Segmentation and nonlinear pricing in the postal sector

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1 Introduction

Network operators typically use quite sophisticated pricing policies. For instance, nonlinear pricing (second degree price discrimination) has a long tradition in the telecommunication and energy sectors. Similarly, in these and other sectors, prices are often differentiated according to market segments (third degree discrimination). Even though it is not the most prominent textbook example to illustrate such pricing policies, the postal sector is in fact no exception. Ramsey pricing is effectively a form of third degree price discrimination, based on price elasticities. In addition, most postal operators have volume discount programs (a form of nonlinear pricing). While regulators and competition authorities are often reluctant to accept these practices, economists tend to view differentiated pricing policies in a more positive way. For a regulated operator, they are an effective way to cover fixed cost while mitigating distortions that would be associated with straight linear pricing. When the regulatory policy is well designed and the operator is welfare maximizing, sophisticated pricing policies can of course only be welfare enhancing. However, when the operator is profit maximizing, this is no longer necessarily true: the scenario where a profit-maximizing operator may engage in pricing policies that are detrimental to welfare can then no longer be ruled out. Still, the literature has shown that departures from linear pricing are often welfare enhancing, even when the operator is profit maximizing.

The implications of differentiated pricing policies for the postal sector have received little attention in the literature. A notable exception is Crew and Kleindorfer (2011) who study the implications of volume discounts. They show that nonlinear pricing can contribute to the financial viability of an operator who faces market opening and intermodal competition (in particular from electronic media).

In this paper we study pricing policies, which combine market segmentation (third degree price discrimination) with nonlinear pricing (second degree price discrimination) and which are referred to as segmented nonlinear pricing (SNLP). More precisely, we assume that the operator can group customers into a certain number of categories on the basis of an exogenously observable characteristic at no cost. Under standard third degree discrimination, the operator charges linear prices that differ across categories. Under SNLP, the operator can use category specific nonlinear tariffs. This is a common practice for telecommunications (particularly mobile phone) and in airlines where different menus of tariffs are offered to the young, the students, or the elderly or where professional customers are treated differently from households. It is also often referred to as tagging in particular in the public economics literature.[1] In the postal sector, SNLP has a number of potential applications. In particular, it could be used to improve the design of volume discounts by targeting them to the categories of customer who are most likely to switch to electronic substitutes. In a context of decreasing mail volume, such pricing policies could mitigate this decrease and boost volumes. Consequently, the could become a crucial factor in safeguarding universal postal service. While our analysis provides a methodological contribution that goes beyond the postal sector, we focus our attention on specific postal issues. This is reflected in our illustrative examples but also in our modeling assumptions. In particular, we include the feature that customers always have the option to use the public stamp rate (reservation utility is determined by a linear tariff and is thus type specific).

The design of nonlinear tariffs is first and foremost an issue of asymmetric information. The operator does not observe the individual customer's willingness to pay. Under full information, the operator would induce an efficient allocation. This is because it can extract the entire surplus and thus chooses a policy to maximize it. Under asymmetric information, there is a trade-off between rent extraction and efficiency and quantities are distorted (typically downward) to mitigate informational rents. Under SNLP, the operator observes the customer's category, which provides some information about the distribution of his type (willingness to pay). More precisely, it can use the intra category distribution of types (rather than the overall distribution) to offer segment-specific tariffs.

We characterize the solution under SNLP in a simple four types, two categories

[1]See Cremer *et al.* (2010). This paper (and a number of others which are referenced there) studies the design of nonlinear tax schedules that can be conditioned on some observable characteristic (like age or gender). While this appears at first to be a very different question from the one we are considering here, the formal problem is very similar.

setting and compare it to the standard NLP (nonlinear pricing) solution. We provide a set of sufficient conditions under which SNLP (used by a profit maximizing operator) is welfare improving. When these conditions are violated, a welfare improvement continues to be possible but whether it materializes depends on the preference and cost structure. For either case, we use numerical examples to study the impact of the pricing schemes on the various customer groups and on the operator's profit.

We start with the basic model, wherein the willingness to pay in the two groups does not overlap and where individual's reservation utility is exogenously given. This is the simplest setting and yields the most clear-cut results. We then consider two extensions. In the first, we consider the case of overlapping groups (Subsection 6.1). In the second, we assume that a simple linear tariff (the stamp price) is available to all customers and determines their reservation utility levels (Subsection 6.2). We study how this specific feature of the postal sector affects our results. Finally we provide some illustrative numerical examples in Section 7.

2 Basic model

The utility of a customer of type i who consumes quantity q_i and pays t_i is given by

$$\theta_i u(q_i) - t_i,$$

where u is increasing and strictly concave ($u' > 0$ and $u'' < 0$) while θ_i is a parameter reflecting the valuation of the good. When the consumer does not buy the good he has a given utility level u_0 (the same for all types) which we can normalize to zero without loss of generality. There are two categories of customers, a and b, and each category consists of two types of customers, 1 and 2, differing in their willingness to pay, with a higher willingness to pay for customer 2. Overall there are thus 4 types of individuals characterized by their valuation of the good: $\theta_{1a}, \theta_{2a}, \theta_{1b}, \theta_{2b}$, with $\theta_{1j} < \theta_{2j}$, $j = a, b$. Assume for the time being that $\theta_{2a} < \theta_{1b}$ so that the two groups do not overlap. In other words the consumer with the highest valuation in group a has a lower willingness to pay than the consumer with the lowest valuation in group b. The distribution of types is common knowledge and the number of type i individuals is denoted n_i, where

$i = 1a, 2a, 1b, 2b$.

There is a single operator whose cost function is given by

$$C(\theta_{1a}, \theta_{2a}, \theta_{1b}, \theta_{2b}) = F + \sum_i n_i c_i q_i,$$

where c_i is the constant marginal cost which may or may not be the same for all types.

To make the case for nonlinear pricing and segmentation as difficult as possible we assume that the operator maximizes profit. When the operator is welfare maximizing it is plain that more sophisticated pricing policies can only lead to a welfare improvement. However, when the operator is profit maximizing, there is a potential conflict between private and social objectives. A more sophisticated pricing policy then necessarily leads to a higher profit, but its impact on welfare is *a priori* ambiguous.

3 Nonlinear pricing without tagging (NLP)

With the assumption that groups are separate we have $\theta_{1a} < \theta_{2a} < \theta_{1b} < \theta_{2b}$. We assume here and throughout the paper that incentive constraints are binding according to decreasing (and adjacent) θ.[2] To simplify notation (and without loss of generality) we normalize n_i's to 1.

The profit-maximizing problem of the operator can then be written as follows

$$\max \quad t_{1a} + t_{2a} + t_{1b} + t_{2b} - F - (c_{1a}q_{1a} + c_{2a}q_{2a} + c_{1b}q_{1b} + c_{2b}q_{2b}) \tag{1}$$

$$\text{s.t.} \quad \theta_{2b}u(q_{2b}) - t_{2b} = \theta_{2b}u(q_{1b}) - t_{1b}, \tag{2}$$

$$\theta_{1b}u(q_{1b}) - t_{1b} = \theta_{1b}u(q_{2a}) - t_{2a}, \tag{3}$$

$$\theta_{2a}u(q_{2a}) - t_{2a} = \theta_{2a}u(q_{1a}) - t_{1a}, \tag{4}$$

$$\theta_{1a}u(q_{1a}) - t_{1a} = 0. \tag{5}$$

The decision variables are (t_i, q_i), $i = 1a, 2a, 1b, 2b$. To set up this problem we have used the standard textbook result that the participation constraint is binding for the

[2] With two types this is necessarily the case. With more than two types this involves some assumptions on the distribution of types (similar to the monotone hazard rate assumption in the continuum case). Without such a regularity assumption we could get solutions with "bunching" (two types receive the same consumption bundles). This would make the analysis more cumbersome without affecting the results.

low type (equation 5), while the utility level of the other types is determined by a series of binding incentive constraints (equations 2–4). The easiest way to solve this problem is to substitute the t_i's in the objective function from the constraints. Combining the last two constraints we obtain

$$t_{2a} = \theta_{2a}u(q_{2a}) - (\theta_{2a} - \theta_{1a})u(q_{1a}), \tag{6}$$

which together with (3) yields

$$t_{1b} = \theta_{1b}u(q_{1b}) - (\theta_{1b} - \theta_{2a})u(q_{2a}) - (\theta_{2a} - \theta_{1a})u(q_{1a}). \tag{7}$$

Combining this equation with (2) yields

$$t_{2b} = \theta_{2b}u(q_{2b}) - (\theta_{2b} - \theta_{1b})u(q_{1b}) - (\theta_{1b} - \theta_{2a})u(q_{2a}) - (\theta_{2a} - \theta_{1a})u(q_{1a}). \tag{8}$$

Substituting these expressions into the objective function (1) yields the following reformulated problem

$$\begin{aligned}\max_{q_{1a},q_{2a},q_{1b},q_{2b}} \quad & \theta_{1a}u(q_{1a}) + \theta_{2a}u(q_{2a}) + \theta_{1b}u(q_{1b}) + \theta_{2b}u(q_{2b}) - F - (c_{1a}q_{1a} + c_{2a}q_{2a} + c_{1b}q_{1b} + c_{2b}q_{2b}) \\ & - (\theta_{2b} - \theta_{1b})u(q_{1b}) - 2(\theta_{1b} - \theta_{2a})u(q_{2a}) - 3(\theta_{2a} - \theta_{1a})u(q_{1a}). \end{aligned} \tag{9}$$

The first line of this expression represents total surplus, while the second line measures the total informational rents that have to be conceded. Under full information (first degree price discrimination), where no informational rents are conceded, the operator maximizes total surplus which it can then totally absorb. The objective function then reduces to the first line of (9). This would yield a (first-best) optimal solution which is given by

$$\theta_{2b}u'(q^*_{2b}) = c_{2b}, \tag{10}$$

$$\theta_{1b}u'(q^*_{1b}) = c_{1b}, \tag{11}$$

$$\theta_{2a}u'(q^*_{2a}) = c_{2a}, \tag{12}$$

$$\theta_{1a}u'(q^*_{1a}) = c_{1a}. \tag{13}$$

When the customers' willingness to pay is private information the operator cannot extract the full surplus and this introduces a wedge between private and social surplus.

Differentiating (9) with respect to the consumption levels of the different types yields the following first-order conditions (FOCs)

$$\theta_{2b}u'(q_{2b}^{nt}) = c_{2b}, \tag{14}$$

$$(2\theta_{1b} - \theta_{2b})u'(q_{1b}^{nt}) = c_{1b}, \tag{15}$$

$$(3\theta_{2a} - 2\theta_{1b})u'(q_{2a}^{nt}) = c_{2a}, \tag{16}$$

$$(4\theta_{1a} - 3\theta_{2a})u'(q_{1a}^{nt}) = c_{1a}, \tag{17}$$

where the superscript nt is used to refer to the non-tagged (or NLP) solution.[3]

Comparing the NLP solution defined by (14)–(17) with the first-best allocation given by (10)–(13) yields:

$$q_{2b}^{nt} = q_{2b}^{*}, \tag{18}$$

$$q_{1b}^{nt} < q_{1b}^{*}, \tag{19}$$

$$q_{2a}^{nt} < q_{2a}^{*}, \tag{20}$$

$$q_{1a}^{nt} < q_{1a}^{*}. \tag{21}$$

These expressions show that we have the first-best solution ("no distortion at the top") for the type with the highest willingness to pay, and lower than (socially) optimal consumption levels for all other types. These are all standard properties of NLP models. In particular, the downward distortion on the consumption level of any given group mitigates the informational rents of all types with higher willingness to pay.[4]

4 Nonlinear pricing with tagging (SNLP)

We now assume that the group a or b to which a customer belongs is observable and that the pricing schedule (price-quantity bundle) can be conditioned on $j = a, b$. Observe

[3] The solution described by these expressions is of course only meaningful when all the q's are positive and when the consumption level increases with θ. This is where the assumptions on the distribution of θ's we mentioned above are needed. If either of these conditions is violated the solution involves bunching of two or more types. We would then have to consider a number of different cases. This does not involve any major difficulty but it would complicate the exposition and distract attention away from the main point we want to make.

[4] This can be seen from the second line of expression (9). This effect is not relevant for the type with the highest willingness to pay which explains that $q_{2b}^{nt} = q_{2b}^{*}$.

that while categories a and b are observable, the position (1 or 2) of the customer inside the category is *not* observable. For example, if an individual is of type a, the operator knows that his willingness to pay parameter is either θ_{1a} or θ_{2a} , but it cannot distinguish between these two types. Observe that since the distribution of types is common knowledge, the operator also knows the distribution of willingness to pay, conditional on the group. Roughly speaking the tagging (or segmentation) thus amounts to using some observable information to get more "precise" (albeit still incomplete) information on the customers' willingness to pay.

To determine the optimal policy in tag $j = a, b$, we solve

$$\begin{aligned} \max_{q_{1j},q_{2j},t_{1j},t_{2j}} \quad & t_{1j} + t_{2j} - F - (c_{1j}q_{1j} + c_{2j}q_{2j}) \\ \text{s.t.} \quad & \theta_{2j}u(q_{2j}) - t_{2j} = \theta_{2j}u(q_{1j}) - t_{1j}, \\ & \theta_{1j}u(q_{1j}) - t_{1j} = 0. \end{aligned}$$

Within each group the participation constraint of the low type and the incentive constraint of the high type are binding. Observe that we no longer have to worry about incentive constraints "between groups". For instance individuals of type $2b$ can take benefit from the pricing scheme designed for type $1b$, but they cannot benefit from any of the pricing schemes designed for group a (because the category to which the individual belongs is observable).

Combining the constraints like in the non-tagged case and substituting into the objective function we obtain in each tag $j = a, b$

$$\begin{aligned} \max_{q_{1j},q_{2j}} \quad & \theta_{1j}u(q_{1j}) + \theta_{2j}u(q_{2j}) - F - (c_{1j}q_{1j} + c_{2j}q_{2j}) \\ & - (\theta_{2j} - \theta_{1j})u(q_{1j}). \end{aligned}$$

Once again, the first line of this expression represents the total surplus (within the group), while the second line represents the informational rent that has to be conceded.

The FOCs are given by

$$\theta_{2j}u'(q^t_{2j}) = c_{2j} \tag{22}$$

$$(2\theta_{1j} - \theta_{2j})u'(q^t_{1j}) = c_{1j}, \tag{23}$$

where the superscript t is used to refer to the tagged (*i.e.,* SNLP) solution. Comparing this solution with the first-best allocation given by (10)–(13) yields

$$q_{2j}^{t} = q_{2j}^{*}, \tag{24}$$

$$q_{1j}^{t} < q_{1j}^{*}. \tag{25}$$

These expressions show that we now have the first-best solution for the type with the highest willingness to pay within each group, and lower than (socially) optimal consumption level for the other types.

5 Tagged vs. non-tagged solution

We are now in a position to compare the two solutions (NLP and SNLP). It is plain that the solution with tagging yields a higher level of profit for the operator. More precisely, profits cannot be lower because the operator continues to have the option to offer the same pricing scheme as under NLP. This is of course just a fallback option for the operator and in general we can expect the solution to differ in which case profits will be strictly larger with tagging. The less straightforward question is how welfare is affected by tagging. To deal with this issue, recall that total surplus in our setting (expressed by the first line of (9)) only depends on the consumption levels of the various types. Comparing these consumption levels across solutions by making use of expressions (10)–(13), (14)–(17) and (22)–(23) yields

$$q_{2b}^{nt} = q_{2b}^{t} = q_{2b}^{*},$$

$$q_{1b}^{nt} = q_{1b}^{t} < q_{1b}^{*},$$

$$q_{2a}^{nt} < q_{2a}^{t} = q_{2a}^{*},$$

$$q_{1a}^{nt} < q_{1a}^{t} < q_{1a}^{*}.$$

In words, when tagging is introduced, the solution in the highest group does not change, but we move closer to the optimal solution in the lower tag. More precisely, the high type in the low group now also consumes the efficient level, while the consumption level of the low type increases but remains below the optimal level. These results (along with the property that total surplus, S, is a concave function of any of the consumption levels) immediately implies that welfare (total surplus) is higher with tagging than without.[5] In other words, *in our setting the profit-maximizing solution under tagging (SNLP) always yields a higher level of welfare than the profit-maximizing solution without tagging (NLP).*

Intuitively, this result can be explained as follows. As shown by equations (6)–(8) the consumption level of any type, i, affects the informational rents of all the "higher" types (with a higher willingness to pay) to which type i individuals are "connected" directly or indirectly via binding incentive constraints. The optimal policy strikes a balance between surplus (which can potentially be extracted) and information rents and to mitigate these rents, consumption levels are distorted downwards. From this perspective, we can easily understand that for group b nothing changes when tagging is introduced. Specifically, the q_{1b} equally affects the rents of type $2b$ in both cases (while q_{2b} does not affect any rents). Now in the untagged case q_{2a} and q_{1a} affect the rents of all types in group b and q_{1a} additionally affects the rents of type $2a$. When tagging is introduced, the link between groups is cut; inter-group mimicking is no longer possible. Consequently, q_{2a} no longer affects any rents (and is left undistorted) while q_{1a} solely influences the rents of type $2a$. This argument not only explains the result but it also indicates that it is quite robust and could easily be generalized to more general distributions of the taste parameter. In particular, it immediately follows for the case where the tags constitute a partition of a continuous distribution into two (or more intervals).

Note that this is exactly similar to age related discounts in the airline sector. Airlines can give major discounts to young passenger without worrying that this offer be used by top level executives with a high willingness to pay simply because there are few such

[5] To see this, differentiate the first line of (9) with respect to q_i which yields

$$\frac{\partial S}{\partial q_i} = \theta_i u'(q_i) - c_i,$$
$$\frac{\partial^2 S}{\partial q_i^2} = \theta_i u''(q_i) < 0.$$

executives in that age group.

6 Variations and extensions

To assess the robustness of this result we now consider two variations and extensions. In the first, we consider the case of overlapping groups. In the second, we assume that a simple linear tariff (the stamp price) is available to all customers and determines their reservation utility levels.

6.1 Overlapping groups

Assume now that $\theta_{1a} < \theta_{1b} < \theta_{2a} < \theta_{2b}$ so that the groups overlap in the sense that (as far as θ is concerned) the high type of the low group is above the low type of the high group. It is plain that this has no impact on the rules that determine tagged solution which continues to be given by expressions (22)–(23) in Section 4. Note however that the actual levels change (θ's are different so that the solution will differ). Similarly, the first-best solution is determined by the same expression as above, namely equations (10)–(13). However, the untagged solution will change. Continuing to assume that incentive constraints are binding according to decreasing θ, we now get a different pattern of binding constraints. Accordingly the profit maximizing problem of the operator is now given by

$$\max \quad t_{1a} + t_{2a} + t_{1b} + t_{2b} - F - (c_{1a}q_{1a} + c_{2a}q_{2a} + c_{1b}q_{1b} + c_{2b}q_{2b}) \tag{26}$$

$$\text{s.t.} \quad \theta_{2b}u(q_{2b}) - t_{2b} = \theta_{2b}u(q_{2a}) - t_{2a}, \tag{27}$$

$$\theta_{2a}u(q_{2a}) - t_{2a} = \theta_{2a}u(q_{1b}) - t_{1b}, \tag{28}$$

$$\theta_{1b}u(q_{1b}) - t_{1b} = \theta_{1b}u(q_{1a}) - t_{1a}, \tag{29}$$

$$\theta_{1a}u(q_{1a}) - t_{1a} = 0. \tag{30}$$

Proceeding exactly like in Section (3), namely substituting the t_i's in the objective function from the constraints and then differentiating with respect to q_i's, we obtain the following FOCs

$$\theta_{2b} u'(q_{2b}^{nto}) = c_{2b},$$
$$(2\theta_{2a} - \theta_{2b}) u'(q_{2a}^{nto}) = c_{2a},$$
$$(3\theta_{1b} - 2\theta_{2a}) u'(q_{1b}^{nto}) = c_{1b},$$
$$(4\theta_{1a} - 3\theta_{1b}) u'(q_{1a}^{nto}) = c_{1a},$$

where the superscript *nto* is used to denote the non-tagged (NLP) solution with overlapping groups. Comparing the different solutions we get

$$q_{2b}^{nto} = q_{2b}^{t} = q_{2b}^{*}, \tag{31}$$
$$q_{2a}^{nto} < q_{2a}^{t} = q_{2a}^{*}, \tag{32}$$
$$q_{1b}^{nto} \gtreqless q_{1b}^{t} < q_{1b}^{*}, \tag{33}$$
$$q_{1a}^{nto} \gtreqless q_{1a}^{t} < q_{1a}^{*}. \tag{34}$$

These expressions show that the three solution yield the same level of q_{2b}, while the move to tagging increases q_{2a} and brings it closer to its optimal level. Consequently, the total surplus generated by the high type in each group increases when tagging is introduced. On the other hand, the impact of tagging on the consumption level of low types (and thus on the surplus generated by them) is ambiguous. Depending on the distribution of θ's tagging can thus increase as well as decrease consumption and total surplus of low types. To sum up, with overlapping groups the impact on welfare is ambiguous. We shall revisit this case (and show that a welfare enhancement continues to be possible) in the numerical Section 7.

6.2 Linear pricing as outside option

So far we have assumed that the reservation utility was the same for all and normalized to zero. The normalization to zero is not important and has no impact on the results. However, the assumption that the reservation utility is the same for all agents is important. In the postal sector we can think about the reservation utility as being determined by the stamp price; every agent has the option of not accepting the contract and paying the normal stamp price for its mail. The reservation utility of an agent with parameter θ is then given by

$$v(p, \theta) = \max_{q} \quad \theta u(q) - pq, \tag{35}$$

where p is the stamp price.

To determine the optimal pricing scheme (optimal contract) both without and with tagging, we now have to solve a principal-agent problem with type-dependent participation constraints which does not necessarily yield nice and simple solutions; see *e.g.*, Laffont and Martimort (2001, Section 3.3.). Depending on the profile of binding incentive and participation constraints many different regimes can occur.[6] Here, the profile of participation constraints is not arbitrary but determined by (35) so that we can obtain some results. In particular, we can look for conditions under which the various solutions considered in Sections 3 and 4 remain valid under the reformulated participation constraints. More precisely, we examine when we will obtain the same quantities for everyone while payments are shifted by a constant.

The participation constraint will be binding for individual θ_{1a} and his utility is given by $v(p, \theta_{1a})$ (rather than zero). This positive utility level adds a constant to the (profit) maximizing problem which has no impact on the FOCs and on the optimal quantities.

With the consumption profile determined in Section 3 the utility of type θ_{2a} is determined by the incentive constraint and now given by

$$(\theta_{2a} - \theta_{1a})u(q_{1a}^{nt}) + v(p, \theta_{1a}).$$

The solution we derived continues to be valid if

$$(\theta_{2a} - \theta_{1a})u(q_{1a}^{nt}) + v(p, \theta_{1a}) \geq v(p, \theta_{2a}). \tag{36}$$

In words, (36) says that the incentive constraint implies a utility level for θ_{2a} that is sufficiently large that the (reformulated) participation constraint is automatically satisfied. We can derive similar conditions for the other individuals as well as for the

[6] When the reservation utility is the same for all types, a binding participation constraint for the low type along with the incentive constraints for the other types ensures that no further participation constraints are binding. When the reservation utility level depends on the type this is no longer necessarily true.

tagged solution. Note that even when the optimal quantities are not affected the sharing of the surplus is affected.

To get a better understanding of these conditions let us consider an illustration with a simple specification of preferences, namely

$$u(q) = 2\sqrt{q}. \tag{37}$$

The indirect utility is defined by

$$v(p,\theta) = \max_{q} \quad 2\theta\sqrt{q} - pq,$$

which yields

$$q = \frac{\theta^2}{p^2},$$

and

$$v(p,\theta) = \frac{\theta^2}{p}.$$

Further, we obtain from (17) that

$$q_{1a}^{nt} = \frac{(4\theta_{1a} - 3\theta_{2a})^2}{c_{1a}^2},$$

so that condition (36) can be rewritten as

$$2(\theta_{2a} - \theta_{1a})\frac{(4\theta_{1a} - 3\theta_{2a})}{c_{1a}} + \frac{\theta_{1a}^2}{p} \geq \frac{\theta_{2a}^2}{p}.$$

Rearranging yields

$$2\frac{(4\theta_{1a} - 3\theta_{2a})}{c_{1a}} \geq \frac{\theta_{1a} + \theta_{2a}}{p}.$$

This condition will certainly be satisfied when p is sufficiently large. Note that p has to be "much" larger than c because it includes some markup to cover the fixed cost (or part of it).

To sum up, the solutions derived above, as well as all the welfare comparisons remain valid when the linear tariff becomes available provided that the stamp price is sufficiently large.

	NLP	SNLP
q_{1a}	0,01	1,21
q_{2a}	0,09	4,41
q_{1b}	1	1
q_{2b}	25	25
CS_{1a}	0	0
CS_{2a}	0,01	0,11
CS in a	0,01	0,11
CS_{1b}	0,064	0
CS_{2b}	0,464	0,4
CS in b	0,528	0,4
CS Total	0,538	0,51
Profit in a	0,138	0,562
Profit in b	2,472	2,6
Total profit	2,61	3,162
Welfare in a	0,148	0,672
Welfare in b	3	3
Welfare	3,148	3,672

Table 1: Solution without and with tagging in the separate group case $\theta_{1a} = 0.16 < \theta_{2a} = 0.21 < \theta_{1b} = 0.3 < \theta_{2b} = 0.5$.

7 Numerical example

We now present some illustrative numerical examples. Like in the previous example we assume that utility is given by (37). Marginal costs are the same for all types and given by $c_{1a} = c_{2a} = c_{1b} = c_{2b} = 0.1$. The distribution of θ's is specified as $\theta_{1a} = 0.16 < \theta_{2a} = 0.21 < \theta_{1b} = 0.3 < \theta_{2b} = 0.5$, which corresponds to the separate group case.

Results are reported in Table 1. The numerical results (of course) confirm the analytical ranking of quantities when non linear prices are used. Specifically, we obtain the same ranking for welfare and for profit: tagging is better than no tagging.

We now move to the numerical examples with overlapping groups. The easiest way to proceed is to take the same four values of θ, but to swap the values of θ_{2a} and θ_{1b}, to obtain $\theta_{1a} = 0.16 < \theta_{1b} = 0.21 < \theta_{2a} = 0.3 < \theta_{2b} = 0.5$. The results are reported in Table 2.

The solutions without tagging are not affected by this change, except for the renam-

	NLP	SNLP
q_{1a}	0,01	0,04
q_{2a}	1	9
q_{1b}	0,09	0,64
q_{2b}	25	25
CS_{1a}	0	0
CS_{2a}	0,064	0,056
CS in a	0,064	0,056
CS_{1b}	0,01	0
CS_{2b}	0,464	0,464
CS in b	0,474	0,464
CS Total	0,538	0,52
Profit in a	0,467	0,904
Profit in b	2,143	2,308
Total profit	2,61	3,212
Welfare in a	0,531	0,96
Welfare in b	2,617	2,772
Welfare	3,148	3,732

Table 2: Solution without and with tagging in the overlapping group case with $\theta_{1a} = 0.16 < \theta_{1b} = 0.21 < \theta_{2a} = 0.3 < \theta_{2b} = 0.5$.

ing of θ_{1b} into θ_{2a}, and vice versa. Total consumer surplus, profit and welfare are not affected by this renaming of θs, but of course their distribution among the two categories a and b is affected. As for the separate groups case, we still obtain a higher welfare level with tagging than without.

We have also studied how the non-linear optimal solutions resist to the introduction of a linear segment (the stamp price) as an outside option in the separate group case. Intuitively, the larger the value of p, the lower this outside utility. Starting with the solution without tagging, we investigate the minimum stamp price p that is such that the optimal solution we identify above gives more utility to the four types of consumers than the outside option which consists of using stamped mail. By definition, the lowest type θ_{1a} receives exactly this outside utility level in the optimal non-linear solution with stamps. The minimum value of p is then $p = 1.850$ for θ_{2a}, $p = 1.006$ for θ_{1b} and $p = 0.484$ for θ_{2b}. We then obtain that, even though the outside utility (with the stamp) increases with θ, the utility (or rent) level at the optimum non-linear allocation we have identified increases even faster with θ, so that higher θs resist better (i.e., to smaller values of p) to the introduction of the stamp than lower θs.

We have performed the same exercise for the non-linear tagged solution. Due to the tag, we only have to check two participation constraints, those of individuals 2 in both groups (we have kept the separate group assumption here). We obtain that the minimum value of p satisfying the participation constraint with a linear outside option is $p = 0.168$ for θ_{2a} and $p = 0.4$ for θ_{2b}. So, unlike in the tagging solution, the value of p actually increases with θ. Observe that the value of p is lower, for a given θ_{2j}, with tagging than without tagging. We can see from Table 1 that the tagged solution gives more utility than the non-tagged to agents 2 in group a, allowing this solution to better resist the introduction of a linear outside opportunity. As for group b, the utility level of θ_{2b} is actually lower with tagging than without, but the participation constraint is also different, since we add the larger $v(p, \theta_{1b})$ to the utility with tagging, compared to $v(p, \theta_{1a})$ without tagging. This latter effect is larger than the former in our setting, resulting in a (slightly) lower minimum value of p with tagging than without.

8 Conclusion

We have studied pricing policies which combine market segmentation (tagging) with nonlinear pricing. More precisely, we have assumed that the operator can group customers into a certain number of categories on the basis of an exogenously observable characteristic at no cost. We have characterized the solution under SNLP in a simple four types, two categories setting and compared it to the standard NLP solution. We have shown that when the groups do not overlap (the highest willingness to pay in the low group is smaller than the lowest willingness to pay in the high group) the profit maximizing SNLP solution yields a higher welfare (total surplus) than the standard NLP solution. While we have restricted our attention to the simplest possible setting, this result could easily be generalized to more general distributions of the taste parameter (and in particular to the case of a continuous distributions). This result shows that the use of nonlinear pricing within categories may lift the well-known ambiguity regarding the welfare impact of third-degree price discrimation.

We then have considered two extensions. In the first, we have analyzed the case of overlapping groups. We have shown that the total surplus generated by the high type in each group increases when tagging is introduced. On the other hand, the impact of tagging on the consumption level of low types (and thus on the surplus generated by them) is ambiguous. We have provided an example where the total welfare impact continues to be positive, but this exercise has merely illustrative value. Consequently, the welfare impact of tagging in that case is essentially an empirical question which hinges on the distribution of the taste parameter. In the second extension, we have assumed that a simple linear tariff (the stamp price) is available to all customers and determines their reservation utility levels. We have shown that this specific feature of the postal sector does not affect our results provided that the stamp price is sufficiently large.

To sum up, this paper shows that conceding more pricing flexibilty to a profit-maximizing operator who is able to categorize its customers according to their valuation of postal services, can be welfare enhancing. In a context of declining mail volumes, the positive impact of such discriminatory pricing policy on volume should be taken into account and considered as an instrument to safeguard universal postal service.

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