The Demand for Attention and Mail[1]

Michael D. Bradley

George Washington University

Jeff Colvin

United States Postal Service

Office of the Inspector General

Mary K. Perkins

Howard University

Eighth Biennial Postal Economics Conference on

E-Commerce, Digital Economy and Delivery Services

Toulouse, France

April 3 - 4, 2014

[1] The views expressed in this paper are solely those of the authors and should not be construed to represent the views of George Washington University, Howard University, the USPS Office of the Inspector General, or any other institution.

The Demand for Attention and Mail

Michael D. Bradley
George Washington University

Jeff Colvin
USPS Office of Inspector General

Mary Perkins
Howard University

1. INTRODUCTION

Over the most recent decade, Postal Operators (POs) have suffered financially from the dramatic reduction in single piece communication mail. This has put Universal Service Obligation (USO) funding in jeopardy. However, to some extent, especially in the US, single piece mail has been partially compensated for by continued strength in bulk mail, which is less profitable because of lower prices. Electronic substitution has been weaker for presentment mail than for remittance mail. Further, direct mail has held on to its share of the overall advertising market.

If the trend in which bulk mail, and especially direct mail, contributes significantly to the principal financial base for postal operations continues, it will be important to consider more deeply the demand for direct mail and its role in postal sustainability.[2] Specifically, it may be of

[2] Such considerations bear with greatest force upon those POs for whom direct mail is a greater fraction of the business. Standard Mail, under which most direct mail is sent in the US, constitutes over half of USPS volumes (see USPSOIG, 2013b). In Europe, direct mail is a substantial fraction of mail volume in Germany, Austria, the Netherlands and Switzerland, but a much smaller fraction in other countries (see Dieke, et al, 2013)

value to expand the consideration of mail demand from a focus on the physical mail piece to the demand for consumer attention.

In a sense, POs have been in this market for a long time, procuring attention by offering free delivery (free to recipients) of content (bills, magazines, parcels) and providing (reselling) attention to anyone else who is willing send mail. Of course the emphasis has been on the content less than on the resell of attention, but this may be changing as POs expand their role as offline attention seekers (along with TV, radio, billboards, etc.).

In this paper, we seek to deepen the understanding of the demand for advertising mail by casting light on a key characteristic that differentiates advertising messages from other communications. This characteristic, *targeting strength*, is certainly a vital characteristic of online advertising, but it is shared by direct mail and, to one degree or another, other advertising media as well. Targeting refers to efforts to direct messages to potential buyers in such a way as to increase the likelihood of a match of the message (a suggestion to buy a product) and buyers with an interest in that product, and targeting strength is greater where the likelihood of such a match increases. Toward this end, we adapt a recent model of targeting (Bergemann and Bonatti, 2011) to a postal market and investigate the implications of the demand for direct mail.

The implications of the demand for attention for the postal industry have been examined in De Donder, *et al* (2011), which presents a model of an incumbent facing competition from an 'alternative media' (online). They analyze the implications of positive 'sender' utility for messages, whether delivered by the PO or alternative media. The focus on price is taken up as well by Crew and Kleindorfer (2012), who discuss nonlinear pricing as a way of retaining customers who would otherwise be lost to e-substitution.

A key question: do other things besides price, and sensitivity to price, differentiate online from offline advertisements? What makes internet advertising different from any non-postal competition a PO might face? The increasingly popular answer is behavioral targeting, and economists have begun to study this phenomenon and its connection to competition between online and offline media.

Targeting is not new. With reference to John Wanamaker's famous remark, "Half my advertising is wasted, I just don't know which half," targeting seems to hold out the promise of finding that wasted half and eliminating it. Recent interest in economic analysis of targeting arises from the expanded capacity for it found in online advertising.

Much of the economic analyses of behavioral targeting have been of the impact of targeting on product markets as in Iyer, *et al* (2005). However, several recent articles have studied the implications of decisions taken by advertisers for the market for advertising. This literature has focused on online advertising, as it is that media with which behavioral advertising is most often associated. The general strategy pursued in this work involves specifying the probability that a potential buyer will receive a message that induces him/her to make a purchase of a given item. The profit maximizing behavior of advertising firms, who must weigh the probability of a match (and sale) against the price of the message, establishes the demand for messages from media outlets.

For example, Athey and Gans (2009) address media competition by comparing general outlets, with targeting, to local news outlets. Their analysis shows that targeting only takes on value for general outlets if ad space is constrained. Athey and Ellison (2008) and Chen and He (2006) offer models that focus on matching products and buyers via internet advertising.

Bergemann and Bonatti (2011) analyze targeted advertising in the market for advertisements, including competition among media outlets. In their model, the probability of an advertisement resulting in a sale depends crucially upon the distribution of buyers across the products they buy. Targeting takes advantage of the distribution of buyers across ad markets to improve the odds of a sale. Bergemann and Bonatti (2011) model the demand for attention through specifying profit maximizing advertising firms that obtain a sale every time one of their messages reaches a consumer interested in buying their product. These firms' goal is to maximize the number of effective matches at the lowest cost. They find that targeting leads to both an increasing effectiveness of advertising and a higher concentration of firms advertising in each market. One drawback of the Bergemann and Bonatti (2011) model is that is assumes that offline media engage in no targeting.

While the recent dramatic growth in targeting has been related to online advertisements, POs have been actively engaged in directing ads to customers for a very long time. Mail is fundamentally a physical message to an individual or a household. Ad mail, especially catalogs, directed at existing customers is the predominant form of direct mail advertising. Mail can be, and is, used as a broadcast media, of course, but even broad mailings are typically "targeted" geographically, and thereby demographically. USPSOIG (2013a) discusses ZIP Code targeting, and suggest that finer geographic partition might enhance the value of the ZIP Code in a variety of uses, including direct mail. USPSOIG (2013b) focuses on technological ways to elicit feedback on ad mailings from recipients. With regard to the future targeting of direct mail, it is clear that POs possess an immense store of data of potential use in this way.

To analyze the potential impacts of this important characteristic of mail, we adapt the Bergemann and Bonatti (2011) model of targeting to a postal application. This means we model the demand for mail as the demand for customers' attention by profit maximizing firms.

In the next section, we develop demand conditions for direct mail by appeal to assumptions on the distribution of potential buyers across products. Section 3 discusses the equilibrium in the mail market and presents two methods for investigating that equilibrium. Section 4 provides conclusions and suggestions for future research.

2. DERIVING THE DEMAND FOR DIRECT MAIL

We consider the demand for ad mail to be driven by the desire of advertising firms to match ad messages to potential customers. In specifying that demand, we follow Bergemann and Bonatti (2011) by specifying that if a message to a group of consumers about product 'x' reaches a consumer with an interest in product 'x', then a sale is made and the advertiser earns revenue of \$1. These assumptions allow us to write the revenue of the representative firm as:

$$\$1 * \begin{bmatrix} number\ of\ households \\ interested\ in\ product\ x \end{bmatrix} * \begin{bmatrix} probability\ that\ a\ mail\ piece\ is \\ received\ by\ a\ household \\ with\ an\ interest\ in\ product\ x \end{bmatrix}$$

We further assume, as in Bergemann and Bonatti (2011), that the only cost of the message to the advertising firm is the price it pays to send the message. We model the demand for two types of messages: saturation advertising mail and targeted advertising mail. Saturation mail is defined as sending the same mailing to each household in a geographic area. Targeted mail is directed to specific subsets of households, on the basis of information about the location of potential buyers of a specific product. Targeted mail depends upon identifying relevant subgroups of households

that may hold a higher affinity for the product being sold. Once this subgroup is identified, they all receive the same mailing. Finally, we assume that firms do not have a fixed budget for advertising. This means a firm will continue to spend on either saturation advertising mail or targeted advertising mail as long as the additional spending increases profit. Thus, the demands for the two types of mail can be derived independently.[3]

The profit function for the representative firm sending saturation messages is given by:

$$\pi_A = [number\ of\ households] * \begin{bmatrix} proportion\ of\ households \\ interested\ in\ product\ x \end{bmatrix}$$

$$* \begin{bmatrix} probability\ of\ a\ household \\ receiving\ a\ mail\ piece \\ about\ product\ x \end{bmatrix}$$

$$- [cost\ per\ mail\ piece] * [number\ of\ mail\ pieces\ sent]$$

Following Bergemann and Bonatti (2011), we assume that the distribution of buyers of product x follows an exponential distribution. Thus, the proportion of households interested in product x is given by:

$$S_x = \lambda e^{\frac{-\lambda}{x}}$$

where λ measures the concentration households' interest in product x. In other words, a high value for λ means that a relatively high proportion of consumers are interested in the product. Also, x indicates the "size" of the firm providing product x, in terms of an index of historical

[3] In essence, this approach assumes that consumers make a purchase of an item if they receive a targeted message even if they also receive a saturation mail about the same product. In other words, receiving a saturation mailing about a product does not reduce the consumer's receptiveness to a targeted advertisement. It would be of interest to relax this assumption in future work.

sales. The larger those sales, the larger the index and the more consumers are interested in the product.

We can now consider the profit maximization problem for the representative firm. Using the above expression the profit function takes the following form:

$$\pi_A = HS_x\left(1 - e^{\frac{-M_A}{H}}\right) - P_A M_A,$$

where

H is the number of households,

$\left(1 - e^{\frac{-M_A}{H}}\right)$ is the probability that a household will receive a mail piece about product x,

M_A is the number of mail pieces, and

P_A is the price of a mail piece.

To gain some insight into the nature of this profit function, we consider its shape for a given sized firm. Suppose that we consider a single firm of fixed size,"x," and investigate how profit varies as the number of messages sent increases. We can use a numerical simulation of the profit function to gain insight into its shape. We calibrate the profit function with the following values:

Parameter	Value
H	1000
Λ	0.4
X	500
Pa	0.2

We then vary the amount of mail pieces to determine the effect on profits. The result is shown in Figure 1:

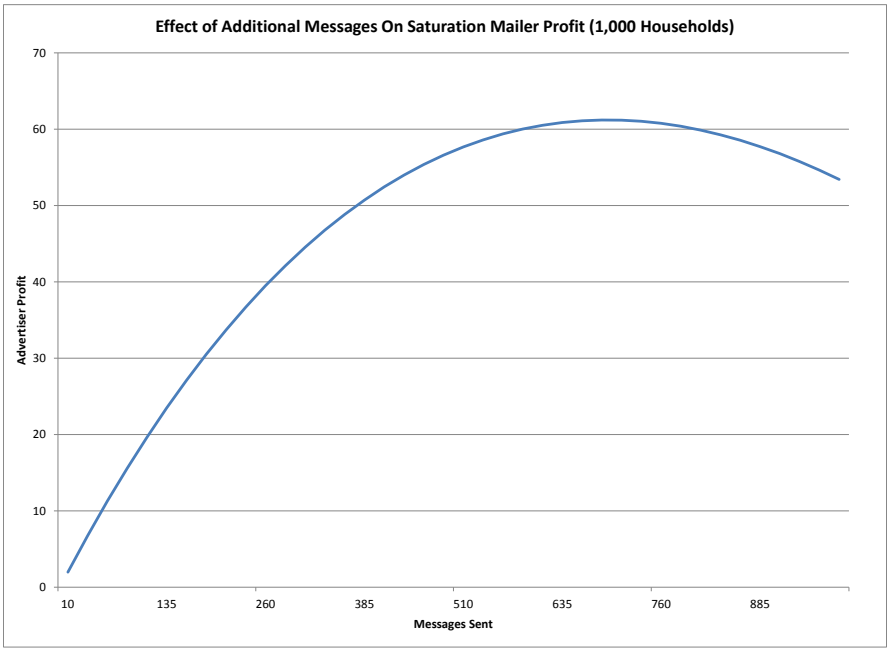

Figure 1

As shown in Figure 1, advertiser profits from saturation mail are increasing and concave in messages. The concavity is due to the increasing likelihood of redundancy as messages increase relative to the number of household. It is also interesting to investigate the impact of concentration of households on potential profitability. We do this by varying the values for λ. Note that as λ increases, so does S_x the proportion of households interested in product x. This, in turn, increases the potential profitability of advertising as shown in Figure 2. Because each message is more effective at reaching a potential customer, the advertising firm's profit, for any level of messages sent, increases.

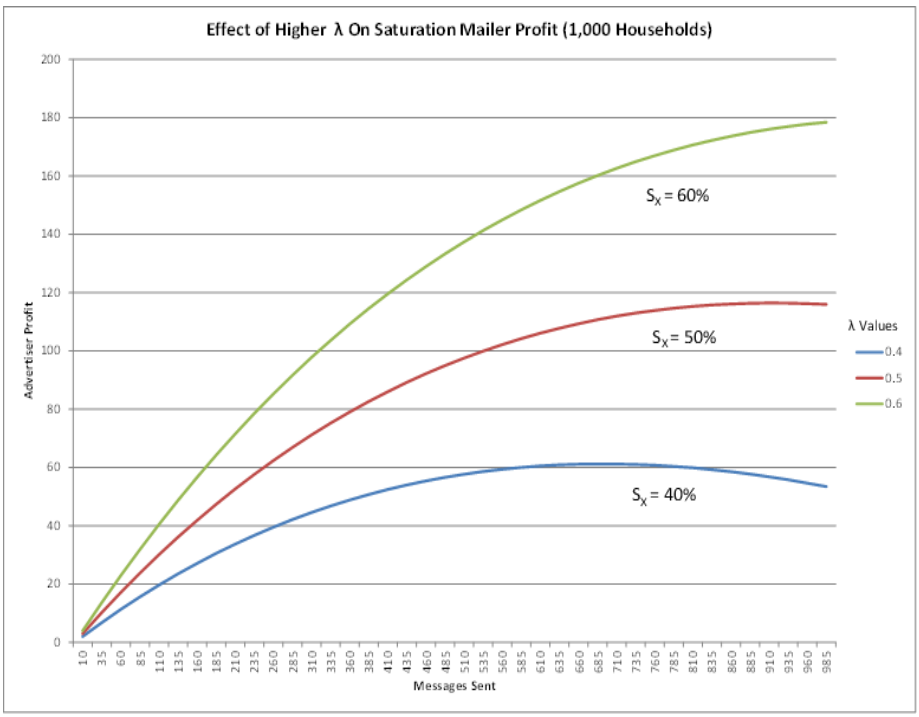

Figure 2

We now derive the demand for saturation mail by finding the profit maximizing level of messages given the household characteristics and the price of saturation mail. The first order condition for profit maximization provides the following demand function:

$$M_A = H\left\{ln\left(\lambda e^{\frac{-\lambda}{x}} / P_A\right)\right\}, \text{ or}$$

$$M_A = H\left\{ln(\lambda) - \left(\frac{\lambda}{x}\right) - lnP_A\right\}.$$

This shows that the demand for saturation mail is increasing in the number of households and decreasing in the price. The demand is also increasing in size of the firm's sales (x). Finally,

consider an impact of an increase in concentration of households on the demand for saturation mail.

$$\frac{\partial M_A}{\partial \lambda} = \frac{1}{\lambda} - \frac{1}{x}, \text{ but } \frac{1}{\lambda} - \frac{1}{x} > 0 \; iff \; \frac{\lambda}{x} < 1.$$

This condition comes from the exponential distribution and guarantees that an increase in λ leads to the expected increase in S_x. As the proportion of households interested in the firm's product rises, so does the demand for saturation mail.

We now turn attention to the demand for targeted messages. The general formulation of the representative firm still holds, but it must be modified to account for the fact that concentration of consumers will allow for more effective distribution of messages to a subpopulation of households. This process is called 'targeting.' In the case of saturation mail demand depends partly on the distribution of buyers across products: some simply have more potential buyers among the population than others. The PO can do nothing to influence this characteristic of the product market. However, it (or its mailing customers) can use information, demographic or behavioral, to direct product x messages to subsets of H where buyers of x read messages and make buying decisions. If the PO has a list of the addresses of potential buyers of x, it can offer to send mail to just those addresses. Of course, that does not imply that a sale will be made at each household but such targeting increases the probability of a match and reduces the frequency of messages sent to buyers with no interest in x.

As with saturation mail, we use the exponential distribution to define the proportion of targeted households and the proportion of targeted households interested in product x.

The proportion of targeted households is:

$$S_T = \frac{\gamma\lambda}{\gamma+\lambda}e^{-\gamma}$$

where γ is a measure of the concentration of the consumers in the targeted mail market and $\frac{\partial S_T}{\partial \gamma} > 0$. [4] The share of households in the target group that are interested in product x is given by:

$$S_{Tx} = \lambda\gamma e^{\frac{-(\lambda+\gamma)}{x}}e^{\gamma}.$$

Using the same formulation as above, we see that the representative firm's profit function for targeted messages is given by:

$$\pi_T = HS_{Tx}\left(1 - e^{-\frac{M_T}{S_T H}}\right) - P_T M_T,$$

where:

$\left(1 - e^{-\frac{M_T}{S_T H}}\right)$ is the probability that a targeted household will receive a message about product x, and M_T is the number of targeted messages.

As we did with the saturation mail profit function, we calibrate and simulate the targeted mail profit function. We use the following values for the calibration:

Parameter	Value
H	1000
λ	0.4
γ	0.25
x	500
P_T	0.3

[4] Note the difference between λ and γ. The former relates to the concentration of consumers interested in purchasing the product in the product market and the latter relates to the concentration of consumers interested in the product in the targeted mail market.

Not surprisingly, the targeted profit function is also concave in messages. As the firm increases the number of messages to the targeted subpopulation, the probability of making an additional match starts to decrease and the resulting expected addition to revenue is eventually less than the cost of sending the mail piece. At that point sending additional messages reduces profit.

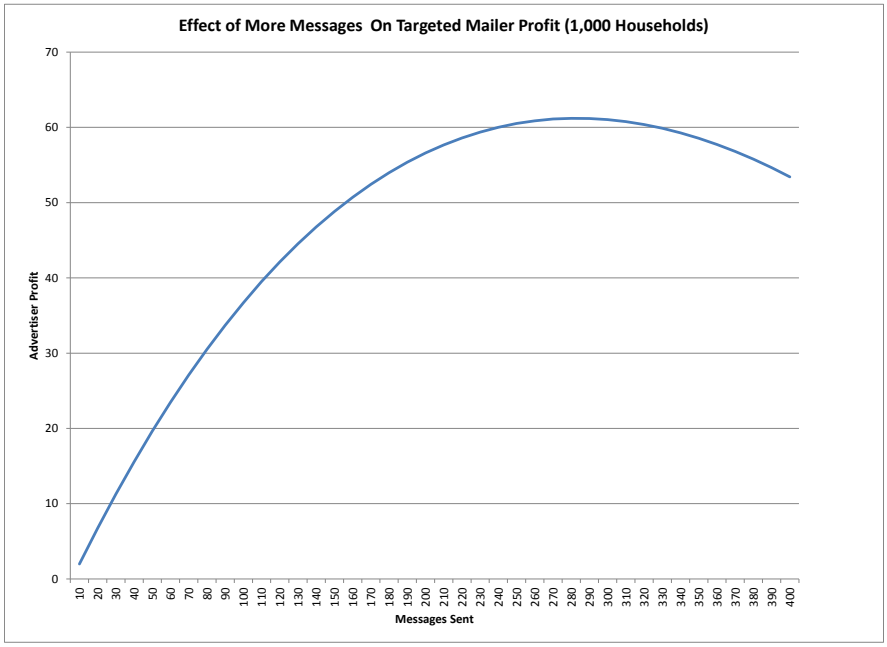

Figure 3

Because we are examining targeted mail, we can investigate the impact of concentration of consumers in the mail market on potential profit. We do this by varying the value of γ while keeping all of the other parameters constant. This alters the relationship between the number of mail pieces purchased and profit. This effect is shown in Figure 2, below:

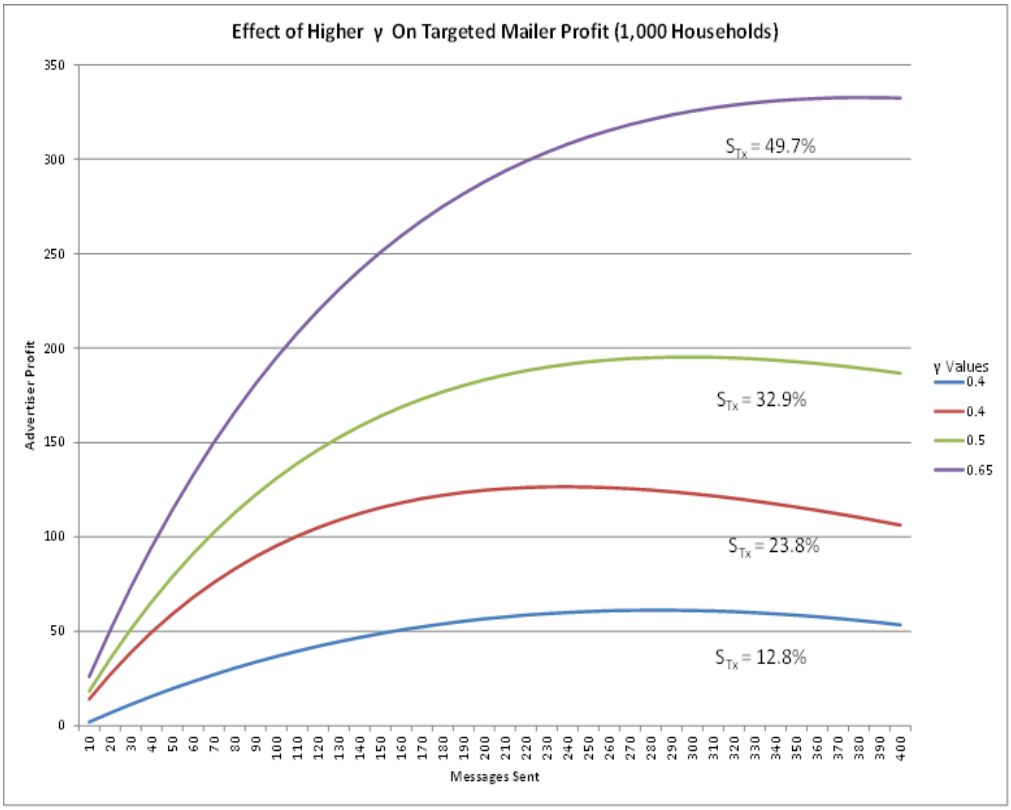

Figure 4

The figure shows that a higher concentration of consumers in the mail market permits better targeting and thus a higher level of profit for any given amount of targeted mail sent. This implies that it postal operators could improve the mails targeting capability, then its demand would increase. We now derive the demand for targeted mail from the first order conditions for profit maximization.

$$M_T = \left(\frac{\gamma\lambda}{\gamma+\lambda}\right) e^{-\lambda H} \ln\left[\frac{(\gamma+\lambda)e^{\left(-\frac{(\gamma+\lambda)}{x}+(\gamma+\lambda)\right)}}{P_T}\right]$$

or

$$M_T = \left(\frac{\gamma\lambda}{\gamma+\lambda}\right) e^{-\lambda} H \left[ln(\gamma+\lambda) - \frac{\gamma+\lambda}{x} + \gamma + \lambda - lnP_T\right]$$

The demand for targeted messages is increasing in the number of households and the firm size index but it is decreasing in the price of targeted mail. To determine the sign of an increase in targeting on the demand for targeted messages we find the derivative of the demand function with respect to γ:

$$\frac{\partial M_T}{\partial \gamma} = \left(\frac{\gamma\lambda}{\gamma+\lambda}\right) e^{-\lambda} H \left[\frac{1}{\gamma+\lambda} - \frac{1}{x} + 1\right] + \left(\frac{\lambda}{(\gamma+\lambda)^2}\right) e^{-\lambda} H \left[ln(\gamma+\lambda) - \frac{\gamma+\lambda}{x} + \gamma + \lambda - lnP_T\right].$$

While this derivative is potentially ambiguous, the impact is likely to be positive for large firms, as $\frac{1}{x}$ goes to 0 as x gets large. In the case of saturation mail, the largest firms buy the messages; in the targeted mail market, the largest firms within a targeted subpopulation will buy messages. However, a firm that is small by national standards may be large in a targeted subpopulation of households. Anderson (2006) argues that the advent of behavioral targeting raises the opportunity for niche firms to reach buyers.

3. INVESTIGATING ADVERTISING MAIL MARKET EQUILIBRIUM

To investigate the nature of mail market equilibrium in a model in which the demand for mail is driven by the demand for attention, we need to specify a model for the postal operator. Because we are focusing on a new approach to specifying postal demand functions, we will specify the simplest model of a postal operator that will capture the flavor of the advertising mail market. For this reason we focus only on saturation advertising mail and targeting advertising mail and ignore single piece and bulk transactions mail. Furthermore, we specify that the postal operator attempts to maximize profit on its two advertising products. This specification can be thought of as a postal operator attempting to maximize the contribution from its advertising products to support both its USO costs and its transactional mail product. In an environment of declining single piece and transactions mail volume, this is a scenario that deserves investigation. Accordingly, the profit function of the postal operator is:

$$\pi = P_A M_A + P_T M_T - C_A M_A - C_T M_T - F_A - F_T,$$

where the C_A and F_A refer to marginal and fixed costs respectively for saturation mail and C_T and F_T, are the same costs for targeted mail.

The complete model is given by the postal operator profit function and the demand functions for saturation and targeted advertising mail, derived above. The postal operator takes the demand functions as constraints and finds the profit maximizing prices for both types of advertising mail. Given those prices, firms that are advertising choose their profit maximizing volumes of the two types of ad mail.

While the approach to solving for equilibrium is straightforward in concept, the nonlinearity of the demand equations makes it difficult to find a closed form solution. We propose two approaches to investigating the equilibrium.

In the first approach, we linearize the demand equations using a first order Taylor series expansion. Although this procedure does not provide equilibrium values, it allows us to examine the impacts of changes in the variables of interest, in so far as they cause deviations from those equilibrium values. The details of these computations are presented in the appendix, but the results for the two mail product prices are presented below:

$$\tilde{P}_A = \left(\frac{\left\{ ln(\lambda^*) - \left(\frac{\lambda^*}{x^*}\right) - lnP_A^* \right\}}{2} \right) \tilde{H} + \frac{\lambda^* P_A^*}{2(x^*)^2} \tilde{x} + \frac{1}{2} \tilde{C}_A$$

$$\tilde{P}_T = \left[\frac{P_T^* \left(ln(\lambda^* + \gamma^*) + \left(1 - \frac{1}{x^*}\right)(\lambda^* + \gamma^*) - lnP_T^* \right)}{2\tilde{H}} \right] \widetilde{H} + + \left[\frac{P_T^* (\lambda^* + \gamma^*)}{2(x^*)^2} \right] \tilde{x}$$

$$+ \frac{1}{2} \tilde{C}_T + \left[P_T^* \gamma^* (\lambda^* + \gamma^*) \left(ln(\lambda^* + \gamma^*) + \left(1 - \frac{1}{x^*}\right)(\lambda^* + \gamma^*) - lnP_T^* \right) \right.$$

$$\left. + P_T^* \left(\frac{1}{\lambda^* + \gamma^*} + \left(1 - \frac{1}{x^*}\right) \right) \right] \tilde{\gamma}$$

The results suggest that our simple equilibrium model provides sensible result as the equilibrium prices are increasing in the number of households, the size of the firms and the products' marginal costs. However the results do not provide a definitive answer for the impact of additional targeting on the price of targeted mail, so we pursue a second approach. In this approach we calibrate the model with numerical values and then solve it numerically using Generalized Reduced Gradient (GRG2) nonlinear optimization algorithm.

The following table provides our initial parameter estimates:

Parameter	Value
H	1000
Λ	0.25
Γ	0.4
X	500
c_T	0.7
c_A	0.2

We then vary key parameters to investigate the impact on the endogenous variables. Those variables are the proportions of households interested in the products, the prices for the two advertising mail products, the amounts of the two advertising mail products send, the profits generated for the postal operator and the profits earned by the advertising firm by sending saturation and targeted mail.

The next table shows the equilibrium solutions given our initial calibration.

Numerical Solutions for Equilibrium Values

Endogenous Solutions

Sx	25.0%
S_T	12.0%
S_{Tx}	14.9%
M_A	108.2
M_T	31.8
P_A	$0.22
P_T	$0.95
π Postal A	$2.6
π Postal T	$8.1
π Adv. A	$1.4
π Adv T	$4.4

Several points about the equilibrium solution are worth noting. First, given our parameter values, 25 percent of households are interested in the product meaning they will buy it if they receive an advertising mail piece about the product. Due to concentration, just 12 percent of households are in the targeted subpopulation. The share of that targeted group interested in the product, as defined above is 14.9 percent.

The marginal costs for targeted mail are assumed to be above the marginal costs for saturation mail, so it is not surprising that the equilibrium price for targeted mail is above the equilibrium price for saturation mail. More interesting is the fact that the ratio of price to marginal cost is 12 percent for saturation mail and 36 percent for targeted mail. This reflects the increased value of targeted mail to advertisers. The higher markup makes targeted mail more profitable for the PO despite the fact that the volume of targeted mail sent is just a third of the amount of saturation mail sent. In addition, because of its higher effectiveness, targeted mail is more profitable for advertising firms despite its much higher price. This result shows the potential importance of targeting.

We now perform sensitivity analyses to see how changes in the parameter values affect equilibrium values. As shown in Table 1 an increase in the concentration of households interested in product x (in the product market) will increase the willingness to pay for saturation mail, and will increase the profits of both senders (advertisers) and the PO. As λ increases, S_x increases so the proportion of households interested in the product rises.[5] This increases the value of sending a piece of saturation mail and the equilibrium price and the equilibrium volume both rise.

[5] In this calibration, x is sufficiently large so that λ and S_x take on the same values to two digits. This is not the case for all calibrations.

Table 1

Effects of Changing λ								
Postal Profit Max			Postal Profit Max			Postal Profit Max		
Exogenous			Exogenous			Exogenous		
H	1000		H	1000		H	1000	
λ	0.25		λ	0.3		λ	0.4	
X	500		X	500		X	500	
Ca	0.2		Ca	0.2		Ca	0.2	
Endogenous			Endogenous			Endogenous		
Sx	0.250		Sx	0.300		Sx	0.400	
Ma	108.167		Ma	191.853		Ma	314.598	
Pa	0.224		Pa	0.247		Pa	0.292	
π Postal A	2.624		π Postal A	9.109		π Postal A	28.880	
π Adv. A	1.361		π Adv. A	4.860		π Adv. A	16.081	

Next, we examine the impact of an increase in the marginal cost of sending a piece of saturation mail. As shown in Table 2, this increases the price of saturation mail and reduced the quantity demanded. As a result, both postal profits and advertisers' profits fall.

Table 2

Effects of Changing MC								
Postal Profit Max			Postal Profit Max			Postal Profit Max		
Exogenous			Exogenous			Exogenous		
H	1000		H	1000		H	1000	
λ	0.3		λ	0.3		λ	0.3	
X	500		X	500		X	500	
Ca	0.1		Ca	0.15		Ca	0.2	
Endogenous			Endogenous			Endogenous		
Sx	0.300		Sx	0.300		Sx	0.300	
Ma	467.618		Ma	314.679		Ma	191.853	
Pa	0.188		Pa	0.219		Pa	0.247	
π Postal A	41.073		π Postal A	21.674		π Postal A	9.109	
π Adv. A	24.150		π Adv. A	12.069		π Adv. A	4.860	

We turn our attention to targeted mail. The first experiment is to increase the degree of targeting while holding all other parameters constant. This is done by increasing the value of γ. Table 3, below, shows that an increase in targeting has beneficial effects for both advertisers and the postal operator. The PO is able to increase the price of targeted mail as the volume sent rises. This is because an increase in targeting creates an increase in the demand for targeted mail. Because they are sending more pieces of effective mail, advertiser profits also rise.

Table 3

Effects of Changing γ					
Postal Profit Max		Postal Profit Max		Postal Profit Max	
Exogenous		Exogenous		Exogenous	
H	1000	H	1000	H	1000
λ	0.25	λ	0.25	λ	0.25
γ	0.3	γ	0.4	γ	0.5
X	500	X	500	X	500
CT	0.7	CT	0.7	CT	0.7
Endogenous		Endogenous		Endogenous	
ST	0.106	ST	0.120	ST	0.130
STx	0.101	STx	0.149	STx	0.206
MT	15.697	MT	31.838	MT	47.294
PT	0.821	PT	0.953	PT	1.101
π Postal T	1.906	π Postal T	8.065	π Postal T	18.977
π Adv T	1.002	π Adv T	4.415	π Adv T	10.754

The effect of targeting on profit is unambiguously positive, when considered within the framework of the model. Both PO and advertiser profits are increasing in γ. In some rough sense, an increase in targeting can be thought of as increasing welfare.

A fuller analysis would require inclusion of other postal customers, such as single piece mailers as in De Donder *et al* (2011), and an understanding of the impact on surplus maximizing prices subject to some constraint. This is left to future research, as is the question of privacy

concerns that looms large in the general issue of behavioral targeting. However, it would seem that increasing the profitability of direct mail would enable a reduction of the single piece mail price at the same level of overall PO profit. In that sense, an optimal targeting level, for any given distribution of consumers across product markets, would maximize the contribution from ad mail to the PO bottom line.

4. CONCLUSIONS AND FUTURE RESEARCH

This paper extends the analysis of advertising demand to the demand for postal direct mail products. Such work is of interest in a world where changes in postal markets have threatened the viability of reliance upon single piece and bulk transaction volumes. Drawing upon complementary literature in studies of internet advertising, we derive direct mail demand from the behavior of advertising firms and key characteristics of the consumers seeking information to make purchases. We offer preliminary investigation of postal market equilibrium in this framework, showing the impact of mail targeting on postal profits. Future research in this area might include analysis of the allocation of fixed ad budgets across channels, including direct mail, as an improved representation of observed firm behavior, as well as a fuller treatment of the welfare implications of using information to target direct mail to consumers.

REFERENCES

Anderson, Chris (2006), *The Long Tail*, Hyperion, New York.

Athey, Susan and G. Ellison (2008), "Position Auctions with Consumer Search," the *Quarterly Journal of Economics*, Oxford University Press, Vol 126(3):1213-1270.

Athey, Susan and Joshua S. Gans (2009), "The impact of targeting technology on advertising markets and media competition, *American Economic Review*, 100(2): 608-13.

Bergemann, Dirk and Alessandro Bonatti (2011), "Targeting in advertising markets: implications for offline and online media," *Rand Journal of Economics*, Vol. 42, No.3, Fall 2011, 47-443.

Chen, Y and C. He (2006), "Paid placement: Advertising and search on the internet," NET Institute Working Paper No. 06-02.

Crew, Michael and Paul Kleindorfer (2012), "Non-linear pricing, volume discounts and the USO under entry," in M.A. Crew and P.R. Kleindorfer (eds) *Multi-model Competition and the Future of Mail,"* Cheltenham, UK and Northampton, MA, USA: Edward Elgar.

De Donder, P., H Cremer, P. Dudley, and F. Rodriguez (2011), "Welfare and pricing of mail in a communications market," *Review of Network Economics*, 10 (3).

Dieke, Alex, Christian Bender, James Campbell, Robert Cohen, Christine Müller, Antonia Niederprüm, Alexandre de Streel, Sonja Thiele, and Claus Zanke (2013), *Main Developments in the Postal Sector (2010 – 2013)*, WIK.

Evans, David, S. (2008) "Attention rivalry among online platforms," http://ssrn.com/abstract=2195340.

Iyer, Ganesh, David Soberman, and Miguel villas-Boas (2005), "The Targeting of Advertising," *Marketing Science* 24 (2005).

United States Postal Service Office of Inspector General (2013a), "The untold story of the ZIP Code," RARC-WP-13-006.

United States Postal Service Office of Inspector General (2013b), "Strengthening advertising Mail by building a digital information market," RARC-WP-14-002.

APPENDIX LINEARIZING THE DIRECT MAIL DEMAND FUNCTIONS

The demand equations for saturation and targeted mail, respectively, are:

$$M_A = H\left\{ln(\lambda) - \left(\frac{\lambda}{x}\right) - lnP_A\right\}.$$

and

$$M_T = \left[\left(\frac{\gamma\lambda}{\gamma+\lambda}\right)e^{-\lambda}H\right] ln(\gamma+\lambda) - \frac{\gamma+\lambda}{x} + \gamma + \lambda - lnP_T$$

In addition to the quantities demanded, of interest is the effects of changes in P_A, P_T, x, H and γ. Unfortunately, demand is highly nonlinear in these variables, making a solution difficult to obtain, e.g., solutions include a price variable as well as the natural log of price.

To overcome this difficulty, we linearize the demand equations using a first order Taylor series expansion. Although this procedure does not provide equilibrium values, it allows us to examine the impacts of changes in the variables of interest, in so far as they cause deviations from those equilibrium values. The general structure we propose for this purpose is shown by the following approximation:

$$f(x,y) \cong f(x^*,y^*) + f_x(x^*,y^*)(x-x^*) + f_y(x^*,y^*)(y-y^*),*$$

in which it is assumed that higher order derivatives are small around the point of evaluation, which point is defined as the steady state equilibrium in the Postal market.

We employ this approximation to linearize the demand for saturation mail:

$$(M_A - M_A^*) = \left\{ln(\lambda^*) - \left(\frac{\lambda^*}{x^*}\right) - lnP_A^*\right\}(H-H^*) + \frac{\lambda^* H^*}{(x^*)^2}(x-x^*) - \frac{H^*}{P_A^*}(P_A - P_A^*),$$

written in equation in deviation form,

$$\widetilde{M}_A = \left\{ln(\lambda^*) - \left(\frac{\lambda^*}{x^*}\right) - lnP_A^*\right\}\widetilde{H} + \frac{\lambda^* H^*}{(x^*)^2}\tilde{x} - \frac{H^*}{P_A^*}\tilde{P}_A,$$

$where\ \widetilde{M}_A = (M_A - M_A^*),$

$\tilde{x} = (x - x^*),$

$\widetilde{H} = (H - H^*), and$

$\tilde{P}_A = (P_A - P_A^*).$

Using the same procedure for the demand for targeted mail, and expressing it in deviation notation,

$$\widetilde{M}_T = \left[\frac{\gamma^*\lambda^*}{\lambda^*+\gamma^*}e^{-\lambda^*}\left\{ln(\lambda^*+\gamma^*)+\left(1-\frac{1}{x^*}\right)(\lambda^*+\gamma^*)-lnP_T^*\right\}\right]\widetilde{H}$$

$$+\left[\left(\frac{\gamma^*\lambda^*}{\lambda^*+\gamma^*}e^{-\lambda}H\right)\frac{(\lambda^*+\gamma^*)}{(x^{*2})}\right]\tilde{x}-\left[\frac{\gamma^*\lambda^*}{P_T^*(\lambda^*+\gamma^*)}e^{-\lambda}H\right]\tilde{P}_T$$

$$+\left[\left(\frac{\gamma^*\lambda^*}{\lambda^*+\gamma^*}e^{-\lambda}H\right)\left\{\begin{matrix}\gamma^*(\lambda^*+\gamma^*)ln(\lambda^*+\gamma^*)\\+\left(1-\frac{1}{x^*}\right)(\lambda^*+\gamma^*)-\gamma^*(\lambda^*+\gamma^*)lnP_T^*+\frac{1}{\lambda^*+\gamma^*}+\left(1-\frac{1}{x^*}\right)\end{matrix}\right\}\right]\tilde{\gamma}$$

Renaming the coefficients, for simplification, yields a demand equation for each product which is linear in the deviations of the key variables.

$\widetilde{M}_A = \alpha_1\widetilde{H}+\alpha_2\tilde{x}-\alpha_3\tilde{P}_A$, and $\widetilde{M}_T = \beta_1\widetilde{H}+\beta_2\tilde{x}-\beta_3\tilde{P}_T+\beta_4\tilde{\gamma}$

To complete the model, we write the PO's profit function in deviation form:

$$\tilde{\pi} = \tilde{P}_A\widetilde{M}_A+\tilde{P}_T\widetilde{M}_T-\tilde{C}_A\widetilde{M}_A-\tilde{C}_T\widetilde{M}_T-\tilde{F}_A-\tilde{F}_T$$

and substitute the demand equations:

$$\tilde{\pi} = \left(\tilde{P}_A-\tilde{C}_A\right)\left[\alpha_1\widetilde{H}+\alpha_2\tilde{x}-\alpha_3\tilde{P}_A\right]+\left(\tilde{P}_T-\tilde{C}_T\right)\left[\beta_1\widetilde{H}+\beta_2\tilde{x}-\beta_3\tilde{P}_T+\beta_4\tilde{\gamma}\right]-\tilde{F}_A-\tilde{F}_T.$$

To maximize profit, the PO will set prices to meet the following first order conditions:

$\tilde{P}_A = \frac{\alpha_1\widetilde{H}+\alpha_2\tilde{x}-\alpha_3\tilde{C}_A}{2\alpha_3}$ and

$\tilde{P}_T = \frac{\beta_1\widetilde{H}+\beta_2\tilde{x}+\beta_3\tilde{C}_T+\beta_4\tilde{\gamma}}{2\beta_3}$.

Finally, substituting the original coefficients for the α and the β yields the solution for each of the two products.

$$\tilde{P}_A = \left(\frac{\left\{ln(\lambda^*)-\left(\frac{\lambda^*}{x^*}\right)-lnP_A^*\right\}}{2}\right)\widetilde{H}+\frac{\lambda^*P_A^*}{2(x^*)^2}\tilde{x}+\frac{1}{2}\tilde{C}_A \; and$$

$$\tilde{P}_T = \left[\frac{P_T^*\left(ln(\lambda^*+\gamma^*)+\left(1-\frac{1}{x^*}\right)(\lambda^*+\gamma^*)-lnP_T^*\right)}{2\widetilde{H}}\right]\widetilde{H}$$

$$+\left[\frac{P_T^*(\lambda^*+\gamma^*)}{2(x^*)^2}\right]\tilde{x}+\frac{1}{2}\tilde{C}_T+\left[P_T^*\gamma^*(\lambda^*+\gamma^*)\left(ln(\lambda^*+\gamma^*)+\left(1-\frac{1}{x^*}\right)(\lambda^*+\gamma^*)-lnP_T^*\right)+P_T^*\left(\frac{1}{\lambda^*+\gamma^*}+\left(1-\frac{1}{x^*}\right)\right)\right]\tilde{\gamma}$$

For saturation mail, we get the expected result that an increase in the number of households increases the price. In addition, when firms sell more output and/or marginal delivery cost increases price of saturation mail rises. We have a potential ambiguity regarding the impact of an increase in households on the price of targeted mail. But can solve the ambiguity by recognizing it depends on the condition that determine that the overall demand for targeted mail is positive.

Specifically, a positive effect on the mail price from an increase in households requires

$$ln(\gamma+\lambda)-\frac{\gamma+\lambda}{x}+\gamma+\lambda-lnP_T>0,$$

But from the linearization above, we see this is just the condition ensuring that overall demand is positive.

A high value of sales by firms and an increase in marginal delivery cost both lead to higher prices for targeted mail. Finally, if the above condition holds, then it is also true that an increase in targeting will lead to a higher targeted mail price.