Potential Competition in Preemption Games

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Introduction

Innovation and R&D activities are often modeled as a race to be first in discovering or adopting a new product/technology:

- usually assumed that at time 0, all the players are present and they know each other (see Reinganum (1981) and Fudenberg and Tirole (1985) for instance),
- however, these two assumptions are not very realistic and it is likely that, at time 0, all the competitors are not known.

Aim of this work: solve such a timing game when these assumptions are relaxed.

Introduction

We model a timing game of entry where the competitors are not identified:

- either there are no other competitors,
- or they did not invest yet.

We show that uncertainty on the presence of competitors reduces the effect of rent dissipation.

Hendricks (1992): competition between two firms, a firm can be an innovator or an imitator. The latter cannot be the first to adopt.

In our setting, the more a player waits, the more likely she faces an opponent. Therefore the risk of being preempted increases.
$\Rightarrow$ Reintroduction of a temporal dimension into preemption games,
$\Rightarrow$ link between the models of Reinganum (1981) and Fudenberg and Tirole (1985).

Introduction: Timing Games

Hoppe and Lehmann-Grubbe (2005): extension of the initial model of Fudenberg and Tirole (1985) by introducing the possibility to have access to a better technology in case investment occurs late. A second-mover advantage may exist.

Hopenhayn and Squintani (2008): preemption games when players' private information states stochastically increase over time. At equilibrium, agents end the game when their state is above a time-decreasing threshold. Over time, each player is afraid of being preempted and end the game for lower payoffs.

Brunnermeier and Morgan (2009): timing game in which each player must decide how long to delay his move after having received a signal. They derive a pure strategy equilibrium.

Introduction: Timing Games

Fudenberg and Tirole (1985) underline the difficulty to solve continuous-time timing games as a limit of discrete time versions of the game:

- the set of equilibrium outcomes in the two representations may differ (for instance in the "grab-the-dollar" game),
- moreover, coordination failure may exist.

$\Rightarrow$ They thus enlarge the strategy spaces by introducing the "intensity" of an interval of "consecutive atoms".

Other authors choose different solutions:

- use of a randomization device to avoid coordination failure (Simon and Stinchcombe (1989), Dutta et al. (1995), Hoppe and Lehmann-Grube (2005)...),
- alternate moves (Harris and Vickers (1985), Riordan (1992)...).

We use a standard definition of strategy (Laraki, Solan and Vieille (2005)) and coordination failure play little role in the derivation of equilibrium.

Outline of the talk

The Model: Actions and Payoffs

Two players, each of them coming into play at some random date $\tau^i \geq 0$.
τ^i: discovery of a new idea or a new market, existence of a new investment opportunity for player i, possibility to adopt a new technology...

Each player i has a single opportunity to make a move at some time $t \geq \tau^i$:

- if player i moves first at time t, she obtains a payoff $L(t)$ evaluated in terms of time 0 utilities, while player j obtains a payoff 0,
- if players i and j move simultaneously, they both obtain a strictly negative payoff,
- note that the payoff structure does not depend on τ_i.

The Model: Payoffs

Assumption. L is twice continuously differentiable, and there are times $T_2 > T_1 > 0$ such that the following holds:

$$L(t) < 0 \quad \text{if} \quad t \in [0, T_1),$$

$$L(t) > 0 \quad \text{if} \quad t \in (T_1, \infty),$$

$$\dot{L}(t) > 0 \quad \text{if} \quad t \in [0, T_2),$$

$$\dot{L}(t) < 0 \quad \text{if} \quad t \in (T_2, \infty),$$

$$\ddot{L}(t) \leq 0 \quad \text{if} \quad t \in [T_1, T_2].$$

Thus L vanishes at T_1 only, $L(T_1) = 0$, reaches its unique maximum at T_2, $\dot{L}(T_2) = 0$, and is concave over $[T_1, T_2]$.

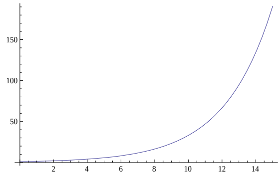
The Model: Payoffs

With $P(t)$ the date t monopoly profit flow and $C(t)$ the date t investment cost

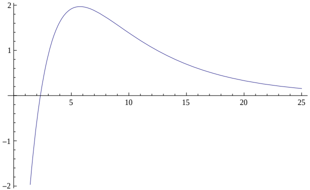
$$L(t) = \int_t^{+\infty} e^{-rs} P(s)\, ds - e^{-rt} C(t).$$

For instance:

- monopoly profit flow $P(t) = e^{\mu t}$ for all $t \geq 0$,
- investment cost $C > 1/(r - \mu)$.



Monopoly instantaneous profit



Leader's payoff

The Model: Information

$\tau_i \hookrightarrow F$, τ_i and τ_j are independent:

- τ_i is the private information or *type* of each player,
- we suppose that F has a non-decreasing hazard rate.

A player never knows whether she has an opponent, except when it is too late and she has been preempted. As time goes by, each player finds it less and less likely that she has no opponent.

The Model: Strategies (Laraki, Solan, Vieille (2005))

A *mixed plan* for player i of type τ^i is given by a cumulative distribution function with support in $[\tau^i, \infty]$.

A *strategy* for player i of type τ^i is a collection $(G_t^i(\cdot\,;\tau^i))_{t \geq \tau^i}$ of mixed plans such that:

- for each $t \geq \tau^i$, the support of $G_t^i(\cdot\,;\tau^i)$ is a subset of $[t, \infty]$: $G_t^i(\cdot\,;\tau^i)$ is a mixed plan in the subgame that starts at date t (properness condition), and

- for each $t' > t \geq \tau^i$ and every Borel set $A \subset [t', \infty]$,

$$\int_A dG_t^i(s;\tau^i) = [1 - G_t^i(t';\tau^i)] \int_A dG_{t'}^i(s;\tau^i).$$

  As long as a plan does not require player i to act with probability 1, the continuation plans can be computed by Bayes' rule (consistency condition).

Notation: $G_{\tau^i}^i(\cdot\,;\tau^i) = G^i(\cdot\,;\tau^i)$.

Necessary Conditions: Late Types

Consider first some type $\tau^i \geq T_2$ of player i, and suppose that player j has not moved yet at time τ^i,
$\Rightarrow$ type τ^i has no incentive to delay her move, if she believes that there is a 0 probability that she will thereby tie with player j.

This is the case if the following properties hold:

P1 The types $\tau^j < T_2$ move with probability 1 before time T_2, that is:

$$\mathbf{P}[j \text{ moves over } [0, T_2) \,|\, \tau^j < T_2] = 1,$$

P2 Each type $\tau^j \geq T_2$ moves immediately at time τ^j.

Necessary Conditions: Late Types

Lemma

In any equilibrium, properties P1 and P2 hold for each player.

With, for each $t \geq 0$,

- $\mathfrak{G}^i(t)$ the unconditional probability that player i moves by time t in equilibrium, and,
- $d\mathfrak{G}^i$ the corresponding Lebesgue–Stieltjes measure that describes the equilibrium distribution of player i's moving,

the lemma implies that

$$\mathfrak{G}^1(t) = \mathfrak{G}^2(t) = F(t) \text{ for all } t \geq T_2.$$

Moreover:

Lemma

In any equilibrium, $\mathrm{Supp}\, d\mathfrak{G}^1 = \mathrm{Supp}\, d\mathfrak{G}^2 = [\tilde{T}_1, \infty)$ *for some* $\tilde{T}_1 \in [T_1, T_2]$.

Necessary Conditions: Intermediate Types

What happens when $\tau_i < T_2$?
Suppose player j also moves immediately at birth when $\tau_j < T_2$,
$\Rightarrow$ expected payoff of any type $\tau^i \in [0, T_2]$ who decides to invest at some time $t \in [\tau^i, T_2]$

$$\frac{1 - F(t)}{1 - F(\tau_i)} L(t).$$

T_2^* implicitly defined by

$$\frac{\dot{L}(T_2^*)}{L(T_2^*)} = \frac{\dot{F}(T_2^*)}{1 - F(T_2^*)}$$

is uniquely defined, lies in (T_1, T_2) and achieves the maximum of the expected payoff.

Necessary Conditions: Intermediate Types

We have:

$$\dot{L}(t)/L(t) \gtreqless \frac{\dot{F}(t)}{1 - F(t)} \text{ if } t \lesseqgtr T_2^*,$$

$\Rightarrow$ at $t = T_2^*$, the marginal benefit of delaying one's move by an infinitesimal amount of time dt, $\dot{L}(t)dt$, exactly compensates the corresponding expected marginal loss, $\frac{\dot{F}(t)dt}{1-F(t)}L(t)$.

Necessary Conditions: Intermediate Types

Consider any type $\tau^i \in [T_2^*, T_2)$ of player i, and suppose that player j has not moved yet at time τ^i.

If the following properties hold:

P1* Each type $\tau^j < T_2^*$ moves with probability 1 before time T_2^*, that is:

$$\mathbf{P}[j \text{ moves over } [0, T_2^*) \,|\, \tau^j < T_2^*] = 1,$$

P2* Each type $\tau^j \geq T_2^*$ moves immediately at time τ^j,

then type τ^i has no incentive to delay her move.

On $[T_2^*, T_2)$, the gains of delaying investment are offset by the probability that the other invests.

Necessary Conditions: Intermediate Types

An intermediary step is necessary to prove that this continuation equilibrium is unique.

Lemma

In any equilibrium, the distributions $d\mathfrak{G}^1$ and $d\mathfrak{G}^2$ have no atom over $(T_2^*, T_2]$.

It follows that:

- for each player i and any time $t \in [T_2^*, T_2)$, the function $(1-\mathfrak{G}^i)L$ necessarily attains a maximum over $[t, T_2]$,
- the function $(1-\mathfrak{G}^i)L$ is necessarily nonincreasing over the interval $(\max\{\tilde{T}_1, T_2^*\}, T_2]$.

Necessary Conditions: Intermediate Types

Lemma
In any equilibrium, properties P1* and P2* hold for each player.

It follows that:

- $\mathfrak{G}^1(t) = \mathfrak{G}^2(t) = F(t)$ for all $t \geq T_2^*$, and
- $\tilde{T}_1$ is lower than T_2^*.

Necessary Conditions: Early Types

Immediately moving at birth cannot be part of an equilibrium before T_2^*.

Conjecture: in any equilibrium, all the types born before time T_2^* are indifferent to move at any time between some time T_1^* and time T_2^*. Their equilibrium payoff is thus

$$[1 - F(T_2^*)]L(T_2^*) = L(T_1^*)$$

in terms of time 0 utilities.

Necessary Conditions: Early Types

Lemma

In any equilibrium, the distributions $d\mathfrak{G}^1$ and $d\mathfrak{G}^2$ have no atom over $[\tilde{T}_1, T_2^*]$.

This implies that for each player i, the function $(1 - \mathfrak{G}^i)L$ is necessarily non-increasing over the interval $[\tilde{T}_1, T_2^*]$. It is in fact constant:

Lemma

In any equilibrium, $\mathfrak{G}^1(t) = \mathfrak{G}^2(t) = 1 - \frac{L(T_1^*)}{L(t)}$ for all $t \in [T_1^*, T_2^*]$.

Hence:

$$\mathfrak{G}(t) = \begin{cases} 0 & \text{if} \quad t < T_1^*, \\ 1 - \frac{L(T_1^*)}{L(t)} & \text{if} \quad T_1^* \leq t < T_2^*, \\ F(t) & \text{if} \quad t \geq T_2^*. \end{cases}$$

Necessary Conditions: Early Types

Each player born before T_2^* is indifferent between moving at any time $t \in [T_1^*, T_2^*]$:

$$[1 - \mathfrak{G}(t)]L(t) = L(T_1^*) \text{ for all } t \ \in [T_1^*, T_2^*],$$

leading to

$$\frac{\dot{L}(t)}{L(t)} = \frac{\dot{\mathfrak{G}}(t)}{1 - \mathfrak{G}(t)} \text{ for all } t \in [T_1^*, T_2^*].$$

Equilibrium Existence

- a unique candidate for the equilibrium distribution of each player's moving time,

- construction of the equilibrium strategies that actually generate this distribution:
  - we have proved there exists a unique continuation equilibrium from time T_2^* on: at any time $t \geq T_2^*$ at which no player has moved yet, it is optimal for any player who was born before or at time t to move immediately,

  - therefore, since no player moves before time T_1^*, constructing an equilibrium requires only specifying the players' behavior over $[T_1^*, T_2^*]$.

Equilibrium Existence

The only condition to be satisfied is that the unconditional distribution of each player's moving time be consistent with the postulated equilibrium strategies:

$$\int_0^t G^i(t;\tau^i)\,dF(\tau^i) = \mathfrak{G}(t) \text{ for all } t \in [T_1^*, T_2^*].$$

Equilibrium Existence: An Equilibrium in Pure Strategy

As $F(t) > \mathfrak{G}(t)$ for all $t < T_2^*$, one can thus define a strictly increasing mapping $\sigma : [0, T_2^*] \to [T_1^*, T_2^*]$ by

$$F(\sigma^{-1}(t)) = \mathfrak{G}(t) \text{ for all } t \in [T_1^*, T_2^*].$$

By construction, $\sigma(0) = T_1^*$, $\sigma(\tau) > \tau$ for all $\tau < T_2^*$ and $\sigma(T_2^*) = T_2^*$. Let then each type $\tau^i \leq T_2^*$ of player i move at time $\sigma(\tau^i)$, that is:

$$G^i(t; \tau^i) = 1_{\{\sigma(\tau^i) \leq t\}} \text{ for all } t \geq \tau^i.$$

It then follows that

$$\int_0^t G^i(t; \tau^i)\, dF(\tau^i) = \int_0^t 1_{\{\tau^i \leq \sigma^{-1}(t)\}}\, dF(\tau^i) = F(\sigma^{-1}(t)) = \mathfrak{G}(t)$$

for all $t \in [T_1^*, T_2^*]$.

Equilibrium Existence: An Equilibrium in Pure Strategy

Proposition

There exists a pure strategy equilibrium in which

- *each player's type* $\tau \leq T_2^*$ *moves at time* $\sigma(\tau)$, *and*
- *each player's type* $\tau > T_2^*$ *moves immediately at time* τ.

Equilibrium Existence: Simple Equilibrium

All players' types have the same underlying payoff function once they are born, so that the players' preferences do not satisfy a strict single-crossing condition.

In the equilibrium in pure strategy we derived, each type $\tau < T_1^*$ moves at time $\sigma(\tau)$, where σ is a strictly increasing function.

Moreover, this equilibrium is not Markov perfect, since players condition their behavior on a payoff irrelevant variable, namely their date of birth.

We therefore introduce a *Simple perfect Bayesian equilibrium*.

Equilibrium Existence: Simple Equilibrium

For each player i, the strategies $(G_t^i(\cdot\,;\tau^i))_{t\geq\tau^i}$ of types $\tau^i > 0$ are themselves derived using Bayes' rule from the strategy $(G_t^i(\cdot\,;0))_{t\geq 0}$ that player i adopts when born at date 0, and therefore ultimately from the mixed plan $G^i(\cdot\,;0)$.

That is, for each $t \geq \tau^i > 0$ such that $G^i(\tau^i;0) < 1$,

$$G^i(t;\tau^i) = \frac{G^i(t;0) - G^i(\tau^i;0)}{1 - G^i(\tau^i;0)}.$$

Equilibrium Existence: Simple Equilibrium

It follows

$$G^i_{\tau^i}(t;0) = \frac{G^i(t;0) - G^i(\tau^i;0)}{1 - G^i(\tau^i;0)} = \frac{G^i(t;\tilde{\tau}^i) - G^i(\tau^i;\tilde{\tau}^i)}{1 - G^i(\tau^i;\tilde{\tau}^i)} = G^i_{\tau^i}(t;\tilde{\tau}^i)$$

for all $t \geq \tau^i \geq \tilde{\tau}^i > 0$ such that $G^i(\tau^i;0) < 1$.

Therefore, $G^i(t;\tau^i)$ represents both

- the probability that type τ^i of player i moves by time $t \geq \tau^i$, and
- the probability that a type $\tilde{\tau}^i \leq \tau^i$ of player i moves between times τ^i and t conditional on not having moved before time τ^i.

$\Rightarrow$ If type $\tilde{\tau}^i$ of player i has not yet moved by time $\tau^i \geq \tilde{\tau}^i$, she need not behave differently from type τ^i after time τ^i.

Equilibrium Existence: The Simple Equilibrium

The simple equilibrium is symmetric and completely characterized by the mixed plan $G(\cdot\,;0)$ of type 0 of each player:

- $d\mathfrak{G}$ has no atoms, and each type $\tau < T_2^*$ moves with probability 1 before time T_2^*,
- therefore, the function $G(\cdot\,;0)$ must be continuous over $[0, T_2^*]$, with $G(T_1^*;0) = 0$ and $G(T_2^*;0) = 1$ and satisfies the following lemma.

Lemma

$\mathrm{Supp}\, dG(\cdot\,;0) = [T_1^*, T_2^*]$.

By Bayes'rule, the consistency condition reads:

$$\int_0^t \left[\frac{G(t;0) - G(\tau;0)}{1 - G(\tau;0)}\right] dF(\tau) = \mathfrak{G}(t) \text{ for all } t \in [T_1^*, T_2^*].$$

Equilibrium Existence: The Simple Equilibrium

Proposition

There exists a unique simple equilibrium, in which

- *each player's type $\tau < T_2^*$ moves according to the mixed plan $G(\cdot\,;\tau) = \frac{G(\cdot\,;0) - G(\tau;0)}{1 - G(\tau;0)}$, where $G(\cdot\,;0) = 0$ over $[0, T_1^*]$ and*

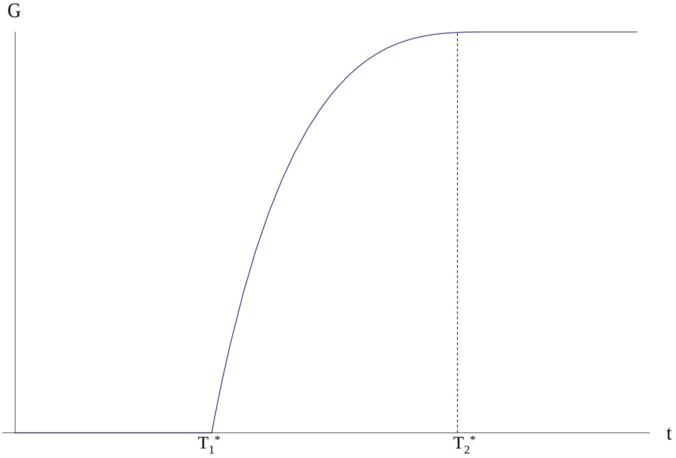
$$G(t;0) = 1 - \left[\frac{F(t) - \mathfrak{G}(t)}{F\left(T_1^*\right)}\right] \exp\left(-\int_{T_1^*}^{t} \frac{dF(\tau)}{F(\tau) - \mathfrak{G}(\tau)}\right)$$

  for all $t \in [T_1^*, T_2^*]$.

- *each player's type $\tau \geq T_2^*$ moves immediately at time τ.*

The Simple Equilibrium Strategy

Function $G(\cdot\,; 0)$.



Discussion: Strength of Competition

Analysis of the hazard rate of the first moving time:

- the distribution function of the first moving time is $1 - (1 - \mathfrak{G})^2$,
- over $[T_1^*, T_2^*]$, the corresponding hazard rate is $2\,\frac{\dot{L}}{L}$, which is strictly decreasing,
- over $[T_2^*, \infty)$, the hazard rate of the first moving time is $2\,\frac{\dot{F}}{1-F}$, which is non-decreasing.

Empirical prediction: the hazard rate of the first moving time, $2\max\left\{\frac{\dot{L}}{L}, \frac{\dot{F}}{1-F}\right\}$, is non-monotonic over $[T_1^*, \infty)$, and tends to increase after having reached a minimum at T_2^*.

Discussion: Strength of Competition

Two results in the case of the simple equilibrium:

- $G(\cdot\,; 0)$ is strictly concave over $[T_1^*, T_2^*]$, and
- $\dot{G}(T_2^*\,; 0) = 0$.

Players born between T_1^* and T_2^* are more likely to move close to their birth dates than close to T_2^*.
Overall, competition is fiercer close to T_1^*, and then tends to decrease.

Discussion: Coordination Failures

In the simple equilibrium, both the distribution of player's types and that of their moving times conditional on their types have no atoms:

- the probability of a joint move by both players is nil, and
- thus coordination failures play no role in the derivation of this equilibrium.

The possibility of coordination failures play a role in our analysis: the assumption that they are always detrimental to both players is key to the proof that the equilibrium distributions of players' moving times have no atoms, and are actually uniquely determined.

Discussion: Eroding Reputation

Let $q(t)$ be the probability that a given player is not yet born by time t given that no player has moved yet by this time:

$$q(t) = \begin{cases} 1 - F(t) & \text{if} \quad t \leq T_1^*, \\ \frac{[1-F(t)]L(t)}{L(T_1^*)} & \text{if} \quad T_1^* < t \leq T_2^*, \\ 1 & \text{if} \quad t > T_2^*. \end{cases}$$

Over the interval (T_1^*, T_2^*), the mapping $t \mapsto q(t)$ continuously increases from $1 - F(T_1^*)$ to 1.

One can interpret $q(t)$ as the reputation of each player at time t if she has not moved by then (Hendricks (1992)). The reputation of each player i evolves according to

$$\tilde{q}(t) = \begin{cases} q(t) & \text{if player } i \text{ has not moved by time } t, \\ 0 & \text{if player } i \text{ has moved by time } t. \end{cases}$$

Discussion: Eroding Reputation

One can prove that the reputation of each player follows a submartingale. Reputations tend to erode:

- as time elapses, it becomes increasingly difficult to maintain a reputation of not being born,
- therefore, players are less ready to wait before moving and players move immediately from time T_2^* onwards.

This is an important difference with Hendricks (1992) where the reputation of a firm is the posterior belief that it is an imitator given that no firm has adopted yet. In this case, the reputation of a firm follows by construction a martingale. Therefore an innovator may in equilibrium wait until time $T_2 > T_2^*$ to adopt.

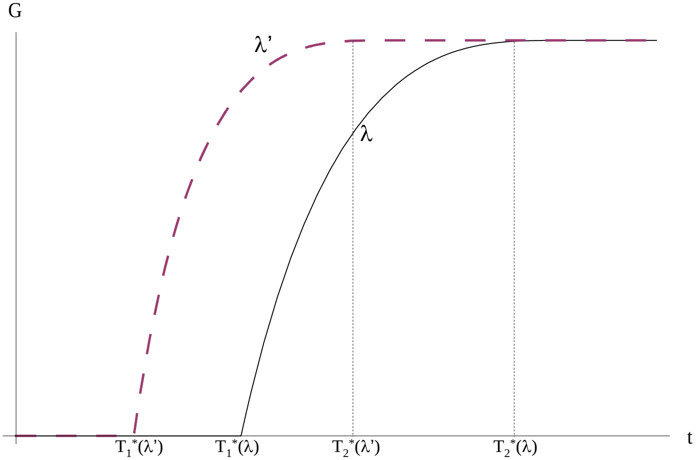
Consider a family of cumulative distribution functions F_θ parameterized by $\theta \in \mathbb{R}$, which satisfies the monotone likelihood ratio property (MLRP); that is, the mapping $t \mapsto \frac{\dot{F}_{\theta_1}(t)}{\dot{F}_{\theta_0}(t)}$ is increasing whenever $\theta_1 > \theta_0$:

- if the family $\{F_\theta\}_{\theta \in \mathbb{R}}$ satisfies MLRP, both $T^*_{1,\theta}$ and $T^*_{2,\theta}$ are increasing functions of θ, and
- if the family $\{F_\theta\}_{\theta \in \mathbb{R}}$ satisfies MLRP, so does the family $\{G_\theta(\cdot\,; 0)\}_{\theta \in \mathbb{R}}$.

Illustration with the case of the family of exponential distributions $\{F_\theta\}_{\theta \in \mathbb{R}_{++}}$ with parameters $\lambda = \frac{1}{\theta}$.

Exponential distribution function

Function $G(\cdot\,;0)$ with $\lambda' > \lambda$.



Concluding remarks

- Analysis of the adoption of a new technology by a player that does not know when she will face a competitor,

- derivation of the unique simple perfect Bayesian equilibrium of the game: coordination failures play not role in the derivation of the equilibrium since players never simultaneously move,

- potential competition does not completely dissipate rents,

- this model links the two canonical models of Reinganum (1981) and Fudenberg and Tirole (1985),

- extension to the case where the payoff function depends on the player's type.