Pricing and imperfect competition in the postal sector[1]

Etienne Billette de Villemeur
University of Toulouse (IDEI and GREMAQ)
21 Allée de Brienne
31000 Toulouse

Helmuth Cremer,[2]

University of Toulouse (IDEI and GREMAQ)
21 Allée de Brienne
31000 Toulouse

Bernard Roy and Joëlle Toledano[3]

La Poste
44 Boulevard de Vaugirard,
75757 Paris Cedex 15

October 2003

Preliminary and incomplete
Please do not quote

[1]We would like to thank Philippe De Donder for his insightful comments and suggestions.

[2]Corresponding author; e-mail: helmut@cict.fr

[3]The views expressed in this paper are those of the authors and do not necessarily reflect the views of La Poste.

1 Introduction

In many network industries like telecommunication, electricity gas, etc., the ongoing liberalization process has spurred an intense debate on the phenomenon of "downstream access". Accordingly this subject has been extensively studied in the economic literature.[1] In the postal sector the issue of "access" has been relevant long before the debate on liberalization was launched. However, it has appeared under a different form namely, the phenomenon of worksharing. Processing workshared mail at a discounted rate is effectively like providing the client with access to one or several segments of the postal network. Like in the case of downstream access, we have a situation where the postal operator sells some products which use only part of its network, while other products use the entire network. Put differently, the postal sector has the specific feature that access is a relevant issue even when there are no competing operators in the market. This is reflected in the existing literature on worksharing which typically considers a monopolistic sector; see Billette de Villemeur et al. (2002, 2003).[2] The structure of prices derived in this literature has to be reconsidered when the market opens. This problem arises for two reasons. First, there is the standard problem that pricing rules under (perfect or imperfect) competition typically differ from those under monopoly. Second, once entry has occurred, the demand for workshared mail may in part emanate from the competing operators.

The regulatory design of postal prices including those for workshared mail is essentially a Ramsey-Boiteux pricing problem. The underlying issues are very simple. The incumbent operator offers different products to different types of customers and we can think of workshared mail as one of these products. While some of these products are final goods, some like worhshared mail may be intermediate goods which are used as inputs

[1] See for instance Laffont and Tirole (1996, 2000) and Armstrong (2002).

[2] See also Panzar (2002), Mitchel (1999) and Sherman (2001).

by other firms. The pricing of this intermediate good then indirectly determines the prices paid by the final consumers of these products. In a "perfect" (first-best) world the pricing rules for all these products are very simple. We know from microeconomic economic theory that the appropriate rule is simply (long-run) marginal cost for all products. This provides consumers with the correct signals and ensures that the decentralized outcome is efficient.

In an industry like the postal sector, where technology involves "fixed" costs (like the cost of maintaining the delivery network) it is however, typically the case that marginal cost (even long-run marginal costs) are well below average costs.[3] Strict marginal cost pricing is then problematic because it implies that the operator cannot break even, which is usually considered as not acceptable for a number of reasons (including political economy considerations).[4] Consequently, one would have to impose positive markups on at least some products in order to meet the break-even constraint. The determination of these markups is precisely what the Ramsey-Boiteux problem is all about. The question is simply how to distort the different prices away from marginal cost in order to break-even while keeping the efficiency cost of these distortions as small as possible. The exact specification of this problem and hence the specific results depend on the characteristics of the industry (costs, technology and demand) the general regulatory environment (e.g., the presence of a uniform pricing constraint) and on the type of competition there is between the incumbent and the entrant(s) (competitive fringe, monopolistic competition, some form of oligopoly, etc.). The literature so far has concentrated on the competitive fringe case for which a number of interesting results have been obtained.[5] Pricing rules for final goods and for

[3]Like most of the regulation literature we use the term fixed cost for the part of cost which is independant of output, *even in the long run.*

[4]In a first-best setting this problem can be overcome by a lump-sum tranfer to the operator covering its fixed cost.

[5]See e.g., Billette de Villemeur et al. (2003), Cremer et al. (1995, 1997), Crew and Kleindorfer (2002), De Donder et al. (2002, 2003).

workshared mail are typically inverse-elasticity rules, properly amended to account for cross-price effects (if any). Consequently, optimal prices depend on demand consideration and not just on cost considerations. This is different from the first best setting where prices simply reflect marginal costs. This analysis has to be completed with studies of imperfect competition settings. This is an ambitious research program. In the current paper we aim at taking a step in that direction by considering a fully fledged model of a differentiated oligopoly where operators interact strategically. Our findings suggest that while the specific results have to be amended, many of the general principles governing pricing in competitive fringe settings remain applicable under imperfect competition.

Summing up, it appears that the economists' toolbox regarding pricing in the postal sector, though still in need to be expanded, does already have the potential to offer valuable guidance in the regulatory debate. In many instances, however, all these studies are ignored by the various parties involved, regulators and postal representatives alike. Instead, the debate concentrates on the relative merits of two essentially ad hoc rules (or classes of rules). The first of these ad hoc approaches is the so called Efficient Component Pricing Rule (ECPR) which (roughly speaking) consists in applying the same (per unit) markup on workshared that is applied on the corresponding final product offered by the incumbent operator. Consequently, worksharing per se does not appear to affect the incumbent's profits and more generally its ability to cover its fixed costs. This rule has the theoretical merit that it leads (under some conditions) to efficient entry decisions: entry occurs if and only if the entrant is more efficient. However, these apparent properties often do not stand under closer scrutiny. Further, the rule is incomplete and does not explain how the markup on the incumbent's relevant final product ought to be determined. In addition and most significantly, it does not in general yield an efficient Ramsey-Boiteux pricing, even when one ignores the determination of all the other prices.

Another *ad hoc* rule which is sometimes advocated by regulators (for instance in the UK) is a simple "cost-plus" rule, where the access price is obtained from the long run marginal cost by applying some *ad hoc* markup. This may at first sound similar to a Ramsey-Boiteux approach and there exists of course a specific level of the markup for which we obtain the Ramsey price (or alternatively the ECPR level). The crucial difference, however, is that no effort in made to optimize the markup by accounting for instance for demand considerations. Instead the markup is set in a ad hoc way. For instance a small markup is added in order to cover the cost of universal service which in turn is calculated in a questionable way (and thus often significantly underestimated; see Cremer et al. (2000)). There is no reason to expect that such a procedure can yield anything which comes close to the optimal prices.

A fully fledged model of postal sector pricing would have to account for both the clients and the competitors demand for workshared mail. This is, however, a rather ambitious endeavor which will occupy our research agenda in the near future (and maybe beyond). We proceed with this project by addressing the problems one at a time. In this paper we continue our quest for simple and intuitive but yet theoretically founded optimal pricing rules. Like in our earlier papers Billette de Villemeur et al. (2002, 2003a, 2003b) we adopt a deliberately simple and stylized setting to focus on what we consider the essential feature of the postal sector (in particular with regard to cost and demand). Here we focus on the ramifications brought about by imperfect competition in a setting where all the demand for workshared mail emanates from competitors (rather than the clients). To make this clear and to distinguish this paper from our earlier paper where workshared mail was demanded solely by clients we shall refer to the price charged for workshared mail as "access price".

The main features of our setting are as follows. There are two types of operators: the incumbent universal service operator on the one hand and the potential entrants on the other hand. There are two postal products: single piece mail and bulk or "commercial" mail. Single piece mail is offered solely by the incumbent operator, at a uniform price. Entry, if any, thus occurs in the market for commercial mail. In this market there is one representative sender and two types of addressees: residents of low cost (urban) areas and residents of high cost (rural) areas.

The incumbent uses the same delivery network to process both types of mail. There is a fixed cost associated with maintaining the delivery network in any given area. Fixed costs per addressee are higher in the rural than in the urban area. In addition, delivery (of any type of mail) involves an identical and constant marginal cost. Entrants have their own delivery network in the urban area. The fixed cost of setting up a delivery network in the rural area, on the other hand, may or may not be prohibitive. In this paper we assume that it is prohibitive so that entrants are able to accept mail to rural addressees only if they can gain access to the incumbent's delivery network. Access is always possible at the commercial mail rate charged to the incumbent's clients (which in turn cannot exceed its single piece rate). However, the entrant may also be able to access the network by buying workshared mail from the incumbent operator at a discounted price.

We study the determination of optimal (regulated) prices under the assumptions that the commercial mail market consists of a differentiated duopoly in which the incumbent and an entrant compete. We consider an asymmetric (sequential) setting in which the incumbent (or alternatively the regulating authority) acts as a Stackelberg leader. We concentrate on cases where the regulator sets all of the incumbent's prices at their optimal level but settings where some prices are given and/or determined competitively are also discussed. Furthermore, upon entry, the incumbent's uniform pricing constraint may or may not continue to extend to the commercial mail market. This latter feature allows us to pinpoint the impact of uniform pricing on pricing rules. In either of these settings, we first derive the pricing rules under general demand functions and then consider two special cases for illustrative purposes. The special cases are obtained by making extreme assumptions about the degree of substitutability between the entrant's and the incumbent products: perfect substitutes on the one hand and independent demands on the other hand.

2 Model

As far as preferences and technologies are concerned, we continue to use the specification considered by Billette de Villemeur et al. (2003). Much of this section is thus drawn from that paper. This is done to make the current paper self-contained and to avoid cumbersome cross referencing.

2.1 Products, preferences and demand

There are two postal products: single piece mail, X, and commercial mail, Y. Single-piece mail is supplied by a single operator, namely the incumbent, I, at a uniform price p_X. The net surplus generated by this product is given by

$$U(X) - p_X X. \tag{1}$$

The aggregate demand function, obtained by maximizing (1) is denoted $X(p_X)$.

There are two operators in the market for Y: the incumbent I and an entrant E. There is one representative sender who sends commercial mail to N addressees. A fraction α of these addressees is located in area u (urban) while the remaining fraction $(1-\alpha)$ is located in r (rural). Utility (net surplus) of the representative sender is given by:[6]

$$\begin{aligned}&\alpha N v(y_I^u, y_E^u) + (1-\alpha) N v(y_I^r, y_E^r) - \alpha N p_I^u y_I^u - \alpha N p_E^u y_E^u \\ &- (1-\alpha) N p_I^r y_I^r - (1-\alpha) N p_E^r y_E^r,\end{aligned} \tag{2}$$

[6] The specification of preferences is inspired by Cremer et al. (2001).

$y_k^j \geq 0$ is the number of units sent to each addressee in area $j = u, r$ through operator $k = I, E$, while p_k^j is the price for y of operator $k = I, E$ for a unit sent to an addressee located in area $j = u, r$. We consider two cases depending on whether or not operator I faces a uniform pricing constraint requiring $p_I^u = p_I^r$. The function $v(\cdot, \cdot)$ represents utility per addressee or, more precisely, the contribution of an addressee to the sender's utility. Observe that the specification of v reflects the degree of substitutability of the incumbent's and the entrant's products. The extreme case of perfect substitutes obtains when utility depends only on $y_I^j + y_E^j$.

Maximizing (2) with respect to $y_I^u, y_E^u, y_I^r, y_E^r$, yields the demand functions. Aggregate (market) demands are obtained by summing up demands per-addressee. They are given by

$$
\begin{aligned}
&Y_I^u(p_I^u, p_E^u) = \alpha N y_I^u(p_I^u, p_E^u) \quad && Y_E^u(p_I^u, p_E^u) = \alpha N y_E^u(p_I^u, p_E^u) \\
&Y_I^r(p_I^r, p_E^r) = (1-\alpha) N y_I^r(p_I^r, p_E^r) \quad && Y_E^r(p_I^r, p_E^r) = (1-\alpha) N y_E^r(p_I^r, p_E^r).
\end{aligned}
$$

There is also the implicit constraint that $p_I^j \leq p_X$. Clearly, the price of commercial mail (in any area) cannot exceed the single piece rate. To limit the number of cases to be considered, we shall assume that this constraint is not binding.

2.2 Cost, profits and welfare

The stylized postal network we consider consists of two segments. Segment 2 corresponds to a composite activity including collecting, sorting and transportation. This activity implies a constant marginal cost of c_2. Segment 1 is delivery with marginal cost of c_1. These *marginal* costs are the same for all operators and they are independent of the location of the addressee. However, there is also a *fixed* cost associated with the delivery network which differs across areas and operators. Single piece mail uses both segments of the network while commercial mail only uses the delivery network. Operator I 's delivery network extends to both areas; it is used to deliver the two types of mail. Operator E, on the other hand, may have part (or all) of its mail in a given area delivered through operator I. Let Z^j denote the quantity of operator E's mail which is delivered by I in area j. We have

$$0 \leq Z^j \leq Y_E^j.$$

The corresponding access charge is $a^j \leq p_I^j$. It cannot exceed the operators price (for commercial mail) in that area, which in turn can of course not exceed the single piece rate.

The cost structure is represented by the following cost function for operator I :

$$C^I = (c_1 + c_2)X + c_1(Y_I^u + Y_I^r + Z^u + Z^r) + \alpha N F^u + (1 - \alpha) N F^r + F,$$

where F^u et F^r are the fixed costs *per addressee* of the delivery network in the urban and in the rural area, with $F^r > F^u > 0$, while $F \geq 0$ includes all the remaining fixed costs (including common costs). The profit of operator I is then given by

$$\pi^I = p_X X + p_I^u Y_I^u + p_I^r Y_I^r + a^r Z^r + a^u Z^u - C^I. \tag{3}$$

Turning to operator E, let f^j denote the fixed delivery network cost (per addressee) in area $j = u, r$, with $f^r > f^u \geq 0$. This fixed cost is only incurred when $Y_E^j > Z^j$ that is, when operator E does not deliver all its mail in the area (if any) by accessing I's network. In the main part of the paper, we shall assume that f^r is prohibitive so that $Z^r = Y_E^r$. In other words, the entrant will never find it optimal to set up a delivery network in area r.

In the urban area, the linearity of the cost structure implies that we have either $Z^u = 0$ or $Z^u = Y_E^u$. We shall for simplicity concentrate on the case where $Z^u = 0$. The other case can, however, be easily accommodated. We then obtain the following cost function for operator E.

$$C^E = a^r Y_E^r + c_1 Y_E^u + \alpha N f^u.$$

Profits of the entrant are given by:

$$\pi^E = p_E^u Y_E^u + p_E^r Y_E^r - C^E. \tag{4}$$

Finally, welfare is measured by (unweighted) total surplus i.e., the sum of consumer surplus and profits

$$\begin{aligned} W = {} & U(X) - p_X X + \alpha N v(y_I^u, y_E^u) + (1-\alpha) N v(y_I^r, y_E^r) \\ & - \alpha N p_I^u y_I^u - \alpha N p_I^u y_I^u - (1-\alpha) N p_I^r y_I^r - (1-\alpha) N p_E^r y_E^r \\ & + \pi^E + \pi^I \end{aligned} \tag{5}$$

Observe that decision variables are prices, so that all the quantities in (5) are given by the corresponding demand functions.

So far we have not specified how the prices of the entrant will be determined. It is clear that they will somehow depend on the incumbent's prices and particularly on the access charge. However, the specific relationship will depend on the entry scenario which is considered. Billette de Villemeur et al. (2003) have considered the case of a competitive fringe, which is the setting on which most of the access pricing literature has concentrated. The main advantage of this approach is its simplicity. With the entrant pricing at marginal cost, the impact of the incumbent's pricing strategy on the entrant is easily assessed. The main drawback is that competitive behavior is not consistent with the presence of a positive fixed cost: when prices simply reflect the (constant) marginal cost, the entrant will make a loss equal to the fixed costs and will thus not be viable. In the current paper we allow the entrant to have some market power in the sense that it can realize a markup over its marginal cost. This market power is, however, limited in that we consider a Stackelberg type game in which the incumbent plays a leading role in that it can commit to its pricing policy. We shall study the solution without and with a uniform pricing constraint imposed on the incumbent. Most of the formal derivations are relegated to the Appendix.

2.3 The entrant's pricing strategy

We assume that the entrant maximizes its profits given the incumbent's prices p_I^u, p_I^r and given the access charge a^r. Formally, it thus solves

$$\max_{p_E^u, p_E^r} \pi_E = (p_E^u - c_1)\, Y_E^u(p_I^u, p_E^u) + (p_E^r - a^r)\, Y_E^r(p_I^r, p_E^r) - \alpha N f^u. \tag{6}$$

The FOC for profit maximization are:

$$Y_E^u + (p_E^u - c_1)\, \frac{\partial Y_E^u}{\partial p_E^u} = 0 \tag{7}$$

$$Y_E^r + (p_E^r - a^r)\, \frac{\partial Y_E^r}{\partial p_E^r} = 0 \tag{8}$$

Optimal prices (reaction functions) are then given by (with some abuse of notation)

$$p_E^u = p_E^u(p_I^u) \tag{9}$$

$$p_E^r = p_E^r(a^r, p_I^r) \tag{10}$$

These pricing functions are a crucial ingredient of our problem. They represent the major channel through which imperfect competition affects the formal analysis. To see this recall that under (perfectly) competitive behavior the relevant pricing rules are given by:[7]

$$p_E^u = c_1, \tag{11}$$

$$p_E^r = a^r. \tag{12}$$

Consequently, with perfect competition, p_E^u is not affected by the incumbent's pricing behavior, while p_E^r is solely determined by the access price, with $dp_E^r/da^r = 1$. In other words, any variation in the access charge is entirely shifted to the final consumer.

With imperfect competition and pricing rules (9)–(10), on the other hand, the incumbent's prices do directly affect the entrant's prices. Without further restrictions on demand, the comparative statics of these pricing rules

[7] See Billette de Villemeur et al. (2003).

are ambiguous. We shall assume $dp_E^u/dp_I^u > 0$ and $\partial p_E^u/\partial p_I^u > 0$ which can be considered as the "normal case" which arises under some additional conditions; see the Appendix A.1. For instance, these conditions are satisfied for linear demand functions. As to the impact of the access charge, we have (in general) $\partial p_E^r/\partial a^r \neq 1$. For instance, a linear demand curve implies $\partial p_E^r/\partial a^r = 1/2$; again, see the Appendix A.1. However, $\partial p_E^r/\partial a^r > 1$ (more than complete shifting) is also possible.[8] Note also the separability: the entrant's price in any area (r or u) do not depend on the incumbent's prices in the other area. For future reference note that (9)–(10) always imply $p_E^u \geq c_1$ and $p_E^r \geq a^r$. As can be shown very easily, it is never optimal for the entrant to set any of its prices below *marginal* cost.[9]

Before proceeding, let us observe that rather than adopting a fully fledged setting of strategic interaction we could have adopted an intermediate approach by assuming

$$p_E^u = c_1(1 + \eta^u), \tag{13}$$

$$p_E^r = a^r(1 + \eta^r), \tag{14}$$

where η^u and η^r are positive constants. In other words, with these pricing rules the entrant simply applies constant markups $(1 + \eta^u)$ and $(1 + \eta^r)$ to its marginal costs in the two areas. These kind of pricing rules are often considered in monopolistic competition models (see Tirole 1988). They are of course simply special case of our rules (9)–(10). To avoid repetitions, we have decided not to consider this special case separately. While these alternative pricing rules yield a simpler problem, the simplification is not significant enough to justify a detailed analysis.

To simplify notation in the remainder of the paper, it is convenient to

[8] This is a standart result from tax incidence theory. In particular, it is well known that when demand elasticity and marginal costs are constant we necessarily have $\partial p_E^r/\partial a^r > 1$; see

[9] This property relies on the separability of demand between u and r and on the one shot specification of the game. In a dynamic setting this would no longer be necessarily be true in all periods.

redefine the demand functions to account for the reaction of the entrant:

$$\widetilde{Y}_I^u(p_I^u) = Y_I^u[p_I^u, p_E^u(p_I^u)] \quad \widetilde{Y}_E^u(p_I^u) = Y_E^u[p_I^u, p_E^u(p_I^u)]$$
$$\widetilde{Y}_I^r(p_I^r, a^r) = Y_I^r[p_I^r, p_E^r(a^r, p_I^r)] \quad \widetilde{Y}_E^r(p_I^r, a^r) = Y_E^r[p_I^r, p_E^r(a^r, p_I^r)]. \tag{15}$$

The properties of these redefined demand functions are presented in the Appendix A.2.

2.4 The optimal pricing problem

Substituting (3), (4), (9) and (10) in (5), while imposing the constraints $\pi^I \geq 0$ and $a^r \leq p_I^r$ yields to the following Lagrangian expression for the optimal pricing problem:

$$\begin{aligned}\Gamma = {} & U(X) - p_X X + \alpha N v(y_I^u, y_E^u) + (1-\alpha) N v(y_I^r, y_E^r) \\ & - \alpha N p_I^u y_I^u - \alpha N p_E^u y_E^u - (1-\alpha) N p_I^r y_I^r - (1-\alpha) N p_E^r y_E^r \\ & + (p_E^u - c_1)\, Y_E^u + (p_E^r - a^r)\, Y_E^r - \alpha N f^u \\ & + (1+\lambda)[p_X X + p_I^u Y_I^u + p_I^r Y_I^r + a^r Y_E^r] \\ & - (1+\lambda)[(c_1 + c_2) X + c_1 (Y_I^u + Y_I^r + Y_E^r)] \\ & - (1+\lambda)[\alpha N F^u + (1-\alpha) N F^r + F] \\ & - \mu(a^r - p_I^r)\end{aligned} \tag{16}$$

where $\lambda > 0$ is the multiplier of the (incumbent) operator's break-even constraint, while $\mu \geq 0$ is the multiplier of the constraint that the access price cannot exceed operator I's price in the relevant market. When $\mu > 0$ the constraint is binding and $a^r = p_I^r$ (Regime II below) while $\mu = 0$ corresponds to the case where the constraint is not binding (Regime I). The decision variables are p_X, p_I^u, p_I^r and a^r. The entrant's prices are then determined according to (9)–(10) and demand levels are specified by (15). When there is no uniform pricing constraint, p_I^u and p_I^r can be set independently. When the incumbent is subject to a uniform pricing requirement, the constraint $p_I^u = p_I^r$ has to be added and we effectively have only three instruments, namely p_X, p_I $(= p_I^u = p_I^r)$ and a^r.

3 No uniform-pricing constraint

The constraint $a^r \leq p_I^r$ may or may not be binding. Accordingly, there are two possible regimes with $\mu = 0$ or $\mu > 0$.

3.1 General demand functions

3.1.1 Regime I: $a^r \leq p_I^r$ not binding ($\mu = 0$).

As shown in the Appendix, differentiating Γ with respect to the instruments and combining and rearranging the first-order conditions, yields the following optimal pricing rules for this case:

$$\frac{p_X - (c_1 + c_2)}{p_X} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon_X}, \tag{17}$$

$$\frac{p_I^u - c_1}{p_I^u} = \frac{\lambda}{1+\lambda}\frac{1}{\sigma_{Y_I^u}} - \frac{(p_E^u - c_1)}{p_I^u}\frac{\left(d\widetilde{Y}_E^u/dp_I^u\right)}{\left(d\widetilde{Y}_I^u/dp_I^u\right)}, \tag{18}$$

$$\frac{p_I^r - c_1}{p_I^r} = \frac{\lambda}{1+\lambda}\frac{1}{\widehat{\sigma}_{Y_I^r Y_E^r}}, \tag{19}$$

$$\frac{a^r - c_1}{a^r} = \frac{\lambda}{1+\lambda}\frac{1}{\widehat{\sigma}_{Y_E^r Y_I^r}} - \frac{1}{1+\lambda}\frac{p_E^r - a^r}{a^r}, \tag{20}$$

where ε_X and $\sigma_{Y_I^u}$ are (absolute values of) "ordinary" price elasticities of demand for X and $\widetilde{Y}_I^u$ respectively.[10] Formally we have:

$$\varepsilon_X = \frac{p_X}{X}\left(\frac{-dX}{dp_X}\right) \qquad \text{and} \qquad \sigma_{Y_I^u} = \frac{p_I^u}{Y_I^u}\left(\frac{-d\widetilde{Y}_I^u}{dp_I^u}\right). \tag{21}$$

Furthermore, $\widehat{\sigma}_{Y_I^r Y_E^r}$ and $\widehat{\sigma}_{Y_E^r Y_I^r}$ are the superelasticities of $\widetilde{Y}_I^r$ and $\widetilde{Y}_E^r$ respectively; see below for further details.

We shall now have a closer look at this expressions by considering the different markets separately. Doing this we shall focus on the impact of imperfect competition. In other words we examine how these expression differ from their counterparts in the competitive fringe case (namely expressions

[10] Troughout the paper ε is used for elasticities (or superelasticities) defined on the basis of the original demand functions. The notation σ, on the other hand, is used for elasticities (or superelasticities) defined on the basis of the redefined demand functions $\widetilde{Y}$ (specified by (15)) which account for the induced adjustments of the entrant's prices.

(8)–(11) in Billette de Villemeur et al. (2003)). Observe that the pricing *rule* in for single piece mail X is not affected by the presence of imperfect competition in the market for Y.[11] We continue to have a simple Ramsey-type inverse elasticity rule. This separability also implies that the rest of he analysis is not affected if we assume that the price of X is exogenously given and cannot be adjusted upon entry.[12] Let us now turn to the two segments in the market for Y.

Rural area The most obvious impact of imperfect competition is the presence of an additional term in the access-pricing formula (20), namely the second term on the RHS. This term is negative when $p_E^r > a^r$, it vanishes when $p_E^r = a^r$ in which case we return to the competitive fringe case. This term thus tends to reduce the access charge. This is because under imperfect competition the price of the entrant is set above marginal cost and is thus too high from an efficiency perspective. Reducing the access charge is then a way to reduce the entrants price and mitigate the inefficiency implied by imperfect competition. This reduction in the access charge implies of course also a cost in that it reduces revenue and thus makes budged balance harder to achieve (some other price has to be increased which brings about other efficiency losses). The optimal pricing rule strikes a balance between these two effects. The revenue considerations are formally reflected by the presence of λ in the denominator of the term. The smaller is λ, the larger will be the imperfect competition adjustment is access charge. This becomes most apparent when we look at the extreme cases. When $\lambda = 0$ (20) simplifies to

$$a^r - c_1 = -(p_E^r - a^r) \leq 0 \tag{22}$$

so that the access charge is set below marginal cost (of access). Put differently the entrant is subsidized. Further simplifying (22) yields $p_E^r - a^r$

[11] The optimal price *level* will, however, in general be different because the value of λ will change

[12] As long as the constraint that $p_I^j \leq p_X$ is not violated.

indicating that this subsidy is set to induce the entrant to price at marginal cost. At the other extreme when $\lambda \to \infty$ the imperfect competition term vanishes altogether. Any reduction in the access charge would be too costly.

The *structure* of the first terms in both (19) and (20) is not affected by the presence of imperfect competition. Like in the competitive fringe these are simply inverse superplasticity term, where the "super" comes in because the cross price effects. Recall that the demand functions of the two operators are not independent. The specific definition of the superelasticity is different, though. This is because demand levels are given by the redefined demand functions $\widetilde{Y}_I^r(p_I^r, a^r)$ and $\widetilde{Y}_E^r(p_I^r, a^r)$ which do take the induced impact on operator E's prices into account. As shown in the Appendix, we then obtain de generalization of the usual definition of superelasticities.

Urban area We now turn to the pricing rule for the urban market, namely (18). The second term on the RHS of this expression is an imperfect competition term; it vanishes in the competitive fringe case (when $p_E^u = c_1$). With $p_E^u - c_1 > 0$, assuming $\partial\widetilde{Y}_E^u/\partial p_I^u > 0$ and $\partial\widetilde{Y}_I^u/\partial p_I^u < 0$, this term is positive and thus tends to increase the incumbent's price in the urban area. The intuition behind this property is once again related to the inefficiencies implied by imperfect competition. When the entrant's price is above marginal cost, its output lower than the efficient level. When goods are "substitutes" ($\partial\widetilde{Y}_E^u/\partial p_I^u > 0$) increasing the incumbent's price then brings about an extra benefit, namely that it *increases* the *entrant's* output.[13] It does, however, also *decrease* the *incumbent's* output (as long as $\partial\widetilde{Y}_I^u/\partial p_I^u < 0$) which in turn is inefficient. The second term in the pricing rule strikes a balance between these two effects.

The first term on the RHS of (18) is the Ramsey term, which has, once again the same structure as in the competitive fringe case. However, it is

[13] The definition of "substitutes" underlying this statement is not standard for it relies on the derivative of $\widetilde{Y}$ rather than of Y. See Appendix A.2 for a study of the properties of $\widetilde{Y}$.

the elasticity of $\widetilde{Y}_I^u$ and not that of Y_I^u which matters and we have

$$\sigma_{Y_I^u} = \frac{p_I^u}{Y_I^u}\left(\frac{-d\widetilde{Y}_I^u}{dp_I^u}\right) = \frac{p_I^u}{Y_I^u}\left(\frac{-\partial Y_I^u}{\partial p_I^u}\right) - \frac{p_I^u}{Y_I^u}\frac{\partial Y_I^u}{\partial p_E^u}\frac{\partial p_E^u}{\partial p_I^u} \leq \varepsilon_{Y_I^u}.$$

Consequently when the reaction of the entrant is accounted for, the incumbent's demand is less elastic than when the entrants price is held fixed. This is because when p_I^u increases, the price of the substitute offered by the entrant p_E^u will also increase which in turn will mitigate the decrease in the incumbent's demand.

The solution discussed so far is valid when it yields a level of the access charge that does not exceed the incumbent's price level. From the pricing rules we see that this is the case when

$$\frac{\lambda}{1+\lambda}\frac{1}{\widehat{\sigma}_{Y_E^r Y_I^r}} - \frac{1}{1+\lambda}\frac{p_E^r - a^r}{a^r} \leq \frac{\lambda}{1+\lambda}\frac{1}{\widehat{\sigma}_{Y_I^r Y_E^r}}. \tag{23}$$

A *sufficient* condition for (23) is $\widehat{\sigma}_{Y_I^r Y_E^r} \leq \widehat{\sigma}_{Y_E^r Y_I^r}$, that is when the superelasticity of the incumbent's demand is smaller than the superelasticity of the entrant's demand.[14] Observe that it is the *super*elasticities (of the appropriately defined demand functions) rather than the elasticities per se which must be ranked properly. When the solution described by (17)–(20) does not automatically satisfy $a^r \leq p_I^r$, this constraint will be binding and we will have Regime II.

3.1.2 Regime *II*: $a^r \leq p_I^r$ binding ($\mu > 0$)

With $a^r = p_I^r \equiv p^r$ the two first-order conditions with respect to these variables are replaced by a single one, which is the sum of the two separate conditions (namely (35c) and (35d) in the appendix). Rearranging and solving then yields:

$$\frac{a^r - c_1}{a^r} = \frac{\lambda}{1+\lambda}\frac{1}{\sigma_{Y^r}} + \frac{1}{1+\lambda}\left(\frac{p_E^r - a^r}{p_I^r}\right)\left(\frac{-\left(\partial\widetilde{Y}_E^r/\partial p_I^r\right) - \left(\partial\widetilde{Y}_E^r/\partial a^r\right)}{\left(\frac{-\partial\widetilde{Y}_I^r}{\partial p_I^r}\right) + \left(\frac{-\partial\widetilde{Y}_E^r}{\partial p_I^r}\right) + \left(\frac{-\partial\widetilde{Y}_I^r}{\partial a^r}\right) + \left(\frac{-\partial\widetilde{Y}_E^r}{\partial a^r}\right)}\right), \tag{24}$$

[14] Where demand levels are once again defined by $\widetilde{Y}_I^r(p_I^r, a^r)$ and $\widetilde{Y}_E^r(p_I^r, a^r)$.

In this expression, the "total elasticity" σ_{Y^r} is defined by

$$\begin{aligned}\sigma_{Y^r} &= \frac{p_I^r}{Y_I^r + Y_E^r}\left(\left(\frac{-\partial \widetilde{Y}_I^r}{\partial p_I^r}\right) + \left(\frac{-\partial \widetilde{Y}_E^r}{\partial p_I^r}\right) + \left(\frac{-\partial \widetilde{Y}_I^r}{\partial a^r}\right) + \left(\frac{-\partial \widetilde{Y}_E^r}{\partial a^r}\right)\right)\\ &= \frac{Y_I^r}{Y_I^r + Y_E^r}\left(\sigma_{Y_I^r} - \sigma_{Y_I^r Y_E^r}\right) + \frac{Y_E^r}{Y_I^r + Y_E^r}\left(\sigma_{Y_E^r} - \sigma_{Y_E^r Y_I^r}\right),\end{aligned}$$

where the direct elasticities are defined by (21) and by

$$\sigma_{Y_E^r} = \frac{a^r}{\widetilde{Y}_E^r}\left(\frac{-\partial \widetilde{Y}_E^r}{\partial a^r}\right) = \frac{a^r}{Y_E^r}\left(\frac{-\partial Y_E^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial a^r}\right) = \left(\frac{a^r}{p_E^r}\frac{\partial p_E^r}{\partial a^r}\right)\varepsilon_{Y_E^r},$$

while the cross-elasticities $\sigma_{Y_I^r Y_E^r}$ and $\sigma_{Y_E^r Y_I^r}$ are given by:

$$\begin{aligned}\sigma_{Y_I^r Y_E^r} &= \frac{a^r}{\widetilde{Y}_I^r}\left(\frac{\partial \widetilde{Y}_I^r}{\partial a^r}\right)\\ \sigma_{Y_E^r Y_I^r} &= \frac{p_I^r}{\widetilde{Y}_E^r}\left(\frac{\partial \widetilde{Y}_E^r}{\partial p_I^r}\right)\end{aligned}$$

We use the term "total" to reflect the property that when $p^r = a^r$ changes, the two arguments of the demand functions $\widetilde{Y}_I^r(p_I^r, a^r)$ and $\widetilde{Y}_E^r(p_I^r, a^r)$ follow the exact same change. Observe that the pricing *rules* for p_X and p_I^u are unchanged and continue to be given by (17) and (18).

To interpret the pricing rule (24) let us compare it to (20), its counterparts in Regime I. The first notable difference is that we return to a simple inverse elasticity rule. There are no superelasticities in the expression anymore because there are no cross price effects per se. To be more precise, the cross price effects disappear through the aggregation of Y_I^r and Y_E^r which are now sold at the same price.[15] The second difference lies in the imperfect competition term which has now a slightly more complex structure. Specifically there is an additional multiplicative term which measures the variation in the incumbent's demand relative to the variation of total demand (in the rural segment).

[15]The aggregation is legitimate because the two products have the same marginal delivery costs. This is not particularly restrictive here because delivery occurs though I's network in any case. Demand functions for the two products are of course different, but this does not prevent us from aggregating their demand.

3.2 Specific demand functions: benchmark cases

Our results so far depend crucially on demand elasticities (and superelasticities), which in turn depend on the degree of product differentiation between the entrant's and the incumbent's product. For instance, the incumbent's demand will tend to be more elastic when operator E's products are close substitutes to operators I's products. To understand the impact of the degree of substitutability on the structure of prices and, in particular on access charges Billette de Villemeur et al. (2003) have considered two extreme special cases within the competitive fringe framework. The first one obtains when the incumbent and the entrants products are *perfect* (and one for one) substitutes while the second one occurs when demands are *independent*. In the current setting of imperfect competition, the degree of substitution between products will have an even more complex impact on the results. This is because under imperfect competition, the pricing strategies of the entrant (9) and (10) are also affected (implicitly) by substitution and by the degree of product differentiation. To see this observe first that when the products are perfect substitutes, we are in a homogenous Bertrand setting where equilibrium prices have to equal marginal costs. Formally, pricing strategies are then given by (12) and (11) and we essentially return to the competitive fringe case. To make this paper self-contained, let us briefly review the major results obtained for that case.

3.2.1 Perfect substitutes

The case of perfect one by one substitutes is obtained by assuming that the utility of the representative consumer is given by:

$$\begin{aligned}&\alpha N v(y_I^u + y_E^u) + (1-\alpha) N v(y_I^r + y_E^r) - \alpha N p_I^u y_I^u - \alpha N p_I^u y_I^u \\ &- (1-\alpha) N p_I^r y_I^r - (1-\alpha) N p_E^r y_E^r,\end{aligned} \tag{25}$$

which is a special case of (2). In this case demand behavior is "extreme" in the sense that the consumer only buys the cheapest product. Interior solutions can only arise when the two operators charge the same price. As in the general case, we assume for the time being that there is no uniform pricing constraint.

To determine the pricing rules, first observe that in the rural market we must have either $p_I^u = p_E^u = c_1$ or $p_I^u > p_E^u = c_1$ with $Y_I^u = 0$. With perfect substitutes operator I can only have a positive market share when its price does not exceed that of operator E. Either way, no profit (contribution towards fixed costs) can be generated in that market.[16] Second, in the rural area, we must have Regime II with $a^r = p_I^r$.[17] Per our above results, the solution is then given by expressions (17) and (24):

$$\frac{p_X - (c_1 + c_2)}{p_X} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon_X} \tag{26}$$

$$\frac{p_I^r - c_1}{p_I^r} = \frac{a^r - c_1}{a^r} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon_{Y^r}}, \tag{27}$$

where it should be recalled that ε_{Y^r} is the total elasticity of $Y^r = Y_I^r + Y_E^r$. Observe that with perfect substitutes, we return to "regular" demand elasticities and we have $\sigma_{Y^r} = \varepsilon_{Y^r}$. Furthermore, with $p_E^r = a^r$ the imperfect competition term in (24) vanishes. We thus have simple Ramsey problem over two prices: p_X (single piece mail) and $p_I^r = a^r$(commercial mail price and access charge in area r). The other price p_I^u is given and the operator has no positive markup on that market. In other words, only single piece mail and rural commercial mail contribute towards the fixed costs. And how the cost coverage translates into markups depends on demand elasticities.

[16] Setting $p_I^u < p_E^u = c_1$ yields negative profits and is even worse.

[17] To see this, observe that $a^r < p_I^r$ implies that operator I's market share is zero. This in turn can never be optimal, as long as $\lambda > 0$. Consequently, we must have $a^r = p_I^r$.

3.2.2 Independent demands

The case of independent demand functions (no cross-price effects) is obtained by assuming that the utility of the representative consumer is given by:

$$\alpha N[v_I(y_I^u) + v_E(y_E^u)] + (1-\alpha)N[v_I(y_I^r) + v_E(y_E^r)] - \alpha N p_I^u y_I^u - \alpha N p_I^u y_I^u$$
$$- (1-\alpha)N p_I^r y_I^r - (1-\alpha) N p_E^r y_E^r, \tag{28}$$

which is again a special case of (2).[18] This amounts to assuming that the entrant offers a different product which "creates its own demand" in the sense that its existing demand (for the incumbent's product) is not affected. In this case the pricing policy of the entrant is considerably simplified. In particular, we have

$$\frac{dp_E^u}{dp_I^u} = 0 \qquad \text{and} \qquad \frac{\partial p_E^r}{\partial p_I^r} = 0,$$

In words, the entrants price do not depend on the incumbent's (final goods) prices. This is like in the competitive fringe setting. However, one has to keep in mind that (the entrant's) prices are *not* equal to marginal costs here.

The solution under Regime I , which was given by expression (18), (19) and (20), then reduces to:

$$\frac{p_I^u - c_1}{p_I^u} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon_{Y_I^u}} \tag{29}$$

$$\frac{p_I^r - c_1}{p_I^r} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon_{Y_I^r}}, \tag{30}$$

$$\frac{a^r - c_1}{a^r} = \frac{\lambda}{1+\lambda}\frac{1}{\sigma_{Y_E^r}} - \frac{1}{1+\lambda}\frac{p_E^r - a^r}{a^r}, \tag{31}$$

where the elasticity σ_{Ea}^r is defined by:

$$\sigma_{Y_E^r} = \frac{a^r}{\widetilde{Y}_E^r}\left(\frac{-\partial \widetilde{Y}_E^r}{\partial a^r}\right) = \frac{a^r}{Y_E^r}\left(\frac{-\partial Y_E^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial a^r}\right) = \varepsilon_{Y_E^r}\left(\frac{a^r}{p_E^r}\frac{\partial p_E^r}{\partial a^r}\right).$$

Simplifications arise because absent of cross-price effects, super-elasticities reduce to ordinary price elasticities and the distinction between $\widetilde{Y}_k^j$ and Y_k^j

[18] We set $v(y_I^j, y_E^j) = v_I(y_I^j) + v_E(y_E^j)$, $j = u, r$.

becomes irrelevant. Consequently, we return to a standard Ramsey problem with simple inverse elasticity rules for the urban segment and for the final product in the rural market. Observe that the imperfect competition term in the urban segment has vanished because with independent demand functions we have $\partial\widetilde{Y}_E^u/\partial p_I^u = 0$. The imperfect competition distortion in the urban segment (associated with $p_E^u > c_1$) continues to exist, but it is now independent of the incumbent's prices. The imperfect competition term in access charge rule, however continues to be present in (31) and tends to decrease the access charge.

Finally note that $\sigma_{Y_E^r} > \varepsilon_{Y_I^r}$ is a *sufficient* condition for Regime I to prevail. The interpretation of this condition is, however, now more complex than that of is counterpart in the competitive fringe case; see Billette de Villemeur et al. (2003).

3.2.3 From specific cases to the general setting: main lessons

The results obtained for these special cases are very interesting. They suggest that the *difference* between the incumbent's price and the access charge decreases with the degree of substitutability between the two operators' products. When there is no substitution, cross price effects and hence competition essentially play no role. The entrant's product is treated like an entirely different good whose consumers contribute to the incumbent's fixed costs (through the implicit tax on access) on the basis of a simple inverse elasticity rule (and Regime I prevails). When goods become substitutes, pricing rules have to account for the impact of access charges on the incumbent demand (via the entrant's price). This effect will tend to increase access charges. At the extreme case of perfect substitutes, the cross-price effects are so significant that access at a rate which is lower than the price would effectively reduce the incumbent's market share to zero (and Regime II prevails).

The benchmark cases are also interesting when it come to assess the welfare implications of entry and/or the provision of access. Recall that for the general case the welfare impact of entry is ambiguous in this model. On the benefit side, a new product variety is offered to consumers. On the cost side, there is the distortion associated with imperfect competition and the fact that the incumbent's budget constraint (which has a positive shadow price) will be harder to satisfy (this leads to more distortions and lower welfare). When the entrant's technology involves a fixed cost, this duplication of fixed costs introduces an additional negative effect. The interesting feature is that for the special cases we can assess the welfare impact.

Consider first the independent demand case. There it is plain that entry can only be welfare enhancing. To see this observe that with independent demands, the monopoly policy for the incumbent remains feasible after entry. The entrant "creates" its own demand and this will generate additional surplus (as long as the entrant manages to break even). Because the entrant prices above marginal cost, this additional surplus is not as large as it could be.[19] This mitigates the positive impact of entry but it will not eliminate it altogether.

The perfect substitute case gives exactly the opposite conclusion. Here entry does not add anything (the product is exactly the same as the incumbent's). However, its presence complicates budget balancing for the incumbent and thus exacerbates distortions. Put differently, whatever solution is feasible with entry is feasible also under monopoly (but at lower cost). Imperfect competition, on the other hand, is irrelevant in this case. With homogenous products, the entrant's prices are set exactly like in the competitive fringe case.

Extrapolating from these special cases we can then conjecture (by continuity) that whenever the goods are close substitutes entry necessarily reduces welfare. Or, in other words, demands must be sufficiently independent for a welfare improvement to be possible. Yet another way to phrase this

[19] If the entrant were to price at marginal cost.

is that the entrant's product must create a sufficient amount of additional demand (and hence additional surplus).

4 Uniform pricing constraint

We now turn to the case where operator I is subject to a (geographically) uniform pricing constraint which requires $p_I^u = p_I^r = p_I$. We have two possible regimes.

4.1 Regime I: $a^r \leq p_I^r$ not binding ($\mu = 0$).

We now have to differentiate Γ with respect to p_X, p_I and a^r. The pricing rule for single piece mail does not change and the expression for p_X continues to be given by (17). Turning to the commercial mail market, rearranging the first-order conditions and defining $Y_I = Y_I^u + Y_I^r$ we obtain the following pricing rules :

$$\frac{p_I - c_1}{p_I} = \frac{\lambda}{1+\lambda}\frac{1}{\widehat{\sigma}_{Y_I Y_E^r}}\left(1 + \frac{1}{\lambda}\left(\frac{p_E^u - c_1}{p_E^u}\right)\frac{\left(\frac{p_E^u Y_E^u}{p_I Y_I}\right)\sigma_{Y_E^u Y_I}\sigma_{Y_E^r}}{\sigma_{Y_E^r} + \frac{a^r Y_E^r}{p_I Y_I}\sigma_{Y_E^r Y_I}}\right), \tag{32}$$

$$\frac{a^r - c_1}{a^r} = \frac{\lambda}{1+\lambda}\frac{1}{\widehat{\sigma}_{Y_E^r Y_I}} - \frac{1}{1+\lambda}\frac{p_E^r - a^r}{a^r} \tag{33}$$

where $\widehat{\sigma}_{Y_I Y_E^r}$ and $\widehat{\sigma}_{Y_E^r Y_I}$ are the superelasticities of $\widetilde{Y}_I$ and $\widetilde{Y}_E^r$ respectively (See appendix A.4).

It thus appears that in our setting the uniform pricing constraint does not change the general form of the access pricing rules. We continue to have an inverse (super)elasticity rules, with an imperfect competition term for the access charge. Further, the definitions of superelasticities (given in the Appendix A.4) are exactly the same as in Section 3. Comparing (33) and (20) reveals that the access pricing rules differ in the "scope" of the second product with which cross-price effects are accounted for. Formally, we now have $\widehat{\sigma}_{Y_E^r Y_I}$ rather than $\widehat{\sigma}_{Y_E^r Y_I^r}$ so that with uniform pricing constraint the second product relevant for the superelasticity includes delivery in urban as well as in rural areas. Without uniform pricing constraint, only the rural segment of the market was relevant.

The next interesting step would be to compare $\widehat{\sigma}_{Y_E^r Y_I}$ and $\widehat{\sigma}_{Y_E^r Y_I^r}$, for this would give us the impact of the uniform pricing constraint on the access charge (at least for a given level of λ). At this point we can only concede that the comparison appears to be ambiguous, but we have no conclusive evidence yet that both cases can effectively arise.

Finally, the pricing rule for the incumbent's (final) product becomes more complex under the uniform pricing constraint. This is because the incumbent's price now pertains to two markets so that (32) combines the effects which were captured separately by (18) and (19).

4.2 Regime II: $a^r \leq p_I^r$ binding ($\mu > 0$)

The solution we have just discussed is valid only when it implies $a^r \leq p_I^r = p_I$. Otherwise, the constraint will be binding and we will have Regime II with $a^r = p_I = p_I^r = p_I^u$. We are then effectively left with only two instruments p_X and p_I. The pricing rule for X remains unchanged. For p_I we obtain:

$$\begin{aligned}\frac{p_I - c_1}{p_I} &= \frac{a^r - c_1}{a^r} \\ &= \frac{\lambda}{1+\lambda}\frac{1}{\sigma_Y} + \frac{1}{1+\lambda}\frac{(p_E^u - c_1)\left(d\widetilde{Y}_E^u/dp_I\right) + (p_E^r - a^r)\left(d\widetilde{Y}_E^r/dp_I\right)}{p_I\left(-d\widetilde{Y}/dp_I\right)} \qquad (34)\end{aligned}$$

where σ_Y is the "total" elasticity of the aggregate good $\widetilde{Y} = \widetilde{Y}_I^u + \widetilde{Y}_I^r + \widetilde{Y}_E^r$. Consequently, like its counterpart in the non uniform pricing case (Section 3), Regime II yields an inverse elasticity pricing rules which also includes an imperfect competition term.

To study the impact of uniform pricing on access charge we have to compare (34) with (24). For a given level of λ this amounts to comparing first of all σ_Y and σ_{Y^r}. Recalling that $\widetilde{Y}^r = \widetilde{Y}_I^r + \widetilde{Y}_E^r$ so that $\widetilde{Y} = \widetilde{Y}_I^u + \widetilde{Y}^r$, it is plausible to assume that $\widetilde{Y}$ has a more elastic demand then $\widetilde{Y}^r$.[20] This is because $\widetilde{Y}^r$ covers the entire demand in the rural area while$\widetilde{Y}_I^u$ covers only the incumbent's market share. Recall that the entrant offers a substitute to the incumbent's product in area u which is priced at c_1. Consequently when p_I increases, consumers in area r can only adjust their demand; substitution with E is not an effective threat since this operators price also increase (through the access charge). Consumers in u, on the other hand can switch to E whose price is not affected. Summing up, it appears that the uniform pricing constraint tends to decrease the first term in the access charge expression, at least for a *given level of* λ.

A Appendix

A.1 Characterization of the Entrant Pricing Strategy

The entrant's optimal pricing (reaction functions) $p_E^u = p_E^u(p_I^u)$ and $p_E^r = p_E^r(a^r, p_I^r)$ are defined implicitly by the equations (7) and (8). Differentiating the first equation with respect to p_I^u and the second with respect to p_I^r and a^r gives

$$\frac{\partial Y_E^u}{\partial p_I^u} + 2\frac{\partial Y_E^u}{\partial p_E^u}\frac{dp_E^u}{dp_I^u} + (p_E^u - c_1)\left[\frac{\partial^2 Y_E^u}{\partial p_I^u \partial p_E^u} + \frac{\partial^2 Y_E^u}{\left(\partial p_E^u\right)^2}\frac{dp_E^u}{dp_I^u}\right] = 0$$

$$\frac{\partial Y_E^r}{\partial p_I^r} + 2\frac{\partial Y_E^r}{\partial p_E^r}\frac{dp_E^r}{dp_I^r} + (p_E^r - a^r)\left[\frac{\partial^2 Y_E^r}{\partial p_I^r \partial p_E^r} + \frac{\partial^2 Y_E^r}{\left(\partial p_E^r\right)^2}\frac{dp_E^r}{dp_I^r}\right] = 0$$

$$\left(2\frac{dp_E^r}{da^r} - 1\right)\frac{\partial Y_E^r}{\partial p_E^r} + (p_E^r - a^r)\left[\frac{\partial^2 Y_E^r}{\left(\partial p_E^r\right)^2}\frac{dp_E^r}{da^r}\right] = 0$$

[20] The (price) elasticity of Y is a weighted sum of the elasticities of Y_I^u and and of Y^r:

$$\varepsilon_Y = \frac{Y_I^u}{Y}\varepsilon_{Y_I^u} + \frac{Y^r}{Y}\varepsilon_{Y^r}.$$

Consequently, $\varepsilon_{Y_I^u} > \varepsilon_{Y^r}$ implies $\varepsilon_Y > \varepsilon_{Y^r}$.

These expression can be re-arranged as:

$$\frac{dp_E^u}{dp_I^u} = \left(-\frac{\partial^2 \pi_E}{\partial \left(p_E^u\right)^2}\right)^{-1} \left[\frac{\partial Y_E^u}{\partial p_I^u} + \left(p_E^u - c_1\right) \frac{\partial^2 Y_E^u}{\partial p_I^u \partial p_E^u}\right]$$

$$\frac{\partial p_E^r}{\partial p_I^r} = \left(-\frac{\partial^2 \pi_E}{\partial \left(p_E^r\right)^2}\right)^{-1} \left[\frac{\partial Y_E^r}{\partial p_I^r} + \left(p_E^r - a^r\right) \frac{\partial^2 Y_E^r}{\partial p_I^r \partial p_E^r}\right]$$

$$\frac{\partial p_E^r}{\partial a^r} = \left(-\frac{\partial^2 \pi_E}{\partial \left(p_E^r\right)^2}\right)^{-1} \left(-\frac{\partial Y_E^r}{\partial p_E^r}\right)$$

where $\left(-\partial^2 \pi_E / \partial \left(p_E^u\right)^2\right)$ and $\left(-\partial^2 \pi_E / \partial \left(p_E^r\right)^2\right)$ are usually assumed to be positive (*i.e.* profits as a function of prices are assumed to be concave). In the linear case, we get:

$$0 \leq \frac{dp_E^u}{dp_I^u} = \frac{\left(\partial Y_E^u / \partial p_I^u\right)}{2\left(-\partial Y_E^u / \partial p_E^u\right)} \leq \frac{1}{2}$$

$$0 \leq \frac{\partial p_E^r}{\partial p_I^r} = \frac{\left(\partial Y_E^r / \partial p_I^r\right)}{2\left(-\partial Y_E^r / \partial p_E^r\right)} \leq \frac{1}{2}$$

and $\left(\partial p_E^r / \partial a^r\right) = 1/2$. More generally, if the demand $Y_E^r\left(p_E^r\right)$ is a decreasing and convex function of its price (*i.e.* $\left(\partial Y_E^r / \partial p_E^r\right) \leq 0$ and $\left(\partial^2 Y_E^r / \partial \left(p_E^r\right)^2\right) \geq 0$) one can show that $\left(\partial p_E^r / \partial a^r\right) \geq 1/2$.

A.2 Properties of the redefined demand function $\widetilde{Y}_j^k$

The demand functions $\widetilde{Y}_j^k$ incorporate the entrant reaction and thus depend only on the incumbent's prices p_I^u, p_I^r and a^r:

$$\begin{array}{ll} \widetilde{Y}_I^u(p_I^u) = Y_I^u[p_I^u, p_E^u(p_I^u)] & \widetilde{Y}_E^u(p_I^u) = Y_E^u[p_I^u, p_E^u(p_I^u)] \\ \widetilde{Y}_I^r(p_I^r, a^r) = Y_I^r[p_I^r, p_E^r(a^r, p_I^r)] & \widetilde{Y}_E^r(p_I^r, a^r) = Y_E^r[p_I^r, p_E^r(a^r, p_I^r)]. \end{array}$$

This give rise to the derivatives

$$\begin{array}{ll} \frac{d\widetilde{Y}_I^u}{dp_I^u} = \frac{\partial Y_I^u}{\partial p_I^u} + \frac{\partial Y_I^u}{\partial p_E^u}\frac{dp_E^u}{dp_I^u}, & \\ \frac{d\widetilde{Y}_E^u}{dp_I^u} = \frac{\partial Y_E^u}{\partial p_I^u} + \frac{\partial Y_E^u}{\partial p_E^u}\frac{dp_E^u}{dp_I^u}, & \\ \frac{\partial\widetilde{Y}_I^r}{\partial p_I^r} = \frac{\partial Y_I^r}{\partial p_I^r} + \frac{\partial Y_I^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial p_I^r}, & \frac{\partial\widetilde{Y}_I^r}{\partial a^r} = \frac{\partial Y_I^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial a^r}, \\ \frac{\partial\widetilde{Y}_E^r}{\partial p_I^r} = \frac{\partial Y_E^r}{\partial p_I^r} + \frac{\partial Y_E^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial p_I^r}, & \frac{\partial\widetilde{Y}_E^r}{\partial a^r} = \frac{\partial Y_E^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial a^r}. \end{array}$$

A.3 Proof of expressions (17)–(20)

The first -order conditions are given by:

$$\frac{\partial \Gamma}{\partial p_X} = -X + (1+\lambda)[X + (p_X - c_1 - c_2)\frac{\partial X}{\partial p_X}], \tag{35a}$$

$$\begin{aligned}\frac{\partial \Gamma}{\partial p_I^u} &= -Y_I^u + (p_E^u - c_1)\,\frac{\partial Y_E^u}{\partial p_I^u} + (1+\lambda)[Y_I^u + (p_I^u - c_1)\frac{d\widetilde{Y}_I^u}{dp_I^u}] \\ &\quad + \frac{dp_E^u}{dp_I^u}\,[-Y_E^u]\end{aligned} \tag{35b}$$

$$\begin{aligned}\frac{\partial \Gamma}{\partial p_I^r} &= -Y_I^r - Y_E^r\frac{\partial p_E^r}{\partial p_I^r} + (p_E^r - a^r)\,\frac{\partial Y_E^r}{\partial p_I^r} \\ &\quad + (1+\lambda)[Y_I^r + (p_I^r - c_1)\frac{\partial \widetilde{Y}_I^r}{\partial p_I^r} + (a^r - c_1)\frac{\partial \widetilde{Y}_E^r}{\partial p_I^r}],\end{aligned} \tag{35c}$$

$$\begin{aligned}\frac{\partial \Gamma}{\partial a^r} &= -Y_E^r\frac{\partial p_E^r}{\partial a^r} + (1+\lambda)[(p_I^r - c_1)\frac{\partial Y_I^r}{\partial p_E^r} + (a^r - c_1)\frac{\partial Y_E^r}{\partial p_E^r}]\frac{\partial p_E^r}{\partial a^r} \\ &\quad + \left(\frac{\partial p_E^r}{\partial a^r} - 1\right) Y_E^r + (p_E^r - a^r)\,\frac{\partial Y_E^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial a^r} + (1+\lambda)Y_E^r.\end{aligned} \tag{35d}$$

Setting equal to zero directly yields (17) from (35a). As to the urban market, making use of (7) which follows from profit maximization we can rearrange (35b) with the notations and formalism detailed in section (A.2) above to get:

$$\frac{\partial \Gamma}{\partial p_I^u} = \lambda Y_I^u + (p_E^u - c_1)\,\frac{d\widetilde{Y}_E^u}{dp_I^u} + (1+\lambda)(p_I^u - c_1)\frac{d\widetilde{Y}_I^u}{dp_I^u} = 0$$

and (18).

For the rural market, we obtain from the remaining conditions:

$$\frac{p_I^r - c_1}{p_I^r} = \frac{\lambda}{1+\lambda}\frac{1}{\sigma_{Y_I^r}} + \frac{(p_E^r - c_1) + \lambda(a^r - c_1)}{(1+\lambda)p_I^r}\frac{\left(\partial \widetilde{Y}_E^r/\partial p_I^r\right)}{\left(-\partial \widetilde{Y}_I^r/\partial p_I^r\right)} \tag{36}$$

$$\frac{a^r - c_1}{a^r} = \frac{\lambda}{1+\lambda}\frac{1}{\sigma_{Y_E^r}} + \frac{p_I^r - c_1}{a^r}\frac{\left(\partial \widetilde{Y}_I^r/\partial a^r\right)}{\left(-\partial \widetilde{Y}_E^r/\partial a^r\right)} - \frac{1}{1+\lambda}\frac{p_E^r - a^r}{a^r} \tag{37}$$

with

$$\sigma_{Y_I^r} = \frac{p_I^r}{Y_I^r}\left(\frac{-\partial \widetilde{Y}_I^r}{\partial p_I^r}\right) = \frac{p_I^r}{Y_I^r}\left(\frac{-\partial Y_I^r}{\partial p_I^r}\right) - \frac{p_I^r}{Y_I^r}\frac{\partial Y_I^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial p_I^r} \leq \varepsilon_{Y_I^r}$$

$$\sigma_{Y_E^r} = \frac{a^r}{\widetilde{Y}_E^r}\left(\frac{-\partial \widetilde{Y}_E^r}{\partial a^r}\right) = \frac{a^r}{Y_E^r}\left(\frac{-\partial Y_E^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial a^r}\right) = \left(\frac{a^r}{p_E^r}\frac{\partial p_E^r}{\partial a^r}\right)\varepsilon_{Y_E^r}.$$

This system can be rearranged to:

$$(p_I^r - c_1)\frac{\partial \widetilde{Y}_I^r}{dp_I^r} + \frac{[(p_E^r - a^r) + (1+\lambda)(a^r - c_1)]}{(1+\lambda)}\frac{\partial \widetilde{Y}_E^r}{\partial p_I^r} = -\frac{\lambda}{1+\lambda}Y_I^r$$
$$(p_I^r - c_1)\frac{\partial \widetilde{Y}_I^r}{\partial a^r} + \frac{(p_E^r - a^r) + (1+\lambda)(a^r - c_1)}{1+\lambda}\frac{\partial \widetilde{Y}_E^r}{\partial a^r} = -\frac{\lambda}{1+\lambda}Y_E^r$$

that yields

$$p_I^r - c_1 = \frac{\lambda}{1+\lambda}\frac{-Y_I^r\left(\partial \widetilde{Y}_E^r/\partial a^r\right) + Y_E^r\left(\partial \widetilde{Y}_E^r/\partial p_I^r\right)}{\left(\partial \widetilde{Y}_I^r/\partial p_I^r\right)\left(\partial \widetilde{Y}_E^r/\partial a^r\right) - \left(\partial \widetilde{Y}_I^r/\partial a^r\right)\left(\partial \widetilde{Y}_E^r/\partial p_I^r\right)}$$
$$\frac{[(p_E^r - a^r) + (1+\lambda)(a^r - c_1)]}{(1+\lambda)} = \frac{\lambda}{1+\lambda}\frac{-Y_E^r\left(\partial \widetilde{Y}_I^r/\partial p_I^r\right) + Y_I^r\left(\partial \widetilde{Y}_I^r/\partial a^r\right)}{\left(\partial \widetilde{Y}_I^r/\partial p_I^r\right)\left(\partial \widetilde{Y}_E^r/\partial a^r\right) - \left(\partial \widetilde{Y}_I^r/\partial a^r\right)\left(\partial \widetilde{Y}_E^r/\partial p_I^r\right)}$$

which reduces to (19)–(20) by defining:

$$\widehat{\sigma}_{Y_I^r Y_E^r} = p_I^r\frac{\left(\partial \widetilde{Y}_I^r/\partial p_I^r\right)\left(\partial \widetilde{Y}_E^r/\partial a^r\right) - \left(\partial \widetilde{Y}_I^r/\partial a^r\right)\left(\partial \widetilde{Y}_E^r/\partial p_I^r\right)}{-Y_I^r\left(\partial \widetilde{Y}_E^r/\partial a^r\right) + Y_E^r\left(\partial \widetilde{Y}_E^r/\partial p_I^r\right)} \tag{38}$$
$$\widehat{\sigma}_{Y_E^r Y_I^r} = a^r\frac{\left(\partial \widetilde{Y}_I^r/\partial p_I^r\right)\left(\partial \widetilde{Y}_E^r/\partial a^r\right) - \left(\partial \widetilde{Y}_I^r/\partial a^r\right)\left(\partial \widetilde{Y}_E^r/\partial p_I^r\right)}{-Y_E^r\left(\partial \widetilde{Y}_I^r/\partial p_I^r\right) + Y_I^r\left(\partial \widetilde{Y}_I^r/\partial a^r\right)} \tag{39}$$

which can also be written as:

$$\widehat{\sigma}_{Y_I^r Y_E^r} = \frac{\sigma_{Y_I^r}\sigma_{Y_E^r} - \sigma_{Y_I^r Y_E^r}\sigma_{Y_E^r Y_I^r}}{\sigma_{Y_E^r} + \frac{a^r Y_E^r}{p_I^r Y_I^r}\sigma_{Y_E^r Y_I^r}} = \sigma_{Y_I^r}\frac{\sigma_{Y_E^r} - \frac{\sigma_{Y_I^r Y_E^r}\sigma_{Y_E^r Y_I^r}}{\sigma_{Y_I^r}}}{\sigma_{Y_E^r} + \frac{a^r Y_E^r}{p_I^r Y_I^r}\sigma_{Y_E^r Y_I^r}} < \sigma_{Y_I^r},$$
$$\widehat{\sigma}_{Y_E^r Y_I^r} = \frac{\sigma_{Y_I^r}\sigma_{Y_E^r} - \sigma_{Y_I^r Y_E^r}\sigma_{Y_E^r Y_I^r}}{\sigma_{Y_I^r} + \frac{p_I^r Y_I^r}{a^r Y_E^r}\sigma_{Y_I^r Y_E^r}} = \sigma_{Y_E^r}\frac{\sigma_{Y_I^r} - \frac{\sigma_{Y_I^r Y_E^r}\sigma_{Y_E^r Y_I^r}}{\sigma_{Y_E^r}}}{\sigma_{Y_I^r} + \frac{p_I^r Y_I^r}{a^r Y_E^r}\sigma_{Y_I^r Y_E^r}} < \sigma_{Y_E^r},$$

where

$$\sigma_{Y_I^r Y_E^r} = \frac{a^r}{Y_I^r}\left(\frac{\partial \widetilde{Y}_I^r}{\partial a^r}\right)$$
$$\sigma_{Y_E^r Y_I^r} = \frac{p_I^r}{Y_E^r}\left(\frac{\partial \widetilde{Y}_E^r}{\partial p_I^r}\right)$$

are the cross price elasticities.

A.4 Proof of expressions (32)–(33)

With a uniform pricing constraint, conditions (35b) and (35c) are replaced by

$$\begin{aligned}\frac{\partial \Gamma}{\partial p_I} &= -Y_I^u + (p_E^u - c_1)\,\frac{\partial Y_E^u}{\partial p_I} + (1+\lambda)[Y_I^u + (p_I - c_1)\frac{d\widetilde{Y}_I^u}{dp_I}] + \frac{dp_E^u}{dp_I}\,[-Y_E^u] \\ &\quad - Y_I^r - Y_E^r\frac{\partial p_E^r}{\partial p_I} + (p_E^r - a^r)\,\frac{\partial Y_E^r}{\partial p_I} \\ &\quad + (1+\lambda)[Y_I^r + (p_I - c_1)\frac{\partial \widetilde{Y}_I^r}{\partial p_I} + (a^r - c_1)\frac{\partial \widetilde{Y}_E^r}{\partial p_I}], \qquad (40)\end{aligned}$$

Note that (35d)—as well as (35a)—continue to apply, so that the pricing rule for p_I and a^r are determined by solving (35d) and (40) that can be rewritten as:

$$\begin{aligned}\frac{p_I - c_1}{p_I} &= \frac{\lambda}{1+\lambda}\frac{1}{\sigma_{Y_I}} + \frac{1}{1+\lambda}\frac{p_E^u - c_1}{p_I}\frac{\left(d\widetilde{Y}_E^u/dp_I\right)}{\left(-\partial\widetilde{Y}_I/\partial p_I\right)} \\ &\quad + \frac{1}{1+\lambda}\left[\frac{(p_E^r - c_1) + \lambda(a^r - c_1)}{p_I}\right]\frac{\partial \widetilde{Y}_E^r}{\partial p_I}, \\ \frac{a^r - c_1}{a^r} &= \frac{\lambda}{1+\lambda}\frac{1}{\sigma_{Y_E^r}} + \frac{p_I^r - c_1}{a^r}\frac{\left(\partial\widetilde{Y}_I^r/\partial a^r\right)}{\left(-\partial\widetilde{Y}_E^r/\partial a^r\right)} - \frac{1}{1+\lambda}\frac{p_E^r - a^r}{a^r},\end{aligned}$$

where

$$\sigma_{Y_I} = \frac{p_I}{Y_I}\left(\frac{-\partial\widetilde{Y}_I}{\partial p_I}\right) = \frac{p_I}{Y_I}\left(\frac{-\partial Y_I}{\partial p_I}\right) - \frac{p_I}{Y_I}\left(\frac{\partial Y_I^u}{\partial p_E^u}\frac{\partial p_E^u}{\partial p_I} + \frac{\partial Y_I^r}{\partial p_E^r}\frac{\partial p_E^r}{\partial p_I}\right) \leq \varepsilon_{Y_I}.$$

This system can be rearranged as:

$$\begin{aligned}(p_I - c_1)\frac{\partial \widetilde{Y}_I}{\partial p_I} + \left[\frac{(p_E^r - c_1) + \lambda(a^r - c_1)}{1+\lambda}\right]\frac{\partial \widetilde{Y}_E^r}{\partial p_I} &= -\frac{\lambda}{1+\lambda}Y_I \\ &\quad - \frac{1}{1+\lambda}\,(p_E^u - c_1)\,\frac{d\widetilde{Y}_E^u}{dp_I} \\ (p_I - c_1)\frac{\partial \widetilde{Y}_I^r}{\partial a^r} + \left[\frac{(p_E^r - a^r) + \lambda(a^r - c_1)}{1+\lambda}\right]\frac{\partial \widetilde{Y}_E^r}{\partial a^r} &= -\frac{\lambda}{1+\lambda}Y_E^r\end{aligned}$$

and the solution is given by:

$$p_I - c_1 = \frac{\lambda}{1+\lambda}\frac{-\left[Y_I + \frac{1}{\lambda}\left(p_E^u - c_1\right)\frac{d\widetilde{Y}_E^u}{dp_I}\right]\left(\partial\widetilde{Y}_E^r/\partial a^r\right) + Y_E^r\left(\partial\widetilde{Y}_E^r/\partial p_I\right)}{\left(\partial\widetilde{Y}_I/\partial p_I\right)\left(\partial\widetilde{Y}_E^r/\partial a^r\right) - \left(\partial\widetilde{Y}_I^r/\partial a^r\right)\left(\partial\widetilde{Y}_E^r/\partial p_I\right)}$$

$$\frac{\left[(p_E^r - a^r) + (1+\lambda)(a^r - c_1)\right]}{(1+\lambda)} = \frac{\lambda}{1+\lambda}\frac{-Y_E^r\left(\partial\widetilde{Y}_I/\partial p_I\right) + Y_I\left(\partial\widetilde{Y}_I^r/\partial a^r\right)}{\left(\partial\widetilde{Y}_I/\partial p_I\right)\left(\partial\widetilde{Y}_E^r/\partial a^r\right) - \left(\partial\widetilde{Y}_I^r/\partial a^r\right)\left(\partial\widetilde{Y}_E^r/\partial p_I\right)}$$

which yields simplifies to (32)–(33) by defining the superelasticities in the usual way:

$$\widehat{\sigma}_{Y_I Y_E^r} = \frac{\sigma_{Y_E^r}\sigma_{Y_I} - \left(Y_I^r/Y_I\right)\sigma_{Y_I^r Y_E^r}\sigma_{Y_E^r Y_I}}{\sigma_{Y_E^r} + \frac{a^r Y_E^r}{p_I Y_I}\sigma_{Y_E^r Y_I}}, \tag{41}$$

$$= \frac{\sigma_{Y_E^r}\sigma_{Y_I} - \left(Y_I^r/Y_I\right)\sigma_{Y_I Y_E^r}\sigma_{Y_E^r Y_I}}{\sigma_{Y_E^r} + \frac{a^r Y_E^r}{p_I Y_I}\sigma_{Y_E^r Y_I}}. \tag{42}$$

Note that to go from (41) to (42), we have used the property $\left(\partial\widetilde{Y}_I/\partial p_E^r\right) = \left(\partial\widetilde{Y}_I^r/\partial p_E^r\right)$ which obtains because $\left(\partial\widetilde{Y}_I^u/\partial p_E^r\right) = 0$. The other superelasticity is defined as follows

$$\widehat{\sigma}_{Y_E^r Y_I} = \frac{\sigma_{Y_E^r}\sigma_{Y_I} - \left(Y_I^r/Y_I\right)\sigma_{Y_I^r Y_E^r}\sigma_{Y_E^r Y_I}}{\sigma_{Y_I} + \frac{p_I Y_I^r}{a^r Y_E^r}\sigma_{Y_I^r Y_E^r}},$$

$$= \frac{\sigma_{Y_E^r}\sigma_{Y_I} - \left(Y_I^r/Y_I\right)\sigma_{Y_I Y_E^r}\sigma_{Y_E^r Y_I}}{\sigma_{Y_I} + \frac{p_I Y_I^r}{a^r Y_E^r}\sigma_{Y_I Y_E^r}}.$$

Remark also that $\widehat{\sigma}_{Y_I Y_E^r} < \sigma_{Y_I}$ and that $\widehat{\sigma}_{Y_E^r Y_I} < \sigma_{Y_E^r}$.

References

Armstrong, M., (2002), "The theory of access pricing and interconnection", in *Handbook of Telecommunication,* vol 1., edited by M. Cave et al., 295–386.

Armstrong, M., C. Doyle and J. Vickers (1996), "The access pricing problem: a synthesis", *Journal of Industrial Economics,* **44,** 131–150**.**

Baumol, W and J. Sidak (1994a), "The pricing of inputs sold to competitors", *Yale Journal on Regulation*, **11**, 171–207.

Baumol, W and J. Sidak (1994b), *Toward Competition in Local Telephony*, MIT Press, Cambridge, MA.

Billette de Villemeur, E, H. Cremer, B. Roy. and J. Toledano (2002), "Pricing and worksharing discounts in the postal sector" in *Postal and Delivery Services: Delivering on Competition*, edited by M. A. Crew and P. R. Kleindorfer, Boston : Kluwer Academic Publishers, 33–48.

Billette de Villemeur, E, H. Cremer, B. Roy. and J. Toledano (2003), "Optimal pricing and global price-cap in the postal sector", *Journal of Regulatory Economics,* **24**, 49-62.

Cremer, H., M. De Rycke and A. Grimaud (1995), "Alternative scenarios for the reform of postal services: optimal pricing and welfare", in *Commercialization of Postal and Delivery Services*, edited by M. A. Crew and P. R. Kleindorfer, Boston: Kluwer Academic Publishers, 247–267.

Cremer, H., M. De Rycke and A. Grimaud (1997), "Costs and benefits of universal service obligations in the postal sector" in *Managing Change in the Postal and Delivery Industries*, edited by M. A. Crew and P. R. Kleindorfer, Boston : Kluwer Academic Publishers, 22–41.

Cremer, H., A. Grimaud and J.-J. Laffont (2000), "The cost of universal service in the postal sector" in *Current Directions in Postal Reform*, edited by M. A. Crew and P. R. Kleindorfer, Boston : Kluwer Academic Publishers, 47–68.

Cremer, H., J.P. Florens, A. Grimaud, S. Marcy, B. Roy. and J. Toledano. (2001), "Entry and competition in the postal market : foundations for the construction of entry scenarios", *Journal of Regulatory Economics*, **19**, 107-121.

Crew M. and P. Kleindorfer (2002), "Balancing access and the universal service obligation", in *Postal and Delivery Services: Delivering on Competition*, edited by M. A. Crew and P. R. Kleindorfer, Boston : Kluwer Academic Publishers, 3–32.

Laffont, J.J. and J. Tirole (1996), "Creating competition through interconnection: Theory and practice", *Journal of Regulatory Economics*, **10**, 227–256.

Laffont, J.J. and J. Tirole (2000), *Competition in Telecommunications,* MIT Press, Cambridge MA.

Law, P. (1997), *Welfare effects of pricing in anticipation of Laspeyres*, Bulletin of Economic research, **49**, 17–27.

Mitchell, R. (1999), Postal worksharing: welfare, technical efficiency, and Pareto optimality, in *Emerging Competition in Postal and Delivery Services*, edited by M. A. Crew and P. R. Kleindorfer, Boston: Kluwer Academic Publishers,

Panzar, J.(2002), "Reconciling competition, downstream access and universal service and universal service in postal markets", in *Postal and Delivery Services: Delivering on Competition*, edited by M. A. Crew and P. R. Kleindorfer, Boston : Kluwer Academic Publishers, 93–115..

Sherman, R. (2001), "Optimal worksharing discounts", *Journal of Regulatory Economics*, **19**, 81–92.