EFFICIENT TUITION & FEES, EXAMINATIONS, AND SUBSIDIES*

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**Abstract.**
A student's future log-wage is given by the sum of a skill premium and a random personal "ability" term. Students observe only a private, noisy signal of their ability, and universities can condition admission decisions on the results of noisy tests. We assume first that universities are maximizing social surplus, and contrast the results with those obtained when they maximize rents. If capital markets are perfect, and if test results are public knowledge, then, there is no sorting on the basis of test scores. Students optimally self-select as a result of pricing only. In the absence of externalities generated by an individual's higher education, the optimal tuition is then greater than the university's marginal cost. If capital markets are perfect but asymmetries of information are bilateral, *i.e.*, if universities observe a private signal of each student's ability, or if there are borrowing constraints, then, the optimal policy involves a mix of pricing and pre-entry selection based on the university's private information. Optimal tuition can then be set below marginal cost, and can even become negative, if the precision of the university's private assessment of students' abilities is high enough.

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## 1. Introduction

The public universities' state of financial crisis is an acute problem in many countries. Public funds shortages being recurrent, increases in tuition appear as a natural remedy, but are understandably met with fierce resistance on the part of citizens. In the United States, many public schools have faced the hard choice of either cutting educational spending and quality, or increasing price, and tuition has gone up in the recent years (see Winston (1999), and his references). In Europe, the situation is almost tragic. A journalist recently described Britain's universities as "depressingly threadbare, overcrowded and politicized"[1]. In the UK, Tony Blair's 2004 reform of tuition charges has been very unpopular, in spite of its apparently limited financial impact[2]. Tuition reform is also discussed in Germany, in Switzerland, and other countries. Yet, it seems that the formal economic theory of university pricing, the question of the optimal balance of fees and subsidies has not been studied with enough precision, paying attention to its relation with the use of entrance examinations as sorting instruments, in a world of incomplete information. Admission criteria have consequences on the universities' optimal mode of financing, and the tuition-vs-exams debate is related to the question of the desirable degree of higher-education decentralization in Public Finance.

The present article proposes an approach to optimal fees, paying special attention to informational asymmetries between higher education institutions and students, and to the incomplete information of students themselves. Academic policies are examined under opposite assumptions; we assume first that universities are non-profit institutions, and contrast the results with those obtained under the assumption that they are rent-seeking, or "for-profit" organizations. The relevance of normative "marginal cost pricing" theories, as well as the possibility of regulating, or providing correct incentives to rent-seeking universities, is also considered.

In our model, heterogeneous students are characterized by a "talent" or "ability" parameter, but they observe a noisy, private signal of their own talent only. A Mincerian wage equation gives future wages as the sum of a skill premium and of the unknown ability parameter. Prospective students form a rational expectation of their future earnings, and apply for higher education on the basis of this forecast, using this Mincerian model. The skill premium depends on both the quality of education, and the number of graduates. Academic authorities cannot observe individual talent directly, but they can condition admission decisions on the results of tests, or entrance examinations, also viewed as noisy signals of ability. The cases in which test results are, and are not publicly disclosed are both analyzed. The latter case is one in which informational asymmetries between a student and a university are bilateral. Higher education is costly, and total cost depends on quality, quantity, and the average ability of recruits (the well-known peer effect). A university has the right to set fees, and to set admission standards in the form of a minimal grade or test score; they also choose a quality variable and total enrollment.

Non-profit universities are assumed to choose their policy in order to maximize social surplus. This provides us with a useful benchmark. But non-profit academic managers

[1] The Economist (2003a)

[2] The Economist (2003b)

have no concern for equity, or no aversion for inequality, so that our results will depend on efficiency considerations only. In contrast, the rent-seeking university simply maximizes profit, that is, tuition revenues minus costs, which provides a clear description of the other extreme. We consider both the case of "perfect capital markets", and the situation in which there are borrowing constraints, that is, when tuition can bar talented students with low-income family backgrounds from higher education.

With these tools, we construct a theory of public university-regulation which is entirely based on observable labour market facts, and combines economic theory with econometric practice. Our results are the following.

First, if capital markets are perfect, if test results are public knowledge, and rational students condition their application decisions on private as well as public signals, then, an optimal policy involves a positive fee, and no sorting on the basis of test scores. Students optimally self-select as a result of pricing only. In addition, under the assumption that there are no externalities associated with an individual's higher education, the optimal tuition fee would typically be greater than marginal cost.

Second, if capital markets are perfect but asymmetries of information are bilateral, in the sense that universities observe a signal of each student's ability which is not disclosed or not taken into account by students, (i.e., admission interviews), then, the optimal policy of a non-profit university involves a mix of pricing and pre-entry selection on the basis of test results. On the assumption that, again, there are no externalities of higher education, the optimal policy then entails a direct student subsidy: optimal tuition is typically set below marginal cost, and can even become negative, if the precision of the university's private assessment of students' abilities is good enough. This result is not due to redistribution or equity motives on the part of the benevolent university (and of course not due to external effects); it is driven solely by efficiency considerations.

Thirdly, the rent-seeking university's policies are inefficient: tuition & fees tend to be too high, and admission standards tend to be too low. An incentive transfer schedule, depending on enrollment and knowledge of the wage distributions, can fully correct the inefficiencies due to rent-seeking or for-profit behaviour.

Fourthly, when students face borrowing constraints, even if test scores are publicly disclosed, the university's optimal policy must be a combination of pricing and selection, with the possibility that, again, tuition be set below marginal cost, to alleviate inefficiencies due to the fact that some good students are deterred by price. Again, this result is not due to assumed redistribution motives on the part of benevolent university managers; it follows from surplus maximization, or efficiency considerations only.

To sum up, we find that university pricing is a socially efficient policy, but that it should be mixed with pre-entry selection of students, either because the university has private information on students' abilities, or because, due to financial market imperfections, some students face borrowing constraints, or both. Now, the optimal tuition fee can be optimally set below marginal cost, even if there are no externalities justifying subsidization of higher education. Optimal tuition will be a decreasing function of the entrance test's precision as a signal of student ability. The more accurate pre-entry assessments are, the larger the discount on tuition.

The industrial organization of the higher education sector remains difficult to model, and in particular, the various forms of price and non-price competition among universities are still very much unexplored[3]. Among recent contributions, Winston (1999) remarkably summarizes the intuitions, and provides a non-technical description, of university economics. The modelling of university behaviour is generally also considered as difficult, and any choice of an objective function is open to criticism[4]. We have kept Winston's vision of the "industry" in mind and arrive at results which, we think, do not contradict his observations. Our choice has been to "cut the Gordian knot" and to consider two extreme, probably unrealistic cases: the purely benevolent, and the purely greedy, for-profit (or rent-seeking) university managers[5]. The perfect competition case has been studied by Rothschild and White (1995), who emphasize the important idea that students are inputs in the production of their own human capital. In the present contribution, the university is endowed with some form of market power. This could represent a private university with a substantial market share or a leadership position, or a dominant network of public universities.

Related research on the political economy of higher education subsidies and on the tuition-vs-taxes debate goes back at least to Hansen and Weisbrod (1969). The redistribution effects of higher education have been studied by Green and Sheshinski (1975). The complementarity of skilled and unskilled labour in the aggregate production function has been studied as a possible justification for public subsidies to universities (and to students) by Johnson (1984). Creedy and François (1990) analysed majority voting on income taxes to fund higher education. Barr (1993) advocates the development of student loans and tuition fees in the UK. Chapman (1997) discusses the related question of income contingent contributions to higher education, and the important Australian experience.

Finally, our analysis of optimal pricing is not independent of an economic theory of examination procedures. Pioneering work on the economic theory of exams is due to Costrell (1994) and Betts (1998). But our approach is closer to that of Fernández and Galí (1999), and Fernández (1998). In the latter work of Fernández, the student population is described by a joint distribution of ability and wealth. The problem is to allocate students to high or low-quality schools, knowing that high-quality school capacity is fixed and that student ability and school quality are complementary inputs in the production of future earnings. For efficiency reasons, high ability types should be allocated to high quality schools. A costly (and socially wasteful) test technology can be used to decide which students will be admitted to high quality schools. Each student can produce a given, deterministic test result at the personal cost of a given amount of effort (which varies with ability). Fernández (1998) then compares a publicly regulated, test-based allocation system with a competitive equilibrium allocation in which schools set prices. Tests and prices are equivalent when students can borrow against future income. We would find the same equivalence of tests and fees as screening devices in a complete information version of our model. She then shows that markets and exams are not equivalent when students cannot borrow.

[3] However, Del Rey (2001) and De Fraja and Iossa (2002), study asymmetric duopoly models; Gary-Bobo and Trannoy (2002) contains an attempt at modelling monopolistic competition, assuming that a country's universities form a symmetric oligopoly.

[4] See e.g., Borooah (1994).

[5] In our model, rent-seeking and profit maximization behaviours are formally equivalent.

Examination-led allocations dominate market equilibria because, under zero-borrowing, some able-but-poor students take the place of some wealthy-but-less-able students, which improves the overall matching of students to schools, and therefore, aggregate output. The main differences of our work with that of Fernández consist in introducing incomplete information under more radical forms (students observe noisy signals of their ability, test results are random), in comparing various forms of informational asymmetry (unilateral and bilateral), and studying variable enrollment and education quality. With the addition of these elements, we show under which circumstances optimal policies should balance selection on the basis of grades and self-selection by means of the price mechanism; we compute the optimal tuition and show how the amount of subsidy (or tuition rebate) is related to the informational properties of the examination technology; we finally sketch the analysis of the role of borrowing constraints in the determination of tuition rebates.

In the following, Section 2 presents the model and basic assumptions; Section 3 presents the results in the asymmetric information setting; Section 4 is devoted to a variant of the model in which informational asymmetries are bilateral; and Section 5 analyses the impact of borrowing constraints on the optimal policy.

## 2. The Model and Basic Assumptions

To simplify the analysis, we assume that there exists a university (or college), with a single department, exerting market power as a provider of higher education and skilled workers.

### *2.1. The Skill Premium and Preferences*

Our point of departure is the skill premium earned by means of higher education. To fix ideas, we stick to the classic human capital theory of education, assuming that education improves productivity, and that productivity, for simplicity again, will be observed by employers[6]. We suppose that there exist two categories of workers only, the skilled, who are graduates from the university (or college), and the unskilled, who did not study. The unskilled workers' wage rate is a random variable $w_u$ and the university (or college) graduates' wage is a random variable denoted $w_s$. Each student is characterized by a pair of independent random variables denoted $(\widehat{\theta}, \widehat{\eta})$, where the first of these variables describes the individual's talent in skilled activities, while the second represents the talent (or luck) in unskilled work. We will explore below different assumptions relative to the information of agents about "talents". Talent in skilled activities will simply be called "ability"; this variable is not observed, neither by the university, nor by the student, and each student receives a noisy private signal relative to her (his) own ability. The university and the students are supposed to know the probability distribution of ability.

The future random wage $w_u$ of an unskilled worker is defined as follows.

$$\ln(w_u) = \ln(\hat{w}_0) + \widehat{\eta}, \tag{1}$$

[6] Our theory would be compatible with imperfect observations of productivity by employers, and thus with a more general signalling theory of education. On the debate between human-capital and signalling views, see Weiss (1995).

where $\hat{w}_0$ is a constant, and $\widehat{\eta}$ is normally distributed with mean 0 and variance $\sigma^2_{\widehat{\eta}}$. We then assume that the following relation describes the future wage $w_s$ of a skilled worker.

$$\ln(w_s) = \ln(\hat{w}_0) + \widehat{\Delta}(x, e) + \widehat{\theta}, \tag{2}$$

where $\widehat{\Delta}$, the skill premium, is a function of the number of graduates, denoted $x$, and of the quality of studies provided by the university, denoted $e$.

The random variable $\widehat{\theta}$ is normally distributed with a zero mean, and captures the fact that graduates are not all equal, due to differing talents, that are reflected in more or less brilliant grades, as well as more or less lucrative perspectives on the labour market. We assume that the skill premium is decreasing with respect to $x$, and increasing with respect to $e$ (and continuously differentiable with respect to both variables); these assumptions are compatible with a partial equilibrium model of the labour market. In a simple static general equilibrium model, the skill premium would still be a decreasing function of the amount of skilled labour. But, if the number of educated people generates an economy-wide externality of the kind described by endogenous growth theorists (cf. Aghion and Howitt (1997)), then, our skill premium function could become an increasing function of $x$. We will discuss the implications of both cases in the following.

An important, and probably strong —although very common— assumption is that all agents observe and (or) form correct expectations about the skill premium $\widehat{\Delta}$.

The student's preferences are represented by the same inter-temporal, infinite horizon, additively separable utility function. Higher education takes place in the first period of the student's life-cycle, at time 0. Utility is assumed to be quasi-linear with respect to consumption at 0. Formally, for a consumption profile $c = (c_0, c_1, c_2, ...)$ we define utility $u(c)$ as follows,

$$u(c) = c_0 + \sum_{\tau=1}^{\infty} \left(\frac{1}{1+r}\right)^{\tau} \ln(c_\tau), \tag{3}$$

where $r$ is a psychological interest rate, used by agents to discount future utility.

To simplify the analysis, we assume that a worker's wage is constant during her (his) entire working life. Agents do not save, and consume their wages, which are expressed in real terms. With the help of these assumptions, an agent who doesn't study remains an unskilled worker for life; his or her utility is random and can be written, using (1) and (3),

$$u_0 = \hat{w}_0 \exp(\hat{\eta}) + \frac{\ln(\hat{w}_0)}{r} + \frac{\hat{\eta}}{r}. \tag{4}$$

Let $p$ denote the tuition charges, paid at time zero. The utility of a skilled worker is random, and using (2) and (3), we get,

$$u_1 = -p + \frac{\ln(\hat{w}_0)}{r} + \frac{\widehat{\Delta}(x, e)}{r} + \frac{\widehat{\theta}}{r}. \tag{5}$$

In the following, we denote,

$$\Delta(x, e) = \frac{\widehat{\Delta}(x, e)}{r}, \qquad \theta = \frac{\widehat{\theta}}{r}, \qquad \eta = \frac{\widehat{\eta}}{r}, \tag{6}$$

the capitalized values of the skill premium, and of ability terms, respectively. The cost of human capital investment is the sum of direct costs $p$ and random opportunity costs $\hat{w}_0 \exp(\hat{\eta})$.

*2.2. University Teaching Technology and Peer Group Effects*

To describe higher education costs, we assume that there exists a continuously differentiable university cost function, denoted $C$, depending on the number of enrolled students $x$, on chosen education quality $e$, and on average student ability, denoted $v$. In other words, total university cost is given by $C(x, e, v)$. Quality $e$ can be viewed as the result of the combined efforts of faculty, staff and students in the higher education technology. We do not model the moral hazard and incentives problems associated with effort[7]. The cost function also captures the impact of the often discussed "peer group effects". Average student ability $v$ affects the quality of education for a given cost, or equivalently, affects the cost of providing a given quality of education. We provide a formal expression for $v$ below. It is natural to assume that $C$ is increasing with respect to $x$ and $e$, and non-increasing with respect to $v$. This latter assumption, which is not the most general way of modelling the fact that students are inputs in the production process of their own human capital (e.g., Rothschild and White (1995)), nevertheless captures the essential idea behind peer effects.

The theory of peer group effects, or local interactions in education has been studied, among other contributions, by Arnott and Rowse (1987), de Bartolome (1990), Benabou (1993), and Epple and Romano (1998). There have been many attempts at testing for the presence of these effects in education, from school to college. The common view is that these effects are important in higher education, even if it is difficult to estimate their magnitude[8].

At this stage, it will probably clarify the discussion if we show that the model described above is formally equivalent to one in which the the peer group effect directly affects the skill premium through average student ability $v$. To see this, assume that the job market values worker quality, denoted $e$, and that worker quality is produced with the help of teacher effort $\tau$ and peer effects so that $e = \phi(\tau, v)$, where $\phi$ is a kind of production function, which is increasing with respect to $\tau$ for all $v$. Now, if the teaching technology is represented by a cost function depending on teacher effort and the number of students only, that is, if $C = \tilde{C}(x, \tau)$, and if the skill premium depends on quality and the number of graduates only, that is, $\Delta = \Delta(x, e)$, then, the latter can be expressed as a function of effort $\tau$ as follows,

$$\tilde{\Delta}(x, \tau, v) \equiv \Delta(x, \phi(\tau, v)).$$

This shows reasonable conditions under which the approach in which the skill premium would be a function of average peer ability is formally equivalent to one in which average peer ability is an argument of the university cost function. It happens that the latter approach is much simpler than the former from the technical point of view[9].

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[7] Recent empirical studies show the importance of education quality on future wages (e.g., Card and Krueger (1992)), on test scores (e.g., Angrist and Lavy (1999)), and the effect of teacher incentives on student achievement (e.g., Lavy (2002)).

[8] Among recent contributions, see Betts and Morell (1999), Hoxby (2000), Sacerdote (2001), Angrist and Lang (2002), Zimmerman (2003).

[9] The alternative modelling strategy leads to a fixed point problem.

*2.3. The Philanthropic and Cynic Views*

We define a higher education institution as "philanthropic" when its objective is to maximize the social value, or social surplus, of its education activity. It is of course a strong assumption to assume that a university is philanthropic, but this approach will provide us with a very clear benchmark, equivalent to the idea of Pareto optimum. Since utilities are quasi-linear, social surplus maximization yields efficient policies and allocations.

The philanthropic university will be financed by subsidies (or by donations), in the case of a deficit. The required amount of public resources (or donations) is simply,

$$D = C(x, e, v) - px. \tag{7}$$

The amount $D$ will be subtracted from social surplus to balance the university budget at time 0. We therefore suppose that the share of total cost that the students of a given cohort do not pay for in the form of tuition fees will be paid in the form of (lump-sum) taxes, or contributions, made by the same (or other) agents, such as alumni donations, etc. There are of course more subtle relations between public pricing, public subsidies voluntary contributions, and the tax system, involving political economy and redistribution problems that will not be studied in the present analysis. In particular, we assume that the social justice problems are solved by means of other redistributive tools, in the hands of independent public authorities. More precisely, we assume that equality of opportunity problems are solved in the sense that no student with the necessary talents is barred from studying because of a financial constraint. We are perfectly aware of the fact that this picture is a bit too rosy, and that more sophisticated modelling work on the case in which imperfections of credit markets and family-background differences create unequal opportunities is needed. We address the borrowing constraints question in the final section of this paper. In the present section and the following ones, our analysis aims at showing the simple structure of the optimal higher education pricing problem in a pure efficiency case, which can again be seen as a benchmark.

We will of course contrast the philanthropic view with another, less optimistic theory of the university objective: the "cynic view". Under this latter view, the university is assumed to maximize its rent $R$, which is the difference between the total sum of tuition charges and total university cost, that is,

$$R = px - C(x, e, v). \tag{8}$$

This rent can be understood as the amount of resources made available to finance faculty activities other than teaching. In academic systems in which teacher's careers essentially depend on research achievement, it is likely that research will be a major faculty objective (although there is of course no guarantee), so that $R$ might as well stand for research. But $R$ can of course easily be interpreted as profit, and our model is then that of a for-profit university.

Under both views, the university is endowed with market power; this is an approximation for a situation in which, say, a centrally governed public network of universities has a quantitatively important share of the higher education market, but the model can as well capture the behaviour of a private university with a dominant position.

## 3. Asymmetric Information and Entrance Examinations

### *3.1. Assumptions about Information*

We can now start the study of philanthropic and cynic views of the university, under the assumption that the students, public regulator, and Faculty, are asymmetrically informed. We assume that students do not observe their ability variables $(\theta, \eta)$. They are endowed with incomplete knowledge of their own talent, formed with the help of noisy informative signals. Students are assumed to observe a private signal of capitalized ability $\theta$ only. Variable $\eta$ is not observed at time zero, and we do not model the information of students about this kind of ability, because it would play no role in the following. Let $s$ denote the unbiased private signal of students. By definition,

$$s = \theta + \varepsilon, \tag{9}$$

where $\varepsilon$ is an independent normal random variable with a zero mean.

In addition, a costless examination technology provides an estimation of ability which is publicly observable. The examination grade is a random variable denoted $z$, and is defined as follows:

$$z = \theta + \nu, \tag{10}$$

where $\nu$ is normally distributed, with a zero mean, and is independent from $\theta$ and $\varepsilon$.

In this section, the grade $z$ is known to the student and to the Faculty (*i.e.*, the university authorities). This examination can be interpreted as a national high school exam, (such as baccalauréat in France, or Abitur in Germany), or $z$ can be viewed as an entrance-test score. The university can set a pass mark $\bar{z}$; a student is then admitted for registration only if his or her grade $z$ is greater than $\bar{z}$.

We assume that students are rational, Bayesian, expected utility maximizers. In other words, using a term coined by Manski (1993), our students are "adolescent econometricians". The value of higher education can now be expressed in expected terms, conditional on the two signals $(s, z)$. Using (4), and the assumptions on $\hat{\eta}$ made above, we get

$$E(u_0 \mid s, z) = \bar{u}_0 \equiv w_0 + \frac{\ln(\hat{w}_0)}{r}, \tag{11}$$

where we use the fact that $\exp(\hat{\eta})$ is log-normal, and where $w_0 = \hat{w}_0 \exp[\sigma_\eta^2/2]$.

Using (5) and (6), the expected utility of higher education is obtained as,

$$E(u_1 \mid s, z) = -p + \frac{\ln(\hat{w}_0)}{r} + \Delta(x, e) + E\left(\theta \mid s, z\right). \tag{12}$$

An individual then applies for higher education if and only if $E(u_1 - u_0 \mid s, z) \geq 0$, that is, equivalently, if and only if,

$$y \equiv E\left(\theta \mid s, z\right) \geq \bar{y} \equiv p + w_0 - \Delta(x, e). \tag{13}$$

The random signal $y = E(\theta \mid s, z)$ is the student's rational expectation of her own ability, and $\bar{y}$ is the total expected cost of education $p + w_0$, including the opportunity cost, minus the discounted utility of the skill premium $\Delta$.

Given that $\theta$, $\varepsilon$ and $\nu$ are normal, $y$ is itself normal. Let $\sigma_\theta^2$, $\sigma_\varepsilon^2$, $\sigma_\nu^2$, be the variances of $\theta$, $\varepsilon$, and $\nu$, respectively. Some computations, using the normality assumption yield the classic result,

$$E(\theta \mid s, z) = \frac{s\sigma_\theta^2\sigma_\nu^2 + z\sigma_\theta^2\sigma_\varepsilon^2}{\sigma_\theta^2\sigma_\nu^2 + \sigma_\theta^2\sigma_\varepsilon^2 + \sigma_\nu^2\sigma_\varepsilon^2}. \tag{14a}$$

This expression being linear with respect to $s$ and $z$, we have,

$$y \equiv \alpha s + \beta z, \tag{14b}$$

where the values of $\alpha$, and $\beta$ are defined by identifying the latter expression with (14a).

With this specification, a student is enrolled if she is willing to apply and if she satisfies the requirements of the entrance selection process based on $z$, that is, if and only if,

$$y \geq \bar{y} \quad \text{and} \quad z \geq \bar{z}. \tag{15}$$

If the potential student population is of size $N$, effective demand can be written,

$$x = q(\bar{y}, \bar{z}) = N \Pr(y \geq \bar{y},\ z \geq \bar{z}). \tag{16}$$

### *3.2. The Philanthropic View: Optimal Examination and Tuition Fees*

Expected social surplus can be written,

$$W = xE\left[u_1 \mid y \geq \bar{y}, z \geq \bar{z}\right] + (N - x)\bar{u}_0 + px - C(x, e, v). \tag{17}$$

Expression (17) can be interpreted as the total sum of the $x$ graduates' expected utilities, and of the remaining $(N-x)$ unskilled workers' utilities, less the university's deficit, which must be covered by public subsidies, or donations. Define,

$$v(\bar{y}, \bar{z}) \equiv E\left[\theta \mid y \geq \bar{y}, z \geq \bar{z}\right]. \tag{18}$$

Function $v$ is the expected ability of enrolled students, knowing that their grade is greater than $\bar{z}$ and that their private assessment $y$ is greater than $\bar{y}$. $W$ can be maximized with respect to $(e, \bar{y}, \bar{z})$, instead of $(e, p, \bar{z})$, given that $\bar{y} = p + w_0 - \Delta$. After some simplifications, we get,

$$W(x, e, \bar{y}, \bar{z}) = x\left[\Delta(x, e) - w_0 + v(\bar{y}, \bar{z})\right] - C[x, e, v(\bar{y}, \bar{z})] + N\bar{u}_0, \tag{19}$$

which must be maximized with respect to $(x, e, \bar{y}, \bar{z})$ subject to the constraint $x = q(\bar{y}, \bar{z})$.

This maximization problem can be decomposed into two sub-problems. A first sub-problem is to maximize $xv(\bar{y}, \bar{z}) - C(x, e, v(\bar{y}, \bar{z}))$ with respect to $(\bar{y}, \bar{z})$, for given $(e, x)$. The second sub-problem is then to maximize $W$ with respect to $(x, e)$, given that the optimal $(\bar{y}, \bar{z})$ have been expressed as functions of $x$.

Given that $C_v \leq 0$, the first sub-problem is tantamount to maximizing $v(\bar{y}, \bar{z})$ subject to $x = q(\bar{y}, \bar{z})$, with respect to $(\bar{y}, \bar{z})$, for fixed $x$. The necessary conditions for an optimal pair $(\bar{y}, \bar{z})$ (if it is finite!) are simply

$$\frac{v_{\bar{y}}}{v_{\bar{z}}} = \frac{q_{\bar{y}}}{q_{\bar{z}}}, \qquad \text{and} \qquad x = q(\bar{y}, \bar{z}), \tag{20}$$

where subscripts denote partial derivatives. The interpretation of condition (20) is easy if it is reminded that, $v_{\bar{y}} = \partial v/\partial \bar{y} = \partial v/\partial p$, $q_{\bar{y}} = \partial q/\partial \bar{y} = \partial q/\partial p$; it says that the marginal rate of substitution between $p$ and $\bar{z}$ should equal its marginal rate of transformation, conditional on the fixed production target $x$.

To study the existence of solutions to the system of equations (20), we state two technical Lemmata. The first one is a key to what follows.

**Lemma 1.** *$y$ is a sufficient statistic for $\theta$, and,*

$$E[\theta \mid y, z] = y \tag{21}$$

*For proof, see the Appendix*

Intuitively, Lemma 1 says that, being a statistically optimal combination of the two signals $s$ and $z$, $y$ conveys all the useful (private and public) information about an individual's ability. Lemma 1 is proved without assuming normality, as can be seen in the Appendix. The next result will be useful for the study of equations (20).

**Lemma 2.** *$v_y/q_y \geq v_z/q_z$ is equivalent to $h_y(\bar{y}, \bar{z}) \geq h_z(\bar{y}, \bar{z})$, where, by definition,*

$$h_y(\bar{y}, \bar{z}) \equiv E(\theta \mid y = \bar{y}, z \geq \bar{z}), \tag{22$a$}$$

$$h_z(\bar{y}, \bar{z}) \equiv E(\theta \mid y \geq \bar{y}, z = \bar{z}), \tag{22$b$}$$

$$\frac{v_y}{q_y} = \frac{1}{q}(h_y - v), \tag{22$c$}$$

*and (22c) also holds with $z$ instead of $y$.*

*For proof, see the Appendix*

From Lemma 2, we must solve $h_y = h_z$ to solve (20). From Lemma 1, we derive,

$$h_y = E[\theta \mid y = \bar{y}, z \geq \bar{z}] = E[E(\theta \mid y, z) \mid y = \bar{y}, z \geq \bar{z}] = \bar{y}, \tag{23}$$

and we get,

$$h_z = \; E[\theta \mid y \geq \bar{y}, z = \bar{z}] = E[E(\theta \mid y, z) \mid y \geq \bar{y}, z = \bar{z}] = E[y \mid y \geq \bar{y}, z = \bar{z}]. \tag{24}$$

These results have the following striking consequence.

**Proposition 1.** *If the grade or test result z is publicly observed and s is privately observed by the students, then, the optimal solution involves $\bar{z} = -\infty$, i.e., admission standards are the lowest possible; optimal screening is performed by means of the tuition fee only. There does not exist a finite solution to (20).*

*For proof, see the Appendix*

The meaning of Proposition 1 can be rephrased as follows. In a world in which economic agents are perfectly rational (i.e., if they are good enough Bayesian statisticians), the university can safely rely on student self-selection through the pricing mechanism only. An optimal tuition $p^*$ is therefore the only useful tool, and selection by means of an admission standard is superfluous, provided that students can assess their ability by conditioning on the publicly disclosed grade $z$.

Thus, according to the philanthropic view, social value maximization doesn't lead the university to make use of an optimal "policy mix" involving pricing and selection on the basis of test scores. We study variants of our model leading to less radical conclusions in the following sections[10]. Let us now examine the pricing behaviour of the philanthropic university.

Given Proposition 1, define

$$\hat{q}(\bar{y}) = \lim_{\bar{z}\to-\infty} q(\bar{y}, \bar{z}) \tag{25$a$}$$

and

$$\hat{v}(\bar{y}) = \lim_{\bar{z}\to-\infty} v(\bar{y}, \bar{z}) = E(\theta \mid y \geq \bar{y}), \tag{25$b$}$$

and rewrite the philanthropic objective (21) as

$$W = \hat{q}(\bar{y})\,[\Delta(\hat{q}(\bar{y}), e) - w_0 + \hat{v}(\bar{y})] - C[\hat{q}(\bar{y}), e, \hat{v}(\bar{y})] + N\bar{u}_0 \qquad (25c),$$

to be maximized with respect to $(e, \bar{y})$. The first-order conditions for the maximization of (25c) yield,

$$\hat{q}\Delta_e = C_e \tag{26$a$}$$

with obvious notations, as a condition "determining" optimal quality $e$, and, after some rearrangement of terms,

$$\Delta - w_0 + \hat{v} + \hat{q}\Delta_x + \hat{q}\frac{\hat{v}_y}{\hat{q}_y} = C_x + C_v\frac{\hat{v}_y}{\hat{q}_y}. \tag{26$b$}$$

Conditions (26a) and (26b) are easily interpreted. Equation (26a) says that the social value of quality $x\Delta_e$ must be equal to its marginal cost $C_e$, while equation (26b) says that

10 In a complete information setting, i.e., $\sigma_\nu^2{=}\sigma_\varepsilon^2{=}0$, it is possible to show that pricing and pre-entry screening on the basis of (observable) ability are perfect substitutes in the philanthropic university's problem, because the optimum is defined in terms of a threshold $\bar{\theta}$, and quality $e$ only. In this case, the model becomes formally analogous to that of a monopoly choosing both quantity and quality, as analysed by Spence (1975), and Sheshinski (1976).

the marginal social value of a graduate must be equal to its marginal cost, at the optimum. The marginal value of a graduate is the sum of three terms: first, the average value of a graduate, $\Delta - w_0 + \hat{v}$, which is the marginal social value of the skills produced by the university; second, the effect $\hat{q}\Delta_x$ of an additional graduate on the total skilled workers' skill premium, which might be positive or negative[11], depending on the sign of $\Delta_x$; and third, the total effect of an additional graduate on average quality $\hat{v}$, which is $\hat{q}\hat{v}_y/\hat{q}_y \leq 0$, because $\hat{q}_y \leq 0$ while $\hat{v}_y \geq 0$. The marginal cost is itself the sum of two terms: the first is the direct marginal cost $C_x$, the second, which is also positive (being the product of two negative terms) is the marginal peer effect. Increasing $x$ by one unit reduces the average quality of students; hence, it reduces the peer-group effect, which increases the cost by

$$C_v \hat{v}_y \frac{\partial \bar{y}}{\partial x} \geq 0,$$

where $\partial \bar{y}/\partial x = 1/\hat{q}_y$.

From (26), with some reworking, we get the next result.

**Proposition 2.** *If $z$ is publicly observed, and if $\Delta_x \leq 0$, the optimal tuition fee $p^*$ of the philanthropic university is higher than marginal cost $C_x$. The optimal tuition is thus positive.*

*Proof:* To prove Proposition 2, remark first that, using the result of Lemma 2, i.e., $\hat{v}_y/\hat{q}_y = (1/\hat{q})(\hat{h}_y - \hat{v})$, where, by definition,

$$\hat{h}_y = \lim_{\bar{z} \to -\infty} E(\theta \mid y = \bar{y}, z \geq \bar{z}) = E(\theta \mid y = \bar{y}),$$

and using $p = \bar{y} + \Delta - w_0$, equation (26b) can be rewritten as,

$$p^* - C_x = -\hat{q}\Delta_x + \frac{C_v}{\hat{q}}(\hat{h}_y - \hat{v}) + (\bar{y} - \hat{h}_y). \tag{27}$$

Remark then that the first term on the right-hand side of (27) is nonnegative since by assumption, $\Delta_x \leq 0$. We show next that the last term is zero. Using the properties of conditional expectations, we get, $E(\theta \mid y) = E[E(\theta \mid y, s, z) \mid y] = E(y \mid y) = y$. Thus $\hat{h}_y = \bar{y}$. Finally, we get,

$$\hat{h}_y = \bar{y} \leq E[y \mid y \geq \bar{y}] = E[E(\theta \mid y) \mid y \geq \bar{y}] = E(\theta \mid y \geq \bar{y}) = \hat{v},$$

and therefore, since $C_v \leq 0$, the second term on the right-hand side of (27) is nonnegative. The right-hand side of (27) is therefore a sum of nonnegative terms; we conclude that $p^* \geq C_x > 0$.
*Q.E.D.*

[11] Intuitively, an additional skilled worker lowers the wage of all other graduates on the labour market. At the optimum, the university must take this effect into account and should not flood the market with too many skilled workers, insofar as there are no important externalities associated with higher education.

Tuition is equal to the marginal social cost of graduates, which is higher than $C_x$, because it internalizes two external effects: $-\hat{q}\Delta_x$ and the peer effect $(C_v/\hat{q})(\hat{h}_y - \hat{v})$.

If we assume $\Delta_x \geq 0$, because we believe that higher education produces strong externalities, and say, $C_v = 0$, then, of course, we get $p^* \leq C_x$ from (27): it could be optimal to subsidize students.

Now, if we are willing to assume that the marginal cost of higher education is non-decreasing, and that the fixed cost is well defined, we get a little more. Define the fixed cost $K$ as $K(e, v) = \lim_{x \to 0^+} C(x, e, v)$. Then, given the result of Proposition 2, and the convexity of $C$ with respect to $x$, we get $p^*\hat{q} \geq C_x(\hat{q}, e, v)\hat{q} \geq C(\hat{q}, e, v) - K(e, v)$, and we can state the following.

**Corollary.** *If $\Delta_x \leq 0$, and if the university cost function is convex with respect to the number of enrolled students, then, at a social optimum, university revenues $px$ must be greater than variable costs $C - K$.*

It may then be the case that part of the university fixed cost will be covered by a public subsidy, or another source of revenue.

*3.3. The Cynic Approach: the Rent-Seeking University's Policy*

Let us now study the rent-seeking university in the same asymmetric information framework, where $z$ is publicly observed. The rent-seeking university will try to maximize $R = pq(\bar{y}, \bar{z}) - C(q(\bar{y}, \bar{z}), e, v(\bar{y}, \bar{z}))$, subject to $p = \bar{y} - w_0 + \Delta(q(\bar{y}, \bar{z}), e)$. The fee $p$ can be eliminated from the expression of rent, which becomes

$$q(\bar{y}, \bar{z})\left[\bar{y} + \Delta(q(\bar{y}, \bar{z}), e) - w_0\right] - C(q(\bar{y}, \bar{z}), e, v(\bar{y}, \bar{z})), \tag{28}$$

and must be maximized with respect to $(e, \bar{y}, \bar{z})$. It would be easy to prove that the rent-seeking university would not like to ration students (*i.e.,* choose to set $x < q$), for it would then always benefit from an increase of the tuition $p$. The rent maximization problem can be decomposed into two steps. For fixed values of $(x, e)$, the thresholds $(\bar{y}, \bar{z})$ can first be set so as to maximize $x\bar{y} - C(x, e, v(\bar{y}, \bar{z}))$ subject to $x = q(\bar{y}, \bar{z})$. This yields the first-order conditions,

$$\frac{x}{C_v q_y} = \frac{v_y}{q_y} - \frac{v_z}{q_z}. \tag{29}$$

Given that $C_v q_y > 0$, due to peer effects, and by Lemma 2, (29) implies that the rent-seeking optimum should satisfy $h_y > h_z$, but, for finite values of $\bar{z}$, this is again impossible. We can state,

**Proposition 3.** *If $z$ is publicly observed, the optimal rent-seeking (or for-profit) university policy is to set $\bar{z} = -\infty$, i.e., admission standards are the lowest possible and tuition does all the screening job.*

*For proof, see the Appendix*

If the peer effects were negligible, *i.e.,* $C_v = 0$, it would be easy to provide a proof of the latter result. It would then always be profitable to increase $\bar{y}$ by $d\bar{y} > 0$ and to reduce $\bar{z}$ by $d\bar{z} = -(q_y/q_y)d\bar{y}$, keeping $x = q(\bar{y}, \bar{z})$ (and thus the cost $C$) constant, for that would increase the rent $R$ by $dR = xdp = xd\bar{y} > 0$. The matter is slightly more complicated in the presence of a peer effect, and the result is driven by the fact that $h_y < h_z$ holds for every finite value of $\bar{z}$.

With the help of definitions (25a)-(25b), rewrite the rent as

$$R = (\bar{y} + \Delta(\hat{q}(\bar{y}), e) - w_0)\hat{q}(\bar{y}) - C(\hat{q}, e, \hat{v}(\bar{y})),$$

to be maximized with respect to $(e, \bar{y})$. A comparison of (25c) and the above expression of rent obviously shows that the rent-seeking policy is not optimal in the philanthropic sense. We can at least say that the rent seeker's quality $e$ would be socially optimal if the rent seeker chose $\bar{y}$ optimally, since (26a) is a first-order condition for social welfare as well as rent maximization. But the rent-seekers become dedicated philanthropists if they are subjected to a certain public incentive transfer.

**Proposition 4.** *The rent-seeking university chooses a socially optimal policy if it is subjected to the following transfer $T$, defined as,*

$$T(e, x, \bar{y}) = x[\hat{v}(\bar{y}) - \bar{y}] + T_0, \tag{30}$$

*where $T_0$ is any constant.*

The proof of this result is obvious since $R + T \equiv W +$ Constant.

In practice, we know that the rent-seeking faculty will tend to underestimate the social value of educating students, because they take $\bar{y}$, the marginal student's value into account, instead of $\hat{v}(\bar{y})$, the average student's value, and that $\bar{y} \leq \hat{v}(\bar{y})$ (as shown by the proof of Proposition 2 above). The transfer function can be approximated by a per capita subsidy, which aims at correcting the gap between marginal and average values, minus a lump sum tax. Equation (30) shows that the transfer $T$ depends on $\bar{y}$, $x$ and knowledge of $\hat{v}$, but the variable part of $T$ can be computed without any reference to the university cost function $C$.

The public authority or public regulator can compute the (variable part of the) transfer with the help of an econometric model with selectivity à la Heckman, if they observe wages $w$, $w_0$, tuition $p$ and the public signal $z$ (see Heckman (1979)). In addition, this method allows complete identification of our theoretical model. To see this, consider first the Mincerian regression function $\ln(w/w_0) = \widehat{\Delta} + r\theta$. A latent variable $y$ determines student self-selection. By (14b), $y = \beta z + \zeta$, where $\zeta = \alpha s$, but the private signals $s$ are not observed. The error term $\zeta$ has a zero mean and a variance $\sigma_\zeta^2$, to be estimated. Let $\sigma_{\theta\zeta}$ denote the covariance of error terms. Let $\Phi$ be the normal c.d.f., and $\phi$ be the normal density. Now, using classic results, we can write,

$$\hat{v} = E(\theta \mid y \geq \bar{y}) = \left(\frac{\sigma_{\theta\zeta}}{\sigma_\zeta}\right) E\left[\frac{\zeta}{\sigma_\zeta} \,\middle|\, \frac{\zeta}{\sigma_\zeta} \geq \frac{\bar{y} - \beta z}{\sigma_\zeta}\right] = \left(\frac{\sigma_{\theta\zeta}}{\sigma_\zeta}\right) \frac{\phi((\bar{y} - \beta z)/\sigma_\zeta)}{1 - \Phi\left((\bar{y} - \beta z)/\sigma_\zeta\right)}.$$

We can first estimate $\bar{y}/\sigma_\zeta$ and $\beta/\sigma_\zeta$ by means of a Probit. Then, the estimated $\phi(.)/(1 - \Phi(.))$ variable can be added to the right-hand side of the log-wage regression, to estimate $\widehat{\Delta}$ and $r\sigma_{\theta\zeta}/\sigma_\zeta$, which is the coefficient of $\phi(.)/(1 - \Phi(.))$. The problem is then to retrieve the 7 parameters $\bar{y}$, $\alpha$, $\beta$, $\sigma_\zeta$, $\sigma_{\theta\zeta}$, $\sigma_\theta$ and $r$ from these estimates. This can be done if we use (13), (14) and an identification restriction derived from student rationality as follows. Since $y$ is given by (14), with the help of (9), we get $\sigma_{\theta\zeta} = \text{Cov}(\theta, \alpha s) = \alpha\sigma_\theta^2$. It can be shown that this additional restriction is sufficient to identify all the model's parameters. The model could of course be enriched by the introduction of other observable student characteristics in the definition of $y$, and various job characteristics or other controls in the wage equation, without any problem[12].

The conclusions reached in this section are somewhat surprising, because the social optimum doesn't rely on exams or test scores, but only on money. Yet, these results are probably less robust than it seems, for they depend on the property that the university's information set is strictly included in that of the student. A balance between the two screening tools appears to be an optimum if we assume a form of *bilateral asymmetric information*, in which the university knows something that the student doesn't take into account about his (her) own ability. We now turn to the study of such a setting, in a variant of our model.

## 4. Bilateral Asymmetric Information

We now assume that the university is endowed with information about student abilities that students themselves do not have, while continuing to assume that students observe a noisy private signal of ability $s$. We have in mind that the Faculty's pre-entry evaluation of an applicant is a kind of "soft" information, that cannot be disclosed easily, due to the examination technology. To fix ideas, assume that $z$ is the assessment of an application file, and (or) the grading of an interview, assumed costless for simplicity, and observed by academic authorities only. In other words, students do not know the test result $z$, and do not know how this result is used to make an admission decision. Our favorite interpretation is that if $z$ is not revealed to students, it is not because the Faculty don't want to disclose the information, but because they can't. If we interpret $z$ as a private signal of the university about the student's ability, we can easily construct a variant of our model in which informational asymmetries are bilateral. It then seems to us that such a bilateral asymmetric-information context is more "generic", and therefore more realistic and robust, than the case studied above, insofar as it is characterized by a clear-cut nesting of information sets. The teachers are, say, more able to assess some of the student's talents (and their labour market value), but the student remains better informed about her own ambitions and tastes[13].

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[12] In a complete information framework, $\hat{v}(\bar{y}) - \bar{y}$ would be the difference between the average wage of a graduate and its minimal value.

[13] The same kind of bilateral asymmetry of information may hold on insurance markets, where insurers can usually make more accurate risk assessments than the insured themselves. A driver knows a number of things, relevant to assess her risk of accident, that the insurer doesn't know, but might at the same time not be aware

Let $\pi(s)$ be the student's subjective probability of admission. Using the same notation as above, unless specified otherwise, an individual will apply for registration if and only if $E(u_1 - u_0 \mid s) \geq 0$, where,

$$E(u_1 \mid s) = \pi(s)\left[-p + \frac{\ln(\hat{w}_0)}{r} + \Delta + E(\theta \mid s)\right] + (1 - \pi(s))\left[w_0 + \frac{\ln(\hat{w}_0)}{r}\right], \qquad (31a)$$

and

$$E(u_0 \mid s) = E(u_0) = w_0 + \frac{\ln(\hat{w}_0)}{r}. \qquad (31b)$$

It follows[14] that $E(u_1 - u_0 \mid s) \geq 0$ if and only if

$$y \equiv E(\theta \mid s) \geq \bar{y} \equiv w_0 + p - \Delta. \qquad (32)$$

Conditional expectations are linear, and coincide with the notion of theoretical regression. As a consequence of normality, we get,

$$y = \alpha_0 s, \qquad \text{where} \qquad \alpha_0 = \frac{\mathrm{Cov}(\theta, s)}{\mathrm{Var}(s)} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2}. \qquad (33)$$

The number of enrolled (admitted) students is still $q(\bar{y}, \bar{z}) = N \Pr(y \geq \bar{y}, z \geq \bar{z})$.

### *4.1. Philanthropic Optimum under Bilateral Asymmetric Information*

Using the new definition of $y$, the philanthropic objective of the university has the same expression as above, that is, $W = x(\Delta(x,e) - w_0 + v(\bar{y}, \bar{z})) - C[x, e, v(\bar{y}, \bar{z})] + Nu_0$, where $v = E[\theta \mid y \geq \bar{y}, z \geq \bar{z}]$, and of course, $x = q(\bar{y}, \bar{z})$.

It follows that the optimal $(\bar{y}, \bar{z})$ for given $(x, e)$ are still a solution of system (20), i.e., $v_y/q_y = v_z/q_z$, and $x = q$, and by Lemma 2, which still applies, (20) is equivalent to $h_y = h_z$, where the $h$ functions are still defined by (22a)-(22b).

There are differences with the version of the model studied in Section 3, starting from this point. Intuitively, $z$ and $y$ now play a symmetric informational part, and $y$ is no longer a sufficient statistic for $(y, z)$. We can state,

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of other risk determinants that the insurer, who processes a lot of observations, knows how to translate into accident probabilities with much higher precision.

[14] If the student knew $\bar{z}$, and knew that $z \geq \bar{z}$ implies admission, then, even if $z$ is not observed, the admission decision would convey information about ability. The expression $E(\theta|s)$ should be replaced with $E(\theta|s, z \geq \bar{z})$, and $\pi(s)$ should be replaced with $\Pr(z \geq \bar{z}|s)$ in (31a). If the students knew that there is an admission region of the form $\{z \geq \bar{z}\}$, but did not knew the threshold $\bar{z}$ itself, then we should average the resulting expressions with respect to some subjective prior distribution of $\bar{z}$. The rigorous analytical treatment of these cases, needless to say, is much more complex, but our approach is that neither $z$ nor $\bar{z}$ can be communicated.

**Lemma 3.**

$$E[\theta \mid y, z] = a_0 y + (1 - a_0) z, \tag{34a}$$

*where,*

$$a_0 = \frac{\sigma_\nu^2}{\sigma_\nu^2 + (1 - \alpha_0)\sigma_\theta^2} \tag{34b}$$

$\alpha_0$ *being defined by (33).*

*For proof, see the appendix.*

In this new setting, the system of equations (20) has a (finite) solution. Let $\phi(x) = (2\pi)^{-1/2} e^{-x^2/2}$ and $\Phi(x) = \int_{-\infty}^{x} \phi(u) du$ denote the normal density and c.d.f. We can state the following.

**Proposition 5.** *If $z$ is a private information of the university and $s$ is a private information of the student, the optimal policy of the philanthropic university involves a mix of non-trivial admission standards $\bar{z}^*$ and tuition $p^*$. Formally, equations (20) possess a solution $(\bar{y}^*, \bar{z}^*)$ for every given $x > 0$. More precisely, this solution is fully characterized as follows:*

$$\bar{y}^* = \alpha_0(\bar{z}^* - \sigma_0 \xi^*), \tag{35a}$$

*where $\xi^*$ solves the equation,*

$$\xi = \left( \Phi(\xi) - \frac{\sigma_\nu^2}{\sigma_\epsilon^2 + \sigma_\nu^2} \right) \frac{\phi(\xi)}{(1 - \Phi(\xi))\Phi(\xi)}, \tag{35b}$$

*$\bar{z}^*$ solves,*

$$x = q[\alpha_0(\bar{z} - \sigma_0 \xi^*), \bar{z}], \tag{35c}$$

*and finally, $\sigma_0 = \sqrt{\sigma_\epsilon^2 + \sigma_\nu^2}$.*

*For Proof, see the Appendix.*

Proposition 5 provides us with a much more reasonable description of the world than Proposition 1. To clarify its meaning, assume for instance that $\sigma_\epsilon^2 = \sigma_\nu^2$ (*i.e.*, signals $z$ and $s$ are "equally noisy"). Then, by (35b), we find that $\xi^* = 0$ is the only solution (because $\Phi(0) = 1/2$), and by (35a), we get $\bar{y}^* = \alpha_0 \bar{z}^*$: the first order condition in (20) has provided us with a linear relationship between $\bar{y}$ and $\bar{z}$. The "production level" $x$ pins down the appropriate value of $\bar{z}$, as indicated by (35c).

The distance between $\bar{y}$ and $\alpha_0 \bar{z}$ depends on the ratio $\sigma_\epsilon^2/\sigma_\nu^2$. To see this, define,

$$\lambda = \frac{\sigma_\nu^2}{\sigma_\epsilon^2 + \sigma_\nu^2}.$$

Then, a simple application of the Implicit Function Theorem shows that

$$\frac{\partial \xi^*}{\partial \lambda} < 0,$$

*i.e.,* $\xi^*$ decreases when $\sigma_\nu^2/\sigma_\epsilon^2$ increases. Using (35a), it is then easy to see that the distance $\bar{y}^* - \alpha_0\bar{z}^*$ increases when $\lambda$ increases, that is, formally,

$$\frac{\partial(\bar{y}^* - \alpha_0\bar{z}^*)}{\partial\lambda} = -\alpha_0\sigma_0\frac{\partial\xi^*}{\partial\lambda} > 0.$$

This result is intuitive, if test scores $z$ become more noisy than private signals $s$, then, less weight should be placed on selection by means of test scores, *i.e.,* $\bar{y}^* - \alpha_0\bar{z}^*$ should increase. This can be achieved if admission standards $\bar{z}^*$ are lowered and (or) tuition fees are raised conditional on $(x, e)$.

Now, maximization of the philanthropic objective $W$ with respect to $(e, \bar{y}, \bar{z})$ yields — after some rearrangement of terms — the following first-order necessary conditions,

$$q\Delta_e = C_e, \qquad h_y = h_z, \tag{36$a$}$$

$$q\Delta_x + \Delta - w_0 + v - C_x = \left(1 - \frac{C_v}{q}\right)(v - h_y). \tag{36$b$}$$

But it is no longer possible to show that (36a) and (36b) jointly imply $p^* > 0$. It happens that $p^*$ could be negative, a personal subsidy instead of a fee, even if $\Delta_x \leq 0$. More precisely, we get the following result.

**Proposition 6.** *Assume that peer effects are negligible, i.e., $C_v = 0$, and that $\Delta_x \leq 0$. Then, in the bilateral asymmetric information version of the model, the optimal tuition is smaller than marginal cost, or even negative if the test score $z$ is accurate enough as a measure of ability. Formally, for sufficiently small $\sigma_\nu$, $p^* < 0$.*

*Proof:* From (36b) and $C_v = 0$, we get, using $\bar{y} = p + w_0 - \Delta$,

$$p^* = -q\Delta_x + C_x + (y - h_y).$$

Using the results obtained in the proof of Proposition 5, if $\sigma_\nu \to 0$, then $a_0 \to 0$, $h_z \to \bar{z}$ and $h_y \to E[z \mid z \geq \bar{z}, y = \bar{y}]$. At the optimum, since $h_z = h_y$, it must then be true that

$$E[z \mid z \geq \bar{z}^*, y = \bar{y}^*] \to \bar{z}^*.$$

But this is possible only if $\bar{z}^* \to +\infty$ and (or) $\bar{y}^* \to -\infty$. If $\sigma_\nu$ is sufficiently close to 0, we get $p^* < 0$ because $(\bar{y}^* - h_y) \sim (\bar{y}^* - \bar{z}^*) \to -\infty$ (the argument is the same as in Proposition 1).
*Q.E.D.*

Proposition 6 captures in part the idea that some higher education institutions would at the same time be highly selective, and subsidize talented students to lure them into their classrooms. Winston (1999) shows that top universities and colleges in the US do indeed at the same time seem to be those who offer the highest subsidy to students, in view of unit cost information. Intuitively, if the student's private signals $s$ are very poor as indicators of talent, but if the university admission test technology is very precise, it could be optimal to select only the very best and to subsidize them heavily, to be sure that no good element is deterred by the price. Remark that the result does not depend on a redistribution motive of the philanthropic university (because their objective $W$ is quasi-linear with respect to period 0 income). Proposition 6 is also independent of the existence of peer group effects since it is proved under $C_v = 0$. Proposition 6 shows that negative fees can improve selection when the university can condition admission on sufficiently accurate information about student's talents. Let us now compare the philanthropic and rent-seeking universities under the same bilateral asymmetric information assumptions.

### *4.2. The Cynic View under Bilateral Asymmetric Information*

The rent $R$ is still $R = (\bar{y} + \Delta(x, e) - w_0)x - C[x, e, v(\bar{y}, \bar{z})]$, with $x = q(\bar{y}, \bar{z})$. Decomposing the problem again, it is easy to see that for fixed $(x, e)$, we must have (29) again. This implies $v_y/q_y > v_z/q_z$, and, by Lemma 2, $h_y > h_z$. The screening policy of the rent-seeking university will therefore not be socially optimal, because optimality requires equality of the latter two terms.

To see what happens in this case, use condition $h_y > h_z$, and with the help of the statement and proof of Proposition 5 above, it can be shown that the solution will be as that given by (35a) above, except that $\xi^*$ is replaced with a value $\xi^{**} < \xi^*$. It follows that $\bar{y} - \alpha_0 \bar{z}$ will be greater under rent-seeking than at the (philanthropic) optimum. From this, and the above remark, we conclude that the rent-seeking university will set higher fees and (or) lower selection standards $\bar{z}$ than the philanthropic university, for any given value of $(x, e)$.

## 5. Borrowing Constraints and Asymmetric Information

We now come back to the incomplete information setting of Section 3 (where $z$ is publicly observed), and address the question of financial constraints in human capital accumulation. Some students, due to borrowing constraints, would not be able to pay the tuition fee, in spite of having received very good signals relative to their future ability and earnings. Due to asymmetric information, a banker would not lend enough money to a student without collateral.

For simplicity, assume that the student's "initial financial endowment" (or "asset") is a normal random variable $b$ with mean $\bar{b}$ and variance $\sigma_b^2$. Assume that $b$ is independent of every other random source in the model. We will now use the simplest of all models of a banker, but this will be sufficient to show how results change with borrowing constraints. Assume that if $p > b$, a student must borrow, and that the lender attaches a "score" to each student, where the score is defined as $(\iota + \Delta)$, and $\iota$ is an independent normal random noise with mean 0 and variance $\sigma_\iota^2$. This random noise reflects the errors of appreciation made by the banker. Now, we assume that the lender will finance the student's education project if and only if

$$p \leq b + \kappa(\Delta + \iota), \tag{37}$$

where $\kappa$ is a coefficient satisfying $0 \le \kappa \le 1$. Defining a new random variable $t = b + \kappa\iota$, the liquidity constraint (37) can be expressed as,

$$t \equiv b + \kappa\iota \ge p - \kappa\Delta \equiv \bar{t}, \tag{38}$$

or simply $t \ge \bar{t}$. Assume that, as in the asymmetric information model above, $t$, $z$ and $s$ are observed by the students and that the university observes $z$ only. Given independence, the effective demand for education is now

$$\tilde{q}(\bar{y}, \bar{z}, \bar{t}) = N \Pr(y \ge \bar{y}, z \ge \bar{z}) \Pr(t \ge \bar{t}) = q(\bar{y}, \bar{z}) \Pr(t \ge \bar{t}). \tag{39}$$

With the addition of the borrowing constraint, again because of independence, the average ability of students does not change, that is,

$$v(\bar{y}, \bar{z}) = E(\theta \mid y \ge \bar{y}, z \ge \bar{z}, t \ge \bar{t}) = E(\theta \mid y \ge \bar{y}, z \ge \bar{z}). \tag{40}$$

It follows that the philanthropic objective $W$ can still be expressed as $W = x(\Delta(x, e) - w_0 + v) + N\bar{u}_0 - C(x, e, v)$, where $x = \tilde{q}(\bar{y}, \bar{z}, \bar{t})$. A difference with the analysis of Section 3 above is that $\bar{t}$ depends on $\bar{y}$. To see this, recall that $p = \bar{y} + \Delta - w_0$, so that

$$\bar{t} \equiv \bar{y} + (1 - \kappa)\Delta - w_0, \tag{41}$$

and one should keep in mind that $\partial\bar{t}/\partial\bar{y} = 1$.

The optimal screening policy $(\bar{y}, \bar{z})$ maximizes $xv - C(x, e, v)$, for fixed $(e, x)$, subject to $x = \tilde{q}(\bar{y}, \bar{z}, \bar{t})$. We get the following system of first-order optimality conditions, using the fact that $\tilde{q}_y = q_y \Pr(t \ge \bar{t})$:

$$\left(\frac{\tilde{q}_y}{\tilde{q}_y + \tilde{q}_t}\right) \frac{v_y}{q_y} = \frac{v_z}{q_z}.$$

Using then (22c), we get the equivalent expression,

$$\left(\frac{\tilde{q}_y}{\tilde{q}_y + \tilde{q}_t}\right) h_y + \left(\frac{\tilde{q}_t}{\tilde{q}_y + \tilde{q}_t}\right) v = h_z \tag{42}$$

Since by Lemma 1 we have $h_y = \bar{y}$ when $z$ is publicly observed, we get the still equivalent,

$$\frac{\tilde{q}_t}{\tilde{q}_y}(v - h_z) = h_z - \bar{y}. \tag{43}$$

In this case, by Lemmata 1 and 2 again, we also have $h_z = E(y \mid y \ge \bar{y}, z = \bar{z})$.

Let $\phi(t; \bar{b}, \sigma_t)$ be the Gaussian density of $t$, and $\Phi(t; \bar{b}, \sigma_t)$ its c.d.f (recall that $t \sim \mathcal{N}(\bar{b}, \sigma_t^2)$). Using $\tilde{q}_y = (1 - \Phi(\bar{t}; \bar{b}, \sigma_t))q_y$, and $\tilde{q}_t = -\phi(\bar{t}; \bar{b}, \sigma_t)q$, we can rewrite again (43) as follows,

$$\frac{\phi(\bar{t}; \bar{b}, \sigma_t)}{(1 - \Phi(\bar{t}; \bar{b}, \sigma_t))} \big(v(\bar{y}, \bar{z}) - h_z(\bar{y}, \bar{z})\big) = \frac{-q_y(\bar{y}, \bar{z})}{q(\bar{y}, \bar{z})} \big(h_z(\bar{y}, \bar{z}) - \bar{y}\big). \tag{44}$$

Equation (44) is a generalization of (20) above, it is quite complex and hard to study, but it cannot be trivially solved with $\bar{z} = -\infty$.

**Proposition 7.** *If $x > 0$, the solution $(\bar{y}, \bar{z})$ of the system comprising equation (44), and $x = \tilde{q}(\bar{y}, \bar{z}, \bar{t})$, where $\bar{t}$ is given as a function of $\bar{y}$ by (41), if it exists, is finite ($\bar{z} = -\infty$ is not a solution of the system).*

*For proof, see the Appendix*

The addition of borrowing constraints justifies the recourse to selection on the basis of test scores and a form of direct student subsidization through tuition rebates.

We have found two cases in which a policy mix of pre-entry selection and pricing can be justified: it is either because universities have knowledge about student abilities that students themselves do not have (the case of bilateral asymmetric information), or because some good students are liquidity constrained, even if students and the university exploit the information conveyed by test scores rationally. In the latter case, it will also be optimal to raise admission standards, and hence, simultaneously decrease tuition (*i.e.*, decrease $\bar{y}$), to reduce the number of poor-but-talented students inefficiently deterred by price. (This is true since $\bar{z}$ is finite, and $q$ is decreasing with respect to both $\bar{y}$ and $\bar{z}$.)

## 6. Conclusion

We have shown that tuition & fees and selection on the basis of test scores are both used as screening instruments in an optimal university policy, if some students are liquidity constrained, or if asymmetric information is bilateral (in the sense that both the university and students possess useful, non-redundant private information about students' abilities). Our theory describes how the two screening instruments should be combined. We also showed that optimal tuition can entail an element of direct subsidy, a rebate, which is an increasing function of the precision of the university's information about student's abilities. Optimal tuition & fees can therefore be smaller than marginal cost, even if higher education doesn't generate global positive externalities. If there are no such externalities, price covers marginal cost only if the student's information set includes the university's information set, in which case entrance examinations are also useless. This means that a social optimum will often be characterized by the need for outside resources in the form of public money or donations, which are thus fully justified, to balance the university budget. Rent-seeking or for-profit teaching institutions will typically set prices too high and enroll too few students; nothing guarantees that the quality of the education they provide will be better than elsewhere. We have also shown how labour market information can be used to regulate, or compute the optimal transfer to universities.

To sum up, it does not seem that tuition fees alone constitute a solution to the university budget problem. We have provided a framework of analysis that can help thinking about tuition reform and university admission policies. Fairness considerations deserve further research.

## 7. Appendix

*Proof of Lemma 1:* Using the properties of conditional expectation, we have $E(\theta \mid y,z) = E[E(\theta \mid s,y,z) \mid y,z]$. Since $y = E[\theta \mid s,z]$ is a function of $s$ and $z$, we also get $E(\theta \mid s,y,z) = E(\theta \mid s,z) = y$ (because the information generated by $(y,s,z)$ is not different from that generated by $(s,z)$). Hence, $E(\theta \mid y,z) = E(y \mid y,z) = y$. *Q.E.D.*

*Proof of Lemma 2:* Note first that,

$$v(\bar{y},\bar{z}) = \frac{\int_{\bar{y}}^{\infty}\int_{\bar{z}}^{\infty}\int_{-\infty}^{\infty}\theta\psi(y,z,\theta)dydzd\theta}{\int_{\bar{y}}^{\infty}\int_{\bar{z}}^{\infty}\int_{-\infty}^{\infty}\psi(y,z,\theta)dydzd\theta},$$

where $\psi$ is the joint normal density of $(y,z,\theta)$. The denominator of $v$ is just $q/N = \int_{\bar{y}}^{+\infty}\int_{\bar{z}}^{+\infty}\int_{-\infty}^{+\infty}\psi(y,z,\theta)dydzd\theta$, so it will be convenient to define a mapping $g$ as $v(\bar{y},\bar{z}) = g(\bar{y},\bar{z})/q(\bar{y},\bar{z})$. Now we get,

$$v_y \equiv \frac{\partial v}{\partial \bar{y}} = \frac{g_y}{q} - \frac{gq_y}{q^2},$$

which is equivalent to, $v_y/q_y = (1/q)\left((g_y/q_y) - v\right)$. Likewise, $v_z/q_z = (1/q)\left((g_z/q_z) - v\right)$. Now remark that,

$$\frac{g_y}{q_y} = \frac{-N\int_{\bar{z}}^{\infty}\int_{-\infty}^{\infty}\theta\psi(\bar{y},z,\theta)dzd\theta}{-N\int_{\bar{z}}^{\infty}\int_{-\infty}^{\infty}\psi(\bar{y},z,\theta)dzd\theta} = E[\theta \mid y=\bar{y}, z \geq \bar{z}] = h_y.$$

Likewise, $g_z/q_z = E[\theta \mid y \geq \bar{y}, z = \bar{z}] = h_z$. We can therefore conclude that $v_y/q_y > v_z/q_z$ if and only if $h_y > h_z$. *Q.E.D.*

*Proof of Proposition 1:* From Lemmata 1 and 2, we get (23) and (24), that is, $h_y = \bar{y}$ and $h_z = E[y \mid y \geq \bar{y}, z = \bar{z}]$. But $E[y \mid y \geq \bar{y}, z = \bar{z}] > \bar{y}$, except possibly in the limit if $\bar{y} = +\infty$ or if $\bar{z} = -\infty$, because $E(y \mid y \geq \bar{y}, z = \bar{z}) \to \bar{y}$ as $\bar{z} \to -\infty$. We must prove this limiting property to conclude that we cannot find a finite solution $(\bar{y},\bar{z})$ to $h_z = h_y$.

Using (14b), we get,

$$\begin{aligned}
h_z &= E\big[\alpha s + \beta z \mid y \geq \bar{y}, z = \bar{z}\big] = \alpha E[s \mid y \geq \bar{y}, z = \bar{z}] + \beta\bar{z} \\
&= \alpha E\left[s \,\middle|\, s \geq \frac{\bar{y} - \beta\bar{z}}{\alpha}\right] + \beta\bar{z} = \alpha\sigma_s E\left[\frac{s}{\sigma_s} \,\middle|\, \frac{s}{\sigma_s} \geq \frac{\bar{y}-\beta\bar{z}}{\alpha\sigma_s}\right] + \beta\bar{z} \\
&= \alpha\sigma_s\big[E(\zeta \mid \zeta \geq \zeta_0) - \zeta_0\big] + \bar{y},
\end{aligned}$$

where $\zeta \sim \mathcal{N}(0,1)$ and, $\zeta_0 = (\bar{y} - \beta\bar{z})/\alpha\sigma_s$. It follows that $h_y = h_z$ is equivalent to, $E(\zeta \mid \zeta \geq \zeta_0) = \zeta_0$, which is impossible, except if $\zeta_0 = +\infty$.

If $\bar{y} = +\infty$, then $x = 0$. But $\lim_{\bar{z}\to-\infty} q(\bar{y},\bar{z})$ is well defined and positive. It is now routine work to show that if there existed an optimum in which $\bar{z}$ is finite, then, decreasing $\bar{z}$ slightly would increase $W$, so that the optimal philanthropic solution involves $\bar{z} = -\infty$. *Q.E.D.*

*Proof of Proposition 3:* Since $C_v < 0$ and $q_y < 0$, by (29), we know that $v_y/q_y \geq v_z/q_z$ at a maximum of rent $R$. By Lemma 2, this is equivalent to $h_y \geq h_z$, but the proof of Proposition 1 above shows that this is equivalent to $\bar{y} \geq E(y \mid y \geq \bar{y}, z = \bar{z})$, which is impossible, except if $\bar{z} = -\infty$ or $\bar{y} = +\infty$. If $\bar{y} = +\infty$, then $x = 0$. The limits of $q$ and $v$ as $\bar{z} \to -\infty$ are well defined. It is again routine work to show that if a rent maximum involved a finite value of $\bar{z}$, then $R$ could be increased slightly by decreasing $\bar{z}$ slightly. It follows that $\bar{z} = -\infty$ is an optimal solution for the rent-seeking university. *Q.E.D.*

*Proof of Lemma 3:* As in the proof of Lemma 1, the classic result on conditional expectations under normality assumptions yields,

$$E[\theta \mid y, z] =(\sigma_{\theta y}, \sigma_{\theta z}) \begin{pmatrix} \sigma_y^2 & \sigma_{yz} \\ \sigma_{yz} & \sigma_z^2 \end{pmatrix}^{-1} \begin{pmatrix} y \\ z \end{pmatrix} = \frac{(\sigma_{\theta y}, \sigma_{\theta z})}{(\sigma_y^2 \sigma_z^2 - \sigma_{yz}^2)} \begin{pmatrix} \sigma_z^2 & -\sigma_{yz} \\ -\sigma_{yz} & \sigma_y^2 \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix}.$$

The coefficients of $y$ and $z$ in the expression of $\hat{\theta}$ are, respectively,

$$a_0 = \frac{\sigma_z^2 \sigma_{\theta y} - \sigma_{\theta z} \sigma_{yz}}{\sigma_y^2 \sigma_z^2 - \sigma_{yz}^2}, \qquad \text{and} \qquad 1 - a_0 = \frac{\sigma_y^2 \sigma_{\theta z} - \sigma_{\theta y} \sigma_{yz}}{\sigma_y^2 \sigma_z^2 - \sigma_{yz}^2}.$$

Simple algebra then yields the stated result, using $\alpha_0 = \sigma_\theta^2/\sigma_s^2$, because $\sigma_y^2 = \alpha_0^2 \sigma_s^2 = \alpha_0 \sigma_\theta^2$; $\sigma_z^2 = \sigma_\theta^2 + \sigma_\nu^2$; and $\sigma_{\theta y} = \mathrm{Cov}(\theta, \alpha_0 s) = \alpha_0 \sigma_{\theta s} = \alpha_0 \sigma_\theta^2$; $\sigma_{yz} = \mathrm{Cov}(\alpha_0 s, z) = \alpha_0 \mathrm{Cov}(\theta, \theta) = \alpha_0 \sigma_\theta^2$; and finally $\sigma_{\theta z} = \sigma_\theta^2$. *Q.E.D.*

*Proof of Proposition 5:* We must solve $h_y = h_z$ in the bilateral asymmetric information version of the model.

$$\begin{aligned} h_y =& E\big[E(\theta \mid z, y) \bigm| y = \bar{y}, z \geq \bar{z}\big] = E[a_0 y + (1 - a_0) z \mid y = \bar{y}, z \geq \bar{z}] \\ =& a_0 \bar{y} + (1 - a_0) E[z \mid y = \bar{y}, z \geq \bar{z}]. \end{aligned}$$

Define $\delta = \nu - \epsilon$, and denote $\sigma_0 = \sigma_\delta = \sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}$. Using (33) and $z = s + \delta$, we get,

$$\begin{aligned} E[z \mid y = \bar{y}, z \geq \bar{z}] =& E\left[s + \delta \;\middle|\; s = \frac{\bar{y}}{\alpha_0},\ s + \delta \geq \bar{z}\right] = \frac{\bar{y}}{\alpha_0} + E\left(\delta \;\middle|\; \delta \geq \frac{\alpha_0 \bar{z} - \bar{y}}{\alpha_0}\right) \\ =& \sigma_0\big(-\xi_0 + E(\xi \mid \xi \geq \xi_0)\big) + \bar{z} = \sigma_0 \left[-\xi_0 + \frac{\phi(\xi_0)}{1 - \Phi(\xi_0)}\right] + \bar{z}. \end{aligned}$$

where $\xi \sim \mathcal{N}(0, 1)$, and $\xi_0 = (\alpha_0 \bar{z} - \bar{y})/\alpha_0 \sigma_0$. Likewise,

$$\begin{aligned} h_z =& E\big[E(\theta \mid y, z) \mid y \geq \bar{y}, z = \bar{z}\big] = E[a_0 y + (1 - a_0) z \mid y \geq \bar{y}, z = \bar{z}] \\ =& a_0 E[y \mid y \geq \bar{y}, z = \bar{z}] + (1 - a_0) \bar{z}, \text{ and} \end{aligned}$$

$$\begin{aligned} E[y \mid y \geq \bar{y}, z = \bar{z}] =& \alpha_0 E\left(s \;\middle|\; s \geq \frac{\bar{y}}{\alpha_0},\ s + \delta = \bar{z}\right) = \alpha_0 \bar{z} - \alpha_0 E\left(\delta \;\middle|\; \delta \leq \frac{\alpha_0 \bar{z} - \bar{y}}{\alpha_0}\right) \\ =& \alpha_0 \sigma_0\big(\xi_0 - E(\xi \mid \xi \leq \xi_0)\big) + \bar{y} = \alpha_0 \sigma_0 \left[\xi_0 + \frac{\phi(\xi_0)}{\Phi(\xi_0)}\right] + \bar{y}. \end{aligned}$$

Therefore, $h_y = h_z$ is equivalent to,

$$b_0\left(-\xi_0 + \frac{\phi(\xi_0)}{1-\Phi(\xi_0)}\right) = a_0\alpha_0\left(\xi_0 + \frac{\phi(\xi_0)}{\Phi(\xi_0)}\right).$$

Since $a_0\alpha_0/(1-a_0) = \sigma_\nu^2/\sigma_\epsilon^2$, the above equation can be rewritten as (35b), that is,

$$0 = (\Phi(\xi) - \lambda)\frac{\phi(\xi)}{(1-\Phi(\xi))\Phi(\xi)} - \xi \equiv f(\xi;\lambda),$$

where $\lambda = \sigma_\nu^2/(\sigma_\nu^2 + \sigma_\epsilon^2)$.

It remains to show that $f(x;\lambda) = 0$ has a solution for every $\lambda \in (0,1)$. To perform this task, we study $f$'s limiting behaviour.

$$\lim_{x\to+\infty} f(x;\lambda) = \lim_{x\to+\infty}\left[\frac{\phi(\Phi-\lambda) - x\Phi(1-\Phi)}{\Phi(1-\Phi)}\right].$$

Now, this ratio goes to $0/0$ since $x(1-\Phi(x)) \to 0$. Using l'Hôpital's rule, and $\phi'(x) = -x\phi(x)$, we get

$$\begin{aligned}\lim_{x\to+\infty} f(x;\lambda) &= \lim_{x\to+\infty}\left[\frac{-x\phi(1-\lambda-\Phi) + \phi^2 - \Phi(1-\Phi)}{\phi(1-2\Phi)}\right]\\ &= \lim_{x\to+\infty}\left[\frac{x(1-\lambda-\Phi)-\phi}{(2\Phi-1)}\right] + \lim_{x\to+\infty}\left[\frac{(1-\Phi)}{(2\Phi-1)}\right]\lim_{x\to+\infty}\left(\frac{\Phi}{\phi}\right)\\ &= \frac{(+\infty)(-\lambda)-0}{1} + \frac{0}{1}.0 = -\infty.\end{aligned}$$

This is because, using l'Hôpital's rule again, $\lim_{x\to+\infty}(\Phi/\phi) = \lim_{x\to+\infty}(-\phi/x\phi) = 0$. The same type of reasoning would show that, $\lim_{x\to-\infty} f(x;\lambda) = +\infty$. Therefore, $f$ being continuous, by the Intermediate Value Theorem, there exists a point $\xi_0(\lambda)$ such that $f(\xi_0(\lambda),\lambda) = 0$ for every $\lambda$ in $(0,1)$. In addition, it is not difficult to check that $\xi_0(\lambda) \to +\infty$ if $\lambda \to 0$ and $\xi_0(\lambda) \to -\infty$ if $\lambda \to 1$. Equations (35a) and (35c) are immediate consequences of the definition of $\xi_0^* = (\alpha_0\bar{z}^* - \bar{y}^*)/\alpha_0\sigma_0$, of the condition $x = q$, and of the fact that $q$ is strictly decreasing with respect to its arguments $\bar{y}$ and $\bar{z}$. *Q.E.D.*

*Proof of Proposition 7:* Assume that $\bar{z} \to -\infty$, then, we get $h_z \to \bar{y}$; $v \to E(y \mid y \geq \bar{y})$; $-q_y/q$ tends towards a finite, positive limit $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\psi(\theta,\bar{y},z)d\theta dz/\Pr(y \geq \bar{y})$; and finally $\phi/(1-\Phi) > 0$ doesn't change, because it doesn't depend on $\bar{z}$. It follows that when $\bar{z} \to -\infty$, the left-hand side of (44) tends towards a positive limit, while the right-hand side tends towards 0. This is impossible for a finite value of $\bar{y}$. If in addition to $\bar{z}$, we let $\bar{y} \to +\infty$, then $\phi/(1-\Phi) \to 0$, and (44) boils down to $0 = 0$, but this also implies $\tilde{q} = 0$, which contradicts the assumption $x > 0$. *Q.E.D.*

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