

Buyer Group Without Exclusive Purchase: With Applications to Library Consortium*

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Abstract

We study how the formation of a buyer group affects the group members' total payoffs. We first consider a two-seller-two-buyer model (relying on the common agency analysis of Bernheim and Whinston 1986, 1998), and show that under standard conditions the buyers never gain from building a group, contrary to what happens in Inderst and Shaffer (2007) and Dana (2012). Second, for the application to library consortium, we introduce buyers' budget constraint. Contrary to the conventional wisdom to build a consortium around groups of homogenous institutions (Davis, 2002), we find that libraries with similar (opposite) preferences are likely to lose (gain) from building a consortium.

Keywords: Common agency, Buyer Group, Library Consortium, Academic Journals, Budget Constraint, Correlation, Multimarket Contact.

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1 Introduction

In many sectors, buyers purchasing multiple products can form a buyer group. Examples of buyer groups abound in retailing, health care, agriculture, academic journals, etc. In addition, existing buyer groups can expand their size by merging with other groups. Understanding how buyer group affects buyer power is very important given the increasing policy makers' concerns about buyer power (European Commission, 1999 and OECD, 2008).¹ In this paper, we study the conditions under which a buyer group increases (or reduces) the total payoffs of its members. We investigate this question in situations when multiple sellers compete by offering personalized non-linear tariffs and each buyer (and buyer group) is interested in buying a positive quantity from each seller. After analyzing this question in a general setting, analogous to the models considered in Bernheim and Whinston (1986, 1998) and Prat and Rustichini (2003), we specialize it to study library consortium.

In the case of the market for academic journals, each library (or library consortium) buys subscriptions to distinct academic journals from multiple publishers, and publishers provide personalized prices to different libraries (or library consortia).² Similarly, hospitals (or groups of hospitals) buy different drugs or vaccines from multiple pharmaceutical firms, and the latter often offer different non-linear tariffs to different hospitals (or co-operatives).³ As long as each buyer participating in a buyer group is a firm, it is very natural to consider that each buyer buys multiple products from multiple sellers, and that these sellers compete by offering personalized non-linear tariffs.⁴ However, the existing literature on buyer group only studies situations in which each buyer or buyer group buys exclusively from one among competing sellers: see in particular Inderst and Shaffer (2007) and Dana (2012). Although exclusive purchase is often practiced, non-exclusive purchase should be the norm when buyers buy diverse products which are not close substitutes as in the case of libraries, hospitals, etc. We add some novel insights to the literature on buyer group by analyzing the situation of non-exclusive purchases.

¹In US, the Federal Trade Commission organized a workshop on slotting allowances in 2000, a major buyer power issue in grocery retailing. See Chen (2007) for a survey of the literature on buyer power and antitrust policy implications.

²Actually, publishers' price offers are tailored directly to individual characteristics of libraries or consortia. According to Edlin and Rubinfeld (2004), "Here, the price that a buyer is quoted depends upon the buyer's observable characteristics. ... Moreover, in practice, the price of the Big Deal is often individually negotiated with a given library or with groups of libraries called "consortia," offering further opportunities for the publisher to price based on individual characteristics."

³See for instance the French antitrust case against GlaxoSmithKline France (Autorité de Concurrence, 2007) in which GlaxoSmithKline offered different non-linear prices to different hospitals or groups of hospitals.

⁴Note that contracts for this kind of B2B transactions are often secret.

In the first part of the paper, We study buyer group in a two-seller-two-buyer model in which the buyers are in separate markets and view products as substitutes, and firms have convex cost functions. In this model, if buyer group is formed, we are in the common agency setting of Bernheim and Whinston (1986, 1998) and therefore focus on the sell-out equilibrium considered in Bernheim and Whinston (1986, 1998), which implements the first best allocation. For the case without buyer group, we identify a particular equilibrium in which sellers use two part tariffs with slope given by the marginal cost at the first best allocation. Whereas in the setting of Inderst and Shaffer (2007) and Dana (2012) with exclusive purchase formation of a buyer group can never reduce the total payoffs of its members, in our setting we find that the formation of a buyer group can never increase the total payoffs of the members, and in fact it strictly harms buyers except in the extreme cases with independent products and/or linear cost functions. Furthermore, we show that such a result holds for any efficient equilibrium the sellers may play on the absence of buyer group since a necessary condition for equilibrium existence turns out to be equivalent to the condition that each firm prefers to trade with the buyer group rather than with separate buyers. We extend our results to the case of complementary products, provided that cost functions are linear. Precisely, buyer group has no effect on any player's payoff if the products sold by the sellers are complements to both buyers, but if the products are strict substitutes for buyer 1, say, and strict complements for buyer 2, then buyer group reduces the buyers' total payoff. This is because the sellers have some residual market power with respect to buyer 2, as each seller charges less than the incremental value of its product because charging the incremental value leads to a strictly negative payoff for buyer 2. Buyer group allows the sellers to transfer the residual market power to buyer 1 in the same way as multi-market contact facilitates collusion by transferring residual collusive power from one market to another (Bernheim and Whinston, 1990).⁵

In the second part of the paper, we introduce some special features into the model for application to library consortium. More precisely, we assume that each publisher sells a bundle of the own electronic academic journals, the bundles have independent values and each buyer faces a budget constraint. Since we assume each of the two sufficient conditions for buyer group neutrality, buyer group has no effect without the budget constraint. Therefore, our analysis isolates the role of the budget constraint on buyer group. We find that libraries with similar (opposite) preferences are likely to lose (win) from building a consortium, regardless of whether the budget is exogenously given or endogenously determined by a benevolent funding agency. The intuition can be given again in terms of multi-market contact (Bernheim and Whinston, 1990). Consider the case in which library 1 consumes both bundles A and B but library 2 consumes only bundle A in the absence of

⁵Inderst and Shaffer (2007) and Dana (2012) do not find this result because they do not allow for complementary products.

consortium: library 2 likes bundle A so much that publisher A monopolizes the market for library 2. Then, building a consortium can either increase or decrease the total surplus of the libraries depending on the size of its budget. If the budget is small enough, publisher A can export its residual monopoly power from library 2 to library 1 and monopolize the market for the consortium. On the contrary, if the budget is large enough, publisher A cannot monopolize the entire market and thus the consortium consumes both bundles. In particular, in the extreme case of perfectly negative correlation, building a consortium creates a level-playing field between the two publishers each of whom monopolized a library's market such that no publisher monopolizes the consortium's market.

There are two lessons from our analysis of two different models. First, in many situations, buyer group may reduce buyer power instead of increasing it. Second, what matters for buyer group to increase buyer power is not the sheer size of the group, but its composition as is stressed by Dana (2012): for instance, in both models, we find that buyers with similar preferences are unlikely to gain from building a buyer group. This goes against what Philip M. Davis (2002), a bibliographer at Cornell University, recommends regarding library consortium.

"It is recommended that institutions consider their consortia membership and organize themselves into groups of homogenous institutions with similar missions".

If one thinks that publishers propose menu of prices with quantity discounts based on the number of potential users, then it might be desirable to build consortia with libraries having similar preferences as is recommended by Philip M. Davis.⁶ However, this reasoning based on quantity discounts implicitly assumes that publishers' price schedules do not change much after a consortium is formed, which can be true in a short-run but cannot be true in a long-run. In fact, Dewatripont et al. (2006) point out that "we may fear that consortia in fact strengthen the possibility for publishers to charge a high price for their electronic collection (p.52)." We take a long-term view in the sense that publishers change their prices after libraries form a consortium and find that libraries with similar preferences have almost nothing to gain from building a consortium ⁷

⁶In fact, Gatten and Sanville (2004) compute the Spearman's correlation coefficients between each pair of member institutions of OhioLINK, a well-known library consortium, which varies between -1 and 1 and find that overall relative use of Big Deal titles between member institutions correlates highly.

⁷Since we consider that publishers make price offers (simultaneously) before libraries make purchase decisions, our model does not capture any gain from increase in the bargaining power of libraries. However, Dewatripont et al. (2006) argue that "since researchers do not see the various publishers as good substitutes and need access to all journals, consortia only introduce a relatively weak 'buyer power' (p.8)." Dewatripont et al. (2006) also write, "This 'buyer concentration' remains however modest in comparison with publisher concentration: the largest library consortium represents 2 or 3% of global journal purchases, while the largest publisher represents more than 20% of journal sales (p.8)".

The paper is organized as follows. Section 1.1 presents a review of the related literature. Section 2 studies buyer group in a framework without the budget constraint. Section 3 studies library consortium when each library faces a budget constraint. Section 4 provides concluding remarks.

1.1 Literature review

To our knowledge, we are the first to analyze the effect of buyer group in a framework that extends the common agency (Bernheim and Whinston, 1986, 1998) to multiple buyers. Prat and Rustichini (2003) analyze a more general framework than ours that has multiple sellers and buyers and prove the existence of an efficient equilibrium, under the assumptions that buyers' utility functions are concave, sellers' cost functions are convex, by allowing for more complex tariffs than ours. Precisely, Prat and Rustichini (2003) allow seller j to make buyer i 's payment depend on the whole vector of i 's purchases such that the payment depends on q_j^i and also on q_h^i for each seller h different from j . However, they do not specify the equilibrium strategies and do not compare the case of buyer group with the case of no group. In our two-buyer-two-seller setting, under the assumption of substitute goods and convex cost functions, we spell out a profile of tariffs which constitute an equilibrium, and such equilibrium has the property that the payment of each buyer i to each seller j only depends on q_j^i , the quantity buyer i buys from seller j . Note also that the well-known result that competition among sellers under common agency achieves the outcome that maximizes the joint payoff of all sellers and the buyer fails to hold in the presence of budget constraint.⁸

Our paper is related to the papers that study buyer group when sellers compete: Inderst and Shaffer (2007), Marvel and Yang (2008), Dana (2012), Chen and Li (2013). Among those paper, our paper is more closely related to Inderst and Shaffer (2007) and Dana (2012) since they assume that sellers have complete information on buyers' preferences and hence can offer personalized tariffs, regardless of whether or not buyers form a group.⁹ Although the two papers differ in the way they generalize their results,¹⁰ the

⁸In contrast, Jeon and Menicucci (2012) find that the result holds when the buyer faces a slot constraint instead of the budget constraint.

⁹For instance, in Marvel and Yang (2008) and Chen and Li (2013), buyers are located on the Hotelling line and each seller makes the same price offer to all buyers in the absence of buyer group. However, Marvel and Yang (2008) and our paper are similar in one aspect: when buyers form a group, sellers propose non-linear tariffs and the group can buy a positive quantity from both sellers.

¹⁰Inderst and Shaffer (2007) consider competition in non-linear tariff between the two sellers and extend their result to a bargaining setup. They also make each seller's choice of product characteristics endogenous. Dana (2012) considers n sellers, a continuum of buyers, and allows that different buyer groups are formed. He proves that the grand coalition is a coalition-proof subgame perfect equilibrium when there are two sellers.

main insight can be obtained by considering a two-seller-two-buyer setting in which each buyer buys one unit from only one of the two sellers. They assume that the buyer group makes exclusive purchase commitment and that sellers make a take-it-or-leave-it offer. In this setup, a buyer group never decreases its members' total payoffs: it strictly increases the members' total payoffs unless they have identical preferences; it has no impact on the members' payoffs in case of identical preferences. There are two main differences between our paper and Inderst and Shaffer (2007) and Dana (2012): in our paper, the buyer group does not make exclusive purchase commitment and each buyer (or buyer group) can buy multiple units from both sellers. Contrary to what happens in their papers, forming a buyer group can reduce the members' total payoffs in our two different models.

Some papers study buyer coalition in a monopoly setting. Chipty and Snyder (1999) and Inderst and Wey (2007) consider some specific bargaining models and find that a convex cost function for the seller helps to make profitable the formation of a buyer group. This occurs because the incremental costs to serve the group is lower than the sum of the incremental cost to serve each member, and buyers are assumed to have some bargaining power. Since we consider take-it-leave-it offers from sellers, this effect is absent in our setting. Precisely, if the products have independent values and there is no budget constraint, the demand faced by each seller is independent of the quantity sold by the other seller; in this monopoly setting, buyer group has no effect. Innes and Sexton (1993, 1994) analyze the case in which a monopolist is facing identical consumers who may form coalitions. They show that even though consumers' characteristics are homogeneous, the monopolist may price discriminate in order to deter the formation of coalitions, whereas price discrimination is unprofitable in the absence of coalitions. Alger (1999) studies a monopolist's optimal menu of price-quantity pairs when (a continuum of) consumers can purchase multiple times and/or jointly in a two-type setting. While the previous papers consider buyer coalition formation under complete information, Jeon and Menicucci (2005) study a monopolist's optimal menu of price-quantity pairs when buyers form a coalition under asymmetric information between themselves.

Our paper builds on the literature on the market for academic journals which studies issues raised by the move to electronic publishing.¹¹ The literature has studied bundling and/or price discrimination (McCabe, 2004, Jeon and Menicucci, 2006, Armstrong 2010), interoperability (Jeon and Menicucci, 2011), open access journals (McCabe and Snyder, 2007, Jeon and Rochet, 2010). We contribute to the literature by studying the issue of library consortium. Our model of competition between libraries builds on our previous papers (Jeon and Menicucci, 2006 and 2011).

Our result that libraries with opposite preferences (instead of libraries with similar preferences) should form a library consortium is reminiscent of a classic paper in the

¹¹See Bergstrom (2001) and Dewatripont et al. (2006) for an introduction.

bundling literature, Adams and Yellen (1976), that shows that pure bundling of two products gives a monopolist a higher (resp. a lower) profit than independent pricing if buyers' valuations of the products are negatively correlated (resp. positively correlated). However, the two papers differ in many aspects. Adams and Yellen consider bundling of two products sold by a monopolist to a mass of heterogenous consumers whereas we study library consortium when publishers compete by offering personalized prices to each buyer.

Our paper also belongs to the emerging literature on personalized pricing (Chen and Iyer, 2002, Choudhary et al., 2005, Ghose and Huand, 2009, Shaffer and Zheng, 2002, Thisse and Vives, 1988). Personalized pricing refers to the practice that firms offer customized prices on a one-to-one basis to each customer (an individual or a firm), which has become possible since advances in information technologies and the Internet allow firms to identify each customer with greater accuracy and cost-effectiveness.

2 Buyer group without budget constraint

In this section, we consider a model in which each buyer has no budget constraint. It is a natural generalization of the common agency model under complete information of Bernheim and Whinston (1988) to two buyers instead of one buyer.

2.1 Model

There are two sellers, A and B , and two buyers, 1 and 2. Let $q_j^i \geq 0$ represent the quantity of the product that buyer i ($= 1, 2$) buys from seller j ($= A, B$). For each buyer i , the gross utility from consuming q_A^i, q_B^i is given by $U^i(q_A^i, q_B^i)$, with U^i strictly increasing and strictly concave in (q_A^i, q_B^i) (precisely, we suppose that the Hessian matrix of U^i is negative definite at any (q_A^i, q_B^i)). Moreover, we assume that the two goods are substitute for buyer i : $\partial^2 U^i / \partial q_A^i \partial q_B^i \leq 0$ at any point. For each seller j , the cost of serving the buyers is $C_j(q_j^1 + q_j^2)$, with C_j convex and strictly increasing.¹²

When there is no buyer group, we allow each seller to offer a personalized non-linear tariff: the tariff offered by seller j to buyer i is denoted by $p_j^i(q)$, and can be different from the tariff seller j offers to buyer k . After seeing the tariffs, buyer i chooses q_A^i, q_B^i in order to maximize $U^i(q_A^i, q_B^i) - p_A^i(q_A^i) - p_B^i(q_B^i)$.

When the buyers form a buyer group, the sellers compete for serving the group. Let C denote the buyer group (i.e., buyer consortium), q_j^C the quantity C buys from seller j , and $p_j^C(q)$ the non-linear tariff that seller j offers to C . Given tariffs, the group's optimization

¹²We consider a case with complementary goods in Proposition 4.

problem is given by:

$$\max_{q_A^1, q_B^1, q_A^2, q_B^2, q_A^C, q_B^C} U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) - p_A^C(q_A^C) - p_B^C(q_B^C)$$

subject to

$$q_A^1 + q_A^2 \leq q_A^C, \quad q_B^1 + q_B^2 \leq q_B^C.$$

Since $q_j^1 + q_j^2 \leq q_j^C$ is satisfied with equality in any equilibrium, $p_j^C(q_j^C)$ can be equivalently written as $p_j^C(q_j^1 + q_j^2)$.

Both with buyer group and without buyer group, we consider a game with the same timing, described as follows:

- Stage 1: When there is no buyer group, each seller j ($= A, B$) simultaneously chooses $p_j^i(q)$ for $i = 1, 2$. When the buyer group is formed, each seller j ($= A, B$) simultaneously chooses $p_j^C(q)$.
- Stage 2: Each buyer i , or the buyer group, makes a purchase decision.

2.2 Analysis

2.2.1 Buyer group

The setting with buyer group is essentially an environment with a unique buyer. This allows to focus on the so-called *sell-out equilibrium* (Bernheim and Whinston 1986, 1998), which we define after introducing suitable notation. Precisely, we define $\mathbf{q}^* \equiv (q_A^{1*}, q_B^{1*}, q_A^{2*}, q_B^{2*})$ as the unique allocation vector that maximizes social welfare,¹³ and V_{AB}^C as the social welfare in the first-best allocation \mathbf{q}^* :

$$\mathbf{q}^* \equiv \arg \max_{(q_A^1, q_B^1, q_A^2, q_B^2)} U^1(q_A^1, q_B^1) + U^2(q_A^2, q_B^2) - C_A(q_A^1 + q_A^2) - C_B(q_B^1 + q_B^2) \quad (1)$$

$$V_{AB}^C \equiv U^1(q_A^{1*}, q_B^{1*}) + U^2(q_A^{2*}, q_B^{2*}) - C_A(q_A^{1*} + q_A^{2*}) - C_B(q_B^{1*} + q_B^{2*}). \quad (2)$$

We also need to study the case in which the group trades only with one seller, for instance only with seller j :

$$(\hat{q}_j^1, \hat{q}_j^2) \equiv \arg \max_{q_j^1, q_j^2} U^1(0, q_j^1) + U^2(0, q_j^2) - C_j(q_j^1 + q_j^2) \quad (3)$$

$$V_j^C \equiv U^1(0, \hat{q}_j^1) + U^2(0, \hat{q}_j^2) - C_j(\hat{q}_j^1 + \hat{q}_j^2) \quad (4)$$

Hence V_j^C is the maximal social welfare when the group trades with seller j , but not with the other seller.

¹³Our assumptions imply that the objective function for each maximization problem defined in this section is strictly concave, and therefore a unique maximizer exists.

In the competition for the buyer group, a sell-out strategy for seller j is

$$p_j^C(q) = F_j^C + C_j(q) \quad \text{for any } q > 0.$$

Precisely, the group is required to pay a fixed fee F_j^C for the right to buy any quantity from seller j at cost. When each seller uses a *sell-out strategy*, the first-best allocation is achieved (provided that the group chooses to buy a positive amount of good from each seller) since the group will buy $q_j^{1*} + q_j^{2*}$ from each seller j and allocate q_j^{1*}, q_j^{2*} to the buyers. In equilibrium it is necessary to have¹⁴

$$V_{AB}^C - F_A^C - F_B^C = V_A^C - F_A^C = V_B^C - F_B^C. \quad (5)$$

The sell-out equilibrium is given by sell-out strategies with the fixed fees obtained from (5):

$$F_A^{C*} = V_{AB}^C - V_B^C, \quad F_B^{C*} = V_{AB}^C - V_A^C$$

The group's payoff is $V_A^C + V_B^C - V_{AB}^C$, which is non-negative since goods are substitutes for each buyer. Each seller j 's profit is equal to F_j^{C*} , which is the marginal contribution to social welfare of seller j , that is the first best social welfare minus the social welfare that is generated when the group trades only with the other seller.

2.2.2 No buyer group

When there is no buyer group, we restrict attention to non-linear tariffs such that what buyer i pays to seller j depends only on what i buys from j but not on what i buys from $h(\neq k)$. It seems natural to focus on a generalization of the sell-out equilibrium to the case of two buyers, such that the tariff offered by seller j to buyer i is given by a fixed fee plus the cost for seller j of serving buyer i . This is straightforward to do for linear cost functions, but when the cost functions are strictly convex there is a cost externality such that the marginal cost seller j incurs to serve a buyer depends on the quantity he sells to the other buyer. We are interested in an efficient equilibrium, that is an equilibrium whose outcome is the first best allocation. Hence, one possibility is to consider the following tariffs:

$$p_j^i(q) = F_j^i + C_j(q + q_j^{k*}) - C_j(q_j^{k*}) \quad \text{for any } q > 0, \text{ for each } i, j \quad (6)$$

The term $C_j(q + q_j^{k*}) - C_j(q_j^{k*})$ is the incremental cost for seller j to produce q for buyer i , given that j already accepted to produce q_j^{k*} for buyer k . The tariff in (6) is such that buyer i is required to pay a fixed fee F_j^i for the right to buy any quantity from seller j at

¹⁴For instance, if $V_{AB}^C - F_A^C - F_B^C > V_A^C - F_A^C$ then firm B can increase the own profit by slightly increasing F_B^C , as the buyer will still buy from firm B .

the incremental cost incurred by j . If the other seller uses a tariff analogous to (6), then each buyer i buys q_A^{i*}, q_B^{i*} and the first-best allocation \mathbf{q}^* is realized. Although we could determine the fixed fees by applying the same arguments used for the buyer group, we would not obtain an equilibrium because a profitable deviation would exist for each seller. Precisely, it would be profitable for seller j to deviate making suitable take-it-or-leave-it offers to buyers in such a way to induce one buyer, i , to buy only from seller j , and the other buyer, k , to buy from both sellers. Seller j can obtain the "equilibrium" payoff by offering buyer i a quantity \tilde{q}_j^i which maximizes $U^i(q_j^i, 0) - C_j(q_j^i + q_j^{k*})$, and offering buyer k the quantity q_j^{k*} (with payments which leave each buyer indifferent between accepting the offer and buying only from seller h). However, substitute products for buyer i imply that \tilde{q}_j^i is greater than q_j^{i*} , and this increases the marginal cost for j of serving buyer k , which makes it profitable for seller j to reduce the quantity offered to buyer k below q_j^{k*} . This yields a payoff higher than the equilibrium payoff, and establishes that no equilibrium exists with tariffs as in (6).

In this subsection we prove that there exists an efficient equilibrium with tariffs as follows:

$$p_j^i(q) = F_j^i + \alpha_j q \quad \text{for any } q > 0, \quad \text{with } \alpha_j = C_j'(q_j^{i*} + q_j^{k*}), \quad \text{for any } i, j \quad (7)$$

This tariff is such that buyer i is required to pay a fixed fee F_j^i for the right to buy any quantity from seller j at the marginal price α_j , which is the marginal cost for seller j at the first best allocation. When both sellers use tariffs of this kind, each buyer i will buy q_A^{i*}, q_B^{i*} ; thus the first-best allocation \mathbf{q}^* is achieved.

In order to describe the equilibrium fees we define $V_{AB}^i, \tilde{q}_h^i, V_h^i$ as follows:

$$V_{AB}^i = U^i(q_A^{i*}, q_B^{i*}) - \alpha_A q_A^{i*} - \alpha_B q_B^{i*} \quad (8)$$

$$\tilde{q}_h^i \equiv \arg \max_{q_h^i} (U^i(q_h^i, 0) - \alpha_h q_h^i) \quad \text{and} \quad V_h^i \equiv U^i(\tilde{q}_h^i, 0) - \alpha_h \tilde{q}_h^i \quad (9)$$

Thus V_{AB}^i is the payoff of buyer i (gross of the fixed fees) if he buys q_A^{i*}, q_B^{i*} , V_h^i is the payoff of buyer i (gross of the fixed fee) if he trades only with seller h , and \tilde{q}_h^i is the payoff maximizing quantity in that case.

As in the case of a single buyer, in equilibrium it is necessary that $V_{AB}^i - F_A^i - F_B^i = V_A^i - F_A^i = V_B^i - F_B^i$, and from these equalities we obtain

$$F_A^{i*} = V_{AB}^i - V_B^i, \quad F_B^{i*} = V_{AB}^i - V_A^i \quad \text{for } i = 1, 2. \quad (10)$$

Next proposition establishes that in this way we have obtained an equilibrium:

Proposition 1 (*equilibrium with two-part tariffs*) Assume substitute goods for each buyer and convex cost functions. Then, for the setting without buyer group, the tariffs described by (7) and (10) constitute an efficient equilibrium.

Given the tariffs described by (7) and (10), no deviation like the one described above is profitable for any seller because, with respect to the case of tariffs in (6), the equilibrium payoff is reduced but the payoff from the deviation is reduced even more. More precisely, the profit of seller j is determined, among other things, by the payoff buyer i can obtain by trading only with seller h : see (9). Since goods are substitutes, \tilde{q}_h^i is greater than q_h^{i*} and the tariff in (7) has a lower marginal cost than the tariff in (6) for $q > q_h^{i*}$. This increases buyer i 's payoff and decreases the equilibrium payoff of seller j . However, in case seller j deviates, he needs to leave to each buyer the buyer's equilibrium payoff, and this generates a reduction in j 's deviation profit which is greater than the reduction in his equilibrium payoff.

In this equilibrium, the payoff of each buyer i is $V_A^i + V_B^i - V_{AB}^i$, which is non-negative since goods are substitutes. The profit of seller j is $\pi_j^{nC} \equiv F_j^{1*} + F_j^{2*} + \alpha_j(q_j^{1*} + q_j^{2*}) - C_j(q_j^{1*} + q_j^{2*})$, and it can be interpreted about in the same way as when there is a unique buyer. Precisely, $F_j^{i*} = V_{AB}^i - V_h^i$ is analogous to the first best welfare (considering only buyer i) minus the social welfare generated when buyer i trades only with seller h , after each cost function is replaced by a linear function. Then the correction term $\alpha_j(q_j^{1*} + q_j^{2*}) - C_j(q_j^{1*} + q_j^{2*})$ is added in order to take into account the real cost borne by firm j .

Comparing π_j^{nC} with F_j^{C*} we see that seller j 's profit under buyer group is higher than the profit in the absence of group if the following inequality holds

$$\begin{aligned} \Delta\pi_j &\equiv F_j^{C*} - \pi_j^{nC} > 0 \\ \Leftrightarrow (V_{AB}^C - V_h^C) - (V_{AB}^1 + V_{AB}^2 - V_h^1 - V_h^2 + \alpha_j(q_j^{1*} + q_j^{2*}) - C_j(q_j^{1*} + q_j^{2*})) &> 0. \end{aligned} \tag{11}$$

Next proposition describes our main result in this subsection

Proposition 2 (*result in two-part tariff equilibrium*) *Assume substitute goods for each buyer, and convex cost functions. Then buyer group weakly increases the profit of each seller, and weakly reduces the payoff of each buyer. Precisely, if the products have independent values and/or cost functions are linear, buyer group has no effect on any player's payoff; otherwise, it strictly reduces the buyers' aggregate payoff.*

Proposition 2 shows that, contrary to Inderst and Shaffer (2007) and Dana (2012), in our setting the buyers can never gain from building a group, and in fact buyer group strictly reduces their total payoff except in very special cases. An intuition for this result can be obtained by referring to (11). Recall that F_j^{C*} is equal to $V_{AB}^C - V_h^C$, the marginal contribution to social welfare of firm j under buyer group, and in particular the presence of seller j entails that the cost of firm h changes from $C_h(\hat{q}_h^1 + \hat{q}_h^2)$ to $C_h(q_h^{1*} + q_h^{2*})$. Since $\hat{q}_h^1 + \hat{q}_h^2 > q_h^{1*} + q_h^{2*}$, it follows that a cost saving $C_h(\hat{q}_h^1 + \hat{q}_h^2) - C_h(q_h^{1*} + q_h^{2*})$ occurs, which

contributes to the profit of seller j . Regarding π_j^{nC} , notice first that if we replace \hat{q}_h^1, \hat{q}_h^2 in V_h^1, V_h^2 with \hat{q}_h^1, \hat{q}_h^2 , then we obtain a weakly higher value for j 's profit under no buyer group. Hence, proving that F_j^{C*} is greater or equal than the "modified" π_j^{nC} implies $\Delta\pi_j \geq 0$. Second, notice that π_j^{nC} has an interpretation similar to F_j^{C*} , except that the cost function of seller h is replaced by a linear function with slope α_h . Therefore the cost saving of seller h due to the presence of seller j is $\alpha_h(\hat{q}_h^1 + \hat{q}_h^2) - \alpha_h(q_h^{1*} + q_h^{2*})$. Given that $C'_h(q) > \alpha_h$ for $q > q_h^{1*} + q_h^{2*}$, it follows that the cost saving is greater under buyer group than under no group. Since these cost savings are the only difference between F_j^{C*} and the modified π_j^{nC} , we conclude that $\Delta\pi_j$ is positive, unless C_h is linear (in this case $C'_h(q) = \alpha_h$ for $q > q_h^{1*} + q_h^{2*}$), or unless products are independent (in this case $\hat{q}_h^1 + \hat{q}_h^2 = q_h^{1*} + q_h^{2*}$). In short, marginal contribution to social welfare of firm j under buyer group is more valuable than under no group because it generates higher cost savings.

The result that buyer group always weakly reduces the buyers' payoff is somewhat surprising, and at a first sight it may seem to depend on the particular equilibrium we are considering for the setting without buyer group, as the equilibrium described by (7) and (10) is just one of many different equilibria. However, next proposition establishes that the result of Proposition 2 holds for any efficient equilibrium under no buyer group.

Proposition 3 (*result in any efficient equilibrium*) *Assume substitute goods for each buyer, convex cost functions, and suppose that an efficient equilibrium is played under no buyer group. Then buyer group weakly increases the profit of each seller and weakly reduces the buyers' aggregate payoff.*

This result relies on a property of each equilibrium under no buyer group. For each seller j it must be the case that the equilibrium strategies are (weakly) more profitable than a deviation which induces both buyers to buy only from seller j . The best deviation of this kind for seller j generates a profit equal to $\pi_j^d = V_j^C - \gamma^i - \gamma^k$ (see (4)), where γ^i, γ^k are the buyers' equilibrium payoffs, which seller j must leave to buyers in order to induce them to buy only from j .¹⁵ As a consequence, $F_h^{C*} + \pi_j^d = V_{AB}^C - \gamma^i - \gamma^k$, that is the sum between the profit of h under buyer group and π_j^d is equal to the first best social welfare, minus buyers' total payoff under no group. Since we consider efficient equilibria and V_{AB}^C is social welfare in any efficient equilibrium, $V_{AB}^C = \pi_j^{nC} + \pi_h^{nC} - \gamma^i - \gamma^k$. Therefore, we have

$$F_h^{C*} + \pi_j^d = \pi_j^{nC} + \pi_h^{nC}$$

equivalently

$$\pi_j^{nC} - \pi_j^d = F_h^{C*} - \pi_h^{nC}.$$

¹⁵ Actually, here we need to consider the buyers' payoffs from trading only with seller h , but in equilibrium these are equal to the equilibrium payoffs.

This equality establishes that the considered deviation is unprofitable for seller j , that is $\pi_j^{nC} - \pi_j^d \geq 0$, if and only if buyer group is weakly more profitable than no buyer group for seller h , that is $F_h^{C*} - \pi_h^{nC} \geq 0$.

Including the case of complementary products as well We conclude this section by examining a setting in which we relax the assumption of substitute goods. For this purpose, we suppose that both cost functions are linear, and that products are complements for buyer 2. This implies $V_A^2 + V_B^2 < V_{AB}^2$, and in the competition for buyer 2 there exist infinitely many equilibria in which sellers use the tariffs described in (7), with $F_A^2 \geq V_A^2$, $F_B^2 \geq V_B^2$, and $F_A^2 + F_B^2 = V_{AB}^2$; the latter equality implies that buyer 2's payoff is zero. The multiplicity is about how the sellers share the surplus of $V_{AB}^2 - V_A^2 - V_B^2$ generated by products complementarity, but that is not very relevant for us, as in every equilibrium buyer 2's payoff is zero. As we have already mentioned, linear costs imply $V_{AB}^C = V_{AB}^1 + V_{AB}^2$, $V_A^C = V_A^1 + V_A^2$ and $V_B^C = V_B^1 + V_B^2$. Therefore, if products are complements for each buyer, that is if $V_A^1 + V_B^1 \leq V_{AB}^1$ and $V_A^2 + V_B^2 \leq V_{AB}^2$, then both buyers have zero payoff under no group, and also the group has zero payoff since $V_A^C + V_B^C \leq V_{AB}^C$. Therefore buyer group has no effect in this case.

Suppose now that the products are strict substitutes for 1 and strict complements for 2: $V_A^1 + V_B^1 > V_{AB}^1$ and $V_A^2 + V_B^2 < V_{AB}^2$. Then, if the group is not formed the sum of the buyers' payoffs coincides with the payoff of buyer 1, which is equal to $V_A^1 + V_B^1 - V_{AB}^1 > 0$. If the group is formed, then its payoff is given by

$$\max \{0, V_A^1 + V_B^1 + V_A^2 + V_B^2 - V_{AB}^1 - V_{AB}^2\},$$

which is always strictly less than $V_A^1 + V_B^1 - V_{AB}^1$ because $V_A^2 + V_B^2 < V_{AB}^2$. Therefore, in this case the group strictly reduces the sum of the payoffs of its members. The intuition is pretty simple. When the products are complements for buyer 2, each seller j has some residual market power with respect to buyer 2 in the sense that because of the constraint that the buyer's payoff cannot be negative, seller j charges less than the marginal value created by its product. Integrating the two markets through buyer group allows the sellers to transfer this residual market power to the market of buyer 1.

Proposition 4 (*complements and substitutes*) *Assume linear cost functions. Then the formation of the buyer group*

- (i) *has no effect on any player's payoff if the products are complements to both buyers;*
- (ii) *strictly reduces the joint payoffs of the buyers if the products are strict complements for one buyer but strict substitutes for the other.*

3 Common agency under budget constraint: application to the market for academic journals.

Electronic publishing has brought fundamental changes in the market for academic journals. It allowed large publishers to practice ‘Big Deal’¹⁶ pricing strategies by bundling a large collection of journals. At the same time, it induced libraries to form consortia, whereby libraries of a given geographical area join forces in order to share acquisition of electronic academic journals licensed through the Big Deal. Academic library consortia are widespread. North American examples include OhioLINK, the Triangle Research Libraries Network of North Carolina (TRLN), the Greater Western Library Alliance (GWLA), the Colorado Alliance of Research Libraries (CARL) and the Ontario Council of University Libraries (OCUL). Some well-known European groups include HEAL-LINK (Greek academic libraries including the National Library) and CBUC (academic libraries of Catalonia in Spain).¹⁷

In this section, we study library consortium in order to identify strategies to make a library consortium successful from a long-term perspective as is suggested by Thomas A. Peters (2001a), director of center for library initiatives:

“One challenge for academic library consortia is to shift gears and engage in more deliberate strategic planning with an eye to positive long-terms outcomes”.

For this purpose, we use the model of our previous papers (Jeon and Menicucci, 2006 and 2011), which is a common agency model with complete information in which the buyer faces a budget constraint. In the previous section, we showed that in the absence of the budget constraint, the buyer group has no effect on any player’s payoff if products have independent values or costs are linear. The model we study in this section satisfies each of the two sufficient conditions for neutrality of buyer group. Therefore, we are isolating the effect of the budget constraint on buyer group. In the baseline model, the budget of each library is exogenously given. In Section 3.4, the budget of each library is endogenously determined.

¹⁶Big Deal is defined as “any online aggregation of e-content that a publisher, aggregator, or vendor offers for sale or lease at prices and/or terms that substantially encourage acquisition of the entire corpus” (Peters, 2001b).

¹⁷Other examples include: CAUL CEIRC (Australia), ANSF (Brazil), CALIS (China), MALMAD (Israel), INFER (Italy).

3.1 Model: publishers, libraries, and consortium

There are two (for-profit) publishers, A and B , and $n \geq 2$ libraries. Without loss of generality, we assume that each publisher offers only the pure bundle of its own journals.¹⁸ Let B_j represent the bundle offered by publisher j ($= A, B$). The monetary utility of library i ($= 1, \dots, n$) from consuming B_j is denoted by $U_j^i > 0$ (and is independent of whether the library also consumes the other bundle) and the budget of library i is $M^i > 0$. The payoff of a library is given by the utility it obtains from the bundles of journals it buys minus the money it spends for the purchases.

Let C represent the consortium of the n libraries. The utility of the consortium C from consuming B_j and the budget of the consortium are given by:

$$U_j^C = \sum_{i=1}^n U_j^i, \quad M^C = \sum_{i=1}^n M^i.$$

As for each member library, the payoff of the consortium is the utility it obtains from the bundles of journals it buys minus the money it spends for the purchases.

Let $P_j^i > 0$ represent the price that publisher j ($= A, B$) charges to library i ($= 1, \dots, n, C$) for bundle B_j . We assume that the fixed cost of producing the first copy of each journal in B_j has already been incurred and that the marginal cost of distributing a journal is zero. Therefore, publisher j 's profit is equal to publisher j 's revenue.

Social welfare is equal, up to a constant, to the total payoff the libraries obtain from consuming bundles of journals, and therefore it is maximized when all libraries consume both bundles.

We consider a game analogous to the one described in Section 2. At stage one, each publisher j simultaneously chooses $P_j^i > 0$ for $i = 1, \dots, n$ ($P_j^C > 0$) if there is no buyer group (if the buyer group is formed). At stage two, each library (the consortium) decides the bundle(s) to buy. Notice that we require $P_j^i > 0$, and exclude $P_j^i = 0$, because in some cases a publisher j earns a library's entire budget, and thus there is no money left for publisher $j' \neq j$. Then our assumption of positive prices rules out the possibility that publisher j' gives away $B_{j'}$ for free. Thus, in a sense we suppose that each publisher prefers not selling its bundle to selling it at zero price, which can be justified if there is an epsilon cost of contracting or billing.

Consider competition in the market for a given library i ($= 1, \dots, n, C$). We eliminate the superscript i and without loss of generality we assume $U_A \geq U_B$. Then, from our previous papers, we have¹⁹

¹⁸Arguing as in the proof of Proposition 2(i) in Jeon and Menicucci (2006), we can prove that, for each publisher, pure bundling of its journals weakly dominates any alternative to pure bundling.

¹⁹In fact, in Jeon and Menicucci (2006, 2011) we assume that publishers play a sequential game in

Lemma 1 (*Jeon and Menicucci, 2006 and 2011*) Consider competition between the two publishers in the market for a given library:

- (i) if $M \leq U_A - U_B$, then publisher A charges $P_A = M$, publisher B charges an arbitrary $P_B > 0$, and the library buys only B_A ;
- (ii) if $U_A - U_B < M < U_A + U_B$, then publishers charge $P_A = \frac{1}{2}(M + U_A - U_B)$, $P_B = \frac{1}{2}(M + U_B - U_A)$, and the library buys both bundles;
- (iii) if $U_A + U_B \leq M$, then publishers charge $P_A = U_A$, $P_B = U_B$, and the library buys both bundles.

Obviously, the budget constraint affects the pricing strategies, and we obtain a different equilibrium with respect to the one described in Section 2. Precisely, when $M \leq U_A - U_B$, only publisher A succeeds in selling its bundle because even when A charges $P_A = M$ (the highest feasible price) the library's payoff from buying only B_A , $U_A - M$, is larger than the payoff from buying only B_B , $U_B - P_B$, for any $P_B > 0$. On the other hand, if $M > U_A - U_B$ then the library buys both bundles and it is simple to see that $P_A = U_A$, $P_B = U_B$ when $M \geq U_A + U_B$: in this case the budget does not matter and each bundle is sold to the library at its marginal value. When instead $U_A - U_B < M < U_A + U_B$, prices are determined by the indifference condition

$$U_A - P_A = U_B - P_B \quad (12)$$

and by the binding budget constraint

$$P_A + P_B = M. \quad (13)$$

In particular, (12) implies that the library is indifferent between purchasing only B_A and purchasing only B_B . Thus no publisher j has an incentive to increase its price above P_j since then the library can not afford to buy both bundles (because of the binding budget constraint) and would buy only the bundle of the rival publisher.

Lemma 1 applies both to each library without the consortium, and also to the consortium. In the next subsections, we compare the outcome without the consortium and the outcome with the consortium.

3.2 Consortium of n libraries for exogenous budgets

In this section, we consider the model of n libraries introduced in Section 3.1. We assume

Assumption B1: $M^i \leq U_A^i + U_B^i$ for $i = 1, \dots, n$.

which first each publisher decides whether to be active or not, and then only active publishers compete in prices (libraries cannot buy from inactive publishers). However, when there are only two publishers, this sequential game yields the same outcome that is described by Lemma 1 for a simultaneous move game.

If Assumption B1 is not satisfied for library i , there is no competition between the two publishers in the market for library i since each publisher extracts the full surplus. Hence, this assumption implies that the two publishers compete, because of the budget constraint, in the market for any given library $i = 1, \dots, n$. As a consequence, every library i ends up spending its whole budget to purchase the journals of the two publishers.²⁰ B1 also implies that $M^C \leq U_A^C + U_B^C$, and thus also the consortium spends its whole budget to buy bundle(s). Therefore, in order to determine the effects of building a consortium on libraries' payoffs, we only need to study how libraries' consumption of bundles is affected.

Without loss of generality, we assume that $\Delta^C \equiv U_A^C - U_B^C$ is non-negative and that there exists an n' between 1 and n such that $\Delta^i \equiv U_A^i - U_B^i \geq 0$ for $i = 1, \dots, n'$ and $\Delta^i < 0$ for $i = n' + 1, \dots, n$. Libraries $1, \dots, n'$ are called *type A* libraries (there is a non-empty set of type A libraries since $U_A^C \geq U_B^C$); the other libraries (if any) are called *type B* libraries.

Lemma 1 makes clear that the only characteristics of library i which matter are Δ^i and M^i . Without the consortium, library i of type j buys only B_j if $M^i \leq |\Delta^i|$, buys both bundles if $M^i > |\Delta^i|$, for $j = A, B$. Likewise, the consortium buys only B_A if $M^C \leq \Delta^C$ (recall that $\Delta^C \geq 0$), buys both bundles if $M^C > \Delta^C$. These remarks deliver the following results.

Proposition 5 (*exogenous budget*) Suppose that the $n(\geq 2)$ libraries form a consortium, that Assumption B1 holds, and (without loss of generality) that $\Delta^C \equiv U_A^C - U_B^C \geq 0$.

(i) When $\Delta^C < M^C$, the consortium buys both bundles and hence the payoff of each library is weakly larger than without the consortium. The consortium strictly increases the total payoff of the libraries unless each library buys both bundles without the consortium.

(ii) When $M^C \leq \Delta^C$, the consortium buys only B_A and hence the payoff of each library is weakly smaller than without the consortium. The consortium strictly reduces the total payoff of the libraries unless each library buys only B_A without the consortium.

It is simple to see why this proposition is true. Without the consortium, each library with type j either buys only B_j or both bundles. When the consortium is formed and $M^C \leq \Delta^C$, each library consumes only B_A and therefore (i) a type B library is worse off; (ii) a type A library is unaffected if it buys only B_A without consortium, otherwise is worse off. On the other hand, when the consortium buys both bundles, each library enjoys maximal consumption and this strictly increases the payoff of each library which does not buy both bundles without the consortium.

Proposition 5 implies that a key issue is whether or not the inequality $\Delta^C < M^C$ holds. This condition is most easily satisfied when the preferences of libraries over bundles are quite heterogenous, that is in the consortium the intensity of the preferences of type

²⁰In Section 3.4 in which we make the budget choice endogenous, B1 is always satisfied

A libraries for B_A over B_B are more or less counterbalanced by the intensity of the preferences of type B libraries for B_B over B_A . The ideal case is such that $\Delta^1 + \dots + \Delta^{n'} = -(\Delta^{n'+1} + \dots + \Delta^n)$, that is $\Delta^C = 0$, which makes $\Delta^C < M^C$ hold for any level of budget of the consortium. For instance, when $n = 2$ this occurs if $\Delta^1 = -\Delta^2$. If instead $\Delta^1 + \dots + \Delta^{n'}$ is much larger than $-(\Delta^{n'+1} + \dots + \Delta^n)$, then Δ^C is much larger than zero and the consortium buys only B_A if its budget is small. Therefore, forming a consortium is more likely to be beneficial for libraries the more they are heterogeneous in terms of preferences for bundles.

In a sense, the key mechanism described in Inderst and Shaffer (2007) and Dana (2012) is at work here. When libraries with heterogenous preferences form a buyer group, the publishers are on a more similar footing, which elicits more aggressive competition between them. This makes it more difficult for the stronger publisher to absorb all the budget.

3.3 Consortium of two libraries for exogenous budgets

In this subsection we analyze our model for the case of $n = 2$ in order to obtain more precise results by focusing on the correlation between the two libraries' preferences. For this purpose, we assume $M^1 = M^2 \equiv M$ and maintain Assumption B1.

As in the previous subsection, we define $\Delta^i \equiv U_A^i - U_B^i$ for $i = 1, 2$, and without loss of generality we assume $\Delta^1 \geq |\Delta^2| \geq 0$ (with at least one strict inequality). In words, library 1 prefers B_A to B_B . If also library 2 prefers B_A , then library 1 prefers B_A more than library 2. If conversely library 2 prefers B_B , then library 1 prefers B_A more than library 2 prefers B_B .

In order to simplify notation, let $\rho \equiv \Delta^2/\Delta^1 \in [-1, 1]$ and $\Delta \equiv \Delta^1$. Notice that ρ is a measure of the correlation between the two libraries' preferences. With this notation we have

$$U_A^1 - U_B^1 = \Delta, \quad U_A^2 - U_B^2 = \rho\Delta, \quad U_A^C - U_B^C = (1 + \rho)\Delta (\geq 0).$$

From Lemma 1 and Proposition 5, in the absence of the consortium, library 1 buys both bundles if and only if $M > \Delta$, library 2 buys both bundles if and only if $M > |\rho|\Delta$, and the consortium buys both bundles if and only if $M > (1 + \rho)\Delta/2$. Hence, we have:

Observation: If every single library buys both bundles in the absence of the consortium (i.e., if $M > \Delta$), then the consortium buys both bundles, and thus the consortium has no effect.

From now on we restrict attention to the case of $M \leq \Delta$ and therefore library 1 buys only B_A in the absence of the consortium. We can further simplify notation by considering a normalized budget $M' \equiv M/\Delta \in (0, 1]$. Hence, in what follows, the model has only two

parameters: $M' \in (0, 1]$ and $\rho \in [-1, 1]$. For instance, in the absence of the consortium, if $\rho = 1$ then both libraries buy only B_A ; if $\rho = -1$, library 1 buys only B_A and library 2 buys only B_B ; if $\rho = 0$, library 1 buys only B_A and library 2 buys both bundles. From these remarks and Proposition 5 we obtain next lemma.

Lemma 2 *Suppose that Assumption B1 holds, $M' \equiv M/\Delta \leq 1$, and the two libraries form a consortium. Then library 1 buys only B_A in the absence of the consortium and*

- (i) *if $M' > (1 + \rho)/2$, the consortium buys both bundles, which strictly increases the libraries' aggregate payoff;*
- (ii) *if $M' \leq (1 + \rho)/2$, the consortium buys only B_A . This reduces the libraries' total payoff if $M' > \rho$, but it does not affect neither any library's consumption nor its payoff if $M' \leq \rho$.*

Figure 1 represents the sets of (ρ, M') which satisfy the conditions in Lemma 2(i) and Lemma 2(ii). The region denoted by $+$ is such that $M' > (1 + \rho)/2$; the region denoted by $-$ is such that $\rho < M' \leq (1 + \rho)/2$; the region denoted by 0 is such that $M' \leq \rho < (1 + \rho)/2$. For each $\rho \in [-1, 1]$, let $L^+(\rho) \in [0, 1]$ represent the length of the interval of values of M' such that the consortium strictly increases the total payoff of the libraries. Similarly, let $L^-(\rho) \in [0, 1]$ (resp., $L^0(\rho) \in [0, 1]$) represents the length of the interval of values of M' such that the consortium strictly reduces the libraries' total payoff (resp., does not affect the total payoff). Using Lemma 2 it is possible to compute each length, and thus we obtain:

Proposition 6 (*exogenous budget and correlation*) *Suppose that the two libraries form a consortium, and that $M' \leq 1$. Under Assumption B1:*

- (i) *The length of the interval of values of M' such that the consortium strictly increases the libraries' total payoff, $L^+(\rho)$, satisfies $L^+(-1) = 1$, $L^+(1) = 0$ and linearly decreases with ρ , that is $L^+(\rho)$ linearly shrinks as the degree of correlation increases.*
- (ii) *The length of the interval of values of M' such that the consortium strictly reduces the libraries' total payoff, $L^-(\rho)$, satisfies $L^-(0) = 1/2$, $L^-(1) = L^-(-1) = 0$ and linearly decreases with $|\rho|$, that is $L^-(\rho)$ linearly shrinks as the absolute degree of correlation increases.*

Corollary 1 *Under Assumption B1 and $M' \leq \Delta$.*

- (i) *In the case of perfectly negative correlation, $\rho = -1$, the consortium always strictly increases the libraries' total payoff.*
- (ii) *In the case of perfectly positive correlation, $\rho = 1$, the consortium has no impact on the libraries' total payoff.*

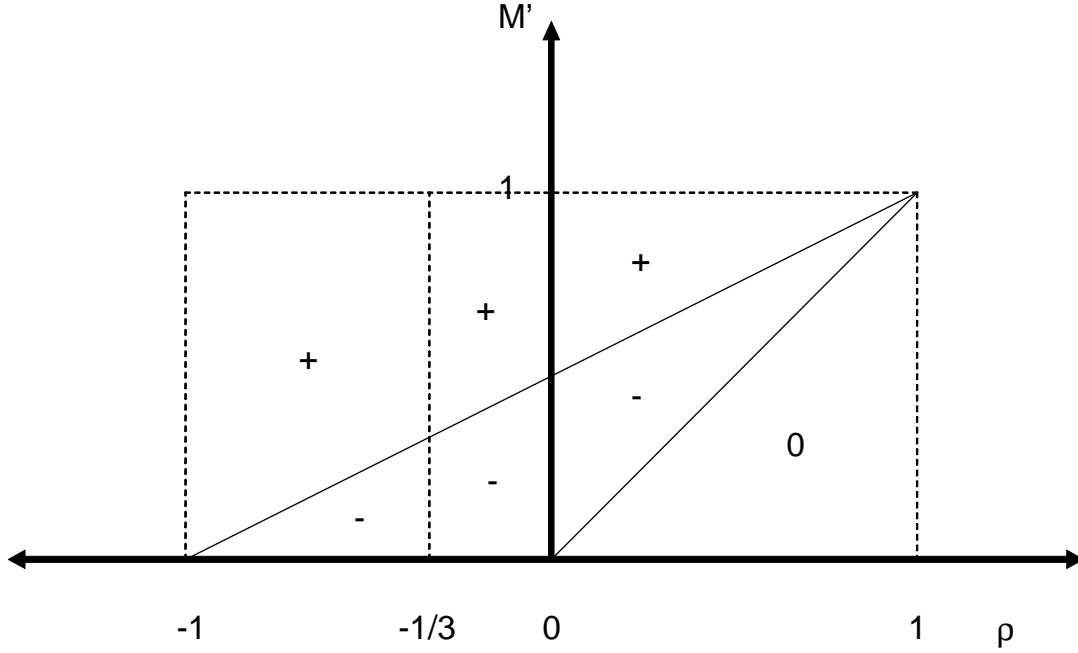


Figure 1: $A^+(\rho)$, $A^-(\rho)$ and $A^0(\rho)$.

In order to provide an intuition, let us first consider the extreme case of two identical libraries. Then, the consortium has no impact since the payment and the consumption of each library (and each publisher's profit) are just like in the absence of the consortium. More generally, Lemma 2(ii) and Figure 1 show that the consortium has no impact as long as the degree of positive correlation is strong enough with respect to the budget, i.e. if $M' \leq \rho$. Then, every library consumes only B_A regardless of whether the two libraries form the consortium or not.

Let us now consider the other extreme case of perfectly negative correlation (i.e. $\rho = -1$). Then, in the absence of the consortium, each library consumes only its preferred bundle: library 1 consumes only B_A and library 2 consumes only B_B . On the contrary, after they form the consortium, the consortium buys both bundles. This occurs because the opposite preferences of the libraries make the market power of each publisher symmetric in the case of the consortium, and this creates a level-playing field for the two publishers (without affecting the profit of any publisher).

Now let us consider the middle case of no correlation (i.e. $\rho = 0$). Then, in the absence of the consortium, library 1 consumes only B_A and library 2 consumes both bundles. In this case, the consortium increases (resp. reduces) the libraries' payoff if its budget is large enough, i.e. if $M' > 1/2$ (resp. small enough, i.e. $1/2 \geq M'$). If the budget is small, publisher A can export its residual monopoly power from library 1 to library 2 in order to monopolize the market for the consortium (and increase its profit). On the contrary, if the

budget is large enough, publisher A 's market power is not strong enough to monopolize the entire market of the consortium and therefore the consortium buys both bundles (but the profit of publisher A still increases).²¹

Another way to see that a lower ρ makes it more likely that a consortium is beneficial consists in noticing that in order to buy both bundles, the consortium needs to have a budget larger than $(1+\rho)\Delta$, which is increasing in ρ . Therefore, if for instance each library buys only one bundle without consortium and the libraries form a consortium aimed at buying both bundles, the required budget for the consortium is smaller the smaller is ρ in $[-1, 1]$.

3.4 Endogenous budget

Up to now, we assumed that each library's budget is given. In this subsection, we continue to analyze the case of two libraries but relax this assumption. Instead, we assume that a public authority perfectly internalizing each library's payoff determines each library's budget before publishers choose prices. For instance, a state authority determines the budget of the libraries of the state's public universities. The timing of the game we consider is as follows:

- Stage 1: A public authority determines the budget for each library i (or the budget for the consortium).
- Stage 2: Each publisher simultaneously chooses a personalized price for its bundle of journals to each library (or the consortium).
- Stage 3: Each library i (or the consortium) decides which bundle(s) to buy.

Consider the market for library i , for instance, with $U_A^i \geq U_B^i$. According to Lemma 1, any positive M^i smaller than $U_A^i - U_B^i$ allows the library to consume B_A and any M^i higher than $U_A^i - U_B^i$ allows the library to consume both bundles. The library's payoff is $U_A^i - M^i$ in the first case, is $U_A^i + U_B^i - M^i$ in the second case. Since the authority wants to minimize the payment to publishers given the consumption of the library, the Supremum of the library's payoff when its budget is endogenous is given by U_A^i in the first case, and by $2U_B^i$ in the second case. Therefore, we have:

²¹As the analysis of the three cases $\rho = -1, \rho = 0, \rho = 1$ suggests, under the consortium the profit of A (the profit of B) is weakly higher (weakly smaller) than without the consortium, for any $M' \leq 1$ and any $\rho \in [-1, 1]$, because the consortium allows A to export its residual monopoly power from library 1 to library 2.

Lemma 3 *Consider competition between two publishers in the market for a given library (or for the consortium) when its budget is endogenously chosen by an authority who perfectly internalizes the library's payoff. Assume $U_A \geq U_B$ without loss of generality. Then, the Supremum of the library's payoff is $\max \{U_A, 2U_B\}$.*

In the equilibrium without consortium, each library consumes only its preferred bundle or both bundles. Let (M,D),²² for instance, represent the situation in which the authority induces library 1 to consume only one bundle (i.e., B_A) and library 2 to consume both bundles in the absence of consortium; (M,M), (D,M) and (D,D) are similarly defined. As in the previous subsection, we define $\Delta \equiv U_A^1 - U_B^1 > 0$, $\rho \equiv \frac{U_A^2 - U_B^2}{\Delta}$, and without loss of generality we assume that $\rho \in [-1, 1]$.

In next lemma we consider the case of $\rho \geq 0$ (positive correlation). We have:

Lemma 4 *(positive correlation) Consider competition between two publishers under endogenous budget, given $\rho > 0$.*

(i) In the case of (M,M) or (D,D), building a consortium has no effect on the bundle(s) consumed and on the payoffs of the libraries.

(ii) In the case of (M,D) or (D,M), building a consortium affects the bundle(s) consumed and strictly reduces the total payoffs of the libraries.

Consider first the case of (M,M), which is such that $U_A^i \geq 2U_B^i$ holds for $i = 1, 2$, and therefore $U_A^C \geq 2U_B^C$. As a consequence, the authority induces the consortium to consume only B_A ; thus building a consortium has no effect on the bundle consumed and on the libraries' payoffs. The same logic applies to the case of (D,D), since then $U_A^i \leq 2U_B^i$ holds for $i = 1, 2$ and $U_A^C \leq 2U_B^C$.

Consider now for instance the case of (M,D). Note first that the authority cannot achieve this pattern of consumption through a consortium, since under a consortium both libraries consume either the single bundle B_A or both bundles. Moreover, given B_A or (B_A, B_B) that the consortium consumes in equilibrium, the authority can achieve the same consumption pattern without the consortium *at the same total price*. This implies that under the consortium the authority chooses between the alternatives (M,M) and (D,D), a subset of the alternatives available without the consortium. Since the authority chooses (M,D) in the absence of the consortium, a revealed preference argument implies that (M,D) gives a higher payoff than (M,M) or (D,D). Therefore building a consortium reduces the total payoffs of the libraries.

Now we consider the case of $\rho < 0$ (negative correlation). In order to reduce the number of cases, we assume that both libraries obtain the same total utility from consuming both bundles:

²²M refers to monopoly and D refers to duopoly.

Assumption B2: $(U_A^1 + U_B^1)/2 = (U_A^2 + U_B^2)/2 \equiv U$.

In the assumption, U represents the average utility from the two bundles. Hence, we have

$$\begin{aligned}(U_A^1, U_B^1) &= (U + \Delta/2, U - \Delta/2), & (U_A^2, U_B^2) &= (U - |\rho| \Delta/2, U + |\rho| \Delta/2), \\ (U_A^C, U_B^C) &= (2U + (1 - |\rho|)\Delta/2, 2U - (1 - |\rho|)\Delta/2).\end{aligned}$$

Then, we can normalize the utilities by dividing them by Δ . Let $U' \equiv U/\Delta$, which must be larger than $1/2$ since $U_B^1 > 0$. Let $U_j^{i'} = U_j^i/\Delta$ for $i = 1, 2, C$ and $j = A, B$. Hence

$$\begin{aligned}(U_A^{1'}, U_B^{1'}) &= (U' + 1/2, U' - 1/2), & (U_A^{2'}, U_B^{2'}) &= (U' - |\rho|/2, U' + |\rho|/2), \\ (U_A^{C'}, U_B^{C'}) &= (2U' + (1 - |\rho|)/2, 2U' - (1 - |\rho|)/2).\end{aligned}$$

Given this normalization, we have only two parameters: $U' > 1/2$ and $\rho \in [-1, 0)$. We have:

Lemma 5 (*negative correlation*) Suppose that Assumption B2 holds, and consider competition between the two publishers under endogenous budget with $\rho < 0$.

(i) Case of $U' \geq \frac{3}{2}$. For any $\rho < 0$, (D, D) arises in the absence of the consortium. Under the consortium, the libraries consume both bundles and obtain greater total payoffs.

(ii) Case of $3/2 > U' > 1/2$.

(a) For $-1/3 \leq \rho < 0$: In the absence of the consortium, only (M, D) arises. Under the consortium, the libraries consume both bundles if and only if $U' \geq 3(1 - |\rho|)/4$. The consortium strictly increases the total payoffs of the libraries if and only if $U' > (3 - 4|\rho|)/2$.

(b) For $-1 \leq \rho < -1/3$: In the absence of the consortium, (M, D) arises if $U' > 3|\rho|/2$ and (M, M) arises otherwise. Under the consortium, the libraries always consume both bundles. The consortium strictly increases the total payoffs of the libraries if and only if $U' > \frac{1}{2} \max\{3 - 4|\rho|, \frac{3 - |\rho|}{2}\}$.

This lemma reveals first that, for any $\rho < 0$, whenever the average value of the bundles is large enough (i.e. $U' \geq 3/2$) such that (D, D) arises without consortium, then the consortium strictly increases the total payoffs of the libraries. This is because building the consortium does not affect consumption but reduces the gap between the willingness to pay for bundle of A and the one for bundle of B ; this in turn increases competition between the two publishers and allows the libraries to consume both bundles at a lower total price. Precisely, without the consortium the total price paid is $1 + |\rho|$ but the consortium pays only $1 - |\rho|$.

When the average value of the bundles is not large (i.e. $3/2 > U' > 1/2$), either (M, D) or (M, M) occurs without consortium. To sharpen the intuition, let us consider

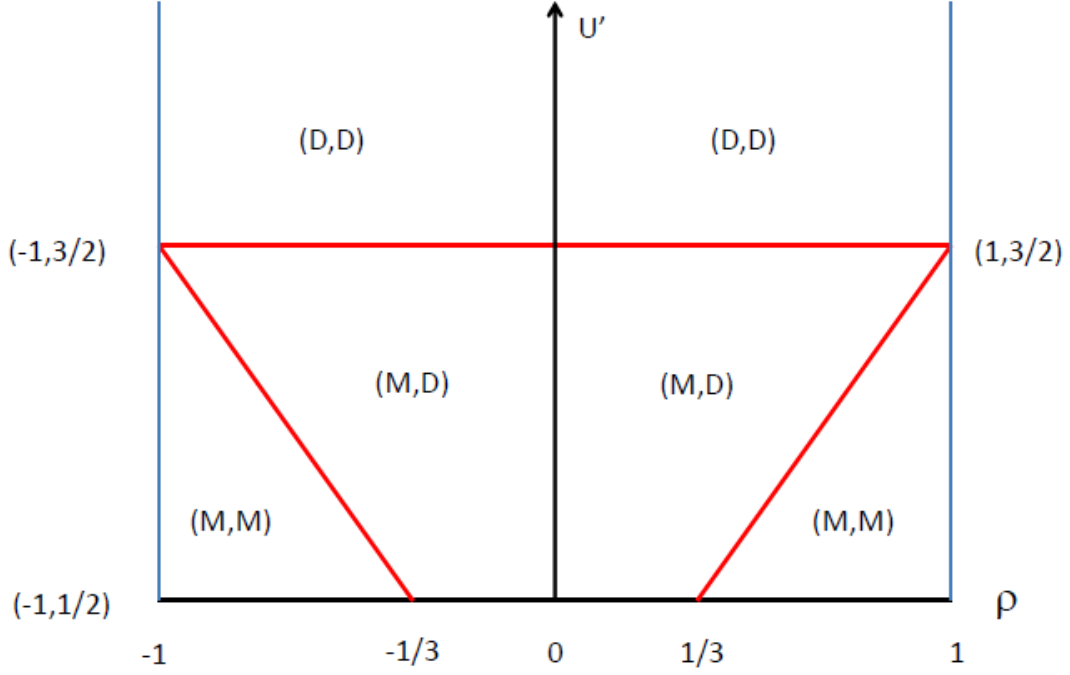


Figure 2: The consumption patterns without consortium under Assumption B2 when the budget is endogenous

the two extreme cases of perfect negative correlation and no correlation. Under perfect negative correlation, building a consortium always strictly increases the total payoffs of the libraries. In this case, only (M,M) arises in the absence of the consortium: library 1 consumes only the bundle of A and library 2 consumes only the bundle of B . Then, building a consortium creates a level playing field between the two publishers such that the consortium can consume both bundles *at almost zero price*. In contrast, in the extreme case of no correlation, only (M,D) occurs without consortium. Then, for the revealed preference argument explained right after Lemma 4, building a consortium always strictly reduces the total payoffs of the libraries. For the general case of negative correlation (i.e. $0 > \rho > -1$), there exists a cut-off value of $U' \equiv U/\Delta$ above which building a consortium strictly increases the sum of the libraries' payoffs. This cut-off strictly decreases with the degree of the negative correlation $|\rho|$.

Figure 2 describes the consumption patterns in the absence of the consortium under Assumption B2. Figure 3 shows the region (marked with +) in which building the consortium strictly increases the sum of the libraries' payoffs, the region (marked with 0) in which building the consortium does not affect it, and the region (marked with -) in which building the consortium strictly reduces it. Summarizing, we have:

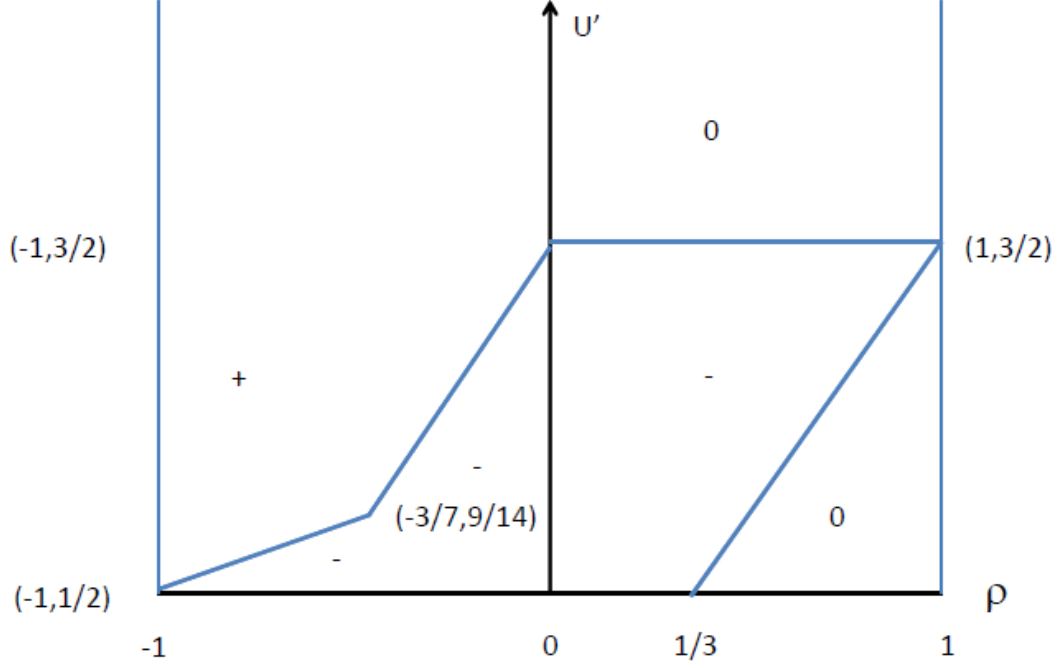


Figure 3: The effect of the consortium under Assumption B2 when the budget is endogenous

Proposition 7 (*endogenous budget and correlation*) Consider competition between two publishers under endogenous budget.

(i) When the two libraries' preferences are positively correlated (i.e. $\rho \geq 0$), building a consortium either has no effect on the sum of the libraries' payoffs, or strictly reduces it. Under Assumption B2, the range of U' for which the consortium is harmful shrinks with the degree of correlation such that it disappears for perfect positive correlation.

(ii) When the two libraries' preferences are negatively correlated (i.e. $\rho < 0$), under Assumption B2 there exists a cut-off value of $U' \equiv U/\Delta$ above which building a consortium strictly increases the sum of the libraries' payoffs. This cut-off strictly decreases with the degree of the correlation $|\rho|$ such that the consortium certainly increases the sum of libraries' payoffs for perfect negative correlation.

Corollary 2 Under Assumption B2:

(i) In the case of perfectly negative correlation, $\rho = -1$, the consortium always strictly increases the libraries' total payoff;

(ii) In the case of perfectly positive correlation, $\rho = 1$, the consortium has no impact on the libraries' total payoff.

When we compare Figure 1 of exogenous budget and Figure 3 of endogenous budget, it is remarkable that they share a number of features even if the parameter represented on the vertical axis is different in the two figures. First, Corollary 1 and Corollary 2 have the identical predictions for the two extreme cases of perfect positive and perfect negative correlation. Second, given negative correlation, the parameter range for which consortium is beneficial increases with the absolute degree of correlation both in Proposition 6(i) and Proposition 7(ii). Third, given positive correlation, the parameter range for which consortium is harmful decreases with the degree of correlation both in Proposition 6(ii) and Proposition 7(i). The key differences arise for the case of positive correlation: while building a consortium is strictly beneficial in a certain parameter range when the budget is exogenously given, it can never be strictly beneficial in the case of endogenous budget.

4 Policy implications

Although we analyzed two different models, two common messages emerge from our results. First, buyers can lose from forming a group in many cases. Second, what determines whether buyers gain or lose from forming a group is not the mere size of the group but its composition. In both models, buyers of similar preferences are unlikely to gain from forming a group. However, in the case of buyers of dissimilar preferences, depending on whether buyers face a budget constraint, they can gain or lose from forming a group.

Our results suggest that there could be a strong tension between a short-term strategy and a long-term strategy as long as the former dictates forming a buyer group among buyers with similar preferences to benefit from quantity discounts. For instance, in the case of library consortium, if publishers are strategic with foresight while buyers are myopic, publishers with strong market power might have incentives to provide quantity discounts to buyers of similar preferences in order to induce them to form a group. In particular, if forming a group requires to incur some sunk cost, which in turn makes undoing the group very costly, publishers could have an incentive to subsidize such cost through quantity discounts only in order to extract more surplus in the long run when they can adjust their tariffs. In the case of library consortium, we cannot exclude such possibility of "consortium trap". This calls for an empirical study about long term effect of library consortium.

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4.1 Appendix

4.1.1 Proof of Proposition 1

In order to prove that the strategies described in (7) and (10) constitute an equilibrium, we need to prove that no profitable deviation exists for any seller. Without loss of generality, we consider deviations of seller j in the form of take-it-or-leave-it offers: (q_j^i, t_j^i) , (q_j^k, t_j^k) .

Deviations such that both buyers buy from j and h First we consider deviations of seller j which induce both buyers to buy from both sellers. The payoff of buyer i from accepting (q_j^i, t_j^i) and choosing to buy q_h^i from seller h is $U^i(q_j^i, q_h^i) - t_j^i - F_h^i - \alpha_h q_h^i$, and it must be not smaller than $V_h^i - F_h^i$, the payoff i can obtain by trading only with seller h . Therefore $\max_{q_h^i} (U^i(q_j^i, q_h^i) - \alpha_h q_h^i - V_h^i)$ is the highest revenue seller j can obtain from buyer i , and likewise $\max_{q_h^k} (U^k(q_j^k, q_h^k) - \alpha_h q_h^k - V_h^k)$ is the highest revenue j can obtain from buyer k . Hence the highest profit j can earn from this kind of deviations is

$$\pi_j^d = \max_{q_h^i} (U^i(q_j^i, q_h^i) - \alpha_h q_h^i - V_h^i) + \max_{q_h^k} (U^k(q_j^k, q_h^k) - \alpha_h q_h^k - V_h^k) - C_j(q_j^i + q_j^k)$$

which is a function of q_j^i, q_j^k . This profit is maximized at $q_j^i = q_j^{i*}, q_j^k = q_j^{k*}$, and then π_j^d is equal to $U^i(q_j^{i*}, q_h^{i*}) + U^k(q_j^{k*}, q_h^{k*}) - \alpha_h(q_h^{i*} + q_h^{k*}) - V_h^i - V_h^k - C_j(q_j^{i*} + q_j^{k*})$, which coincides with π_j^{nC} . Hence no profitable deviation exists for seller j such that it induces both buyers to buy from both sellers.

Deviations such that both buyers buy only from j Now we consider deviations of seller j which induce both buyers to buy only from j . This requires that $U^i(q_j^i, 0) - t_j^i \geq V_h^i - F_h^{i*}$ and $U^k(q_j^k, 0) - t_j^k \geq V_h^k - F_h^{k*}$. Therefore the deviation profit of j is $U^i(q_j^i, 0) - (V_h^i - F_h^{i*}) + U^k(q_j^k, 0) - (V_h^k - F_h^{k*}) - C_j(q_j^i + q_j^k)$, which is maximized at $q_j^i = \hat{q}_j^i, q_j^k = \hat{q}_j^k$: see (3). The inequality $U^i(\hat{q}_j^i, 0) - (V_h^i - F_h^{i*}) + U^k(\hat{q}_j^k, 0) - (V_h^k - F_h^{k*}) - C_j(\hat{q}_j^i + \hat{q}_j^k) \leq \pi_j^{nC}$ is equivalent to

$$U^i(\hat{q}_j^i, 0) - \alpha_j(\hat{q}_j^i - q_j^{i*}) + U^k(\hat{q}_j^k, 0) - \alpha_j(\hat{q}_j^k - q_j^{k*}) \geq U^i(\hat{q}_j^i, 0) + U^k(\hat{q}_j^k, 0) - C_j(\hat{q}_j^i + \hat{q}_j^k) + C_j(q_j^{i*} + q_j^{k*}) \quad (14)$$

- In the case of C_j linear, from (3) and (9) we find that $\hat{q}_j^i = \tilde{q}_j^i$ for any i, j , therefore (14) holds with equality.
- In the case of independent products, from (1), (3) and (9) we see that $\tilde{q}_j^i = \hat{q}_j^i = q_j^{i*}$, therefore (14) holds with equality.
- In the case of C_j convex, we see that $U^i(\hat{q}_j^i, 0) - \alpha_j(\hat{q}_j^i - q_j^{i*}) + U^k(\hat{q}_j^k, 0) - \alpha_j(\hat{q}_j^k - q_j^{k*}) \geq U^i(\hat{q}_j^i, 0) - \alpha_j(\hat{q}_j^i - q_j^{i*}) + U^k(\hat{q}_j^k, 0) - \alpha_j(\hat{q}_j^k - q_j^{k*})$ by definition of $\tilde{q}_j^i, \tilde{q}_j^k$, and $U^i(\hat{q}_j^i, 0) - \alpha_j(\hat{q}_j^i - q_j^{i*}) + U^k(\hat{q}_j^k, 0) - \alpha_j(\hat{q}_j^k - q_j^{k*}) \geq U^i(\hat{q}_j^i, 0) + U^k(\hat{q}_j^k, 0) - C_j(\hat{q}_j^i + \hat{q}_j^k) + C_j(q_j^{i*} + q_j^{k*})$ reduces to

$$C_j(\hat{q}_j^i + \hat{q}_j^k) - C_j(q_j^{i*} + q_j^{k*}) \geq \alpha_j(\hat{q}_j^i + \hat{q}_j^k - q_j^{i*} - q_j^{k*}) \quad (15)$$

Substitute products imply $\hat{q}_j^i + \hat{q}_j^k \geq q_j^{i*} + q_j^{k*}$, and convexity of C_j implies $C_j'(q_j^{i*} + q_j^{k*}) > \alpha_j \equiv C_j'(q_j^{i*} + q_j^{k*})$ for any $q_j^i + q_j^k \geq q_j^{i*} + q_j^{k*}$, which makes (15) satisfied (strictly so if $\hat{q}_j^i + \hat{q}_j^k > q_j^{i*} + q_j^{k*}$ and C_j is strictly convex).

Deviations such that buyer i buys only from j , buyer k buys from both sellers

Now we consider deviations of seller j such that buyer i buys only from j and buyer k buys from both sellers. Arguing as in the previous cases we infer that the profit of j from this kind of deviations is

$$U^i(q_j^i, 0) - (V_h^i - F_h^i) + \max_{q_h^k} (U^k(q_j^k, q_h^k) - \alpha_h q_h^k - V_h^k) - C_j(q_j^i + q_j^k)$$

In the problem $\max_{q_h^k} (U^k(q_j^k, q_h^k) - \alpha_h q_h^k - V_h^k)$, the solution depends on q_j^k and is characterized by $U_h^k(q_j^k, q_h^k) = \alpha_h$. We let $g(q_j^k)$ denote the optimal q_h^k , and notice that $g(q_j^{k*}) = q_h^{k*}$, $g'(q_j^k) = -U_{hj}^k/U_{hh}^k < 0$. Therefore the deviation profit of seller j for given q_j^i, q_j^k is $\pi_j^d = U^i(q_j^i, 0) - (V_h^i - F_h^i) + U^k(q_j^k, g(q_j^k)) - V_h^k - \alpha_h g(q_j^k) - C_j(q_j^i + q_j^k)$, and the inequality $\pi_j^d \leq \pi_j^{nC}$ reduces to

$$\begin{aligned} & U^i(q_j^i, 0) + U^k(q_j^k, g(q_j^k)) - \alpha_h(g(q_j^k) - q_h^{k*}) - C_j(q_j^i + q_j^k) \\ & \leq U^k(q_j^{k*}, q_h^{k*}) - C_j(q_j^{i*} + q_j^{k*}) + U^i(\tilde{q}_j^i, 0) - \alpha_j(\tilde{q}_j^i - q_j^{i*}) \end{aligned} \quad (16)$$

This implies that no deviation of this kind is profitable for seller j if and only if

$$\max_{q_j^i, q_j^k} L(q_j^i, q_j^k) \leq \max_{q_j^i} R(q_j^i) \quad (17)$$

with $L(q_j^i, q_j^k)$ equal to the left hand side of (16), a strictly concave function,²³ and $R(q_j^i) \equiv U^k(q_j^{k*}, q_h^{k*}) - C_j(q_j^{i*} + q_j^{k*}) + U^i(q_j^i, 0) - \alpha_j(q_j^i - q_j^{i*})$. We let $(\bar{q}_j^i, \bar{q}_j^k) \equiv \arg \max_{q_j^i, q_j^k} L(q_j^i, q_j^k)$, and using

$$\frac{\partial L}{\partial q_j^i} = U_j^i(q_j^i, 0) - C_j'(q_j^i + q_j^k), \quad \frac{\partial L}{\partial q_j^k} = U_j^k(q_j^k, g(q_j^k)) - C_j'(q_j^i + q_j^k) \quad (18)$$

we show that (17) holds if $\bar{q}_j^k \leq q_j^{k*}$, whereas $\bar{q}_j^k > q_j^{k*}$ is impossible.

Case of $\bar{q}_j^k = q_j^{k*}$. Then $\bar{q}_j^i = q_j^{i*}$ and $L(\bar{q}_j^i, \bar{q}_j^k) = U^i(q_j^{i*}, 0) + U^k(q_j^{k*}, q_h^{k*}) - C_j(q_j^{i*} + q_j^{k*})$. Since $R(q_j^{i*}) = L(\bar{q}_j^i, \bar{q}_j^k)$, it follows that (17) is satisfied.

Case of $\bar{q}_j^k < q_j^{k*}$. Then $U_j^k(\bar{q}_j^k, g(\bar{q}_j^k)) > U_j^k(q_j^{k*}, q_h^{k*})$ and $C_j'(\bar{q}_j^i + \bar{q}_j^k) > C_j'(q_j^{i*} + q_j^{k*})$, hence $\bar{q}_j^i + \bar{q}_j^k > q_j^{i*} + q_j^{k*}$ and $\bar{q}_j^i > q_j^{i*}$. In maximizing L with respect to q_j^i, q_j^k , it is convenient as a first step to maximize L with respect to q_j^k , for given q_j^i ; we write the maximum point as $f(q_j^i)$. Then, as a second step, we maximize $L(q_j^i, f(q_j^i)) \equiv \ell(q_j^i)$ with respect to q_j^i . We find that $f(q_j^{i*}) = q_j^{k*}$, hence $\ell(q_j^{i*}) = L(q_j^{i*}, q_j^{k*}) = U^i(q_j^{i*}, 0) + U^k(q_j^{k*}, q_h^{k*}) - C_j(q_j^{i*} + q_j^{k*})$ and $\ell'(q_j^i) = U_j^i(q_j^i, 0) - C_j'(q_j^i + f(q_j^i))$. Regarding $\max_{q_j^i} R(q_j^i)$, notice that $R(q_j^{i*}) = \ell(q_j^{i*})$ and $R'(q_j^i) = U_j^i(q_j^i, 0) - \alpha_j$. Since $q_j^i + f(q_j^i)$ is strictly increasing with respect to q_j^i ,²⁴ it

²³ Indeed, $dU_j^k(q_j^k, g(q_j^k))/dq_j^k = U_{jj}^k(q_j^k, g(q_j^k)) + U_{jh}^k(q_j^k, g(q_j^k))g'(q_j^k) = (U_{jj}^k U_{hh}^k - U_{hj}^k U_{jh}^k)/U_{hh}^k < 0$.

²⁴ The optimal q_j^k given q_j^i satisfies $U_j^k(q_j^k, g(q_j^k)) - C_j'(q_j^i + q_j^k) = 0$, thus $f'(q_j^i) = C_j'' U_{hh}^k / (U_{jj}^k U_{hh}^k - U_{jh}^k U_{hj}^k) > 0$, and $1 + f'(q_j^i) = (U_{jj}^k U_{hh}^k - U_{jh}^k U_{hj}^k) / (U_{jj}^k U_{hh}^k - U_{jh}^k U_{hj}^k - C_j'' U_{hh}^k) > 0$.

follows that $q_j^i + f(q_j^i) > q_j^{i*} + q_j^{k*}$ for $q_j^i > q_j^{i*}$ and $\ell'(q_j^i) \leq U_j^i(q_j^i, 0) - C_j'(q_j^{i*} + q_j^{k*}) = R'(q_j^i)$ for $q_j^i > q_j^{i*}$. Therefore (17) is satisfied.

Case of $\bar{q}_j^k > q_j^{k*}$. Then $U_j^k(\bar{q}_j^k, g(\bar{q}_j^k)) < U_j^k(q_j^{k*}, q_h^{k*})$, and in view of (18), $C_j'(\bar{q}_j^i + \bar{q}_j^k) < C_j'(q_j^{i*} + q_j^{k*})$; hence $\bar{q}_j^i + \bar{q}_j^k < q_j^{i*} + q_j^{k*}$. The latter inequality implies $\bar{q}_j^i < q_j^{i*}$, but this is impossible because of (18) as $U_j^i(\bar{q}_j^i, 0) > U_j^i(q_j^{i*}, 0) \geq U_j^i(q_j^{i*}, q_h^{i*}) = C_j'(q_j^{i*} + q_j^{k*}) > C_j'(\bar{q}_j^i + \bar{q}_j^k)$.

4.1.2 Proofs of Propositions 2 and 3

First we prove Proposition 3. We consider here general tariffs as follows

$$p_j^i(q) = F_j^i + a_j^i(q) \quad \text{s.t.} \quad a_j^i(0) = 0 \quad \text{and} \quad \frac{da_j^i(q_j^{i*})}{dq} = C_j'(q_j^{i*} + q_j^{k*}) \quad \text{for each } i, j$$

The condition $\frac{da_j^i(q_j^{i*})}{dq} = C_j'(q_j^{i*} + q_j^{k*})$ is imposed in order to guarantee that facing p_j^i, p_h^i , buyer i chooses $q_j^i = q_j^{i*}, q_h^i = q_h^{i*}$, so that the first best allocation is the outcome. In order to shorten notation, in the proof we define $U^{i*} \equiv U^i(q_j^{i*}, q_h^{i*})$, $a_j^{i*} \equiv a_j^i(q_j^{i*})$, $a_h^{i*} \equiv a_h^i(q_h^{i*})$, $p_j^{i*} \equiv p_j^i(q_j^{i*})$, $p_h^{i*} \equiv p_h^i(q_h^{i*})$, $C_j^* \equiv C_j(q_j^{i*} + q_j^{k*})$.

If i trades only with seller h , then i 's payoff is $\gamma_h^i \equiv \max_{q_h^i} U^i(q_h^i, 0) - a_h^i(q_h^i) - F_h^i$. If i buys from both sellers, then his payoff is $\gamma^i \equiv U^{i*} - p_j^{i*} - p_h^{i*}$. In an efficient equilibrium it is necessary that $\gamma^i = \gamma_h^i = \gamma_j^i$. After letting , from $U^{i*} - p_j^{i*} - p_h^{i*} = \gamma_h^i$ we obtain

$$p_j^{i*} = U^{i*} - p_h^{i*} - \gamma_h^i = U^{i*} - w_h^i$$

with $w_h^i \equiv U^i(\tilde{q}_h^i, 0) - (a_h^i(\tilde{q}_h^i) - a_h^{i*})$; thus $F_j^{i*} = U^{i*} - a_j^{i*} - w_h^i$. Likewise, $p_j^{k*} = U^{k*} - w_h^k$ and $F_j^{k*} = U^{k*} - a_j^{k*} - w_h^k$. Hence the buyers' payoffs are

$$\gamma^i = w_j^i + w_h^i - U^{i*}, \quad \gamma^k = w_j^k + w_h^k - U^{k*}$$

The profit of seller j is $\pi_j^{nC} \equiv p_j^{i*} + p_j^{k*} - C_j^*$, hence

$$\pi_j^{nC} = U^{i*} + U^{k*} - C_j^* - w_h^i - w_h^k, \quad \pi_h^{nC} = U^{i*} + U^{k*} - C_h^* - w_j^i - w_j^k$$

Now we consider a deviation of seller j , with take-it-or-leave-it offers: $(q_j^i, t_j^i), (q_j^k, t_j^k)$, which induce both buyers to buy only from seller j . This requires that $U^i(q_j^i, 0) - t_j^i \geq \gamma_h^i$ and $U^k(q_j^k, 0) - t_j^k \geq \gamma_h^k$. Therefore the deviation profit of j is $U^i(q_j^i, 0) - \gamma_h^i + U^k(q_j^k, 0) - \gamma_h^k - C_j(q_j^i + q_j^k)$, which is maximized at $q_j^i = \hat{q}_j^i, q_j^k = \hat{q}_j^k$. The inequality $U^i(\hat{q}_j^i, 0) - \gamma_h^i + U^k(\hat{q}_j^k, 0) - \gamma_h^k - C_j(\hat{q}_j^i + \hat{q}_j^k) \leq \pi_j^{nC}$ is equivalent to

$$V_j^C + C_j^* \leq w_j^i + w_j^k \quad (19)$$

and it must be satisfied in order to rule out profitable deviations for seller j .

Now recall that the profit of seller h under buyer group is $F_h^{C*} = V_{AB}^C - V_j^C$, and notice that $\pi_h^{nC} \leq F_h^{C*}$ is equivalent to (19). Therefore the deviation is unprofitable for seller j if and only if buyer group is weakly profitable for h . Likewise, an analogous deviation is unprofitable for seller h if and only if buyer group is weakly profitable for j . Therefore any efficient equilibrium is such that buyer group is unprofitable for buyers.

Regarding Proposition 2, the arguments above establish the first claim in the statement of Proposition 2. Regarding the other claim, notice that (14) in proof of Proposition 1 holds with equality if cost functions are linear and/or in the case of independent products. This implies that sellers are indifferent between deviating and not deviating, and therefore they are indifferent also between buyer group and no group. If at least one cost function is strictly convex and products are not independent, then (15) is strictly satisfied and therefore buyers' total payoff is smaller under buyer group.