}}{t_b^{\xi,S,b,P} + t_b^s + 2t_{c,b}^r + t_b^{b,S,b,P}} \tag{93}$$

With the time required to perform a single full vehicle capacity tour with items to be picked up with a vehicle of type g_b given the density of intermediate depots across the city area being defined as:

$$t_b^{\xi,S,b,P} = \xi_b^{n,S,b,P} \Psi_b \tag{94}$$

With the cummulative time required to unload the full vehicle capacity along the tour for a vehicle of type $g_b$given the density of intermediate depots across the city area for items to be picked up being defined as:

$$t_b^{b,S,b,P} = \xi_b^{n,S,b,P} t^{b,S} \tag{95}$$

With the nominal full vehicle capacity in terms of the average number of intermediate depots that can be served with items to be picked up on a single tour of a vehicle of type g_b a given density of intermediate depots across the city area being defined as:

$$\xi_b^{n,S,b,P} = \frac{\xi_b^{\nu}}{\frac{\Delta P^{S,b}}{N^d}} \tag{96}$$

With the average physical volume of all small items to be picked up across the entire city area being defined as:

$$\Delta P^{S,b} = \sum_i \left[A_i \gamma_i \rho_i^P p_i^{P,S} E_i^{\nu,P,S} \right] \tag{97}$$

Transportation cost contributions for large item pickup and delivery The cost of transportation of large items between the city hub and demand nodes can be defined as:

$$K^{a,L} = \sum_c \sum_i \sum_l S_{c,i,l}^L k_{c,i,l}^{a,L} \tag{98}$$

Where the cost contribution of serving city segment u_i from a given city hub location l_c using a given vehicle type g_l for large item pickup and delivery is defined as:

$$\begin{aligned} k_{c,i,l}^{a,L} = q_{c,i,l}^L \Bigg[& m_{c,i,l}^L t_l^s w_l + 2m_{c,i,l}^L r_{d,i} \frac{w_l + c_l^v}{s_l^{v,o}} \\ & + m_{c,i,l}^L n_{d,i,l}^L \Psi_{i,l} \left(w_l + c_l^v \right) + f_l^v \Bigg] \end{aligned} \tag{99}$$

In the expression above, the number of vehicles of type g_l that is required to serve a given city segment u_i from a given city hub location l_c for large item delivery is defined as:

$$q_{c,i,l}^L = \frac{A_i \gamma_i \left(\rho_i^P p_i^{P,L} + \rho_i^D p_i^{D,L} \right)}{\xi_{c,i,l}^{r,L}} \tag{100}$$

Here, the real vehicle capacity of a vehicle of type g_l in terms of average number of stops that can be performed within the maximum allowed service time when serving city segment u_i from city hub location l_c for large item pickup and delivery is defined as:

$$\xi^{r,L}_{c,i,l} = \xi^{n,L}_{i,s} \delta^{L}_{c,i,l} \tag{101}$$

Here, the average number of full vehicle capacity utilization tours that a single vehicle of type g_l can perform within the maximum allowed service time when serving city segment u_i from city hub location l_c for large item pickup and delivery is defined as:

$$\delta^{L}_{c,i,l} = \frac{T^{L}_{max}}{t^{\xi,L}_{i,l} + t^{s}_{l} + 2t^{r}_{c,i,l} + t^{b,L}_{i,l}} \tag{102}$$

With the time required to perform a single full vehicle capacity tour with a vehicle of type g_l given the stop density of city segment u_i for large item pickup and delivery being defined as:

$$t^{\xi,L}_{i,l} = \xi^{n,L}_{i,l} \Psi_{i,l} \tag{103}$$

With the average time required by a vehicle of type g_l to serve a single stop and move on to the next stop given the stop density of city segment u_i being defined as:

$$\Psi_{i,l} = t^{a}_{l} + \frac{\kappa_{i,l}}{s^{v,i}_{l} \sqrt{\gamma_i}} \tag{104}$$

With the average time required by a vehicle of type g_l to overcome the line-haul distance between intermediate city hub location l_c and city segment u_i being defined as:

$$t^{r}_{c,i,l} = \frac{r_{c,i} \kappa^{H}_{l}}{s^{v,o}_{l}} \tag{105}$$

With the cummulative time required to unload the full vehicle capacity along the tour for a vehicle of type g_s given the stop density of city segment u_i for large item pickup and delivery being defined as:

$$t^{b,L}_{i,l} = \xi^{n,S}_{i,l} t^{b,L}_{l} \tag{106}$$

With the nominal full vehicle capacity in terms of the average number of stops that can be performed on a single tour of a vehicle of type g_l when jointly considering large item pickup and delivery for a given the stop density of city segment u_i being defined as:

$$\xi^{n,L}_{i,l} = \frac{\xi^{\nu}_{l}}{\max\left[\Delta D^{L}_{i}, \Delta P^{L}_{i}\right]} \tag{107}$$

Here the average vehicle capacity in terms of volume that is filled up by the average number of large items that are picked up at a single stop in city segment u_i is defined as:

$$\Delta P^{L}_{i} = E^{\nu,P,L}_{i} \rho^{P}_{i} p^{P,L}_{i} \tag{108}$$

Accordingly, the average vehicle capacity in terms of volume that is freed by the average number of large items that are delivered at a single stop in city segment u_i is defined as:

$$\Delta D^{L}_{i} = E^{\nu,D,L}_{i} \rho^{D}_{i} p^{D,L}_{i} \tag{109}$$

Here, the expected size in terms of volume of a large item to be picked up in city segment u_i is defined as:

$$E^{\nu,P,L}_{i} = \sum_{h} \left[\nu_h p^{P,L}_{i,h}\right] \tag{110}$$

Accordingly, the expected size in terms of volume of a large item to be delivered in city segment u_i is defined as:

$$E_i^{\nu,D,L} = \sum_h \left[\nu_h p_{i,h}^{D,L}\right] \tag{111}$$

With the probability of a large item that is to be picked up in city segment u_i belonging to item size class ν_h being defined as:

$$p_{i,h}^{P,L} = \begin{cases} \frac{p_{i,h}^P}{p_i^{P,L}} & \forall h \mid \nu_h > \alpha^\nu \\ 0 & \forall h \mid \nu_h \leq \alpha^\nu \end{cases} \tag{112}$$

With the probability of a large item that is to be delivered in city segment u_i belonging to item size class ν_h being defined as:

$$p_{i,h}^{D,L} = \begin{cases} \frac{p_{i,h}^D}{p_i^{D,L}} & \forall h \mid \nu_h > \alpha^\nu \\ 0 & \forall h \mid \nu_h \leq \alpha^\nu \end{cases} \tag{113}$$

With the probability of an item to be picked up in city segment u_i being a large item:

$$p_i^{P,L} = \sum_{h \mid \nu_h > \alpha^\nu} \left[p_{i,h}^P\right] \tag{114}$$

With the probability of an item to be delivered in city segment u_i being a large item:

$$p_i^{D,L} = \sum_{h \mid \nu_h > \alpha^\nu} \left[p_{i,h}^D\right] \tag{115}$$

The actual number of stops per tour that a vehicle of type g_l can perform when serving city segment u_i from city hub location l_c for large item pickup and delivery is defined as:

$$\begin{aligned} n_{c,i,l}^L &= \min \left[\xi_{c,i,l}^{r,L}, \xi_{i,l}^{n,L}\right] \\ &= \begin{cases} \xi_{c,i,l}^{r,L} & \delta_{c,i,l}^L < 1 \\ \xi_{i,l}^{n,L} & \delta_{c,i,l}^L \geq 1 \end{cases} \end{aligned} \tag{116}$$

The average number of tours that a vehicle of type g_l starts within the maximum allowed service time when serving city segment u_i from city location l_c for large item pickup and delivery is defined as:

$$m_{c,i,l}^L = \max \left[\delta_{c,i,l}^L, 1\right] \tag{117}$$

Capacity related facility cost The cost of space required at the intermediate depots for sorting, vehicle parking, and loading can be calculated as follows:

$$K^{S,a,\zeta} = \sum_d \sum_i \sum_s S_{d,i,s}^S k_{d,i,s}^{S,a,\zeta} \tag{118}$$

Where the contribution of serving city segment u_i from a given intermediate depot location l_d using a given vehicle type g_s for small item pickup and delivery to the total cost of space is defined as:

$$k_{d,i,s}^{S,a,\zeta} = q_{d,i,s}^S \zeta_s^v c_d^\zeta \tag{119}$$

The cost of space required at the city hub for sorting, vehicle parking, and loading related to the vehicles performing the second-tier transportation of small items between the city hub and the intermediate depots can be calculated as follows:

$$K^{S,b,\zeta} = \sum_c \sum_b a_{c,b}^{C,H} k_{c,b}^{S,b,\zeta} \tag{120}$$

Where the contribution of serving the intermediate depots from a given city hub location l_c using a given vehicle type g_b for small items to the total cost of space is defined as:

$$k_{c,b}^{S,b,\zeta} = \max\left[q_{c,b}^{b,S,D}, q_{c,b}^{b,S,P}\right]\zeta_b^v c_c^\zeta \tag{121}$$

The cost of space required at the city hub for sorting, vehicle parking, and loading related to the vehicles performing the transportation of large items between the city hub and the demand nodes can be calculated as follows:

$$K^{L,\zeta} = \sum_c \sum_i \sum_l S_{c,i,l}^L k_{c,i,l}^{L,\zeta} \tag{122}$$

Where the contribution of serving city segment u_i from a given city hub location l_c using a given vehicle type g_l for large item pickup and delivery to the total cost of space is defined as:

$$k_{c,i,l}^{L,\zeta} = q_{c,i,l}^L \zeta_l^v c_c^\zeta \tag{123}$$

Total cost Given the definitions above, the total average daily cost of transportation for the described urban logistics network sums up to:

$$K^T = K^h + K^f + K^{a,S} + K^{b,S} + K^{a,L} + K^{S,a,\zeta} + K^{S,b,\zeta} + K^{L,\zeta} \tag{124}$$

Appendix C: Optimization Model Formulation

Additional Notation

The following additional parameters are exogenously given for our optimization model.

ζ_d^D: Maximum physical space available at intermediate depot location l_d [m²].

ζ_c^C: Maximum physical space available at city hub location l_c [m²].

$z_p^{a,D}$: Binary variable defining whether facility location l_p should be forced to accommodate an active intermediate depot.

$z_p^{a,C}$: Binary variable defining whether facility location l_p should be forced to accommodate an active city hub.

$b^{D,l}$: Lower bound to the number of active intermediate depots to be located [].

$b^{D,u}$: Upper bound to the number of active intermediate depots to be located [].

The Simplified Cost Model For Solving The Facility Location Sub-Problem

The minimum cost contribution of serving city segment u_i from a given intermediate depot location l_d with small item transportation between the intermediate depot and the demand nodes is given by:

$$\tilde{k}_{d,i}^{S,a} = min_{s|(l_d,u_i,g_s)\in F_S}\left[k_{d,i,s}^{a,S} + k_{d,i,s}^{S,a,\zeta}\right] \tag{125}$$

The minimum cost contribution of serving the intermediate depots from a given city hub location l_c with small item transportation between the city hub and the intermediate depots is given by:

$$\tilde{k}_c^{S,b} = min_b\left[k_{d,i,s}^{a,S} + k_{c,b}^{S,b,\zeta}\right] \tag{126}$$

The minimum cost contribution of serving city segment u_i from a given city hub location l_c with large item transportation between the city hub and the end customers is given by:

$$\tilde{k}_c^{L,a} = min_{l|(l_c,u_i,g_l)\in F_L} \left[\sum_i \left[k_{c,i,l}^{a,L} + k_{c,i,l}^{L,\zeta} \right] \right] \tag{127}$$

The minimum total daily cost of transportation can thus be defined as:

$$\tilde{K}^T = \sum_d \sum_i \tilde{S}_{d,i}^S \tilde{k}_{d,i}^{S,a} + \sum_c a_c^C \left(\tilde{k}_c^{S,b} + \tilde{k}_c^{L,a} \right) \tag{128}$$

Here, $\tilde{S}_{d,i}^S$ denotes a binary variable that defines whether or not city segment u_i is served from intermediate depot location l_d. There is no choice with regards to the vehicle type because in this relaxed optimization problem it is always feasible to choose the cost optimal vehicle type which has already been determined in Equation 125.

Handling cost and facility fixed cost are not considered here as they have no effect of the actual optimization result of this reduced optimization model.