Anticompetitive Vertical Merger Waves[*]

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Abstract

This paper develops an equilibrium model of vertical mergers. We show that competition on an upstream market between integrated firms only is less intense than in the presence of unintegrated upstream firms. Indeed, when an integrated firm supplies the upstream market, it becomes a soft downstream competitor to preserve its upstream profits. This benefits other integrated firms, which may therefore choose not to cut prices on the upstream market. This mechanism generates waves of vertical mergers in which every upstream firm integrates with a downstream firm, and the remaining unintegrated downstream firms obtain the input at a high upstream price.

1   Introduction

The satellite navigation industry has a two-tier structure. The upstream market is the market for navigable digital map databases, where only Tele Atlas and Navteq are active, with market shares of approximately 50% each. At the downstream level, firms embed digital maps in the devices they manufacture in order to provide their customers with navigation solutions. Downstream firms include portable navigation device manufacturers such as TomTom, and manufacturers of mobile handsets that incorporate navigation possibilities, such as Nokia. In October 2007, TomTom notified the European Commission that it would acquire Tele Atlas; four months later Nokia responded by announcing its planned acquisition of Navteq. The European Commission eventually

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cleared these mergers without conditions, and the only two upstream producers in the industry became vertically integrated.[1]

Riordan (2008) describes a similar case in the molded doors market in 2001. Initially, the upstream market was populated with two firms: one vertically integrated firm, which Riordan calls the non-party firm, and an unintegrated upstream producer, Masonite. In the downstream market, the non-party firm was competing with a non-integrated downstream producer, Premdor, and a competitive fringe of small suppliers. In 2001, the United States Department of Justice challenged Premdor's acquisition of Masonite. The DOJ eventually cleared the merger, but it required Masonite to divest part of its upstream capacity to a new upstream entrant.

In this paper, we will argue that such (waves of) vertical mergers, when they effectively eliminate all unintegrated upstream producers, can have strongly anticompetitive effects. In our model there are initially M upstream firms and $N > M$ downstream firms. The game starts with a merger stage, in which downstream firms bid sequentially to acquire upstream firms. Next, upstream firms (integrated or not) compete in prices to sell the intermediate input to the remaining unintegrated downstream firms. Finally, downstream firms (integrated or not) compete in prices with differentiated products. The upstream market exhibits the usual ingredients of tough competition: upstream firms compete in prices, produce a perfectly homogeneous upstream good, and incur the same constant marginal cost. When fewer than M mergers have taken place, the standard Bertrand logic applies and upstream competition drives the upstream price to the marginal cost.

Things are different when all upstream firms are vertically integrated. Suppose a vertically integrated firm, call it $U_i - D_i$, supplies the input to all unintegrated downstream firms at a positive price-cost margin. In the textbook Bertrand model, another integrated firm, call it $U_j - D_j$, could profitably deviate by offering a price slightly lower than $U_i - D_i$'s to take over the entire market. In our framework, $U_j - D_j$ may not want to do that. This is because the upstream supplier does not adopt the same behavior on the downstream market as other vertically integrated firms. When $U_i - D_i$ increases its downstream price, it understands that some of the downstream consumers it loses will eventually purchase from unintegrated downstream firms, thereby increasing upstream demand and revenues. It follows that the upstream supplier, $U_i - D_i$ charges a higher downstream price than its integrated rivals. Firm $U_j - D_j$ benefits from this *softening effect*, since it faces a less aggressive rival on the downstream market. Now, if $U_j - D_j$ undercuts $U_i - D_i$ on the upstream market and becomes the upstream supplier, then the roles are reversed, and $U_i - D_i$ stops being a soft competitor on the downstream market. To sum up, integrated firm $U_j - D_j$ faces the following trade-off when deciding whether to undercut: on the one hand, undercutting yields upstream profits; on the other hand, it makes integrated firm $U_i - D_i$ more aggressive on

[1]See European Commission COMP M.4854 *TomTom/Tele Atlas* and COMP M.4942 *Nokia/Navteq*.

the downstream market. When the latter effect dominates, incentives to undercut disappear.

We obtain equilibria in which each upstream firm integrates vertically with a downstream firm. The downstream firms which remain non-integrated end up purchasing the input from only one vertically integrated firm which sets its monopoly upstream price, whereas other integrated firms make no upstream offer. We show that this kind of anticompetitive vertical merger wave, leading up to a *monopoly-like equilibrium* on the upstream market, is more likely to arise when downstream products are close substitutes.

When competition authorities decide whether to clear a vertical merger, they often compare its potential foreclosure effects with the efficiency gains it may generate. In our model, mergers can generate synergies, in that they can reduce the cost the vertically integrated firm has to pay to transform its input into the final product. We show that the flip side of these synergies is that they also make monopoly-like equilibria easier to sustain. An implication of this result is that the optimal decision of a competition authority may be non-monotonic in the strength of synergies. For instance, the competition authority may want to clear a merger when synergies are weak or strong, but to block it when synergies are intermediate.

Monopoly-like equilibria are the most asymmetric equilibria of our model. At the other extreme, lie *symmetric collusive-like equilibria*, in which all vertically integrated firms offer the same input price above marginal cost, and end up obtaining equal market shares in the upstream market. These equilibria look like collusion, but we show that repeated interactions are not needed to sustain them. This comes once again from the trade-off between the softening effect and the upstream profit effect: if an integrated firm cuts its price, then it obtains the entire upstream profit, but it makes all its rivals more aggressive on the downstream market; conversely, if an integrated firm withdraws its offer, then it gives up upstream profits, but it makes its integrated rivals softer. While there is a large multiplicity of equilibria in our model, we prove two results which make this multiplicity less problematic. First, when neither monopoly-like nor symmetric collusive-like equilibria exist, the input is sold at marginal cost in equilibrium. Second, symmetric collusive-like equilibria are more likely to arise when synergies are intermediate, whereas monopoly-like equilibria are easier to sustain when synergies are strong.

Our paper contributes to the literature on the competitive effects of vertical mergers. The traditional vertical foreclosure theory, which was widely accepted by antitrust practitioners until the end of the 1960s, was seriously challenged by Chicago school authors in the 1970s, notably Bork (1978) and Posner (1976), on the ground that firms cannot leverage market power from one market to another. A more recent strategic approach, initiated by Ordover, Saloner and Salop (1990), has established conditions under which vertical integration can relax competition. This strand of the literature can be summarized in a common framework with two upstream firms, two downstream

firms, and price competition on both markets.[2] The main message conveyed in these papers is that vertical mergers can lead to input foreclosure because upstream competition between vertically integrated firms and unintegrated upstream firms is softer than upstream competition between unintegrated upstream firms only. This result, however, holds true only under specific assumptions, including extra commitment power for vertically integrated firms (Ordover, Saloner and Salop, 1990; Reiffen, 1992), choice of input specification (Choi and Yi, 2000), switching costs (Chen, 2001), tacit collusion (Nocke and White, 2007; Normann, 2009), exclusive dealing (Chen and Riordan, 2007). In this paper we argue that a wave of vertical mergers that eliminates all unintegrated upstream firms can have severe anticompetitive effects, even in the absence of the above-mentioned specific assumptions leading to input foreclosure. This is because upstream competition between vertically integrated firms only – a market structure the literature has surprisingly overlooked – can be very ineffective.

The softening effect that is the key driving force behind our results has been unveiled by Chen (2001).[3, 4] He shows that when there is one vertical merger, the remaining downstream firm prefers purchasing the input from the integrated firm than from the unintegrated upstream firm in order to benefit from the softening effect. If there are upstream cost asymmetries and upstream switching costs, then the unintegrated upstream firm is unable to undercut the integrated firm on the upstream market and there is partial foreclosure in equilibrium. Our result is different. We show that, when several integrated firms are competing against each other, integrated firms are *able* to undercut since we assume away any cost differential or switching cost, but they are not *willing* to do so.

In another paper with an additional coauthor (Bourreau, Hombert, Pouyet and Schutz, 2011), we apply our analysis of the softening effect and monopoly-like equilibria to a telecommunications context. The novelty of the present paper is that we endogenize the market structure by developing an endogenous merger game, which allows us to analyze the optimal decision of a competition authority. We also provide a complete characterization of equilibria in the general case with M upstream firms and $N > M$ downstream firms.

The rest of the paper is organized as follows. We describe the model in Section 2 and solve it in a special case with two upstream firms and three downstream firms in Section 3. The general case with M upstream firms and N downstream firms is solved in Section 4. In Section 5, we show that our results are robust to alternative timing assumptions, third-degree price discrimination

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[2]Exceptions include an early contribution by Salinger (1988) who considers Cournot competition on both markets, and the strand of the literature initiated by Hart and Tirole (1990) which analyzes the consequences of upstream secret offers and focuses mainly on the commitment problem faced by an upstream monopolist.

[3]Chen (2001) refers to it as the collusive effect. We adopt a different terminology to make it clear that the softening effect does not involve any form of tacit or overt collusion.

[4]See also Fauli-Oller and Sandonis (2002) for an application of the softening effect in a licensing context.

in the input market, two-part pricing and secret contracting. Section 6 concludes. Unless stated otherwise, the proof of results involving general demand functions are contained in Appendix A. Results involving linear demands are proven in a separate technical appendix (Hombert, Pouyet and Schutz, 2012).

2   Model

2.1   Setup

We consider a vertically related industry with $M \geq 2$ identical upstream firms, U_1, U_2, ..., U_M, and $N \geq M + 1$ symmetric downstream firms, D_1, D_2, ..., D_N. In the following, we will refer to the case with M upstream firms and N downstream firms as the (M, N) case. The upstream firms produce a homogeneous input at constant marginal cost m and sell it to the downstream firms. The downstream firms can also obtain the input from an alternative source at constant marginal cost $\overline{m} > m$.[5] The downstream firms transform the intermediate input into a final product on a one-to-one basis at a constant unit cost, which we normalize to zero without loss of generality. Downstream products are imperfect substitutes.

Downstream firms will be allowed to merge with upstream producers. When D_k merges with U_i, it produces the intermediate input in-house at unit cost m, its downstream unit transformation cost drops by $\delta \in [0, m]$, and its downstream marginal cost therefore becomes $m - \delta$. We say that the merger involves downstream synergies whenever $\delta > 0$. Synergies are not merger-specific, i.e., δ does not depend on which downstream firm merges with which upstream firm. The model also includes the case of no synergies, when $\delta = 0$.

The demand for downstream firm D_k's product is $q_k = q(p_k, \boldsymbol{p_{-k}})$, where p_k denotes D_k's price and $\boldsymbol{p_{-k}}$ denotes the vector of prices charged by D_k's rivals.[6] The demand function $q(.,.)$ is twice continuously differentiable, and it is the same for all downstream firms. The demand addressed to a firm is decreasing in its own price ($\partial q_k / \partial p_k \leq 0$ with a strict inequality whenever $q_k > 0$) and increasing in its competitors' prices ($\partial q_k / \partial p_{k'} \geq 0$, $k \neq k'$, with a strict inequality whenever q_k, $q_{k'} > 0$), and the total demand is decreasing in each firm's price ($\sum_{k=1}^{N} \partial q_k / \partial p_{k'} \leq 0$, with a strict inequality whenever $q_{k'} > 0$).

The model has three stages. Stage 1 is the merger stage. All N downstream firms first bid simultaneously to acquire upstream firm U_1, and U_1 decides which bid to accept, if any. Next, the remaining unintegrated downstream firms bid simultaneously to acquire U_2. This process goes on up to firm U_M.[7] Firms cannot merge horizontally, and downstream firms cannot acquire more

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[5]The alternative source of supply can come from a competitive fringe of less efficient upstream firms.

[6]We use bold fonts to denote vectors.

[7]To avoid trivial solutions, we assume that an upstream firm never accepts a bid of zero even if it is indifferent

than one upstream firm. Without loss of generality, we relabel firms as follows at the end of stage 1: if $K \in \{0, \ldots, M\}$ vertical mergers have taken place, then for all $1 \leq i \leq K$, upstream firm U_i is acquired by downstream firm D_i to form integrated firm $U_i - D_i$, while upstream firms $U_{K+1}, \ldots,$ U_M, and downstream firms $D_{K+1}, \ldots, D_N$ remain unintegrated.

In the second stage, each upstream firm (integrated or not) $U_i(-D_i)$ announces the price w_i at which it is willing to sell the input to any unintegrated downstream firm.[8]

In the third stage, downstream firms (integrated or not) set their prices and, at the same time, each unintegrated downstream firm chooses from which upstream producer to purchase the input. We denote firm D_k's choice of upstream supplier by $U_{s_k}(-D_{s_k}$ if it is integrated), $s_k \in \{0, \ldots, M\}$, with the convention that U_0 refers to the alternative source of input and that $w_0 \equiv \overline{m}$. Next, downstream demands are realized, unintegrated downstream firms order the amount of input needed to supply their consumers, and make payments to their suppliers. The assumption that downstream pricing decisions and upstream supplier choices are made simultaneously simplifies the analysis by ensuring that unintegrated downstream firms always buy the input from the cheapest supplier.[9]

We look for subgame-perfect equilibria in pure strategies.

2.2 Equilibrium of stage 3

We solve the game by backward induction, and start with stage 3. Denote by $\boldsymbol{w} = (w_0, \ldots, w_M)$ the vector of upstream offers. The profit of unintegrated downstream firm D_k is

$$\pi_k = (p_k - w_{s_k}) \, q(p_k, \boldsymbol{p_{-k}}). \tag{1}$$

The profit of integrated firm $U_i - D_i$ is given by

$$\pi_i = (p_i - (m - \delta)) \, q(p_i, \boldsymbol{p_{-i}}) + (w_i - m) \sum_{s_k = i} q(p_k, \boldsymbol{p_{-k}}), \tag{2}$$

where the first term is the profit obtained in the downstream market and the second term is the profit earned from selling the input to unintegrated downstream firms D_k such that $s_k = i$.

An equilibrium of stage 3 is a pair of vectors $(\boldsymbol{p}, \boldsymbol{s})$ such that every integrated firm $U_i - D_i$ maximizes its profit in p_i given $(\boldsymbol{p_{-i}}, \boldsymbol{s})$, and every unintegrated downstream firm D_k maximizes its profit in p_k and s_k given $(\boldsymbol{p_{-k}}, \boldsymbol{s_{-k}})$. Consider first the upstream supplier choice strategy of firm D_k. Given $(\boldsymbol{p}, \boldsymbol{s_{-k}})$, s_k is consistent with subgame-perfection if and only if $s_k \in \arg\min_{0 \leq i \leq M} w_i$, i.e., if

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between accepting and rejecting. This would obviously be the case whenever mergers involve transaction costs.

[8]Upstream prices are public, discrimination is not possible, and only linear tariffs are used. We relax these assumptions in Section 5.

[9]In Section 5.2 we show that our results carry over to the maybe more natural timing in which the choice of upstream supplier is made before downstream competition.

and only if D_k chooses (one of) the cheapest offer(s). Next, we turn our attention to downstream pricing strategies. For any profile of supplier choices $\boldsymbol{s}$ consistent with subgame-perfection, we assume that:

(i) Firms' best responses in prices are unique and defined by the first order conditions $\partial \pi_k / \partial p_k = 0$.

(ii) Prices are strategic complements: for all $k \neq k'$, $\partial^2 \pi_k / \partial p_k \partial p_{k'} \geq 0$.

(iii) There exists a unique profile of downstream prices $\boldsymbol{p}(\boldsymbol{s})$ such that $(\boldsymbol{p}(\boldsymbol{s}), \boldsymbol{s})$ is a Nash equilibrium of stage 3.

When several upstream firms (integrated or not) are offering the lowest upstream price, $\min(\boldsymbol{w}) = \min_{0 \leq i \leq M}\{w_i\}$, there are multiple equilibria in stage 3, since any distribution of the upstream demand between these upstream firms can be sustained in equilibrium. Assumption (iii) states that, for any such distribution of the upstream market between the cheapest suppliers, there is a unique vector of equilibrium downstream prices.

To streamline the exposition, we adopt the following (partial) selection criterion. When several input suppliers offer $\min(\boldsymbol{w})$, and when at least one of these suppliers is vertically integrated, firms play a Nash equilibrium of stage 3 in which no downstream firm purchases from an unintegrated upstream firm. None of our results on anticompetitive vertical mergers depend on this equilibrium selection. Instead, as we show formally in Section 5.1, this assumption simplifies the analysis by ruling out other potential anticompetitive equilibria with vertical mergers.

We assume that comparative statics are well-behaved, as is standard in the vertical relations literature. Formally, consider an auxiliary game, in which all downstream firms are purchasing the input at price m. Demands are still given by function $q(.,.)$, but firms have heterogenous marginal costs, contained in vector $\boldsymbol{c}$. Firms compete in prices, and we assume that, for any $\boldsymbol{c}$, this auxiliary game has a unique Nash equilibrium. Denote by $\widetilde{\pi}_k(\boldsymbol{c})$ the payoff of firm D_k at this Nash equilibrium. We make the following assumption:

(iv) An increase in a firm's marginal cost lowers its equilibrium profit, $\partial \widetilde{\pi}_k / \partial c_k < 0$.

This assumption merely says that direct effects dominate indirect ones. Finally, we assume that $\overline{m}$ is a relevant outside option: whatever the market structure, an unintegrated downstream firm earns positive profits if it buys the intermediate input at a price lower than or equal to $\overline{m}$.

We will later illustrate our results using the Shubik and Levitan (1980) linear demand system, which satisfies the above assumptions and is frequently used in oligopoly models.

Example 1. *A unit mass of identical consumers have utility function*

$$U = q_0 + \sum_{k=1}^{\overline{N}} q_k - \frac{1}{2} \left(\sum_{k=1}^{\overline{N}} q_k \right)^2 - \frac{\overline{N}}{2(1+\gamma)} \left(\sum_{k=1}^{\overline{N}} q_k^2 - \frac{1}{\overline{N}} (\sum_{k=1}^{\overline{N}} q_k)^2 \right), \tag{3}$$

where q_0 is consumption of the numeraire, q_k is consumption of D_k's product, $\gamma > 0$ parameterizes the degree of differentiation between final products, and $\overline{N} \geq N$ is the number of varieties of the final product.

The demands derived from utility function (3) can be written as:

$$q(p_k, \boldsymbol{p_{-k}}) = \frac{1+\gamma}{\frac{\overline{N}}{N} + \gamma} \frac{1}{N} \left(1 - p_k - \gamma \frac{N}{\overline{N}} \left(p_k - \frac{\sum_{k'=1}^{N} p_{k'}}{N} \right) \right).$$

γ parameterizes the degree of differentiation between final products. Products become homogeneous as γ approaches ∞, and independent as γ approaches 0. $\overline{N}$ is the number of varieties of the final good. N varieties are sold by the downstream firms while the other $\overline{N} - N$ are not available to consumers. Allowing the *potential* number of varieties to differ from the *actual* number of varieties will be helpful in Section 4.4, as this will allow us to perform comparative statics on the number of downstream firms without arbitrarily changing consumers' preferences.

2.3 The Bertrand outcome

We define the Bertrand outcome (in the K-merger subgame) as the situation in which all downstream firms, integrated or not, receive the input at marginal cost and set the corresponding downstream equilibrium prices. It follows from equations (1) and (2) that this profile of downstream prices does not depend on who supplies whom in the upstream market, since upstream profits are all zero. We say that the Bertrand outcome is an equilibrium when there exists a subgame-perfect equilibrium in stage 2, in which at least two upstream firms, integrated or not, set prices equal to marginal cost, and no other upstream firm sets a price below marginal cost.

Lemma 1. *After $K \in \{0, \ldots, M\}$ mergers have taken place in the first stage:*

(i) *The Bertrand outcome is always an equilibrium.*

(ii) *If $K < M$, then there is no equilibrium of the upstream competition subgame in which the input is sold above marginal cost.*

There might exist equilibria of the upstream competition subgame in which the input is sold below marginal cost by integrated firms, for reasons that will become clear in Section 3 (see footnote 12). However, when they exist, these equilibria are Pareto-dominated by the Bertrand

equilibrium from the point of view of upstream players. In the following, we restrict attention to equilibria of the entire game in which the upstream margin is non-negative in every subgame. With this refinement, we are left with only the Bertrand outcome when $K < M$ mergers have taken place in the first stage.

It remains to look for equilibria in which the input is sold above marginal cost when all M upstream firms are integrated. We start with $(2,3)$ case postpone the treatment of the general case until Section 4.

3 Merger Waves: The $(2,3)$ Case

3.1 Partial foreclosure after two mergers

Assume two mergers have taken place. We investigate whether there can be equilibria in which the upstream market is supplied above marginal cost. Suppose unintegrated downstream firm D_3 obtains the input at price $w > m$ from integrated firm $U_i - D_i$, and call the other integrated firm $U_j - D_j$. We denote by $p(1,w)$, $p(0,w)$, and $p(d,w)$, the equilibrium downstream prices of $U_i - D_i$, $U_j - D_j$ and D_3, respectively, and by $\Pi(1,w)$ and $\Pi(0,w)$ the equilibrium profits of $U_i - D_i$ and $U_j - D_j$, respectively.

$U_i - D_i$'s first-order condition is given by:

$$0 = q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} + (w - m)\frac{\partial q_3}{\partial p_i}. \tag{4}$$

The first-order condition of its integrated rival, $U_j - D_j$, is:

$$0 = q_j + (p_j - m + \delta)\frac{\partial q_j}{\partial p_j}. \tag{5}$$

Since the last term in the right-hand side of equation (4) is positive, $U_i - D_i$ has more incentives to increase its downstream price than $U_j - D_j$. Intuitively, when $U_i - D_i$ increases its downstream price, some of the consumers it loses in the final market start buying from unintegrated downstream firm D_3. The downstream firm therefore needs to purchase more input, which eventually increases $U_i - D_i$'s profit in the upstream market. It follows that, in equilibrium, $p(1,w) > p(0,w)$. Integrated firm $U_j - D_j$ benefits from $U_i - D_i$'s being a soft downstream competitor, and therefore, by a standard revealed profitability argument, earns a larger downstream profit than $U_i - D_i$. We summarize these insights in the following lemma:

Lemma 2. *If the unintegrated downstream firm obtains the input at price $w > m$, the integrated firm which does not sell the input sets a lower downstream price and makes higher downstream profits than the integrated firm which sells the input.*

Consider now the incentives of $U_j - D_j$ to set its upstream price at $w_j = w - \epsilon$ to take over the upstream market. Undercutting brings in profits from the upstream market. But on the other hand, $U_j - D_j$'s *downstream* profit jumps discontinuously downward, since $U_i - D_i$ no longer has incentives to be a soft downstream competitor. The decision to undercut therefore trades off the *upstream profit effect* against the loss of the *softening effect*. The change in profit if $U_j - D_j$ undercuts is equal to:

$$\Pi(1, w) - \Pi(0, w) = \underbrace{(w - m)q(p(d, w), p(1, w), p(0, w))}_{\text{Upstream profit effect } (>0)}$$

$$- \underbrace{[(p(0, w) - m + \delta) \, q\, (p(0, w), p(1, w), p(d, w)) - (p(1, w) - m + \delta) \, q\, (p(1, w), p(0, w), p(d, w))]}_{\text{Softening effect } (>0 \text{ by Lemma 2})}$$

If the softening effect dominates the upstream profit effect, then $U_j - D_j$ does not undercut.

For this outcome to be an equilibrium, $U_i - D_i$ should not be willing to change its upstream price either. We denote by $w_m \equiv \arg \max_{w \leq \overline{m}} \Pi(1, w)$ the monopoly upstream price.

Lemma 3. w_m *exists, and it is larger than* m.

Lemma 3 states that monopoly power generates a positive markup in the input market. w_m is only constrained by the alternative source of input to be no larger than $\overline{m}$, and we assume for simplicity that this price is unique. It is straightforward to check that, if $U_j - D_j$ stays out of the market, then $U_i - D_i$ is better off offering w_m rather than letting the alternative source of input supply D_3.

Lemmas 2 and 3 imply that the monopoly outcome may be sustained at the equilibrium of the upstream competition subgame:

Proposition 1. *When two mergers have taken place:*

(i) *There is a* monopoly-like *equilibrium in which one integrated firm sets* w_m *and the other integrated firm makes no offer*[10] *if and only if*

$$\Pi(1, w_m) \leq \Pi(0, w_m). \tag{6}$$

(ii) *In all other equilibria, upstream prices are equal,* $w_1 = w_2 \equiv w$, *and satisfy* $w \leq w_m$ *and* $\Pi(1, w) = \Pi(0, w)$.

(iii) *When condition* (6) *holds strictly, monopoly-like equilibria Pareto-dominate all other equilibria from the integrated firms' point of view, and they are the only equilibria which do not involve weakly dominated strategies.*

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[10]By "makes no upstream offer", we mean that the other integrated firm offers an input price above $\bar{m}$.

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When the softening effect is strong enough so that condition (6) holds, the outcome in which one of the integrated firm exits the upstream market, granting a monopoly position to the other integrated firm, is an equilibrium. Note also that monopoly-like equilibria come by pairs since the upstream supplier can be either $U_1 - D_1$ or $U_2 - D_2$.

Proposition 1 gives foundations to the classical analysis of Ordover, Saloner and Salop (1990), in which a vertically integrated firm commits to exiting the upstream market in order to let the upstream rival charge the monopoly price. We show that no commitment is actually necessary when the upstream rival is integrated, provided that the softening effect is strong enough.

All other equilibria feature both integrated firms setting the same upstream price: $w_1 = w_2 \equiv w$. Obviously, $w \leq w_m$, otherwise an integrated firm would rather undercut to w_m. Such outcomes are part of an equilibrium only if the softening effect and the upstream profit effect exactly cancel out, so that the upstream supplier earns as much profit as its integrated rival: $\Pi(1, w) = \Pi(0, w)$.[11] The Bertrand outcome is one such equilibrium. There can also be other equilibria with an upstream price above m.[12]

When inequality (6) holds strictly, this multiplicity of equilibria can be resolved using standard selection criteria. First, it follows immediately from the definition of w_m that monopoly-like equilibria Pareto-dominate all other equilibria. Second, we show in the proof of Proposition 1 that playing w is weakly dominated by playing w_m. Therefore, it seems reasonable to think that integrated firms will coordinate on one of the monopoly-like equilibria.

3.2 Merger wave with no synergies

To assess the welfare impact of vertical mergers, we define the following market performance measure. We fix $\lambda \in [0, 1]$ and define market performance as $W(\lambda) = $ (Consumer surplus) $+ \lambda \times$ (Industry profit). Notice that $W(0)$ is consumer surplus, and $W(1)$ is social welfare.

Proposition 2. *With no synergies ($\delta = 0$):*

(i) *There always exists an equilibrium with no merger and the Bertrand outcome in the upstream market.*

(ii) *If condition (6) is satisfied, and integrated firms do not play weakly dominated strategies in the upstream market or do not play equilibria that are Pareto-dominated by another equilib-*

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[11]Necessary conditions $w \leq w_m$ and $\Pi(1, w) = \Pi(0, w)$ are also sufficient if $\Pi(1, .)$ is quasi-concave.

[12]As well as with $w < m$. In that case, upstream profits are negative and the softening effect is reversed, with the upstream supplier adopting an aggressive stance on the downstream market to limit its upstream losses. As explained in Section 2.3 we abstract away from these equilibria because they are Pareto-dominated by the Bertrand equilibrium from the point of view of the integrated firms. Indeed, the upstream supplier makes upstream losses and both integrated firms suffer from the upstream supplier's aggressive behavior in the downstream market.

rium, then, in equilibrium, there are two mergers and the upstream market is supplied at the monopoly price.

(iii) For any $\lambda \in [0,1]$, a merger wave leading up to the monopoly outcome degrades market performance.

When the Bertrand outcome arises in every subgame of the upstream competition stage, the absence of synergies implies that unintegrated downstream firms and integrated firms earn the same profit in every subgame. As a result, downstream firms have no incentives to integrate backwards. There always exists an equilibrium with no merger and a Bertrand outcome in the upstream market.

When condition (6) is satisfied, given our selection criterion, a monopoly-like outcome emerges after two mergers. In this situation, the profits of the two merged entities are larger than unintegrated downstream firms' profits when zero or one merger occurs. Since unintegrated upstream firms always make zero profit, each merger raises the joint profits of the merging parties. Therefore, a wave of vertical mergers occurs, which eliminates all unintegrated upstream firms and implements the monopoly-like outcome.

The merger wave comes from the strategic complementarity between vertical mergers. The second merger is profitable only if the first upstream firm has merged before. By the same token, the first merger is profitable only because the merging firms anticipate that it will be followed by a counter-merger.

A wave of vertical mergers that does not generates efficiency gains and leads to a monopoly-like outcome in the upstream market degrades market performance. When the upstream price increases above marginal cost in the two-merger subgame, the best response functions of both the upstream supplier and the unintegrated downstream firm shift upwards, which props up all downstream prices by strategic complementarity. This is clearly detrimental to all consumers. This also degrades social welfare, since the total demand is already too low in the Bertrand outcome because of positive markups in the downstream market, and because the downstream outcome becomes more asymmetric.

We illustrate Proposition 2 using the demand system of Example 1.[13]

Proposition 3. *In Example 1 with no synergies, there exists $\overline{\gamma} > 0$, such that, if $\gamma \geq \overline{\gamma}$, then, in equilibrium, there are two mergers and a monopoly-like outcome in the upstream market if integrated firms do not play weakly dominated strategies on the upstream market, or do not play equilibria that are Pareto-dominated by another equilibrium.[14] If $\gamma < \overline{\gamma}$, there is no merger and*

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[13]To avoid the proliferation of cases to be considered, we assume that $\bar{m}$ is high enough, so that it does not constrain the monopoly upstream price.

[14]For completeness: When $\gamma \geq \overline{\gamma}$, the only equilibria are the monopoly-like ones, an equilibrium with $w_1 = w_2 \in (m, w_m)$, and the Bertrand equilibrium. If $\gamma < \overline{\gamma}$, then the Bertrand outcome is the only equilibrium.

the Bertrand outcome on the upstream market.

Interestingly, there is a tension between competition in the downstream market and competition in the upstream market. When the downstream market features fierce competition (high γ), there exist monopoly-like equilibria in the upstream market, while the input is priced at marginal cost when downstream competition is weak (low γ). To see the intuition, suppose that the upstream market is supplied at the monopoly upstream price. When the substitutability between final products is strong, the integrated firm which supplies the upstream market is reluctant to set too low of a downstream price since this would strongly contract its upstream profit. The other integrated firm benefits from a substantial softening effect and, as a result, is not willing to take over the upstream market. The reverse holds when downstream products are strongly differentiated.

In its non-horizontal merger guidelines (EC, 2007), the European Commission argues that, when assessing the potential anti-competitive effect of a vertical merger, the competition authority should distinguish the vertically integrated firm's *ability* to foreclose from its *incentives* to foreclose. The Commission also claims that integrated firms' incentives to foreclose are weak when pre-merger downstream margins are low. The idea is that integrated firms would not find it profitable to forego upstream revenues to preserve low downstream profits.[15] In our model, integrated firms always have *incentives* to foreclose. Starting from the Bertrand equilibrium, if $U_1 - D_1$ and $U_2 - D_2$ could somehow commit to raise the input price from m to w_m, they would do so. This is true no matter what the downstream margins are. What is key here is the *ability* to foreclose. If pre-merger downstream margins are low because downstream products are close substitutes, then the softening effect is strong and input foreclosure can be sustained in equilibrium. In other words, low pre-merger downstream margins indicate that integrated firms are better able to foreclose.

3.3   Merger wave with synergies

Now, assume $\delta > 0$. When vertical integration generates synergies, firms are willing to integrate to lower their costs, and the first merger of the wave always improves market performance.

Proposition 4. *With synergies ($\delta > 0$):*

   (i) There are always two mergers in equilibrium.

   (ii) Under the additional assumption that more productive firms charge higher markups in equilibrium, the first merger improves market performance.

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[15]Inderst and Valletti (2008) question the EC's reasoning. They argue that low downstream margins are indicative of closely substitutable final products and that, in this situation, the integrated firms' incentives to raise their rivals' costs are strong.

The first merger of the wave always improves performance, since productive efficiency improves while the input price remains at marginal cost. The welfare effect of the second merger depends on the outcome in the upstream market. If the upstream market is supplied at marginal cost, then the second merger also improves market performance. By contrast, when partial input foreclosure arises in the two-merger subgame, there is a tradeoff between efficiency gains and anticompetitive effects. From an antitrust perspective, it is therefore the last integration of the merger wave that calls for scrutiny. Using again the specification of Example 1, we compare $W(\lambda)$ at the unique equilibrium outcome of the one-merger subgame (the Bertrand outcome), and at the equilibrium outcome of the two-merger subgame (the Bertrand outcome or the monopoly-like outcome):[16]

Proposition 5. *In Example 1, in the two-merger subgame, there exists δ_m such that monopoly-like equilibria exist if and only if $\delta \geq \delta_m$. If $\delta < \delta_m$, then the Bertrand outcome is the only equilibrium.*

As shown in Figure ??, there exist γ_1, γ_2 and δ_W such that the second merger degrades market performance if and only if (i) $\gamma_1 < \gamma \leq \gamma_2$ and $\delta \in [\delta_m, \delta_W)$, or (ii) $\gamma > \gamma_2$ and $\delta \geq \delta_m$.[17]

<div align="center">INSERT FIGURE</div>

The first part of Proposition 5 says that monopoly-like equilibria exist when synergies are strong enough. Intuitively, as the cost differential between the unintegrated downstream firm and its integrated counterparts widens, the market share of the former declines and profits in the upstream market shrink. The magnitude of the softening effect, which works at the margin and reflects the willingness of the upstream supplier to raise its upstream demand, is not directly affected. Because undercutting decisions trade off the upstream profit effect with the softening effect, it becomes more and more attractive to stay out of the market as δ increases.

According to the second part of Proposition 5, the optimal policy response to the second merger is quite different from the one which conventional wisdom would suggest. In particular, following a simple rule-of-thumb, where the competition authority should be more favorable towards a vertical merger when synergies get stronger, may lead to welfare losses here. To illustrate this point, assume that $\gamma \in (\gamma_1, \gamma_2)$. Then, the competition authority should clear the merger when $0 < \delta < \delta_m$, challenge it when $\delta \in [\delta_m, \delta_W)$, and clear it again when $\delta \geq \delta_W$. So the optimal merger control policy is non-monotonic in δ. This follows from the fact that, while larger efficiency gains improve welfare *for a given outcome in the input market*, they also increase the likelihood of input

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[16]Again, to avoid the proliferation of cases, we assume that δ is not too high, so that the unconstrained maximization problem $\max_w \Pi(1, w)$ has an interior solution. This defines a $\delta_{max} > 0$. As in footnote 13, we assume that $\bar{m}$ is high enough, so that it does not constrain the monopoly upstream price. We make similar assumptions to prove all propositions involving linear demands and synergies.

[17]In this proposition, as well as in all other propositions involving linear demands, the thresholds for δ and γ are functions of the other parameters of the model.

foreclosure. This highlights that foreclosure and efficiency effects are intertwined and should be considered jointly when investigating the competitive effects of a vertical merger.

4 Merger Waves: The (M, N) Case

4.1 Preliminaries

In the M-merger subgame, there may be more than one unintegrated downstream firm purchasing the input on the upstream market. Therefore, there may be equilibrium candidates in which several integrated firms share the upstream market. Given our focus on equilibria in pure strategies, many distributions of upstream market shares may not be feasible because of integer constraints. For instance, a vertically integrated cannot supply half a downstream firm. To avoid unnecessary complications due to those integer constraints, we allow downstream firms to randomize when they choose their upstream suppliers. To do so, we assume that each unintegrated downstream firm D_k privately observes a non-payoff relevant random variable θ_k after upstream prices have been set, but before upstream suppliers are chosen. Those $N - M$ random variables are independently and uniformly distributed on some interval of the real line. The supplier choice strategy of firm D_k is now denoted by $s_k(\theta_k)$.[18]

Before solving for equilibria in the M-merger subgame, we note that the assumptions and preliminary results laid out in Sections 2.2 and 2.3 extend easily to this new environment. In the following, we assume firms are risk neutral, and we focus on equilibria in which downstream firms do not condition their downstream prices on the realization of their (non-payoff relevant) θ_k's. The profit of downstream firm D_k at the beginning of stage 3 is still given by equation (1) if we replace s_k by $s_k(\theta_k)$, while the expected profit of integrated firm $U_i - D_i$ becomes

$$\pi_i = (p_i - (m - \delta)) \, q(p_i, \boldsymbol{p_{-i}}) + (w_i - m) \, E\left[\sum_{s_k(\theta_k)=i} q(p_k, \boldsymbol{p_{-k}})\right]$$

Sequentially rational supplier choice strategies consist in any randomization between the cheapest upstream suppliers: if $w_i > \min(\boldsymbol{w})$, then $s_k(\theta_k) \neq i$ for all k and for all θ_k. Assumptions (i)–(iv) are easily extended, and we still select equilibria in which if at least one integrated firm offers the cheapest input price, then downstream firms never purchase from an unintegrated upstream firm.

In stage 2, it is straightforward to extend the proof of Lemma 1 to show that the Bertrand outcome is always an equilibrium, and that it is the only one in the $K < M$-merger subgames.

---

[18]Since the θ_k's are private information, we use perfect Bayesian equilibrium as our solution concept from now on. Since private information does not kick in before the last stage of the game, signalling considerations are ruled out, and a player's posterior on other players' types at the beginning of stage 3 is just its prior. Since the θ_k's are not payoff relevant, perfect bayesian equilibrium is needed only to ensure sequential rationality.

From now on, we focus on the M-merger subgame. For a given profile of upstream offers $\boldsymbol{w}$, there exists a continuum of equilibria of stage 3, in which the integrated firms offering $w = \min(\boldsymbol{w})$ share the upstream market. Fix such an equilibrium. Then, we can define $U_i - D_i$'s (expected) upstream market share: $\alpha_i \equiv \frac{1}{N-M} \sum_{k=M+1}^{N} Pr(s_k(\theta_k) = i)$, $i = 1, \ldots, M$. The following lemma states it is sufficient to know the input price and the upstream market shares to calculate the equilibrium profits of vertically integrated firms:

Lemma 4. *In the M-merger subgame, when the input price is w, the profit of integrated firm $U_i - D_i$ at the unique equilibrium with supplier choices $\boldsymbol{s}(.)$ can be written as $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$. Besides, if $\tilde{\boldsymbol{\alpha}}_{-i}$ is a permutation of $\boldsymbol{\alpha}_{-i}$, then $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w) = \Pi(\alpha_i, \tilde{\boldsymbol{\alpha}}_{-i}, w)$.*

The following notation will be useful to write down non-deviation constraints in the next section:

$$S_Y(Z) = \left\{ \boldsymbol{\alpha} \in [0,1]^Y : \sum_{i=1}^{Z} \alpha_i = 1, \text{ and } \alpha_i = 0 \ \forall i > Z \right\},$$

where $1 \leq Z \leq Y \leq M$. In words, $S_Y(Z)$ is the set of feasible equilibrium market shares in an industry with Y integrated firms, when only the first Z firms offer the cheapest input price. It will be useful to keep in mind that, when exactly Z firms are offering the cheapest price, given a feasible profile of market shares $\boldsymbol{\alpha}$, there exists a permutation of $\boldsymbol{\alpha}$ which belongs to $S_M(Z)$.

4.2 Partial foreclosure after M mergers

We first establish the robustness of monopoly-like equilibria. By analogy with the two-by-three case, we call a monopoly-like outcome a situation in which an integrated firm, call it $U_i - D_i$, supplies all $N-M$ downstream firms at the monopoly upstream price $w_m \equiv \arg\max_{w \leq \overline{m}} \Pi(1, \boldsymbol{0}, w)$, where $\boldsymbol{0}$ denotes the $M-1$-tuple $(0, \ldots, 0)$, while all other integrated firms make no upstream offer. Assuming that w_m is unique, it is straightforward to adapt the proof of Lemma 3 to show that $w_m > m$.

As in the two-by-three case, it follows from the comparison of first-order conditions that $U_i - D_i$ has more incentives to increase its downstream price than any of its vertically integrated rivals. Those integrated rivals therefore make higher downstream profits than $U_i - D_i$, which means that the softening effect is still at work. As a result, an integrated rival $U_j - D_j$'s decision to undercut trades off the gain of upstream profits against the loss of the softening effect. When the softening effect dominates, $U_j - D_j$ does not want to take over the upstream market by setting a price lower than w_m. This happens when

$$\Pi(1, \boldsymbol{0}, w_m) \leq \Pi(0, \boldsymbol{1}, w_m), \tag{7}$$

where $\boldsymbol{1}$ denotes the $M - 1$-tuple $(1, 0, \ldots, 0)$. We also need to check that $U_j - D_j$ does not want to match w_m. If it does, then the deviation leads to multiple equilibria at stage 3, since

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any distribution of the upstream demand between $U_i - D_i$ and $U_j - D_j$ is possible. In particular, the equilibrium in which $\alpha_j = 0$ is such that $U_j - D_j$'s profit does not increase. It follows that monopoly-like equilibria exist if and only if condition (7) is satisfied.

Next, we look for partial foreclosure equilibria in which several (maybe all) integrated firms offer the lowest upstream price $w = \min(\boldsymbol{w}) > m$. In that case, any distribution of the upstream demand between the integrated firms offering w is possible. Those upstream market outcomes are referred to as *collusive-like*. Indeed, several integrated firms share the upstream demand at a price above marginal cost and no one is willing to deviate, as in models of collusion with repeated interactions. Nocke and White (2007) obtain similar upstream outcomes in a repeated games framework with a market structure close to our model's.[19] In the following, we show that these outcomes can actually be sustained in a one-shot game when all upstream firms are vertically integrated.

Consider a collusive-like outcome in which $Z \in \{2, \ldots, M\}$ integrated firms offer $w = \min(\boldsymbol{w}) > m$. Assume without loss of generality that the distribution of market shares is given by $\boldsymbol{\alpha} \in S_M(Z)$, and consider integrated firm $U_i - D_i$'s incentives to deviate. If $U_i - D_i$ has an upstream market share $\alpha_i \in (0, 1)$, it is making some upstream profits, while at the same time benefiting from the other integrated upstream suppliers' relatively soft behavior in the downstream market. $U_i - D_i$ therefore faces the following tradeoffs. If it undercuts, it takes over the entire upstream profit but its integrated rivals become relatively more aggressive in the downstream market. Undercutting is not profitable if and only if

$$\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w) \geq \max_{\tilde{w} \leq w} \Pi(1, \boldsymbol{0}, \tilde{w}). \tag{8}$$

Alternatively, if $U_i - D_i$ withdraws its offer and lets its integrated rivals supply the upstream market, it benefits from a stronger softening effect but gives up its upstream revenues. In that case, the unintegrated downstream firms select a supplier among the remaining integrated firms which offer w, and again, any distribution of the upstream demand between those integrated firms can be sustained at the equilibrium of stage 3. There exists an equilibrium of stage 3 in which $U_i - D_i$'s profit does not increase if and only if

$$\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w) \geq \min_{\boldsymbol{\beta} \in S_{M-1}(Z-1)} \Pi(0, \boldsymbol{\beta}, w), \tag{9}$$

where the minimum is taken over all the distributions of market shares among $U_i - D_i$'s integrated rivals which also offer w.

If $U_i - D_i$ has a market share of $\alpha_i = 0$, or a market share of $\alpha_i = 1$, the non-deviation conditions are still given by (8) and (9). Therefore, there exists a collusive-like equilibrium with

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[19]However, Nocke and White (2007)'s downstream outcome is different from ours, since they focus on equilibria in which overall industry profit is maximized.

market shares $\boldsymbol{\alpha}$ if and only if

$$\min_{1\leq i\leq M}\Pi(\alpha_i,\boldsymbol{\alpha}_{-i},w)\geq\max\left\{\max_{\tilde{w}\leq w}\Pi(1,\mathbf{0},\tilde{w}),\min_{\boldsymbol{\beta}\in S_{M\text{-}1}(Z\text{-}1)}\Pi(0,\boldsymbol{\beta},w)\right\}. \tag{10}$$

Given the equilibrium multiplicity in stage 3, there are many candidate collusive-like equilibria to check for. However, if the integrated firms' profit function is quasi-concave in the market shares, then the *symmetric collusive-like equilibria*, in which every integrated firm $U_i - D_i$ offers the same input price $w_i = w > m$ and supplies a fraction $\alpha_i = 1/M$ of the upstream market, are the easiest to sustain:

Lemma 5. *Assume that, for all $w > m$, $(\alpha_i, \boldsymbol{\alpha}_{-i}) \mapsto \Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$ is quasi-concave. If there exists a collusive-like equilibrium at upstream price $w > m$, then there also exists a symmetric collusive-like equilibrium at upstream price w.*

Proof. Suppose there exists a collusive-like equilibrium with an input price $w > m$ offered by $Z \in \{2,\ldots,M\}$ integrated firms, and a profile of upstream market shares $\boldsymbol{\alpha} \in S_M(Z)$. Define $\boldsymbol{\beta}_{-i} = (\alpha_{i+1},\ldots,\alpha_M,\alpha_1,\ldots,\alpha_{i-1})$. Then,

$$
\begin{aligned}
\Pi(\frac{1}{M},\ldots,\frac{1}{M}) \ &= \ \Pi(\frac{\sum_{i=1}^M\alpha_i}{M},\ldots,\frac{\sum_{i=1}^M\alpha_i}{M}) \\
&= \ \Pi(\sum_{i=1}^M\frac{1}{M}(\alpha_i,\boldsymbol{\beta}_{-i}),w) \\
&\geq \ \min_{1\leq i\leq M}\Pi(\alpha_i,\boldsymbol{\beta}_{-i},w) \text{ by quasi-concavity} \\
&= \ \min_{1\leq i\leq M}\Pi(\alpha_i,\boldsymbol{\alpha}_{-i},w) \text{ by Lemma 4} \\
&\geq \ \max\left\{\max_{\tilde{w}\leq w}\Pi(1,\mathbf{0},\tilde{w}),\min_{\boldsymbol{\beta}\in S_{M\text{-}1}(Z\text{-}1)}\Pi(0,\boldsymbol{\beta},w)\right\} \\
&\geq \ \max\left\{\max_{\tilde{w}\leq w}\Pi(1,\mathbf{0},\tilde{w}),\min_{\boldsymbol{\beta}\in S_{M\text{-}1}(M\text{-}1)}\Pi(0,\boldsymbol{\beta},w)\right\},
\end{aligned}
$$

where the penultimate inequality follows from condition (10), and the last inequality follows from the fact that $S_{M-1}(Z-1) \subseteq S_{M-1}(M-1)$. $\qquad\square$

The condition that the profit function $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$ is quasi-concave in $(\alpha_i, \boldsymbol{\alpha}_{-i})$ is natural. It merely states that, starting from a given distribution of the upstream market shares, making it more asymmetric cannot increase simultaneously the profits of all the integrated firms. This property sounds intuitive in an environment in which consumers have symmetric and convex preferences, and in which firms have symmetric and constant unit costs. Below, we show that this property is satisfied in Example 1.

Lemma 5 implies that, when looking for a partial foreclosure equilibrium other than the monopoly-like outcome, it is enough to focus on the symmetric collusive-like outcome.[20] We summarize the results of this section in the following proposition.

Proposition 6. *When M mergers have taken place:*

(i) *There is a monopoly-like equilibrium if and only if condition (7) is satisfied.*

(ii) *There is a symmetric collusive-like equilibrium with upstream price $w > m$ if and only if*

$$\Pi(1/M, 1/M, \ldots, 1/M, w) \geq \max\left\{ \max_{\tilde{w} \leq w} \Pi(1, \mathbf{0}, \tilde{w}), \min_{\boldsymbol{\beta} \in S_{M\text{-}1}(M\text{-}1)} \Pi(0, \boldsymbol{\beta}, w) \right\}. \quad (11)$$

(iii) *If condition (7) is not satisfied, condition (11) is not satisfied for any $w > m$, and $\Pi(., ., w)$ is quasi-concave for all $w > m$, then the Bertrand outcome is the only equilibrium.*

We now illustrate Proposition 6 with the specification of Example 1. When downstream demands are linear, it turns out that the equilibrium profit functions $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$ do not depend on $\boldsymbol{\alpha}_{-i}$. In words, an integrated firm only cares about the upstream price and its upstream market share, but the distribution of the remaining upstream market shares does not affect its profits. This property greatly simplifies the characterization of collusive-like equilibria, because the deviation profit after an integrated firm withdraws its upstream offer no longer depends on the distribution of the upstream demand in the subgame following the deviation. Therefore, the second term on the right-hand side of equilibrium condition (11) is just equal to $\Pi(0, \mathbf{1}, w)$. Besides, with linear demands, $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$ is concave in α_i. Therefore, Proposition 6 applies and we have necessary and sufficient conditions for the existence of monopoly-like and collusive-like equilibria:

Proposition 7. *In Example 1, there exist three thresholds $\underline{\delta}_c \leq \delta_m < \overline{\delta}_c$ such that, in the M-merger subgame:*

(i) *Monopoly-like equilibria exist if and only if $\delta \geq \delta_m$.*

(ii) *If $\delta \in [\underline{\delta}_c, \overline{\delta}_c)$, then the set of prices which can be sustained in a symmetric collusive-like equilibrium is a non-empty interval. Otherwise, there are no collusive-like equilibria.*

(iii) *If $\delta < \underline{\delta}_c$, then the Bertrand outcome is the only equilibrium.*

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[20]There is one last type of equilibrium candidate of the M-subgame that we need to inspect: only one integrated firm offers the lowest upstream price $w \in (m, w_m)$, and one or several other integrated firms make upstream offers between w and w_m that prevent the upstream supplier to increase its upstream price up to w_m. In Appendix A.8 we show that this situation cannot happen if $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$ is quasi-concave in $(\alpha_i, \boldsymbol{\alpha}_{-i})$.

The condition stated in (i) is the same as in Proposition 4. The two thresholds in (ii) come from the two terms in the right-hand side of non-deviation condition (11). The first term tells us that the upstream profit effect should not be too strong compared to the softening effect to make undercutting not profitable. This arises when synergies are strong enough. The second term tells us that the softening effect should not be too strong compared to the upstream profit effect to make exit not profitable. This arises when synergies are not too strong.

Proposition 7 also shows that there exist a continuum of symmetric collusive-like equilibria parameterized by the input price. This comes from the fact that the existence condition (11) for collusive-like equilibria is an inequality. Therefore, if it is satisfied strictly for a given w, then, by continuity, it is also satisfied in a neighborhood of w.

Proposition 7 also ranks the thresholds on δ for the existence of monopoly-like and collusive-like equilibria: $\underline{\delta}_c \leq \delta_m < \overline{\delta}_c$. This ranking follows once again from the quasi-concavity of Π. To see this, suppose that the monopoly-like equilibrium condition is just satisfied, $\delta = \delta_m$, which implies that the no-undercut condition is binding, $\Pi(1, \mathbf{0}, w_m) = \Pi(0, \mathbf{1}, w_m)$. Quasi-concavity implies that

$$\Pi(1/M, 1/M, \ldots, 1/M, w_m) \geq \min \{\Pi(1, \mathbf{0}, w_m), \Pi(0, \mathbf{1}, w_m)\} = \Pi(1, \mathbf{0}, w_m) = \Pi(0, \mathbf{1}, w_m).$$

Therefore, there also exists a symmetric collusive-like equilibrium with input price w_m. From this, we can conclude that collusive-like equilibria are easier to sustain when δ is intermediate, whereas monopoly-like equilibria are easier to sustain when δ is high.

4.3   Outcome of the merger game and equilibrium bids

Combining Lemma 1, and Propositions 6 and 7, we obtain the following result:

Proposition 8. *Assume $\Pi(., ., w)$ is quasi-concave for all $w > m$. There exists an equilibrium with an anticompetitive vertical merger wave[21] if and only if Condition (7) is satisfied or Condition (11) is satisfied for some $w > m$. In Example 1, there exists an equilibrium with an anticompetitive vertical merger wave if and only if $\delta \geq \underline{\delta}_c$.*

As in Section 3.3, when the existence condition for monopoly-like or symmetric collusive-like equilibria is satisfied, firms merge to implement a partial foreclosure equilibrium and, when $\delta > 0$, to benefit from efficiency gains. It is tedious to provide a complete characterization of equilibrium bids, because those bids depend on which equilibrium is selected in each of the M-merger subgames. In the following, we will focus on a special case, in which firms anticipate that a symmetric collusive-like equilibrium at price $w > m$ will be implemented in all M-merger subgames. This will be helpful to identify who is likely to gain or to lose from the vertical merger wave.

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[21]By anticompetitive, we mean that the merger wave raises unintegrated downstream rivals' costs. As in Section 3.3, it may or may not improve overall market performance, depending on parameters' values.

Denote by $\Pi_d(1/M, \ldots, 1/M, w)$ the profit of an unintegrated downstream firm in M-merger subgames. In equilibrium, all winning bids are equal to $\Pi(1/M, \ldots, 1/M, w) - \Pi_d(1/M, \ldots, 1/M, w)$. It follows that the owners of downstream firms end up with net payoff $\Pi_d(1/M, \ldots, 1/M, w)$, whereas the initial owners of upstream firms end up with payoff $\Pi(1/M, \ldots, 1/M, w) - \Pi_d(1/M, \ldots, 1/M, w)$. Therefore, upstream firms' owners clearly gain from the merger wave, whereas *all* downstream firms' owners suffer from it.[22] The reason is that the sequence of auctions which takes place in stage 1 involves negative externalities between buyers. This result does not depend on the particular bargaining structure we are assuming: if we allow instead the upstream firms to bid to acquire the downstream firms, then it is possible to show that the equilibrium payoffs are the same as when downstream firms bid.

4.4 Impact of upstream and downstream entry

In this section, we use the linear demand functions of Example 1 to analyze the impact of variations in the number of upstream or downstream firms on the emergence of an equilibrium anticompetitive vertical merger wave. First, we fix $\overline{N}$ and γ. This pins down consumers' preferences in equation (3). We know from Proposition 8 that an anticompetitive vertical merger wave occurs if and only if $\delta \geq \underline{\delta}_c$. Therefore, the problem boils down to analyzing the behavior of $\underline{\delta}_c$ as a function of M and N.

Consider first the impact of upstream entry. According to the conventional wisdom, when more firms compete on the upstream market, this market should end up being more competitive. In our model, the impact of upstream entry can be decomposed as follows. Consider the M-merger subgame. On the one hand, the number of unintegrated downstream firms in this subgame decreases. In addition, unintegrated downstream firms suffer from the presence of an additional vertically integrated firm, which benefits from synergies, and which gets access to the input at marginal cost. The overall output of unintegrated downstream firms falls down, which weakens the upstream profit effect. On the other hand, upstream entry dilutes the softening effect, because when an integrated firm increases its downstream price, a smaller fraction of the consumers it loses end up purchasing from unintegrated downstream firms. So upstream entry weakens both the upstream profit effect and the softening effect, and the upstream market may or may not become more competitive. We confirm this intuition by proving the following result numerically:

Result 1. *In Example 1, $M \mapsto \underline{\delta}_c(M, N, \gamma)$ is (i) non-increasing when γ is low, (ii) non-decreasing when γ is high and N is small, (iii) hump-shaped when γ and N are high.*

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[22]Here, we assume implicitly that $\Pi_d(1/M, \ldots, 1/M, w)$ is smaller than the profit of an unintegrated downstream firm in the disintegrated industry. This means that direct effects (firms dislike marginal cost increases) dominate indirect ones (higher marginal costs soften downstream competition).

Now, consider the impact of downstream entry. Here, it is a bit more difficult to state what the conventional wisdom would be, so let us rely this time on the European Commission's vertical mergers guidelines, which we discussed in Section 3.2. When more firms compete on the downstream market, pre-merger downstream markups are lower, and the guidelines predict that foreclosure is unlikely to arise. In our model, an increase in the number of downstream firms strengthens both the softening effect and the upstream profit effect, and downstream entry may or may not make the upstream market more competitive:

Result 2. *In Example 1, $N \mapsto \underline{\delta}_c(M, N, \gamma)$ is (i) non-increasing when $M \geq 4$ or when $M < 4$ and γ is low, (ii) U-shaped when $M = 2$ and γ is intermediate, or when $M = 3$ and γ is high, (iii) non-decreasing when $M = 2$ and γ is high.*

5 Extensions

5.1 Equilibrium selection in stage 3

Throughout the paper, we have maintained the assumption that, when several firms offer the lowest upstream price, and when at least one of these firms is vertically integrated, no downstream firm purchases from an unintegrated upstream firm. Without this equilibrium selection, the Bertrand outcome may not be the only equilibrium of stage 2 when fewer than M mergers have taken place. To see the intuition, consider the $(3, 5)$ case, assume two mergers have taken place, and start from an equilibrium candidate in which the three upstream firms offer the same input price $w > m$, and each of these firms supplies exactly one downstream firm. Then, it could be that the integrated firms want neither to exit nor to undercut as in a collusive-like equilibrium. The unintegrated upstream firm may not want to undercut, because if it did so, then integrated firms would become more aggressive on the downstream market, and this would reduce the input demand coming from the downstream firm it already supplies.

While we have not been able to construct such equilibria, we cannot rule them out either. If such equilibria exist, then there can be subgame-perfect equilibria of the whole game with fewer than M (anticompetitive) mergers. In this case, anticompetitive vertical integration still takes place because of the trade-off between the softening effect and the upstream profit effect, and therefore, the main message of the paper is preserved.[23]

One way to motivate our selection criterion is to allow downstream firms to pre-commit *ex ante* to their supplier choices, as in Chen (2001). Consider the following modification of our timing:

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[23]A similar remark applies to the extensions laid out in Sections 5.2– 5.5. In those extensions, the Bertrand outcome may not be the only equilibrium in subgames with fewer than M mergers, because of the trade-off between the softening effect and the upstream profit effect.

in stage 2, after input prices have been set, each downstream firm elects one upstream supplier. In stage 3, each downstream firm is allowed to switch to another supplier if it pays a fixed cost ε. Then, we can show that, as ε goes to zero, the equilibria of this family of auxiliary games converge towards equilibria of our original game which satisfy our equilibrium selection criterion. The reason is that downstream firms want to pre-commit to purchasing from integrated firms, so as to make them softer competitors on the downstream market.

5.2   Timing

Suppose now that unintegrated downstream firms choose their input supplier (at stage 2.5) after upstream prices have been set (at stage 2) but before downstream competition takes place (at stage 3).[24]

Then, supplier choices made in stage 2.5 have an impact on equilibrium downstream prices in the continuation subgame. Because of this, the choices of upstream suppliers become a strategic game between unintegrated downstream firms. Therefore, some market share distributions may not be equilibria of the supplier choice subgame. We sidestep this difficulty by using linear demands. In Example 1, in the M-merger subgame, an unintegrated downstream firm's profit at the equilibrium of stage 3 only depends on the prices at which it and the other unintegrated downstream firms obtain the input, but not on the identity of the suppliers. Therefore, any distribution of upstream market shares between the cheapest suppliers is an equilibrium of the upstream supplier choice subgame. It follows that our previous analysis of monopoly-like equilibria and collusive-like equilibria still applies:

Proposition 9. *Assume the choice of supplier is made before downstream competition takes place. In Example 1:*

(i) *There exists an equilibrium with M mergers and a monopoly-like outcome in the upstream market if and only if $\delta \geq \delta_m$.*

(ii) *There exists an equilibrium with M mergers and a collusive-like outcome in the upstream market if and only if $\delta \in [\underline{\delta}_c^t, \overline{\delta}_c)$, where $\underline{\delta}_c^t \in [\underline{\delta}_c, \delta_m]$.*

(iii) *If $\delta < \underline{\delta}_c^t$ then there is no anticompetitive M-merger wave.*

δ_m and the monopoly upstream price are the same as in Section 4. When M divides $N - M$, symmetric collusive-like equilibria can be implemented, and $\underline{\delta}_c^t$ is the same as in Section 4. Otherwise, a symmetric distribution of market shares is not feasible, even with a randomization device, since upstream suppliers are known before downstream competition takes place. In that case $\underline{\delta}_c^t$ is higher than with the original timing.

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[24]The θ_k's are now realized between stages 2 and 2.5.

5.3 Discrimination

Now, suppose upstream firms can third-degree price discriminate in the input market. Under general demands, we cannot rule out that the monopoly outcome may be asymmetric (i.e., it may be optimal to offer different w_m's to different downstream firms), or that optimal deviations from a monopoly-like equilibrium or from a symmetric collusive-like equilibrium may be asymmetric too. We resolve these difficulties in Example 1:

Proposition 10. *Assume upstream producers can price-discriminate in the input market. In Example 1:*

(i) *There exists an equilibrium with M mergers and a monopoly-like outcome in the upstream market if and only if $\gamma > \gamma_d$ and $\delta \geq \delta_m^d$, where $\delta_m^d \geq \delta_m$.*

(ii) *There exists an equilibrium with M mergers and a symmetric collusive-like outcome in the upstream market if $\delta \in [\underline{\delta}_c^d, \overline{\delta}_c]$, and only if $\delta \in [\underline{\underline{\delta}}_c^d, \overline{\delta}_c]$, where $\underline{\delta}_c^d \geq \underline{\underline{\delta}}_c^d \geq \underline{\delta}_c$.*

Although upstream sellers can charge different prices to different downstream buyers, we show that, in the monopoly-like outcome, the integrated supplier sets a uniform monopoly upstream price w_m which is the same as under non-discrimination. Intuitively, the upstream supplier has no reason to charge different prices to symmetric downstream firms.

Inequalities $\delta_m^d \geq \delta_m$ and $\underline{\delta}_c^d \geq \underline{\delta}_c$ mean that monopoly-like and symmetric collusive-like equilibria are more difficult to sustain when upstream discrimination is allowed. This is because, under discrimination, integrated firms can cut their prices selectively when they deviate from a monopoly-like or a collusive-like equilibrium. For instance, when deviating from a monopoly-like equilibrium, an integrated firm may choose not to attract all the downstream firms, which was not feasible under non-discrimination. This suggests that allowing price discrimination in input markets may actually make these markets more competitive.[25]

5.4 Two-part tariff competition

Assume that firms compete in two-part tariffs on the upstream market, and denote by (w_i, T_i) the contract offered by firm U_i. We allow the variable part w_i to take any value, but we restrict the analysis to non-negative fixed parts: $T_i \geq 0$.[26] We also assume that upstream suppliers are chosen

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[25]In addition, we show that the maximum input price which can be sustained in a symmetric collusive-like equilibrium under discrimination is no larger than under non-discrimination. This is also consistent with the idea that discrimination can be pro-competitive.

[26]The reason is that, in the absence of exclusive dealing contracts, downstream buyers would have incentives to accept the offers of several upstream producers in order to receive the negative fixed fees, while actually buying a positive quantity from only one of them (see Chen, 2001). Since exclusive dealing contracts combined with

before downstream competition takes place as in Section 5.2.[27]

As explained in Section 5.2, when upstream suppliers are chosen before stage 3, the choices of upstream suppliers become a strategic game between downstream firms. We sidestep this difficulty by focusing first on the $(M, M+1)$ case. The fact that there is only one unintegrated downstream firm left after a merger wave makes it easier to derive an existence condition for monopoly-like equilibria in the M-merger subgame. In all subgames, we assume that the equilibrium profit of a downstream firm (gross of the fixed fee) is strictly decreasing in the variable part of its the contract, i.e., direct effects dominate indirect ones. In M-merger subgames, we denote by $\Pi_d(1, \mathbf{0}, w)$ and $\Pi_d(0, \mathbf{0}, \overline{m})$ the profit (gross of the fixed fee) of the unintegrated downstream firm when it buys the input from an integrated supplier at price w and when it buys from the alternative source at price $\overline{m}$, respectively. Assume that $\Pi(1, \mathbf{0}, w)$ and $\Pi(1, \mathbf{0}, w) + \Pi_d(1, \mathbf{0}, w)$ are strictly quasi-concave in w.

Then, the monopoly upstream contract, (w_m^{tp}, T_m^{tp}), which solves

$$\max_{(w,T)} \Pi(1, \mathbf{0}, w) + T \text{ subject to } \Pi_d(1, \mathbf{0}, w) - T \geq \Pi_d(0, \mathbf{0}, \bar{m}) \text{ and } T \geq 0,$$

exists and is unique, and we can prove the following lemma:

Lemma 6. $m < w_m^{tp} \leq w_m$.

The second inequality comes from the fact that two-part tariffs alleviate double-marginalization. The first inequality, however, states that double marginalization does not vanish completely. This is because the upstream supplier is willing to increase the marginal cost of the unintegrated downstream firm to reduce the cannibalization of its own downstream sales, and to soften downstream competition as in Bonanno and Vickers (1988).

Since $w_m^{tp} > m$, the softening effect is still at work, and the integrated firms which do not supply the upstream market earn larger downstream profits than the upstream supplier. Those firms may therefore not be willing to take over the upstream market. We define a monopoly-like outcome under two-part pricing as a situation in which the unintegrated downstream firm accepts a contract with a variable part equal to w_m^{tp}.

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vertical integration are already known to harm competition (Chen and Riordan, 2007), we assume that they are not enforceable, which implies that negative fixed parts would not survive in equilibrium.

[27]If we were to stick to our original timing, we would face the following problem. Assume U_i offers a low variable part and a high fixed part, whereas U_j offers a high w and a low T. Then, a downstream firm's optimal choice of supplier would depend on the downstream price it sets at the same time. If it sets a low downstream price, then the demand it receives is high, incentives to minimize marginal cost are strong, and the downstream firm should pick U_i's offer. Conversely, if it sets a high price, then it should go for U_j's offer. The fact that a downstream firm's marginal cost can depend on its downstream price may make the best response in downstream price discontinuous, which would jeopardize equilibrium existence in stage 3.

Proposition 11. *In the $(M, M+1)$ case, when firms compete in two-part tariffs, there exists an equilibrium with M mergers and a monopoly-like outcome if and only if $\Pi(1, \mathbf{0}, w_m^{tp}) \leq \Pi(0, \mathbf{1}, w_m^{tp})$. In Example 1, this condition is equivalent to $\delta \geq \delta_m^{tp}$, where $\delta_m^{tp} > \delta_m$.*

Compared to linear tariff competition, the monopoly-like outcome is both less harmful to consumers ($w_m^{tp} \leq w_m$) and more difficult to sustain ($\delta_m^{tp} > \delta_m$) under two-part pricing. The intuition for $\delta_m^{tp} > \delta_m$ is that when w becomes very large, the upstream demand and therefore the upstream profit shrink to zero. By continuity, it follows that the softening effect dominates when w is large. Since $w_m^{tp} \leq w_m$, the softening effect is more likely to dominate under linear tariff competition than under two-part pricing.

In the $(M, M+1)$ case, there is only one unintegrated downstream firm left in M-merger subgames, and since we assume upstream suppliers are chosen in stage 2.5, we cannot use a private randomization device to get rid of integer constraints. To investigate the robustness of collusive-like equilibria to two-part pricing, we solve the model in another special case, the $(2, 4)$ case with linear demands, in which integer constraints do not matter. In the following, we show that an equilibrium with two mergers and a symmetric collusive-like outcome on the upstream market (in which, say, $D3$ purchases from $U_1 - D_1$ and D_4 purchases from $U_2 - D_2$ at (w, T) with $w > m$) exists under conditions similar to those derived in the linear pricing case:

Proposition 12. *Consider the $(2, 4)$ case in Example 1. When firms compete in two-part tariffs, there exists an equilibrium with two mergers and a collusive-like outcome in the upstream market if $\delta \in [\underline{\delta}_c^{tp}, \overline{\delta}_c^{tp}]$.*

5.5 Secret offers

We modify the timing and the information structure as follows. At the beginning of stage 2, upstream firms offer secret, linear and discriminatory contracts to the downstream firms.[28] Next, each downstream firm decides which offer to accept, if any. In stage 3, acceptance decisions are publicly observed (i.e., everybody knows who purchases from whom), and downstream firms set their prices simultaneously. Since upstream offers are private information, we use perfect Bayesian equilibrium as our solution concept.

We look for monopoly-like equilibria in the $(M, M+1)$ case; collusive-like equilibria and the general (M, N) case will be discussed later on. The first step is to define the monopoly upstream price under secret offers. Suppose $U_i - D_i$ supplies D_{M+1} at price w, but all the other integrated

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[28] We allow upstream firms to third-degree price discriminate as in Section 5.3, since non-discriminatory and secret offers would be *de facto* observed by all the downstream firms. Notice that we also use the more natural timing introduced in Section 5.2, where upstream suppliers are chosen before downstream competition takes place. We discuss what happens in the original timing at the end of the section.

firms believe the upstream price is w^b. Those integrated firms set a downstream price, denoted by $p(0, w^b)$, which is the price they would charge under public offers when $U_i - D_i$ supplies the upstream market at price w^b. In this branch of the game tree, everything works as if $U_i - D_i$ and D_{M+1} were playing a two-player game with common knowledge of the upstream price (w) and of the prices set by other integrated firms $(p(0, w^b))$. We assume that this game has a unique Nash equilibrium, which determines $U_i - D_i$ and D_{M+1}'s downstream prices. By strategic complementarity, these equilibrium prices are increasing in $p(0, w^b)$. We assume that $U_i - D_i$ and D_{M+1}'s equilibrium quantities are also increasing in $p(0, w^b)$, which means as usual that direct effect dominate indirect ones. Denote by $\Pi^s(1, w, w^b)$ and $\Pi^s_d(1, w, w^b)$ the upstream supplier's and the downstream firm's equilibrium profits. We assume that $\Pi^s(1, w, w^b)$ and $\Pi(1, w)$ are strictly quasi-concave in w, and that $\Pi^s_d(1, w, w^b)$ and $\Pi_d(1, w)$ are strictly decreasing in w.

w^s_m is a monopoly upstream price under secret offers if and only if $U_i - D_i$ indeed wants to set w^s_m when other integrated firms believe the upstream price is w^s_m. Formally, $w^s_m = \arg\max_w \Pi^s(1, w, w^s_m)$ subject to $\Pi^s_d(1, w, w^s_m) \geq \Pi_d(0, \bar{m})$.

Lemma 7. *There exists a monopoly upstream price under secret offers. Any monopoly upstream price under secret offers belongs to the interval $(m, w_m]$.*

To streamline the analysis, we assume that w^s_m is unique, and that $\bar{m}$ is not too high, which ensures that $\Pi(1, w^s_m) \geq \Pi(0, \bar{m})$, i.e., $U_i - D_i$ prefers supplying the market at w^s_m rather than letting D_{M+1} purchase from the alternative source. The intuition for $w^s_m \leq w_m$ is that, under public offers, when $U_i - D_i$ cuts its upstream price, other integrated firms understand that both $U_i - D_i$ and D_{M+1} will become more aggressive on the downstream market. By strategic complementarity, those other integrated firms lower their downstream prices too, which hurts $U_i - D_i$. Under private contracting, those firms do not observe the deviation, and this mechanism therefore disappears.[29]

As usual, we define a monopoly-like outcome as a situation in which $U_i - D_i$ offers w^s_m, and other integrated firms make not upstream offer. Since $w^s_m > m$, the softening effect is still at work, and other integrated firms may not want to undercut. When investigating whether undercutting is profitable for, say, integrated firm $U_j - D_j$, the difficulty is that D_{M+1}'s acceptance decision and $U_j - D_j$'s choice of upstream price depend on how other integrated firms update their beliefs when they find out that $U_j - D_j$ has become the upstream supplier. Since the perfect Bayesian equilibrium concept does not put any restrictions on such out-of-equilibrium beliefs, it is not hard to imagine beliefs which would ruthlessly "punish" $U_j - D_j$'s deviation. The argument we use to

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[29]This is reminiscent of the opportunism problem identified by Hart and Tirole (1990), O'Brien and Shaffer (1992) and McAfee and Schwartz (1994) in that, starting from the optimal public contract, the upstream supplier has incentives to offer a secret "sweetheart deal" to the downstream firm, to increase their profits at the expense of other firms in the industry.

refine out-of-equilibrium beliefs is related to forward induction and to the concept of wary beliefs.[30] The idea is that, when firms observe that D_{M+1} takes an out-of-equilibrium action, they should not perceive this as an involuntary tremble, but rather, as a consequence of D_{M+1}'s optimizing behavior. In turn, D_{M+1}'s deviation should come from the fact that $U_j - D_j$ also deviated, and was also trying to maximize its profit.

The implications of this concept in terms of beliefs formation are the following. Assume that $U_j - D_j$ deviates by offering w_j, that D_{M+1} accepts this offer, and that other integrated firms believe that $U_j - D_j$ offered w_j^b to D_{M+1}. Using the same reasoning and notations as above, $U_j - D_j$ earns a profit of $\Pi^s(1, w_j, w_j^b)$. Under forward induction, the other integrated firms expect $U_j - D_j$ to maximize its deviation profit. Therefore, beliefs are consistent with forward induction if and only if $w_j^b \in \arg\max_{w_j} \Pi^s(1, w_j, w_j^b)$ subject to $\Pi_d^s(1, w_j, w_j^b) \geq \Pi_d(1, w_m^s, w_m^s)$. Therefore, $w_j^b = w_m^s$. It follows that there exists a monopoly-like equilibrium with beliefs consistent with forward induction if and only if $\Pi(1, w_m^s) \leq \Pi(0, w_m^s)$.

In subgames with fewer than M mergers, we make the usual assumption that the profit of an unintegrated downstream firm is decreasing in the price it pays for the input, and we show that the Bertrand outcome is an equilibrium in passive beliefs. In terms of behavior, passive beliefs have the following (appealing) implications: (a) a downstream firm never accepts an upward deviation, and (b) when a downstream firm receives a deviating offer below marginal cost, it always accepts this offer and cuts its downstream price. It is easy to see that the Bertrand outcome would also be an equilibrium with any beliefs system generating those two properties. We prove the following proposition:

Proposition 13. *In the $(M, M+1)$ case, when upstream offers are secret, there exists an equilibrium with M mergers and a monopoly-like outcome if and only if $\Pi(1, w_m^s) \leq \Pi(0, w_m^s)$. In Example 1, this condition is equivalent to $\delta \geq \delta_m^s$, where $\delta_m^s > \delta_m$.*

Under secret offers, monopoly-like equilibria are less harmful to consumers than under public offers $(w_m^s \leq w_m)$. As explained in Section 5.4, this implies that they are also less likely to arise $(\delta_m^s > \delta_m)$.

Extending Proposition 13 to the general (M, N) case is far from trivial. In the M-merger subgame, when a downstream firm receives an unexpected offer, it updates its beliefs about the offers made to other downstream firms. Starting from a monopoly-like outcome, a downstream firm which receives an out-of-equilibrium offer from $U_j - D_j$ must form beliefs about the number of other downstream firms to which $U_j - D_j$ made offers and about the prices of these other unexpected

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[30]See Fudenberg and Tirole (1991) for a discussion of forward induction. Wary beliefs were first introduced by McAfee and Schwartz (1994) in a vertical relations model with an upstream bottleneck. See also Rey and Vergé (2004) for a thorough treatment of wary beliefs in an upstream monopoly framework.

offers. We have not been able to refine these beliefs using forward induction. For the same reasons, it is difficult to establish the robustness of collusive-like equilibria to secret offers. Nevertheless, we prove the following proposition, which provides a necessary and sufficient condition for symmetric collusive-like equilibria in passive beliefs to exist in the $(2, 4)$ case:

Proposition 14. *Consider the $(2, 4)$ case in Example 1. In the two-merger subgame, there exist two thresholds, $\underline{\delta}_c^s < \overline{\delta}_c^s$ such that:*

(i) *If $\delta \in [\underline{\delta}_c^s, \overline{\delta}_c^s)$, then the set of input prices which can be sustained in a symmetric collusive-like equilibrium in passive beliefs is an interval.*

(ii) *Otherwise, there are no symmetric collusive-like equilibria in passive beliefs.*

To close this section, we would like to highlight the minimal set of assumptions required to generate the softening effect and the ensuing non-competitive equilibria. All we need is that, when downstream competition takes place, integrated firms know whether they are going to supply the input to downstream firms. This condition is satisfied when upstream offers are public, because integrated firms can anticipate downstream firms' supplier choices. It also holds when upstream contracts are secret and signed before downstream competition takes place. The softening effect disappears only in the case where upstream offers are secret *and* upstream suppliers are chosen after the downstream competition stage. In this light, we believe it is safe to say that the softening effect and our anti-competitive waves of vertical mergers are robust.

6   Conclusion

The main message conveyed in this paper is that competition between vertically integrated firms on an upstream market can be much softer than competition between vertically integrated firms and upstream firms, or than competition between upstream firms only. The reason lies in the softening effect, which links changes in the upstream market shares of vertically integrated firms to changes in downstream pricing strategies. The softening effect may induce a vertical merger wave, which effectively eliminates all unintegrated upstream firms and leads to the partial foreclosure of the remaining unintegrated downstream firms. Throughout the paper, we have maintained the assumption that the number of upstream firms was smaller than the number of downstream firms. Given the stylized features of our upstream market (price competition with homogeneous products), if this assumption did not hold, then a vertical merger wave would not be anticompetitive, because of the competitive pressure coming from unintegrated upstream firms. We do not think this should be taken at face value. Most input market are not frictionless, and it seems very unlikely that a single unintegrated upstream firm would have the ability or the willingness to entirely take over the

market when prices are above costs. With frictions, we therefore believe that the main message of our paper would survive: a vertical merger wave, by increasing the proportion of vertically integrated firms in the input market, can make upstream competition softer.

A Proofs

A.1 Proof of Proposition 1

Proof. Let us start with result (ii). Assume that $K < M$ mergers have taken place, and suppose that the input is supplied at a price w above marginal cost. If $K = 0$, then an unintegrated upstream firm can profitably deviate by setting $w - \varepsilon$ as in the textbook Bertrand model. If $K > 0$, given our equilibrium selection in stage 3, either the upstream market is supplied by unintegrated upstream firms only (and vertically integrated firms are setting prices above w), or it is supplied by vertically integrated firms only. In the latter case, an unintegrated upstream firm can profitably deviate by setting $w - \varepsilon$. In the former case, we claim that a vertically integrated firm, call it $U_i - D_i$ can profitably deviate by matching price w. If $U_i - D_i$ does not deviate, then its first order condition is given by:

$$0 = q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i}.$$

If it matches w, and becomes the sole input supplier, its first order condition becomes:

$$0 = q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} + (w - m)\sum_{k=K+1}^{N}\frac{\partial q_k}{\partial p_i}.$$

Since the last term in the right-hand side is positive, firm $U_i - D_i$'s first order condition shifts upward when it matches w. It follows that all downstream prices increase. For all $1 \leq j \leq N$, let $\tilde{\pi}_j^0(.)$ the payoff function of downstream firm D_j (where D_j may or may not be integrated), when the upstream market is supplied by unintegrated upstream firms at price w. Similarly, let $\tilde{\pi}_j^1(.)$ the payoff function of downstream firm D_j when the upstream market is supplied by $U_i - D_i$ at price w. The game $(\mathbb{R}; \tilde{\pi}_j^{(k)}(.), k = 0, 1; j = 1, 2, 3)$ is supermodular. For all j, $\tilde{\pi}_j^{(k)}(p_j, \boldsymbol{p}_{-j})$ has increasing differences in (p_j, k), and $\tilde{\pi}_i^{(k)}(p_i, \boldsymbol{p}_{-i})$ has strictly increasing differences in (p_i, k). Given Assumption (iii) in Section 2.2, supermodularity theory (see Vives, 1999, p.35) tells us that the equilibrium price vector is increasing in k, i.e., all downstream prices increase when $U_i - D_i$ matches w. It follows that $U_i - D_i$ wants to match to soften downstream competition and to make positive upstream profits.

The proof of result (i) is standard. The only twist is for integrated firms' downward deviations. If a vertically integrated firm deviates downward, it makes upstream losses. On top of that, the

deviator wants to price more aggressively to limit its upstream losses. Using the same supermodularity argument as above, downstream equilibrium prices all fall down which further hurts the deviator. The deviation is therefore not profitable. □

A.2 Proof of Lemma 2

Proof. Let $w > m$. To show that $p(1, w) > p(0, w)$, we denote by $BR_1(.,.,w)$ (respectively $BR_0(.,.,w)$) firm $U_i - D_i$ (resp. $U_j - D_j$)'s best response when the upstream market is supplied by $U_i - D_i$ at price w. The first order conditions (4) and (5) indicate that $BR_1(.,.,w) > BR_0(.,.,w)$. It follows that

$$p(1, w) = BR_1(p(0, w), p(d, w), w) > BR_0(p(0, w), p(d, w), w).$$

Besides,

$$p(0, w) = BR_0(p(1, w), p(d, w), w).$$

By strategic complementarity, BR_0 is increasing in its first argument, and $p(1, w) > p(0, w)$. The second part of the lemma follows from revealed profitability. □

A.3 Proof of Lemma 3

Proof. First, we claim that $\Pi(1, w) < \Pi(1, m)$ for $w < m$. When the upstream price decreases from m to $w < m$, following the supermodularity argument used in Section A.1, all equilibrium downstream prices decrease, which makes the upstream supplier worse off. On top of that, the upstream supplier makes upstream losses. Therefore, the maximization problem becomes $\max_{w \in [m, \bar{m}]} \Pi(1, w)$. Since $[m, w_m]$ is compact and $\Pi(1, .)$ is continuous, w_m exists.

Now, we claim that $\Pi'(1, m) > 0$. Denote by $U_i - D_i$ the upstream supplier, and by $U_j - D_j$ its integrated rival. Using the envelope theorem, we get:

$$\Pi'(1, m) = (p_i - m + \delta) \left(\left. \frac{dp_j}{dw} \right|_{w=m} \frac{\partial q_i}{\partial p_j} + \left. \frac{dp_3}{dw} \right|_{w=m} \frac{\partial q_i}{\partial p_3} \right) + q_3 > 0,$$

since, using once again a supermodularity argument, the downstream prices are increasing in w. We conclude that $w_m > m$.

Notice finally that, if both integrated firms offer prices above $\bar{m}$, then one integrated firm can profitably deviate by matching $\bar{m}$. When it does so, all downstream prices go up, and the deviator starts making upstream profits. It follows that the upstream market will never be supplied by the alternative source in equilibrium. □

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A.4 Proof of Proposition 1

Proof. (i) is immediate.

Proof of (ii). Notice first that $\Pi(0, .)$ is strictly increasing since, by the envelope theorem,

$$\Pi'(0, w) = (p_j - m + \delta) \left(\frac{\partial q_j}{\partial p_i} \frac{dp_i}{dw} + \frac{\partial q_j}{\partial p_3} \frac{dp_3}{dw} \right).$$

This is strictly positive since, using a supermodularity argument, all downstream prices are increasing in w.

Fix some equilibrium of stage 2, let (w_1, w_2) the input prices offered in this equilibrium, and assume this equilibrium is not monopoly-like. We know from the proof of Lemma 3 that at least one of these prices is no smaller than $\bar{m}$. Suppose $U_i - D_i$ is the upstream supplier in this equilibrium. Then, $w_i \leq \min(w_j, \bar{m})$.

Assume first that $w_i < w_j \leq \bar{m}$. For this to be an equilibrium, the following inequalities have to hold: $\Pi(0, w_j) \leq \Pi(1, w_i) \leq \Pi(0, w_i)$. But this is impossible, since $w_i < w_j$ and $\Pi(0, .)' > 0$. Next, assume $w_i \leq \bar{m} < w_j$. Then, by definition of w_m, $w_i = w_m$, and the equilibrium is monopoly-like.

Last, assume $(w =) w_i = w_j \leq \bar{m}$. Clearly, $w \leq w_m$, otherwise an integrated firm can undercut to w_m to increase its profits. $\Pi(1, w) = \Pi(0, w)$ must also hold, otherwise an integrated firm would rather undercut or exit the upstream market.

Proof of (iii). Assume condition (6) holds strictly, and fix an equilibrium which is not monopoly-like. We know that in this equilibrium, $w_i = w_j = w$, and $\Pi(0, w) = \Pi(1, w)$. Besides, since condition (6) holds strictly, $w \neq w_m$. By definition of w_m, $\Pi(0, w) = \Pi(1, w) < \Pi(1, w_m) < \Pi(0, w_m)$, which proves that monopoly-like equilibria Pareto-dominate all other equilibria. Now, we show that offering $w_i = w_m$ weakly dominates offering $w_i = w$ for integrated firm $U_i - D_i$. If the integrated rival offers $w_j \leq w$, then both strategies are equivalent. If $w < w_j < w_m$, then offering w_m yields a payoff $\Pi(0, w_j)$, which is larger than the payoff it would get from offering w, $\Pi(0, w)$, because $\Pi'(0, .) > 0$. If $w_j > w_m$, then offering w_m yields a payoff $\Pi(1, w_m)$, which is larger than the payoff when offering w, $\Pi(1, w)$, by definition of w_m. If $w_j = w_m$, then the integrated firm gets either $\Pi(0, w_m)$ or $\Pi(1, w_m)$. These payoffs are larger than $\Pi(1, w)$. $\qquad \square$

A.5 Proof of Proposition 2

Proof. (i) and (ii) are immediate.

Proof of (iii). We compare an equilibrium with two mergers in which D_3 purchases the input from $U_1 - D_1$ at $w > m$ (in which case downstream equilibrium prices are $\hat{p}_1$, $\hat{p}_2$ and $\hat{p}_3$), with an equilibrium with no merger in which all downstream firms obtain the input at marginal cost (in which case all equilibrium prices are equal to p^*). First, by supermodularity, $(\hat{p}_1, \hat{p}_2, \hat{p}_3) > (p^*, p^*, p^*)$, therefore consumers are strictly worse off in the foreclosure equilibrium.

Second, we show that social welfare in also strictly lower in the partial foreclosure equilibrium. Assume that there exists a representative consumer with a quasi-linear, continuously differentiable and quasi-concave utility function $q_0 + u(q_1, q_2, q_3)$, where q_0 denotes consumption of the numeraire and q_k denotes consumption of product $k \in \{1, 2, 3\}$. We can then write the social welfare as

$$SW(\boldsymbol{p}) = u\big(q(p_1, \boldsymbol{p}_{-1}), q(p_2, \boldsymbol{p}_{-2}), q(p_3, \boldsymbol{p}_{-3})\big) - m \sum_{k=1}^{3} q(p_k, \boldsymbol{p}_{-k}).$$

Since the welfare function is symmetric in its arguments, we can relabel the downstream prices in the partial foreclosure equilibrium so that $p^* \leq \hat{p}_1 \leq \hat{p}_2 \leq \hat{p}_3$.[31] We want to show that $W(\hat{p}_1, \hat{p}_2, \hat{p}_3) - W(p^*, p^*, p^*)$, the variation in social welfare when one shifts from the Bertrand outcome from the partial foreclosure outcome, is strictly negative. The variation in welfare can be written as:

$$\int_{p^*}^{\hat{p}_1} \sum_{k=1}^{3} \frac{\partial SW}{\partial p_k}(r, r, r) dr + \int_{\hat{p}_1}^{\hat{p}_2} \sum_{k=2}^{3} \frac{\partial SW}{\partial p_k}(\hat{p}_1, r, r) dr + \int_{\hat{p}_2}^{\hat{p}_3} \frac{\partial SW}{\partial p_3}(\hat{p}_1, \hat{p}_2, r) dr.$$

All the integrands are strictly negative. For instance,

$$\frac{\partial SW}{\partial p_k}(p_1, p_2, p_3) = \sum_{k'=1}^{3} \left(\frac{\partial u}{\partial q_{k'}} - m \right) \frac{\partial q_{k'}}{\partial p_k} = \sum_{k'=1}^{3} (p_{k'} - m) \frac{\partial q_{k'}}{\partial p_k}$$

is strictly negative when $p_k \geq p_{k'}$, $k' \neq k$, since $\partial q_k / \partial p_k < -\sum_{k' \neq k} |\partial q_{k'}/\partial p_k|$. It follows that the variation in welfare is negative. Since $W(\lambda) = CS + \lambda W$, we can conclude that industry performance degrades when the industry shifts from the Bertrand outcome to a partial foreclosure equilibrium. $\square$

A.6 Proof of Proposition 4

Proof. (i) is immediate using Assumption (iv) in Section 2.2.

Proof of (ii). After the first merger, the marginal cost of the merging parties goes down, and the input is still priced at marginal cost. Using a supermodularity argument, it follows that downstream prices all fall down, which increases consumer surplus. Next, as in the proof of Proposition 2-(iii), we look at the variation in social welfare between the zero-merger subgame (in which all firms set downstream prices equal to p^* and have marginal cost m) and the one-merger subgame (in which $U_1 - D_1$ sets $\hat{p}_1$ and has marginal cost $m - \delta$, and firms D_2 and D_3 set $\hat{p}_d$ and still have marginal cost m). Notice that $p^* \geq \hat{p}_d > \hat{p}_1$. The negative of the variation in social

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[31]It is straightforward to show, using again supermodularity, that at least one of these inequalities is strict.

welfare is given by:

$$\Delta SW = \int_{\hat{p}_d}^{p^*} \left\{ (\delta + 3(p - m)) \left(\frac{\partial q(p, p, p)}{\partial p_1} + 2\frac{\partial q(p, p, p)}{\partial p_2} \right) \right\} dp$$

$$+ \int_{\hat{p}_1}^{\hat{p}_d} \left\{ (\hat{p}_d - m) \left(\frac{\partial q(p, \hat{p}_d, \hat{p}_d)}{\partial p_1} + 2\frac{\partial q(\hat{p}_d, p, \hat{p}_d)}{\partial p_2} \right) + ((p - m + \delta) - (\hat{p}_d - m))\frac{\partial q(p, \hat{p}_d, \hat{p}_d)}{\partial p_1} \right\} dp.$$

The first integral and the first term in the second integral are negative, since total demand is decreasing in prices. The second term in the second integral is negative too, because of the assumption that more productive firms charge higher markups. It follows that the first merger improves social welfare and market performance. □

A.7 Proof of Lemma 4

Proof. Fix an input price vector $\boldsymbol{w}$ and a profile of supplier choices $(\theta_k \mapsto s_k(\theta_k))_{M+1 \leq k \leq N}$ consistent with sequential rationality. Let $i \leq M$ and $j \geq M + 1$. Since all downstream firms end up purchasing at price $w = \min(\boldsymbol{w})$, and since there exists a unique profile of equilibrium downstream prices associated with this profile of supplier choices, all unintegrated downstream firms set the same price. It follows in particular that, for all $k \geq M + 1$, $\partial q_k/\partial p_i = \partial q_j/\partial p_i$ and $q_k = q_j$, where the functions are evaluated at the equilibrium price vector. The first-order condition of firm $U_i - D_i$ is given by:

$$0 = q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} + (w - m) \sum_{i=M+1}^{N} E[\mathbb{1}_{s_k(\theta_k)=i}]\frac{\partial q_k}{\partial p_i},$$

$$= q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} + (w - m)\alpha_i(N - M)\frac{\partial q_j}{\partial p_i}.$$

The first-order condition of firm D_k is given by:

$$0 = q_k + (p_k - w)\frac{\partial q_k}{\partial p_k}.$$

It follows that the equilibrium downstream prices and depend only on w and $\boldsymbol{\alpha}$. The profit of firm $U_i - D_i$ is equal to

$$\pi_i = (p_i - m + \delta)q_i + (w - m)\alpha_i(N - M)q_j.$$

Therefore, the equilibrium profit of $U_i - D_i$ only depends on w and $\boldsymbol{\alpha}$. It follows from symmetry that this profit can be written as $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$, and is invariant to permutations of $\boldsymbol{\alpha}_{-i}$. □

A.8 Proof of Proposition 6

Proof. What remains to be shown is that there is no equilibrium of the M-merger subgame with only one integrated firm offering the lowest upstream price w, with $w > m$ and $w \neq w_m$. Assume,

by contradiction, that there is one such equilibrium and call the upstream supplier $U_i - D_i$. If the other integrated firms make no upstream offer, or make offers above w_m, then $U_i - D_i$ has a profitable deviation: set w_m. Otherwise, we denote the second lowest upstream price by $w' = \min(\boldsymbol{w}_{-i}) \leq w_m$. A first equilibrium condition is that $U_i - D_i$'s integrated rivals do not want to slightly undercut in the upstream market, or

$$\Pi(0, \mathbf{1}, w) \geq \Pi(1, \mathbf{0}, w).$$

A second equilibrium condition is that $U_i - D_i$ does not want to withdraw its upstream offer and let the integrated firms which set a price of w' supply the upstream market. In that case, any repartition of the upstream market shares between those integrated firms can be sustained at the equilibrium of stage 3. However, whatever the distribution $\boldsymbol{\alpha}_{-i}$ of the market shares, the deviation is strictly profitable for $U_i - D_i$. Indeed, it follows from the quasi-concavity of $\Pi(., ., w')$ that

$$\Pi(0, \boldsymbol{\alpha}_{-i}, w') \geq \Pi(0, \mathbf{1}, w').$$

Using a supermodularity argument,

$$\Pi(0, \mathbf{1}, w') > \Pi(0, \mathbf{1}, w).$$

Combining the three above equations we obtain that the deviation is strictly profitable. $\qquad\square$

A.9 Proof of Lemma 6

Proof. Denote by $\hat{w}_m^{tp}$ the unique solution of the following maximization problem:

$$\max_w \Pi(1, \mathbf{0}, w) + \Pi_d(1, \mathbf{0}, w).$$

Assume first that $\Pi_d(1, \mathbf{0}, \hat{w}_m^{tp}) \leq \Pi_d(0, \mathbf{0}, \bar{m})$. Then, it follows from the quasi-concavity of the joint profit and from the definition of w_m that $(w_m^{tp}, T_m^{tp}) = (w_m, 0)$, which implies that the two inequalities in the statement of Lemma 6 are satisfied. Conversely, assume that $\Pi_d(1, \mathbf{0}, \hat{w}_m^{tp}) > \Pi_d(0, \mathbf{0}, \bar{m})$. Then, the monopoly contract is $w_m^{tp} = \hat{w}^{tp}$, and $T_m^{tp} = \Pi_d(1, \mathbf{0}, \hat{w}_m^{tp}) - \Pi_d(0, \mathbf{0}, \bar{m})$.

$\hat{w}_m^{tp} > m$ follow from quasi-concavity and from the fact that, when $w = m$, the derivative with respect to w of the joint profit of the input seller $Ui - Di$ and the input buyer Dk,

$$\frac{d(\Pi(1, \mathbf{0}, w) + \Pi_d(1, \mathbf{0}, w))}{dw}\bigg|_{(w=m)} = (p_i - m + \delta)\sum_{j \neq i} \frac{\partial q_i}{\partial p_j}\frac{dp_j}{dw} + (p_k - m)\sum_{j \neq k} \frac{\partial q_k}{\partial p_j}\frac{dp_j}{dw},$$

is strictly positive, since, as usual, $dp_j/dw > 0$ for all j. Now, notice that

$$\frac{d\Pi(1, \mathbf{0}, w)}{dw}\bigg|_{(w=\hat{w}_m^{tp})} > \frac{d(\Pi(1, \mathbf{0}, w) + \Pi_d(1, \mathbf{0}, w))}{dw}\bigg|_{(w=\hat{w}_m^{tp})} = 0,$$

where the inequality follows from $d\Pi_d(1, \mathbf{0}, w)/dw < 0$. Therefore, since $\Pi(1, \mathbf{0}, w)$ is strictly quasi-concave in w, $w_m > w_m^{tp}$. $\qquad\square$

A.10 Proof of Proposition 11

Proof. First, we prove that the Bertrand outcome is an equilibrium in subgames with fewer than M mergers. Assume that all upstream firms, integrated or not, offer $(m, 0)$. Consider first that firm U_i (which may or may not be integrated) deviates upward, and offers (w, T), with $w > m$ and $T \geq 0$. Then, since, by assumption, the equilibrium profit of a downstream firm is decreasing in the variable part of its contract, there is still an equilibrium in the continuation subgame in which all downstream firms stick to the Bertrand outcome offers. We assume that this equilibrium is played in those subgames, which makes this deviation unprofitable.

Now, assume that unintegrated upstream firm U_i offers (w, T) with $w < m$, and that this deviation attracts a set S of unintegrated downstream firms. Let $j \in S$. Denote by $\boldsymbol{p}^S$ the equilibrium downstream price vector when all firms in S (and only in S) accept the deviating offer. Denote also by $\boldsymbol{p}^{S \backslash \{j\}}$ the equilibrium downstream price vector when the deviating offer is only accepted by firms in $S \backslash \{j\}$. When j accepts the deviating offer, its best-response function shifts down. Supermodularity theory implies that all downstream prices decrease, i.e., $\boldsymbol{p}^{S \backslash \{j\}} > \boldsymbol{p}^S$, where the inequality is taken component by component. Notice that, if $p_j^S < m$, then the joint profit of firm U_i and of the firms in S is negative. Therefore, one of these firms has to make negative profits. This implies that, either the deviation is not profitable, or it cannot be accepted by all firms in S, which is a contradiction. Now, assume that $p_j^S \geq m$. A necessary condition for the deviating offer to be accepted by firm j in an equilibrium of the continuation subgame is that

$$(p_j^S - w)q(p_j^S, \boldsymbol{p}_{-j}^S) - T \geq (p_j^{S \backslash \{j\}} - m)q(p_j^{S \backslash \{j\}}, \boldsymbol{p}_{-j}^{S \backslash \{j\}})$$

Rearranging terms, we get:

$$
\begin{aligned}
(w - m)q(p_j^S, \boldsymbol{p}_{-j}^S) + T &\leq (p_j^S - m)q(p_j^S, \boldsymbol{p}_{-j}^S) - (p_j^{S \backslash \{j\}} - m)q(p_j^{S \backslash \{j\}}, \boldsymbol{p}_{-j}^{S \backslash \{j\}}) \\
&\leq (p_j^S - m)q(p_j^S, \boldsymbol{p}_{-j}^{S \backslash \{j\}}) - (p_j^{S \backslash \{j\}} - m)q(p_j^{S \backslash \{j\}}, \boldsymbol{p}_{-j}^{S \backslash \{j\}}) \\
&< 0, \text{ by definition of } p_j^{S \backslash \{j\}}
\end{aligned}
$$

This implies that firm U_i is making negative profits on each of the firms in S: this deviation is not profitable.

If the deviator were a vertically integrated firm, then, by the same token, the deviation would generate upstream losses. On top of that, it would make the downstream market more competitive, which would degrade the deviator's downstream profits. Again, such a deviation would not be profitable. Therefore, the Bertrand outcome is an equilibrium in all subgames with fewer than M mergers.

Next, we look for monopoly-like equilibria in the M-merger subgame. Assume there exists a monopoly-like equilibrium. Then, the firms which do not supply the upstream market should not

be willing to undercut:

$$\Pi(0, \mathbf{1}, w_m^{tp}) \geq \Pi(1, \mathbf{0}, w_m^{tp}) + T_m^{tp} \geq \Pi(1, \mathbf{0}, w_m^{tp}),$$

since $T_m^{tp} \geq 0$.

Conversely, suppose that $\Pi(1, \mathbf{0}, w_m^{tp}) \leq \Pi(0, \mathbf{1}, w_m^{tp})$. We distinguish two cases. Assume first that $\Pi(1, \mathbf{0}, w_m^{tp}) + T_m^{tp} \leq \Pi(0, \mathbf{1}, w_m^{tp})$. Then, the monopoly-like outcome in which $U_1 - D_1$ offers contract (w_m^{tp}, T_m^{tp}) and other integrated firms do not make any offer is an equilibrium. Second, assume that $\Pi(1, \mathbf{0}, w_m^{tp}) \leq \Pi(0, \mathbf{1}, w_m^{tp}) < \Pi(1, \mathbf{0}, w_m^{tp}) + T_m^{tp}$. Then, the monopoly-like outcome in which all integrated firms offer contract $(w_m^{tp}, \Pi(0, \mathbf{1}, w_m^{tp}) - \Pi(1, \mathbf{0}, w_m^{tp}))$ and the unintegrated downstream firm accepts $U_1 - D_1$'s contract is an equilibrium. $\qquad\square$

A.11 Proof of Lemma 7

Proof. We begin the proof by introducing some notations. Fix some w^b and let $\bar{w}(w^b)$ (resp. $\bar{w}$) such that $\Pi_d^s(1, w, w^b) \geq \Pi_d(0, \bar{m})$ (resp. $\Pi_d(1, w) \geq \Pi_d(0, \bar{m})$) if and only if $w \leq \bar{w}(w^b)$ (resp. $w \leq \bar{w}$). These thresholds are well-defined, since $\Pi_d^s(1, w, w^b)$ and $\Pi_d(1, w)$ are strictly decreasing in w. By strict quasi-concavity, $\hat{w}(w^b) = \arg\max_{w \leq \bar{w}(w^b)} \Pi(1, w, w^b)$ exists and it is unique. Let $f(w^b) = \hat{w}(w^b) - w^b$, and notice that f is continuous. Then, $\tilde{w}$ is a monopoly upstream price under secret offers if and only if it is a zero of f. The proof proceeds in several steps.

Step 1 : $f(w^b) > 0$ for all $w^b \leq m$.

Let $w^b \leq m$. Clearly, $\Pi_d^s(1, w^b, w^b) > \Pi_d(1, \bar{m}) > \Pi_d(0, \bar{m})$, so $w^b < \bar{w}(w^b)$. Next, we show that the upstream supplier, call him $U_i - D_i$, has incentives to slightly increase w, starting from $w = w^b$:

$$\left.\frac{\partial \Pi(1, w, w^b)}{\partial w}\right|_{w=w^b} = \left[(p_i - m + \delta)\frac{\partial q_i}{\partial p_{M+1}} + (w^b - m)\frac{\partial q_{M+1}}{\partial p_{M+1}}\right]\frac{\partial p_{M+1}}{\partial w} + q_{M+1}.$$

The first and third terms on the right-hand side are positive, and, since $w^b \leq m$, the second term is non-negative. Therefore, by quasi-concavity, $f(w^b) > 0$.

Step 2: When $\tilde{w} > m$, $\left.\frac{d\Pi(1,w)}{dw}\right|_{w=\tilde{w}} > \left.\frac{\partial\Pi^s(1,w,w^b)}{\partial w}\right|_{w=w^b=\tilde{w}}$.

Since $\Pi(1, w) = \Pi^s(1, w, w)$ for all w, $\left.\frac{d\Pi(1,w)}{dw}\right|_{w=\tilde{w}} = \left.\frac{\partial\Pi^s(1,w,w^b)}{\partial w}\right|_{w=w^b=\tilde{w}} + \left.\frac{\partial\Pi^s(1,w,w^b)}{\partial w^b}\right|_{w=w^b=\tilde{w}}$. Therefore, all we need to show is that $\left.\frac{\partial\Pi^s(1,w,w^b)}{\partial w^b}\right|_{w=w^b=\tilde{w}} > 0$.

When the upstream price is w and other integrated firms believe that it is equal to w^b, equilibrium downstream prices solve the fixed point problem: $p_i = BR(p_{-i}; 1, w)$, $p_j = p(0, w^b)$ $(j \neq i, M+1)$, and $p_{M+1} = BR(p_{-(M+1)}; d, w)$, where $BR(.; i, w)$ and $BR(.; d, w)$ denote $U_i - D_i$ and D_{M+1}'s best responses in downstream prices, respectively, when the upstream price is w, and $p(0, w^b)$ is the downstream price set by other integrated firms when they believe the upstream price

is w^b. When w^b increases, $p(0, w^b)$ and the equilibrium prices of $U_i - D_i$ and D_{M+1} go up, following a standard supermodularity argument. By assumption, the equilibrium quantities of $U_i - D_i$ and D_{M+1} increase as well, which implies that $\frac{\partial \Pi^s(1, w, w^b)}{\partial w^b} > 0$ provided that $w \geq m$.

Step 3: If $w_m = \bar{w}$, then for all $w^b > w_m$, $f(w^b) < 0$.
When $w^b > \bar{w}$, $\Pi_d(1, w^b) < \Pi_d(0, \bar{m})$. Therefore, $\bar{w}(w^b) < w^b$, and $f(w^b) < 0$.

Step 4: If $w_m < \bar{w}$, then for all $w^b \geq w_m$, $f(w^b) < 0$.
By quasi-concavity, for all $w^b \geq w_m$, $\left. \frac{d\Pi(1, w)}{dw} \right|_{w=w^b} \leq 0$. It follows from step 2 that $\left. \frac{\partial \Pi^s(1, w, w^b)}{\partial w} \right|_{w=w^b} < 0$, which implies that $\hat{w}(w^b) < w^b$, i.e., $f(w^b) < 0$.

Combining steps 1, 3 and 4, we can conclude that f has at least one fixed point in interval $(m, w_m]$, and does not have any fixed point outside this interval. □

A.12 Proof of Proposition 13

Proof. We already prove the necessary and sufficient condition for monopoly-like equilibria in the M-merger subgame in Section 5.5. All that is left to show is that the Bertrand outcome is an equilibrium in passive beliefs in subgames with fewer than M mergers.

Assume that $K < M$ mergers have taken place, and that all upstream firms, integrated or not, offer the input at marginal cost to all downstream firms. Notice that an unintegrated downstream firm would never accept an upward deviation. To see this, suppose that downstream firm D_k receives the out-of-equilibrium offer $\hat{w} > m$ from upstream firm $U_i(-D_i)$. Denote by $\tilde{\pi}_d(\hat{w})$ the equilibrium profit of firm D_k when it purchases the input from firm U_i at price $\hat{w}$, and all other downstream firms (except D_i, if it is vertically integrated with U_i) believe that D_k purchases at marginal cost. Given passive beliefs, D_k expects to earn $\tilde{\pi}_d(m)$ if it rejects the offer, and $\tilde{\pi}_d(\hat{w})$ if it accepts it. Since $\tilde{\pi}_d(.)$ is decreasing, it follows that D_k will decline the offer. By the same token, it is straightforward to show that an unintegrated downstream firm always accepts a downward deviation.

The above paragraph implies that unintegrated upstream firms have no way to make positive profits, and therefore, that they do not have any profitable deviation. Assume integrated firm $U_i - D_i$ deviates, and offers a price below marginal cost to $L \geq 1$ downstream firms, and a price no smaller than marginal cost to the other downstream firms. Then, the L downstream firms accept the deviation and, under passive beliefs, cut their downstream prices, while the other downstream firms do not change their downstream prices. This hurts the deviator. On top of that, $U_i - D_i$ starts making losses, which lowers its profit even more. The deviation is therefore not profitable. □

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