

Partial Communication and Collusion with Demand Uncertainty

Heiko Gerlach*

May 2005

Abstract

This paper analyzes the role of communication between firms in an infinitely repeated Bertrand game in which firms receive an imperfect private signal of a common value i.i.d. demand shock. Communication allows firms to coordinate on the most collusive price and it eliminates the possibility of undetectable price cuts. It is shown that firms can use stochastic intertemporal market sharing as a perfect substitute for communication in low demand states. Therefore, partial communication in high demand states is sufficient to achieve the first-best, full communication outcome. And partial communication in low demand state does not improve on the equilibrium without communication. Communication is most valuable to firms if signal frequency is intermediate, demand is characterized by upward shocks and the number of firms is neither too small nor too large.

JEL-classification: L41, L13, D82

Keywords: Communication, Collusion, Repeated Games, Competition Policy,

*Department of Economics, University of Auckland, email: h.gerlach@auckland.ac.nz. I would like to thank Begoña Domínguez, Michihiro Kandori, Matthew Ryan, Julian Wright and seminar participants at Melbourne, Tokyo and at the Summer Workshop of Industrial Organization 2005 in Auckland for helpful discussions and comments.

1 Introduction

The detection and prosecution of collusive agreements is the most daunting task of competition policy. Taking collusion at face value, i.e. as market outcomes worse than some competitive benchmark, competition authorities could in principle try to infer collusion from price, quantity and cost data in a given industry. However, as several authors recently argued, inferring collusion from market data is virtually impossible.¹ In practice, the relevant market information is - for strategic or technical reasons - never fully available to competition authorities. Moreover, quantitative studies of allegedly collusive behaviour have proven to be highly sensitive to the specification of the functional forms of the empirical model and therefore not very useful in court.² In some cases it was pointed out that rather than looking at price levels an analysis of the evolution of prices in an industry would reduce the data requirements. However, as evidenced in the famous *Wood Pulp* case³, price parallelism is at most a necessary - not a sufficient - condition for the existence of collusion.

Consequently, most competition authorities around the world have adopted the so-called *parallelism plus rule*. This policy allows prosecution of collusive behavior only in cases where well-founded suspicion can be supported by hard evidence of facilitating practices like communication between firms, resale price maintenance or other institutionalized market design features. The advantage of this approach is that it is based on court-proof, hard evidence. The downside is that its effectiveness crucially hinges on two factors. First, the parallelism plus rule is unable to prosecute collusive outcomes that do not require facilitating practices. And therefore, the less important facilitating practices are to sustain collusion, the less effective is this policy. Second, competition authorities need to be able to observe the use of facilitating practices. While it seems less obvious to detect information sharing or communication between firms, recent high-profile cartel cases such as *Citric Acids* or *Vitamins* suggest that communication typically leaves hard evidence in the form of memos, email etc. and that this evidence can potentially be seized.

In this paper I focus on the first condition and analyze the importance of communication between firms for the sustainability of collusion. While there is some consensus about the fact that communication facilitates collusion, the question here is rather how much communication is actually needed and in which circumstances firms have stronger incentives to communicate. To this end, I consider an infinitely repeated Bertrand game with independent, common value demand fluctuations. At the beginning of each period, each firm receives a private, independent (over time and firms) though imperfect signal

¹These authors include Kühn (2001), Motta (2004) and Rey (2001).

²An often cited example are the diametrically opposed conclusions based on the same data set of the US railroad cartel in the 1880's in Porter (1983) and Ellison (1994).

³Preparation of Wood Pulp, Case IV/29.725, L85/1, 26.3.85 ECJ Cases C-89, 104, 114, 116, 117 and 125 to 129/85, see Motta (2004) for a comprehensive summary

about the current demand level. The resulting asymmetric information between firms implies that firms do no longer agree on the most collusive industry price and have an incentive to engage in communication. At the following communication stage, firms are allowed to send simultaneously messages to all other firms in the industry. Then firms set prices and profits are realized.

In the absence of an informative message, it is impossible for a firm to infer the signal that its rival has received at the beginning of the stage game. As a consequence, the private history of a firm does not coincide with the public history. Following Fudenberg, Levine & Maskin (1994) I restrict my analysis to symmetric perfect public equilibria (PPE). A PPE is a Nash equilibrium in public strategies, i.e. strategies in which firms condition on the public history of the game and not on their own private history. In particular, I analyze and compare the optimal organization of collusion with three different modes of communication: no communication, full communication (i.e. communication in all states of demand) and partial communication (i.e. communication in one demand state only).

The unobservability of private signals introduces the possibility of opportunistic price cuts. Consider for example a high demand period in which all firms receive an informative signal but do not communicate. In such a situation a firm might have an incentive to choose the lower equilibrium price of a firm with an uninformative signal and undercut its rivals. Such *on-schedule* price deviations are not detectable and therefore not punishable. Consequently, they impose static incentive compatibility constraints on the firms' collusive scheme, i.e. incentive constraints that do not depend on the patience (discount factor) of firms. Additionally, the optimal organization of collusion has to take into account *partially on-schedule* price deviations, i.e. deviations that are detectable with a certain probability. For example, if a firm with an uninformative signal deviates to the equilibrium price of a firm with a low demand signal, this deviation is only detected if demand is actually high. However, contrary to completely on-schedule deviations, these deviations are punishable but they might impose strong conditions on the patience of firms.

The analysis shows that the optimal organization of collusion in the absence of communication is characterized by a semi-pooling price strategy. In order to avoid opportunistic price cuts by firms with a high demand signal, the industry has to incur an informational cost by distorting the equilibrium price downwards and setting it equal to the price of firms with an uninformative signal. By contrast, with full communication, firms always achieve common knowledge about demand and the on-schedule constraints to prevent opportunistic price cuts are replaced by incentive constraints for communication. Consequently, the optimal collusive price scheme with full communication is a fully separating strategy. The role of communication is therefore twofold: it helps firms to coordinate on the most collusive price and it eliminates undetectable, opportunistic price cuts.

It is then shown that the most collusive equilibrium with partial communication in

low demand states cannot achieve higher industry profits than collusion without communication. The reason for this equivalence is that firms can use stochastic market sharing as a substitute for communication in low demand states. With communication in low demand states, firms share the market evenly in every low demand period. Without communication, firms with an informative signal undercut their rivals with an uninformative signal and win the whole market, i.e. firms share the market stochastically over time. Nevertheless, since communication in low demand states eliminates the possibility of partially on-schedule deviation, firms need to be more patient without communication to realize the same industry profits.

For a similar reason it turns out that partial communication in high demand states can perfectly replicate the full communication outcome. In fact, communication in high demand states solves the coordination and the opportunistic pricing problem while stochastic market sharing allows full collusion profits in low demand states. Again, for this equivalence to hold, firms need to be more patient in the equilibrium with less communication because an additional partially on-schedule constraint has to be satisfied.

When further comparing the different communication equilibria I find that the ex ante expected price in any period is the same independent of how often firms communicate. However, since consumer surplus is decreasing and convex in prices, consumers actually prefer collusive equilibria without communication. Finally, towards identifying industries with a stronger need for communication to sustain maximum collusion, I calculate the value of communication as a function of the parameters of the model. It is demonstrated that communication is more likely to occur in industries with a higher demand variance, an intermediate signal frequency, a demand with upward shocks and an intermediate number of firms.

The basic set-up of this paper is based on the seminal work of Rotemberg & Saloner (1986). They consider an industry with observable i.i.d. demand fluctuations and show that the optimal collusive arrangement might involve countercyclical price movements. Firms reduce the collusive price in high demand states to counterbalance the stronger incentive to deviate for cartel member. In this paper, I replace the perfect public demand signal with imperfect, independent, private signals and add a communication stage before the pricing decisions. Therefore, my analysis is close to the work of Athey & Bagwell (2001). Their paper considers a repeated game duopoly with inelastic demand in which firms' costs can either be high or low, with independent draws in each period. Each firm knows its own cost realization but not the cost level of its rival. They find an asymmetric perfect public equilibrium that implements first-best profits in which firms communicate their cost level. Productive (firm) efficiency is achieved by allocating high cost firms a higher future market share. In a similar set-up, Athey, Bagwell & Sanchirico (2004) consider a continuum of cost types and show that the optimal symmetric PPE sacrifices productive efficiency by using a rigid, non-sorting price scheme in order to deter high-cost firms from mimicking low-cost types. The present paper differs from these two seminal contributions in two important ways. First, firms have private information about a

common demand shift and the same cost structure. Thus, the firms' main concern is to coordinate on allocative (firm) efficiency rather than productive efficiency. And secondly, the main focus here is how much communication is necessary to achieve first-best, rather than whether communication can achieve maximum collusive profits.

This paper is also related to the "moral hazard" literature of collusion following the work of Stigler (1964) and Green & Porter (1984). As opposed to the "adverse selection" assumption in this paper, these authors consider situations in which symmetrically informed firms are unable to perfectly observe the behavior of their rivals. If firms receive public signals generated by their price or output choices, the continuation play is always an equilibrium of the repeated game and the dynamic programming technique of Abreu, Pearce & Stacchetti (1986, 1990) and Fudenberg, Levine & Maskin (1994) can be applied to establish folk theorems. If, by contrast, firms receive private signals, this recursive structure is destroyed. In this context Kandori & Matsushima (1998) and Compte (1998) stress the role of communication by generating publicly observable history on which the continuation play can be conditioned. This recovers the recursive structure and allows the proof of folk theorems.

The paper is organized as follows. The next section introduces the model. The following sections analyze collusion with no, full and partial communication. Section 6 presents the main results and the last section concludes.

2 The Model

Consider an infinitely repeated game with n firms, labelled $i \in N = \{1, 2, \dots, n\}$. Firms compete in prices in a market for a homogenous good with stochastic demand. Market demand \tilde{D} in any period is a linear function of the market price p and an i.i.d. random variable $\tilde{\theta}$ such that⁴

$$D(\tilde{\theta}, p) = \tilde{\theta} - p.$$

The random variable $\tilde{\theta}$ has two possible realizations. In demand state $j = H$ it takes the value $\theta^H = a + \Delta$; in state $j = L$ it is $\theta^L = a - \Delta$, with $a > 0$ and $0 \leq \Delta \leq a/3$. The upper bound on Δ ensures positive sales in all demand states. The probability of demand being in state $j = H$ in any period is given by $Pr(\tilde{\theta} = \theta^H) = \rho$ and the probability of a low demand state is $Pr(\tilde{\theta} = \theta^L) = 1 - \rho$. The variance of demand is increasing in Δ^2 which makes the parameter Δ a measure for the degree of demand uncertainty. I shall refer to situations with $\rho < 1/2$ as demand with upward shocks and to constellations with $\rho > 1/2$ as demand with downward shocks. At the beginning of each period, firm i receives a private signal $s_i \in S \equiv \{L, H, \emptyset\}$ about the state of demand. A firm's signal can either be perfectly informative or not informative at all.

⁴The linearity assumption is necessary to derive closed-form solutions in the equilibrium without communication.

The probability for firm i to learn the true state of demand $j \in \{L, H\}$ is given by

$$Pr\{s_i = j | \tilde{\theta} = \theta^j\} = \sigma,$$

and the probability of getting an uninformative signal in demand state j is

$$Pr\{s_i = \emptyset | \tilde{\theta} = \theta^j\} = 1 - \sigma.$$

Firms' signals are uncorrelated across firms and independent over time. The parameter σ , $0 \leq \sigma \leq 1$, measures the availability of private demand information for firms.

After observing their private signal, firms communicate by simultaneously announcing a message $m_i \in M = \{L, H, \emptyset\}$ to all other firms.^{5,6} The extent to which communication is possible is determined by the degree to which information is verifiable. I assume that firm i with a signal $s_i \in \{L, H\}$ can verifiably report this information to its rivals. In other words, firms can prove that they received a high demand or low demand signal but they cannot prove that they didn't receive any information at all.⁷

The communication of the message vector $m = (m_1..m_n)$ allows firms to update their belief b_i defined as the probability that firm i assigns to the event that demand is high,

$$b_i(s_i, m) = Pr_i\{\tilde{\theta} = \theta^H | \{s_i, m\}\}.$$

It will be useful to refer to (s_i, m) as defining a private information state \mathcal{I}_i of firm i . As a function of their information state firms choose simultaneously prices $(p_1, ..p_n)$. The strategy space of firm i for the stage game is given by

$$\Omega_i = \{\mu_i | \mu_i : S \rightarrow M\} \times \{p_i | p_i : M \rightarrow \mathfrak{R}\},$$

and a given strategy ω_i is specified as

$$\omega_i = (\mu_i(s_i), p_i(s_i, \mu_i)),$$

i.e. a function that maps each possible signal into a message and a function that maps the firm's private signal and the firms' messages into a price.

For analytical convenience, firms' marginal costs are assumed to be constant and normalized to zero. Industry profits in demand state $j \in \{L, H\}$ are defined as

$$\Pi^j(p) \equiv pD(\theta^j, p),$$

⁵I prefer to interpret a \emptyset -message as no communication between firms as the main focus of this paper is to investigate the extent of communication which is both necessary to sustain collusion and potentially detectable by competition authorities.

⁶Public communication between all industry members precludes the formation of information coalitions within the industry. Throughout the paper I restrict attention to the optimal organization of collusion amongst all firms in the industry.

⁷The main results of the paper do not depend on this assumption. Cheap talk communication imposes additional restrictions on the discount factor, i.e. the patience of firms, but does not affect the optimal organization of collusion.

where the market price p is the lowest price any firm charges in a given period, i.e.

$$p = \min\{p_1, p_2, \dots, p_n\}.$$

This stage game is repeated in each period $t=1,2,\dots$. Firms discount future profits with a common discount factor δ and maximize the discounted sum of the stage profits. At the end of each period firms can observe the price charged by their rivals and the demand level. However, in the absence of a (verifiable) informative message, it is impossible for a firm to infer the signal that a rival has received at the beginning of the stage game. As a consequence, the private history of a firm does not coincide with the public history. Following Fudenberg, Levine & Maskin (1994) I restrict my attention to symmetric perfect public equilibria (PPE). A PPE is a Nash equilibrium in public strategies, i.e. strategies in which firms condition on the public history of the game but not on their own private history (here their past private signals).

In the next three sections, I analyze the optimal organization of collusion with three different modes of communication: (i) equilibria without communication, (ii) equilibria with full communication, and (iii) equilibria with partial communication.

3 Collusion without Communication

As a benchmark this section analyzes equilibria without communication between firms, i.e.

$$\mu_i(s_i) = \emptyset, \forall s_i \in S, i \in N.$$

This implies that an individual firm i 's information state \mathcal{I}_i at the price setting stage is uniquely determined by its private signal. Denote the state in which firm i received signal s_i by $\mathcal{I}_i = s_i$ and firm i 's price vector as

$$p_i(\mathcal{I}) \equiv (p_i(\mathcal{I}_i = L), p_i(\mathcal{I}_i = H), p_i(\mathcal{I}_i = \emptyset)).$$

Without communication firms collude on prices that maximize *ex ante* expected industry profits. For symmetric price vectors $p_i(\mathcal{I}) = p_j(\mathcal{I}) = p(\mathcal{I}), i \neq j; i, j \in N$ the expected industry profits in a high demand state without communication can be written as

$$\begin{aligned} E\Pi_{\emptyset}^H(p(\mathcal{I})) &\equiv \begin{cases} \Pr\{N_H = n\}\Pi^H(p(H)) + \Pr\{N_{\emptyset} \geq 1\}\Pi^H(p(\emptyset)) & \text{if } p(\emptyset) \leq p(H), \\ \Pr\{N_{\emptyset} = n\}\Pi^H(p(\emptyset)) + \Pr\{N_H \geq 1\}\Pi^H(p(H)) & \text{if } p(\emptyset) > p(H) \end{cases} \\ &= \begin{cases} \sigma^n \Pi^H(p(H)) + (1 - \sigma^n) \Pi^H(p(\emptyset)) & \text{if } p(\emptyset) \leq p(H), \\ (1 - \sigma)^n \Pi^H(p(\emptyset)) + (1 - (1 - \sigma)^n) \Pi^H(p(H)) & \text{if } p(\emptyset) > p(H) \end{cases} \end{aligned}$$

where $Pr\{N_x = n\}$ denotes the probability that N_x firms are in information state $x \in S$. Expected industry profits in the low demand state are similarly

$$\begin{aligned} E\Pi_{\emptyset}^L(p(\mathcal{I})) &\equiv \begin{cases} Pr\{N_{\emptyset} = n\}\Pi^L(p(\emptyset)) + Pr\{N_L \geq 1\}\Pi^L(p(L)) & \text{if } p(L) \leq p(\emptyset), \\ Pr\{N_L = n\}\Pi^L(p(L)) + Pr\{N_{\emptyset} \geq 1\}\Pi^L(p(\emptyset)) & \text{if } p(L) > p(\emptyset), \end{cases} \\ &= \begin{cases} (1 - \sigma)^n \Pi^L(p(\emptyset)) + (1 - (1 - \sigma)^n) \Pi^L(p(L)) & \text{if } p(L) \leq p(\emptyset), \\ \sigma^n \Pi^L(p(L)) + (1 - \sigma^n) \Pi^L(p(\emptyset)) & \text{if } p(L) > p(\emptyset). \end{cases} \end{aligned}$$

The overall *ex ante* expected industry profit is then given by

$$E\Pi_{\emptyset}(p(\mathcal{I})) \equiv \rho E\Pi_{\emptyset}^H(p(\mathcal{I})) + (1 - \rho) E\Pi_{\emptyset}^L(p(\mathcal{I})).$$

To be implementable, a price vector has to resist three types of deviations. Firms can deviate to out-of-equilibrium prices; these deviations are always detectable by rivals. Second, firms can choose prices that are out-of-equilibrium with a positive probability, e.g. a firm with a \emptyset -signal could charge $p(L)$. This semi-detectable deviation would be uncovered if demand is actually high. And thirdly, firms can choose on-schedule deviations, i.e. deviations to the equilibrium price of a firm in a different information state. To sustain an equilibrium, the first two types of deviation induce with strictly positive probability the worst possible punishment, i.e. eternal reversion to marginal cost pricing, and firms have to trade off short-term gains with long-run reduced profits from the break-down of collusion. Sufficiently patient firms would never deviate off-schedule or partially off-schedule. By contrast, on-schedule deviations do not entail punishment and are independent of the firms' patience. Consequently, firms have to devise collusive price vectors that are robust to on-schedule deviation incentives.

In an equilibrium without communication, there exist two feasible on-schedule deviations: a firm with a H -signal could deviate to $p(\emptyset)$ and a firm with a L -signal could deviate to $p(\emptyset)$. First, consider the deviation incentives of a firm with a H -signal if $p(H) \neq p(\emptyset)$. If firm i with $\mathcal{I}_i = H$ plays its equilibrium price $p(H)$, it can expect a stage profit of

$$\Pi_i(p(H)|\mathcal{I}_i = H) = \begin{cases} Pr\{N_H = n - 1 | n - 1\} \Pi^H(p(H))/n & \text{if } p(\emptyset) < p(H), \\ \sum_{j=0}^{n-1} Pr\{N_H = j\} \Pi^H(p(H))/(j + 1) & \text{if } p(\emptyset) > p(H). \end{cases}$$

For $p(H) > p(\emptyset)$, a firm with a high demand signal can only get its share of the industry profit if all the other $n - 1$ firms also get a H -signal. With the opposite price ranking, the firm's profit depends on how many other firms also get a H -signal. Deviating to the equilibrium strategy of a firm with a \emptyset -signal yields an expected stage profit of

$$\Pi_i(p(\emptyset)|\mathcal{I}_i = H) = \begin{cases} \sum_{j=0}^{n-1} Pr\{N_{\emptyset} = j\} \Pi^H(p(\emptyset))/(j + 1) & \text{if } p(\emptyset) < p(H), \\ Pr\{N_{\emptyset} = n - 1 | n - 1\} \Pi^H(p(\emptyset))/n & \text{if } p(\emptyset) > p(H). \end{cases}$$

Deviation to a lower $p(\emptyset)$ allows firm i to undercut all firms with a high demand signal and to share total industry profits with all firms that received an uninformative signal. A firm with a high demand signal does not deviate if

$$\Pi_i(p(H)|\mathcal{I}_i = H) \geq \Pi_i(p(\emptyset)|\mathcal{I}_i = H). \quad (\text{OSH})$$

This condition is satisfied if either⁸

$$p(H) = p(\emptyset), \quad (\text{OSH-0})$$

or, for $p(\emptyset) < p(H)$,

$$\frac{\sigma^{n-1}}{n} \Pi^H(p(H)) \geq \frac{1 - \sigma^n}{n(1 - \sigma)} \Pi^H(p(\emptyset)), \quad (\text{OSH-1})$$

or, for $p(\emptyset) > p(H)$,

$$\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^H(p(H)) \geq \frac{(1 - \sigma)^{n-1}}{n} \Pi^H(p(\emptyset)). \quad (\text{OSH-2})$$

A firm with a low demand signal playing its equilibrium price $p(L)$ expects a stage profit of

$$\Pi_i(p(L)|\mathcal{I}_i = L) = \begin{cases} \sum_{j=0}^{n-1} \Pr\{N_L = j\} \Pi^L(p(L))/(j+1) & \text{if } p(L) < p(\emptyset), \\ \Pr\{N_L = n-1|n-1\} \Pi^L(p(L))/n & \text{if } p(L) > p(\emptyset). \end{cases}$$

Again, a firm only makes profits with the higher price if no firm receives a signal that implies a lower equilibrium price. When charging the lower of the two prices, a firm's profit depends on the number of competitors that receive the same signal. A firm with a low demand signal could deviate without detection to the equilibrium price a firm with an uninformative signal would have charged and get profits of

$$\Pi_i(p(\emptyset)|\mathcal{I}_i = L) = \begin{cases} \Pr\{N_\emptyset = n-1|n-1\} \Pi^L(p(\emptyset))/n & \text{if } p(L) < p(\emptyset), \\ \sum_{j=0}^{n-1} \Pr\{N_\emptyset = j\} \Pi^L(p(\emptyset))/(j+1) & \text{if } p(L) > p(\emptyset). \end{cases}$$

And therefore the on-schedule constraint for a firm with a L-signal is given by

$$\Pi_i(p(L)|\mathcal{I}_i = L) \geq \Pi_i(p(\emptyset)|\mathcal{I}_i = L), \quad (\text{OSL})$$

which holds either if

$$p(L) = p(\emptyset), \quad (\text{OSL-0})$$

or, for $p(L) < p(\emptyset)$,

$$\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^L(p(L)) \geq \frac{(1 - \sigma)^{n-1}}{n} \Pi^L(p(\emptyset)). \quad (\text{OSL-1})$$

⁸See the appendix of Lemma 1 for the derivation of these expressions.

or, for $p(L) > p(\emptyset)$,

$$\frac{\sigma^{n-1}}{n} \Pi^L(p(L)) \geq \frac{1 - \sigma^n}{(1 - \sigma)n} \Pi^L(p(\emptyset)). \quad (\text{OSL-2})$$

The possibility of undetectable price deviations requires that firms devise pricing strategies that satisfy the two on-schedule deviation constraints, i.e. firms maximize expected industry profits subject to (OSL) and (OSH),

$$\max_{p(\mathcal{I})} E\Pi_\emptyset(p(\mathcal{I})) \text{ s.t. } (\text{OSH}) \text{ and } (\text{OSL}) \quad (1)$$

The solution to this problem is summarized in the following lemma. Denote $p^*(L)$ and $p^*(H)$ the monopoly price under complete information with low and high demand respectively.

Lemma 1 *The solution $p_\emptyset(\mathcal{I})$ to maximization problem (1) is characterized by:*

$$p_\emptyset(L) = p^*(L) < p_\emptyset(\emptyset) = p_\emptyset(H) = \pi p^*(H) + (1 - \pi)p^*(L) \leq p^*(H)$$

with

$$\pi = \frac{\rho}{\rho + (1 - \rho)(1 - \sigma)^n}.$$

The only strictly binding constraint in the optimum is (OSH-0).

Figure 1 illustrates maximization problem (1) in two diagrams with $p(\emptyset)$ on the vertical axis and $p(H)$ and $p(L)$ on the horizontal axis. The shaded areas represent the admissible sets of prices as defined by (OSH) and (OSL). The unconstrained solution (points A_1 and A_2) is not feasible since the optimal price $p(H) = p^*(H)$ would require a lower $p(\emptyset)$ to avoid opportunistic price cuts from H -signal firms. The iso-profit curves around A_1 and A_2 indicate that there are two likely candidates as the second-best solution. Either firms impose (OSH-0) and set $p(\emptyset) = p(H)$ or they create a sufficiently large wedge between the two prices in order to satisfy (OSH-1). Lemma 1 states that the former option always dominates for demand variances $\Delta < a/3$. Intuitively, the lower the variance, the less costly are deviations for H -signal firms to $p(\emptyset)$. Therefore, condition (OSH-1) becomes harder to satisfy and the second option less attractive.

The most collusive pricing strategy without communication is a semi-pooling strategy. If no firm receives a L -signal the effective industry price is $p_\emptyset(\emptyset) = p_\emptyset(H)$. Otherwise, firms with a low demand signal set the complete information monopoly price and undercut all firms who received an uninformative signal. This means that the optimal organization of collusion without communication implies that in some periods some firms make zero profits while the remaining firms share the market at the most collusive price.

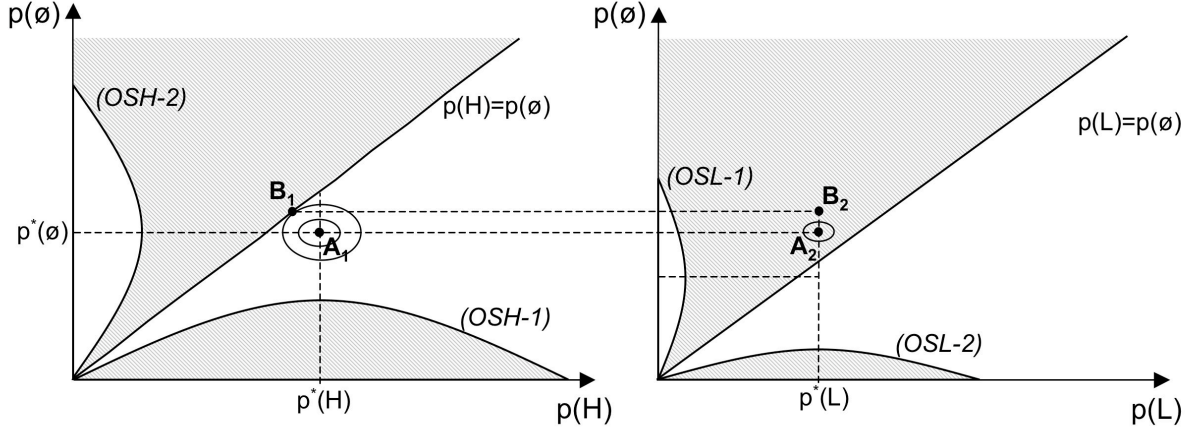


Figure 1: Optimal collusion without communication

The optimal price for firms with \emptyset - and H -signals is a weighted average of the monopoly price with low and high demand with the conditional probability of a high demand state given no firm receives a L -signal as the weight for the high demand monopoly price. Thus, the pooled price increases in the probability of a high demand (ρ), the signal frequency (σ) and the number of firms (n). The effect of the demand variance on the optimal price is ambiguous. For a low conditional probability that demand is high (i.e. for ρ , σ , n low), a higher variance leads to a lower price, otherwise the price increases in Δ .

With the solution of the static industry profit maximization problem in place, one can state the incentive constraints for equilibrium communication and off-schedule price deviations. Denote $\delta_\emptyset(H)$ the threshold value above which a firm with a H -signal would not deviate from its equilibrium price. Denote $\delta_\emptyset(\emptyset)$ the threshold above which a firm with \emptyset -signal would not deviate to $p_\emptyset(L)$.

Proposition 1 *If $\delta \geq \max\{\delta_\emptyset(H), \delta_\emptyset(\emptyset)\}$, then the price vector $p_\emptyset(\mathcal{I})$ from Lemma 1 is sustainable in a PPE of the repeated game without communication.*

To ensure that firms have no incentive to send L or H messages, it suffices to make them a trigger for eternal reversion to marginal cost pricing from the current period onwards. In the price subgame, deviations are most profitable for either a H -signal firm or a \emptyset -signal firm. A firm with a high demand signal knows that all its rival are setting the same price $p_\emptyset(H) = p_\emptyset(\emptyset)$ and the optimal deviation is to undercut this price slightly. If a firm with a \emptyset -signal deviates to $p_\emptyset(L)$ it can expect to be on-schedule with probability $1 - \rho$ and undercut all firms with an uninformative signal. Consequently, $\delta_\emptyset(H)$ ($\delta_\emptyset(\emptyset)$) is the binding threshold for high (low) ρ and high (low) σ .

4 Collusion with Full Communication

In this section I consider equilibria in which firms communicate all information they receive, i.e.

$$\mu_i(s_i) = s_i, \forall s_i \in S.$$

Based on their own signal and the messages received from their competitors, firms update their beliefs about the state of demand. In particular, after the communication stage, firms are in one of three possible information states and for notational convenience I denote the information state of firm i by $\mathcal{I}_i \in S = \{L, H, \emptyset\}$. With an *ex ante* probability of $1 - (1 - \sigma)^n$, at least one firm receives an informative signal and sends a verifiable message to the other firms.⁹ Thus, firms have perfect common knowledge about the state of demand they are in, i.e. $\mathcal{I}_i = L$, or $\mathcal{I}_i = H$, $\forall i$ with corresponding beliefs of $b_i(\mathcal{I}_i = L) = 0$ and $b_i(\mathcal{I}_i = H) = 1$. With the remaining probability $(1 - \sigma)^n$ all firms receive uninformative signals and send \emptyset -messages to their competitors. In this case firms know that no one knows the demand state, i.e. $\mathcal{I}_i = \emptyset$, $\forall i$ and $b_i(\mathcal{I}_i = \emptyset) = \rho$.¹⁰ Denote a firm's price as function of its information state by

$$p_i(\mathcal{I}) \equiv (p_i(\mathcal{I}_i = L), p_i(\mathcal{I}_i = H), p_i(\mathcal{I}_i = \emptyset)).$$

Then, for a given symmetric price vector $p_i(\mathcal{I}) = p_j(\mathcal{I}) = p(\mathcal{I})$, $i \neq j$; $i, j \in N$, the expected industry profits in a high demand period with communication can be written as

$$\begin{aligned} E\Pi_c^H(p(\mathcal{I})) &= Pr\{N_H \geq 1\}\Pi^H(p(H)) + Pr\{N_\emptyset = n\}\Pi^H(p(\emptyset)) \\ &= (1 - (1 - \sigma)^n)\Pi^H(p(H)) + (1 - \sigma)^n\Pi^H(p(\emptyset)), \end{aligned}$$

where N_x refers to the number of firms in information state x . The expected industry profits in a low demand period are given by

$$\begin{aligned} E\Pi_c^L(p(\mathcal{I})) &= Pr\{N_L \geq 1\}\Pi^L(p(L)) + Pr\{N_\emptyset = n\}\Pi^L(p(\emptyset)) \\ &= (1 - (1 - \sigma)^n)\Pi^L(p(L)) + (1 - \sigma)^n\Pi^L(p(\emptyset)), \end{aligned}$$

and *ex ante* expected industry profits with full communication are

$$E\Pi_{LH}(p(\mathcal{I})) \equiv \rho E\Pi_c^H(p(\mathcal{I})) + (1 - \rho)E\Pi_c^L(p(\mathcal{I})).$$

Full communication creates common knowledge and the highest possible degree of coordination among firms. More importantly, full equilibrium communication also implies

⁹To avoid uncertified L or H messages to be sent, assume that firms revert - upon reception of an uncertified message - to marginal cost pricing from the current period onward.

¹⁰For simplicity, I shall use the same notation for information states in all four classes of equilibria, although in each type of communication equilibrium an information state is defined in a different way.

that the on-schedule price deviation constraints are being replaced by the incentive constraints for communication. In other words, if it is incentive-compatible to report a H - or L -signal, deviations to the equilibrium price for an uninformative signal become off-schedule and detectable. This means that firms do not need to take into account static incentive constraints and the industry profit maximizing price vector $p(\mathcal{I})$ simply solves

$$\max_{p(\mathcal{I})} E\Pi_{LH}(p(\mathcal{I})). \quad (2)$$

The next lemma characterizes the solution of this maximization problem and compares it to the solution of Lemma 1.

Lemma 2 *The most collusive price strategy with full communication, $p_c(\mathcal{I})$, satisfies*

$$p_c(L) = p^*(L) < p_c(\emptyset) = (1 - \rho)p^*(L) + \rho p^*(H) < p_c(H) = p^*(H) \quad (i)$$

and

$$p_c(\emptyset) < p_\emptyset(\emptyset) = p_\emptyset(H) < p_c(H). \quad (ii)$$

The optimal price strategy with full communication implies that the price an individual firm sets increases with its private demand signal or the messages from other firms. If at least one firm receives an informative signal, all firms set the complete information monopoly price for the respective demand state and share the market. If no firm received a signal, firms share the market at an intermediate price equal to the *ex ante* monopoly price without demand signals.

The effective market price with and without communication is the same if at least one firm receives a L -signal. In the first case all firms quote the same price, in the second firms with a L -signal undercut uninformed firms. The market price without communication is lower in high demand states in which at least one firm receives an informative signal. Communication allows firms to coordinate on the most collusive price while without communication firms have to pool the informed and uninformed price to avoid opportunistic price cuts. On the other hand, the market price is higher without communication if all firms receive an uninformative signal since the on-schedule constraint for high demand signal firms distorts the industry price upwards.

The price vector in Lemma 2 is sustainable if firms have no incentive to deviate at the communication stage and at the pricing stage. The following proposition discusses the conditions for which this holds true. Denote $\delta_c(H)$ the threshold value above which a firm with a H -signal would not deviate from its equilibrium price.

Proposition 2 *If $\delta \geq \delta_c(H)$, then the price vector $p_c(\mathcal{I})$ from Lemma 2 can be supported in a PPE with full communication.*

Similar to the case with complete information in Rotemberg & Saloner (1986) firms have the strongest incentive to deviate in price subgames in which it is common knowledge that demand is high, i.e. after at least one H -message has been sent. Proposition

2 implies that if firms are sufficiently patient no to deviate in these price subgames then they also have no incentive to deviate at the communication stage. The reason for this is that price deviations are most profitable if all rival firms hold at least as strong a belief in high demand than the potential deviator does. Suppose a firm receives a high demand signal but deviates to a \emptyset -message and no other firms sends a H -message. Then all of the deviating firm's rivals are in information state \emptyset and set the uninformed price. However, undercutting this price is less profitable than undercutting the monopoly price with high demand and the deviating firm would have been better off sending a H -message at the communication stage. A similar argument applies to the communication incentives of L -signal firms.

The role of communication in this model is twofold: it helps firms to coordinate on the most-collusive price and it eliminates opportunistic price behavior due to unobservable demand signals.

5 Collusion with Partial Communication

5.1 Collusion with Communication in Low Demand State

I refer to equilibria with partial communication as situation where firms communicate in one state of demand but not in the other. First, I analyze equilibria where firms report a low demand signal but not a high demand signal, i.e.

$$\mu_i(L) = L \wedge \mu_i(H) = \mu_i(\emptyset) = \emptyset.$$

This implies that after the communication stage, there are two types of price subgames and any firm i can be in one of three information states $\mathcal{I}_i \in \{L, H, \emptyset\}$. With an *ex ante* probability of $(1 - \rho)(1 - (1 - \sigma)^n)$ at least one firm receives a L -signal, communicates it and firms have common knowledge that they are in a low-demand period, i.e. $\mathcal{I}_i = L \forall i$ and $b_i(\mathcal{I}_i = L) = 0$. With the remaining probability firms receive an H - or \emptyset -signal and send uninformative messages. A firm with a H -signal has private information that demand is high, i.e. $\mathcal{I}_i = H$ and $b_i(\mathcal{I}_i = H) = 1$. A firm i receiving an uninformative private signal and \emptyset -messages from all other firms ($\mathcal{I}_i = \emptyset$) updates its belief according to

$$b_i(\mathcal{I}_i = \emptyset) = \frac{\rho}{\rho + (1 - \rho)(1 - \sigma)^{n-1}}.$$

Note that $p_i(\emptyset)$ and $p_i(H)$ occur in the price subgame following \emptyset -messages while $p_i(L)$ only occur in the price subgame following at least one L -message. Therefore, maximizing *ex ante* and *interim* (i.e. after communication) expected industry profits is equivalent and for expositional reasons I shall use the former. Without communication in the high demand state expected industry profits are the same - modulo the definition of information states - as in section 3, i.e. $E\Pi_\emptyset^H$. Meanwhile, expected industry profits in

the low demand state with communication are identical to section 4, i.e. $E\Pi_c^L$. Thus, *ex ante* expected industry profits with partial communication in the L-state are

$$E\Pi_L(p(\mathcal{I})) \equiv \rho E\Pi_\emptyset^H(p(\mathcal{I})) + (1 - \rho) E\Pi_c^L(p(\mathcal{I})).$$

Moreover, if it is (in equilibrium) incentive-compatible for firms to communicate a private L -signal, any price deviation afterwards becomes detectable. This means that the industry profit maximizing price strategy does not have to take into account the on-schedule constraint (OSL). However, on-schedule deviations for firms with a H -signal are still possible. Thus, the most collusive price strategy with partial communication in the L-state solves

$$\max_{p(\mathcal{I})} E\Pi_L(p(\mathcal{I})) \text{ s.t. } (OSH), \quad (3)$$

and the next lemma follows directly.

Lemma 3 *The price vector that maximizes ex ante expected industry profits with partial communication in low demand states is given by $p_\emptyset(\mathcal{I})$.*

Two observations explain this equivalence result. First, the expected industry profits in the low demand state are the same with no communication and partial communication as long as $p(L) \leq p(\emptyset)$. To see this, note that $p(L)$ becomes the effective industry price in both situations if at least one firm receives a L -signal, otherwise the industry price is $p(\emptyset)$. With communication, all firms share the market evenly in every low demand period; without communication, all firms with a L -signal share the market by undercutting their rivals with an uninformative signal. In other words, without communication firms share the market stochastically over time and this stochastic market sharing is a perfect substitute for communication in low demand states. Secondly, the global maximizer of the maximization problem (1) without communication satisfies $p(L) \leq p(\emptyset)$ and the on-schedule constraint (OSL) was not strictly binding. Therefore, the solution to (1) and to (3) have to be identical.

The next proposition gives the condition under which partial communication and collusion on the price from Lemma 3 can be sustained in the repeated game.

Proposition 3 *If $\delta \geq \delta_\emptyset(H)$, then the most collusive price strategy from Lemma 3 is sustainable in a PPE with partial communication in the low demand state.*

This proposition states that while the most collusive price is the same with partial communication in L-states and with no communication at all, collusion is harder to sustain in the absence of communication. The reason for this is that communication destroys the possibility of semi-detectable price deviations. In particular, in situations in which all firms send \emptyset -messages, the deviation of a \emptyset -signal to $p_\emptyset(L)$ becomes off the equilibrium path since a L -signal firm should have announced the low demand state at the communication stage. The condition in Proposition 3 ensures that a H -signal

firm is not deviating from its equilibrium price in the price subgame in which all firms sent uninformative messages at the communication stage. If this condition is in place it follows that firms with a L -signal have an incentive to communicate their information at the first stage and share the market with its rivals. Withholding a L -signal either leads to inefficient pricing at the price stage or in case of price undercutting it triggers reversion to marginal cost pricing.

5.2 Collusion with Communication in High Demand State

Consider the class of equilibria in which firms only communicate if they receive a high demand signal, i.e.

$$\mu_i(H) = H \wedge \mu_i(L) = \mu_i(\emptyset) = \emptyset.$$

Following the communication stage, there are two types of price subgames and firms can be in one of three information states. With an *ex ante* probability of $\rho(1 - (1 - \sigma)^n)$ at least one firm receives a H -signal, communicates it and firms have common knowledge that they are in a high-demand period, i.e. $\mathcal{I}_i = H \forall i$ and $b_i(\mathcal{I}_i = H) = 1$. With the remaining probability firms receive an L - or \emptyset -signal and send uninformative messages. In this subgame a firm with a L -signal has private information that demand is low, i.e. $\mathcal{I}_i = L$ and $b_i(\mathcal{I}_i = L) = 0$. Finally, firm i receiving an uninformative private signal and \emptyset -messages from all other firms ($\mathcal{I}_i = \emptyset$) updates its belief according to

$$b_i(\mathcal{I}_i = \emptyset) = \frac{\rho(1 - \sigma)^{n-1}}{1 - \rho + \rho(1 - \sigma)^{n-1}}.$$

It is again sufficient to maximize *ex ante* expected industry profits since $p_i(\emptyset)$ and $p_i(L)$ are only charged in the subgame with \emptyset -messages while $p_i(H)$ is only set in the price subgame following at least one H -message. Without communication in the low demand state expected industry profits are the same - modulo the definition of information states - as in section 3, i.e. $E\Pi_\emptyset^L$. Expected industry profits in the high demand state with communication are given by $E\Pi_c^H$. And *ex ante* expected industry profits with partial communication in the H -state are

$$E\Pi_H(p(\mathcal{I})) \equiv \rho E\Pi_c^H(p(\mathcal{I})) + (1 - \rho) E\Pi_\emptyset^L(p(\mathcal{I})).$$

In this partial communication equilibrium, the on-schedule price constraint for a H -signal firm is replaced with the information revelation constraint. In other words, communication in the high demand state makes opportunistic price cuts for firms with a H -signal detectable and punishable. Therefore, to find the most collusive price strategy, firms solve

$$\max_{p(\mathcal{I})} E\Pi_H(p(\mathcal{I})) \text{ s.t. } (OSL). \quad (4)$$

The following lemma gives the solution to this problem.

Lemma 4 *The price vector that maximizes ex ante expected industry profits with partial communication in the high demand state is given by $p_c(\mathcal{I})$.*

Partial communication in the high demand state implies that the profits with high demand are the same as with full communication. In low demand states firms can substitute stochastic market sharing for communication. This yields the same profit structure as with full communication. Moreover, we know from Lemma 2 that $p_c(\emptyset) > p_c(L) = p^*(L)$, i.e. (OSL-1) is satisfied for the unconstrained maximization problem. From this Lemma 4 follows. Denote $\delta_c(\emptyset)$ the threshold above which a firm with $s_i = \emptyset$ would not deviate to $p_c(L)$.

Proposition 4 *If $\delta \geq \max\{\delta_c(H), \delta_c(\emptyset)\}$, then the most collusive price vector from Lemma 4 is sustainable in a PPE with partial communication in high demand states.*

Two conditions place lower bounds on the threshold discount factor. Like in the equilibrium with full communication, the discount factor has to be sufficiently high to prevent undercutting after high demand communication ($\delta \geq \delta_c(H)$). Additionally, firms have to be sufficiently patient to resist the partially off-schedule deviation to $p_c(L)$ in case they receive a \emptyset -signal and only \emptyset -messages. This deviation is even more tempting since the absence of a high demand message means that firms believe stronger that demand is actually low. It follows that $\delta_c(\emptyset)$ is the binding threshold whenever $b_i(\mathcal{I}_i = \emptyset)$ is sufficiently small and collusion with partial communication in the high demand state is harder to sustain than with full communication. As long as the two conditions in Proposition 4 hold, firms also have no incentives to deviate at the communication stage. As noted in the equilibrium with full communication, the optimal price deviation for a firm with a H -signal occurs if its rivals know about the high demand state. Therefore, firms prefer to communicate high demand before undercutting at the price stage.

6 The Value of Communication

In this section I compare the four different modes of communication and collusion with respect to expected prices, profits and consumer surplus. Denote $ECS_\emptyset, ECS_{LH}, ECS_L$ and ECS_H as the expected consumer surplus profits with no, full and partial communication in low demand and high demand state respectively.

Proposition 5 *For $\delta \geq \max\{\delta_\emptyset(H), \delta_\emptyset(\emptyset), \delta_c(H), \delta_c(\emptyset)\}$ it holds that*

- (i) *the ex ante expected industry price is the same in all four types of communication equilibria and equal to $p_c(\emptyset)$,*
- (ii) *the profit ranking is*

$$E\Pi_\emptyset(p_\emptyset(s)) = E\Pi_L(p_\emptyset(s)) < E\Pi_H(p_c(\mathcal{I})) = E\Pi_{LH}(p_c(\mathcal{I})),$$

(iii) the consumer surplus ranking is given by

$$ECS_{\phi}(p_{\phi}(s)) = ECS_L(p_{\phi}(s)) > ECS_H(p_c(\mathcal{I})) = ECS_{LH}(p_c(\mathcal{I})).$$

The first part of this proposition states that the expected industry price in each period is the same independent of whether and how much firms communicate and it is equal to the most collusive price firms would set in the absence of informative signals. This result is due to the fact that in all four equilibria *ex ante* and *interim* (i.e. after signals and communication) maximization of industry profits are equivalent.

The second part of Proposition 5 summarizes the main result of the paper. Partial communication in the low demand state cannot improve on the equilibrium without communication and partial communication in high demand states achieves the same profit as full communication. As discussed in the previous section, firms do not require communication in low demand states if they rely on stochastic market sharing. However, communication in high demand states helps firms to coordinate on the most collusive industry and avoids undercutting due to asymmetric information among firms. Therefore, full communication and partial communication in the high demand state dominate the two other communication equilibria.

The above result also implies that whether communication facilitates collusion does not necessarily depend on how much information is exchanged among firms but on the content of information. For examples, in industries with upward demand shocks (i.e. low ρ), firms communicate more often if they partially communicate in low demand states compared to partial communication in high demand states only. Nevertheless, partial communication in high demand states is more collusive in the sense that it leads to higher industry profits.

The third part of the proposition says that - given firms are colluding - consumers are better off without communication (or partial communication in low demand states) between firms. This result follows from the fact that while the expected industry price is the same for all communication equilibria, the price variance is higher with full communication. And since consumers' utility is decreasing and convex in the market price they are best off without communication among firms.

While competition policy is not explicitly modeled in this framework, some implications of the above analysis seem warranted. In application of the "parallelism plus" rule, competition authorities require evidence of communication to prosecute collusion. If there is the possibility that communication might be detected, part (ii) of Proposition 5 suggests that firms would optimally react by either not communicating at all or by communicating in high demand states only. The implications of this for competition authorities are ambiguous. On the one hand, partial communication in high demand states is sufficient to achieve first-best collusion, i.e. less evidence is produced which reduces the scope of the "parallelism plus" rule. On the other hand, the result indicates that communication is most likely to occur in high demand states and this might provide helpful guidance in the search for evidence.

Finally, towards identifying industries with a stronger need for communication to support collusion, the last proposition analyzes the comparative statics of the value of communication for firms, i.e. $E\Pi_H(p_c(s)) - E\Pi_\phi(p_\phi(s))$.

Proposition 6 *The value of communication for firms is (i) increasing in the demand variance Δ^2 , (ii) maximal for a $\hat{\sigma}$ with $0 < \hat{\sigma} < 1$ and a $\hat{\rho}$ with $0 < \hat{\rho} < 1/3$ and (iii), for sufficiently small values of σ , it is first increasing then decreasing in the number of firms.*

Communication is more valuable the higher the demand variance since firms stand to lose more when pooling the uninformed price and the H -signal price in equilibria without communication in high demand states. Part (ii) of Proposition 6 states that the value of communication is highest for intermediate signal frequencies. The need for coordination among firms is low when firms hardly receive any informative signal or when informative signals are very likely. Communication is more valuable when industry demand is characterized by upward demand shocks. Demand uncertainty is maximized for $\rho = 1/2$. However, low values of ρ imply that the price for H -signal firms without communication is low and therefore the value of communication in high demand states is high. Finally, part (iii) implies that communication is more likely to occur in industries with neither too few nor too many firms. The more firms there are the higher the conditional probability that firms are in high demand state if none receives a L -signal. Thus, the optimal price without communication $p_\phi(H)$ approaches $p_c(H)$ and communication becomes less valuable. On the other hand, for a small number of firms, communication is less valuable since there are less private demand signals, i.e. less information, to be shared.

7 Conclusions

This paper analyzes the role of communication for collusion in markets with demand uncertainty and shows that extensive information exchange is not a prerequisite for firms to implement the first-best collusion profits. In particular, in periods of low demand firms do not need to communicate at all as long as they rely on stochastic, intertemporal market sharing. In high demand periods, however, communication is necessary both to coordinate on the most collusive and to prevent opportunistic, undetectable price deviations. The implications of this result for competition policy are ambiguous. On the one hand, less need for communication means that less evidence is produced and it harder to prosecute firms. On the other hand, firms are more likely to communicate in high demand states and this could potentially guide competition authorities in their search for evidence.

Several interesting extensions and generalizations of this result are beyond the scope of this paper and await future research. It seems clear that the results in this depend on the mode of competition. Stochastic, intertemporal market sharing is not feasible with

quantity setting oligopolists. Furthermore, quantity competition should reduce firms' incentives to reveal information in high demand situation because a firm's deviation is more profitable the lower the demand expectations of its rivals. However, the main results should - at least to some extent - carry over to collusive pricing with capacity constraints like in Staiger & Wolak (1992). A second interesting extension of this framework could be to analyze whether firms have incentives to form informational coalitions and to communicate only within a small group of firms. Firms would have to trade off less information within their coalition with a higher market share in case their competitors remain uninformed.

Appendix

Proof of Lemma 1

First I derive the formulas for the on-schedule deviation constraints. Then, I solve maximization problem (1). For $p(\phi) < p(H)$, the right-hand side ('rhs' hereafter) of OSH is equivalent to

$$\begin{aligned}
\sum_{j=0}^{n-1} Pr\{N_\phi = j\} \frac{\Pi^H(p(\phi))}{j+1} &= \Pi^H(p(\phi)) \sum_{j=0}^{n-1} \binom{n-1}{j} (1-\sigma)^j \sigma^{n-1-j} \frac{1}{j+1} \\
&= \Pi^H(p(\phi)) \sum_{j=0}^{n-1} \frac{(n-1)!}{(n-1-j)!(j+1)!} (1-\sigma)^j \sigma^{n-1-j} \\
&= \frac{\Pi^H(p(\phi))}{n(1-\sigma)} \sum_{j=0}^{n-1} \frac{n!}{(n-1-j)!(j+1)!} (1-\sigma)^{j+1} \sigma^{n-1-j} \\
&= \frac{\Pi^H(p(\phi))}{n(1-\sigma)} \sum_{i=1}^n \binom{n}{i} (1-\sigma)^i \sigma^{n-i} + \sigma^n - \sigma^n = \frac{1-\sigma^n}{n(1-\sigma)} \Pi^H(p(\phi))
\end{aligned}$$

and constraint (OSH-1) follows. For $p(\phi) > p(H)$, the left-hand side ('lhs') of (OSH) can be simplified in the same way using the Binomial Theorem,

$$\begin{aligned}
\sum_{j=0}^{n-1} Pr\{N_H = j\} \frac{\Pi^H(p(H))}{j+1} &= \Pi^H(p(H)) \sum_{j=0}^{n-1} \binom{n-1}{j} \sigma^j (1-\sigma)^{n-1-j} \frac{1}{j+1} \\
&= \frac{1 - (1-\sigma^n)}{\sigma n} \Pi^H(p(H)),
\end{aligned}$$

and condition (OSH-2) follows. The corresponding formulas for (OSL) can be derived likewise.

I now solve maximization problem (1) by looking for local maximizers in the four different price spaces resulting from the definition of the objective function. In each case one also has to take into account the non-negativity constraint in the low-demand state with an uninformative price, i.e. $p(\phi) \leq a - \Delta$.

A. $p(L) \leq p(\emptyset) \wedge p(H) \geq p(\emptyset)$. Omitting (OSL-1) for the moment, the Lagrangian of maximization problem (1) can be written as

$$\begin{aligned} \mathcal{L}(p(\mathcal{I}), \lambda) = & E\Pi_{\emptyset}(p(\mathcal{I})) + \lambda_1(p(H) - p(\emptyset)) + \lambda_2[\sigma^{n-1}\Pi^H(p(H)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^H(p(\emptyset))] \\ & + \lambda_3(p(\emptyset) - p(L)). \end{aligned}$$

For $p(\emptyset) \leq a - \Delta$ the resulting Kuhn-Tucker conditions are given by

$$\frac{\partial \mathcal{L}}{\partial p(\emptyset)} = \rho(1 - \sigma^n) \frac{\partial \Pi^H}{\partial p(\emptyset)} + (1 - \rho)(1 - \sigma)^n \frac{\partial \Pi^L}{\partial p(\emptyset)} - \lambda_1 - \lambda_2 \frac{1 - \sigma^n}{1 - \sigma} \frac{\partial \Pi^H}{\partial p(\emptyset)} + \lambda_3 = 0, \quad (\text{A-1})$$

$$\frac{\partial \mathcal{L}}{\partial p(H)} = \rho\sigma^n \frac{\partial \Pi^H}{\partial p(H)} + \lambda_1 + \lambda_2\sigma^{n-1} \frac{\partial \Pi^H}{\partial p(H)} = 0, \quad (\text{A-2})$$

$$\frac{\partial \mathcal{L}}{\partial p(L)} = (1 - \rho)(1 - (1 - \sigma)^n) \frac{\partial \Pi^L}{\partial p(L)} - \lambda_3 = 0, \quad (\text{A-3})$$

$$\lambda_1(p(H) - p(\emptyset)) = 0, \lambda_1 \neq 0, \quad (\text{A-4})$$

$$\lambda_2(\sigma^{n-1}\Pi^H(p(H)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^H(p(\emptyset))) = 0, \lambda_2 \geq 0, \quad (\text{A-5})$$

$$\lambda_3(p(\emptyset) - p(L)) = 0, \lambda_3 \geq 0. \quad (\text{A-6})$$

Only one of the two constraints (OSH-0) or (OSH-1) can be binding at a time, i.e. $\lambda_1\lambda_2 = 0$. This means we have to distinguish six cases.

A.1. $\lambda_1 = \lambda_2 = \lambda_3 = 0$, i.e. none of the three constraints are binding. This yields the unconstrained solution $p^*(s)$ that would result if firms would be able to observe their rivals' signals. From (A-2) and (A-3) we get

$$\begin{aligned} \frac{\partial \Pi^H}{\partial p(H)} = 0 & \iff p^*(H) = \frac{a + \Delta}{2}, \\ \frac{\partial \Pi^L}{\partial p(L)} = 0 & \iff p^*(L) = \frac{a - \Delta}{2} \end{aligned}$$

and from (A-1), $\rho(1 - \sigma^n) \frac{\partial \Pi^H}{\partial p(\emptyset)} + (1 - \rho)(1 - \sigma)^n \frac{\partial \Pi^L}{\partial p(\emptyset)} = 0$, follows

$$p^*(\emptyset) = \frac{a + \Delta}{2} - \frac{\Delta(1 - \rho)(1 - \sigma)^n}{(1 - \rho)(1 - \sigma)^n + \rho(1 - \sigma^n)}$$

To check whether this solution indeed satisfies (OSH-1), note that the maximum value $p(\emptyset)$ for which (OSH-1) holds is at $p(H) = p^*(H)$ and is given by

$$\bar{p}(\emptyset)^{OSH1} \equiv \frac{a + \Delta}{2}(1 - \Upsilon) \text{ with } \Upsilon \equiv \sqrt{\frac{1 - \sigma^{n-1}}{1 - \sigma^n}}.$$

For future reference, verify that $\partial \Upsilon / \partial \sigma < 0$, $\partial \Upsilon / \partial n > 0$ and $\Upsilon \in [1/\sqrt{2}, 1]$. Constraint (OSH-1) holds if $p^*(\emptyset) \leq \bar{p}(\emptyset)^{OSH1}$ or

$$\rho \leq \frac{(1 - \sigma^n)[(2 - \Upsilon)\Delta - a\Upsilon]}{(a + \Delta)(1 - \sigma^n - (1 - \sigma)^n)\Upsilon + 2\Delta(1 - \sigma)^n}.$$

For $\Delta \leq a/3$ the numerator is always negative and the above solution is not feasible.

A.2. $\lambda_1 = \lambda_2 = 0, \lambda_3 > 0$, i.e. (OSH) is slack and (OSL-0) is strictly binding. From (A-2) follows $p(H) = p^*(H)$. From (A-1),(A-3) and $p(\phi) = p(L)$ one gets

$$\rho(1 - \sigma^n) \frac{\partial \Pi^H}{\partial p(\phi)} + (1 - \rho) \frac{\partial \Pi^L}{\partial p(\phi)} = 0,$$

which implies that $p(\phi) = p(L) > p^*(L)$. This means that $\partial \Pi^L / \partial p(\phi) < 0$ and from (A-3) follows that $\lambda_3 < 0$. The solution is thus not feasible.

A.3. $\lambda_1 \neq 0, \lambda_2 = \lambda_3 = 0$, i.e. (OSH-0) is binding and (OSL) is slack. From (A-3) follows $p(L) = p^*(L)$. From (A-1),(A-2) and $p(\phi) = p(L)$ I find

$$\rho \frac{\partial \Pi^H}{\partial p(\phi)} + (1 - \rho)(1 - \sigma)^n \frac{\partial \Pi^L}{\partial p(\phi)} = 0, \quad (5)$$

or

$$p(H) = p(\phi) = \frac{a - \Delta}{2} + \frac{\rho \Delta}{\rho + (1 - \rho)(1 - \sigma)^n}.$$

Since $p_3(\phi) \geq p^*(L)$, this solution is viable for all parameter values. The industry profits in this case are

$$E[\Pi^{A3}] = \frac{1}{4}(a - \Delta)(a - \Delta + 4\rho\Delta) + \frac{\rho^2 \Delta^2}{\rho + (1 - \rho)(1 - \sigma)^n}.$$

A.4. $\lambda_1 \neq 0, \lambda_2 = 0, \lambda_3 > 0$, i.e. (OSL-0) and (OSH-0) are binding. Plugging (A-2) and (A-3) in (A-1), one gets

$$\rho \frac{\partial \Pi^H}{\partial p(\phi)} + (1 - \rho) \frac{\partial \Pi^L}{\partial p(\phi)} = 0,$$

which implies $p(\phi) = p(L) = p(H) > p^*(L)$ and by (A-3) a $\lambda_3 < 0$. The solution is therefore dominated by an interior solution $p(L) < p(\phi)$.

A.5. $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 = 0$, i.e. (OSH-1) is binding and (OSL) is slack. From (A-3) follows $p(L) = p^*(L)$ and from (A-2)

$$(\rho\sigma^n + \lambda_2\sigma^{n-1}) \frac{\partial \Pi^H}{\partial p(H)} = 0 \iff p(H) = p^*(H)$$

Plugging $p(H)$ in (A-5) yields $p(\phi) = \bar{p}(\phi)^{OSH1} = (a + \Delta)(1 - \Upsilon)/2$. In order to satisfy (OSL-1) it has to hold that $\bar{p}(\phi)^{OSH1} \geq p^*(L)$ or

$$\Delta - \frac{(a + \Delta)\Upsilon}{2} \geq 0.$$

The left-hand side of this inequality is increasing in Δ . At $\Delta = a/3$ it has a value of $a(1 - 2\Upsilon)/3 < 0$. Thus, $\bar{p}(\phi)^{OSH1} < p^*(L)$ and the solution is not feasible.

A.6. $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 > 0$, i.e. (OSH-1) and (OSL-0) are strictly binding. From (A-2) follows $p(H) = p^*(H)$ which implies with (A-5) and (A-6) that $p(\phi) = p(L) = \bar{p}(\phi)^{OSH1}$. We

know from case A.5. that $\bar{p}(\phi)^{OSH1} < p^*(L)$. Therefore, (A-3) implies that $\lambda_3 > 0$ and from rewriting (A-1) to

$$\rho(1 - \sigma^n) \frac{\partial \Pi^H}{\partial p(\phi)} + (1 - \rho) \frac{\partial \Pi^L}{\partial p(\phi)} - \lambda_2 \frac{1 - \sigma^n}{1 - \sigma} \frac{\partial \Pi^H}{\partial p(\phi)} = 0$$

follows that $\lambda_2 > 0$. Thus, this solution is a candidate maximizer of the problem. The expected profits in this case are

$$E[\Pi^{A6}] = \frac{a + \Delta}{4} (a - 3\Delta + 4\Delta(\rho + (1 - \rho)\Upsilon) - (1 - \rho\sigma^n)(a + \Delta)\Upsilon^2).$$

A.7. Suppose $p(\phi) \geq a - \Delta$, i.e. $\Pi^L(p(\phi)) = 0$. This changes (A-1) into

$$\frac{\partial \mathcal{L}}{\partial p(H)} = \rho\sigma^n \frac{\partial \Pi^H}{\partial p(H)} + \lambda_1 + \lambda_2\sigma^{n-1} \frac{\partial \Pi^H}{\partial p(H)} = 0.$$

It follows that the solution to the unconstrained maximization problem (i.e. $\lambda_1 = \lambda_2 = \lambda_3 = 0$) is given by $p(H) = p(\phi) = p^*(H)$ and $p(L) = p^*(L)$. Since $\Delta \leq a/3$, it holds that $p(\phi) \geq a - \Delta$. Thus, the unconstrained solution is a candidate maximizer. The corresponding expected industry profits are

$$E[\Pi^{A7}] = (1 - (1 - \rho)(1 - \sigma)^n) \frac{(a - \Delta)^2}{4} + \rho a \Delta$$

Finally, we need to check that the omitted constraint (OSL-1) is satisfied in the three candidate solutions. The minimum value $p(L)$ for which (OSL-1) holds for any $p(\phi)$ is at $p(\phi) = p^*(L)$ and is given by

$$\underline{p}(L)^{OSL1} \equiv \frac{a - \Delta}{2} \left(1 - \sqrt{\frac{1 - \sigma - (1 - \sigma)^n}{(1 - (1 - \sigma)^n)(1 - \sigma)}} \right) \leq p^*(L).$$

Thus, (OSL-1) is always satisfied.

B. $p(L) \leq p(\phi) \wedge p(H) \leq p(\phi)$. Omitting (OSL-1) and (OSH-2) for the moment, the Lagrangian can be written as

$$\mathcal{L}(p(\mathcal{I}), \lambda) = E\Pi_\phi(p(\mathcal{I})) + \lambda_1(p(\phi) - p(H)) + \lambda_2(p(\phi) - p(L)).$$

For $p(\phi) \leq a - \Delta$ the resulting Kuhn-Tucker conditions are given by

$$\frac{\partial \mathcal{L}}{\partial p(\phi)} = \rho(1 - \sigma)^n \frac{\partial \Pi^H}{\partial p(\phi)} + (1 - \rho)(1 - \sigma)^n \frac{\partial \Pi^L}{\partial p(\phi)} + \lambda_1 + \lambda_2 = 0, \quad (\text{B-1})$$

$$\frac{\partial \mathcal{L}}{\partial p(H)} = \rho(1 - (1 - \sigma)^n) \frac{\partial \Pi^H}{\partial p(H)} - \lambda_1 = 0, \quad (\text{B-2})$$

$$\frac{\partial \mathcal{L}}{\partial p(L)} = (1 - \rho)(1 - (1 - \sigma)^n) \frac{\partial \Pi^L}{\partial p(L)} - \lambda_2 = 0, \quad (\text{B-3})$$

$$\lambda_1(p(\phi) - p(H)) = 0, \lambda_1 \geq 0, \quad (\text{B-4})$$

$$\lambda_2(p(\phi) - p(L)) = 0, \lambda_2 \geq 0, \quad (\text{B-5})$$

We have to distinguish four cases. The two cases with (OSH-0) binding are identical to A.3 and A.4 respectively. Moreover, it is straightforward to check that the case of $p(\emptyset) \geq a - \Delta$ is identical to A.7. The two remaining cases are:

B.1. $\lambda_1 = \lambda_2 = 0$, i.e. none of the two constraints are binding. This means $p(L) = p^*(L)$ and $p(H) = p^*(H)$. However, (B-1) requires $p^*(L) < p(\emptyset) < p^*(H)$, which is incompatible with $p(\emptyset) > p(H)$. Thus, this solution is not feasible.

B.2. $\lambda_1 = 0, \lambda_2 > 0$, i.e. (OSL-0) is strictly binding. This implies $p(H) = p^*(H)$. Plugging (B-3) in (B-1) shows that $p(\emptyset) = p(L) > p^*(L)$ which implies from (B-3) that $\lambda_2 < 0$. This solution is not feasible.

C. $p(L) \geq p(\emptyset) \wedge p(H) \geq p(\emptyset)$. The Lagrangian can be written as

$$\begin{aligned} \mathcal{L}(p(\mathcal{I}), \lambda) = & E\Pi_\emptyset(p(\mathcal{I})) + \lambda_1(p(H) - p(\emptyset)) + \lambda_2(\sigma^{n-1}\Pi^H(p(H)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^H(p(\emptyset))) \\ & + \lambda_3(p(L) - p(\emptyset)) + \lambda_4(\sigma^{n-1}\Pi^L(p(L)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^L(p(\emptyset))). \end{aligned}$$

For $p(\emptyset) \leq a - \Delta$ the resulting Kuhn-Tucker conditions are given by

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial p(\emptyset)} = & \rho(1 - \sigma^n) \frac{\partial \Pi^H}{\partial p(\emptyset)} + (1 - \rho)(1 - \sigma^n) \frac{\partial \Pi^L}{\partial p(\emptyset)} - \lambda_1 - \lambda_2 \frac{1 - \sigma^n}{1 - \sigma} \frac{\partial \Pi^H}{\partial p(\emptyset)} \\ & - \lambda_3 - \lambda_4 \frac{1 - \sigma^n}{1 - \sigma} \frac{\partial \Pi^L}{\partial p(\emptyset)} = 0, \end{aligned} \quad (\text{C-1})$$

$$\frac{\partial \mathcal{L}}{\partial p(H)} = \rho\sigma^n \frac{\partial \Pi^H}{\partial p(H)} + \lambda_1 + \lambda_2\sigma^{n-1} \frac{\partial \Pi^H}{\partial p(H)} = 0, \quad (\text{C-2})$$

$$\frac{\partial \mathcal{L}}{\partial p(L)} = (1 - \rho)\sigma^n \frac{\partial \Pi^L}{\partial p(L)} + \lambda_3 + \lambda_4(\sigma^{n-1}) \frac{\partial \Pi^L}{\partial p(L)} = 0, \quad (\text{C-3})$$

$$\lambda_1(p(H) - p(\emptyset)) = 0, \lambda_1 \neq 0, \quad (\text{C-4})$$

$$\lambda_2(\sigma^{n-1}\Pi^H(p(H)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^H(p(\emptyset))) = 0, \lambda_2 \geq 0, \quad (\text{C-5})$$

$$\lambda_3(p(L) - p(\emptyset)) = 0, \lambda_3 \neq 0, \quad (\text{C-6})$$

$$\lambda_4(\sigma^{n-1}\Pi^L(p(L)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^L(p(\emptyset))) = 0, \lambda_4 \geq 0. \quad (\text{C-7})$$

Again it has to hold that $\lambda_1\lambda_2 = 0$ which leaves us with nine possible cases. The cases in which (OSL-0) is strictly binding have been dealt with in A.2, A.4 and A.6 respectively. Thus, we have to consider the remaining cases with $\lambda_3 = 0$.

C.1. $\lambda_1 = \lambda_2 = \lambda_4 = 0$, i.e. none of the three constraints are binding. This means $p(L) = p^*(L)$ and $p(H) = p^*(H)$. However, (C-1) requires $p^*(L) < p(\emptyset) < p^*(H)$, which is incompatible with $p(L) > p(\emptyset)$.

C.2. $\lambda_1 = \lambda_2 = 0, \lambda_4 > 0$, i.e. (OSL-2) is strictly binding. From (C-2) and (C-3) follows $p(H) = p^*(H)$ and $p(L) = p^*(L)$. For (C-7) to hold strictly

$$\sigma^{n-1}\Pi^L(p^*(L)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^L(p(\emptyset)) = 0 \iff p(\emptyset) = \frac{a - \Delta}{2}(1 - \Upsilon).$$

Since $p(\phi) < p^*(L)$ it follows from (C-1) that $\lambda_4 > 0$. Moreover, $p(\phi) < \bar{p}(\phi)^{OSH1}$ and therefore (OSH-1) is satisfied. The corresponding expected industry profits for this solution are

$$E[\Pi^{C2}] = \frac{a - \Delta}{4}(a - \Delta + 4\rho\Delta - 4\rho(1 - \sigma^n)\Delta\Upsilon - (1 - \sigma^n)(a - \Delta)\Upsilon^2).$$

C.3. $\lambda_1 = 0, \lambda_2 > 0, \lambda_4 = 0$, i.e. (OSH-1) is strictly binding. From (C-2) and (C-3) follows $p(H) = p^*(H)$ and $p(L) = p^*(L)$. From (C-5) one gets $p(\phi) = (a + \Delta)(1 - \Upsilon)/2$, which implies that (OSL-2) cannot be satisfied.

C.4. $\lambda_1 = 0, \lambda_2 > 0, \lambda_4 > 0$, i.e. (OSL-2) is strictly binding. From (C-2) and (C-3) follows $p(H) = p^*(H)$ and $p(L) = p^*(L)$. Then (C-5) and (C-7) cannot hold simultaneously.

C.5. $\lambda_1 \neq 0, \lambda_2 = 0, \lambda_4 = 0$, i.e. (OSH-0) is strictly binding. From (C-3) follows $p(L) = p^*(L)$. Conditions (C-1), (C-2) and (C-4) imply that $p(\phi) > p^*(L)$ which violates (OSL-2).

C.6. $\lambda_1 \neq 0, \lambda_2 = 0, \lambda_4 > 0$, i.e. (OSH-0) and (OSL-2) are strictly binding. From (C-3) follows $p(L) = p^*(L)$ and from (C-7) follows $p(\phi) = p(H) = (a + \Delta)(1 - \Upsilon)/2$. This solution, however, is always weakly dominated by the solution in C.2 which has one binding constraint less.

C.7. $p(\phi) > a - \Delta$, i.e. $\Pi^L(p(\phi)) = 0$. For these prices, (OSH-1) and (OSL-2) cannot be satisfied. This means it has to hold that $p(L) = p(H) = p(\phi) = a - \Delta$ which is always dominated by the solution in A.7.

D. $p(L) \geq p(\phi) \wedge p(H) \leq p(\phi)$. The Lagrangian can be written as

$$\begin{aligned} \mathcal{L}(p(\mathcal{I}), \lambda) = & E\Pi_\phi(p(\mathcal{I})) + \lambda_1(p(\phi) - p(H)) + \lambda_2(p(\phi) - p(L)) \\ & + \lambda_3(\sigma^{n-1}\Pi^L(p(L)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^L(p(\phi))) \end{aligned}$$

For $p(\phi) \leq a - \Delta$ the resulting Kuhn-Tucker conditions are given by

$$\frac{\partial \mathcal{L}}{\partial p(\phi)} = \rho(1 - \sigma)^n \frac{\partial \Pi^H}{\partial p(\phi)} + (1 - \rho)(1 - \sigma^n) \frac{\partial \Pi^L}{\partial p(\phi)} + \lambda_1 + \lambda_2 - \lambda_3 \frac{1 - \sigma^n}{1 - \sigma} \frac{\partial \Pi^L}{\partial p(\phi)} = 0, \quad (\text{D-1})$$

$$\frac{\partial \mathcal{L}}{\partial p(H)} = \rho(1 - (1 - \sigma)^n) \frac{\partial \Pi^H}{\partial p(H)} - \lambda_1 = 0, \quad (\text{D-2})$$

$$\frac{\partial \mathcal{L}}{\partial p(L)} = (1 - \rho)\sigma^n \frac{\partial \Pi^L}{\partial p(L)} - \lambda_2 + \lambda_3(\sigma^{n-1}) \frac{\partial \Pi^L}{\partial p(L)} = 0, \quad (\text{D-3})$$

$$\lambda_1(p(\phi) - p(H)) = 0, \lambda_1 \geq 0, \quad (\text{D-4})$$

$$\lambda_2(p(\phi) - p(L)) = 0, \lambda_2 \neq 0, \quad (\text{D-5})$$

$$\lambda_3(\sigma^{n-1}\Pi^L(p(L)) - \frac{1 - \sigma^n}{1 - \sigma}\Pi^L(p(\phi))) = 0, \lambda_3 \geq 0. \quad (\text{D-6})$$

Additionally it has to hold that $\lambda_2\lambda_3 = 0$. The cases in which (OSH-0) is strictly binding have been dealt with in C.5, C.6 and A.4. Thus, we have to consider the remaining cases with $\lambda_1 = 0$.

D.1. $\lambda_1 = 0, \lambda_2 = \lambda_3 = 0$, i.e. none of the three constraints are binding. This means $p(L) = p^*(L)$ and $p(H) = p^*(H)$. However, (D-1) requires $p^*(L) < p(\phi) < p^*(H)$ which is

incompatible with $p(\phi) > p(H)$.

D.2. $\lambda_1 = \lambda_2 = 0, \lambda_3 > 0$, i.e. (OSL-2) is strictly binding. From (D-2) and (D-3) follows $p(H) = p^*(H)$ and $p(L) = p^*(L)$. From (D-6) one gets $p(\phi) = (a - \Delta)(1 - \Upsilon)/2$ which contradicts $p(\phi) > p(H)$.

D.3. $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 = 0$, i.e. (OSL-0) is strictly binding. From (D-2) follows $p(H) = p^*(H)$. Combining (D-1) and (D-3) implies $p^*(L) < p(\phi) < p^*(H)$ which contradicts $p(\phi) > p(H)$.

D.4. $p(\phi) > a - \Delta$, i.e. $\Pi^L(p(\phi)) = 0$. For these prices (OSL-2) cannot be satisfied, i.e. $\lambda_2 \neq 0$. Two cases exist. First, if $\lambda_1 = 0$ then $p(H) = p^*(H)$. But (D-1) and (D-3) imply $p(\phi) < p^*(H)$ which contradicts $p(\phi) > p(H)$. Second, if $\lambda_1 > 0$ then $p(L) = p(\phi) = p(H) = a - \Delta$ which is dominated by A.7.

The preceding analysis leaves us with four candidate solutions: A.3, A.6, A.7 and C.2. In what follows I show that (i) A.7 dominates C.2, (ii) A.7 dominates A.6 and (iii) A.3 dominates A.7.

(i) A.7 dominates C.2 First consider the difference $\Psi_1 \equiv E[\Pi^{A7}] - E[\Pi^{C2}]$ and check that $(\partial^2 \Psi_1)/(\partial \rho)^2 = 0$ which means that Ψ_1 is either monotonically increasing or decreasing. Further calculate the value of Ψ_1 at $\rho = 0$,

$$\Psi_1(\rho = 0) = \frac{(a - \Delta)^2}{4}((1 - \sigma^n)\Upsilon^2 - (1 - \sigma)^n) = \frac{(a - \Delta)^2}{4}(1 - \sigma^{n-1} - (1 - \sigma)^n) \geq 0,$$

and at $\rho = 1$, $\Psi_1(\rho = 1) = \frac{1 - \sigma^n}{4}(\Delta(2 - \Upsilon) + a\Upsilon)^2 \geq 0$. It follows that $E[\Pi^{A7}] \geq E[\Pi^{C2}]$.

(ii) A.7 dominates A.6. Define the difference between the expected industry profits as $\Psi_2 \equiv E[\Pi^{A7}] - E[\Pi^{C2}]$. To show that $\Psi_2 \geq 0$ I proceed in three steps. First, verify that Ψ_2 is either increasing or decreasing in ρ since $(\partial^2 \Psi_2)/(\partial \rho)^2 = 0$. Second, the value of Ψ_2 at $\rho = 1$ is $\Psi_2(\rho = 1) = (a + \Delta)^2(1 - \sigma^n)\Upsilon^2/4 \geq 0$. The last step is to show that

$$\Psi_2(\rho = 0) = \frac{1}{4}[(\Delta(2 - \Upsilon) - a\Upsilon)^2 - (1 - \sigma)^n(a - \Delta)^2] \geq 0.$$

To prove this, I show that the following three sufficient conditions hold: (a) $\Psi_2(\rho = 0)$ is convex in Δ , (b) it has a positive value at its upper bound $\Delta = a/3$ and (c) it has a negative slope at its upper bound $\Delta = a/3$. The second derivative with respect to Δ is

$$\Psi_2'' \equiv \frac{\partial^2 \Psi_2(\rho = 0)}{(\partial \Delta)^2} = \frac{1}{2}((2 - \Upsilon)^2 - (1 - \sigma)^n).$$

Taking the derivative of this expression with respect to σ gives

$$\begin{aligned} \frac{\partial \Psi_2''}{\partial \sigma} &= \frac{1}{2}(n(1 - \sigma)^{n-1} - 2(2 - \Upsilon)\frac{\partial \Upsilon}{\partial \sigma}) \\ &= \frac{1}{2}(n(1 - \sigma)^{n-1} + \frac{s^{n-2}(2 - \Upsilon)(n - 1 - n\sigma + \sigma^n)}{(1 - \sigma^n)^2 \Upsilon}) \geq 0. \end{aligned}$$

Therefore Ψ_2'' takes its lowest value at $\sigma = 0$ which is $\Psi_2''(\sigma = 0) = 0$. Thus, $\Psi_2(\rho = 0)$ is convex in Δ . The value of $\Psi_2(\rho = 0)$ at its upper bound $\Delta = a/3$ is

$$\Psi_2^\Delta \equiv \frac{a^2}{9}(1 - (1 - \sigma)^n - 4(1 - \Upsilon)\Upsilon).$$

Taking the derivative of this expression with respect to n gives

$$\frac{\partial \Psi_2^\Delta}{\partial n} = -\frac{a^2}{9}[(1-\sigma)^n \ln(1-\sigma) - (8\Upsilon - 4)\frac{\partial \Upsilon}{\partial n}] \geq 0,$$

which means that Ψ_2^Δ takes its minimum value at $n = 2$. At $n = 2$ the non-negativity condition for Ψ_2^Δ becomes

$$1 - (1-\sigma)^2 - 4\left(1 - \frac{1}{\sqrt{1+\sigma}}\right)\frac{1}{\sqrt{1+\sigma}} \geq 0,$$

which holds for all $\sigma \in [0, 1]$. Thus, $\Psi_2(\rho = 0)$ has a positive value at $\Delta = a/3$. Finally, the slope of $\Psi_2(\rho = 0)$ at $\Delta = a/3$ is

$$\left.\frac{\partial \Psi_2(\rho = 0)}{\partial \Delta}\right|_{\Delta=a/3} = \frac{a}{3}(2 + (1-\sigma)^n - \Upsilon(5 - 2\Upsilon)).$$

This expression is negative if $\Lambda \equiv \Upsilon(5 - 2\Upsilon) - 2 - (1-\sigma)^n \geq 0$. Check that Λ is increasing in n since

$$\frac{\partial \Lambda}{\partial n} = -(1-\sigma)^n \ln(1-\sigma) + (5 - 4\Upsilon)\frac{\partial \Upsilon}{\partial n} \geq 0.$$

Therefore, Λ takes a minimum value at $n = 2$,

$$\Lambda(n = 2) = (2 - \sigma)\sigma + \frac{5}{\sqrt{1+\sigma}} - 3 - \frac{2}{1+\sigma},$$

which is positive for any $\sigma \in [0, 1]$ since $\Lambda(n = 2)$ is concave in σ and takes a value of 0 at $\sigma = 0$ and of $5/\sqrt{2} - 3 > 0$ at $\sigma = 1$. Thus, the slope of $\Psi_2(\rho = 0)$ at $\Delta = a/3$ is negative.

(iii) **A.3 dominates A.7.** The difference between the expected industry profits is given by

$$E[\Pi^{A3}] - E[\Pi^{A7}] = \frac{(1-\rho)(1-\sigma^n)[(1-\rho)(1-\sigma)^n(a-\Delta)^2 + \rho(a-3\Delta)(a+\Delta)]}{4(\rho + (1-\rho)(1-\sigma)^n)},$$

which is positive for all $\Delta < a/3$.

The only effective constraint for the solution A.3 is (OSH-0). It is straightforward to check that the bordered Hessian for this local maximum is negative definite. Thus, the second-order condition for a global maximum is satisfied and the lemma follows. ■

Proof of Proposition 1

Firms apply a strategy that punishes out-of equilibrium messages and prices with eternal reversion to marginal cost pricing. This implies that at the communication stage sending an uninformative message $m_i \neq \emptyset$ is a (weakly) dominant strategy. Then consider the price setting subgame following the communication of $m_i = \emptyset \forall i$. The best deviation for a firm with a L -signal from its equilibrium price $p_\emptyset(L)$ is to undercut slightly at $p_\emptyset(L) - \epsilon$. Thus, it has to hold that

$$\frac{1 - (1-\sigma)^n}{\sigma n} \Pi^L(p_\emptyset(L)) + \frac{\delta V}{1-\delta} \geq \Pi^L(p_\emptyset(L))$$

or

$$\frac{\delta V}{1 - \delta} \geq (1 - \frac{1 - (1 - \sigma)^n}{\sigma n}) \Pi^L(p_\phi(L)), \quad (6)$$

where $V \equiv E\Pi_\phi(p_\phi(\mathcal{I}))/n$ is the *ex ante* expected, per-period profit in equilibrium.

A firm with a H-signal does not deviate if

$$\frac{\delta V}{1 - \delta} \geq \Pi^H(p_\phi(H)) - \frac{\Pi^H(p_\phi(H))}{n} \iff \delta \geq \delta_\phi(H) \equiv \frac{(n - 1)\Pi^H(p_\phi(H))}{nV + (n - 1)\Pi^H(p_\phi(H))} \quad (7)$$

Condition (7) implies condition (6) if

$$(1 - \frac{1}{n})\Pi^H(p_\phi(H)) \geq (1 - \frac{1 - (1 - \sigma)^n}{\sigma n})\Pi^L(p_\phi(L)).$$

This inequality always holds because $\Pi^H(p_\phi(H)) > \Pi^L(p_\phi(L))$ since $\Pi^H(p_\phi(L)) > \Pi^L(p_\phi(L))$ and $p_\phi(L) < p_\phi(H) < p^*(H)$. Moreover, $(1 - (1 - \sigma)^n)/(\sigma n)$ is decreasing in σ and always greater or equal than $1/n$. Therefore, the incentive-compatibility condition for a L-signal firm is always satisfied for $\delta \geq \delta_\phi(H)$.

A firm with a ϕ -signal expects an equilibrium profit in the current period of

$$\rho \frac{\Pi^H(p_\phi(\phi))}{n} + (1 - \rho)(1 - \sigma)^{n-1} \frac{\Pi^L(p_\phi(\phi))}{n}$$

There are three types of price deviations. First consider prices below $p_\phi(L)$. The most profitable deviation is $p_\phi(L) - \epsilon$ which yields expected deviation profits of

$$\rho \Pi^H(p_\phi(L)) + (1 - \rho)\Pi^L(p_\phi(L)).$$

A deviation to exactly $p_\phi(L)$ is only semi-detectable because if the demand state is indeed low, the firm behaved like it had gotten a low signal and no punishment is triggered. Deviation profits are thus

$$\rho \Pi^H(p_\phi(L)) + (1 - \rho) \left[\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^L(p_\phi(L)) + \frac{\delta V}{1 - \delta} \right]$$

and deviation is not profitable if

$$\begin{aligned} \frac{\delta V}{1 - \delta} &\geq \rho \left[\Pi^H(p_\phi(L)) - \frac{\Pi^H(p_\phi(\phi))}{n} \right] \\ &\quad + (1 - \rho) \left[\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^L(p_\phi(L)) + \frac{\delta V}{1 - \delta} - (1 - \sigma)^{n-1} \frac{\Pi^L(p_\phi(\phi))}{n} \right] \end{aligned} \quad (8)$$

or

$$\delta \geq \delta_\phi(\phi) \equiv \frac{\Theta}{b_i(\mathcal{I}_i = \phi)V + \Theta},$$

with

$$\Theta \equiv \rho \left[\Pi^H(p_\phi(L)) - \frac{\Pi^H(p_\phi(\phi))}{n} \right] + (1 - \rho) \left[\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^L(p_\phi(L)) - (1 - \sigma)^{n-1} \frac{\Pi^L(p_\phi(\phi))}{n} \right].$$

It is easily checked that the deviation to $p_\phi(L)$ is more profitable than the deviation to $p_\phi(L) - \epsilon$ if

$$\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^L(p_\phi(L)) + \frac{\delta V}{1 - \delta} \geq \Pi^L(p_\phi(L)),$$

which is exactly the incentive compatibility condition for a firm with a low signal which holds for $\delta \geq \delta_\phi(H)$. Finally, a firm with a ϕ -signal could deviate to a price p^D with $p(L) < p^D < p(\phi)$. This gives expected profits of

$$\rho \Pi^H(p^D) + (1 - \rho)(1 - \sigma)^{n-1} \Pi^L(p^D).$$

The resulting incentive compatibility constraint is implied by condition (7) if

$$\frac{n-1}{n} \Pi^H(p_\phi(\phi)) \geq \rho [\Pi^H(p^D) - \frac{\Pi^H(p_\phi(\phi))}{n}] + (1 - \rho)(1 - \sigma)^{n-1} [\Pi^L(p^D) - \frac{\Pi^L(p_\phi(\phi))}{n}]$$

or

$$\rho [\Pi^H(p_\phi(\phi)) - \Pi^H(p^D)] + (1 - \rho)(1 - \sigma^{n-1}) [\frac{n-1}{n(1 - \sigma^{n-1})} \Pi^H(p_\phi(\phi)) - \Pi^L(p^D) + \frac{\Pi^L(p_\phi(\phi))}{n}] \geq 0$$

The expression in the first bracket is positive since $p^D < p_\phi(\phi) < p^*(H)$. The second bracket is positive if

$$\frac{n-1}{n} \Pi^H(p_\phi(\phi)) + \frac{\Pi^L(p_\phi(\phi))}{n} - \Pi^L(p^D) \geq 0.$$

This inequality holds since the sum of the first two terms is increasing in the price $p_\phi(\phi)$ and at $p_\phi(\phi) = p^D$ the inequality holds strictly. This leaves us with at most two necessary conditions, (7) and (8). Check that for $\rho \rightarrow 1$, the right-hand side ('rhs') of (8) reduces to $\Pi^H(p_\phi(L)) - \Pi^H(p_\phi(\phi))/n$ which is always smaller than the rhs of (7). On the other hand, for $\delta \rightarrow 1$ the rhs of (8) goes to infinity. Therefore, there exist parameter values for which each of the two conditions is more restrictive than the other one and the proposition follows. ■

Proof of Lemma 2

The optimal prices follow from maximizing $E\Pi_{LH}(p(\mathcal{I}))$ w.r.t. $p(\mathcal{I})$:

$$\begin{aligned} \frac{\partial E\Pi_{LH}}{\partial p(\phi)} &= \rho(1 - \sigma)^n \frac{\partial \Pi^H}{\partial p(\phi)} + (1 - \rho)(1 - \sigma)^n \frac{\partial \Pi^L}{\partial p(\phi)} = 0, \\ \frac{\partial E\Pi_{LH}}{\partial p(H)} &= \rho(1 - \sigma)^n \frac{\partial \Pi^H}{\partial p(H)} = 0, \\ \frac{\partial E\Pi_{LH}}{\partial p(L)} &= (1 - \rho)(1 - \sigma)^n \frac{\partial \Pi^L}{\partial p(L)} = 0. \quad \blacksquare \end{aligned} \tag{9}$$

Proof of Proposition 2

After the communication stage there are three possible types of price subgames on the equilibrium path. Consider the price subgame following a verified message $m_i = H$ of at least one firm. This sets all firms in information state $\mathcal{I}_i = H$ and a firm charges its equilibrium price $p_c(H)$ if

$$\frac{\Pi^H(p_c(H))}{n} + \frac{\delta V}{1 - \delta} \geq \Pi^H(p_c(H)) \quad (10)$$

where $V \equiv E\Pi_{LH}(p_c(\mathcal{I}))/n$ is the *ex ante* expected, per-period profit in equilibrium. With at least one L-message, firms stick to their equilibrium price $p_c(L)$ if

$$\frac{\Pi^L(p_c(L))}{n} + \frac{\delta V}{1 - \delta} \geq \Pi^L(p_c(L)). \quad (11)$$

If all firms receive uninformative signals and messages, firms set $p_c(\emptyset)$ if

$$\frac{\rho\Pi^L(p_c(\emptyset)) + (1 - \rho)\Pi^H(p_c(\emptyset))}{n} + \frac{\delta V}{1 - \delta} \geq \rho\Pi^L(p_c(\emptyset)) + (1 - \rho)\Pi^H(p_c(\emptyset)). \quad (12)$$

It is straightforward to see that (10) implies (11) and (12). Rewriting (10) yields

$$\delta \geq \delta_c(H) \equiv \frac{(n - 1)\Pi^H(p_c(H))}{nV + (n - 1)\Pi^H(p_c(H))}.$$

Additionally suppose for all out-of-equilibrium outcomes of the communication stage (i.e. communication including unverified L - or H -messages) that firms revert to marginal cost pricing from the current period onwards. Then at the communication stage, firms with \emptyset -signals cannot do better than truthfully report their signal. Suppose firm i receives a H -signal and deviates to $m_i = \emptyset$. This is an on-schedule deviation. If no other firm received an informative signal, all remaining firms $j \in N, j \neq i$ are in information state $\mathcal{I}_j = \emptyset$ and set price $p_c(\emptyset)$. The best deviation for firm i is $p_c(\emptyset) - \epsilon$. However, this deviation is always dominated by setting $p_c(\emptyset)$ if

$$\frac{\Pi^H(p_c(\emptyset))}{n} + \frac{\delta V}{1 - \delta} \geq \Pi^H(p_c(\emptyset)).$$

which always holds for $\delta \geq \delta_c(H)$. This means that at the communication stage a firm i with a H -signal sending $m_i = \emptyset$ expects profits of

$$(1 - \sigma)^{n-1} \left(\frac{\Pi^H(p_c(\emptyset))}{n} + \frac{\delta V}{1 - \delta} \right) + (1 - (1 - \sigma)^{n-1}) \left(\frac{\Pi^H(p_c(H))}{n} + \frac{\delta V}{1 - \delta} \right)$$

which is strictly dominated by sending $m_i = H$ and receiving

$$\frac{\Pi^H(p_c(H))}{n} + \frac{\delta V}{1 - \delta}.$$

Similarly if a firm i with a L -signal deviates to $m_i = \emptyset$ and no other firm receives and communicates a L -signal, firm i would never find it optimal to undercut its rivals at the pricing stage since

$$\frac{\Pi^L(p_c(\emptyset))}{n} + \frac{\delta V}{1 - \delta} \geq \Pi^L(p_c(\emptyset))$$

is always satisfied for $\delta \geq \delta_c(H)$. Thus, sending $m_i = \emptyset$ yields

$$(1 - \sigma)^{n-1} \left(\frac{\Pi^L(p_c(\emptyset))}{n} + \frac{\delta V}{1 - \delta} \right) + (1 - (1 - \sigma)^{n-1}) \left(\frac{\Pi^L(p_c(L))}{n} + \frac{\delta V}{1 - \delta} \right)$$

which is always dominated by sending $m_i = L$ and receiving

$$\frac{\Pi^L(p_c(L))}{n} + \frac{\delta V}{1 - \delta}. \quad \blacksquare$$

Proof of Proposition 3

Along the equilibrium path there are two types of subgames at the pricing stage as a function of the communication between firms. First suppose all firms send uninformative messages $m_i = \emptyset \forall i$. Then a firm with a H-signal knows that all firms either received a H-signal or a \emptyset -signal and it sets the equilibrium price $p_\emptyset(\emptyset) = p_\emptyset(H)$ if

$$\frac{\delta V}{1 - \delta} \geq \frac{n - 1}{n} \Pi^H(p_\emptyset(\emptyset)), \quad (13)$$

where $V \equiv E\Pi_L(p_\emptyset(\mathcal{I}))$. This condition is equivalent to (7) and holds if $\delta \geq \delta_\emptyset(H)$. A firm with a \emptyset -signal has an updated belief of $b_i(\mathcal{I}_i = \emptyset)$ and expects an equilibrium profit of

$$b_i(\mathcal{I}_i = \emptyset) \frac{\Pi^H(p_\emptyset(\emptyset))}{n} + (1 - b_i(\mathcal{I}_i = \emptyset)) \frac{\Pi^L(p_\emptyset(\emptyset))}{n} + \frac{\delta V}{1 - \delta}.$$

The optimal deviation price $p^d < p_\emptyset(\emptyset)$ solves

$$\max_{p^d} b_i(\mathcal{I}_i = \emptyset) \Pi^H(p^d) + (1 - b_i(\mathcal{I}_i = \emptyset)) \Pi^L(p^d)$$

which yields the necessary condition

$$\rho \frac{\partial \Pi^H(p^d)}{\partial p^d} + (1 - \rho)(1 - \sigma)^{n-1} \frac{\partial \Pi^L(p^d)}{\partial p^d} = 0.$$

Comparing this condition with the first order condition (5) for $p_\emptyset(\emptyset)$ implies $p^*(L) \leq p^d \leq p_\emptyset(\emptyset)$. Incentive compatibility for a \emptyset -signal firms requires

$$\frac{\delta V}{1 - \delta} \geq b_i(\mathcal{I}_i = \emptyset) \left[\Pi^H(p^d) - \frac{\Pi^H(p_\emptyset(\emptyset))}{n} \right] + (1 - b_i(\mathcal{I}_i = \emptyset)) \left[\Pi^L(p^d) - \frac{\Pi^L(p_\emptyset(\emptyset))}{n} \right]. \quad (14)$$

This condition is satisfied for $\delta \geq \delta_\emptyset(H)$ if the rhs of (13) is larger than the rhs of (14), i.e.

$$(1 - b_i(\mathcal{I}_i = \emptyset)) \left[\frac{n - 1}{n} \Pi^H(p_\emptyset(\emptyset)) - \Pi^L(p^d) + \frac{\Pi^L(p_\emptyset(\emptyset))}{n} \right] + b_i(\mathcal{I}_i = \emptyset) [\Pi^H(p_\emptyset(\emptyset)) - \Pi^H(p^d)] \geq 0$$

The second term is always positive since $p^d \leq p_\emptyset(\emptyset) \leq p^*(H)$. The expression in the first bracket increases in n and is positive for $n = 2$ if

$$\frac{1}{2} \Pi^H(p_\emptyset(\emptyset)) + \frac{1}{2} \Pi^L(p_\emptyset(\emptyset)) \geq \Pi^L(p^d).$$

The right-hand side of this inequality is maximized for $p^d = p^*(L)$. The left-hand side is minimized at $p_\emptyset(\emptyset) = p^*(L)$ and $p_\emptyset(\emptyset) = p^*(H)$ with a minimum of

$$\frac{1}{2}\Pi^H(p^*(L)) + \frac{1}{2}\Pi^L(p^*(L)) \geq \Pi^L(p^*(L)).$$

It follows that the incentive compatibility constraint for a \emptyset -signal firm holds for all $\delta \geq \delta_\emptyset(H)$. In the second type of price subgames at least one firm sends a verifiable L -message at the communication stage. Then, setting the equilibrium price dominates undercutting by a small $\epsilon > 0$ if

$$\frac{\Pi^L(p_\emptyset(L))}{n} + \frac{\delta V}{1 - \delta} \geq \Pi^L(p_\emptyset(L)). \quad (15)$$

Since $\Pi^H(p_\emptyset(\emptyset)) \geq \Pi^L(p_\emptyset(L))$ this condition always holds if $\delta \geq \delta_\emptyset(H)$. Finally, assume that for any off-the-equilibrium outcome of the communication stage, i.e. if any firm sends a H -signal or an unverifiable L -signal, firms revert instantaneously to marginal cost pricing.

Now consider the communication stage. If firm i receives a L -signal and deviates by announcing $m_i = \emptyset$, then with probability $1 - (1 - \sigma)^{n-1}$ at least one other firm receives a L -signal and sends a L -message. This means that all firms are in information state $\mathcal{I}_i = L$ and - as shown above - no firm deviates from $p_\emptyset(L)$ for $\delta \geq \delta_\emptyset(H)$. With probability $(1 - \sigma)^{n-1}$ no other firm receives a L -signal, only uninformative messages are sent and firms set $p_\emptyset(\emptyset)$. If the deviating firm i sets $p_\emptyset(\emptyset)$ it receives

$$\frac{\Pi^L(p_\emptyset(\emptyset))}{n} + \frac{\delta V}{1 - \delta}.$$

If it undercuts at $p_\emptyset(L)$ it obtains $\Pi^L(p_\emptyset(L))$. Sending a L -message at the communication stage dominates sending $m_i = \emptyset$ and pricing at $p_\emptyset(\emptyset)$ since

$$\frac{\Pi^L(p_\emptyset(L))}{n} + \frac{\delta V}{1 - \delta} \geq (1 - \sigma)^{n-1} \frac{\Pi^L(p_\emptyset(\emptyset))}{n} + (1 - (1 - \sigma)^{n-1}) \frac{\Pi^L(p_\emptyset(L))}{n} + \frac{\delta V}{1 - \delta}$$

for $p_\emptyset(L) = p^*(L) < p_\emptyset(\emptyset)$. Sending a L -message dominates announcing $m_i = \emptyset$ and undercutting at $p_\emptyset(L)$ if

$$\frac{\Pi^L(p_\emptyset(L))}{n} + \frac{\delta V}{1 - \delta} \geq (1 - \sigma)^{n-1} \Pi^L(p_\emptyset(L)) + (1 - (1 - \sigma)^{n-1}) \left[\frac{\Pi^L(p_\emptyset(L))}{n} + \frac{\delta V}{1 - \delta} \right].$$

If (15) holds, the rhs is always less or equal than $\Pi^L(p_\emptyset(L))$ and then again by virtue of (15) the inequality holds true. The proposition follows. ■

Proof of Proposition 4

In equilibrium there are two types of subgames at the pricing stage as a function of the communication between firms. First suppose there was at least one H -message. Then any firm sticks to the equilibrium price $p_c(H)$ if

$$\frac{\delta V}{1 - \delta} \geq \frac{n - 1}{n} \Pi^H(p_c(H)) \quad (16)$$

with $V \equiv E\Pi_H(p_c(\mathcal{I}))$. This inequality holds if $\delta \geq \delta_c(H)$. Next suppose all firms sent uninformative messages, $m_i = \emptyset \forall i$. Then a firm with a L -signal knows that all firms either received a L -signal or a \emptyset -signal and it sets the equilibrium price $p_c(L) = p^*(L)$ if

$$\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^L(p_c(L)) + \frac{\delta V}{1 - \delta} \geq \Pi^L(p_c(L)). \quad (17)$$

This condition is always satisfied if condition (16) holds if

$$\frac{n - 1}{n} \Pi^H(p_c(H)) \geq \frac{\sigma n - 1 + (1 - \sigma)^n}{\sigma n} \Pi^L(p_c(L)).$$

The rhs increases in σ and for $\sigma = 1$ it becomes $(n - 1)\Pi^L(p_c(L))/n$ which is strictly smaller than the lhs. Thus, (17) holds for $\delta \geq \delta_c(H)$.

A firm with a \emptyset -signal has an updated belief of $b_i(\mathcal{I}_i = \emptyset)$ and expects equilibrium profits of

$$b_i(\mathcal{I}_i = \emptyset) \frac{\Pi^H(p_c(\emptyset))}{n} + (1 - b_i(\mathcal{I}_i = \emptyset))(1 - \sigma)^{n-1} \frac{\Pi^L(p_c(\emptyset))}{n} + \frac{\delta V}{1 - \delta}.$$

Its rival firms either set $p_c(L)$ or $p_c(\emptyset)$. Profitable deviations are therefore $p^d \in (0, p_c(\emptyset)]$. Consider first the partially on-schedule deviation to $p^d = p_c(L)$ which yields expected profits of

$$b_i(\mathcal{I}_i = \emptyset) \Pi^H(p_c(L)) + (1 - b_i(\mathcal{I}_i = \emptyset)) \left[\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^L(p_c(L)) + \frac{\delta V}{1 - \delta} \right]. \quad (18)$$

This deviation is not profitable if

$$\begin{aligned} \frac{\delta V}{1 - \delta} &\geq b_i(\mathcal{I}_i = \emptyset) \left[\Pi^H(p_c(L)) - \frac{\Pi^H(p_c(\emptyset))}{n} \right] \\ &\quad + (1 - b_i(\mathcal{I}_i = \emptyset)) \left[\frac{1 - (1 - \sigma)^n}{\sigma n} \Pi^L(p_c(L)) - (1 - \sigma)^{n-1} \frac{\Pi^L(p_c(\emptyset))}{n} + \frac{\delta V}{1 - \delta} \right] \end{aligned} \quad (19)$$

or

$$\delta \geq \delta_c(\emptyset) \equiv \frac{\Theta}{b_i(\mathcal{I}_i = \emptyset)V + \Theta}$$

with

$$\begin{aligned} \Theta &= (1 - b_i(\mathcal{I}_i = \emptyset)) \left[\frac{1 - (1 - \sigma)^n}{n\sigma} \Pi^L(p_c(L)) - (1 - \sigma)^{n-1} \frac{\Pi^L(p_c(\emptyset))}{n} \right] \\ &\quad + b_i(\mathcal{I}_i = \emptyset) \left[\Pi^H(p_c(L)) - \frac{\Pi^H(p_c(\emptyset))}{n} \right]. \end{aligned}$$

To show that (16) might be more restrictive than (19), compare their respective rhs for $b_i(\mathcal{I}_i = \emptyset) \rightarrow 1$. The rhs of (16) is larger since

$$\Pi^H(p_c(H)) - \Pi^H(p_c(L)) \geq \frac{\Pi^H(p_c(H))}{n} - \frac{\Pi^H(p_c(\emptyset))}{n}$$

and $p_c(L) \leq p_c(\emptyset) \leq p_c(H)$. To show that (19) might be more restrictive than (16), note that for $b_i(\mathcal{I}_i = \emptyset) \rightarrow 0$, the rhs of (19) goes to infinity for $\delta \rightarrow 1$.

Two more possible price deviation for a \emptyset -signal need to be checked. First deviating to $p^d = p_c(L) - \epsilon$ yields

$$b_i(\mathcal{I}_i = \emptyset)\Pi^H(p_c(L)) + (1 - b_i(\mathcal{I}_i = \emptyset))\Pi^L(p_c(L)).$$

It follows from (17) and (18) that this deviation is always dominated by the deviation to $p^d = p_c(L)$. Secondly, deviating to a p^d with $p_c(L) < p^d < p_c(\emptyset)$ gives

$$b_i(\mathcal{I}_i = \emptyset)\Pi^H(p^d) + (1 - b_i(\mathcal{I}_i = \emptyset))(1 - \sigma)^{n-1}\Pi^L(p^d)$$

Maximizing with respect to p^d yields the same condition as (9) from Lemma 2, i.e. $p^d = p_c(\emptyset) - \epsilon$. Setting the equilibrium price $p_c(\emptyset)$ dominates this deviation if

$$\frac{\delta V}{1 - \delta} \geq b_i(\mathcal{I}_i = \emptyset)\frac{n-1}{n}\Pi^H(p_c(\emptyset)) + (1 - b_i(\mathcal{I}_i = \emptyset))(1 - \sigma)^{n-1}\frac{n-1}{n}\Pi^L(p_c(\emptyset))$$

which always holds if (16) and (17) hold.

Assume that if firms receive any out-of-equilibrium message (any L -message or unverifiable H -signal) at the communication stage, they revert instantaneously to marginal cost pricing. Thus, the only profitable deviation for a firm i with a H -signal is to announce $m_i = \emptyset$. In this case, with probability $1 - (1 - \sigma)^{n-1}$ at least one other firm receives a H -signal and sends a H -message. Then, for $\delta \geq \delta_c(H)$, the deviating firm cannot do better than setting $p_c(H)$, i.e. it receives the same profits as if it had sent $m_i = H$. With probability $(1 - \sigma)^{n-1}$ no other firm receives a H -signal, only uninformative messages are sent and the deviating firm knows that all of its rivals got a \emptyset -signal and set $p_c(\emptyset)$. If the deviating firm charges $p_c(\emptyset) - \epsilon$ it receives $\Pi^H(p_c(\emptyset))$. Setting $p_c(\emptyset)$ yields

$$\frac{\Pi^H(p_c(\emptyset))}{n} + \frac{\delta V}{1 - \delta},$$

which is always larger than $\Pi^H(p_c(\emptyset))$ for $\delta \geq \delta_c(H)$. However, since this maximum deviation profit is less than the equilibrium profits the deviating firm could earn if it announces $m_i = H$, no deviation occurs and the proposition follows. ■

Proof of Proposition 5

(i) The expected price with full communication and partial communication in high demand is identical and equal to

$$(1 - (1 - \sigma)^n)[\rho p_c(H) + (1 - \rho)p_c(L)] + (1 - \sigma)^n p_c(\emptyset) = p_c(\emptyset).$$

The expected price without communication and partial communication in low demand states is identical and equal to

$$\begin{aligned} & [\rho + (1 - \rho)(1 - \sigma)^n]p_\emptyset(\emptyset) + (1 - \rho)(1 - (1 - \sigma)^n)p_\emptyset(L) \\ &= [\rho + (1 - \rho)(1 - \sigma)^n][(1 - \frac{\rho}{\rho + (1 - \rho)(1 - \sigma)^n})p^*(L) + \frac{\rho}{\rho + (1 - \rho)(1 - \sigma)^n}p^*(H)] + \\ & (1 - \rho)(1 - (1 - \sigma)^n)p^*(L) = (1 - \rho)p^*(L) + \rho p^*(H) = p_c(\emptyset). \end{aligned}$$

(ii) The difference between the expected industry profits with full communication (or partial communication in H) and without communication (or partial communication in L) is given by

$$\begin{aligned} E\Pi_H(p_c(\mathcal{I})) - E\Pi_\phi(p_\phi(\mathcal{I})) &= \rho(1 - (1 - \sigma)^n)\Pi^H(p_c(H)) + \rho(1 - \sigma)^n\Pi^H(p_c(\phi)) \\ &\quad + (1 - \rho)(1 - \sigma)^n\Pi^L(p_c(\phi)) - [\rho\Pi^H(p_\phi(\phi)) + (1 - \rho)(1 - \sigma)^n\Pi^L(p_\phi(\phi))] \\ &= \frac{\rho(1 - \rho)^2(1 - (1 - \sigma)^n)(1 - \sigma)^n\Delta^2}{\rho + (1 - \rho)(1 - \sigma)^n} \equiv \Gamma > 0. \end{aligned}$$

(iii) Consumer surplus in a high demand state is $CS^H(p) \equiv (a + \Delta - p)^2$ and in a low demand state it is $CS^L(p) \equiv (a - \Delta - p)^2$. Thus, consumer surplus without communication (or communication in low demand states) is

$$\begin{aligned} ECS_\phi &= ECS_L = \rho CS^H(p_\phi(\phi)) + (1 - \rho)(1 - (1 - \sigma)^n)CS^L(p_\phi(L)) \\ &\quad + (1 - \rho)(1 - \sigma)^nCS^L(p_\phi(\phi)). \end{aligned}$$

Consumer surplus without communication (or communication in high demand states) is

$$\begin{aligned} ECS_{LH} &= ECS_H = (1 - \sigma)^n[\rho CS^H(p_c(\phi)) + (1 - \rho)CS^L(p_c(\phi))] + (1 - \rho)CS^L(p_c(L)) \\ &\quad + (1 - (1 - \sigma)^n)[\rho CS^H(p_c(H))]. \end{aligned}$$

Comparing yields

$$\begin{aligned} ECS_{LH} - ECS_\phi &= \rho(1 - \sigma)^n[CS^H(p_c(\phi)) - CS^H(p_\phi(\phi))] + \rho(1 - (1 - \sigma)^n)[CS^H(p_c(H)) \\ &\quad - CS^H(p_\phi(\phi))] + (1 - \rho)(1 - \sigma)^n[CS^L(p_c(\phi)) - CS^L(p_\phi(\phi))] \\ &= -\frac{3\rho(1 - \rho)^2(1 - (1 - \sigma)^n)(1 - \sigma)^n\Delta^2}{2(\rho + (1 - \rho)(1 - \sigma)^n)} = -\frac{3}{2}\Gamma < 0. \quad \blacksquare \end{aligned}$$

Proof of Proposition 6

(i) follows from inspection of Γ in part (ii) of the proof of Proposition 5. To show (ii) derive

$$\frac{\partial \Gamma}{\partial \rho} = \frac{(1 - \rho)(1 - (1 - \sigma)^n)(1 - \sigma)^n\Delta^2[(1 - \rho)(1 - 2\rho)(1 - \sigma)^n - 2\rho^2]}{(\rho + (1 - \rho)(1 - \sigma)^n)^2} = 0.$$

The only feasible, real root $\rho \in [0, 1]$ of the last bracket in the numerator is given by

$$\hat{\rho} = 2/(3 + \sqrt{\frac{8 + (1 - \sigma)^2}{(1 - \sigma)^2}}),$$

which has to be the global maximizer since $\Gamma \geq 0$, $\Gamma(\rho = 0) = \Gamma(\rho = 1) = 0$, $\partial \Gamma / \partial \rho(\rho = 0) > 0$ and $\partial \Gamma / \partial \rho < 0$ for $\rho \geq 1/2$. Check that $\hat{\rho}$ is decreasing in σ . Thus, the highest value it can take is $\hat{\rho}(\sigma = 0) = 1/3$. Next, derive

$$\frac{\partial \Gamma}{\partial \sigma} = \frac{\rho(1 - \rho)^2n(1 - \sigma)^{n-1}\Delta^2[2\rho(1 - \sigma)^n + (1 - \rho)(1 - \sigma)^{2n} - \rho]}{(\rho + (1 - \rho)(1 - \sigma)^n)^2} = 0.$$

The only feasible real root $\sigma \in [0, 1]$ of the last bracket in the numerator is given by

$$\hat{\sigma} = 1 - \left(\frac{\sqrt{\rho}}{1 + \sqrt{\rho}} \right)^{\frac{1}{n}},$$

which has to be the global maximizer since $\Gamma(\sigma = 0) = \Gamma(\sigma = 1) = 0$, $\partial\Gamma/\partial\sigma(\sigma = 0) > 0$ and $\partial\Gamma/\partial\sigma = 0$ for $\sigma = 1$. Check that $\hat{\sigma} \rightarrow 1$ for $\rho \rightarrow 0$ and that $\hat{\sigma}(\rho = 1) \geq 0$.

To show part (iii) of the corollary derive

$$\frac{\partial\Gamma}{\partial n} = \frac{\rho(1-\rho)^2(1-\sigma)^n \Delta^2 \ln(1-\sigma) [\rho - 2\rho(1-\sigma)^n - (1-\rho)(1-\sigma)^{2n}]}{(\rho + (1-\rho)(1-\sigma)^n)^2} = 0.$$

The only feasible real root of the last bracket in the numerator is given by

$$\hat{n} = \ln(1 - \frac{1}{1 + \sqrt{\rho}}) / \ln(1 - \sigma)$$

which has to be the global maximizer since $\Gamma(n = 0) = 0, \Gamma \rightarrow 0$ for $n \rightarrow \infty$ and $\partial\Gamma/\partial n(n = 0) > 0$. Check that $\hat{n} \geq 2$ if and only if

$$\sigma \leq 1 - \sqrt{\sqrt{\rho}/(1 + \sqrt{\rho})} > 0.$$

The proposition follows. ■

References

- ABREU, D., PEARCE, D. & STACCHETTI, E. (1986). Optimal cartel equilibria with imperfect monitoring. *Journal of Economic Theory* **39**, 251–69.
- ABREU, D., PEARCE, D. & STACCHETTI, E. (1990). Towards a theory of discounted repeated games with imperfect monitoring. *Econometrica* **58**(5), 1041–1063.
- ATHEY, S. & BAGWELL, K. (2001). Optimal collusion with private information. *RAND Journal of Economics* **32**, 428–465.
- ATHEY, S., BAGWELL, K. & SANCHIRICO, C. (2004). Collusion and price rigidity. *Review of Economic Studies* **71**, 317–349.
- COMPTE, O. (1998). Communication in repeated games with imperfect private monitoring. *Econometrica* **66**(3), 597–626.
- ELLISON, G. (1994). Theories of cartel stability and the joint executive committee. *RAND Journal of Economics* **25**, 37–57.
- FUDENBERG, D., LEVINE, D. & MASKIN, E. (1994). The folk theorem with imperfect public information. *Econometrica* **62**, 997–1040.

- GREEN, E. & PORTER, R. H. (1984). Noncooperative collusion under imperfect price information. *Econometrica* **52**, 87–100.
- KANDORI, M. & MATSUSHIMA, H. (1998). Private observation, communication and collusion. *Econometrica* **66**(3), 627–652.
- KÜHN, K.-U. (2001). Fighting collusion by regulating communication between firms. *Economic Policy* **16**, 167–204.
- MOTTA, M. (2004). *Competition Policy: Theory and Practice*. Cambridge University Press.
- PORTER, R. (1983). A study of cartel stability: The joint executive committee. *Bell Journal of Economics* **14**, 301–314.
- REY, P. (2001). Towards a theory of competition policy. *IDEI, University of Toulouse, mimeo* .
- ROTEMBERG, J. & SALONER, G. (1986). A supergame-theoretic model of price wars during booms. *American Economic Review* **76**, 390–407.
- STAIGER, R. & WOLAK, F. (1992). Collusive pricing with capacity constraints in the presence of demand uncertainty. *RAND Journal of Economics* **23**, 203–220.
- STIGLER, G. (1964). A theory of oligopoly. *Journal of Political Economy* **72**, 44–61.