

# Competition Between Mail and Electronic Substitutes in the Financial Sector: A Hotelling Approach<sup>1</sup>

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<sup>1</sup>The analysis contained in this paper reflects the view of the authors and may not necessarily be those of Royal Mail Group.

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## **Abstract**

We build a model where two banks compete for the patronage of consumers by offering them, among other services and products, two forms of transactional media: paper statements and electronic substitutes. Both banks and both products are horizontally differentiated and modeled à la Hotelling(1929). Assuming symmetry of consumer preferences (over banks and, independently, over the two transactional media) and of banks's costs, we obtain that the unique profit-maximizing symmetrical prices reflect both the transactional media marginal costs and the intensity of competition between banks. Most notably, the intensity of consumers preferences for one variant of transactional medium over another has no influence on the profit-maximizing media prices. Also, there is total pass-through of increases in input prices (such as mail price for paper statements) into prices paid by final consumers.

# 1 Introduction

Historically, financial institutions have been large mail users, as they provided financial information and paper statements to their customers. In more recent times, however, their customers have been offered the choice of alternative services through the digital medium on-line. The customers of these institutions not only have a choice of bank, but also a choice in the kind of service they receive within the transactional market. In recent years we have observed a significant switching from transactional mail to the digital alternative. Understanding these market developments is important for policy makers, as they seek to maintain the financial viability of the Universal Service Provider (USP).

In De Donder *et al.* (2012), we tackle this issue by assuming that the USP first sets its prices, including the mail price to be paid by financial institutions, and that several banks then play a Cournot game, choosing simultaneously the quantity of transactional media (both paper statements and electronic substitutes) that maximizes their profit given the choice of others. No effort was made in that paper to provide micro-foundations for the demand functions relating prices and volumes of transactional media that were used by banks.

The objective of this paper is to provide such micro-foundations, in a setting where both banks and transactional media are horizontally differentiated. Assuming symmetry of consumers' tastes for banks and for media, as well as symmetry of marginal costs across banks, we obtain that the profit-maximizing transactional media prices of the banks reflect their marginal cost, plus a mark-up inversely linked to the intensity of competition between banks. These profit-maximizing prices are not affected by the intensity of preferences of consumers for one type of transactional medium versus another; they are characterized by complete pass-through of any increase in input costs (such as mail price for paper statements) on the final consumer price of the transactional medium.

The next section presents the model. The next three sections solve the model by backward induction, starting with the choice of transactional medium within a bank (Section 3), moving to the choice of banks (Section 4) and ending up with the profit-maximizing media prices (Section 5). The final section concludes.

## 2 The model

Consumers choose between two banks ( $A$  and  $B$ ) to satisfy their banking needs. Banking services can be considered as bundles of goods and services (such as checking and savings accounts, loans and credits, savings products, financial counseling, etc.). One of the services provided by banks consists in providing their clients with information regarding their bank accounts and more generally their financial

dealings with the bank. To provide this information, banks can either use paper statements (good 2) or an electronic substitute (good 1). Since the objective of this paper is to concentrate on the determinants of the demand for paper statements, we assume from now on that the characteristics of the other goods and services offered by the two banks are set exogenously and we concentrate on how banks price paper statements and their electronic substitute.

Both banks and both transactional media within banks are modeled as horizontally differentiated. Following Hotelling (1929)'s tradition, we assume that bank  $A$  is located at zero on the  $X = [0, 1]$  axis, while bank  $B$  is located at one. These locations are assumed to be exogenous, and to reflect some characteristics of the banks that are left unmodeled here, such as for instance the location of the banks on Main Street. Likewise, within both banks, paper statements are located at 1 while the electronic substitute is located at zero on the  $Z = [0, 1]$  dimension. Here also, we take these locations as given, and reflecting the exogenous characteristics of these two transactional media.

There is a continuum of consumers who differ according to their intrinsic preferences for banks and for transactional media. We assume for simplicity that the two dimensions of preferences are orthogonal to each other. Formally, consumers are uniformly and independently distributed on the  $Z \times X = [0, 1] \times [0, 1]$  space. The  $Z$  dimension measures the preference for the characteristics of the transactional media, while the  $X$  dimension measures preferences for banks' characteristics.

The location  $x \in [0, 1]$  of a consumer represents his preferences for banks' characteristics. An individual located at  $x$  incurs a disutility of  $x$  (the distance between his most-preferred variant of the bank and the characteristics of bank  $A$ ) times a linear "transportation cost"  $t$  when opening an account with bank  $A$ , and a disutility of  $(1 - x)$  times  $t$  when opening an account with bank  $B$ . As usual in the horizontal differentiation models, the linear cost  $t$  can be given either a geographical or a preference interpretation.

Similarly, we assume that the location  $z \in [0, 1]$  of a consumer represents his preferences for the characteristics of transactional media. That is, an individual located at  $z$  incurs a disutility of  $z$  (the distance between his most-preferred variant of the medium and the characteristic of good 1) times a linear "transportation cost",  $r$ , when consuming good 1, and a disutility of  $(1 - z)$  times  $r$  when consuming good 2. The interpretation of the transportation cost is here exclusively in terms of preferences. For instance, agents located close to one may not be computer literate and thus prefer to receive, handle and file paper, while agents closer to zero may be younger, well-versed in dealing with the Internet, and prefer electronic handling of statements.

Let  $q_i^j$  denote the consumer price of good  $i$  ( $i = 1, 2$ ) in bank  $j$  ( $j = A, B$ ) and assume that banks set their prices in order to maximize profits. Each consumer

chooses one bank and one form of transactional medium in that bank.

The timing of the model we consider is that banks first post (simultaneously) their prices  $q_i^j$ , and that consumers then choose which bank to patronize and which form of transactional medium to use in that bank. To simplify the presentation, we assume that consumers first choose their bank, and then choose their preferred form of transactional medium. This assumption of a sequential choice leads to the same (subgame perfect Nash) equilibrium as the simultaneous choice.<sup>1</sup>

As usual, we solve the game by backward induction, starting with the choice between goods 1 and 2 for a consumer with an account in bank  $j$ .

### 3 The choice between paper statements and electronic substitutes

Consumers all obtain a gross utility level of  $U_i^j$  when they consume one unit of good  $i$  in bank  $j$ . This corresponds to the utility they would obtain if bank  $j$  had the ideal characteristic of the consumer on the  $X$  axis, and if it were offering the consumer's ideal transactional variant of good  $i$  on the  $Z$  axis. Unfortunately, the characteristics of both banks and transactional media usually differ from consumers' ideal ones. To obtain the net utility of a consumer located at  $(x, z)$ , we subtract the price he has to pay for this unit of medium, and the disutility from consuming a good whose characteristics are not exactly what he most prefers, in a bank whose characteristics are in general not optimal for him either. Denoting by  $V_i^j$  the net utility from buying one unit of good  $i$  from bank  $j$ , we obtain that

$$\begin{aligned} V_1^A &= U_1^A - q_1^A - rz - tx, \\ V_2^A &= U_2^A - q_2^A - r(1 - z) - tx, \\ V_1^B &= U_1^B - q_1^B - rz - t(1 - x), \\ V_2^B &= U_2^B - q_2^B - r(1 - z) - t(1 - x). \end{aligned}$$

An individual located at  $(x, z)$  and with an account in bank  $j$  chooses paper statements (good 2) over the electronic substitute (good 1) if

$$\begin{aligned} &V_1^j \leq V_2^j \\ \Leftrightarrow &U_1^j - q_1^j - rz \leq U_2^j - q_2^j - r(1 - z) \\ \Leftrightarrow &z \geq \tilde{z}^j(q_1^j, q_2^j) = \frac{1}{2} + \frac{(U_1^j - q_1^j) - (U_2^j - q_2^j)}{2r}. \end{aligned} \tag{1}$$

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<sup>1</sup>The decisions are made by the same agent and no new information is revealed between the two stages.

Equation (1) is intuitive. Consumers who are located close enough to 1 in the  $Z$  space (*i.e.*, who prefer an ideal form of transactional medium that is close enough to the characteristics of paper statements) buy paper statements rather than their electronic substitutes. The threshold value of  $z$  above which consumers buy paper statements in bank  $j$  (denoted by  $\tilde{z}^j$ ) decreases when the net utility from buying this good ( $U_2^j - q_2^j$ ) increases compared to the net utility of buying the other good ( $U_1^j - q_1^j$ ). The net utility of buying good  $i$  in bank  $j$  can increase either because its price  $q_i^j$  decreases, or because this good becomes intrinsically more attractive ( $U_i^j$  increases). As  $r$  increases, it becomes more difficult to convince consumers by changing price levels to consume a variant of the transactional medium different from the one closer to their most-preferred option. At the limit, when  $r$  tends toward infinity,  $\tilde{z}^j$  is then equal to one half. Also, the preference for bank  $A$  *vs*  $B$  of the client plays no role, once the client has decided which bank to use (since the same disutility from the bank appears whatever the transactional medium used in that bank). Finally, with a uniform distribution of consumers on the  $Z$  axis,  $\tilde{z}^j$  (resp.,  $1 - \tilde{z}^j$ ) also measures the proportion of bank  $j$ 's consumers who buy the electronic substitute (resp., the paper statements), and the fraction of the total demand for transactional medium that takes the form of electronic substitute (resp., paper statements) in bank  $j$ .

We now move to the first stage choice of the bank.

## 4 The choice of a bank

We denote by  $W^j$  the net utility of opening an account with bank  $j$  and we obtain

$$\begin{aligned} W^A &= V_1^A = U_1^A - q_1^A - rz - tx \text{ if } z \leq \tilde{z}^A(q_1^A, q_2^A), \\ W^A &= V_2^A = U_2^A - q_2^A - r(1 - z) - tx \text{ if } z > \tilde{z}^A(q_1^A, q_2^A), \\ W^B &= V_1^B = U_1^B - q_1^B - rz - t(1 - x) \text{ if } z \leq \tilde{z}^B(q_1^B, q_2^B), \\ W^B &= V_2^B = U_2^B - q_2^B - r(1 - z) - t(1 - x) \text{ if } z > \tilde{z}^B(q_1^B, q_2^B). \end{aligned}$$

Observe that the decision of which bank to choose depends on both  $x$  and  $z$ . This is intuitive since both banks may charge different prices for the same product, so that intrinsic preferences both for banks and for the type of transactional medium offered in each bank plays a role when choosing which bank to patronize.

Assume for the moment that  $\tilde{z}^A(q_1^A, q_2^A) \leq \tilde{z}^B(q_1^B, q_2^B)$  —*i.e.*, that paper statements are relatively more attractive in bank  $A$  than in bank  $B$  (the case where  $\tilde{z}^A(q_1^A, q_2^A) > \tilde{z}^B(q_1^B, q_2^B)$  is solved in a similar way). Depending on his preferences for transactional medium (as given by his location  $z$ ), a consumer belongs to one of three groups.

In the first group, characterized by  $z < \tilde{z}^A(q_1^A, q_2^A) < \tilde{z}^B(q_1^B, q_2^B)$ , a consumer located at  $(x, z)$  knows that he will choose electronic statements (good 1) whatever the bank he joins. He chooses bank  $A$  if

$$\begin{aligned} W^A &\geq W^B \Leftrightarrow V_1^A \geq V_1^B \\ \Leftrightarrow U_1^A - q_1^A - rz - tx &\geq U_1^B - q_1^B - rz - t(1-x) \\ \Leftrightarrow x &\leq \tilde{x}_1(q_1^A, q_1^B) = \frac{1}{2} + \frac{(U_1^A - q_1^A) - (U_1^B - q_1^B)}{2t}. \end{aligned} \quad (2)$$

The preference for paper *vs* electronic statements (the value of  $z$ ) plays no role in the choice of banks once it is understood that the consumer would choose the same medium in both banks. The interpretation of (2) is similar to (1): agents whose preferences are close enough to zero on the  $X$  dimension (*i.e.*, who care more for the intrinsic characteristics of bank  $A$  than of bank  $B$ ) patronize bank  $A$ . This is the case for agents whose most-preferred value of  $x$  is lower than the threshold  $\tilde{x}_1$ , where the subscript 1 is used to indicate that the consumer would buy good 1 in both banks. This threshold depends only on the prices and gross utility of good 1 in both banks. A larger value of  $t$  (the utility cost of moving away from your most-preferred bank's characteristics) means that less importance is given to the characteristics of the paper statements chosen in both banks.

Similarly, in the second group, where  $\tilde{z}^A(q_1^A, q_2^A) < \tilde{z}^B(q_1^B, q_2^B) < z$ , the consumer located at  $(x, z)$  knows that he will choose the paper statements (good 2) whatever the bank he joins. He chooses bank  $A$  if

$$\begin{aligned} W^A &\geq W^B \Leftrightarrow V_2^A \geq V_2^B \\ \Leftrightarrow U_2^A - q_2^A - r(1-z) - tx &\geq U_2^B - q_2^B - r(1-z) - t(1-x) \\ \Leftrightarrow x &\leq \tilde{x}_2(q_2^A, q_2^B) = \frac{1}{2} + \frac{(U_2^A - q_2^A) - (U_2^B - q_2^B)}{2t}. \end{aligned} \quad (3)$$

We index the threshold value of  $x$  below which consumers choose bank  $A$  by 2 since the consumer would buy good 2 in both banks. This threshold depends only on net utility provided by paper statements in both banks, and the intensity of preferences for banks (as measured by the cost  $t$ ).

In the third group, defined by  $\tilde{z}^A(q_1^A, q_2^A) < z < \tilde{z}^B(q_1^B, q_2^B)$ , a consumer located at  $(x, z)$  knows that he will choose electronic statements (good 1) if he joins bank  $B$  and paper statements (good 2) if he joins bank  $A$ . He chooses bank  $A$  if

$$\begin{aligned} W^A &\geq W^B \Leftrightarrow V_2^A \geq V_1^B \\ \Leftrightarrow U_2^A - q_2^A - r(1-z) - tx &\geq U_1^B - q_1^B - r - t(1-x) \\ \Leftrightarrow x &\leq \tilde{x}_{21}(q_1^A, q_1^B, z) = \frac{1}{2} + \frac{(U_2^A - q_2^A) - (U_1^B - q_1^B) - r + 2rz}{2t} \end{aligned} \quad (4)$$

We index the threshold value of  $x$  below which consumers choose bank  $A$  by 21 since the consumer would buy good 2 in bank  $A$  and good 1 in bank  $B$ . The choice of bank now depends on the difference in the net utility provided by paper statements in bank  $A$  and electronic substitutes in bank  $B$ . It also depends on both the preference of the individual for digital medium (measured by  $z$ ) and the intensity of this preference (as measured by  $r$ ). An individual with a larger value of  $z$  cares more for paper statements, which will be chosen only if the agent patronizes bank  $A$ . A larger value of  $z$  then increases the threshold  $\tilde{x}_{21}$  below which agents choose bank  $A$ , at a rate equal to the ratio of the transport costs,  $r/t$ . Observe that  $\tilde{x}_{21}$  increases (resp., decreases) with  $r$  if  $z > 1/2$  (resp., if  $z < 1/2$ ). When a consumer's most-preferred variant of transactional medium is closer to paper statements than to electronic substitutes ( $z > 1/2$ ), a larger value of  $r$  means that he puts more emphasis on this dimension of choice, where bank  $A$  dominates bank  $B$ . This in turn increases the attractiveness of bank  $A$  compared to  $B$ , which results in an increase in the threshold  $\tilde{x}_{21}$  below which agents choose bank  $A$ .

Observe that

$$\begin{aligned}\tilde{x}_{21}(q_1^A, q_1^B, \tilde{z}^A(q_1^A, q_2^A)) &= \tilde{x}_1(q_1^A, q_1^B), \\ \tilde{x}_{21}(q_1^A, q_1^B, \tilde{z}^B(q_1^B, q_2^B)) &= \tilde{x}_2(q_2^A, q_2^B),\end{aligned}$$

and that  $\tilde{x}_2(q_2^A, q_2^B) > \tilde{x}_1(q_1^A, q_1^B)$  so that the threshold value of  $x$  below which consumers choose bank  $A$  is a continuous function of  $z$ , even when consumers switch from one of the three afore mentioned groups to another.

We represent the demand for both goods in both banks in the following Figure.

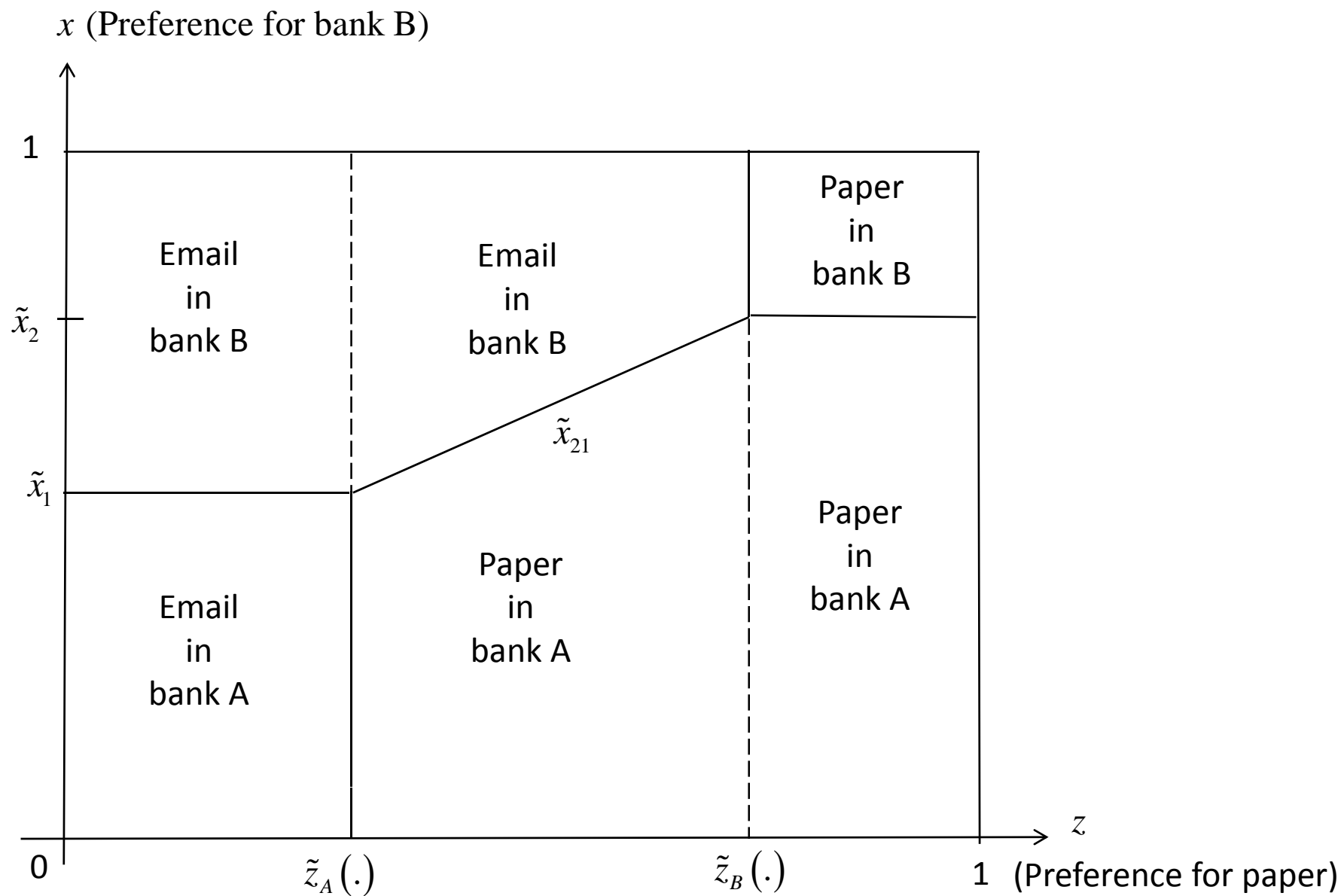
Insert Figure 1 around here.

Consumers with low values of both  $x$  and  $z$  patronize bank  $A$  where they choose electronic media, while consumers with large values of both  $x$  and  $z$  choose bank  $B$  and opt for paper statements there. The rest of the  $Z \times X$  space is divided in two. People with low values of  $z$  and large values of  $x$  prefer electronic medium in bank  $B$ , while agents with large values of  $z$  and small values of  $x$  prefer paper statements in bank  $A$ . Since we have assumed that  $\tilde{z}^A < \tilde{z}^B$ , consumers with intermediate values of  $z$  prefer either electronic medium in bank  $B$  or paper statements in bank  $A$ , as is shown on Figure 1.

From Figure 1, together with the uniform distribution of the unitary mass of consumers over  $Z \times X$ , we obtain the expressions for the demand functions, where



Figure 1: Choice of banks and of medium when  $\tilde{z}_A(\cdot) < \tilde{z}_B(\cdot)$



$Y_i^j$  represents the total demand for good  $i$  in bank  $j$ :<sup>2</sup>

$$\begin{aligned}
Y_1^A(q_1^A, q_2^A, q_1^B, q_2^B) &= \tilde{z}^A(q_1^A, q_2^A) \tilde{x}_1(q_1^A, q_1^B), \\
Y_1^B(q_1^A, q_2^A, q_1^B, q_2^B) &= \tilde{z}^A(q_1^A, q_2^A) (1 - \tilde{x}_1(q_1^A, q_1^B)) + \int_{\tilde{z}^A(q_1^A, q_2^A)}^{\tilde{z}^B(q_1^B, q_2^B)} (1 - \tilde{x}_{21}(q_2^A, q_1^B)) dz, \\
Y_2^A(q_1^A, q_2^A, q_1^B, q_2^B) &= (1 - \tilde{z}^B(q_1^B, q_2^B)) \tilde{x}_2(q_2^A, q_2^B) + \int_{\tilde{z}^A(q_1^A, q_2^A)}^{\tilde{z}^B(q_1^B, q_2^B)} \tilde{x}_{21}(q_2^A, q_1^B) dz, \\
Y_2^B(q_1^A, q_2^A, q_1^B, q_2^B) &= (1 - \tilde{z}^B(q_1^B, q_2^B)) (1 - \tilde{x}_2(q_2^A, q_2^B)).
\end{aligned}$$

We now move to the first stage of the model, namely the profit-maximizing behavior of the two banks.

## 5 Equilibrium transactional media prices

For each good  $i$ , bank  $j$  faces a marginal cost  $c_i^j$ ; fixed costs, if any, do not affect profit maximizing prices (as long as profits are positive). The profit function of bank  $j$  is given by

$$\Pi^j = (q_1^j - c_1^j) Y_1^j(q_1^A, q_2^A, q_1^B, q_2^B) + (q_2^j - c_2^j - p - k^j) Y_2^j(q_1^A, q_2^A, q_1^B, q_2^B).$$

Each bank maximizes its profit by choosing its prices  $q_1^j$  and  $q_2^j$  while taking the prices of the other bank as given (Nash equilibrium).

We concentrate on the case where the banks are totally symmetrical in costs ( $c_i^A = c_i^B = c_i$ ) and in the two transactional products that they offer ( $U_i^A = U_i^B = U_i$ ). On the other hand, the two goods are not symmetrical, because they may differ in marginal cost ( $c_2 \neq c_1$ ) and also in the willingness to pay for them ( $U_1 \neq U_2$ ). We look for a symmetrical equilibrium, where the prices posted by the banks are the same ( $q_i^A = q_i^B, i = 1, 2$ ) so that the marginal consumer indifferent between banks is located at  $x = 1/2$ .

We obtain the following proposition.

**Proposition 1** *There is a unique symmetrical profit-maximizing equilibrium, which is such that*

$$q_i^A = q_i^B = q_i^* = c_i + t.$$

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<sup>2</sup>There is an analogue to Figure 1 where  $\tilde{z}_B < \tilde{z}_A$  when prices and preferences are such that paper is relatively more attractive (compared to its electronic substitute) in bank  $B$  than in bank  $A$ .

**Proof:** See Appendix.

In both banks and for both transactional media, the profit-maximizing symmetrical price is given by the sum of marginal cost and of the transportation cost between banks. This transportation cost reflects the intensity of competition between banks. If  $t = 0$ , banks are seen as perfectly interchangeable by consumers who all patronize the cheapest one. It is then no surprise to obtain marginal cost pricing in that case. As  $t$  increases, the intensity of competition between banks decreases, which leaves more room to increase the transactional media prices.

Observe that  $r$ , the intensity of preferences for one type a transactional medium rather than another, plays no role in the formula of the profit-maximizing media prices. Even if  $r$  is large, banks cannot use this lower sensitivity of the choice of media to their prices to increase these prices, because banks have first and foremost to attract consumers if they want to charge them for transactional media. In other words, how much consumers care for one form of transactional medium versus another has no impact on their prices, which are set by the banks according to the marginal cost of each of these media and intensity of competition between them, the latter as measured in this model by the parameter  $t$ .

Observe also that  $U_i$ , the gross utility obtained from consuming transactional medium  $i$ , does not influence the optimal consumer prices by the banks, for the same reason that  $r$  does not influence them either. On the other hand, the equilibrium volume of good  $i$  is increasing in  $U_i$ , since more consumers buy good  $i$  when  $U_i$  increases, other things remaining equal.

We now make more explicit the link between transactional media costs, mail price and profit-maximizing prices by banks. Assume that  $c_1$  reflects the marginal cost of processing one consumer's information in either bank, and that the marginal cost of making this information available to the consumer by electronic means is zero. Assume as well that this marginal processing costs has also to be borne by a bank in the case of paper statements. In that case, the bank has also to support a marginal mail preparation cost of  $k$  and has to pay the mail price  $p$  for each consumer opting for paper statements. In other words, we assume that

$$c_2 = c_1 + k + p.$$

We then obtain that paper statements should be more expensive than electronic media (since their marginal cost is higher), which results in our model in lower volumes for paper statements than for electronic substitutes. The difference in the prices of the two transactional media should exactly reflect the difference in costs. Also, observe that we have complete pass through of any mail price increase into the final price paid by consumers for mail statements. These observations remain true whatever the intensity of competition between banks. They of course rely heavily on the symmetry assumptions (including the uniform distribution of consumers preferences over both  $X$  and  $Z$ ) made in the paper.

## 6 Conclusion

We have built a model where two banks compete for the patronage of consumers by offering them, among other services and products, two forms of transactional media: paper statements and electronic substitutes. Both banks and both products are horizontally differentiated and modeled à la Hotelling (1929). Assuming symmetry of consumer preferences (over banks and, independently, over the two transactional media) and of banks's costs, we obtain that the unique profit-maximizing symmetrical prices reflect both the transactional media marginal costs and the intensity of competition between banks. Most notably, the intensity of consumers preferences for one variant of transactional medium over another has no influence on the profit-maximizing media prices. Also, there is total pass-through of increases in input prices (such as mail price for paper statements) into prices paid by final consumers.

These results are obtained under the strong assumptions of symmetry in consumers preferences (both for banks and for transactional media) and in banks marginal costs. Bringing this model to empirical studies would then first require to extend our results to non symmetrical environments. We leave this extension for future research.

## Appendix: Proof of Proposition 1

The first-order condition for  $q_1^A$  is given by

$$\begin{aligned} \frac{\partial \Pi^A}{\partial q_1^A} &= \frac{(q_1^A - c_1)((U_1^B - q_1^B) - (U_1^A - q_1^A) - t) + (q_2^A - c_2)((U_1^A - q_1^A) - (U_1^B - q_1^A) + t)}{4rt} \\ &\quad + \frac{(q_1^A - c_1)((U_2^A - q_2^A) - (U_1^A - q_1^A) - r)}{4rt} \\ &\quad + \frac{((U_1^A - q_1^A) - (U_1^B - q_1^B) + t)((U_1^A - q_1^A) - (U_2^A - q_2^A) + r)}{4rt} \\ &= 0. \end{aligned}$$

The first-order condition for  $q_2^B$  is given by

$$\begin{aligned} \frac{\partial \Pi^B}{\partial q_2^B} &= \frac{(q_2^B - c_2)((U_1^B - q_1^B) - (U_2^B - q_2^B) - r) + (q_1^B - c_1)((U_2^B - q_2^B) - (U_2^A - q_2^A) + t)}{4rt} \\ &\quad + \frac{(q_2^B - c_2)((U_2^A - q_2^A) - (U_2^B - q_2^B) - t)}{4rt} \\ &\quad + \frac{((U_2^B - q_2^B) - (U_1^B - q_1^B) + r)((U_2^B - q_2^B) - (U_2^A - q_2^A) + t)}{4rt} \\ &= 0. \end{aligned}$$

When measured at the symmetrical equilibrium ( $q_1^A = q_1^B = q_1$ ,  $q_2^A = q_2^B = q_2$ ), and using the assumptions that  $U_i^A = U_i^B = U_i$  and that  $c_i^A = c_i^B = c_i$  ( $i = 1, 2$ ), these two FOCs simplify to<sup>3</sup>

$$\begin{aligned}
\frac{\partial \Pi^A}{\partial q_1^A} &= \frac{(q_1)^2 + t(U_1 - U_2 - c_2 - 2q_2 + r)}{4rt} \\
&\quad - \frac{c_1(U_1 - U_2 - q_1 + q_2 + r + t)(U_1 - U_2 + q_2 + r + 2t)}{4rt} \\
&= 0, \\
\frac{\partial \Pi^B}{\partial q_2^B} &= \frac{(q_1 - c_1)t + (q_2 - c_2)((U_1 - q_1) - (U_2 - q_2) - r - t)}{4rt} \\
&\quad - \frac{t((U_1 - q_1) - (U_2 - q_2) - r)}{4rt} \\
&= 0.
\end{aligned}$$

Solving these two FOCs simultaneously gives the unique symmetrical equilibrium

$$\begin{aligned}
q_1 &= c_1 + t, \\
q_2 &= c_2 + t.
\end{aligned}$$

## References

- [1] De Donder Ph., Cremer H., Dudley P. and Rodriguez F. (2012a), “Optimal pricing of mail in the transactional market and welfare for the wider communications market”, in Multi-modal competition and the future of mail, edited by M.A. Crew and P.R.Kleindorfer, Cheltenham: Edward Edgar, 206–220.
- [2] Hotelling, H. (1929), Stability in Competition, Economic Journal, 39, 41–57.

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<sup>3</sup>The reader can check that we obtain the exact same formulas if we define  $\Pi^A$  and  $\Pi^B$  using  $\tilde{z}^A(q_1^A, q_2^A) \geq \tilde{z}^B(q_1^B, q_2^B)$ .