

# Comparisons of Alternative Payment Mechanisms for French Treasury Auctions

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## Abstract

Treasury auctions around the world have been typically conducted under either the uniform or discriminatory format. We propose alternative payment mechanisms, including uniform and discriminatory formats as special cases, and also the Spanish format. We compare the properties of these alternative formats in the specific context of French Treasury auctions.

## 1 Introduction

Treasury auctions around the world have been typically conducted under either the uniform or discriminatory format. Beginning with Friedman (1960), the choice between these two payment mechanisms has been often debated among economists.<sup>1</sup> Both theoretical and empirical analyses, however, have yielded ambiguous results, and it appears that the ranking of the two auction formats may only be established on a case-by-case basis.<sup>2</sup> For instance, Armantier and Sbai (2004) (hereafter A&S) find that the uniform pricing rule would have generated higher revenues for the French Treasury. The object of the present paper is to explore whether alternative auction formats may further increase the revenue raised by the French Treasury.

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<sup>1</sup>For surveys of the literature on Treasury auctions, see Bikchandani and Huang (1993), Das and Sundaram (1996), Nandi (1997), or Klemperer (2000).

<sup>2</sup>See e.g. Back and Zender (1993), Wang and Zender (2002) or Ausbel and Cramton (2002) for theoretical analyses. Empirical studies based on the reduced form approach include Umlauf (1993), Tenorio (1993), Simon (1994), Mester (1995), Nyborg and Sundaresan (1996), Malvey and Archibald (1998), and Berg, Boukai and Landsberger (1999); while empirical studies based on the structural approach include Hortaçsu (2002), Fevrier, Preguet and Visser (2004), as well as Armantier and Sbai (2004)

At a Treasury auction, a specific type of security is sold to several accredited financial institutions. The bidders submit simultaneously a sealed bid consisting of a demand schedule. A bid therefore specifies the number of units of the security requested at each possible price. The market-clearing price, also known as the stop-out-price, matches aggregate demand with the available supply of security. As previously mentioned, two basic payment mechanisms are typically considered: under the discriminatory format, the format used by most Treasuries around the world including the French Treasury, the highest bids are filled at the bided price until supply is exhausted; under the uniform-price format, bidders pay the stop-out-price for all units they requested at prices exceeding the stop-out-price. These two auction formats are not the only payment mechanisms to have been considered. For instance, the so-called “Spanish auction”, have also been implemented in practice to sell Treasury securities.<sup>3</sup> Moreover, hybrid formats, such as the  $\alpha$ -auction of Wang and Zender (2002), have recently been proposed in theoretic models.

In the present paper, we investigate the properties of these alternative formats, and we also propose different classes of original payment mechanisms. To facilitate the analysis, we loosely partition these auction formats into either a “discriminatory-type” or a “uniform-type” of pricing rules. As further explained in section 4, the highest price paid for a unit of the security by each bidder at “discriminatory-type” auctions is different, while it is the same at “uniform-type” auctions. The different pricing rules proposed may be further differentiated on the basis of the factors influencing the highest price paid by each bidder. Indeed, we will see that under our “ $\alpha$ -discriminatory” and “ $\alpha$ -uniform” auctions the highest price paid depends on the bid functions submitted by all bidders; under the “ $\alpha$ -price-discriminatory” and “ $\alpha$ -price-uniform” auctions the highest bids submitted influence the highest price bidders will have to pay; under the “ $k^{th}$ -average-price” format only the average price of all winning bids matters; finally, at a “Spanish auction” bidders do not pay more than the weighted average price of all winning bids.

As previously mentioned, auction formats may only be ranked on a case-by-case basis. In fact, Ausubel and Cramton (2002) conclude their theoretical analysis by stating that “determining the better pricing rule is necessarily an empirical question.” In other words, the superiority of an auction format depends on specific empirical factors, such as the bidders’ distribution of types, and/or their levels of risk aversion. Therefore, to compare the pricing rules just mentioned, we concentrate on the case of French Treasury auctions, and we rely on the structural parameters estimated by A&S in that specific context. As a consequence, the conclusions reached in this paper are specific to the French Treasury, and may not immediately extend to other Treasury markets in different countries. Nevertheless, the methodology developed allows one to compare the formats proposed to Treasury auctions in different countries.

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<sup>3</sup>See Álvarez and Mazón (2002), and Álvarez, Mazón and Cerdá (2003) for theoretical analyses of the Spanish auctions, or Abbink, Brandts and Pezanis-Christou (2002) for a laboratory experiment.

When we concentrate on existing payment mechanisms, we find that the revenue of the French Treasury would be higher under the uniform-price format than under the Spanish format, and it is the lowest under the discriminatory-price format, the payment mechanism currently used by the French Treasury. We find also that a new payment mechanism, namely the  $k$ -average-price format, dominates all others. As we shall see, this format may be interpreted as a combination of the best features of the discriminatory and uniform-price formats.

The paper is structured as follows: section 2 briefly describes the market for the French Treasury securities; in section 3, we summarize the structural model used by A&S, as well as their estimation results; section 4 introduces the alternative payment mechanisms; the simulation outcomes for the French Treasury auctions are presented in section 5; finally, section 6 concludes.

## 2 The French Treasury Auction Market

Following A&S, we concentrate on two types of securities, the OAT and the BTAN, sold by the French government at traditional discriminatory-price auctions. The OAT are the government's long-term debt instruments with maturities ranging from seven to thirty years. The BTAN represent medium-term government debts with a maturity of either two or five years.

The timing of these auctions unfold as follows: auctions for OAT and BTAN are held respectively the first and the third Thursday of each month. Four business days before the auction, the "Agence France Trésor", which is in charge of conducting the auction, announces the details of the different "lines" to be auctioned. A line consists of either an OAT or a BTAN with specific characteristics including the nominal yield, the maturity, as well as a bracket for the volume of security to be served. Part of the announced Treasury security may then be traded on a primary (or "when-issued ") market until the date of the auction by a limited number of authorized dealers.

Competitive bidders may submit a demand function. A quantity demanded by a competitive bidder is in fact an amount in Euros representing a share of the quantity sold by the Treasury. Prices are expressed as a percentage (formulated with two decimal digits) of the nominal value of the security (one Euro). Moreover, pre-qualified bidders may submit a non-competitive offer for any line, consisting in a (limited) amount that will be systematically served at a price equal to the (quantity weighted) average price of the awarded competitive bids. Bids by eligible institutions, for all lines to be auctioned that day, must be submitted either electronically or in sealed envelopes at least 10 minutes prior to the auction.

Before the bids are observed, the French Treasury sets the exact quantity that will be supplied to competitive and non-competitive bidders.<sup>4</sup> Competitive bids are then ranked in descending order, and the stop-out-price is determined

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<sup>4</sup>The Agence France Trésor has been known to set the exact quantity to be supplied as a response to exogenous short term shocks, and therefore it not suspected to act strategically.

in such a way that the aggregate competitive and non-competitive demand matches the exact quantity supplied. Auction results are announced within five minutes after the end of the auction, and the Banque de France completes the delivery-versus-payment orders with the auction winners within three business days. The security may then be traded to the general public on a secondary market.

Although occasional bidders may participate, the French State’s policy issuance essentially relies on a network of primary dealers (aka “Spécialistes en Valeurs du Trésor”). The role of these primary dealers is to advise and assist the French Treasury in marketing appropriately its debt. In particular, the primary dealers must be active on the primary market, and maintain a liquid secondary market.<sup>5</sup> During the period studied by A&S (1998 to 2000), the primary dealers were composed of 19 institutions accounting for over 90% of the securities bought. The involvement of the primary dealers to each French Treasury auction may vary notably between financial institutions. In particular, the Agence France Trésor identified five large financial institutions (Crédit Agricole, Deutsch Bank, BNP-Paribas, Morgan Stanley, and Société Générale) who participated in most auctions, and were allocated during our sample period more than 60% of the securities issued.

Finally, note that the common-value assumption finds support in the fact that most of the bonds purchased at French Treasury auctions are eventually resold on the secondary market, or directly to the bidders’ own clients (e.g. mutual funds, insurance companies, individuals).<sup>6</sup> Moreover, the flow of pre-auction orders submitted by their own clients, may partially explain why participants may form different forecasts about the future market value of the bond (see also the argument in Fevrier et al. 2002 who conclude as well that the common-value paradigm best fits the French Treasury auctions).<sup>7</sup>

## 3 Baseline Model

### 3.1 A Share Auction Model with Asymmetric Bidders and Risk Aversion

We now present the structural model estimated by A&S. This model is a generalization of the common-value share auction of Wang and Zender (2002), in which the quantity for sale is not perfectly known at the time of the auction, to account for informational and risk aversion asymmetries across bidders.

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<sup>5</sup>Since 2003, a “Spécialiste en Valeurs du Trésor” is also required to account for at least 2% of the volume auctioned over the last 12 months in order to maintain its status.

<sup>6</sup>In particular, unlike the Turkish Treasury auctions analyzed by Hortaçsu (2002a, 2002b), the primary dealers are not required by the French Treasury to hold a portion of their reserves in the form of Treasury bonds.

<sup>7</sup>It may be argued that the French Treasury auctions also possess a private-values component, since a bank must purchase bonds to fill its clients pre-auction orders. However, these orders may be filled by purchasing bonds either at the auction, or on one of the markets (i.e. primary or secondary). Therefore, as mentioned in section 2, the literature typically considers the common value paradigm more appropriate to describe Treasury auctions.

At a given auction, a specific quantity of a perfectly divisible good is for sale to  $N$  competitive bidders ( $N \geq 2$ ) each maximizing his ex-ante expected utility.<sup>8</sup> A bidder's decision to participate in the auction (i.e., to submit a competitive bid) is assumed to be exogenous and common knowledge. The quantity supplied to the bidders by the auctioneer is unknown at the time of the auction, and it is represented by a random variable  $Q \in \Theta_Q$ , with cumulative distribution function (c.d.f.)  $G(Q)$ .<sup>9</sup> The actual value of the good,  $V \in \Theta_V$ , is random with a c.d.f.  $F_0(V|\delta)$ . This true value is assumed to be the same to each bidder, but unknown at the time of the auction.

As further explained in 3.2, A&S find that participants in each French Treasury auction may be divided in two groups, respectively composed of  $N_1$  and  $N_2$  bidders ( $N_1 + N_2 = N$ ). Bidders within a given group are symmetric, but bidders are asymmetric across groups. Bidder  $i$  in group  $l = 1, 2$  receives a signal,  $s_{i,l} \in \Theta_S$  containing some private information about the value of the good. This signal is generated from a conditional distribution with c.d.f.  $F_l(s_{i,l}|V, \sigma_l)$ . After bidder  $i$  in group  $l$  receives the private signal  $s_{i,l}$ , she submits a sealed bid. This bid consists of a schedule specifying the share of the good demanded  $\varphi_{i,l}(p, s_{i,l})$  for any price  $p > 0$ . The demand schedules are assumed to be (piecewise) continuously differentiable. The stop-out-price  $p^0$  is defined as the non-negative price at which aggregate competitive demand  $\Phi(\cdot)$  equals total supply:

$$\Phi(p^0, s) = \sum_{i=1}^{N_1} \varphi_{i,1}(p^0, s_{i,1}) + \sum_{j=1}^{N_2} \varphi_{j,2}(p^0, s_{j,2}) = Q \quad \text{given } p^0 \geq 0 \quad . \quad (1)$$

Winning bids are those submitted for prices greater than the stop-out-price. In other words, bidder  $i$  in group  $l$  receives a quantity  $\varphi_{i,l}(p^0, s_{i,l})$ . It is important to note at this point that the determination of the stop-out-price  $p^0$ , and the allocation process just described will be common to every auction format we will be discussing in the present paper. The only difference between the auction formats will reside in the pricing mechanism applied to bidders in order to pay for the share they receive.

Although bidders in French Treasury auctions face the traditional discriminatory pricing rule, we adopt the general notation of Viswanathan and Wang (2000) in order to introduce simultaneously the traditional uniform-price mechanism. For the quantity  $\varphi_{i,l}(p^0, s_{i,l})$  received, a bidder  $i$  in group  $l$  is asked to

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<sup>8</sup>Non-competitive bids are assumed to result from exogenous decisions made prior to the auction. Therefore, non-competitive bidding will not be modelled, and it is assumed to affect only the quantity available to competitive bidders. The competitive bidders will be referred, in the remainder, as the "bidders", the "players", or the "participants" unless mentioned otherwise

<sup>9</sup>As previously mentioned, the quantity available to competitive bidders is uncertain because i) non-competitive bids are only revealed after the auction, and ii) the French Treasury announces only a bracket for the total quantity of securities to be supplied to competitive and non-competitive bidders.

pay

$$p^0 \varphi_{i,l}(p^0, s_{i,l}) + \alpha \int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp \quad , \quad (2)$$

where  $p_{i,l}^{\max}$  is the highest price for which the demand for bidder  $i$  in group  $l$  is strictly positive. Within this formulation,  $\alpha = 0$  corresponds to the price paid under the uniform-price format, and  $\alpha = 1$  corresponds to the price paid under the discriminatory format. In this context, we will subsequently refer to the quantity  $\int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp$  as the discriminatory surplus. To illustrate how the payment mechanisms work, consider the following example.

**Example 1** Consider an auction with two bidders (e.g. one in each group), in which the auctioneer sells one unit of fully divisible Treasury bonds. We plotted in Figures 1.1 and 1.2 two possible bid functions for bidders 1 and 2. First, note that at the stop-out-price  $p^0$  aggregate demand and supply are both equal to 1. The quantity allocated to bidder 1 (respectively bidder 2) corresponds to  $q_1 = \varphi_1(p^0, s_1)$  (respectively  $q_2 = \varphi_2(p^0, s_2)$ ). To receive the allocated quantity  $q_1$  (respectively  $q_2$ ), bidder 1 (respectively bidder 2) must pay under the uniform-price auction a total price corresponding to the area denoted 1 in Figure 1.1 (respectively 1.2). In contrast, to receive its allocated quantity, a bidder under the discriminatory format must pay the total price under the uniform-price format (area 1), plus the discriminatory surplus (area 2).

Under this general pricing rule, the profit of bidder  $i$  in group  $l$  may then be written as

$$\Pi_{i,l}(\varphi_{i,l}(\cdot), p^0, V, s_{i,l}) = (V - p^0) \varphi_{i,l}(p^0, s_{i,l}) - \alpha \int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp \quad . \quad (3)$$

Finally, A&S find that a bidder  $i$  in group  $l$  exhibits risk aversion in the form of a CARA utility function:

$$U_{i,l}(\varphi_{i,l}(\cdot), p^0, V, s_{i,l}, \lambda_l) = -\exp[-\lambda_l \Pi_{i,l}(\varphi_{i,l}(\cdot), p^0, V, s_{i,l}, Q)] \quad , \quad (4)$$

where  $\lambda_l > 0$  is the constant level of absolute risk aversion for players in group  $l$ .

To conclude this section, note that the asymmetric share auction model we just presented cannot be solved analytically. Therefore, to estimate the structural parameters, A&S rely on the Constrained Strategic Equilibrium (hereafter CSE) technique developed by Armantier, Florens and Richard (2004) to approximate intractable Bayesian Nash Equilibria. This approximation technique will also be used in section 5 to analyze the properties of the alternative auction formats.

### 3.2 Estimated Structural Parameters

A&S estimated the structural model presented in section 3.1 with a sample of 40,496 observations collected in 118 auctions (60 OAT and 58 BTAN) which took place at 64 different dates between May 1998 and December 2000. We present in Table ?? (CHANGER) summary statistics for the variables in our sample. Participation in French Treasury auctions appears to be dominated by smaller financial institutions (14.6 small banks versus 4.5 large banks). Although the number of bidders within groups may vary, the average number of participants (roughly 19) is rather stable across auctions and lines. Large banks received a larger share of the security (63.8% on average). This result is consistent with the fact that, on average, large banks submit higher prices for the initial units demanded (101.193 versus 100.922 for small banks), and pay slightly more per unit awarded (101.185 versus 100.180). (J'AI SIMPLEMENT REPRIS 2 COMMENTAIRES POUR ESSAYER DE SOULIGNER LA STABILITE DU NOMBRE DE PARTICIPANTS ET LA PERTINENCE DU PARTAGE EN 2 GROUPE DE BIDDERS). We now briefly summarize their estimation results. Remember that these estimated structural parameters will be used in section 5 to compare the different auction formats.

A&S estimate that the quantity offered by the French Treasury at a given auction follows the following relationship:

$$Q_t = 1,426.927 + 0.681MeanBracket_t - 0.193Maturity_t - 376.245Yield_t + 1,208.732Type_t + v_t \quad , \quad (5)$$

where  $MeanBracket_t = (\underline{Q}_t + \overline{Q}_t)/2$ ;  $\overline{Q}_t$  and  $\underline{Q}_t$  are the upper and lower bounds of the quantity bracket announced by the French Treasury before auction  $t$ ;  $Maturity_t$  and  $Yield_t$  are respectively the maturity and the nominal yield associated with the security sold at auction  $t$ ;  $Type_t$  is a dummy variable equal to 1 when the line is an OAT, and 0 when the line is a BTAN; and finally,  $v_t$  is an identically and independently normally distributed error term with mean zero and estimated standard deviation 475.207.

The true value of the security  $V_t$  is found to be normally distributed with mean  $\mu_{V_t} = 81.165 + 4.782Yield_t - 7.223E^{-4}Maturity_t$ , and standard deviation 0.632. The private signals distribution for bidders in group 1 (respectively group 2) is normal with mean  $V_t$  and standard error 0.070 (respectively 0.178). Finally, the levels of absolute risk aversion are estimated at respectively  $5.732E^{-8}$  and  $6.907E^{-6}$  for banks in group 1 and 2<sup>10</sup>.

In other words, the structural parameters estimated in A&S indicate that the participants in French Treasury auctions may be divided in two distinct groups. In contrast with group 1, group 2 consists mostly of smaller financial institutions, characterized by a higher level of risk aversion, and receiving significantly noisier signals about the true value of the security. In this context, A&S find that the

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<sup>10</sup>The estimated absolute risk aversion parameters may appear rather low, if not compared to the profits of the bidders. Indeed, the relative risk aversion levels range between 0.02 and 0.9 when calculated with the profits only. These figures, although still low since they do not include the actual wealth of the participants, appear however more reasonable.

French Treasury would have accumulated during the sample period an additional 8.4 billion Euros (a 4.5% increase), had it used the uniform-price instead of the discriminatory format. In section 6, we will explore whether alternative auction formats could have further increased the revenue of the French Treasury during the sample period.

## 4 Alternative Auction Formats

In this section, we extend the general model proposed in section 3 by specifying alternative payment mechanisms. As previously explained, these alternative auction formats leave the allocation mechanism unaffected. In particular, the stop-out-price  $p^0$  is still determined by equation (1), and the security is still divided among participants submitting bids above  $p^0$ . In other words, for a given bid function, a bidder receives the same share of the security under every auction format, and only the amount he is required to pay will differ.

As previously mentioned, we partition the different payment mechanisms in two classes. Under a “uniform-type” auction, the highest price paid for a unit of the security, hereafter denoted  $\bar{p}$ , is the same for every winning bidder. In contrast, each winning bidder  $i$  in group  $l$  must pay a different highest price, denoted  $\bar{p}_{i,l}$ , at “discriminatory-type” auctions. Therefore, under every auction format discussed in the present paper, the payment of a bidder  $i$  in group  $l$  when he receives a quantity  $\varphi_{i,l}(p^0)$  may be written

$$p^0 \varphi_{i,l}(p^0, s_{i,l}) + \int_{p^0}^{\mathbf{p}} \varphi_{i,l}(p, s_{i,l}) dp \quad , \quad (6)$$

where  $\mathbf{p} = \bar{p}$  at “uniform-type” auctions, and  $\mathbf{p} = \bar{p}_{i,l}$  at “discriminatory-type” auctions. As illustrated in example ???? below, the pricing rule may also be interpreted as follows: a bidder must pay his bid for any winning bid he announced at a price below  $\mathbf{p}$ , but he only needs to pay  $\mathbf{p}$  for all winning bids he made at prices above  $\bar{p}$ . Finally, note that this formulation includes as special cases the traditional discriminatory and uniform pricing rules, when  $\bar{p}_{i,l} = p_{i,l}^{\max}$ , and  $\bar{p} = p^0$  respectively. As we shall see in the section 5, we find that such a partition of auction formats has a significant influence on the bidders’ behavior in the presence of asymmetry.

### 4.1 Discriminatory-Type Auctions

#### 4.1.1 The $\alpha$ -Discriminatory Auction

The  $\alpha$ -discriminatory payment mechanism was in fact introduced in equation (2). Indeed we can generalize the uniform-price format ( $\alpha = 0$ ) and the discriminatory-price format ( $\alpha = 1$ ) to allow  $\alpha$  to vary between 0 and 1. This payment mechanism was initially by Viswanathan and Wang (2000), and Wang



and Zender (2002). Observe that the payment of a winning bidder in an  $\alpha$ -discriminatory auction lays between the amount paid under the uniform and discriminatory formats. In fact,  $\alpha$  represents the fraction of the discriminatory surplus  $\left( \int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp \right)$  extracted from a bidder, in addition to his payment under the uniform-price format.

To be consistent with equation (6), we can re-write the payment mechanism (2) as

$$p^0 \varphi_{i,l}(p^0, s_{i,l}) + \alpha \int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp = p^0 \varphi_{i,l}(p^0, s_{i,l}) + \int_{p^0}^{\bar{p}_{i,l}} \varphi_{i,l}(p, s_{i,l}) dp ,$$

where  $\bar{p}_{i,l}$ , the highest price paid by bidder  $i$  for a unit of the security, is a function of the  $\alpha$  selected by the auctioneer. Note that the  $\alpha$ -discriminatory payment mechanism indeed belongs to the family of “discriminatory-type” auctions, since  $\bar{p}_{i,l}$  differs across bidders . We now illustrate how the payment mechanism can be implemented.

**Example 2** Consider the same auction as in Example 1, except that the auctioneer now selects the  $\alpha$ -discriminatory payment mechanism with  $\alpha = 2/3$  (see Figures 2.1 and 2.2). To receive his winning share  $3/5$ , bidder 1 must pay a total price corresponding to the areas denoted 1 and 2. Note that this amount is in between the payment the bidder would have made under the uniform-price format (area 1) and discriminatory-price format (areas 1, 2 and 3)<sup>11</sup>. Area 2 represents the share  $\alpha$  ( $2/3$  in this case) of the discriminatory surplus (corresponding here to the sum of areas 2 and 3). Figure 2.2 also illustrates a different but equivalent way to interpret the  $\alpha$ -discriminatory payment mechanism. Indeed, the payment of bidder 2 for his share  $2/5$  of the security may be decomposed in two areas. Area 1 indicates that, analogously with a uniform-price auction, bidder 2 pays the highest price  $\bar{p}_2$  for any share between 0 and  $3/10$ ; while, area 2 indicates that, analogously with a discriminatory auction, he pays the price he announced for any share between  $3/10$  and  $2/5$ .

#### 4.1.2 The $\alpha$ -Price-Discriminatory Auction

The  $\alpha$ -Price-Discriminatory format is similar to the payment mechanism we just introduced, except that  $\bar{p}_{i,l}$ , the highest price paid by bidder  $i$ , is now defined

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<sup>11</sup>Note that the bid functions of bidder 1 (respectively 2) are the same in figures 1.1 and 2.1 (respectively 1.2 and 2.2). Indeed, to compare the different payment mechanisms proposed throughout Section 4, When comparing different auction mechanisms throughout Section 4, we consider a given strategy for each bidder. However, we will see in Section 5 that, since the strategic environment differ, the equilibrium behavior of a bidder will be different across payment mechanisms.

as a convex combination between  $p^0$ , the stop-out-price, and  $p_{i,l}^{\max}$ , the highest price he announced.

$$\bar{p}_{i,l} = p^0 + \alpha (p_{i,l}^{\max} - p^0) . \quad (7)$$

Note that once again,  $\bar{p}_{i,l}$  depends on the value of the parameter  $\alpha$  selected by the auctioneer. The main difference with the  $\alpha$ -discriminatory auction resides in the fact that the highest price paid  $\bar{p}_{i,l}$  in (7) is not immediately affected by the shape of the bid function  $\varphi_{i,l}$ . Indeed,  $\bar{p}_{i,l}$  depends directly on  $p_{i,l}^{\max}$ , the highest price announced by bidder  $i$ , while it only depends indirectly on the entire bid function  $\varphi_{i,l}$  through the determination of the stop-out-price  $p^0$  (see equation 1). The distinction between the two auction formats may seem minor at first glance, but as we shall see, it turns out to have a significant strategic impact on the shape of the equilibrium bid functions. To illustrate the different features of this payment mechanism, we now present an example.

**Example 3** Consider the same auction as in Example 1, except that the auctioneer now selects the  $\alpha$ -price-discriminatory payment mechanism with  $\alpha = 2/3$  (see Figures 3.1 and 3.2). In contrast with the  $\alpha$ -discriminatory format,  $\bar{p}_i$  ( $i = 1, 2$ ), the highest price paid by bidder  $i$ , is now located at exactly  $2/3$  of the distance between  $p^0$ , the stop-out-price, and  $p_i^{\max}$ , the highest price announced by bidder  $i$ . The  $\alpha$ -price-discriminatory payment mechanism is then such that bidder  $i$  must pay his bid for any winning share he asked at a price below  $\bar{p}_i$ , and he must pay  $\bar{p}_i$  for any winning share asked at a price above  $\bar{p}_i$ . In other words to receive his winning share of the security  $3/5$  (respectively  $2/5$ ), bidder 1 (respectively bidder 2) must pay a total price corresponding to the areas denoted 1 and 2 in Figure 3.1 (respectively 3.2). The payment of bidder  $i$ , therefore, still lay between the payments under the traditional uniform-price (i.e. area 1) and discriminatory-price (i.e. area 1, 2 and 3) auctions, but observe that unlike the  $\alpha$ -discriminatory format, area 2 does not correspond anymore to  $2/3$  of the discriminatory surplus (i.e. area 2 and 3).

## 4.2 Uniform-Type Auctions

### 4.2.1 The $\alpha$ -Uniform Auction

The  $\alpha$ -uniform payment mechanism is similar to its discriminatory counterpart presented in section 4.1.1, except that each winning bidder pays the same highest price  $\bar{p}$ . We can therefore write the payment mechanism in the  $\alpha$ -uniform auction as

$$p^0 \varphi_{i,l}(p^0, s_{i,l}) + \int_{p^0}^{\bar{p}} \varphi_{i,l}(p, s_{i,l}) dp ,$$

The common highest price  $\bar{p}$  is defined by the following relationship

$$\int_{p_0}^{\bar{p}} \Phi(p, s) dp = \alpha \int_{p_0}^{p^{\max}} \Phi(p, s) dp, \quad (8)$$

where  $\Phi(p, s)$  represents the aggregate demand as defined in (1), and  $p^{\max} = \max(p_{i,l}^{\max})_{i,l}$  is the highest price announced across all bidders, and  $\int_{p_0}^{p^{\max}} \Phi(p, s) dp$  may be interpreted as the aggregate discriminatory surplus. In other words, the parameter  $\alpha$  now represents the fraction of the aggregate discriminatory surplus extracted in addition to the revenue generated under the traditional uniform-price auction.

Observe that the  $\alpha$ -uniform and  $\alpha$ -discriminatory formats share a number of characteristics: first, setting  $\alpha = 0$  (respectively,  $\alpha = 1$ ) yields in both cases the traditional uniform-price (respectively, discriminatory) payment mechanism; second, for a given value  $\alpha$  and the same bid function, both mechanisms generate the same revenue for the Treasury; and third,  $\alpha$  represents in both cases the same fraction of the aggregate discriminatory surplus. Note however that, in contrast with the  $\alpha$ -discriminatory format, the total payment of some bidders may be the same under the  $\alpha$ -uniform and the traditional discriminatory format. Indeed, such an event would occur to a bidder  $i$ , if his highest bid is such that  $p_{i,l}^{\max} \leq \bar{p}$ . Finally, note that unlike the  $\alpha$ -discriminatory format, a slight change in a bid function of a bidder under the  $\alpha$ -uniform format, may not affect seriously the highest price paid by that bidder. Indeed, an individual deviation should only moderately influence the aggregate demand  $\Phi(p, s)$ , and therefore  $\bar{p}$  as defined in (8) should not vary significantly. To better appreciate the similitudes and differences between the two auction formats, consider the following example.

**Example 4** Consider the same example as before, except that the auctioneer now selects the  $\alpha$ -uniform payment mechanism with  $\alpha = 2/3$ . The bid functions of the two bidders are plotted in Figures 4.1 and 4.2, while the aggregate demand is presented in Figure 4.3. As just explained, the highest price  $\bar{p}$  is defined in Figure 4.3, such that area 2 corresponds to  $2/3$  of the aggregate discriminatory surplus (areas 2 and 3). The highest price  $\bar{p}$  may then be reported on the plot of the bid functions in order to determine the exact payment of each bidder (i.e. areas 1 and 2 in Figures 4.1 and 4.2). Finally, a comparison of Figures 2.1 and 4.1 (respectively, 2.2 and 4.2) indicates that, although the two auction formats would generate the same total revenue for the Treasury, bidder 1 (respectively, bidder 2) would pay a higher (respectively, lower) price under the  $\alpha$ -uniform pricing rule compared to the  $\alpha$ -discriminatory pricing rule.

#### 4.2.2 The $\alpha$ -Price-Uniform Auction

The  $\alpha$ -price-uniform format is the analog to its discriminatory counterpart presented in section 4.1.2, except that once again the highest price paid is common to all bidders. This new highest price  $\bar{p}$  is now defined as a convex combination

between the stop-out-price  $p^0$  and  $p^{\max}$ , the highest price announced across all bidders

$$\bar{p} = p^0 + \alpha (p^{\max} - p^0) . \quad (9)$$

As with the other payment mechanisms previously introduced, the revenue of the Treasury under the  $\alpha$ -price-discriminatory format will lay between the revenues generated under the traditional uniform and discriminatory auction formats. Note however, that for a specific value  $\alpha$  and a given set of strategies for bidder 1 and 2, the revenue generated by the Treasury is superior under the  $\alpha$ -price-uniform format compared to the  $\alpha$ -price-discriminatory format. Indeed, since  $\bar{p}$  (as defined in equation 9) is necessarily larger or equal than  $\bar{p}_{i,l}$  (as defined in equation 7) for any bidder  $i$ .

The  $\alpha$ -price-uniform format is a perfect illustration of the fact that with “uniform-type” auctions, the highest price paid by a bidder does not depend directly on its own bid function. Indeed, unless a bidder submits the highest price  $p^{\max}$ , he has virtually no control (except indirectly through  $p^0$ ) over the value of the highest price  $\bar{p}$  he will pay. This obviously contrasts sharply with the  $\alpha$ -price-discriminatory auction in which  $\bar{p}_{i,l}$ , the highest price paid by bidder  $i$  in group  $l$ , depends directly on its own bid function through the highest price he announced  $p_{i,l}^{\max}$  (see equation 7).

**Example 5** Consider the same example as before, except that the auctioneer now selects the  $\alpha$ -price-uniform payment mechanism with  $\alpha = 2/3$  (see Figures 5.1 and 5.2). Since  $p_1^{\max} < p_2^{\max}$ , the highest price paid by each bidder is set to  $\bar{p} = p^0 + 2/3 (p_2^{\max} - p^0)$ . This highest price may then be reported on the two bid functions in order to determine the exact payment of each participant. Note that since  $p_1^{\max} < \bar{p}$ , the payment of bidder 1 is in fact equivalent to what he would have paid under the traditional discriminatory format. Bidder 2 on the other hand, pays exactly the same amount as he would have under the  $\alpha$ -price-discriminatory format (see Figure 3.2).

### 4.2.3 The $k^{\text{th}}$ -Average-Price Auction

This pricing rule is inspired by the second-price payment mechanism initially proposed by Vickrey (1961) to conduct single-unit auction. At a single-unit second-price auction, the good is allocated to the highest bidder, and the price paid equals the second highest bid submitted. Although rarely used in practice, the second-price auction possesses interesting theoretical properties. In particular, the equilibrium bid under the private-values paradigm consists for a bidder in announcing his true private valuation. This pricing rule has been then generalized into a  $k^{\text{th}}$ -price auction in which the highest bidder pays only the  $k^{\text{th}}$  highest price submitted. Note, however, that this generalized model has drastically different theoretical properties than the second-price auction. In particular, truthful bidding is not necessarily an equilibrium strategy (VERIFIE: PAR EXEMPLE UN WORKING PAPER DE Elmar Wolfstetter).

To define the  $k^{\text{th}}$ -average-price auction, consider first  $p_{i,l}^a$ , the average winning price submitted by a winning bidder  $i$  in group  $l$  for the bids he is allocated:

$$p_{i,l}^a = \frac{p^0 \varphi_{i,l}(p^0, s_{i,l}) + \int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp}{\varphi_{i,l}(p^0, s_{i,l})} .$$

We can then define  $(p_{(1)}, p_{(2)}, \dots, p_{(n)})$ , the vector of winning prices ranked in decreasing order, for the  $n \leq N$  bidders who receive a strictly positive share of the security. The common highest price  $\bar{p}$  is then set equal to the  $k^{\text{th}}$  highest average winning price  $p_{(k)}$  (and  $\bar{p} = p_{(n)}$  when  $n < k$ ).

Observe that there is no value of  $k$  for which the  $k^{\text{th}}$ -average-price auction shares with the second-price auction the property that truthful revelation is a dominant strategy. Indeed, just like with the other auction formats presented in this paper, participants have an incentive to shade their bids. Moreover, note that once again a slight change in the bid function of a participant has little bearing on the highest price he will pay, as long as it does not change the identity of the bidder submitting the  $k^{\text{th}}$  highest average winning price.

**Example 6** Consider the same example as before, except that the auction is now conducted under the second-average-price payment mechanism. In figures 6.1 and 6.2, we have plotted  $p_1^a$  and  $p_2^a$ , the average price per unit allocated to bidder 1 and 2. Since  $p_2^a > p_1^a$ , we can set  $\bar{p} = p_2^a$ , which determines the payment of bidder 1 and 2. Observe that in this example, bidder 1's payment is similar to the payment he would have to make under the traditional discriminatory-price format. Note, however, that this payment mechanism cannot be interpreted as special case of the auction formats introduced so far. As a result, the participants' payments, and the Treasury's revenue, cannot be compared directly with those obtained previously with other auction formats.

#### 4.2.4 The Spanish Auction Format

We conclude this section by presenting an additional member of the family of "uniform-type" auctions. This payment mechanism is actually used in practice to sell Spanish Treasury securities. Under this auction format,  $\bar{p}$  is defined as the weighted average price of all winning bids:

$$\bar{p} = \frac{p^0 Q + \int_{p^0}^{p^{\max}} \Phi(p, s) dp}{Q} ,$$

where  $Q$  is the quantity supplied by the Treasury,  $\Phi(p, s)$  represents the aggregate demand as defined in (1), and  $p^{\max} = \max(p_{i,l}^{\max})_{i,l}$  is the highest price announced across all bidders.

The Spanish experience has given rise to a small number of economic analyses. For instance, using simulations of a simplified model in which two units

of Treasury bonds are for sale, Álvarez and Mazón (2002) find that the Spanish format may in some cases generate higher revenues than the traditional discriminatory format. Abbink, Brandts and Pezanis-Christou (2002), using a similar model, confirmed experimentally that the Spanish auction may dominate the traditional discriminatory format. This result has then been slightly generalized by Álvarez, Mazón and Cerdá (2003). Indeed, adopting a share auction model with linear strategies, the authors find that, for a given set of structural parameters, the Spanish format may increase the Treasury’s revenue over the traditional uniform and discriminatory auctions. Our analysis of the Spanish auction format differs in essentially two ways from the papers just mentioned: first, the theoretical model we adopt is significantly richer as it accounts simultaneously for random supply, risk aversion and asymmetry; second, our simulations are conducted with specific structural parameters estimated in the particular context of the French Treasury auctions. The simulations conducted in section 5 will therefore help us establish whether or not the Spanish auction format would have increased the French Treasury revenue during the period spanned by A&S’s sample.

**Example 7** *Consider the same example as before, except that the auction is now conducted under the Spanish format. We plotted in Figures 7.1 and 7.2 the bid functions of bidders 1 and 2, as well as the highest price of payment  $\bar{p}$ , defined here as the weighted average price of all winning bids. Observe that in this example, bidder 1’s payment is similar to the payment he would have to make under the traditional discriminatory-price format. Note that, similarly to the  $k^{\text{th}}$ -average-price auction, this payment mechanism cannot be interpreted as special case of the  $\alpha$ -type payment mechanism introduced in sections ? ? ?.*

## 5 Comparative Results

We now conduct a Monte Carlo simulation to compare the revenue the French Treasury would have generated in the 118 auctions conducted during the sampling period in A&S (May 1998 to December 2000). To do so, we use the structural parameter estimated in A&S for the specific case of the French Treasury, as well as the specific exogenous variables for each of these 118 auctions (i.e. the number of bidders in group 1 and in group 2, the security’s type, yield and maturity, and the announced bracket for the quantity to be served).<sup>12</sup> Moreover, we derive the Bayesian Nash equilibrium bid function under each of the six payment mechanisms we just presented, using the numerical technique developed by Armantier, Florens and Richard (2005).

The simulation results are summarized in Tables 1.a to 1.c and Figures 8.a to 8.e. We report in Table 1.a the per auction and total expected revenue of the

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<sup>12</sup>The Monte Carlo simulations rely on the common random number technique. In other words, the comparison of the payment mechanisms are conducted with the same exogenous variables and the same pseudo-random private signals. As a result, the Monte Carlo simulation may be directly compared across payment mechanisms.

French Treasury, under the  $\alpha$ -type payment mechanisms for different values of  $\alpha$ . Table 1.b corresponds to the revenue of the French Treasury under the  $k$ -average-price auction for different values of  $k$ . Finally, we present in Table 1.c the revenue of the French Treasury under the Spanish format. The evolutions of the total revenue of the French Treasury as a function of  $\alpha$  and  $k$  under the  $\alpha$ -type and  $k$ -average-price payment mechanisms are illustrated in Figures 8.a to 8.e.

## 5.1 Ranking Between the Traditional Uniform-Price, Discriminatory and Spanish Formats

We start by comparing the three auction formats that have been used in practice to sell Treasury securities. Recall that under any of the  $\alpha$ -type payment mechanisms (i.e.  $\alpha$ -discriminatory,  $\alpha$ -price-discriminatory,  $\alpha$ -uniform and  $\alpha$ -price-uniform),  $\alpha = 0$  corresponds to the traditional uniform-price format and  $\alpha = 1$  to the traditional discriminatory format. The first and last columns of Table 1.a illustrate the main result in A&S. Namely, the French government would have raised a higher revenue had it run its Treasury auctions under the uniform-price format instead of the discriminatory format. To understand why the uniform-price auction dominates, it is important to remember that a fundamental characteristic of this auction is that bidders do not pay their bid for each unit they receive. Instead, a winning bidder pays a single price, the stop-out-price, for every units he is allocated. As a result, bidders are inclined to announce higher prices than at discriminatory auctions, for the first units demanded. Indeed, strategic bidders realize that the stop-out-price they will have to pay will be lower than the price they announced for these initial shares. In particular A&S find that the group of small bidders is willing to take more risk under the uniform-price auction. Indeed, small banks, which are more risk averse and less informed about the true value of the security, are willing to buy larger amounts of the security at any relevant price under the uniform-price format, as they know that they will not have to pay the price they announce. As a result of this more aggressive bidding behavior, the price paid by a bank (regardless of type) for a unit of the security increase, which implies that the revenue raised by the French Treasury during our sample period would have been higher under the uniform-price format. As noted by A&S, however, the uniform-price auction has a drawback. Indeed, as illustrated in Table 1.a, the standard deviations of the per auction (PAS DE TRAIT D'UNION, VERIFIE) Treasury revenues indicate that the revenue raised by the Treasury is significantly more variable from one auction to the next under the uniform-price format.<sup>13</sup> In other words, although the Treasury revenue is higher under the uniform-price format, the precise amount of money an auction will generate be-

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<sup>13</sup>The standard deviations in Tables 1.a to 1.c are calculated across auctions in our sample. The standard deviations therefore represent a measure of the variation of the French Treasury revenue from one auction to the next. In other words, these standard deviations should not be confused for a measure of the accuracy of an estimate, as typically presented along the results of a regression.

comes less predictable. Under these circumstances, it may be more difficult for the French government to use the uniform-price Treasury auction as an efficient short-term tool to manage its public debt. Finally, A&S find these results are context specific, and they may not extend directly beyond the French Treasury experience. Indeed, Monte-Carlo simulations suggest that alternative values of the structural parameters may yield different conclusions. In particular, A&S find that the discriminatory format would generate higher revenues for the Treasury than the uniform-price auction, if bidders were considered symmetric, risk neutral and small banks received private signals drawn from the same distribution as their larger counterparts. In other words, accounting for informational and risk aversion asymmetries is crucial to determine accurately the payment mechanism generating the highest revenue.

We now turn to the Spanish format. Table 1.c indicates that, in terms of the revenue raised by the French Treasury, the uniform-price format dominates the Spanish format which in turn dominates the discriminatory format. This result is consistent with the theoretic analysis of Álvarez et al. (2003) and the experimental analysis of Abbink et al. (2002). Note also that the standard deviation of the per auction revenue is notably lower under the Spanish format. In other words, the French government may find in this Spanish format a good compromise between raising the highest possible revenue and maintaining a stable stream of revenues from one auction to the next. Intuitively this result may be explained by the fact that ... (VOIR PAPIERS EXPAGNOLS) (AU MOINS 4-5 lignes, au plus 12 lignes).

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## 5.2 Alternative Formats

### 5.2.1 $\alpha$ -Type Payment Mechanisms

In this section we consider the  $\alpha$ -type auction formats which, to the best of our knowledge, have never been implemented to sell Treasury auctions. We present in Table 1.a the Treasury revenue under the different  $\alpha$ -type payment mechanisms, for different values of  $\alpha$ . Three comments are in order at this point. First, we assume that the value of  $\alpha$  is fixed by the Treasury and common knowledge prior to any auction. Second, the value of  $\alpha$  is assumed to be the same for each of the 118 auctions in our sample. A possible alternative would be for the Treasury to select the value of  $\alpha$  prior to each auction as a function of the exogenous variables, such as the yield, the quantity to be served and possibly the number of bidders. We opted for a fixed  $\alpha$  policy as a simplification, as we suspect it may be easier to implement in practice, and it easier for the bidders to accept. Third, we do not try to determine the exact value of  $\alpha$  that maximizes the revenue of the French Treasury. Instead, we consider a grid consisting in common fractions such as deciles and quartiles. This choice was motivated by the fact that the auctions would be easier to implement in practice with a common fraction rather than with a real number.



Figures 8.a to 8.d indicate that, under the  $\alpha$ -type payment mechanisms, the Treasury's revenue is systematically skewed to the left with a maximum between 0.25 and 0.3. The non-monotonic shape of the Treasury's revenue (i.e. first increasing and then decreasing) may be explained by the following trade off. On one hand, a small  $\alpha$  induces more aggressive behavior. Indeed, recall that the  $\alpha$ -type payment mechanisms get closer to the traditional uniform-price auction format when  $\alpha$  get smaller. As a result, bidders, and in particular small bidders, are more aggressive knowing they are less likely to pay the bids they actually submit. On the other hand, the revenue extracted by the Treasury per bidder increases with  $\alpha$ . Indeed, recall that  $\alpha$ -type payment mechanisms get closer to the traditional discriminatory auction format when  $\alpha$  get close to 1, in which case a bidder must pay, in addition to his payment under the uniform-price auction, almost all of its discriminatory surplus. Our simulations suggest that in the specific case of the French Treasury auctions, the best compromise may be found for low values of  $\alpha$  between 0.25 and 0.3. This result may be explained in part by the influential role played by small bidders in French Treasury auctions. Indeed, as indicated in Table ?? small bidders outweigh large bidders by a ratio of 3 to 1 on average. In such a context, it is reasonable to expect that the primary objective of the French Treasury is to incite small bidders to submit larger bids.

Note also that although it does not dominate for every possible value of  $\alpha$ , the  $\alpha$ -price-discriminatory and  $\alpha$ -price-uniform formats (Figures 8.c and 8.d) may yield higher revenues than respectively the  $\alpha$ -discriminatory and  $\alpha$ -uniform formats (Figures 8.a and 8.b). Likewise, within the class of  $\alpha$ -type formats, the uniform price mechanisms (i.e.  $\alpha$ -uniform and  $\alpha$ -price-uniform formats in Figures 8.b and 8.d) dominate the discriminatory price mechanisms (i.e.  $\alpha$ -discriminatory and  $\alpha$ -price-discriminatory formats in Figures 8.a and 8.c) for some values of  $\alpha$ . In other words, it appears that the best payment mechanism for the French Treasury, within the class of  $\alpha$ -type formats, is the  $\alpha$ -price-uniform payment mechanism, with  $\alpha = 0.25$ . Table 1.a indicates that this auction format would have raised a total revenue for the French Treasury of nearly 204.5 billions Euros during our sampling period, corresponding to an increase of 9.4% compare to the discriminatory format used by the French Treasury. To understand why the  $\alpha$ -price-uniform format dominates, it is important to see that under this payment mechanism a bidder has the least control over the highest price he would have to pay. As a result, bidders, and in particular small bidders, can afford to be more aggressive, which benefit the French Treasury. Indeed, recall that under the  $\alpha$ -discriminatory and  $\alpha$ -uniform formats, the highest price paid by a bidder depends directly on the entire shape of the bid function he submits. In contrast, under the class of  $\alpha$ -price auctions (i.e.  $\alpha$ -price-discriminatory and  $\alpha$ -price-uniform formats) the highest price paid by a bidder depends almost exclusively on the highest price he submits for the first unit of the security. As a result, bidders, and in particular small bidders, have an incentive to behave more aggressively by submitting steeper bid functions. Likewise, we have seen that, by definition, the highest price paid by a bidder under discriminatory-type auctions depends directly on the bids he submits, while

under uniform-type auctions a bidder has very little control over the highest price he will pay through his own bid function.

### 5.2.2 $k^{th}$ -Average-Price Payment Mechanism

We present in Table 1.b the Treasury revenue under the  $k^{th}$ -average-price payment mechanism, for different values of  $k$ . Once again, we assume that the French Treasury fixes and announces a value for  $k$  prior to the 118 auctions in our sample. This implies in particular that although the number of bidders may vary from one auction to the next, the value of  $k$  remains unchanged. As we have seen in Section 3.2, the number of bidders in the French Treasury auctions we analyze is somewhat stable around 19. Therefore, we have calculated in Table 1.b the revenue of the French Treasury for  $k$  varying from 1 to 15. Since one cannot predict exactly the number of winning bidders, the values of  $k$  we consider may turn out to be larger than the number of winning bidders in some of the 118 auctions in our sample. Recall that in such cases, the highest price of payment  $\bar{p}$  is set to the lowest average submitted winning bid across bidders.

Observe first in Figure 8.e that the shape of the French Treasury's revenue as a function of  $k$  is similar to a bell-curve. This result may be explained by the following trade off. On one hand, the French Treasury would like to select  $k$  as small as possible. Indeed, all things being equal otherwise, the French Treasury benefits the most when  $k$  equals one, since in this case the highest price paid  $\bar{p}$  is set equal to the average price of the bidder submitting the highest average bid. On the other hand, a low value of  $k$ , does not incite the group of large banks, which typically submits the highest bids, to be aggressive. Indeed,  $\bar{p}$  is in this case determined by this group of large banks. In contrast, when  $k$  is high, the large banks can afford to submit high bids as these are unlikely to influence  $\bar{p}$ . Table 1.b indicates that the best compromise between setting  $\bar{p}$  high and inciting large bidders to be aggressive is found for  $k = 6$ . The French Treasury revenue would have been 205.61 billions Euros, 10% higher than the revenue actually generated by the French Treasury during the sample period. This  $k^{th}$ -average-price auction format actually dominates the family of  $\alpha$ -type payment mechanisms we just discussed. Intuitively this result may be explained by the fact that bidders, and in particular the bidders submitting the highest bids, can afford to be aggressive as the bid function they submit is likely to have little or no weight on the determination of the highest price paid  $\bar{p}$ . To conclude, it is worth noting that the standard deviation of the per auction revenue is higher under the  $k^{th}$ -average-price mechanism than under the discriminatory auction format used by the French Treasury (see Tables 1.a and 1.b). In other words, the discriminatory format may still be preferred if the objective of the French government is to generate a stable stream of revenues to finance its debt.

## 6 Conclusions

The aim of this work was to propose and investigate alternative Treasury auction formats. We compare them with the traditional uniform-price, discriminatory or Spanish formats. Given the structural model estimated by Armantier and Sbai (2005), counter-factual analyses provide a ranking of these formats, in term of the revenue raised by the Treasury. First, we compare the traditional formats. We find that the uniform-price format should be preferred to the Spanish which in turn should dominate the discriminatory. This is consistent with the analyses conducted by Abbink, Brandts and Pezani-Christou (2002). Second, we see that among all the possible formats, the  $k^{th}$ -average-price format, with  $k = 6$ , should dominate. During a period of 2.5 years, if we compare Tables 1.a and 1.c, we see that the choice of this format may have raised about 10 billions euros more than the choice of the uniform format, and about 19 billions euros more than the discriminatory format which is currently used by the French Treasury.

One may wonder whether the  $k^{th}$ -average-price format would be well understood and accepted by the participants to Treasury auctions. We should note, however, that the  $k^{th}$ -average-price format is not necessarily more complicated than the Spanish format which has been used since January 1987 in Spain.

Our results suggest that the French Treasury could shift its auction's format 1) to the  $6^{th}$ -average-price format if the priority of this institution is to increase the average revenue raised through Treasury auctions, or 2) to the Spanish format if the priority is to raise a less variable revenue. If we consider the average revenue raised, the  $6^{th}$ -average-price format dominates all the other formats, while the Spanish format is dominated by all the other formats except the discriminatory format. If we consider the stability of the revenue raised by the Treasury, the Spanish format dominates all the other formats, while the  $6^{th}$ -average-price format is only dominated by the Spanish and discriminatory formats. This implies in particular that  $6^{th}$ -average-price format should be preferred to the uniform-price format, for both criteria, while the Spanish format only dominates the uniform-price for the stability of the revenue raised. Moreover, although the discriminatory format raises the less revenue on average, we can note that it is the second best format in terms of stability of the revenue raised by the Treasury.

Note that as a result, the conclusions of this paper may not immediately extend to Treasury markets in other countries. Nevertheless, although specific to the French Treasury, our analysis may be considered of general interest. Indeed, just like in France, many Treasury auctions around the world seem to involve asymmetric bidders, and are conducted under the discriminatory format. The methodology developed by Armantier and Sbai (2005) can therefore be applied to other Treasury auctions to estimate the structural parameters of the underlying model. As we do in this work, a counterfactual analysis can be implemented and produce a new ranking of the different formats. Then, one could decide in each individual case which payment mechanism appears to be the most advantageous to the auctioneer.

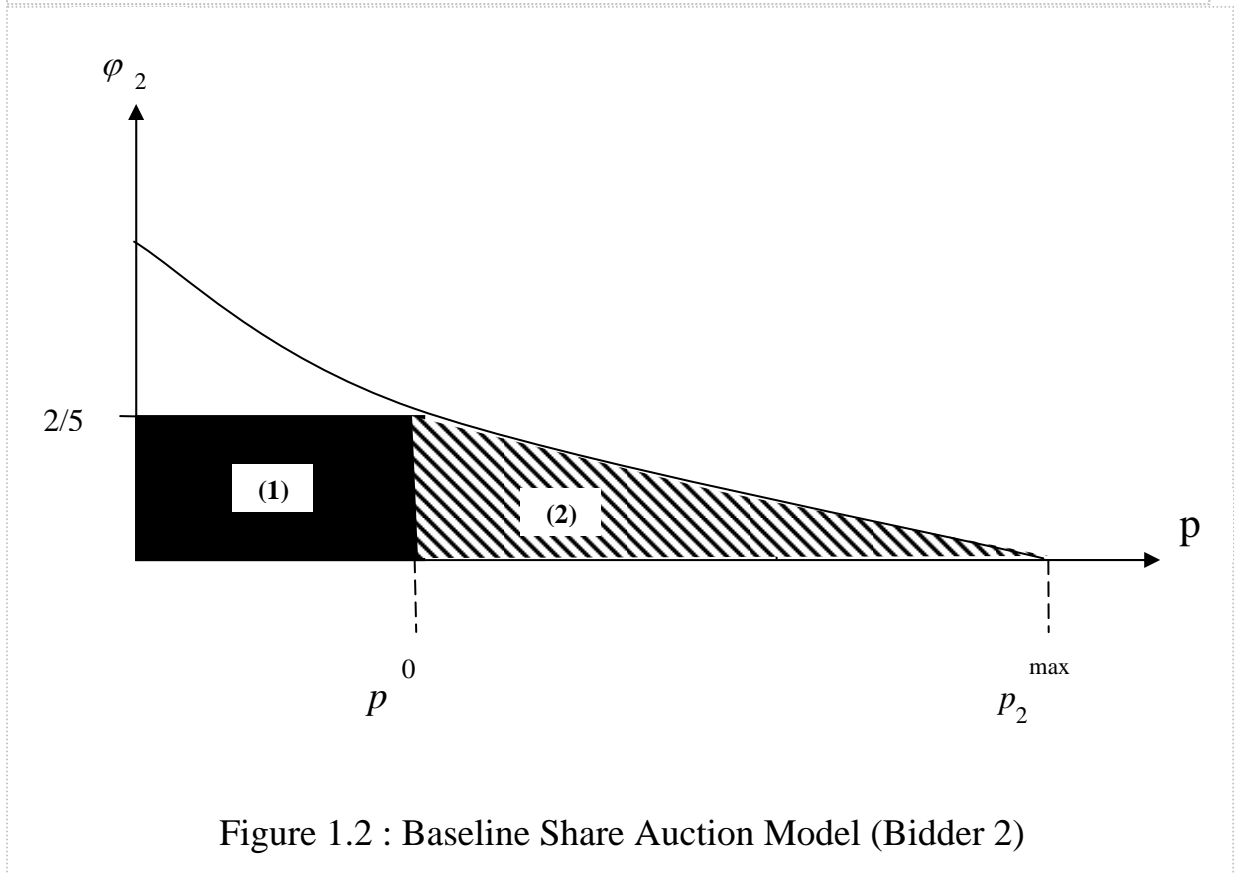
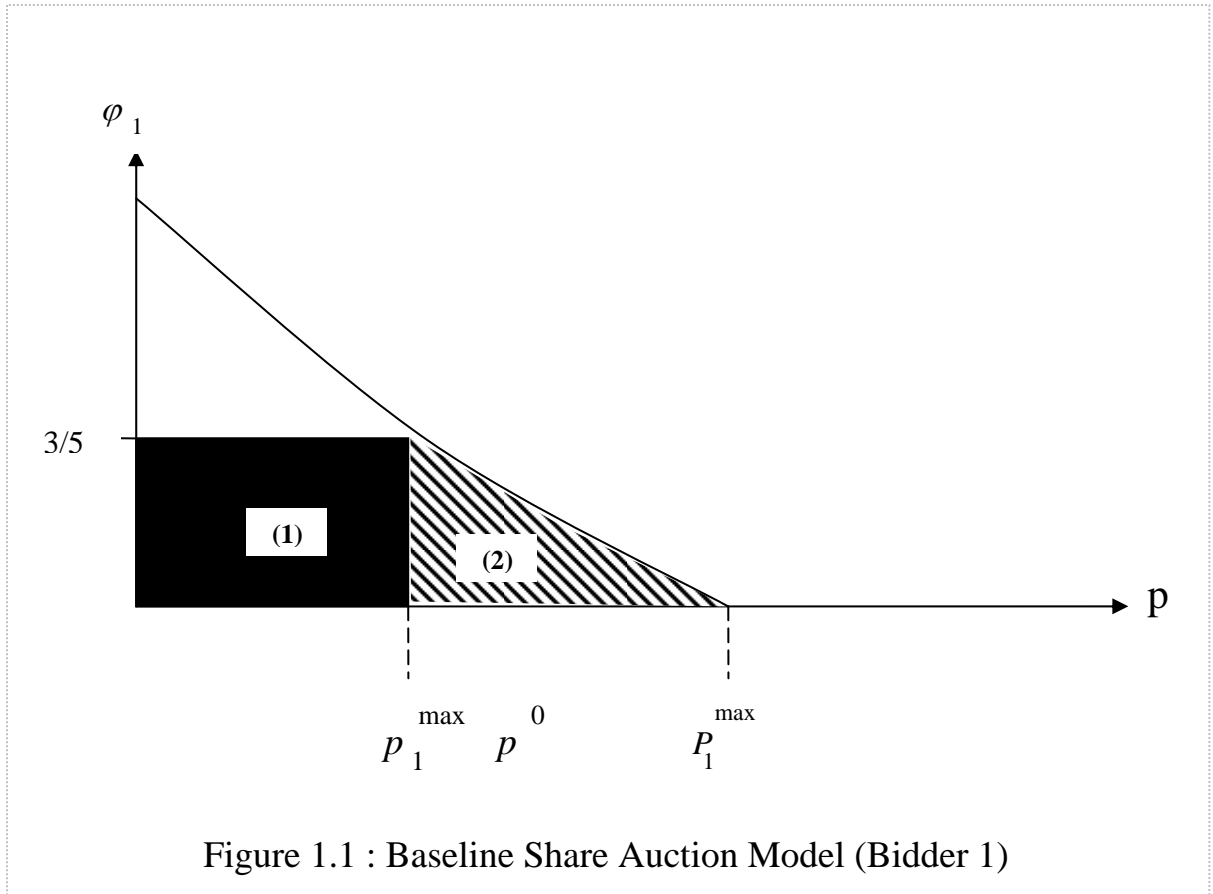
We must also acknowledge the limitation of our approach. Indeed, we have

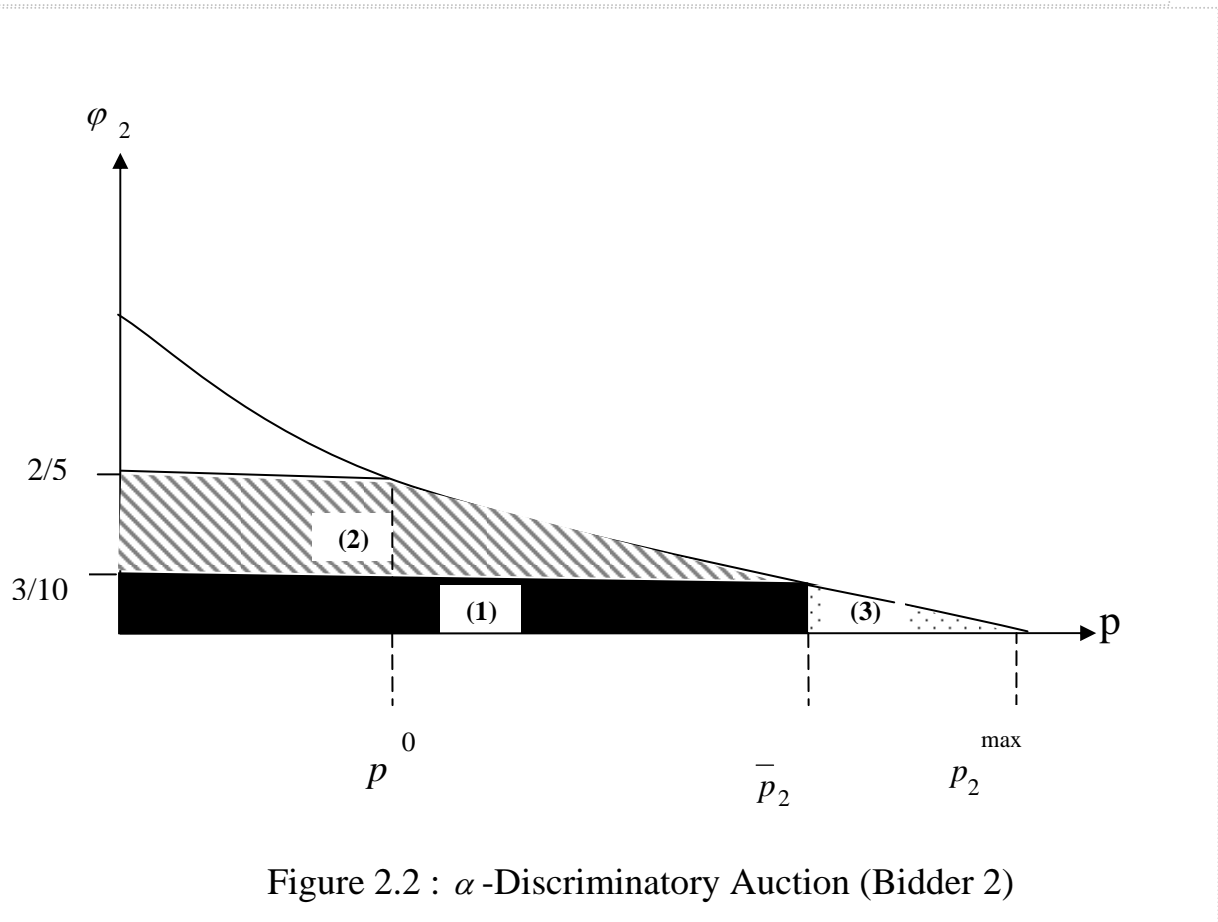
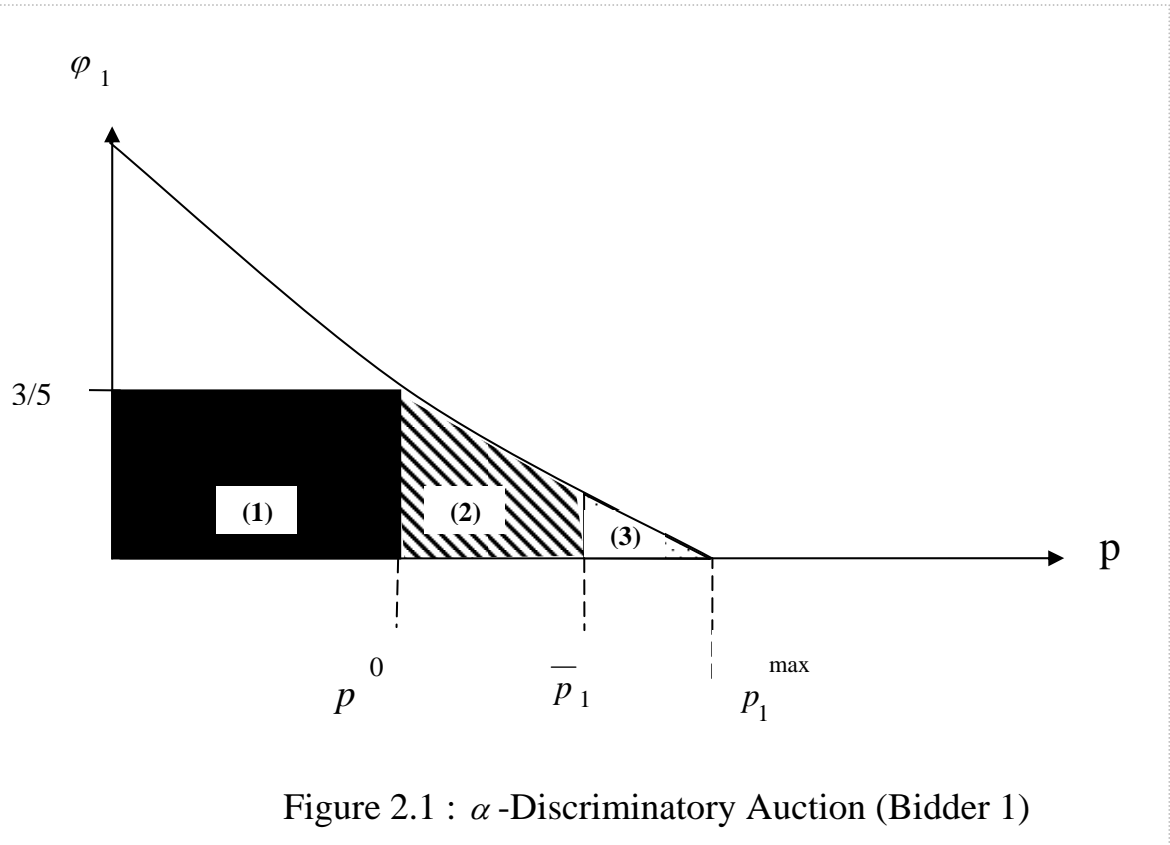
assumed that the participation is exogenous. Alternatively, there exists a possibility that the number of bidders change with the payment mechanism. The analysis of such a model, however is significantly more challenging, as it would require to estimate a model with endogenous participation. To the best of our knowledge, such a model does not exist in the literature of Treasury auctions.

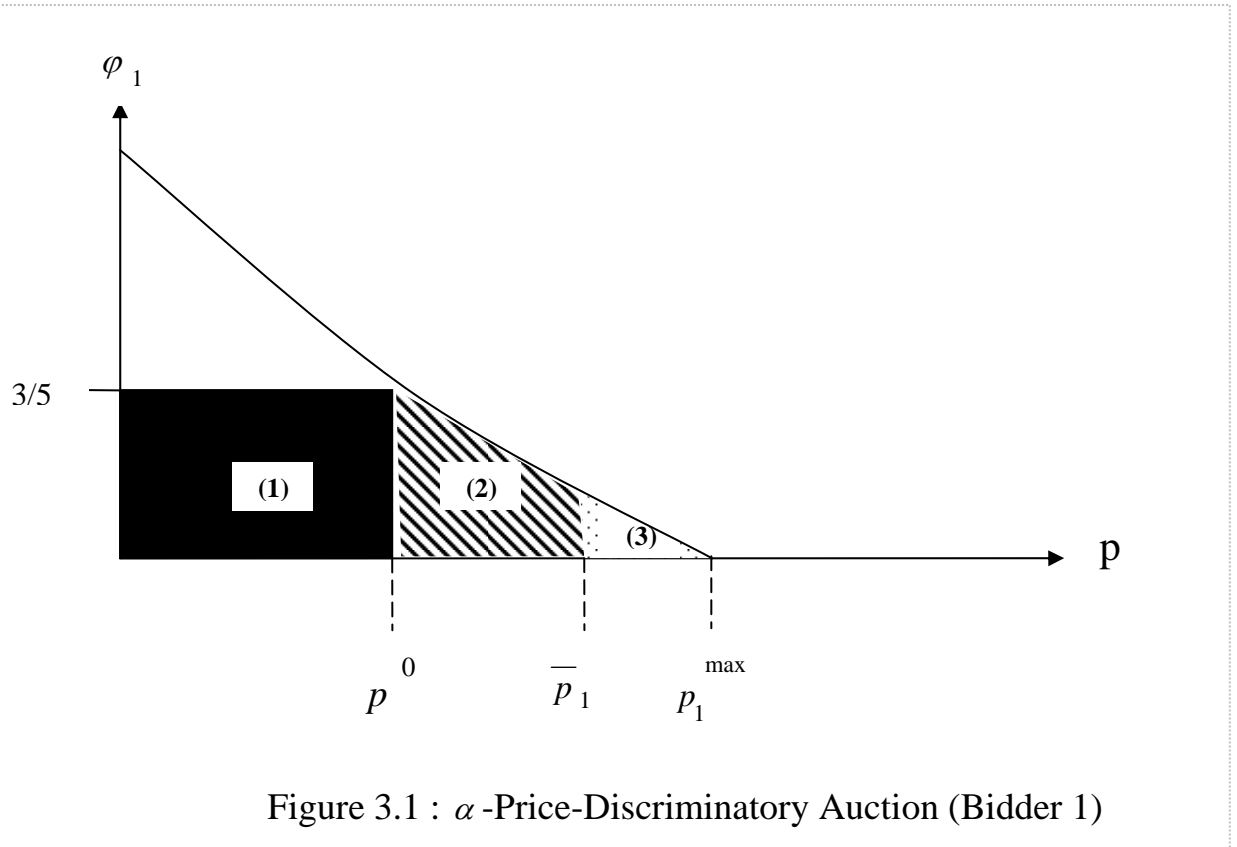
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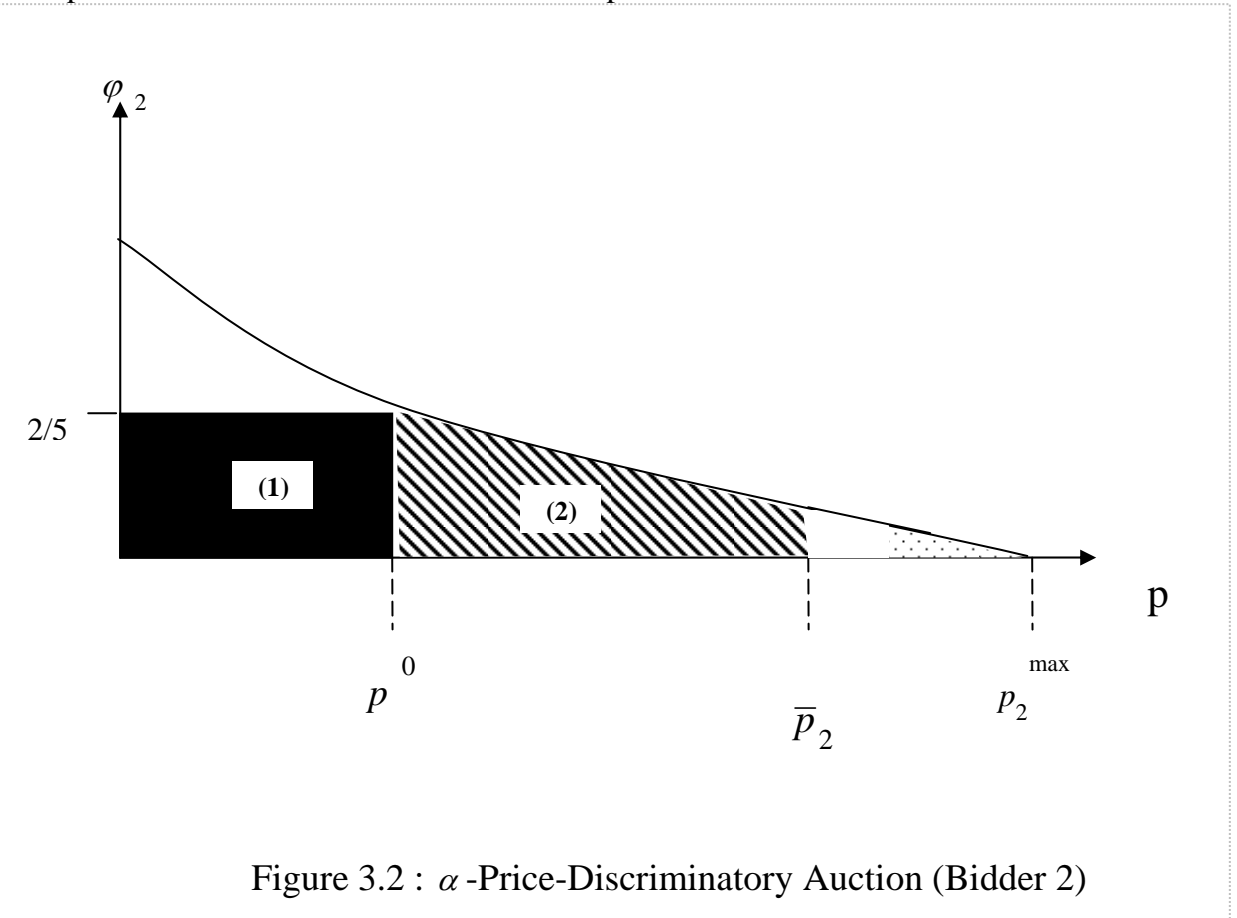
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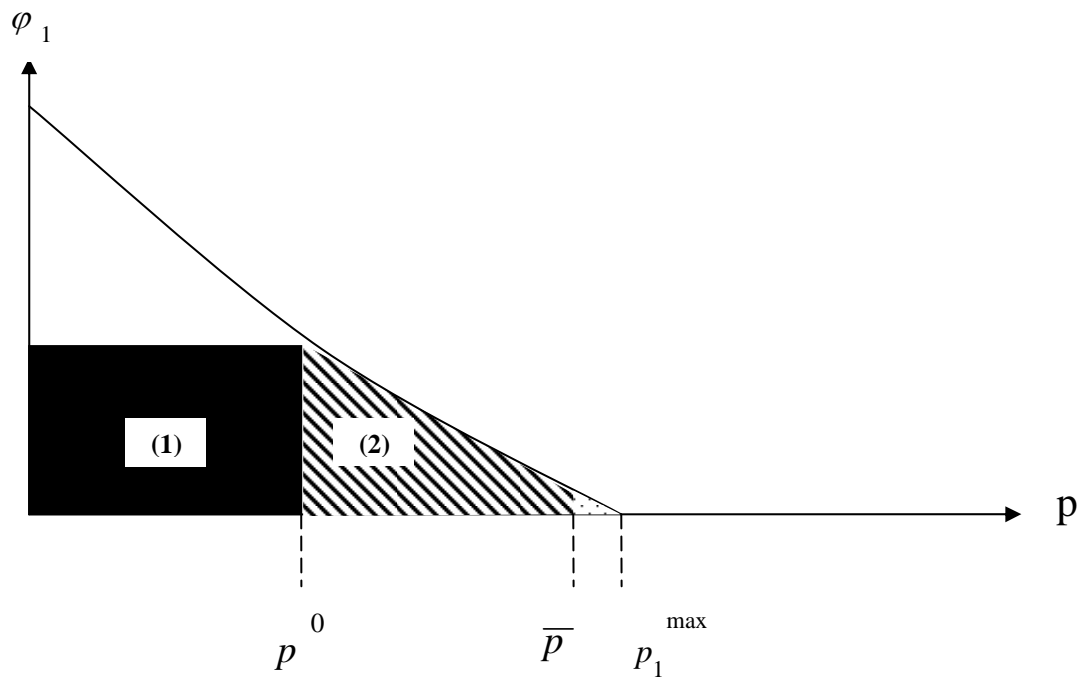


Figure 4.1 :  $\alpha$  - Uniform Auction (Bidder 1)

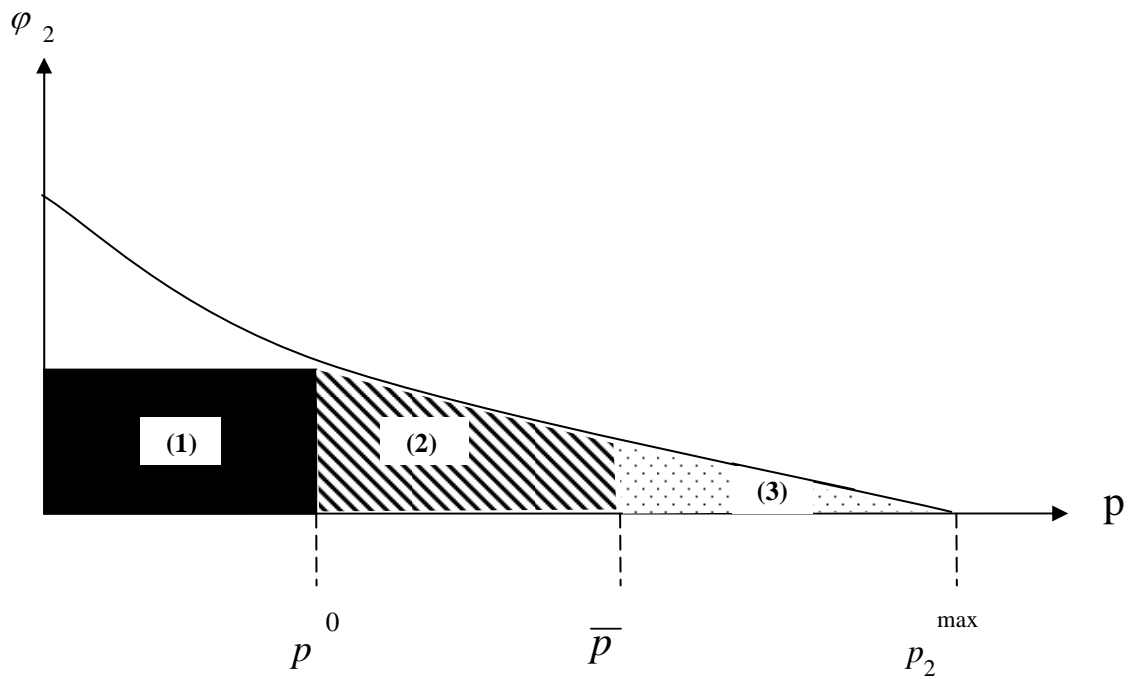


Figure 4.2 :  $\alpha$  - Uniform Auction (Bidder 2)

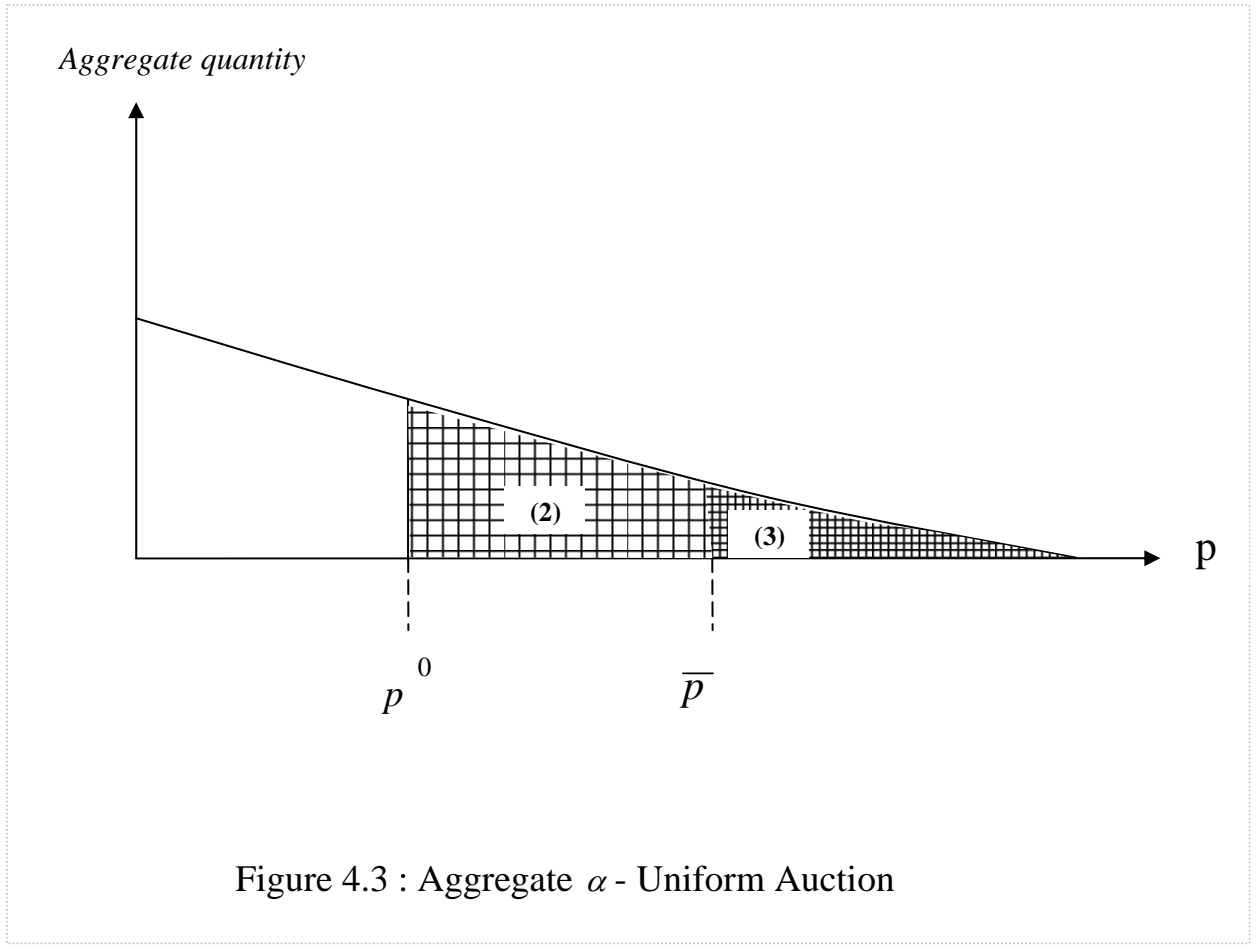


Figure 4.3 : Aggregate  $\alpha$  - Uniform Auction

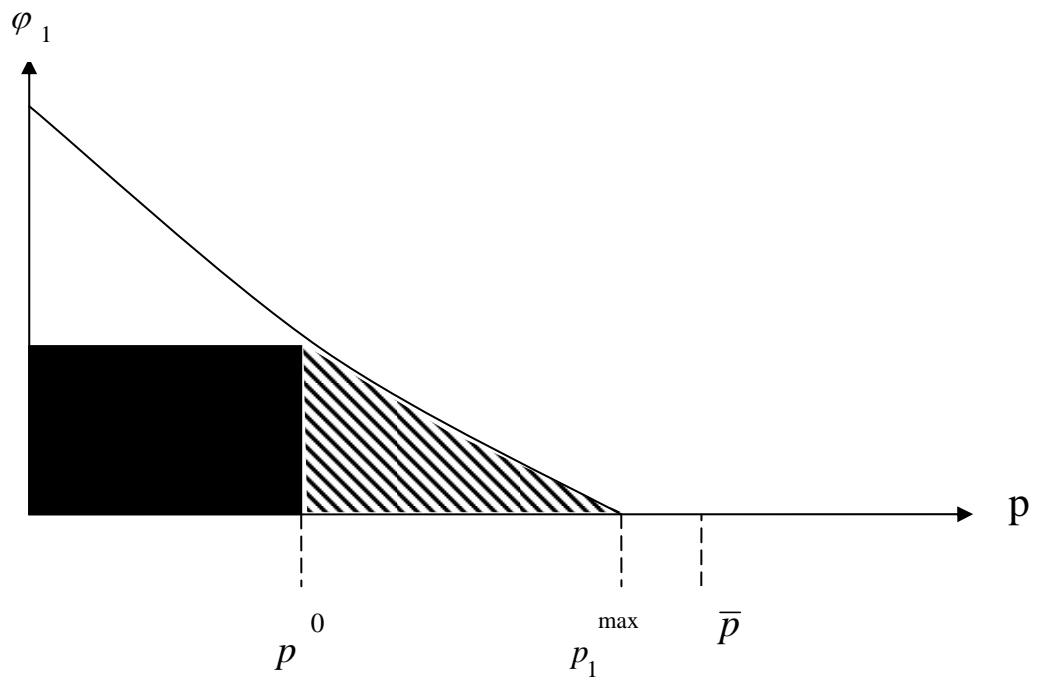


Figure 5.1 :  $\alpha$  - Price-Uniform Auction (Bidder 1)

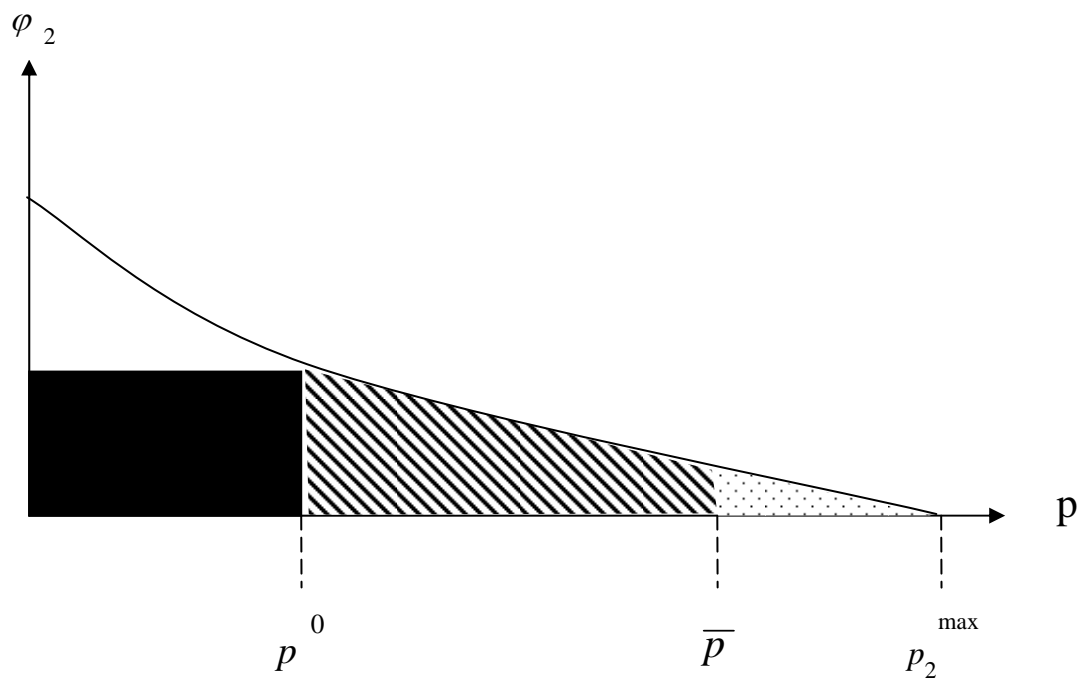


Figure 5.2 :  $\alpha$  - Price-Uniform Auction (Bidder 2)

Aggregate quantity

Aggregate Alpha-Price U  
Alpha = 2/3

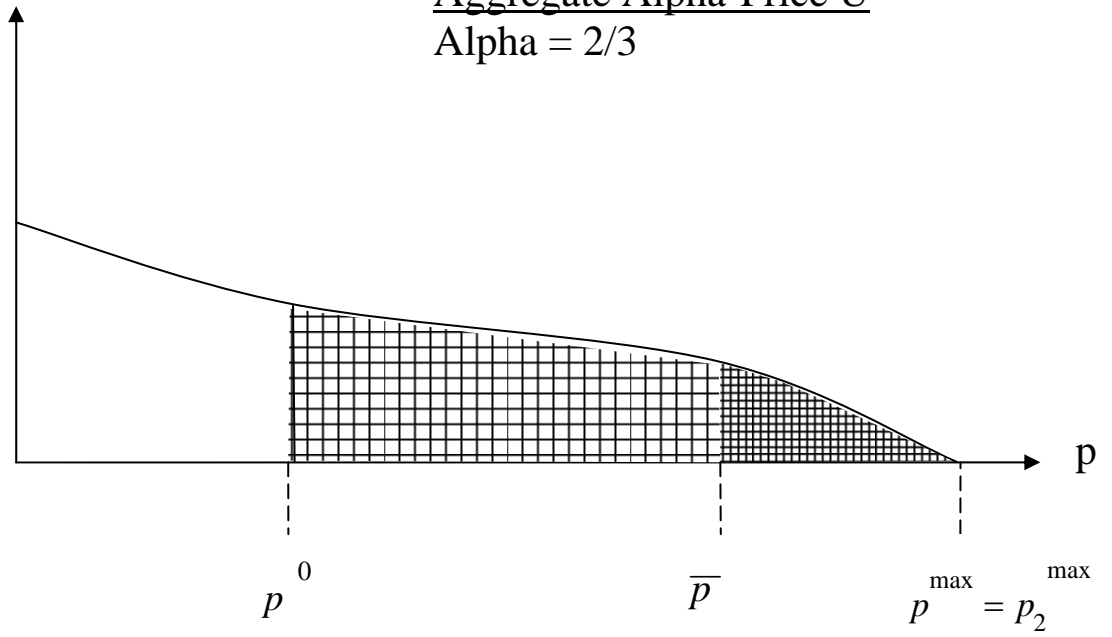


Figure 5.3 : Aggregate  $\alpha$  - Price-Uniform Auction

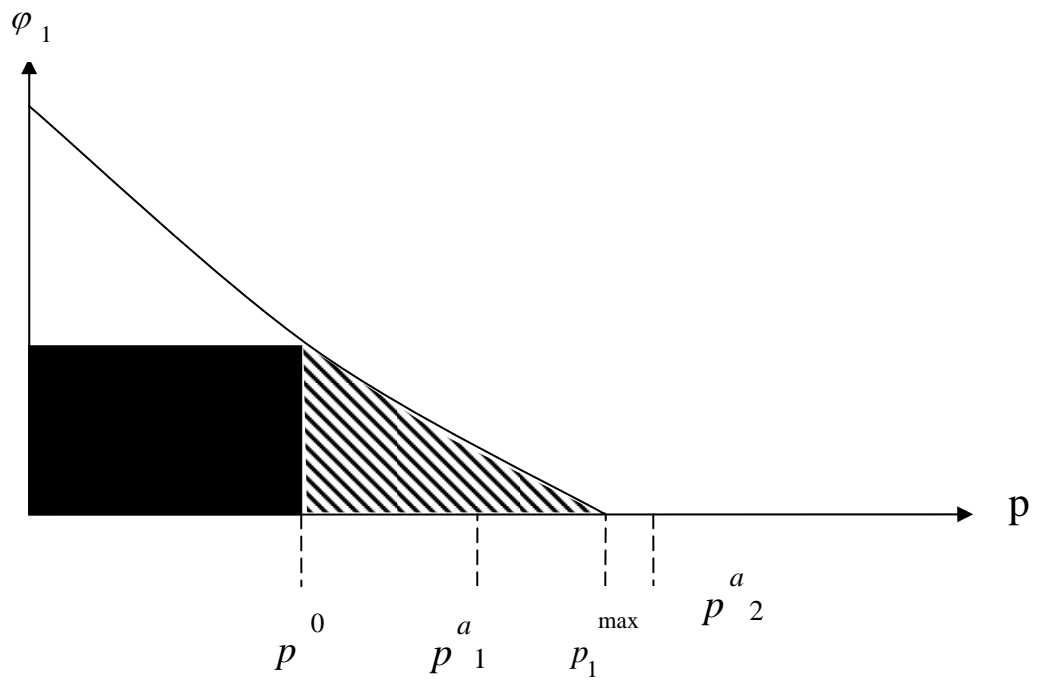


Figure 6.1 : 2<sup>nd</sup>- Average-Price Auction (Bidder 1)

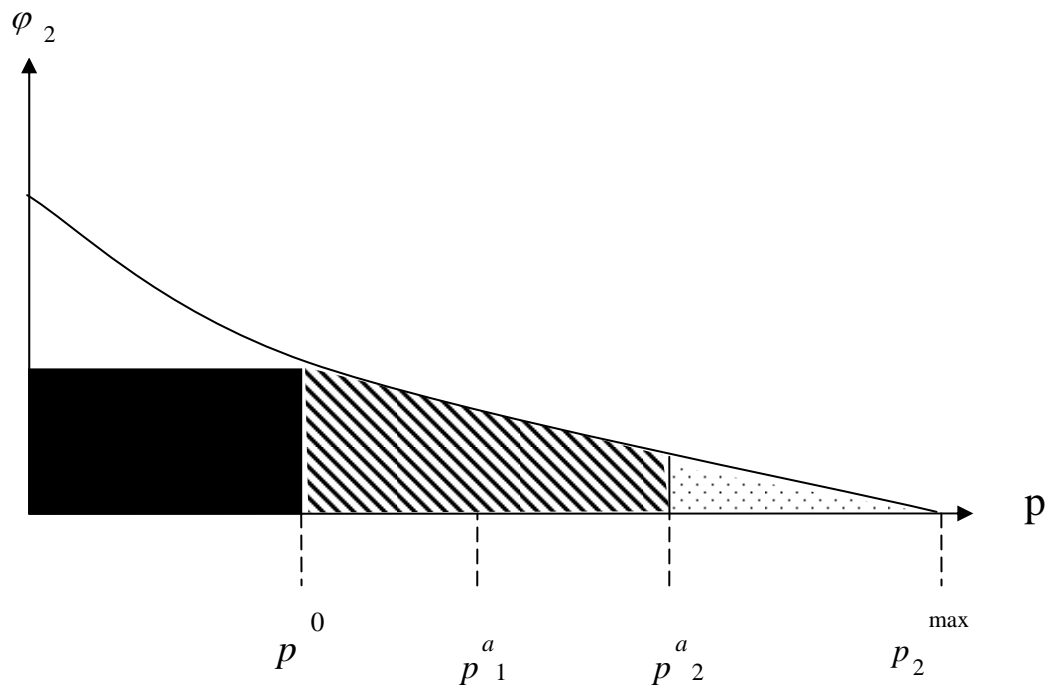


Figure 6.2 : 2<sup>nd</sup>- Average-Price Auction (Bidder 2)

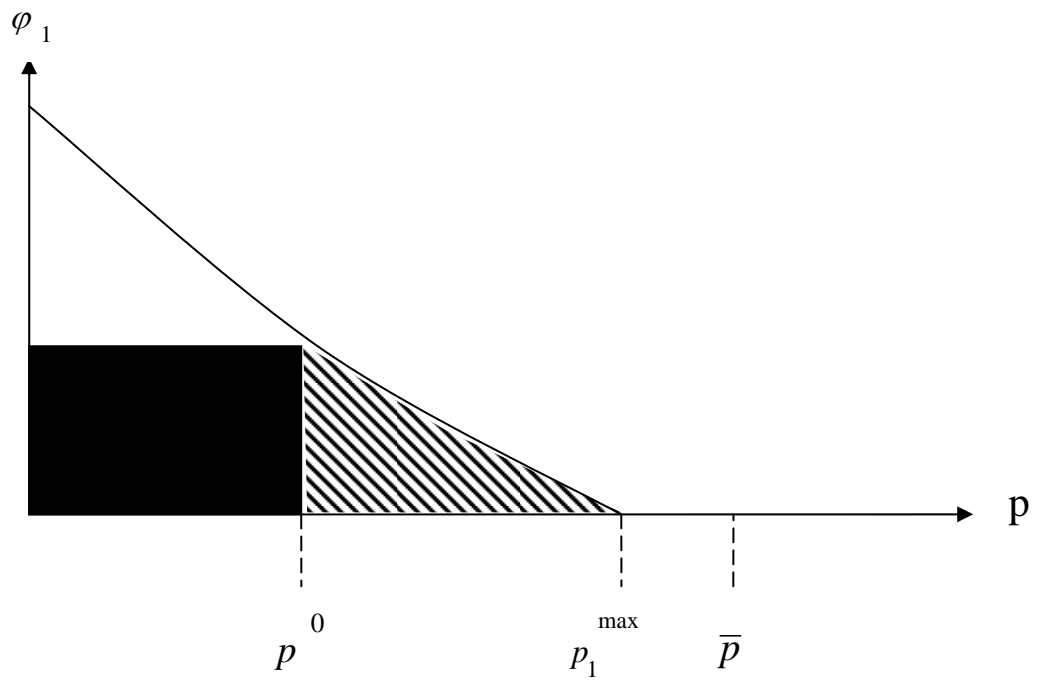


Figure 7.1 : Spanish Auction (Bidder 1)

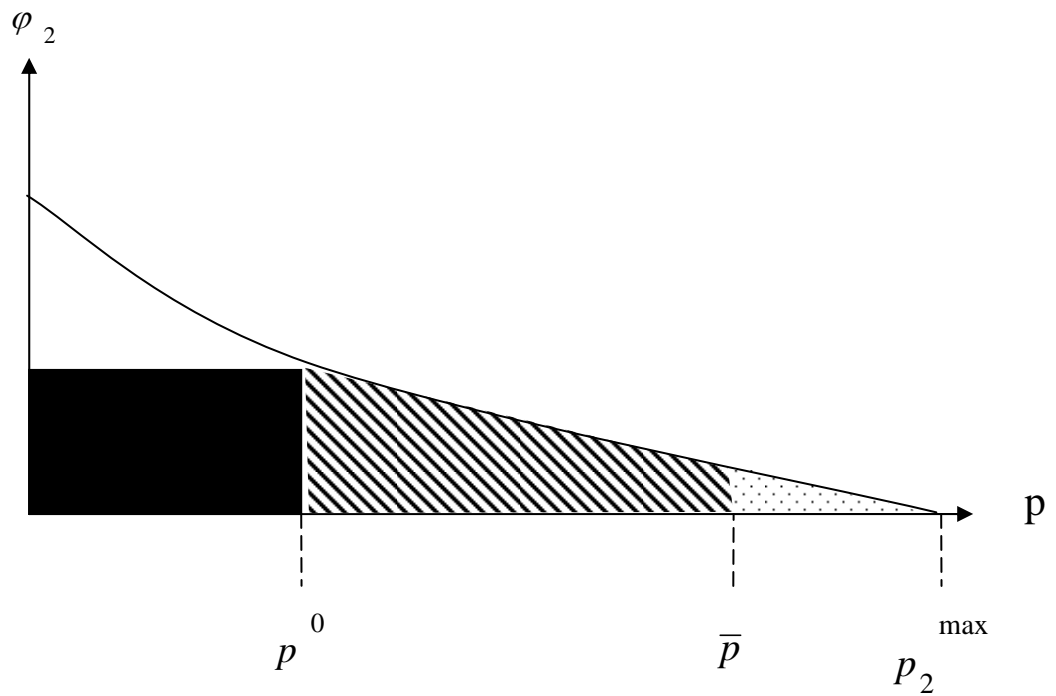


Figure 7.2 : Spanish Auction (Bidder 2)

**Table 1.a**  
**French Treasury Revenue**  
**Alpha Type Auctions**

Alpha	<u>Alpha-Discriminatory</u>		<u>Alpha-Uniform</u>		<u>Alpha-Price Discriminatory</u>		<u>Alpha-Price Uniform</u>	
	<i>Average per auction</i>	<i>Total</i>	<i>Average per auction</i>	<i>Total</i>	<i>Average per auction</i>	<i>Total</i>	<i>Average per auction</i>	<i>Total</i>
<b>0.00</b>	1659.40 (623.41)	195,810.33	1659.41 (623.41)	195,810.33	1659.41 (623.41)	195,810.33	1659.41 (623.41)	195,810.33
<b>0.10</b>	625.81 (625.81)	196,059.44	1670.32 (627.05)	197,098.26	1664.47 (626.66)	196,407.95	1685.67 (628.52)	198,908.48
<b>0.20</b>	1667.83 (627.60)	196,803.60	1691.11 (630.31)	199,551.35	1673.28 (630.67)	197,446.48	1718.70 (635.91)	202,806.20
<b>0.25</b>	1674.12 (629.91)	197,546.66	1710.86 (633.79)	201,881.64	1682.39 (633.81)	198,521.61	1733.32 (636.46)	204,532.29
<b>0.30</b>	1676.97 (627.07)	197,882.06	1720.04 (637.84)	202,964.42	1689.08 (636.25)	199,311.15	1719.07 (633.45)	202,850.08
<b>0.33</b>	1672.72 (624.38)	197,380.58	1709.72 (642.83)	201,746.63	1684.90 (635.64)	198,817.98	1703.41 (628.77)	201,002.16
<b>0.40</b>	1658.77 (618.54)	195,734.75	1678.45 (647.20)	198,057.24	1670.89 (632.69)	197,165.00	1675.39 (621.06)	197,695.64
<b>0.50</b>	1639.73 (612.34)	193,487.70	1644.75 (642.08)	194,080.22	1646.51 (628.54)	194,288.70	1644.09 (611.10)	194,002.90
<b>0.60</b>	1621.56 (604.30)	191,344.53	1622.75 (631.61)	191,484.96	1623.84 (620.27)	191,612.68	1621.34 (604.69)	191,317.66
<b>0.67</b>	1609.36 (601.48)	189,903.90	1606.51 (622.65)	189,567.68	1614.13 (612.53)	190,466.91	1610.41 (600.13)	190,028.69
<b>0.70</b>	1603.95 (599.81)	189,265.63	1601.83 (616.79)	189,016.18	1607.98 (611.26)	189,741.17	1605.44 (598.59)	189,441.53
<b>0.75</b>	1596.24 (597.20)	188,356.27	1594.31 (609.02)	188,128.23	1601.31 (605.36)	188,954.94	1598.80 (597.71)	188,658.25
<b>0.80</b>	1590.34 (596.44)	187,660.32	1588.32 (603.00)	187,421.37	1596.65 (602.05)	188,404.30	1593.93 (597.27)	188,083.29
<b>0.90</b>	1585.78 (595.45)	187,122.27	1583.90 (596.35)	186,900.08	1589.09 (596.63)	187,512.51	1587.28 (594.04)	187,299.20
<b>1.00</b>	1583.60 (594.13)	186,864.94	1583.60 (594.13)	186,864.94	1583.60 (594.13)	186,864.94	1583.60 (594.13)	186,864.94

Revenue expressed in millions Euros.  
Standard deviation in parenthesis.

**Table 1.b**  
**French Treasury Revenue**  
(in millions Euros)  
 **$K^{\text{th}}$ -Average-Price Format**

<i>k</i>	<i>Average per Auction</i>	<i>Total</i>
1	1669.35 (633.66)	196,983.11
2	1676.44 (636.78)	197,820.44
3	1686.09 (634.48)	198,959.10
4	1698.91 (629.20)	200,470.88
5	1724.58 (624.03)	203,500.15
6	1742.46 (620.93)	205,610.76
7	1742.01 (616.90)	205,556.92
8	1733.62 (613.62)	204,567.25
9	1714.34 (611.12)	202,292.00
10	1691.21 (608.62)	199,562.94
11	1675.24 (606.17)	197,678.48
12	1659.80 (604.61)	195,856.46
13	1645.05 (602.82)	194,115.54
14	1634.11 (602.94)	192,825.22
15	1625.81 (601.09)	191,845.20

Revenue expressed in millions Euros.  
Standard deviation in parenthesis.

**Table 1.c**  
**French Treasury Revenue**  
(in millions Euros)  
**Spanish Format**

<i>Average per auction</i>	<i>Total</i>
1623.17 (543.43)	191,534.26

Revenue expressed in millions Euros.  
Standard deviation in parenthesis.