Signaling quality in vertical relationships*

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Abstract

This paper addresses the issue of price signaling in a model of vertical relationship between a manufacturer and a retailer who share the same information about quality, contrary to consumers who do not observe it a priori. We show that delegating the price setting task to a retailer and controlling it through a vertical contract helps to drastically reduce the number of price signaling equilibria available to the retailer. For this, a linear wholesale price would not be sufficient and a two-part tariff contract structure is needed at least. The outcome of a unique price charged to consumers obtains without invoking the consumer sophistication usually required by selection criterions. The vertical contract turns to be the most efficient way for the vertical chain to tie his hands on a unique final price. This price may disclose or not information to consumers depending on their initial optimism about quality.

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1 Introduction

Firms often use prices as a signal of quality when they introduce new products whose quality is not observable to consumers before purchase. Like most signaling behaviors, price signaling entails some inefficiencies: to prove that quality is high, the firm will somewhat distort the price compared to what would prevail under full information. In fact, the signaling literature initiated by Spence shows that, once the conditions for the existence of a separating equilibrium are met, there is always a multiplicity of separating equilibria which range from the least to the most costly ones in terms of information disclosure. This multiplicity has long been viewed as a weakness of signaling models since it harms their predicative power. The standard solution proposed by the literature to this problem is to employ selection criterions which, generally, single out the Riley equilibrium. This equilibrium seems the most “reasonable” or “intuitive” in the set of separating equilibria, in that it entails the minimum distortion needed to disclose information. However, the logic of selection criterions relies on the assumption that uninformed agents – consumers in the present case – are highly sophisticated in the way they build and update their beliefs.

This paper addresses the issue of price signaling in a model of vertical relationship between a manufacturer and a retailer who share the same information about quality, contrary to consumers who do not observe it a priori. We show that delegating the price setting task to a retailer and controlling it through a vertical contract helps to drastically reduce the number of price signaling equilibria available to the retailer. For this, a linear wholesale price would not be sufficient and a two-part tariff contract structure is needed at least. The outcome of a unique price charged to consumers obtains without invoking the consumer sophistication usually required by selection criterions. The vertical contract turns to be the most efficient way for the vertical chain to tie his hands on a unique final price. This price may disclose or not information to consumers depending on their initial optimism about quality.

The paper is organized as follows. In the section 2, we lay down the model and the assumptions. Then we characterize the equilibria under full information and asymmetric information in the case of no delegation. In section 3, we then examine the case of delegation. Section 4 concludes. An appendix contains most of proofs.

2 Signaling quality in the vertically integrated structure: the case of no delegation

2.1 Assumptions and notations

Consider a Hotelling (1929) market with a unit mass of consumers uniformly distributed along the segment $[0, 1]$. Two differentiated products are located at the two extremes of the segment. When purchasing either product, consumers pay transportation cost $t \geq 0$ per unit of distance, which represents the utility loss from buying a product that does not perfectly cater for their needs. Both products are produced with constant marginal costs and provide buyers with the same gross surplus of value r. Product 0 located at the left extreme of the segment is sold by a manufacturer M at the per-unit consumer price p. This product is sufficiently differentiated in taste or quality to provide the manufacturer with some degree of monopoly power. Product 1 located at the right extreme of the segment is an imperfect substitute of product 0, sold by a fringe of competitive producers at price equal to marginal cost. As the parameter t measures the degree of differentiation between products, it will serve as an index for the manufacturer's market power. Both products require the same production technology with constant returns to scale.

Using the terminology of Nelson (1970), product 0 is assumed to be an "experience good" in the sense that consumers cannot observe its actual quality before purchase. We denote i the vertical quality characteristic of product 0, and we assume, for simplicity, that i may be either high ($i = H$) or low ($i = L$) with $H > L \geq 0$. The manufacturer incurs constant marginal costs of production denoted c_i for $i = H, L$, with $c_H = c > 0$ and $c_L = 0$, i.e., higher quality is more costly to produce. We assume that there are no cost of distribution. *A priori*, consumers perceive the quality of product 0 to be high with probability $\mu_0 \equiv prob(i = H)$, and low with probability $1-\mu_0 \equiv prob(i = L)$, $\mu_0 \in (0,1)$.

Unlike i, the alternative quality a of product 1 is perfectly observable to consumers. The provision of a entails marginal cost c or 0 depending on whether $a = H$ or $a = L$. To treat both cases simultaneously, we will use the following notations: $a = \alpha\Delta + L$ with the corresponding marginal cost equals to αc, where $\Delta = H - L$ and α is a dummy variable such that $\alpha = 0$ (resp. 1) is the case where the certain quality of product 1 is L (resp. H).

Consumers are assumed to purchase at most one unit of product. It is also assumed that r is large enough for all consumers to find a product for which their surplus is positive in equilibrium. The surplus from purchasing one unit of product depends on the consumer's location $x \in [0,1]$ according to

$$u_1(x) = r + a - \alpha c - t(1-x)$$

when buying the sure product 1. As the actual quality of product 0 is not observable to consumers, they must rely on their beliefs about this quality when deciding to purchase the good. Observing p, consumers try to infer some information about quality and update their beliefs. Let $\mu(p) : R^+ \to [0,1]$ denote the consumers' posterior belief that quality of product 0 is H upon seeing p. If consumers assign probability $\mu = \mu(p)$ to the high quality, then the expected surplus for a consumer located in x, from purchasing one unit of product 0, is given by:

$$u_0(x) = r + \mu H + (1-\mu)L - p - tx.$$

The market splits in two at the marginal consumer who is indifferent between both products. It follows that the demand for product 0 can be expressed as:

$$D(p,\mu) = \frac{2A + \mu\Delta - p}{2t} \tag{1}$$

with

$$A \equiv \frac{t + \alpha(c - \Delta)}{2}$$

as long as it is non-negative and does not exceed 1.[1]

We will restrict the parameters of the model to satisfy the following assumptions:

$$c - t < \Delta < c + t \tag{2}$$

This parameter configuration ensures that both producers selling different qualities at marginal cost have a positive market share under complete information, whatever the cost advantage. Hence, the presence of both products on the market is socially efficient under complete information. Note that under assumption (2), we have $0 < A$ and $c < A + \Delta$.

In the integrated structure, when the actual quality of his product is $i \in \{H, L\}$ and consumers believe this quality to be high with probability μ, the manufacturer's profit can be written:

$$\pi_i^I (p, \mu) = (p - c_i) D (p, \mu) \,, \text{ for } i = H, L. \tag{3}$$

As long as demand $D(p, \mu)$ is positive, we can compute the optimal price $p_i^I(\mu)$ which maximizes $\pi_i^I (p, \mu)$ with respect to p and we get:

$$p_i^I(\mu) = \frac{c_i + 2A + \mu\Delta}{2} \tag{4}$$

At price $p_i^I(\mu)$, demand is given by $D\left(p_i^I(\mu), \mu\right) = \frac{2A+\mu\Delta-c_i}{4t}$, which is positive for $c_i < 2A + \mu\Delta$. Thus, the maximized profit $\pi_i^I(\mu) \equiv \pi_i^I(p_i^I(\mu), \mu)$ for a given belief μ is:

$$\pi_i^I(\mu) = \frac{1}{2t}\left(\frac{2A + \mu\Delta - c_i}{2}\right)^2 \mathbf{1}_{c_i<2A+\mu\Delta}. \tag{5}$$

2.2 Full information

Our first proposition characterizes the equilibrium outcome of the vertically integrated structure under full information (using the expressions of prices and profits given by (4) and (5)).

[1] Note that the price elasticity of demand for product 0, that is, $\left|\frac{\partial D(p,\mu)}{\partial p}\frac{p}{D(p,\mu)}\right|$ decreases, *ceteris paribus*, with the measure t of monopoly power.

Proposition 1 *The following strategies and outcomes constitute an equilibrium of the integrated structure under full information. The consumer prices $\widehat{p}_i^I$ and the manufacturer's monopoly profits $\widehat{\pi}_i^I$ are*

$$\begin{aligned}\widehat{p}_L^I &= A \text{ and } \widehat{\pi}_L^I = \frac{A^2}{2t} \\ \widehat{p}_H^I &= A + \frac{c+\Delta}{2} \text{ and } \widehat{\pi}_H^I = \frac{(2A+\Delta-c)^2}{8t}\end{aligned}$$

It can be seen that a higher degree of horizontal differentiation makes the neighboring clientele more captive, thereby raising the price set by the manufacturer, which increases his profit. The same effects occur regarding vertical differentiation to the extent that the manufacturer's quality is higher than that of the product supplied by the competitive fringe: when $\alpha = 0$, an increase in Δ reduces the elasticity of demand for product 0 of quality H, thereby raising $\widehat{\pi}_H^I$. In contrast, when $\alpha = 1$, if the manufacturer provides inferior quality relative to the fringe then its profit $\widehat{\pi}_L^I$ declines with Δ because more product differentiation erodes the manufacturer's market power.

2.3 Asymmetric information

Under asymmetric information, the manufacturer may choose either to disclose his private information on quality through separating prices, or to conceal this information by setting pooling prices. The manufacturer maximizes profit with respect to price, given the beliefs held by consumers after observing this price. This objective defines a signaling game similar to that investigated by Bagwell and Riordan (1991). The manufacturer's pricing strategies must be supported as a Perfect Bayesian Equilibrium (PBE). Many of the proof techniques used by Bagwell and Riordan (1991) are readily adapted for characterizing the PBE in our setting. We will consider in turn the issue of separating price equilibria and of pooling price equilibria.

2.3.1 The set of separating outcomes

Consider a putative separating equilibrium $\left(p_H^I, p_L^I\right)$ in the integrated structure. As usual, such an equilibrium must satisfy two kinds of constraints: first, individual rationality (IR) constraints and, second, incentive compatibility (IC) constraints. The IR constraints ensure that the manufacturer finds it profitable to choose an equilibrium price rather than the optimal price associated with the worst belief that consumers can hold from the manufacturer's standpoint, i. e., $\mu = 0$. The IC constraints require that the price set for one quality would not be worth duplicating if the quality were different.

The IR constraints are given respectively by

$$\pi_H^I\left(p_H^I, 1\right) \geq \pi_H^I(0) \tag{IR$_H$}$$

$$\pi_L^I\left(p_L^I, 0\right) \geq \pi_L^I(0). \tag{IR$_L$}$$

Furthermore, the IC constraints are given by

$$\pi_H^I\left(p_H^I, 1\right) \geq \pi_H^I(p_L^I, 0) \tag{IC$_H$}$$

$$\pi_L^I(p_L^I, 0) \geq \pi_L^I(p_H^I, 1). \tag{IC$_L$}$$

By determining the set of prices satisfying the above constraints, we are able to obtain the following Proposition. For this, we introduce the following condition on the parameters (c, Δ, t) :

$$c < \min\left\{2A, \sqrt{\Delta\left(4A+\Delta\right)}\right\} \tag{C}$$

Proposition 2 *Under Assumption (2), there always exists a unique pair of separating equilibrium prices $\left(p_L^I, p_H^I\right)$ robust to the intuitive criterion, given by:*

$$\begin{aligned} p_L^I &= \widehat{p}_L^I = A \\ p_H^I &= \begin{cases} \overline{p}(0) = \widehat{p}_H^I + \frac{1}{2}\left(\sqrt{\Delta\left(4A+\Delta\right)} - c\right) & \textit{if condition } C \textit{ holds} \\ \widehat{p}_H^I = A + \frac{c+\Delta}{2} & \textit{otherwise.} \end{cases} \end{aligned}$$

Proof: See appendix A. ■

Proposition 2 first indicates that the equilibrium price of a low quality good is never distorted compared to the full information situation. On the contrary, when the condition C holds true then the separating price for high quality involves an upward distortion compared to the full information price. Otherwise, the high quality price is not impacted by the presence of asymmetric information.

In the appendix (see appendix B), we characterize the area in which the high quality separating price is upward distorted (that is when condition C holds). Figure 1 depicts in the space (Δ, c) the situation when the fringe is producing a low quality product $(\alpha = 0)$, taking t as a parameter strictly positive, while Figure 2 depicts the case where the fringe is producing a high quality good $(\alpha = 1)$. In both situations, whenever c and/or Δ are sufficiently large, then asymmetric information imposes no distortion on prices. This upward distortion only appears in the areas (a) and (b) depicted in both figures.

Finally, note that the size of the distortion (whenever it is needed for signalling quality) is increasing in A. It follows that it is larger when the high quality manufacturer faces a high quality fringe compared to a low quality fringe if and only if $c > \Delta$. Conversely, a high quality manufacturer has to distort its price to a lower extent when facing a high quality fringe compared to a low quality fringe if and only $c < \Delta$.

2.3.2 The set of pooling outcomes

We now turn to the characterization of pooling equilibria if they exist. Let p^I denote a pooling equilibrium price in the integrated structure. Since the price charged by the manufacturer is the same regardless of quality, buyers' posterior beliefs after observing this price are the same as their prior beliefs μ_0. Hence, the manufacturer earns $\pi_i^I(p^I, \mu_0)$ in equilibrium. To conceal information in equilibrium, any pooling price p^I must yield no less profit than what the manufacturer could get at best if product 0 were thought to be of low quality with certainty, that is:

$$\pi_i^I(p^I, \mu_0) \geq \pi_i^I(0), \text{ for } i = H, L. \tag{6}$$

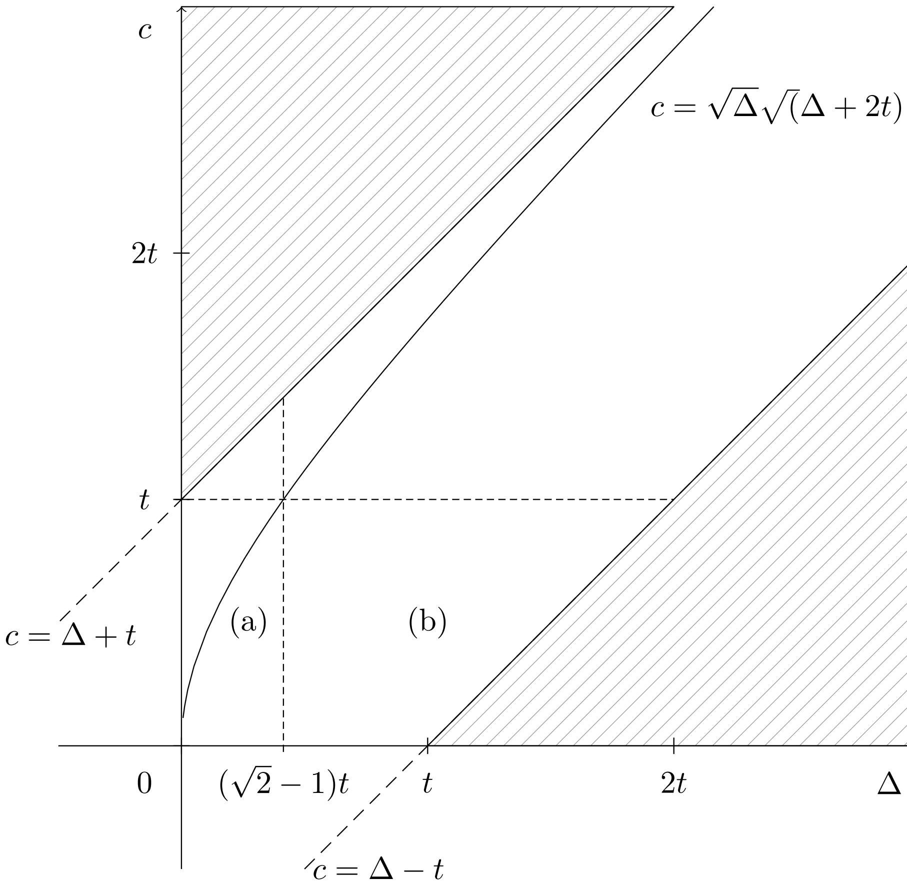

Figure 1: Areas (a) and (b) where the high quality separating price is upward distorted and when the fringe produces a low quality good

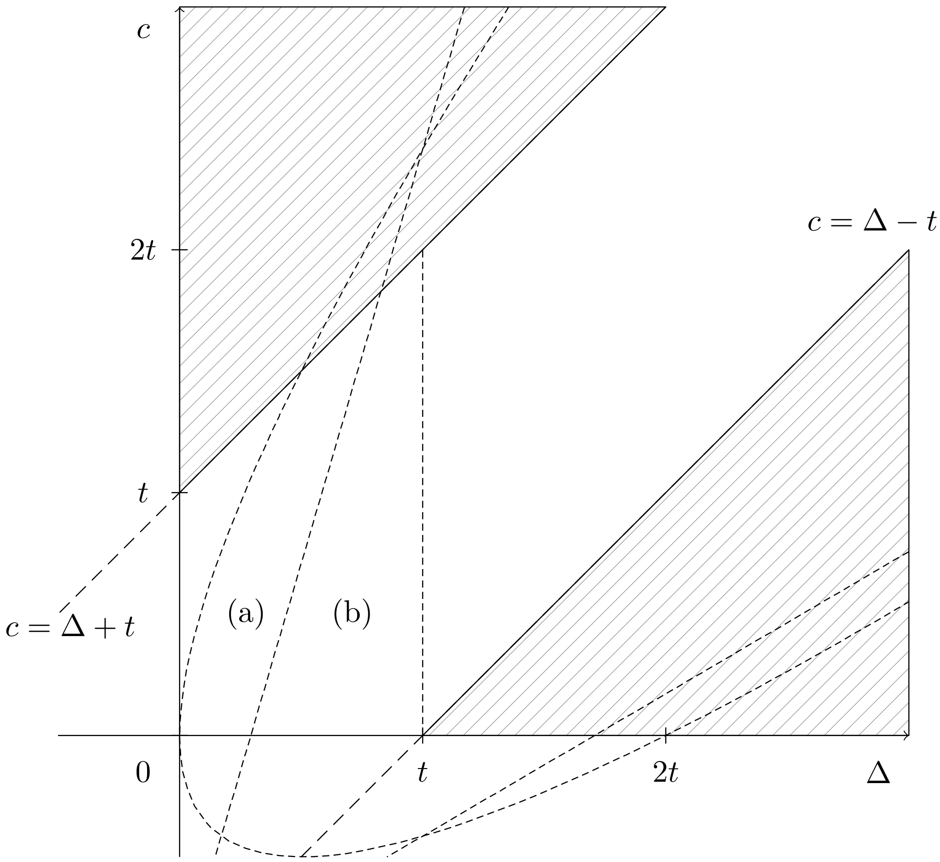

Figure 2: Areas (a) and (b) where the high quality separating price is upward distorted and when the fringe produces a high quality good

The set of prices p^I such that (6) holds is the set of pooling equilibrium prices.

It can easily be shown that any pooling equilibrium fails to survive the intuitive criterion. As a result, the unique equilibrium robust to the intuitive criterion is the least-cost separating one stated by Proposition 2. Surprisingly enough, this is the only equilibrium outcome to be "reasonable" in the sense of the intuitive criterion even if a pooling equilibrium may sometimes Pareto dominate the least-cost separating equilibrium, from the perspective of both types of manufacturer.

Let us determine the parameter configuration in which the latter situation occurs. As $\pi_L^I(p^I, 0) \geq \pi_L^I(0) = \pi_L^I(\hat{p}_L^I, 0)$ by definition of a pooling equilibrium, the low-quality manufacturer that chooses p^I will always make at least the profit secured when separating with $\hat{p}_L^I$.

The same is not always true for the high-quality manufacturer but happens to be true when the two following conditions are met. First, it must be that the price charged for the high-quality product is distorted upward relative to the full information situation, i. e., $p_H^I = \overline{p}(0)$, which arises when condition C holds. Otherwise, signalling quality would entail no cost and the high quality manufacturer would never benefit from pooling because $\max_{p^I} \pi_H^I(p^I, \mu_0) < \pi_H^I(1)$ as long as $\mu_0 < 1$. The second condition is that the high-quality manufacturer is indeed better off with the uninformative price p^I than with the separating price $\overline{p}(0)$:

$$\pi_H^I(p^I, \mu_0) \geq \pi_H^I(\overline{p}(0), 1). \tag{7}$$

Note that condition (7) holds for sufficiently high values of μ_0. Using $\pi_H^I(\overline{p}(0), 1) = (\overline{p}(0) - c)\left(\frac{2A+\Delta-\overline{p}(0)}{2t}\right)$ and $\max_{p^I} \pi_H^I(p^I, \mu_0) = \frac{1}{2t}\left(\frac{2A+\mu_0\Delta-c}{2}\right)^2$, we can define $\overline{\mu}$ as the unique probability μ_0 such that

$$(\overline{p}(0) - c)\left(\frac{2A + \Delta - \overline{p}(0)}{2t}\right) = \frac{1}{2t}\left(\frac{2A + \mu_0\Delta - c}{2}\right)^2. \tag{8}$$

The right-hand side of (8) is the maximum profit that the high-quality manufacturer can make by holding back information, when buyers believe the quality of product 0 to be high with probability μ_0. The critical value $\overline{\mu}$ is the probability level such that the high-quality manufacturer is indifferent between signaling quality with the upward-distorted price $\overline{p}(0)$ and concealing information about quality in the optimal way. For all μ_0 higher than $\overline{\mu}$, there exists at least one pooling equilibrium (actually an infinity of) that Pareto dominates the least-cost separating equilibrium regardless of the actual quality. Using (8), the explicit expression of $\overline{\mu}$ is given by:

$$\overline{\mu} = \frac{1}{\Delta}\left(c - 2A + 2\sqrt{A(A-c) + \frac{c}{2}\left(\sqrt{\Delta\left(4A+\Delta\right)} - \Delta\right)}\right).$$

3 The vertically decentralized structure: the case of delegation

We now consider a similar set-up but where the manufacturer delegates the task of distributing and pricing the product 0 on the market to one retailer R competing with the product 1 (passive) seller. Hence, it is now the retailer R that signals the product quality by choosing its price. Nevertheless, the decision of the retailer will be influenced via the procurement contract signed with the manufacturer. We consider in the following that the set of possible contracts is limited to the set of two-part tariffs. More precisely, product 0 is exchanged between M and R at a per-unit price w, while R is paying a franchise F to M. We also make two simplifying assumptions: i) M has all the bargaining power and hence proposes a take-it-or-leave-it offer (w, F) to R, ii) consumers know that the contract set is the set of two-part tariffs but do not observe the terms of the contract that remain private information shared by M and R. Hence, consumers only observe the final price set up by R. Finally, the rest of the notations and assumptions made in the vertically integrated structure case also holds in the vertically decentralized structure.

We investigate a four-period signaling game which proceeds as follows. In period one, Nature selects quality i from the set $\{H, L\}$ according to the commonly known probability distribution μ_0. In period two, both M and R learn the actual i and M makes a take-it-or-leave-it offer (w_i, F_i) to R. In period three, R accepts or refuses the manufacturer's offer, and, in case of acceptance, charges p. In period four, consumers observe p but not w_i nor F_i, and update their prior beliefs μ_0, thereby making their choice between products on the basis of this observation. Consumers' posterior beliefs will also be denoted by $\mu(p) : \mathbb{R}^+ \to [0,1]$ giving the probability weight consumers attach to the possibility that quality i is H after observing p.

In the spirit of subgame perfection, equilibrium is characterized by first looking for the consumer prices in the subgame $G(w, F)$ that starts after the manufacturer of either type has made an offer, $(w, F) = (w_H, F_H, w_L, F_L)$ being the pair of proposed franchise fees and prices. Lastly, the contract choices are determined.

The profits of the manufacturer and the retailer can be defined as functions of their price, given consumers' beliefs and the actual quality of product 0. The profits of M and R will be denoted respectively, for $i = H, L$:

$$\pi_i^M(w_i, F_i, p, \mu) = (w_i - c_i)D(p, \mu) + F_i \tag{9}$$

$$\pi_i(p, \mu) = (p - w_i)D(p, \mu) - F_i. \tag{10}$$

From the viewpoint of both M and R, $\mu = 0$ is the least favorable belief that consumers can hold, regardless of the actual quality. Indeed, for all $\mu > 0$, we have $\pi_i^M(w_i, p, \mu) > (w_i - c_i)D(p, 0) + F_i$ and $\pi_i(p, \mu) > (p - w_i)D(p, 0) - F_i$. As long as demand $D(p, \mu)$ is positive, we can compute the optimal price $p_i(\mu)$ which maximizes $\pi_i(p, \mu)$ with respect to p and we get:

$$p_i(\mu) = A + \frac{w_i + \mu\Delta}{2}. \tag{11}$$

At this price, demand is given by $D(p_i(\mu), \mu) = \frac{2A+\mu\Delta-w_i}{4t}$, which is positive for $w_i < 2A+\mu\Delta$. Hence, in the presence of perfectly informed consumers, the maximum wholesale price to be acceptable to the retailer is $w_H^{\max} = 2A + \Delta$ and $w_L^{\max} = 2A$ when quality is high and low, respectively.

For a positive demand (i.e. for $w_i < 2A + \mu\Delta$), the maximized profit $\pi_i(\mu)$ for a given belief μ is:

$$\pi_i(\mu) \equiv \pi_i(p_i(\mu), \mu) = \frac{1}{2t}\left(\frac{2A + \mu\Delta - w_i}{2}\right)^2 - F_i \tag{12}$$

3.1 Full information

The next proposition states the equilibrium outcome under full information. For this, we define $\varphi_i(w_i)$ as the retailers's optimal reaction to the wholesale price w_i proposed by the manufacturer when consumers are perfectly informed about quality, that is, $\varphi_H(w_H) = p_H(1)$ and $\varphi_L(w_L) = p_L(0)$. The following characterization of equilibrium provides the benchmark for the subsequent analysis of incomplete information.

Proposition 3 *The following strategies and outcomes constitute an equilibrium of the vertically decentralized structure under full information. The consumer prices $\widehat{p}_i$ and the retailers' profits $\widehat{\pi}_i$ are:*

$$\begin{aligned} \widehat{p}_L &= \varphi_L(\widehat{w}_L) = A \textit{ and } \widehat{\pi}_L = 0 \\ \widehat{p}_H &= \varphi_H(\widehat{w}_H) = A + \frac{\Delta + c}{2} \textit{ and } \widehat{\pi}_H = 0 \end{aligned}$$

The wholesale prices $\widehat{w}_i$, franchises $\hat{F}_i$ and corresponding profits $\hat{\pi}_i^M$ are:

$$\begin{aligned} \widehat{w}_L &= 0 \textit{ and } \widehat{\pi}_L^M = \hat{F}_L = \frac{A^2}{2t} \\ \widehat{w}_H &= c \textit{ and } \widehat{\pi}_H^M = \hat{F}_H = \frac{(2A + \Delta - c)^2}{8t}. \end{aligned}$$

Not surprisingly, this proposition shows that the equilibrium profit of the manufacturer is equal to the profit of the vertically integrated structure under complete information. Indeed, it is well known that, in our context, two-part tariffs are sufficient to eliminate any vertical externality due to double marginalization between the manufacturer and the retailer. Whatever the quality, the manufacturer simply fixes wholesale price to marginal cost which induces the retailer to set final price to its optimal level under the vertically integrated structure and then collects the total profit via the franchise fee, leaving zero rents to the retailer.

3.2 Asymmetric information

Under asymmetric information, the quality of the product is supposed known to both the manufacturer and the retailer, but not to the consumers. We start by defining the equilibrium concept.

3.3 The equilibrium concept

A perfect Bayesian equilibrium of the whole game is a set of price strategies $\{(w_i^*, F_i^*)_{i=H,L}, (p_i^*)_{i=H,L})\}$ and beliefs $\mu^*(p)$ such that, at any period of the game, strategies must be optimal given that the following player, if any, responds optimally, and given consumers' beliefs. Formally, the optimality condition for M requires that, for each $i = H, L$,

$$\begin{aligned} w_i^*, F_i^* &\in \arg\max_{w_i, F_i} \pi_i^M (w_i, F_i, p_i^*, \mu^*(p_i^*)) \\ \text{s.t. } \pi_i (p_i^*, \mu^*(p^*)) &\geq 0. \end{aligned} \tag{FC$_i$}$$

The feasibility constraint (FC$_i$) means that the retailer of type i should accept the contract.[2]

The perfection condition for R is

$$p_i^* \in \arg\max_{p_i} \pi_i (p_i, \mu^*(p)) \tag{13}$$

Bayes' consistency of beliefs demands that consumers form posterior beliefs about q from observing prices only by using Bayes' rule out of equilibrium. It follows that

$$\begin{aligned} \text{if } p_H^* &\neq p_L^*, \text{ then } \mu^*(p_H^*) = 1 \text{ and } \mu^*(p_L^*) = 0, \\ \text{and if } p_H^* &= p_L^*, \text{ then } \mu^*(p_H^*) = \mu^*(p_L^*) = \mu_0. \end{aligned} \tag{14}$$

3.4 Characterizing the set of outcomes in $G(w, F)$

3.4.1 The set of separating outcomes in $G(w, F)$

In this section, we first characterize the set of separating equilibria if any, for a given pair of wholesale prices and franchise fees. Consider a potential separating equilibrium

[2] In equilibrium, we expect the manufacturer of any type to always propose a contract that meets the feasibility constraints. Suppose on the contrary that in equilibrium the contract offered by one type of M is not accepted; then, this type of M can always secure a positive profit with an acceptable contract simply by reducing her franchise without affecting the relationship between R and M of the other type.

$(p_H^*(w,F), p_L^*(w,F))$ of the subgame $G(w,F)$. Without loss of generality, let consumers' beliefs be the least favorable ones out of equilibrium from the retailer's point of view, i. e., $\mu(p) = 0$ for all $p \notin \{p_H^*(w,F), p_L^*(w,F)\}$. Such beliefs will generate all of the possible perfect Bayesian equilibrium paths. Indeed, if the retailer of any type does not have an incentive to charge p when $\mu(p) \neq 0$, then he will not have an incentive when $\mu(p) = 0$, since his profit is lower.

Moreover, considering $G(w,F)$ implicitly means that the contract has been accepted by both types of R. Hence, to achieve separation the two following feasibility constraints must hold:

$$\pi_H(p_H^*(w,F), 1) \geq 0 \tag{FC$_H$}$$

$$\pi_L(p_L^*(w,F), 0) \geq 0 \tag{FC$_L$}$$

In what follows, we assume that these constraints hold and and we will check both of them once separating equilibria are characterized.

We next introduce the two individual rationality constraints. A type H retailer who is clearly identified as a high-quality provider by consumers must find profitable to choose $p_H^*(w,F)$ instead of any other price that fool consumers:

$$\pi_H(p_H^*(w,F), 1) \geq \pi_H(0). \tag{IR$_H$}$$

Similarly, the individual rationality constraint for a type L retailer is:

$$\pi_L(p_L^*(w,F), 0) \geq \pi_L(0) \tag{IR$_L$}$$

which means that $p_L^*(w,F)$ must be an optimal strategy when consumers correctly identify the product as being a low quality one.

Observe that the constraint (IR$_L$) is necessarily binding so that $p_L^*(w,F)$ is the profit-maximizing price for a type L retailer under full information for a given w. This yields our first result.

Lemma 4 *For any contract such that $w_L \leq 2A$ and $F_L \leq \left(\frac{2A-w_L}{2}\right)^2/2t$, the retailer chooses $p_L^*(w,F) = \varphi_L(w_L)$ to separate low quality in equilibrium.*

As the type L retailer's strategy in the separating equilibrium is the same as under full information, such is also the case for the resulting demand $D(p_L^*(w,F),0) = \frac{A-w_L/2}{2t}$ for a given w_L. Hence, to secure a positive market share, the type L retailer will not accept a price higher than $w_L^{\max} = 2A$ from the manufacturer. In addition, to meet the feasibility constraint (FC_L the franchise fee must not be too high for a given w_L, that is:

$$F_L \leq \left(\frac{2A-w_L}{2}\right)^2/2t. \tag{FC$_L$}$$

To lighten notation, $p_i^*(w,F)$ will be written p_i^* from now on.

We now turn to the incentive compatibility constraints. First, the type H retailer must not find profitable to mimick the equilibrium price chosen by the type L retailer, that is:

$$\pi_H(p_H^*,1) \geq \pi_H(p_L^*,0). \tag{IC$_H$}$$

Second, the type L retailer must not find profitable to behave as if he were the type H retailer, that is:

$$\pi_L(p_L^*,0) \geq \pi_L(p_H^*,1). \tag{IC$_L$}$$

Remark that franchise fees do not impact directly incentive compatibility constraints: only the wholesale prices matter. Note also that the individual rationality constraint (IR$_H$) for a type H retailer implies the corresponding incentive compatibility constraint (IC$_H$). We are thus left with the two constraints (IR$_H$) and (IC$_L$), together with the fact that $p_L^* = \varphi_L(w_L)$, and so we are looking for the set of price p_H^* such that these constraints hold simultaneously.[3]

Proposition 5 *Consider a given pair of wholesale prices (w_H, w_L) such that $w_H \in [c, w_H^{\max}]$ and $w_L \in [0, w_L^{\max}]$. For all $F_H \leq \check{F}_H(w_H,w_L)$ together with $w_H \leq w_L$ and all $F_H \leq \hat{F}_H(w_H,w_L)$ together with $w_L \leq w_H$, there exists a pair of separating equilibrium prices $(p_H^*(w,F), p_L^*(w,F))$ that signals the true quality, with $p_L^*(w,F) = \varphi_L(w_L)$.*

[3] As previously mentioned, we will check that the feasibility constraints (FC$_H$) and (FC$_L$) hold for the characterized prices.

(i) *If $w_H \leq w_L$, the price $p_H^*(w,F)$ may be distorted downward relative to the full information price $\varphi_H(w_H)$ and belongs to the interval S_d defined by:*

$$S_d = \begin{cases} \left[\underline{p}_2(w_H,F_H), \underline{p}_1(w_L)\right] & \text{when } \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \leq F_H \leq \check{F}_H(w_H,w_L), \\ \left[\underline{p}_1(w_H), \underline{p}_1(w_L)\right] & \text{when } F_H \leq \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2. \end{cases}$$

(ii) *If $w_L \leq w_H$, the price $p_H^*(w,F)$ may be distorted upward relative to the full information price $\varphi_H(w_H)$ and belongs to the interval S_u defined by:*

$$S_u = \begin{cases} [\overline{p}_1(w_L), \overline{p}_2(w_H,F_H)] & \text{when } \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \mathbf{1}_{w_H \leq 2A} \leq F_H \leq \hat{F}_H(w_H,w_L), \\ [\overline{p}_1(w_L), \overline{p}_1(w_H)] & \text{when } 0 \leq F_H \leq \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \text{ or } F_H \leq 0 \leq \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2, \\ [\overline{p}_1(w_L), w_H^{\max}] & \text{when } F_H \leq \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \leq 0. \end{cases}$$

Proof: See Appendix *C*. ■

We also obtain straightforwardly the following Corollary.

Corollary 6 *The least cost separating price that signals quality H is $p_H^{**}(w,F)$ such that*

$$p_H^{**}(w,F) = \begin{cases} \min\left\{\underline{p}_1(w_L), \varphi_H(w_H)\right\} & \text{if } w_H \leq w_L, \\ \max\left\{\overline{p}_1(w_L), \varphi_H(w_H)\right\} & \text{if } w_L \leq w_H. \end{cases}$$

The insight from Corollary 6 is twofold. First, to signal high quality, the retailer may distort the consumer price relative to the full information situation, either downward ($p_H^{**}(w,F) = \underline{p}_1(w_L)$) or upward ($p_H^{**}(w,F) = \overline{p}_1(w_L)$) depending on whether the wholesale price for high quality falls short or not of the wholesale price for low quality. Second, to the extent that $p_H^{**}(w,F)$ is biased away from $\varphi_H(w_H)$, $p_H^{**}(w,F)$ becomes insensitive to changes in w_H as shown by the expressions of $\underline{p}_1(w_L)$ and $\overline{p}_1(w_L)$ given by (16) and (17) respectively.

The sense of the signaling distortion necessary to signal high quality to consumers, if any, is determined backward by the manufacturer. If the wholesale price discriminates on behalf of low (resp. high) quality, i.e., $w_L \leq w_H$, the manufacturer may induce the retailer to signal high quality with a price higher (resp. lower) than what would prevail under full information. Setting a lower wholesale price to either quality amounts to reduce the retailer's cost of selling this quality. When the retailer considers changing consumer price, he has to take into account a direct and an indirect effects on his profit, which play in opposite directions. First, the retailer directly modifies his gross revenue at the current sale. But second, the retailer changes the demand for his product by making consumers switch from one product to another, thereby modifying indirectly the profit from sales (net from the wholesale price). Thus, a downward deviation from the full information price both reduces the gross revenue and increases the net profit by boosting demand, whereas an upward deviation has the converse effects: it raises the gross revenue but entails a loss in profit due to business switching to the competitive fringe. If the wholesale price for the high quality is lower than that for the low quality, the type H retailer has more incentive than the type L to distort downward the consumer price because business switching toward the retailer's product is more profitable to the high quality due to less expensive sales. Conversely, if the type H retailer must pay a wholesale price higher than that proposed to the type L retailer, distorting upward the consumer price can successfully signal high quality because the global effect of the foregone profit from adverse business switching on one hand, and the higher gain from increased price on the other hand, is less damaging to the type H retailer, but also his gain from increased price is higher.

To get further insight on the impact of the wholesale price choice on the signaling cost entailed by the consumer price distortions, we define $\beta(w_H, w_L)$ as the minimum price bias in equilibrium relative to the full information price $\varphi(w_H)$.

Corollary 7 *In a separating equilibrium, the lowest cost of quality signaling through price is measured by*

$$\beta(w_H, w_L) = \begin{cases} 0 \text{ } \textit{if } 0 \leq w_H \leq w_L - \sqrt{d(w_L)}, \\ \varphi_H(w_H) - \underline{p}_1(w_L) = \left(w_H - w_L + \sqrt{d(w_L)}\right)/2 \text{ } \textit{if } w_L - \sqrt{d(w_L)} < w_H \leq w_L, \\ \overline{p}_1(w_L) - \varphi_H(w_H) = \left(w_L - w_H + \sqrt{d(w_L)}\right)/2 \text{ } \textit{if } w_L \leq w_H < w_L + \sqrt{d(w_L)}, \\ 0 \text{ } \textit{if } w_L + \sqrt{d(w_L)} \leq w_H. \end{cases}$$

For a given w_L, $\beta(w_H, w_L)$ can be read as a function of w_H that is zero when the gap between w_H and w_L is high, and single-peaked in $w_H = w_L$. This shows, first, that *ceteris paribus* the lowest cost of signaling is maximum when w_H and w_L are alike, and second, that the manufacturer is likely to reduce this cost to nothing, either by decreasing or increasing w_H away from w_L. As $D\left(p_H^{**}\left(w,F\right),1\right)$ is insensitive to changes in w_H when $p_H^{**}\left(w,F\right)$ is either $\underline{p}_1(w_L)$ or $\overline{p}_1(w_L)$, we have that $\pi_H^M\left(w_H,F_H,p_H^{**}\left(w,F\right),1\right)=(w_H-c_H)D\left(p_H^{**}\left(w,F\right),1\right)+F_H$ is an increasing function of w_H when signaling high quality to consumers is costly. Thus, for a given F_H, the manufacturer has an incentive to raise w_H when he expects signaling to be costly.

Note also that higher degrees of product differentiation, either horizontal or vertical, entail larger costs of signaling since $d(w_L)=\Delta\left(4A+\Delta-2w_L\right)$ (by (18)) increases both with t and Δ. Signaling high quality through consumer price is more difficult when products are more differentiated.

3.4.2 The set of pooling outcomes in $G(w,F)$

In this section, we characterize the set of pooling equilibria if any, for a given pair of wholesale prices w and franchise fees F. Consider a potential pooling price $p^*\left(w,F\right)$ of the subgame $G\left(w,F\right)$ such that $\mu^*(p^*(w,F))=\mu_0$. Without loss of generality, let consumers' beliefs be the least favorable ones out of equilibrium from the retailer's point of view, i. e., $\mu(p)=0$ for all $p\neq p^*\left(w,F\right)$. As for the separating prices case, feasibility and individual rationality constraints should hold for a potential pooling price, that is:

$$\pi_H(p^*\left(w,F\right),\mu_0)\geq 0 \qquad (\text{FC}_H)$$

$$\pi_L(p^*\left(w,F\right),\mu_0)\geq 0 \qquad (\text{FC}_L)$$

and

$$\pi_H(p^*\left(w,F\right),\mu_0)\geq \pi_H(0) \qquad (\text{IR}_H)$$

$$\pi_L(p^*\left(w,F\right),\mu_0)\geq \pi_L(0) \qquad (\text{IR}_L)$$

The next Proposition characterizes the set of pooling equilibrium.

Proposition 8 *Consider a given pair of wholesale prices (w_H,w_L) such that $w_H\in[c,w_H^{\max}]$ and $w_L\in[0,w_L^{\max}]$. For all $F_H\leq\check{F}_H(w_H,w_L,\mu_0)$ such that $w_H\leq w_L$ and all $F_H\leq$*

$\hat{F}_H(w_H, w_L, \mu_0)$ *such that* $w_L \leq w_H$, *there exists a continuum of pooling equilibrium prices* $p^*(w, F)$ *that conceal information about quality.*

(i) *If* $w_H \leq w_L$, *the price* $p^*(w, F)$ *belongs to the interval* P_d *such that:*

$$P_d = \begin{cases} \left[\max\left\{\underline{p}_2(w_H, F_H, \mu_0), \underline{p}_1(w_L, \mu_0)\right\}, \overline{p}_2(w_H, F_H, \mu_0)\right] \\ \text{when } \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \leq F_H \leq \check{F}_H(w_H, w_L, \mu_0), \\ \left[\underline{p}_1(w_L, \mu_0), \overline{p}_1(w_H, \mu_0)\right] \text{ when } F_H \leq \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2. \end{cases}$$

(ii) *If* $w_L \leq w_H$, *the price* $p^*(w, F)$ *belongs to the interval* P_u *such that:*

$$P_u = \begin{cases} \left[\underline{p}_2(w_H, F_H, \mu_0), \min\left\{\overline{p}_1(w_L, \mu_0), \overline{p}_2(w_H, F_H, \mu_0)\right\}\right] \\ \text{when } \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \mathbf{1}_{2A>w_H} \leq F_H \leq \hat{F}_H(w_H, w_L, \mu_0), \\ \left[\underline{p}_1(w_H, \mu_0), \overline{p}_1(w_L, \mu_0)\right] \\ \text{when } 0 \leq F_H \leq \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \text{ or } F_H \leq 0 \leq \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2, \\ \left[\max\left\{w_H, \underline{p}_1(w_L, \mu_0)\right\}, \overline{p}_1(w_L, \mu_0)\right] \text{ when } F_H \leq \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \leq 0. \end{cases}$$

Proof: See Appendix D. ■

3.5 The optimal contract

We now examine the contract chosen by the manufacturer. The manufacturer follows the same logic as under full information in that he sets wholesale prices equal to marginal costs and, at the same time, uses the franchise fees to fully extract the retailer's rent. The novel insight is that the contract ties the retailer's hands in a context of asymmetric information, which yields a unique price signal on the marketplace.

Proposition 9 *Optimal contracts make it possible for the retailer to commit on a unique final price for each quality level.*

(i) *If* $\mu_0 \leq \overline{\mu}$, *there is a unique optimal contract that achieves separation with the pair of final prices robust to the intuitive criterion, i. e.,* $(p_L^*, p_H^*) = \left(p_L^I, p_H^I\right)$. *This contract involves the following wholesale prices and franchises:*

$$w_L^* = 0, F_L^* = \widehat{\pi}_L^M$$

and

$$w_H^* = c,\ F_H^* = \begin{cases} \pi_H^I(\overline{p}(0), 1) \text{ when condition } C \text{ holds} \\ \widehat{\pi}_H^M \text{ otherwise.} \end{cases}$$

(ii) *If $\mu_0 > \overline{\mu}$, there is a multiplicity of optimal contracts that conceal information with the following wholesale prices and franchises:*

$$w_L^* = 0,\ \ F_L^* \in \left[\pi_L^I(p_H^I(\mu_0), \mu_0), \pi_L^I(\mu_0)\right]$$

and

$$w_H^* = c,\ \ F_H^* \in \left[\pi_H^I(p_L^I(\mu_0), \mu_0), \pi_H^I(\mu_0)\right].$$

Optimal contracts induce the retailer to set the pooling price $p^* = \underline{p}_2(c, F_H^*, \mu_0) = \overline{p}_2(0, F_L^*, \mu_0)$.

4 Conclusion

to be completed.

Appendix

A Proof of Proposition 2

First of all, note that clearly (IR_L) is binding since $\pi_L^I(0) = \max_p \pi_L^I(p,0)$, hence $p_L^I = \widehat{p}_L^I = A$ and thereby signaling the low quality entails no price distortion relative to the full information situation. Moreover, examining the two constraints (IR_H) and (IC_H) reveals that if the former inequality holds then the latter also holds because $\pi_L^I(0) > \pi_H^I(p_L^I, 0)$ for all $p_L^I \neq p_H^I(0)$. It follows that the manufacturer has only to take into account the two following constraints, (IR_H) and (IC_L) that writes as follows:

$$(p_H^I - c)\left(\frac{2A + \Delta - p_H^I}{2t}\right) \geq \frac{1}{2t}\left(A - \frac{c}{2}\right)^2 \mathbf{1}_{2A>c} \tag{IR_H}$$

and

$$\frac{A^2}{2t} \geq p_H^I\left(\frac{2A + \Delta - p_H^I}{2t}\right). \tag{IC_L}$$

Let P_L and P_H denote the sets of p_H^I for which respectively (IC_L) and (IR_H) hold. Thus, $P_L \cap P_H$ contains all the prices that truthfully signal high quality. To characterize this set, it is useful to define the function

$$f(p,c) = (p - c)(2A + \Delta - p)/2t - \frac{1}{2t}\left(A - \frac{c}{2}\right)^2 \mathbf{1}_{2A>c}$$

It follows that the twofold constraint of satisfying simultaneously (IR_H) and (IC_L) is tantamount to requiring that p_H^I solves

$$f(p,c) \geq 0 \geq f(p,0).$$

Consider first that $c < 2A$. Then, demand is positive no matter what consumers believe about quality and we can write $f(p,c) = -p^2 + p(2A + \Delta + c) - A^2 - (A + \Delta)c - \frac{c^2}{4}$. We define $d(c) = \Delta(4A + \Delta - 2c)$ and look for the prices that solve $f(p,c) = 0$. This equation has two real solutions in p if and only if $d(c) \geq 0$, which holds as long as $c < 2A + \frac{\Delta}{2}$. The two roots in p of $f(p,c) = 0$ have the following expressions: $\underline{p}(c) = \left(2A + \Delta + c - \sqrt{d(c)}\right)/2$ and $\overline{p}(c) = \left(2A + \Delta + c + \sqrt{d(c)}\right)/2$. These roots are differentiable with respect to c and the derivatives are $\underline{p}'(c) = 1 + \Delta/\sqrt{d(c)}$ and $\overline{p}'(c) = 1 - \Delta/\sqrt{d(c)}$. It is straightforward that $\underline{p}'(c) = 1 + \Delta/\sqrt{d(c)} > 0$. Moreover, $\overline{p}'(c) > 0$ if and only if $d(c) \geq \Delta^2$. It can be checked that the latter inequality is equivalent to $2A - c > 0$. Thus, when $c < 2A$, we have that $\underline{p}(0) < \underline{p}(c)$ and $\overline{p}(0) < \overline{p}(c)$. It follows that $P_L \cap P_H = [\overline{p}(0), \overline{p}(c)]$ and, when $\overline{p}(0) > \widehat{p}_H^I$, the price $\overline{p}(0)$ is the least-costly price among all the separating equilibrium prices that signal high quality. Easy computations show that $\overline{p}(0) - \widehat{p}_H^I = \left(\sqrt{\Delta\left(4A + \Delta\right)} - c\right)/2$, hence $\overline{p}(0) > \widehat{p}_H^I \iff c < \min\left\{2A, \sqrt{\Delta\left(4A + \Delta\right)}\right\}$. Whenever the difference $\overline{p}(0) - \widehat{p}_H^I$ is positive, it represents an upward distortion in price relative to the full information case, which measures the cost of signaling high quality. In that case, the equilibrium price $\overline{p}(0)$ is the only one to survive selection by the intuitive criterion.

Assuming that $A \leq \Delta$, we now turn to the parameter configuration such that $c \in [2A, A + \Delta]$. Demand is nil when the quality of product 0 is believed to be low for sure, and so $f(p, c) = (p - c)\left(2A + \Delta - p\right)/2t$. Solutions in p of equation $f(p, c) = 0$ are then given by $\underline{p}(c) = c$ and $\overline{p}(c) = 2A + \Delta$. First, it turns out that $\underline{p}(0) = \left(2A + \Delta - \sqrt{\Delta\left(4A + \Delta\right)}\right)/2 < 2A$ for all $A \leq \Delta$; as moreover $2A \leq c = \underline{p}(c)$, we get that $\underline{p}(0) < \underline{p}(c)$. Moreover, easy computations yield that $\overline{p}(0) = \left(2A + \Delta + \sqrt{\Delta\left(4A + \Delta\right)}\right)/2 < \overline{p}(c)$. Thus, for any $c \in [2A, A + \Delta]$, we also have $P_L \cap P_H = [\overline{p}(0), \overline{p}(c)]$ and, when $\overline{p}(0) > \widehat{p}_H^I$, the lowest separating price $\overline{p}(0)$ signals high quality in the unique separating equilibrium robust to the intuitive criterion.

B Characterization of condition C

Condition C writes:

$$c < \min\left\{2A, \sqrt{\Delta\left(4A + \Delta\right)}\right\}.$$

- Assume first that $\alpha = 0$. Then $2A = t$. It follows that:

$$\begin{aligned}\sqrt{\Delta\left(4A+\Delta\right)} &\lesseqgtr 2A \Leftrightarrow \sqrt{\Delta\left(2t+\Delta\right)} \lesseqgtr t \\ &\Rightarrow \Delta\left(2t+\Delta\right) \lesseqgtr t^2 \\ &\Rightarrow \left(\Delta+(1-\sqrt{2})t\right)\left(\Delta+(1+\sqrt{2})t\right) \lesseqgtr 0.\end{aligned}$$

  Hence, whenever $\Delta + (1-\sqrt{2})t > 0$, then condition C reduces to $c < 2A = t$. The corresponding area is area (b) in Figure 1. Conversely, when $\Delta + (1-\sqrt{2})t < 0$ then condition C reduces to $c < \sqrt{\Delta\left(2t+\Delta\right)}$ and yields to area (a) in Figure 1.

- Second assume that $\alpha = 1$. Then $2A = t + c - \Delta$. We have:

$$\begin{aligned}\sqrt{\Delta\left(4A+\Delta\right)} &\lesseqgtr 2A \Leftrightarrow \sqrt{-\Delta^2+2\Delta\left(t+c\right)} \lesseqgtr t+c-\Delta \\ &\Rightarrow -\Delta^2+2\Delta\left(t+c\right) \lesseqgtr \left(t+c-\Delta\right)^2 \\ &\Rightarrow -2\Delta^2+(t+c)(4\Delta-t-c) \lesseqgtr 0.\end{aligned}$$

  The quadratic form $P(\Delta, c) = -2\Delta^2 + (t+c)(4\Delta - t - c)$ is actually a set of two lines crossing in $(0, -t)$, a particular case of a conic. Whenever $P(\Delta, c) > 0$, then condition C reduces to $c < 2A = t + c - \Delta$, that is $\Delta < t$. The corresponding area is area (b) in Figure 2. Conversely, when $P(\Delta, c) > 0 < 0$ then condition C reduces to $c < \sqrt{-\Delta^2 + 2\Delta\left(t+c\right)}$ or equivalently to $(\Delta - c)^2 < 2\Delta t$ (the frontier of this domain is represented by a parabola) and yields to area (a) in Figure 2.

C Proof of Proposition 5

Let us first examine mimicry deterrence under (IC_L). Define the set of p_H^* which solve (IC_L) as P_L. Under the restriction $w_L \leq w_L^{\max}$, the constraint (IC_L) can be rewritten

$$\left(p_H^* - w_L\right)\left(2A + \Delta - p_H^*\right)/2t \leq \left(\frac{2A - w_L}{2}\right)^2 /2t. \tag{15}$$

Thus, $P_L = \left\{p/p \leq \underline{p}_1(w_L) \text{ or } p \geq \overline{p}_1(w_L)\right\}$ where $\underline{p}_1(w_L)$ and $\overline{p}_1(w_L)$ are respectively the lower and upper thresholds of mimicry deterrence for the L type. These thresholds can be explicitely solved as

$$\underline{p}_1(w_L) = \left(2A + \Delta + w_L - \sqrt{d(w_L)}\right)/2, \quad (16)$$

$$\overline{p}_1(w_L) = \left(2A + \Delta + w_L + \sqrt{d(w_L)}\right)/2, \quad (17)$$

$$\text{with } d(w_L) = \Delta\,(4A + \Delta - 2w_L)\,. \quad (18)$$

Note that $d(w_L) > 0$ because $w_L \leq w_L^{\max}$.

Let us now turn to the two constraints (IR_H) and (FC_H). Define the set of p_H^* which solve (IR_H) as P_{1H}, i. e., $P_{1H} = \left\{p/\underline{p}_1(w_H) \leq p \leq \overline{p}_1(w_H)\right\}$ and let $P_{2H} = \left\{p/\underline{p}_2(w_H, F_H) \leq p \leq \overline{p}_2(w_H, F_H)\right\}$ be the set of p_H^* which solve (FC_H). It is useful to define $P_H \equiv P_{1H} \cap P_{2H}$. Then, we have two cases: (i) $\pi_H(0) \geq 0$, or equivalently $F_H \leq \max\left\{0, \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2\right\}$, so that ($\mathrm{IR}_H$) implies ($\mathrm{FC}_H$) and thus $P_H = P_{1H}$, and (ii) $\pi_H(0) < 0$ so that (IR_H) is irrelevant and thus $P_H = P_{2H}$.

Recall that the maximum wholesale price for the demand to be positive, when a type H retailer is perfectly identified, is $w_H^{\max} = 2A + \Delta$. In addition, when the retailer is believed to certainly have a low-quality product, demand is positive for all $w_H \in (c, 2A)$, and nil for all $w_H \in [2A, w_H^{\max}]$.

C.1 The case where $F_H \leq \max\left\{0, \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2\right\}$

Consider first the case where $F_H \leq \max\left\{0, \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2\right\}$ so that $P_H = P_{1H}$. Two subcases must be distinguished depending on whether w_H is lower or higher than $2A$.

- If $w_H \leq 2A$, then $\pi_H(0) = \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2$

For any $w_H \leq 2A$, (IR_H) can be rewritten as

$$(p_H^* - w_H)\,(2A + \Delta - p_H^*)\,/2t \geq \left(\frac{2A - w_H}{2}\right)^2/2t \quad (19)$$

Let us denote

$$f(p, w) = (p - w)\,(2A + \Delta - p)\,/2t - \left(A - \frac{w}{2}\right)^2/2t \quad (20)$$

It follows that the twofold constraint of satisfying simultaneously (19) and (15) is tantamount to requiring that p_H^* solves

$$f(p, w_H) \geq 0 \geq f(p, w_L) \tag{21}$$

in p for all pair of wholesale prices $(w_H, w_L) \in [c, 2A] \times [0, w_L^{\max}]$.

Under the restriction that $w \leq 2A$, equation $f(p, w) = 0$ can be written

$$-p^2 + p(2A + \Delta + w) - A^2 - (A + \Delta)w - \frac{w^2}{4} = 0. \tag{22}$$

Defining $d(w) = \Delta\,(4A + \Delta - 2w)$, there is a solution to (22) if and only if $d(w) \geq 0$, which holds as long as $w < 2A + \frac{\Delta}{2}$.Then, equation (22) has two solutions, namely the lower root $\underline{p}_1(w) = \left(2A + \Delta + w - \sqrt{d(w)}\right)/2$ and the upper root $\overline{p}_1(w) = \left(2A + \Delta + w + \sqrt{d(w)}\right)/2$. These roots are differentiable with respect to w and the derivatives are $\underline{p}_1'(w) = 1 + \Delta/\sqrt{d(w)}$ and $\overline{p}_1'(w) = 1 - \Delta/\sqrt{d(w)}$. Clearly, $\underline{p}_1'(w) = 1 + \Delta/\sqrt{d(w)} > 0$. In addition, straightforward calculations yield that $\overline{p}_1'(w) > 0$ if and only if $2A - w > 0$.

We can summarize the previous discussion as follows:

(i) $w_H < w_L \leq 2A$ implies $\underline{p}_1(w_H) < \underline{p}_1(w_L)$ and $\overline{p}_1(w_H) < \overline{p}_1(w_L)$ Thus, $P_L \cap P_H = \left[\underline{p}_1(w_H), \underline{p}_1(w_L)\right]$ and, when $\underline{p}_1(w_L) > \varphi_H(w_H)$, then the full information price belongs to the set of separating prices.

(ii) $w_L < w_H \leq 2A$ implies $\underline{p}_1(w_L) < \underline{p}_1(w_H)$ and $\overline{p}_1(w_L) < \overline{p}_1(w_H)$. Thus, $P_L \cap P_H = [\overline{p}_1(w_L), \overline{p}_1(w_H)]$ and, when $\overline{p}_1(w_L) < \varphi_H(w_H)$, then the full information price belongs to the set of separating prices.

Note that these results do not depend on whether F_H is positive or negative. Lastly, we have obtained the following expressions from the analysis of (19)

$$\underline{p}_1(w_H) = \begin{cases} \left(2A + \Delta + w_H - \sqrt{d(w_H)}\right)/2 & \text{if } w_H \in \left(c, 2A + \frac{\Delta}{2}\right), \\ w_H & \text{if } w_H \in \left[2A + \frac{\Delta}{2}, w_H^{\max}\right], \end{cases} \tag{23}$$

$$\overline{p}_1(w_H) = \begin{cases} \left(2A + \Delta + w_H + \sqrt{d(w_H)}\right)/2 & \text{if } w_H \in \left(c, 2A + \frac{\Delta}{2}\right), \\ 2A + \Delta & \text{if } w_H \in \left[2A + \frac{\Delta}{2}, w_H^{\max}\right], \end{cases} \tag{24}$$

$$\text{with } d(w_H) = \Delta\,(4A + \Delta - 2w_H). \tag{25}$$

- If $w_H \in [2A, w_H^{\max}]$, then $\pi_H(0) = 0$ and $F_H \leq 0$.

When the retailer is believed to certainly have a low-quality product, demand is nil for all $w_H \in [2A, w_H^{\max}]$. The equality version of constraint (IR$_H$) turns to $(p - w_H)(2A + \Delta - p)/2t = 0$ which admits w_H and $2A + \Delta$ as solutions. First, we show that $\underline{p}_1(w_L) \leq 2A$ for all $w_L \leq 2A$. We define $f(w_L) = 2A - \underline{p}(w_L)$. Using (16), we obtain $f(2A) = 0$ and $f'(w_L) = -\frac{1+\Delta/\sqrt{d(w_L)}}{2} < 0$, thus $f(w_L) \geq 0$ for all $w_L \leq 2A$. It follows that $\underline{p}_1(w_L) \leq 2A < w_H$. Moreover, easy calculations yield that $\overline{p}_1(w_L) < 2A + \Delta$ for all $w_L \leq 2A$. Thus, in this case, we have $P_L \cap P_H = [\overline{p}_1(w_L), 2A + \Delta]$.

C.2 The case where $F_H > \max\left\{0, \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2\right\}$

Consider now the second case where $F_H > \max\left\{0, \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2\right\}$, so that $P_H = P_{2H}$. For any $w_H \leq w_H^{\max}$, the feasibility constraint (FC$_H$) can be rewritten as

$$(p_H^* - w_H)(2A + \Delta - p_H^*)/2t \geq F_H \tag{26}$$

Define $\tilde{d}(w_H, F_H) = (2A + \Delta - w_H)^2 - 8tF_H$. There is a solution to (26) if and only if $\tilde{d}(w_H, F_H) \geq 0$, or equivalently, $F_H \leq \frac{(2A+\Delta-w_H)^2}{8t}$.Then, (26) admits two roots denoted by $\underline{p}_2(w_H, F_H)$ and $\overline{p}_2(w_H, F_H)$.

Consider first that $(w_H, w_L) \in [c, 2A] \times [0, w_L^{\max}]$ so that $\frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2 \geq 0$.

As previously seen for this parameter configuration, there exists two real roots $\underline{p}_1(w_H)$ and $\overline{p}_1(w_H)$ respectively given by (23) and (24). As $F_H > \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2$, we moreover have

$$\begin{aligned} \underline{p}_1(w_H) &< \underline{p}_2(w_H, F_H) \text{ and} \\ \overline{p}_1(w_H) &> \overline{p}_2(w_H, F_H) \end{aligned}$$

We can conclude that, for these values of w_H and w_L, the existence of separating equilibria only depends on the magnitude of F_H.

Let us now turn to the case where $(w_H, w_L) \in [2A, w_H^{\max}] \times [0, w_L^{\max}]$.

Then, we have $\max\left\{0, \frac{1}{2t}\left(\frac{2A-w_H}{2}\right)^2\right\} = 0$. As previously seen, solutions in p of the constraint (IR$_H$) taken as an equality are given by w_H and $2A + \Delta$. Furthermore, we know that $\underline{p}_1(w_L) \leq w_H$ for all $w_L \leq 2A$, and $\overline{p}_1(w_L) < 2A + \Delta$ for all $w_L \leq 2A$. Since $F_H > 0$, we have $w_H < \underline{p}_2(w_H, F_H)$ and $2A + \Delta > \overline{p}_2(w_H, F_H)$. Thus, in this case, we have $P_L \cap P_H = [\overline{p}_1(w_L), \overline{p}_2(w_H, F_H)]$ unless F_H is too high.

To sum up: for all $F_H > \max\left\{0, \frac{1}{2t}\left(\frac{2A - w_H}{2}\right)^2\right\}$, we have that:

(i) $w_H < w_L \leq 2A$ implies $\underline{p}_1(w_H) < \underline{p}_1(w_L)$ and $\overline{p}_1(w_H) < \overline{p}_1(w_L)$. Moreover there exists an upper bound $\check{F}_H(w_H, w_L)$ on F_H such that $\underline{p}_2(w_H, F_H) < \underline{p}_1(w_L)$. Hence for that case, the set of separating prices is $P_L \cap P_H = \left[\underline{p}_2(w_H, F_H), \underline{p}_1(w_L)\right]$;

(ii) $w_L < w_H$

- **(a)** $w_H \leq 2A$ implies $\underline{p}_1(w_L) < \underline{p}_1(w_H)$ and $\overline{p}_1(w_L) < \overline{p}_1(w_H)$.
- **(b)** $2A < w_H \leq w_H^{\max}$ implies $\underline{p}_1(w_L) < w_H$ and $\overline{p}_1(w_L) < 2A + \Delta$.

In both subcases (a) and (b), there exists an upper bound $\hat{F}_H(w_H, w_L)$ on F_H such that $\overline{p}_1(w_L) < \overline{p}_2(w_H, F_H)$. Hence, when $w_L < w_H$, the set of separating prices is $P_L \cap P_H = [\overline{p}_1(w_L), \overline{p}_2(w_H, F_H)]$.

If F_H is greater than either $\check{F}_H(w_H, w_L)$ or $\hat{F}_H(w_H, w_L)$ then the set of separating prices is empty.

Lastly, we have obtained the following expressions from the analysis of (19)

$$\underline{p}_2(w_H, F_H) = \frac{1}{2}\left(2A + \Delta + w_H - \sqrt{\tilde{d}(w_H, F_H)}\right) \text{ if } w_H \in \left(c, 2A + \frac{\Delta}{2}\right), \tag{27}$$

$$\overline{p}_2(w_H, F_H) = \frac{1}{2}\left(2A + \Delta + w_H + \sqrt{\tilde{d}(w_H, F_H)}\right) \text{ if } w_H \in \left(c, 2A + \frac{\Delta}{2}\right), \tag{28}$$

$$\text{with } \tilde{d}(w_H, F_H) = (2A + \Delta - w_H)^2 - 8tF_H, \tag{29}$$

$$\check{F}_H(w_H, w_L) = \frac{1}{8t}\left(4A^2 - w_L^2 + 4Aw_H + 2w_Lw_H + 2w_L\Delta + 2w_H\Delta + 2(w_L - w_H)\sqrt{d(w_L)}\right) \tag{30}$$

$$\hat{F}_H(w_H, w_L) = \frac{1}{8t}\left(4A^2 - w_L^2 + 4Aw_H + 2w_Lw_H + 2w_L\Delta + 2w_H\Delta + 2(w_H - w_L)\sqrt{d(w_L)}\right) \tag{31}$$

D Proof of Proposition 8

Henceforth, we use the simplified notation p^* for $p^*(w, F)$. Let us first examine the individual rationality constraint for the L type. Define the set of p^* which solve (IR_L) as $P_L(\mu_0)$. Under the restriction $w_L \leq w_L^{\max}$, the constraint (IR_L) implies (FC_L) which is thus non binding. In addition, (IR_L) can be rewritten

$$(p^* - w_L)(2A + \mu_0\Delta - p^*)/2t \leq \left(\frac{2A - w_L}{2}\right)^2/2t. \tag{32}$$

Thus, $P_L(\mu_0) = \left\{p/p \leq \underline{p}_1(w_L, \mu_0) \text{ or } p \geq \overline{p}_1(w_L, \mu_0)\right\}$ where $\underline{p}_1(w_L, \mu_0)$ and $\overline{p}_1(w_L, \mu_0)$ are respectively the lower and upper roots for the equality version of (IR_L). These roots can be explicitely solved as:

$$\begin{aligned}
\underline{p}_1(w_L, \mu_0) &= \left(2A + \Delta + w_L - \sqrt{d(w_L, \mu_0)}\right)/2, &(33)\\
\overline{p}_1(w_L, \mu_0) &= \left(2A + \Delta + w_L + \sqrt{d(w_L, \mu_0)}\right)/2, &(34)\\
\text{with } d(w_L, \mu_0) &= \Delta\mu_0(4A + \Delta\mu_0 - 2w_L). &(35)
\end{aligned}$$

Note that $d(w_L, \mu_0) > 0$ because $w_L \leq w_L^{\max}$.

Let us now turn to the two constraints (IR_H) and (FC_H). Define the set of p^* which solve (IR_H) as $P_{1H}(\mu_0)$, i. e., $P_{1H}(\mu_0) = \left\{p/\underline{p}_1(w_H, \mu_0) \leq p \leq \overline{p}_1(w_H, \mu_0)\right\}$ and let $P_{2H}(\mu_0) = \left\{p/\underline{p}_2(w_H, F_H, \mu_0) \leq p \leq \overline{p}_2(w_H, F_H, \mu_0)\right\}$ be the set of p^* which solve (FC_H). We then have two cases: (i) $\pi_H(0) \geq 0$, or equivalently $F_H \leq \max\left\{0, \frac{1}{2t}\left(\frac{2A - w_H}{2}\right)^2\right\}$, so that ($\text{IR}_H$) implies ($\text{FC}_H$) and thus $P_{1H}(\mu_0) \cap P_{2H}(\mu_0) = P_{1H}(\mu_0)$, and (ii) $\pi_H(0) < 0$ so that (IR_H) is irrelevant and thus $P_{1H}(\mu_0) \cap P_{2H}(\mu_0) = P_{2H}(\mu_0)$.